Output Gap, Monetary Policy Trade-offs and Financial Frictions*

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Abstract

This paper investigates how the presence of financial frictions and financial shocks changes the definition and the estimated dynamics of the output gap in a New Keynesian model. Financial shocks absorb explanatory power from efficient labor supply shocks, thus changing radically the dynamics of the economy’s efficient frontier. Despite their large impact on the output gap, financial factors affect the monetary policy trade-offs only to some extent. Nominal stabilization can be achieved at the cost of limited (but non-negligible) fluctuations in real economic activity. Finally, we discuss an alternative measure of the output gap (in deviation from the optimal equilibrium) that is a better measure of imbalances in the economy than the conventional output gap.

Keywords: Financial frictions; output gap; monetary policy.

JEL codes: E32, C51, C52

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1 Introduction

The recent series of boom and bust episodes in stock and house prices and the financial flavor of the Great Recession have renewed the interest in models with financial frictions. The workhorse models in macroeconomic analysis (Christiano, Eichenbaum and Evans, 2005 and Smets and Wouters, 2007) have been extended to include financial frictions as a propagation of standard disturbances (Bernanke, Gertler and Gilchrist, 1999, and Kiyotaki and Moore, 1997) and as a source of shocks, as in Christiano, Motto and Rostagno (henceforth CMR) (2014), Gilchrist and Zakrajsek (2011) and Jermann and Quadrini (2012). However, while the literature has made important advances in terms of specification and estimation, the policy implications stemming from these models have not been investigated in depth. This paper aims at filling this gap.

In particular, we provide two related contributions to the literature. First, we investigate how the presence of financial frictions and financial shocks changes the definition of the output gap and we provide a model-based estimate of it.\(^1\) Second, we compute the optimal Ramsey equilibrium and we analyze the trade-offs between different objectives that emerge under optimal policy. The presence of financial frictions implies that the central bank faces an additional source of inefficiencies, besides monopolistic competition and nominal rigidities in goods and labor markets as in the standard New Keynesian model, which may result in more complicated trade-offs. In this context a second measure of the output gap, in deviation from the optimal equilibrium, emerges. We argue that this second measure of the output gap is a better measure of imbalances, unlike the conventional definition.

We conduct our analysis in the context of an estimated New Keynesian model which is extended to include a financial accelerator mechanism along the lines of Bernanke, Gertler and Gilchrist (1999). The New Keynesian core of the model is taken from Justiniano, Primiceri and Tambalotti (henceforth JPT) (2013), which constitutes our reference for monetary policy trade-offs in New Keynesian models without financial frictions. The financial frictions block of the model is taken from CMR (2014). In particular, as in the latter study we include two financial shocks: a shock to the net worth of firms, which directly affects the availability of credit for the production sector, and a shock to the volatility of the cross-sectional idiosyncratic uncertainty (risk shock), which reflects possible tensions in financial markets (or fluctuations in uncertainty).

\(^1\)We consider the conventional definition of the output gap as the deviation of actual output from potential output (cf. Smets and Wouters, 2007, Justiniano, Primiceri and Tambalotti, 2013, Levin, Onatski, Williams and Williams, 2005, Sala, Söderström and Trigari, 2008, among others). Potential output is defined as the counterfactual level of output that emerges under flexible prices and wages and in the absence of inefficient shocks, i.e. shocks that do not affect the efficient frontier of the economy.
and includes news components.

We find that the presence of financial frictions has a large effect on the estimated output gap. In fact, the output gap derived from our baseline model is more persistent and volatile than the output gap derived in the absence of financial frictions, which constitutes our reference for comparison. In particular, we estimate a long cycle for the output gap that was positive from the mid-1990s until the Great Recession, thus over a period characterized by several asset price boom and bust cycles. A standard New Keynesian model implies instead a negative output gap in the pre-Great Recession period. The main reason for such a different shape for the output gap in the model with financial frictions is that financial shocks absorb explanatory power from efficient labor supply shocks. In fact, neither of the financial shocks propagates in the efficient economy or affects potential output, which closely tracks the efficient frontier of the economy. Potential output in the model with financial frictions is therefore substantially different from its counterpart in the standard New Keynesian model.

The presence of financial frictions also changes the trade-offs faced by the monetary policy authority. While optimal monetary policy is able to stabilize price inflation, wage inflation and output around potential almost completely in the standard New Keynesian model, the trade-offs are more complicated in the model with financial frictions. We find that under optimal monetary policy, although the central bank achieves a good stabilization of price inflation and (especially) wage inflation, it does so at the cost of limited but non-negligible fluctuations in the output gap. The coexistence of several frictions imposes a challenge to the central bank, which cannot stabilize all its intermediate targets at the same time. Notably, the optimal monetary policy prescribed by our estimated model would have been able to avoid a large share of the fluctuations in price inflation, wage inflation and output gap observed in the sample period. Nevertheless, according to our model, a positive output gap over the period 2005-2007 was optimal and consistent with a policy of nominal stabilization.

Based on our analysis of the trade-offs, we define a second measure of the output gap, the "Monetary Policy Score", that, we argue, may be a better measure of imbalances in the economy than the conventional output gap. In fact, a positive (or negative) conventional output gap may be fully consistent with optimal policy as long as it reflects the optimal solution of trade-offs with other objectives. A more useful measure of the monetary policy stance is then given by the difference between actual output and the counterfactual level of output that emerges under optimal policy (i.e. optimal output). Closing the Monetary Policy Score is the right goal for
the monetary policy authority and it is feasible. In the standard New Keynesian model by JPT (2013), stabilizing the Monetary Policy Score is equivalent to stabilizing the conventional output gap. In the presence of financial frictions, it is suboptimal to stabilize the conventional output gap, and the Monetary Policy Score is the right indicator of imbalances that should be stabilized at all times.

This paper contributes to three strands of the literature. The first relates to the behavior of the output gap in Dynamic Stochastic General Equilibrium (DSGE) models. Earlier contributions include Levin, Onatski, Williams and Williams (2005) and Edge, Kiley and Laforte (2008). Sala, Söderström and Trigari (2008) were the first to obtain a cyclical output gap in an estimated DSGE model with unemployment: their model-based output gap exhibits cyclical properties that resemble measures of the output gap obtained using statistical methods. JPT (2013) and Galí, Smets and Wouters (2011) relate the model-based output gap to the stochastic processes driving labor supply shocks and wage mark-up shocks. As far as we know, our paper is the first that derives the output gap from an estimated model with financial frictions driven by a large set of shocks.²

We also contribute to the literature on optimal monetary policy in models with financial frictions. Fendoglu (2014) computes the Ramsey monetary policy in a calibrated financial accelerator model driven by three disturbances (productivity, government spending and risk). Carlstrom, Fuerst and Paustian (2010), De Fiore and Tristani (2013) and Ravenna and Walsh (2006) evaluate optimal monetary policy in simple small-scale models with financial frictions, where they are able to derive analytical expressions for the model-consistent welfare functions. In a similar set-up, Faia and Monacelli (2007) study optimal monetary policy rules in a financial accelerator model driven by technology and government spending shocks, whereas Cúrdia and Woodford (2010) discuss the costs and the benefits of including credit spreads in the standard Taylor rule. De Fiore, Teles and Tristani (2011) analyze optimal monetary policy in a model in which firms’ financial positions are denominated in nominal terms and debt contracts are not state-contingent. We contribute to this literature by conducting our analysis in an estimated (rather than calibrated) model driven by several disturbances, including two financial shocks.

Third, and related to the previous point, we contribute to the literature investigating mon-

²The concept of the output gap in the presence of financial frictions is briefly discussed in Carlstrom, Fuerst and Paustian (2010), Cúrdia and Woodford (2010), De Fiore and Tristani (2013) and Davis and Huang (2013) in calibrated models driven by few shocks. However, these papers do not provide an estimated series for the output gap. The importance of considering financial factors in the computation of the output gap is stressed in Borio, Disyatat and Juselius (2013) in a reduced-form set-up. Our paper considers the same issues in a DSGE model.
etary policy trade-offs by using the New Keynesian model as a study framework. Most central banks perceive a trade-off between stabilizing inflation and stabilizing a measure of capacity utilization. However, Blanchard and Galí (2007) show that within small scale New Keynesian models, there is no such trade-off or, in other words, there is "Divine Coincidence". In this simple set-up, only cost-push shocks (price and wage-markup shocks) can generate trade-offs. Notably, as discussed in Galí, Gertler and Lopez-Salido (2009) and Blanchard and Galí (2007), "Divine Coincidence" holds only under strong assumptions of no capital accumulation and no real rigidities in the form of habit persistence or real wage rigidities. In medium-scale DSGE models, as in Smets and Wouters (2007), where real rigidities and capital accumulation play an important role, all shocks could potentially induce cost-push effects and generate trade-offs. Then, it becomes crucial to estimate the magnitude of these trade-offs, and JPT (2013) provide a quantitative setup to analyze policy trade-offs in a medium size DSGE model similar to Smets and Wouters (2007). They compute the counterfactual level of output that emerges under optimal monetary policy and show that trade-offs between real and nominal stabilization exist but are fairly weak. Using the JPT (2013) terminology, a sort of "Trinity" holds. The monetary policy authority is able to stabilize price inflation, wage inflation and the output gap almost completely, as long as wage mark-up shocks are small. Debortoli, Kim, Lindé and Nunes (2016) show that trade-offs are substantially larger when price and wage mark-up shocks are fairly large and argue that the weight on the output gap should be equal to or larger than that of annualized inflation when designing a loss function for the central bank. Our contribution is to measure the policy trade-offs in an environment where frictions are more pervasive.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 summarizes the details of the Bayesian estimation and the main properties of the estimated model. Section 4 discusses the model-based measure of the output gap and its properties. In Section 5 we present the optimal monetary policy exercise and we introduce the concept of Monetary Policy Score that emerges naturally from our model. Finally, we conclude in Section 6.

2 The Model

Our baseline model of the US economy combines the standard New Keynesian model (cf. Christiano, Eichenbaum and Evans, 2005, and Smets and Wouters, 2007) together with the workhorse model with financial frictions (cf. Bernanke, Gertler and Gilchrist, 1999). More specifically, we
introduce a financial accelerator block in the model estimated by JPT (2013) following the most recent contributions of CMR (2014) and Del Negro and Schorfheide (2013). In this section we present the problems of all agents in non-stationary form, while in Appendixes A and B we report the full set of equilibrium conditions in their stationary form. The notation follows closely JPT (2013).

**Final good producers.** A representative, competitive final good producer combines a continuum of intermediate goods \( Y_t(i) \), indexed with \( i \in [0, 1] \), according to a Dixit-Stiglitz technology to produce the homogenous good \( Y_t \)

\[
Y_t = \left[ \int_0^1 Y_t(i)^{\frac{1}{1+\Lambda_{p,t}}} \, di \right]^{1+\Lambda_{p,t}},
\]

where \( \Lambda_{p,t} \) is related to the degree of substitutability across different intermediates. It is a measure of competitiveness in the intermediate goods markets and its exogenous movements are one of the forces driving the economy away from its efficient frontier. \( \Lambda_{p,t} \) varies exogenously over time in response to its independently and identically distributed \( N(0, \sigma_p) \) innovation \( \varepsilon_{p,t} \) (referred to as price markup shock) according to

\[
\log (1 + \Lambda_{p,t}) \equiv \lambda_{p,t} = (1 - \rho_p) \lambda_p + \rho_p \lambda_{p,t-1} + \varepsilon_{p,t}.
\]

The associated price index \( P_t \) obtained from profit maximization is an aggregate of the intermediate goods prices \( P_t(i) \)

\[
P_t = \left[ \int_0^1 P_t(i)^{-\frac{1}{\Lambda_{p,t}}} \, di \right]^{-\Lambda_{p,t}},
\]

whereas the demand function for each intermediate good \( i \) is given by

\[
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1+\Lambda_{p,t}}{-\Lambda_{p,t}}} Y_t.
\]

**Intermediate goods producers.** The intermediate goods are produced by monopolistically competitive firms using the following production function

\[
Y_t(i) = A_t^{1-\alpha} K_t(i)^{\alpha} L_t(i)^{1-\alpha} - A_t F_t,
\]
where $K_t(i)$ and $L_t(i)$ represent the services of effective capital and labor used by firm $i$ in the production sector. $F$ is a fixed cost of production (indexed to technology) which is set such that profits are zero in steady state. $A_t$ is the Solow residual of the production function. Its growth rate $z_t$ ($z_t = \Delta \log A_t$) is stationary and varies exogenously over time in response to independently and identically distributed $N(0, \sigma_z)$ technology shocks $\varepsilon_{z,t}$, as follows

$$z_t = (1 - \rho_z) \gamma + \rho_z z_{t-1} + \varepsilon_{z,t},$$

where $\gamma$ represents the growth rate of the economy along a balanced growth path. Each producer chooses its price subject to a Calvo (1983) mechanism. Every period a fraction $\xi_p$ does not choose prices optimally but simply indexes their current price according to the rule

$$P_t(i) = P_{t-1}(i) \pi_{t-1}^{\pi-1} \pi_{t-1}^{1-\pi},$$

where $\pi_t$ is the gross inflation rate and $\pi$ represents its steady state value. As explained in JPT (2013), this indexation scheme has the desirable property that the level of steady state inflation does not affect welfare and the level of output in steady state.

Remaining firms set their price $\bar{P}_t(i)$ by maximizing profits intertemporally

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \lambda_{t+s} \left\{ \left[ \bar{P}_t(i) \left( \prod_{j=0}^{s} \pi_{t-1+j}^{\pi-1} \pi_{t-1+j}^{1-\pi} \right) \right] Y_{t+s}(i) - \left[ W_t L_t(i) + P_t r_t K_t(i) \right] \right\},$$

where $\beta^s \lambda_{t+s} / \lambda_t$ represents the household’s discount factor, $\lambda_t$ being the marginal utility of consumption, whereas $W_t$ and $r_t$ indicate the nominal wage and the real rental rate of capital, respectively.

**Employment agencies.** A representative competitive employment agency combines differentiated labor services, indexed by $j \in [0, 1]$, into homogeneous labor using the following technology

$$L_t = \left[ \int_0^1 L_t(j)^{1+\lambda_{w,t}} dj \right]^{1+\Lambda_{w,t}},$$

where $\Lambda_{w,t}$ is the elasticity of substitution across different labor varieties. $\log (1 + \Lambda_{w,t}) = \lambda_{w,t}$ is an independently and identically distributed $N(0, \sigma_w^2)$ wage mark-up shock. As in the
goods market, the demand function for labor of type $j$ is given by

$$L_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\gamma_{w,t}}} L_t,$$

whereas the wage index is

$$W_t = \left[ \left( \frac{1}{0} W_t(j)^{-\frac{1}{\gamma_{w,t}}} \right) \right]^{-\lambda_{w,t}}.$$

For each labor type, we assume the existence of a union representing all workers of that type. Wages are set subject to Calvo lotteries. Every period, a fraction $\xi_w$ of unions index the wage according to the rule

$$W_t(j) = W_{t-1}(j) \left( \pi_{t-1} e^{-\gamma} \right)^{1-w} (\pi e^\gamma)^{1-w}.$$

This indexation scheme implies that output is independent of the steady state value of wage inflation. The remaining unions choose the wage optimally by maximizing the utility of their members subject to labor demand.

**Households.** The household sector is composed of a large number of identical households, each composed of a continuum of family members indexed by $j$. All labor types are represented in each household and family members pool wage income and share the same amount of consumption. After goods production in period $t$, the representative household constructs raw capital by combining investment goods $I_t$ and undepreciated capital $K_{t-1}$ according to the following technology\footnote{The timing convention for the state variables follows JPT (2013) and reflects the end-of-period value of those variables. The stock of raw capital is produced within the household as in CMR (2014). Alternatively, this task could be assigned to competitive capital producers.}

$$K_t = (1 - \delta) K_{t-1} + \mu_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t,$$

where $\delta$ is the depreciation rate, and the function $S \left( \frac{I_t}{I_{t-1}} \right) = \zeta \left( \frac{I_t}{I_{t-1}} - e^{-\gamma} \right)^2$ captures investment adjustment costs, as in Christiano, Eichenbaum and Evans (2005). In steady state $S(\cdot) = S'(\cdot) = 0$ and $S''(\cdot) = \zeta$. $\mu_t$ varies exogenously over time in response to independently and
identically distributed $N \left( 0, \sigma^2_{\mu} \right)$ shocks to the marginal efficiency of investment $\varepsilon_{\mu,t}$, as follows

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t}.$$ 

The representative household takes the price of capital $Q_t$, the price of investment (and consumption) goods $P_t$ and labor income as given. It maximizes the utility function

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[ \log (C_{t+s} - hC_{t+s-1}) - \varphi_t \int_0^1 \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} dj \right] \right\},$$

where $C_t$ stands for consumption, $h$ for the degree of habit formation, $\nu$ for the inverse of labor supply elasticity. $b_t$ varies exogenously over time in response to independently and identically distributed $N \left( 0, \sigma^2_b \right)$ intertemporal preference shocks $\varepsilon_{b,t}$, as follows

$$\log b_t = \rho_b \log b_{t-1} + \varepsilon_{b,t},$$

as does $\varphi_t$ in response to independently and identically distributed $N \left( 0, \sigma^2_\varphi \right)$ intratemporal labor supply shocks $\varepsilon_{\varphi,t}$

$$\log \varphi_t = (1 - \rho_\varphi) \varphi + \rho_\varphi \log \varphi_{t-1} + \varepsilon_{\varphi,t}.$$ 

The representative household maximizes utility subject to the budget constraint

$$P_tC_t + P_tI_t + T_t + Q_t(1-\delta)K_{t-1} + B_t = \int_0^1 W_t(j)L_t(j) dj + R_tB_{t-1} + Q_tK_t + O_t + H_t.$$

Funds are used to buy consumption and investment goods, to pay lump sum taxes ($T_t$), to buy undepreciated capital from entrepreneurs and to save in a one period bond $B_t$ that pays a gross nominal return $R_t$ in each state of nature. This bond is the source of external funds for entrepreneurs and plays a crucial role in the financial accelerator mechanism. Expenses are financed with labor income, revenues from previous period savings and from selling capital to entrepreneurs, profits from ownership of firms in the intermediate good sectors $O_t$ and net transfers from entrepreneurs $H_t$.

**Entrepreneurs.** There is a continuum of entrepreneurs indexed by $l$. Each entrepreneur uses its own net worth $N_{l-1}(l)$ and borrows $B_{l-1}^e(l)$ from a financial intermediary (that channels households’ savings to entrepreneurs) to purchase $K_{l-1}(l)$ units of raw capital from households
at the end of period $t - 1$ according to

$$B_{t-1}^e(l) = Q_{t-1}K_{t-1}(l) - N_{t-1}(l).$$

After purchasing capital, at the beginning of period $t$, each entrepreneur is subject to an idiosyncratic productivity shock ($\omega$) that transforms raw capital into effective capital $\omega_t(l)K_{t-1}(l)$. This shock is assumed to be independently drawn across time and across entrepreneurs and log-normally distributed with mean 1 and standard deviation $\sigma_t$. The latter is the so-called risk shock, modeled exactly as in CMR (2014). In particular

$$\log \sigma_t = (1 - \rho_\sigma) \sigma + \rho_\sigma \log \sigma_{t-1} + \xi_{0,t} + \xi_{1,t-1} + \ldots + \xi_{8,t-8},$$

where $\xi_{\sigma,t}$ is a sum of independently and identically distributed mean zero random variables. It is assumed that in period $t$ agents observe $\xi_{j,t}$, $j = 0, 1, \ldots, 8$ and that $\xi_{0,t}$ is defined as the unanticipated component of $\xi_{\sigma,t}$ and $\xi_{j,t}$ as anticipated, or news, components. It is further assumed that $\xi_{j,t}$'s follow a correlation structure such that for this shock there are four free parameters to be estimated: $\rho_\sigma$, $\sigma_\sigma$, $\sigma_{\sigma,n}$, and $\rho_{\sigma,n}$. They are respectively the autoregressive coefficient of the risk shock, the standard deviation of the unanticipated shock, the standard deviation of the anticipated shock, and the correlations between news, namely

$$\rho_{\sigma,n}^{[i-j]} = \frac{E\xi_{i,t}\xi_{j,t}}{\sqrt{(E\xi_{i,t}^2)(E\xi_{j,t}^2)}} \quad i, j = 0, 1, \ldots, 8,$$

with the extra assumption that $E\xi_{0,t}^2 = \sigma_\sigma^2$, $E\xi_{1,t}^2 = E\xi_{2,t}^2 = \ldots E\xi_{8,t}^2 = \sigma_{\sigma,n}^2$.

After observing the idiosyncratic shock, each entrepreneur chooses the utilization rate $u_t$ of its effective capital and rents an amount of capital services $K_t(l) = u_t(l)\omega_t(l)K_{t-1}(l)$ to intermediate goods-producing firms at the competitive real rental rate $r^k_t$. At the end of the period, each entrepreneur is left with $(1 - \delta)K_{t-1}(l)$ units of capital that are sold to households at price $Q_t$. The overall gross nominal rate of return $R^n_{t,k}$ enjoyed by the entrepreneur in period $t$ is

$$R^n_{t,k} = \frac{P_t\tilde{r}_t^k u_t - P_t a (u_t) + (1 - \delta)Q_t}{Q_{t-1}},$$

where $a(u_t)$ represents the cost of changing capital utilization and where we omit the index.
l, as we take advantage of the fact that the capital utilization decision is common across entrepreneurs. As in Levin, Onatski, Williams, and Williams (2005), \( a(u_t) = \frac{u_t^{1+\chi - 1}}{1+\chi} \) and in steady state \( u = 1 \), \( a(1) = 0 \) and \( \chi \equiv \frac{a''(1)}{a'(1)} \).

To cope with the asymmetric information about entrepreneurs’ idiosyncratic productivity, financial intermediaries enter into a financial contract with entrepreneurs. There is a cutoff value \( \varpi_t(l) \) such that entrepreneurs whose \( \omega_t(l) \) is lower than \( \varpi_t(l) \) declare bankruptcy and the intermediary must pay a monitoring cost \( \mu^e \) proportional to the realized gross payoff to recover the remaining assets. The debt contract undertaken in period \( t - 1 \) consists of a triplet \( \varpi_t(l), B_t^e(l) \) and \( Z_t(l) \) where \( Z_t(l) \) represents the loan rate paid to the financial intermediary. The cut-off value satisfies the following equation:

\[
\varpi_t(l) R_t^{n,k} Q_{t-1} K_{t-1} (l) = Z_t(l) B_t^{e} (l).
\]

Note that the previous expression can be used to express \( Z_t(l) \) in terms of \( \varpi_t(l) \). Entrepreneurs maximize expected profits

\[
E_{t-1} \left\{ [1 - \Gamma_{t-1} (\varpi_t(l))] R_t^{n,k} Q_{t-1} K_{t-1} (l) \right\},
\]

subject to the lender’s participation constraint that must be satisfied in each period \( t \) state of nature:

\[
[\Gamma_{t-1} (\varpi_t(l)) - \mu^e G_{t-1} (\varpi_t(l))] R_t^{n,k} Q_{t-1} K_{t-1} (l) - R_{t-1} B_t^e (l) = 0,
\]

where \( \Gamma_{t-1} (\varpi_t(l)) \) is the share of profits going to the lender and \( \mu^e G_{t-1} (\varpi_t(l)) \) are the expected monitoring costs. As explained in detail by CMR (2014) and Del Negro and Schorfheide (2013), the previous problem can be solved with respect to \( \varpi_t(l) \) and the ratio \( B_t^e(l) / N_{t-1}(l) \), which is related to each entrepreneur’s leverage. Notably, the solution of this program implies that the optimal choices of \( \varpi_t(l) \) and \( B_t^e(l) / N_{t-1}(l) \) are common across entrepreneurs, thus facilitating aggregation.

At the end of period \( t \), after having sold undepreciated capital, collected rental income and paid the contractual rate to the financial intermediary, a fraction \( 1 - \gamma^*_t \) of the entrepreneurs exits the economy, whereas the complementary fraction \( \gamma^*_t \) continues operating in the next period. A fraction of total net worth owned by exiting entrepreneurs is consumed upon exit, while the rest
Aggregate entrepreneurs’ equity $V_t$ evolves as follows

$$V_t = R^{n,k}_t Q_{t-1} K_{t-1} - R_{t-1} \left( Q_{t-1} K_{t-1} - N_{t-1} \right) - \mu^e G_{t-1} (\sigma_{t}) R^{n,k}_t Q_{t-1} K_{t-1}.$$

The evolution of entrepreneurs’ total net worth is

$$N_t = \gamma_t^* V_t + W^e_t,$$
inflation targeting shock $\varepsilon_{\pi^*,t}$, as in Ireland (2007), to account for the low frequency behavior of inflation

$$\log \pi_t^* = (1 - \rho_{\pi^*}) \log \pi_t^* + \rho_{\pi^*} \log \pi_{t-1}^* + \varepsilon_{\pi^*,t}.$$ 

In the optimal policy exercise we will assume that the central bank sets the interest rate to maximize the utility of the representative agent, and thus equation 2 will be substituted by the optimal (Ramsey) decision rule.

Public spending is a time-varying fraction of output

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t,$$

where $g_t$ varies exogenously over time in response to independently and identically distributed $N\left(0, \sigma_g^2\right)$ fiscal shocks $\varepsilon_{g,t}$, as follows

$$\log g_t = (1 - \rho_g) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t}.$$ 

Finally, the resource constraint is given by

$$P_tC_t + P_tI_t + P_t a (u_t) \bar{K}_{t-1} = \frac{1}{g_t} P_t Y_t.$$ 

3 The Bayesian Estimation

This section presents our empirical analysis. In a first step we describe the data and the details of the Bayesian estimation’s procedure. In a second step we discuss the main results of our exercise in terms of posterior distributions for the estimated parameters and variance decompositions.

Data. We use eleven quarterly observables series for the US economy focusing on the sample 1964:Q2 - 2009:Q4. We include the same eight key macroeconomic variables as JPT (2013) and three financial variables also used by CMR (2014). The sample period is the same as the one used by JPT (2013), since we want to nest their results as a special case in our analysis.

The eight macroeconomic variables include the inflation rate, the nominal interest rate, the logarithm of per-capita hours, the log-difference of real per-capita GDP, consumption and investment, and two measures of nominal hourly wage inflation. To match the wage inflation variable in the model, $\Delta \log W_t$, with the two data series, we use the following measurement
equations

\[
\begin{bmatrix}
\Delta \log NHC_t \\
\Delta \log NE_t
\end{bmatrix}
= \begin{bmatrix} 1 \\ \Gamma \end{bmatrix} \Delta \log W_t + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}
\]

\[e_{i,t} \sim i.i.d. \quad N(0, \sigma_{e_i}) \quad i = 1, 2\]

where \(\Delta \log NHC_t\) represents the growth rate of nominal compensation per hour in the total economy, \(\Delta \log NE_t\) represents the growth rate of average hourly earnings of production and nonsupervisory employees, \(\Gamma\) is a loading coefficient of the second wage series, while the first wage series’ loading coefficient is normalized to one, and \(e_{1,t}\) and \(e_{2,t}\) are observation errors.

In addition, we use three financial variables as in CMR (2014), namely the credit spread measured by the difference between the interest rate on BAA-rated corporate bonds and the ten-year US government bond rate (as a proxy for the external finance premium), the log difference of per-capita real net worth, and the log difference of per-capita real credit to firms.\(^4\) Following CMR (2014), an independently and identically distributed observation error, with a zero mean, \(\sigma_{e_i}\) standard deviation Weibull distribution, is also assumed for the net worth series. A detailed description of the data is presented in Appendix D.

**Prior and posterior distributions.** For the parameters that are in common with JPT (2013), we follow their distributional assumptions. We borrow the prior assumptions of the parameters that are related to the financial frictions block from CMR (2014) and Del Negro and Schorfheide (2013). The information on prior distributions is summarized in Table 1 while related figures are provided in Appendix E.

Following the standard practice in the literature, some parameters are fixed in the estimation procedure. The capital depreciation rate is calibrated at 0.025, the steady state ratio of government spending to GDP at 0.2, the steady state net wage mark-up at 25 percent and the persistence of the inflation target shock at 0.995. As for the financial sector, the entrepreneurs’ default probability \(F(\pi)\) is set at 0.0075 (3 percent in annual terms) and the entrepreneurs’ survival rate \(\gamma^*\) at 0.99.\(^5\) We also fix the steady state of technology growth \((100\gamma)\), hours worked \((\log L_{ss})\) and inflation rate \((100(\pi - 1))\) at the JPT (2013) estimated posterior medians, i.e. 0.47, 0, and 0.24 respectively. This implies that sample means of all observed variables have

\(^4\)As also pointed out in CMR (2014), we obtain similar results when we repeat our empirical analysis using the alternative measure of the spread constructed by Gilchrist and Zakrrajsek (2012).

\(^5\)CMR (2014), focusing on the shorter sample 1985-2010, estimate \(F(\pi)\) at 0.0056, and calibrate \(\gamma^*\) at 0.985. Our results are largely unaffected under this alternative parameterization.
been removed before the estimation, with the exception of the credit spread mean \((\hat{S})\) which is estimated as in Del Negro and Schorfheide (2013). This is to prevent low-frequency elements, such as the long-run means, from having counterfactual implications for the model business cycle frequencies. For example, average consumption growth is higher than GDP growth in the data, while in the model the consumption to GDP ratio is stationary.

We estimate the posterior distributions by maximizing the log-posterior function, which combines the prior information on the parameters with the likelihood of the data. In the next step, the Metropolis-Hastings algorithm is used to obtain a complete picture of the posterior distribution and to evaluate the marginal likelihood of the model. We run two Metropolis-Hastings chains of 1 000 000 iterations each, with a 20 percent burn-in. The model is estimated over the full sample period, but our results are robust when we focus on the shorter sample period (1985-2010) used in CMR (2014). Brooks and Gelman (1998)'s multivariate convergence statistics of MCMC are presented in Appendix E together with the full posterior distributions.

We report the estimated posterior medians of our baseline model with financial frictions in Table 1. Some parameters display substantial changes with respect to the standard New Keynesian model and play a key role in explaining our results in terms of output gap and policy trade-offs. The most striking difference is in the estimated process for the labor supply shock. Both its standard deviation and its persistence are found to be much lower in our baseline model. The former is estimated at a value of 0.52 (as opposed to 4.49), the latter at 0.47 (instead of 0.98). The second important difference is in the parameters regulating price and wage dynamics, specifically \(\xi_w\) and \(\epsilon_p\), together with, to a minor extent, the inverse of labor supply elasticity \(\nu\), which imply flatter New Keynesian Phillips curve for prices and wages as further discussed below. All the remaining parameters in common with the standard New Keynesian model are mainly in line with JPT (2013) estimates and, if variations occur, they do not drive our results. Finally, the financial frictions parameters \(\zeta_{sp,b}\) and \(\tilde{S}\), whose posterior medians are 0.04 and 0.43 respectively, are in the ballpark of the estimates provided in Del Negro, Giannoni and Schorfheide (2015) and CMR (2014).

**Variance decompositions.** The difference in the estimated parameters of the labor supply shock process has strong implications for the variance decomposition. In fact, while in the standard New Keynesian model labor supply shocks explain a large share of the low frequency fluctuations in actual output, as shown by the unconditional variance decomposition in Table 2, this is not the case in our baseline model with financial frictions, where actual output is mostly
driven by shocks to the marginal efficiency of investment (62 percent) and financial shocks (35 percent). At business cycle frequencies, financial shocks are dominant. They explain a large fraction of output fluctuations (73 percent) and crowd out the importance of shocks to the marginal efficiency of investment, which instead play a key role in the standard New Keynesian model.

The fact that financial shocks absorb explanatory power from investment shocks is not a new result. This has already been shown and explained in detail in CMR (2014). Here we just extend the validity of this result to a longer sample period. A key contribution of our paper is instead to uncover the minor importance of labor supply shocks at low frequencies in favor of shocks to the marginal efficiency of investment which, despite losing importance at business cycle frequencies, become relevant in the long run (cf. Table 2). The lower importance of labor supply shocks together with the relevance of financial shocks have critical implications for the dynamics of potential output (and as a consequence for the output gap), as we will discuss in detail in the next section.

At this stage it is crucial to understand why the role of labor supply shocks is so marginal in our baseline model. The use of financial variables in the estimation rationalizes this result. Financial variables are positively correlated with price and wage inflation and thus favor a more important role for demand shocks, as emerges clearly from the variance decomposition in Table 2. Demand shocks account for 86 percent of output fluctuations at business cycle frequencies in our model compared to only 62 percent in the standard New Keynesian model, in keeping with previous results in CMR (2014). While positively correlated with price and wage inflation, however, stock market booms (that are a proxy for the evolution of net worth in our model) and credit booms are associated to limited fluctuations in price and wage inflation, as can be seen in the first panel of Figure 1. The shaded areas highlight the US stock market booms, as classified by Christiano, Ilut, Motto and Rostagno (2010). During those periods, the evolution of price and wage indexes does not exhibit any remarkable acceleration. While the fact that stock market and credit booms have been non-inflationary in the US post-war period has already been discussed at length in Christiano, Ilut, Motto and Rostagno (2010), here we emphasize that the same result applies to wages.

\footnote{CMR (2014) do not feature a labor supply shock in the published version of their paper. In previous versions of their paper, however, a wage mark-up shock was included and turned out to be almost irrelevant for economic fluctuations.}

How does our estimated DSGE model with financial frictions rationalize this non-inflationary nature of financial booms? One way for the model to generate this limited correlation is through flat New Keynesian Phillips curves for prices and wages. In fact, we estimate a substantially higher degree of wage rigidity in the model with financial frictions, as the Calvo wage parameter $\xi_w$ is estimated at 0.93 instead of 0.73 in the standard New Keynesian model (while wage indexation remains unchanged). Another parameter affecting the slope of the wage curve is the inverse of the labor supply elasticity $\nu$ which we estimate as somewhat lower in our baseline model, i.e. 2.35 instead of 2.67. A flatter New Keynesian Phillips curve for wages implies that smaller labor supply shocks are necessary to reconcile data on wages and on the marginal rate of substitution (which is a function of consumption and hours worked). As far as the degree of price stickiness, measured by the parameter $\xi_p$, is concerned, we do not observe a relevant difference, but we find a higher indexation parameter $\lambda_p$, 0.53 as opposed to 0.15, which also translates into a flatter Phillips curve. A similar intuition is developed in Del Negro, Giannoni and Schorfheide (2015) to explain how a model with financial frictions accounts for the limited drop in inflation during the Great Recession through a flat New Keynesian Phillips curve for prices. Here, in the context of the same kind of model, but with more observables used in the estimation, we find a similar mechanism acting mainly through the wage equation.

There is, however, another important reason to explain why labor supply shocks lose importance. In fact, as it can be seen in Figure 2, the external finance premium is pro-cyclical conditional on labor supply shocks, while it is strongly countercyclical unconditionally. This property of labor supply shocks does not matter in models that do not include a measure of the spread as an observed variable but it is of course relevant in our case.\textsuperscript{8} Why then a contractionary labor supply shock does lead to a decline in the premium? An exogenous decline in the labor input has a negative effect on the demand for capital, as the two factors of production are complements in the production function. This leads to a decline in the price of capital, a reduction in its utilization rate and a decline in investment. Such a persistent decline in the value of the capital stock (the assets of entrepreneurs), translates into a decline in both the liabilities and the net worth of entrepreneurs. However, the decline in the value of assets is larger than the decline in net worth, thus leading to a reduction in leverage. A lower level of leverage is reflected in a decline in the external finance premium as it can be seen in equation 1. We

\textsuperscript{8}We conjecture that the same mechanism is at play in the CMR (2014) model where wage mark-up shocks are crowded out by financial shocks. We will further explore this conjecture and the empirical validity of these spread’s dynamics generated by financial accelerator models in future research.

We 4.94), (1.07, 3.64, 4.44, 8.06), (2.16, 1.74, 5.19, 15.39), (1.15, 2.98, 0.70, 10.09).
conclude that, as the use of stock market data is crucial to limit the role of investment shocks at business cycle frequencies, the use of data on the external finance premium helps explaining the reduced role of labor supply shocks, both in the short run and in the long run.

4 The Output Gap and Financial Frictions

Summary of the distortions. Our model features three sets of distortions: monopolistic competition (in goods and labor markets), nominal rigidities (in prices and wages) and financial frictions that create a wedge between the interest rate paid by entrepreneurs to finance capital expenditures and the interest rate set by the central bank. In what follows, we analyze each friction in turn.

The monopolistic competition distortion is essentially static by making the steady state level of output inefficiently low. However, as explained in JPT (2013), it also has a minor effect on the dynamics of the log-linear model because of the fixed cost in production, which is calibrated to obtain zero profits in the distorted steady state. In an (efficient) competitive economy, the fixed cost would disappear together with profits.

Nominal rigidities, in the form of sticky prices and wages, distort the transmission of shocks. A classic example is given by a positive technology shock that in most cases increases hours under flexible prices and wages but lowers hours under sticky prices and wages (cf. Galí, 1999). Financial frictions distort the steady state of the economy (the external finance premium is positive in steady state) but also the dynamic responses to shocks through the financial accelerator mechanism.

To summarize, the steady state is distorted by the monopolistic competition distortion and the presence of a positive spread, whereas the dynamics are distorted by the presence of nominal rigidities, the financial accelerator mechanism and the possibly minor effect of steady-state distortions.

The reference level of output. In such a distorted economy as our medium-scale model with financial frictions, it is not obvious what should be the reference level of output to calculate the output gap.

In small-scale New Keynesian models, the reference level of output is efficient output. In that context, the output gap is at the same time i) a measure of welfare (since it enters the microfounded loss function derived as a second order approximation of the utility function) ii) a measure of the economy’s cyclical position (with respect to the efficient frontier of the economy)
and iii) a measure of imbalances and inflationary pressures, as the output gap enters directly in the New Keynesian Phillips curve for prices, as a result of the proportionality between the real marginal cost and the output gap.\footnote{Ravenna and Walsh (2006), Carlstrom, Fuerst and Paustian (2010) and De Fiore and Tristani (2013) derive welfare relevant measures of the output gap in small-scale models with financial frictions. In the three papers the gap is defined in terms of deviation from the efficient level of output.}

In medium-scale models, the choice of the reference level of output is less obvious since an analytical characterization of the welfare function is not available. Moreover, in presence of capital accumulation the output gap (calculated in deviation from efficient output) is no longer proportional to the real marginal cost and thus is not necessarily a measure of imbalances (and inflationary pressures in particular). Nonetheless, the previous literature on medium-scale models has still considered a reference level of output that is a good approximation of the efficient level of output. Smets and Wouters (2007) calculate the gap in deviation from potential output, i.e. the counterfactual level of output that emerges under flexible prices and wages and in the absence of inefficient shocks (i.e. price mark-up and wage mark-up shocks). The level of potential output is lower than efficient output, as it is affected by steady-state distortions (monopolistic competition). However, it approximates the dynamics of efficient output well, since steady-state distortions have only a minor effect on the dynamics of the model. Notably, the literature has concentrated on potential output (and not on efficient output itself) on the basis of the argument that monetary policy is not the right instrument to deal with the steady-state distortions.

In our medium-scale model frictions are more pervasive but potential output may still be a good approximation of variations in the efficient frontier of the economy. Hence, we follow the previous literature and we choose the potential level of output as a reference. However, the definition of potential output is more involved in our model with financial frictions than in simpler medium-scale models. In fact, while in the standard New Keynesian model nominal rigidities are the only distortion that affects the dynamics of the model, here the financial accelerator mechanism distorts the economy’s response to shocks. Therefore, our counterfactual is computed in the absence of both the nominal rigidities and the financial accelerator, with the aim of approximating the dynamics of the efficient frontier. This is achieved by imposing the parametric restrictions $\Lambda_{p,t} = \Lambda_{p,t} = \zeta_{sp,b} = 0$ in the counterfactual. The same approach is taken in JPT (2013), Levin, Onatski, Williams and Williams (2005), Sala, Söderström and Trigari (2008) Cúrdia and Woodford (2010) and Gali, Smets and Wouters (2011). Most of the literature uses the state variables from the allocation in which prices and wages have been flexible forever. We also follow this common practice.
The split between efficient and inefficient shocks is more problematic in our economy than in the standard New Keynesian model. The two financial shocks may be interpreted as inefficient shocks, together with price and wage mark-up shocks. On the one hand, being shocks to financial frictions, these disturbances should not affect the efficient frontier of the economy. Moreover, the risk shock is a shock to the standard deviation of the idiosyncratic technology shock, which may call for an efficient shock interpretation. Notably, however, the interpretation of financial shocks as inefficient or efficient is inconsequential in our model. In fact, both financial shocks do not propagate under flexible prices and wages and in the absence of a financial accelerator mechanism (cf. dashed lines in Figure 8). In the absence of financial frictions, as in the efficient equilibrium, variations in net worth have no impact and the spread is equal to zero, thus making the risk shocks immaterial. Therefore, even if considered as efficient, those shocks affect neither the efficient frontier nor potential output, which turns out to be a close approximation of efficient output. In other words, both financial shocks share the same properties of monetary shocks and do not propagate in our counterfactual exercise.

To sum up, the potential level of output in our economy is defined as the counterfactual level of output that emerges under flexible prices and wages, no inefficient shocks and no dynamic distortion associated to financial frictions. As in the previous literature, steady-state distortions (positive price and wage mark-ups and positive external finance premium) are not closed on the basis of the argument that monetary policy is not the right instrument to deal with those (quantitatively minor) inefficiencies.

Estimated output gap. In the first panel of Figure 3 we plot the output gap derived in our model with financial frictions and the output gap derived in the model without financial frictions that replicates exactly the results in JPT (2013). We note large differences between the two output gaps. The estimated gap is more volatile in the model with financial frictions and exhibits an important low frequency component, such that we observe a long positive cycle in the pre-Great Recession period and a large drop around Volcker’s disinflation period. These large differences are explained by the behavior of potential output, which we plot in the second panel of Figure 3. In the model with financial frictions, potential output is substantially higher in the 1980s and lower from 1993 until the beginning of the Great Recession than in the model without financial frictions.

\[11\] In keeping with this view, Dedola, Karadi and Lombardo (2013) include a financial shock (a shock to the fraction of bank assets that can be diverted) in their model and consider it inefficient. Gilchrist and Leahy (2002) interpret the fluctuations induced by a net worth shock as inefficient, as the shock does not propagate in frictionless models.
On the one hand, the standard New Keynesian model implies a large positive output gap during the Volcker disinflation and the twin recessions that followed it: negative labor supply shocks are responsible for this result, as they lower potential output more than actual output, thus opening a positive output gap.\textsuperscript{12} On the other hand, the model with financial frictions identifies a large output gap during the second half of the 1990s, when the path of potential output is essentially flat, as the boom in actual output in that period is mainly driven by expansionary financial shocks. Importantly, the output gap is still positive in the pre-Great Recession period, but its size is much lower than in the previous decade. In contrast, in the standard New Keynesian model potential output is much higher, sustained by large positive labor supply shocks. Put differently, the output boom is driven by growth in potential output, such that the output gap is almost always negative over the period 1995-2007. Finally, the standard New Keynesian model identifies a large drop in potential during the Great Recession, whereas potential even increases slightly in the model with financial frictions, despite the large decline in actual output, as potential output is unaffected by the large negative financial shocks that lower actual output in that period.

It is important to stress that we do not want to convince the reader that one or the other measure of the output gap is more plausible. Both measures differ in many respects from the "conventional view" of the US business cycle, often summarized by statistical measures of the output gap. Both models, with or without financial frictions, rely on measures of potential output that are volatile and that have an important low frequency component and thus differ from conventional measures of the output gap almost by construction. We rather want to highlight how the mere presence of financial frictions and financial shocks has large effects on the estimated output gap. We re-emphasize here that the difference between the two lines plotted in the first panel of Figure 3 are driven exclusively by the presence of financial frictions and financial shocks, as our model fully nests the JPT (2013) model.

Why then does potential output have such a different shape in our model with financial frictions? Essentially because financial shocks absorb explanatory power from efficient labor supply shocks (and to some extent also from investment-specific technology shocks, at least at business cycle frequencies). Notably, financial shocks do not affect potential output. As can be seen in Figure 4, labor supply shocks are smaller and propagate less in our model than in

\textsuperscript{12}These implications of the standard New Keynesian model have been criticized by Chari, Kehoe and McGrattan (2009) and Walsh (2006) who argue that a joint decline in actual and efficient output during the recessionary period 1979-1984 is implausible given the monetary flavor of those recessions. The presence of large negative labor supply shocks over the period is also in contrast with the dynamics of steadily increasing labor force participation.
the model without financial frictions. Since wage mark-up shocks are small by construction (absorbed by the measurement error in wages, as in JPT, 2013), the role of labor market shocks is limited in our model. Notice that, unlike in statistical measures of the output gap, potential output is very volatile in both models, although for different reasons. In the standard New Keynesian model, volatility is driven by labor supply shocks that are estimated to be very persistent and to propagate more under flexible prices and wages. In contrast, fluctuations in potential output in the model with financial frictions are dominated by investment-specific shocks that are extremely persistent, thus driving the low frequency dynamics (while they are dampened at business cycle frequencies, as in CMR, 2014).

Comparison with other models. At this stage it is instructive to compare our model-based output gap with previous contributions in the literature. As already mentioned, our gap is substantially different from statistical measures, as potential output does not evolve as smoothly as trend output but also because the model identifies some important low frequency components. However, such a non-stationary measure of the output gap with maximum values in the order of 10-15% is not specific to our estimated model. In Figure 5 we report the output gap derived in the original model by Smets and Wouters (2007) and we see how the shape, and to some extent also the magnitude, of that measure of the output gap resemble those of our estimated series. This similarity also emerges when we consider a version of the JPT model in which the labor market shock is interpreted as a persistent wage mark-up shock (and not as a labor supply shock), which corresponds to a version of our model without financial frictions and without labor supply shocks. As we see in Figure 5, also that measure of the gap resembles our gap. Galí, Smets and Wouters (2011) provide a model-based measure of the output gap for a version of their model in which they do not use unemployment as unobservable in the estimation (cf. Figure 8 in their paper).\footnote{The model in Galí, Smets and Wouters (2011) is almost identical to the original Smets and Wouters (2007) model where data on unemployment are not used in the estimation. The use of data on unemployment in Galí, Smets and Wouters (2011) is useful to separately identify (at least in the short run) wage mark-up shocks and labor supply shocks. The use of unemployment data in the estimation can also help obtain more stationary measures of the output gap (cf. also Christiano, Trabandt and Walentin, 2011).} Once again, the shape and also the magnitude of that measure are comparable to our measure of the gap. While the similarity across models is striking, the driving forces for the results are substantially different: in Galí, Smets and Wouters (2011), Smets and Wouters (2007) and in the JPT model with wage mark-up shocks only, the fluctuations induced by labor supply shocks are absorbed by inefficient wage mark-up shocks. In contrast, in our set-up wage mark-up shocks are small by construction and the importance of
labor supply shocks is limited by the presence of financial shocks that do not affect the efficient frontier of the economy. Nevertheless, a common pattern that emerges is that a non-stationary form of the output gap seems to rely on a large role for shocks that do not affect the efficient frontier of the economy: wage mark-up shocks in the previous literature and financial shocks in our model.

As mentioned in the introduction, this is, as far as we know, the first paper that highlights the importance of financial factors for the output gap in the DSGE literature. However, the role of financial factors has long been emphasized in the policy discussion. Borio, Disyatat and Juselius (2013) argue that standard measures of the output gap are unreliable since they do not properly take into account financial factors. They claim that those measures do not identify any imbalances in the pre-Great Recession period, essentially because boom and bust cycles in credit and asset prices are non-inflationary.\textsuperscript{14} In particular, they make their point in the context of an empirical analysis in which they regress the output gap on a lagged measure of the output gap itself and on credit growth and house price growth to purify the output gap from the influence of financial factors ("finance-neutral" output gap). Our point is related, although in a totally different set-up. Our estimated output gap is "finance-neutral" because financial shocks do not affect the efficient frontier of the economy. Notice, however, that Borio, Disyatat and Juselius (2013) use a measure of total credit that increases substantially in the pre-Great Recession period driven by the boom in credit to households. Given the structure of our model, we restrict our attention to credit to firms that is more stable in that period but that still exhibits the typical low frequency dynamics of the credit cycle.

**Alternative reference levels of output.** In keeping with the previous literature, potential output is the reference level of output to compute the output gap in our baseline model. Potential output is affected by the static distortions, whereas it does not respond to the inefficient shocks and the dynamic distortions. While these choices closely follow the previous literature, they are not obvious. Therefore, we now evaluate their impact on the estimation of the output gap.

First, we consider the effect of the static distortions that make the potential level of output inefficiently low. These steady-state distortions have a minor effect on the dynamics of potential output, thus driving a small wedge between potential and efficient output.\textsuperscript{15} We see in Figure

\textsuperscript{14}Empirical evidence on the non-inflationary nature of financial shocks is provided by Furlanetto, Ravazzolo and Sarferaz (2017) in a Vector Autoregression (VAR) model. For a more general discussion on the link between monetary policy, inflation and boom and bust cycles, cf. Christiano, Ilić, Motto and Rostagno (2010).

\textsuperscript{15}The value of the spread in steady state enters in the log-linear system of first order conditions. Moreover, as in JPT (2013), the presence of the fixed cost in the distorted economy affects the elasticity of output with respect to changes in the inputs of production. In the competitive economy, this effect is not present.
6 (dashed-dotted blue lines) that the effect of steady-state distortions on the dynamics is quantitatively minor. In fact, potential output and efficient output (both in deviation from their steady state) follow each other closely, thus showing that the choice of reference level of output to calculate the output gap is largely inconsequential.

We now evaluate the impact of closing the financial accelerator. In Figure 6 we plot (dashed-starred green lines) the output gap when the financial accelerator is left active both in the actual and in the counterfactual economy under flexible prices and wages. Somewhat surprisingly, we notice that the effect on the output gap is relatively minor and consists of a level shift in the middle and at the very end of the sample. These differences are due to the behavior of potential output that is mainly driven by investment-specific shocks, as labor supply shocks play a minor role. We see in Figure 4 that leaving the financial accelerator active or inactive under flexible prices and wages (dashed-starred green line vs dashed-plus red line) has some implications for the propagation of investment shocks. These differences, however, have a limited impact on the shape of the estimated output gap.

In our baseline model, financial shocks do not affect potential output as they do not propagate when the financial accelerator mechanism is inactive. We now evaluate what happens when we allow financial shocks to affect potential output in the model with an active financial accelerator mechanism (cf. dashed purple line in Figure 6). We see that, when we leave the financial shocks open, the effect on output is at most minor. This is due to the fact that financial shocks hardly propagate at all under flexible prices and wages in our model with an active financial accelerator. While a limited propagation under flexible prices and wages is a feature of most demand shocks, this is particularly striking for financial shocks.

To sum up, closing or opening the static distortions, the dynamic distortion associated with the financial accelerator and financial shocks has a very limited impact in our model, thus highlighting that the shape of the estimated output gap does not depend on the choice of the reference level of output, which is arguably debatable.

5 Optimal monetary policy and financial frictions

In the previous section we documented large and persistent deviations of the output gap from its potential level, which constitutes a good approximation of the efficient frontier. In this section, we investigate whether these fluctuations could have been avoided under a different monetary policy or whether they were unavoidable, thus representing the price to pay to stabilize other
policy objectives.

**Monetary policy trade-offs.** In small-scale models (cf. Erceg, Henderson and Levin, 2000), the output gap, price inflation and wage inflation are the only variables entering in the microfounded loss function for the central bank. Furthermore, given the simple structure of the model, a stable output gap is compatible with stability in a weighted average of price and wage inflation (in the absence of cost-push shocks). In a medium-scale model with financial frictions, an analytical expression for the loss function derived as an approximation of the representative agent utility function is not available. Other variables, and financial variables in particular, may be of direct relevance for the central bank and the trade-offs between different objectives may be more complicated.

Our goal here is to investigate how these trade-offs change in the presence of financial frictions with a quantitative perspective. We study the model’s optimal equilibrium, i.e. the welfare-maximizing equilibrium chosen by the central bank under commitment subject to the constraints represented by the behavior of private agents. More specifically, we use the solution of the model under Ramsey monetary policy to compute the counterfactual path of output and other endogenous variables that would have emerged if policy had always been optimal and the economy had been perturbed by the series of shocks estimated in the baseline version of the model under the historical Taylor-type interest rate rule (with the exception of the two shocks entering the Taylor rule that do not affect the optimal equilibrium). We assume that the only instrument available to the central planner is the short-term interest rate.

In Figure 7 we plot with solid blue lines the historical evolution of price inflation, wage inflation and the output gap (as defined in the previous section), while the dashed-dotted red lines refer to the counterfactual evolution of the same variables under optimal policy. We note that under optimal policy, wage inflation is almost perfectly stabilized, which also implies, given the direct link between wages, marginal costs and prices, a low and stable price inflation rate over the sample period. Taken together, these results show the optimality of a strong focus on nominal stabilization to undo the effects of nominal rigidities for the monetary policy authority. More specifically, we confirm previous results in the literature on the optimality of wage inflation targeting in New Keynesian models (cf. JPT, 2013, and Levin, Onatski, Williams and Williams, 2005). A strong (although not exclusive) focus on nominal stabilization turns out to be optimal also in small-scale stylized models with financial frictions in which it is possible to derive a welfare criterion analytically (cf. Cúrdia and Woodford, 2010, Carlstrom, Fuerst and Paustian,
2010 and De Fiore and Tristani, 2013), at least in response to standard disturbances. The advantages of our set-up are that we can quantify the trade-offs between nominal stabilization and alternative objectives and that our results do not rely on one (or a few) specific shocks but on a full set of disturbances, as recovered in the estimation process.

In panel A we compare the evolution of actual and optimal output (both plotted in deviation from potential output). While a substantial share of output gap fluctuations could have been avoided under optimal policy, we see that optimal output does not fully track potential output (cf. dashed line). This means that a non-negligible share of fluctuations (summarized by the difference between the dashed-dotted red line and the zero line) was unavoidable. While in the model without financial frictions the share of unavoidable fluctuations is extremely small and optimal monetary policy can achieve a "Trinity" by stabilizing the output gap, price inflation and wage inflation at the same time (cf. JPT, 2013), in our model optimal monetary policy can achieve only a "weak trinity": some fluctuations in the output gap are the unavoidable price to pay to achieve nominal stabilization. Nevertheless, the trade-offs between nominal and real stabilization remain relatively small under optimal policy (despite the presence of several distortions in the model), whereas the estimated trade-offs are large under the estimated Taylor rule. Our "weak trinity" result places our paper somewhere in between JPT (2013), who estimate negligible trade-offs, and Debortoli, Kim, Lindé and Nunes (2016), who identify large trade-offs even in the absence of financial frictions.

What shocks are responsible for the unavoidable fluctuations, or in other words, what are the shocks responsible for the diverging dynamics between optimal output and potential output? In Figure 8 we plot the impulse responses of potential and optimal output to all shocks. We see that optimal output tracks the response of potential output in response to most shocks. The main discrepancies are found in response to price mark-up, wage mark-up and government spending shocks. Therefore, price and wage mark-up shocks are the main drivers of the unavoidable fluctuations even in the context of a complex medium-scale model with financial frictions, as is the case in small-scale models. The other shocks, that in principle could generate large trade-offs, generate in practice only small trade-offs (with the partial exception of government spending shocks), given the estimated set of parameters. Notably, financial shocks generate small trade-offs under optimal policy, and nominal stabilization turns out to offset the effect on output of risk and net worth shocks. In fact, these disturbances propagate substantially more in presence of nominal rigidities whereas they propagate little under optimal policy and under flexible prices.
and wages. We can say that a "conditional trinity" emerges in response to the two financial shocks (but also to many other shocks), thus showing that a policy of nominal stabilization is close to optimal in most cases.

**The Monetary Policy Score.** Since a non-negligible share of output gap fluctuations was unavoidable (and actually desirable), the conventional output gap cannot be considered as an indicator of imbalances (or of inflationary pressures in particular) in the economy, unlike in small-scale models. A proper measure of imbalances in our model is given by the difference between actual and optimal output, i.e. the difference between the solid blue and the dashed-dotted red line in Figure 7 that we plot in isolation in Figure 9 with dashed-starred green line. We name this gap Monetary Policy Score, since it reflects all fluctuations that could have been avoided under optimal monetary policy. In other words, it can be seen as a measure of policy mistakes due to the suboptimality of the estimated Taylor rule. The Monetary Policy Score identifies large imbalances that build up rapidly over the mid-1990s and vanish abruptly during the Great Recession.

Besides being a proper measure of imbalances, the "Monetary Policy Score" features an additional advantage over the conventional measure of the output gap. In fact, in the computation of the "Monetary Policy Score" there is no need to distinguish between efficient and inefficient shocks, as is the case for the conventional output gap. As noted by Woodford (2003), it is often problematic to determine whether a specific real shock distorts the economy towards inefficiency or simply leads to fluctuations in the efficient frontier. Furthermore, it has proven challenging to distinguish between efficient and inefficient shocks even in the context of theoretical models: efficient labor supply shocks are observationally equivalent to inefficient wage mark-up shocks in standard models, whereas disentangling efficient productivity shocks from inefficient price mark-up shocks is often challenging in empirical exercises. The "Monetary Policy Score", and more specifically both actual and optimal output, is affected by all disturbances, regardless of their nature, and the distinction between efficient and inefficient shocks vanishes.

6 **Conclusion**

We have shown that the presence of financial frictions and financial shocks crucially changes the size and the shape of the estimated output gap. Furthermore, the conventional output gap and a better measure of imbalances (the Monetary Policy Score) are no longer equivalent as in the standard New Keynesian model by JPT (2013) that is nested by our model. Nevertheless, a
policy of nominal stabilization emerges as nearly optimal also in our model, at the cost of limited (although non-negligible) trade-offs with real economic activity stabilization. The key point of our paper is to show that the mere introduction of financial frictions and financial shocks has important policy implications.

This opens up several avenues for future research. First, we have conducted our analysis in the most standard model with financial frictions (the financial accelerator model) that, however, completely ignores frictions in the banking sector and household debt. Extending our analysis to alternative models with different kinds of financial frictions seems of paramount importance for monetary policy analysis. A first step in that direction has been taken by Rabanal and Taheri Sanjani (2015).

Second, our estimated conventional output gap (but also the Monetary Policy Score) features an important low frequency component and exhibits a large magnitude. This is in keeping with alternative models featuring an important role for inefficient shocks (cf. Smets and Wouters, 2007 among others). Nevertheless, Galí, Smets and Wouters (2011) show that modeling unemployment explicitly may be useful to obtain a more stationary measure of the output gap. Investigating whether this result is confirmed in a model that combines labor market and financial frictions also seems an interesting avenue for future research.

Finally, financial shocks play an important role in our model as long as news components are attached to the shock process. While this is also the case in the state-of-the-art estimated model by CMR (2014), the more recent VAR evidence hints that purely unanticipated financial shocks may play an important role on their own (cf. Furlanetto, Ravazzolo and Sarferaz, 2017, and the references therein). Finding alternative theoretical mechanisms (or alternative observable variables) in order to generate a more important role for unanticipated financial shocks also seems to be an urgent challenge for macroeconomic modelers.

References


Borio, C., Disyatat, F. P., Juselius, M., 2013. Rethinking potential output: Embedding infor-


Table 1: Estimated parameters
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Table 2: Variance decompositon
Figure 1: Evolution of wage, GDP deflator, credit to firms and stock prices. Shaded areas represent the periods of stock price booms as identified by Christiano, Ilut, Motto and Rostagno (2010).

Figure 2: Impulse responses of output and external finance premium (EFP) to a negative labor supply shock in the baseline model.
Figure 3: Output gap and potential output in the baseline model and in the standard New Keynesian model. Output gap is computed as the difference between actual output and potential output.
Figure 4: Impulse responses of output and potential output to an adverse labor supply shock and a positive investment-specific shock in the baseline model and in the standard New Keynesian model. The green line represents the response of output to an investment-specific shock in a version of the baseline model under flexible prices and wages and with an active financial accelerator.
Figure 5: Output gap in the baseline model compared with other model-based estimates.
Figure 6: Alternative measures of the output gap in the baseline model. The solid red line refers to a measure of the output gap calculated in deviation from potential output. The dashed-dotted blue line refers to a measure of the output gap calculated in deviation from efficient output. The dashed-starred green line refers to a measure of the output gap calculated in deviation from the counterfactual level of output under flexible prices and wages, with an active financial accelerator mechanism and in the absence of financial shocks. The dashed purple line refers to a measure of the output gap calculated in deviation from the counterfactual level of output under flexible prices and wages, with an active financial accelerator mechanism and in the presence of financial shocks.
Figure 7: Monetary policy trade-offs in the baseline model.
Figure 8: Impulse responses of potential output and optimal Ramsey output in the baseline model.
Figure 9: Conventional output gap and Monetary Policy Score in the baseline model.
Online appendices

A Model without financial frictions

In this appendix we report the set of non-linear equations characterizing the equilibrium dynamics of the following 24 endogenous variables of the model without financial frictions

\[ y_t, D_{p,t}, L_t, s_t, \pi_t, M_{p,t}, N_{p,t}, \bar{p}_t, c_t, \lambda_t, r^k_t, q_t, R^k_t, i_t, k_t, \bar{k}_t, \bar{w}_t, M_{w,t}, N_{w,t}, w_t, D_{w,t}, R_t, x_t, u_t \]

Production function

\[ D_{p,t}y_t = k_t^\alpha L_t^{1-\alpha} - F. \]  \hspace{1cm} (A.3)

Price dispersion

\[ D_{p,t} = (1 - \xi_p) \tilde{p}_t^{-1+\lambda_p} + \xi_p \left[ \left( \frac{\pi_{t-1}}{\pi} \right)^{\psi_p} \left( \frac{\pi_t}{\pi} \right)^{-1} \right]^{-1+\lambda_p} \tilde{p}_{p,t-1}. \]  \hspace{1cm} (A.4)

Capital-labor ratio

\[ \frac{k_t}{L_t} = \frac{w_t}{r^k_t} \alpha \frac{1}{1-\alpha}. \]  \hspace{1cm} (A.5)

Marginal cost

\[ s_t = \frac{1}{\alpha \alpha^\alpha (1-\alpha)^{1-\alpha}} \left( r^k_t \right)^{\alpha} w_t^{1-\alpha}. \]  \hspace{1cm} (A.6)

Phillips curve

\[ \frac{N_{p,t}}{M_{p,t}} = \tilde{p}_t, \]  \hspace{1cm} (A.7)

\[ M_{p,t} = \lambda_t y_t + \xi_p \beta E_t \left\{ \left[ \left( \frac{\pi_t}{\pi} \right)^{\psi_p} \left( \frac{\pi_{t+1}}{\pi} \right)^{-1} \right]^{-1} \frac{1}{\lambda_p} M_{p,t+1} \right\}, \]  \hspace{1cm} (A.8)

\[ N_{p,t} = \lambda_t y_t \lambda_{p,t} s_t + \xi_p \beta E_t \left\{ \left[ \left( \frac{\pi_t}{\pi} \right)^{\psi_p} \left( \frac{\pi_{t+1}}{\pi} \right)^{-1} \right]^{-1} \frac{1+\lambda_p}{\lambda_p} N_{p,t+1} \right\}, \]  \hspace{1cm} (A.9)

\[ \left( 1 - \xi_p \right) \tilde{p}_t^{-\frac{1}{\lambda_p}} + \xi_p \left[ \left( \frac{\pi_{t-1}}{\pi} \right)^{\psi_p} \left( \frac{\pi_t}{\pi} \right)^{-1} \right]^{-1} \frac{1}{\lambda_p} \tilde{p}_{p,t} = 1. \]  \hspace{1cm} (A.10)
Marginal utility of income

\[ \lambda_t = \frac{e^{zt}b_t}{e^{zt}c_t - hc_{t-1}} - h\beta E_t \left( \frac{b_{t+1}}{e^{zt+1}c_{t+1} - hc_{t+1}} \right). \tag{A.11} \]

Euler equation

\[ \lambda_t = \beta R_t E_t \left( \frac{\lambda_{t+1} e^{-zt+1}}{\pi_{t+1}} \right). \tag{A.12} \]

Optimal capital utilization

\[ r_t^k = r^k u_t^x. \tag{A.13} \]

Optimal choice of physical capital

\[ \phi_t = \beta E_t \left( e^{-zt+1} \lambda_{t+1} \left[ r_{t+1}^k u_{t+1} - r^k u_{t+1}^{1+\chi} - 1 \right] \frac{1}{1+\chi} \right) + (1 - \delta) \beta E_t \left( \phi_{t+1} e^{-zt+1} \right), \]

where \( \phi_t = q_t \lambda_t \), and \( q_t \) is the relative price of capital. With the aim of incorporating financial frictions into the model, we need to define the real gross return to capital \( R_t^k \) (and nominal gross \( R_t^{n,k} = R_t^k \pi_t \)). To do that, instead of the previous equation we use the following set of alternative but equivalent equations

\[ R_t^k = \frac{r_t^k u_t - r^k u_t^{1+\chi} - 1}{q_t} (1 - \delta) q_t, \tag{A.14} \]

\[ E_t \left( R_{t+1}^k \right) = E_t \left( \frac{R_t}{\pi_{t+1}} \right). \tag{A.15} \]

Optimal choice of investment \( (S'' = \zeta) \)

\[ \lambda_t = \phi_t \mu_t \left\{ 1 - \frac{S''}{2} \left( \frac{i_t}{i_{t-1}} e^{zt} - e^\zeta \right)^2 - \frac{i_t}{i_{t-1}} e^{zt} S'' \left( \frac{i_t}{i_{t-1}} e^{zt} - e^\zeta \right) \right\} + \beta E_t \left( \phi_{t+1} e^{-zt+1} \mu_{t+1} \left( \frac{i_t+1}{i_t} e^{zt+1} \right)^2 S'' \left( \frac{i_t+1}{i_t} e^{zt+1} - e^\zeta \right) \right). \tag{A.16} \]

Capital input

\[ k_t = u_t k_{t-1} e^{-zt}. \tag{A.17} \]
Physical capital accumulation

\[ \bar{k}_t = (1 - \delta) e^{-\gamma t} \bar{k}_{t-1} + \mu_t \left[ 1 - \frac{S''}{2} \left( \frac{i_t e^{\gamma t} - e^{\gamma t}}{t_{t-1}} \right)^2 \right] i_t. \]  

(A.18)

Wage Phillips curve

\[ \tilde{w}_t = w_t \left( \frac{N_{w,t}}{M_{w,t}} \right)^{\frac{\lambda_w}{\lambda_w + \gamma (1 + \lambda_w)}}, \]  

(A.19)

\[ M_{w,t} = \lambda_t L_t w_t + \xi_w \beta E_t \left\{ \left[ \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{i_w} \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{-1} \left( \frac{w_{t+1}}{w_t} \right)^{-1} \right]^\gamma \right\} N_{w,t+1}, \]  

(A.20)

\[ N_{t,w} = \lambda_w t \varphi b_t L_t^{1+\nu} + \xi_w \beta E_t \left\{ \left[ \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{i_w} \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{-1} \left( \frac{w_{t+1}}{w_t} \right)^{-1} \right]^\gamma \right\} \frac{(1 + \nu) (1 + \lambda_w)}{\lambda_w} \frac{N_{w,t+1}}{N_{w,t}}, \]  

(A.21)

\[ (1 - \xi_w) \left( \frac{\tilde{w}_t}{w_t} \right)^{\frac{1}{\lambda_w}} + \xi_w \left[ \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{i_w} \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{-1} \left( \frac{w_{t-1}}{w_t} \right)^{-1} \right]^\gamma = 1, \]  

(A.22)

\[ D_{w,t} = (1 - \xi_w) \left( \frac{\tilde{w}_t}{w_t} \right)^{\frac{(1 + \lambda_w) (1 + \nu)}{\lambda_w}} + \xi_w \left[ \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{i_w} \left( \frac{\pi_t e^{\gamma t}}{\pi_t e^{\gamma t}} \right)^{-1} \left( \frac{w_{t-1}}{w_t} \right)^{-1} \right]^\gamma \frac{(1 + \nu) (1 + \lambda_w)}{\lambda_w} \frac{D_{w,t+1}}{D_{w,t-1}}. \]  

(A.23)

Monetary policy rule

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{3}{2 - \rho_R} \left( \frac{x_t}{t_{t-1}} \right)^{1/4} \phi_x \left( \frac{x_t}{t_{t-1}} \right)^{1/4} \phi_x \right)^{1 - \rho_R} e^{\xi_{R,t}}. \]  

(A.24)

Definition of GDP

\[ x_t = c_t + i_t + \left( 1 - \frac{1}{\delta} \right) y_t. \]  

(A.25)

Resource constraint

\[ c_t + i_t + r e^{-\gamma t} \bar{r}^t_{t-1} \frac{u_{t-1} + \nu - 1}{1 + \chi} = \frac{1}{\delta} y_t. \]  

(A.26)
B  Financial frictions block

The financial block determines the dynamics of the following 5 variables

$$\overline{\omega}_t, v_t, n_t, b_t^e, S_t$$

The zero profit condition is

$$[\Gamma_{t-1}(\overline{\omega}_t) - \mu^e G_{t-1}(\overline{\omega}_t)] \frac{R_{t}^{n,k}}{R_{t-1}} = \frac{q_{t-1} \overline{k}_{t-1} - n_{t-1}}{q_{t-1} \overline{k}_{t-1}}.$$  \hfill (B.1)

Let’s define the cumulative and marginal distribution ($\text{normcdf}$ and $\text{normpdf}$ in MATLAB respectively) of a variable as follows

$$\Phi \left( \overline{z}_t \right) = \int_{-\infty}^{\overline{z}_t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx,$$

$$\phi \left( \overline{z}_t \right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\overline{z}_t)^2},$$

where

$$\overline{z}_t = \ln \overline{\omega}_t + \frac{1}{2} \sigma_t^2 - \sigma_{t-1}^2.$$  

Then

$$\Gamma_{t-1}(\overline{\omega}_t) = \overline{\omega}_t \left[ 1 - \Phi \left( \overline{z}_t \right) \right] + \Phi \left( \overline{z}_t - \sigma_{t-1} \right),$$

$$G_{t-1}(\overline{\omega}_t) = \Phi \left( \overline{z}_t - \sigma_{t-1} \right).$$

The equity value

$$v_t e^{\overline{z}_t \pi_t} = R_{t}^{n,k} q_{t-1} \overline{k}_{t-1} - R_{t-1} \left( q_{t-1} \overline{k}_{t-1} - n_{t-1} \right) - \mu^e G_{t-1}(\overline{\omega}_t) R_{t}^{n,k} q_{t-1} \overline{k}_{t-1}.$$  \hfill (B.2)

The law of motion for net worth

$$n_t = \gamma_t^e v_t + w^e.$$  \hfill (B.3)
Entrepreneurs debt

\[ b_t^e = q_t k_t - n_t. \]  

(B.4)

The first order condition with respect to capital

\[
E_{t-1} \left[ \left[ 1 - \Gamma_{t-1} (\tilde{\omega}_t) \right] \frac{R^r_{t}^{n,k}}{R_{t-1}} + \frac{\Gamma'_{t-1} (\tilde{\omega}_t)}{\Gamma_{t-1} (\tilde{\omega}_t) - \mu^e G_{t-1} (\tilde{\omega}_t)} \left\{ \left[ \Gamma_{t-1} (\tilde{\omega}_t) - \mu^e G_{t-1} (\tilde{\omega}_t) \right] \frac{R^r_{t}^{n,k}}{R_{t-1}} - 1 \right\} \right] = 0,
\]

(B.5)

where

\[
\Gamma'_{t-1} (\tilde{\omega}_t) = 1 - \Phi \left( \frac{\tilde{\omega}_t}{\sigma} \right).
\]

\[
G'_{t-1} (\tilde{\omega}_t) = \frac{1}{\sigma_{t-1}} \phi \left( \frac{\tilde{\omega}_t}{\sigma} \right).
\]

Equation B.5 replaces equation A.15. In fact, it shows that there is a wedge between the expected return to capital and the real interest rate, the so called external finance premium. If combined with equation B.1 it can be showed that the premium positively depends on the entrepreneurs’ leverage. In the steady state section we will show that using the log-linearized equations. Finally the definition of the external finance premium

\[
S_t = E_t \left\{ \frac{R^r_{t}^{n,k}}{R_{t-1}} \right\} .
\]

(B.6)

C Steady state

In this section we derive the steady state expressions for the endogenous variables. For the financial frictions block we closely follow Del Negro and Schorfheide (2013). From equations A.14 and A.15 we have

\[ R = r^k + (1 - \delta). \]

According to equation A.12 \( R = \frac{e^\gamma}{\beta} \), so

\[ \frac{e^\gamma}{\beta} = r^k + (1 - \delta), \]
\[ r^k = \frac{e^\gamma}{\beta} - (1 - \delta). \quad (C.1) \]

With \( r^k \), equation A.6 and A.7-A.10 imply
\[ w = \left[ \frac{1}{1 + \lambda}\alpha (1 - \alpha)^{1-\alpha} \frac{1}{(r^k)^\alpha} \right]^{\frac{1}{1-\alpha}}. \quad (C.2) \]

With \( r^k \) and \( w \), A.5 can be used to compute
\[ \frac{k}{L} = \frac{w}{r^k} \frac{\alpha}{1 - \alpha}. \quad (C.3) \]

The zero profit condition for intermediate goods producers
\[ y - r^k k - wL = \left( \frac{k}{L} \right)^\alpha L - F - r^k k - wL = 0, \]
implies
\[ \frac{F}{L} = \left( \frac{k}{L} \right)^\alpha - r^k \frac{k}{L} - w. \quad (C.4) \]

Therefore
\[ \frac{y}{L} = \left( \frac{k}{L} \right)^\alpha - \frac{F}{L}. \quad (C.5) \]

From A.18 and A.26
\[ \frac{i}{L} = \left[ 1 - (1 - \delta) e^{-\gamma} \right] \frac{e^\gamma k}{L}, \quad (C.6) \]
\[ \frac{c}{L} = \frac{y}{L} \frac{1}{g} - \frac{i}{L}. \quad (C.7) \]

And from equation A.11
\[ \lambda L = \left( \frac{c}{L} \right)^{-1} \frac{e^\gamma - h\beta}{e^\gamma - h}, \]
so that from A.19-A.23 it is possible to obtain an expression for \( L \)
\[ L = \left[ \frac{w}{(1 + \lambda w) \varphi} \lambda L \right]^{\frac{1}{1+\varphi}}. \quad (C.8) \]
The easiest choice is to parametrize \( L \) (so that \( \varphi \) is uniquely determined). This implies that

\[
k = \frac{k}{L}, \tag{C.9}
\]

\[
y = \frac{y}{L}, \tag{C.10}
\]

\[
i = \frac{i}{L}, \tag{C.11}
\]

\[
c = \frac{c}{L}. \tag{C.12}
\]

All the other steady state values follow straightforwardly. Turning to the financial frictions block, equation C.1 becomes

\[
r^k = S\frac{C}{\beta} - (1 - \delta),
\]

where \( S = 1 + \tilde{S}/100 \). The steady state relationships C.1-C.12 and those following from them remain unchanged. To derive financial variables steady state values it is worth first to log-linearize some equations because some elasticities appearing in those equations are relevant to explain how the steady state is computed. First equation B.5 which yields (where hatted variables represent log-deviation from steady state)

\[
E_t \left\{ \tilde{R}_{t+1}^{n,k} - \tilde{R}_t \right\} + \zeta_{b,x} E_t \left\{ \tilde{\sigma}_{t+1} \right\} + \zeta_{b,\sigma} \tilde{\sigma}_t = 0, \tag{C.13}
\]

where

\[
\zeta_{b,x} = \frac{\partial}{\partial x} \left[ \frac{[1 - \Gamma (\bar{w})] \frac{\partial n}{\partial x} + \frac{\Gamma'(\bar{w})}{\Gamma(\bar{w}) - \mu c' G (\bar{w})} \left\{ [\Gamma (\bar{w}) - \mu c G (\bar{w})] \frac{\partial n}{\partial x} - 1 \right\}} {\left\{ [1 - \Gamma (\bar{w})] + \frac{\Gamma'(\bar{w})}{\Gamma(\bar{w}) - \mu c' G (\bar{w})} [\Gamma (\bar{w}) - \mu c G (\bar{w})] \frac{\partial n}{\partial x} \right\}} \right] x
\]

\[
\Gamma (\bar{w}) = \bar{w} \left[ 1 - \Phi (\bar{z}) \right] + \Phi (\bar{z} - \sigma),
\]

\[
G (\bar{w}) = \Phi (\bar{z} - \sigma),
\]

\[
\Gamma' (\bar{w}) = 1 - \Phi (\bar{z}),
\]

\[
G' (\bar{w}) = \frac{1}{\sigma} \phi (\bar{z}),
\]

\[
\bar{z} = \frac{\ln \bar{w} + \frac{1}{2} \sigma^2}{\sigma}.
\]
Log-linearization of the zero profit condition (expression B.1) yields

\[ \tilde{R}_t^{n,k} = \tilde{R}_{t-1} + \zeta_{z,x} \tilde{\omega}_t + \zeta_{z,\sigma} \tilde{\sigma}_{t-1} = - \left( \frac{\tilde{k}}{n} - 1 \right)^{-1} \left( \tilde{n}_{t-1} - \tilde{q}_{t-1} - \tilde{k}_{t-1} \right), \]

(C.14)

where

\[ \zeta_{z,x} = \frac{\frac{\partial}{\partial x} \left[ \Gamma (\tilde{\omega}) - \mu G (\tilde{\omega}) \right]}{\Gamma (\tilde{\omega}) - \mu G (\tilde{\omega})}, \quad x = \tilde{\omega}, \sigma \]

Combining equations C.13 and C.14 yields the well-known positive relationship between the external finance premium and the entrepreneur’s leverage

\[ E_t \left( \tilde{R}_{t+1}^{n,k} - \tilde{R}_t \right) = \zeta_{sp,b} \left( \tilde{q}_t + \tilde{k}_t - \tilde{n}_t \right) + \zeta_{sp,\sigma} \tilde{\sigma}_t, \]

where

\[ \zeta_{sp,b} = \frac{\zeta_{b,\tilde{\omega}}}{\zeta_{z,\tilde{\omega}}} \frac{n}{\frac{\tilde{k}}{n} - 1}, \]

\[ \zeta_{sp,\sigma} = \frac{\zeta_{b,\tilde{\omega}} - \zeta_{b,\sigma}}{\zeta_{z,\tilde{\omega}}}. \]

Elasticities \( \zeta_{b,\tilde{\omega}}, \zeta_{b,\sigma}, \zeta_{z,\tilde{\omega}}, \) and \( \zeta_{z,\sigma} \) have the following expressions

\[ \zeta_{b,\tilde{\omega}} = \frac{\frac{\mu e_n}{\tilde{k}} \Gamma'' (\tilde{\omega}) G' (\tilde{\omega}) - \Gamma'' (\tilde{\omega}) \Gamma' (\tilde{\omega})}{\left\{ 1 - \Gamma (\tilde{\omega}) \right\} + \frac{\Gamma'' (\tilde{\omega})}{\Gamma (\tilde{\omega}) - \mu G (\tilde{\omega})} \left\{ \Gamma (\tilde{\omega}) - \mu G (\tilde{\omega}) \right\}} \frac{R_{n,k}}{R}, \]

\[ \zeta_{b,\sigma} = \frac{1 - \mu e_n G'' (\tilde{\omega})}{1 - \mu e_n G' (\tilde{\omega})} + \frac{\mu e_n \Gamma'' (\tilde{\omega}) G'' (\tilde{\omega}) - \Gamma' (\tilde{\omega}) G'' (\tilde{\omega})}{\left\{ \Gamma (\tilde{\omega}) - \mu G (\tilde{\omega}) \right\}^2} \frac{R_{n,k}}{R} \]

\[ \zeta_{z,\tilde{\omega}} = \frac{\Gamma'' (\tilde{\omega}) - \mu G'' (\tilde{\omega})}{\Gamma (\tilde{\omega}) - \mu G (\tilde{\omega})}, \]

\[ \zeta_{z,\sigma} = \frac{\Gamma'' (\tilde{\omega}) - \mu G'' (\tilde{\omega})}{\Gamma (\tilde{\omega}) - \mu G (\tilde{\omega})} \Gamma (\tilde{\omega}) - \mu G (\tilde{\omega}), \]

where

\[ G'' (\tilde{\omega}) = - \frac{z_{\tilde{\omega}}}{z_{\sigma}^2} \phi (z_{\tilde{\omega}}), \]
\[ \Gamma''(\varpi) = -\frac{1}{\omega \sigma} \phi \left( z^\varpi \right), \]
\[ G_\sigma(\varpi) = -\frac{z^\varpi}{\sigma} \phi \left( z^\varpi - \sigma \right), \]
\[ G'_\sigma(\varpi) = -\frac{\phi \left( z^\varpi \right)}{\sigma^2} \left\{ 1 - z^\varpi \left[ z^\varpi - \sigma \right] \right\}, \]
\[ \Gamma_\sigma(\varpi) = -\phi \left( z^\varpi - \sigma \right), \]
\[ \Gamma'_\sigma(\varpi) = \left( \frac{z^\varpi}{\sigma} - 1 \right) \phi \left( z^\varpi \right). \]

The strategy to compute financial variables steady states is to start by computing the value of \( \sigma \). To do that, we start from expression C.15 and we make it dependent only on known quantities and on one unknown, i.e. \( \sigma \), for which it is solved for. The elements that have to be set or made function of \( \sigma \) are: \( z^\varpi, \varpi, \frac{R_{n,k}}{R}, \mu^e, \frac{n}{k}, \) and \( \zeta_{sp,b} \). Two of them, i.e. \( \frac{R_{n,k}}{R} = S \) and \( \zeta_{sp,b} \), are estimated from data. Then by calibrating the entrepreneurs default probability \( F(\varpi) \) we can compute \( z^\varpi \) using the inverse cumulative distribution (\textit{norminv} in MATLAB)

\[ z^\varpi = \Phi^{-1} \left( F(\varpi) \right), \]

which we can use to write \( \varpi \) as a function of \( \sigma \) only

\[ \varpi = \exp \left\{ \sigma z^\varpi - \frac{1}{2} \sigma^2 \right\}. \tag{C.16} \]

Then, solving expression B.5 for \( \frac{R_{n,k}}{R} \) yields

\[ S^{-1} = 1 - \mu^e \left\{ \frac{G'(\varpi)}{\Gamma'(\varpi)} \left[ 1 - \Gamma(\varpi) \right] + G(\varpi) \right\}, \]

which can be used to set \( \mu^e \) as a function of \( \sigma \) only

\[ \mu^e = \frac{1 - S^{-1}}{\frac{G'(\varpi)}{\Gamma'(\varpi)} \left[ 1 - \Gamma(\varpi) \right] + G(\varpi)}. \tag{C.17} \]

Finally, from expression B.1 we can set \( \frac{n}{k} \) as a function of \( \sigma \) only

\[ \frac{n}{k} = 1 - \left[ \Gamma(\varpi) - \mu^e G(\varpi) \right] S. \tag{C.18} \]
Once we get the value for $\sigma$, it is straightforward to obtain the values of $\bar{\omega}, \frac{n}{k}$, and $\mu^e$ through equations C.16, C.17, and C.18 respectively. As a consequence

$$n = \frac{n}{k}.$$

Using expressions B.2 and B.3 and calibrating entrepreneurs’ survival rate $\gamma^*$ we can derive a value for $\frac{w^e}{k}$ (and for $w^e$ as a consequence as $w^e = \frac{w^e}{k}$)

$$\frac{w^e}{k} = \left(1 - \frac{\gamma^*}{\beta}\right) \frac{n}{k} - \frac{\gamma^*}{\beta} \left\{ S \left[ 1 - \mu^e G (\bar{\omega}) \right] - 1 \right\}.$$ 

From equation B.3

$$\frac{n}{k} = \frac{\gamma^* v}{k} + \frac{w^e}{k},$$

$$\frac{v}{k} = \frac{1}{\gamma^*} \left( \frac{n}{k} - \frac{w^e}{k} \right),$$

$$v = \frac{v}{k}.$$

Finally the value for the entrepreneurs debt

$$b^e = \bar{k} - n.$$

### D Data

**Inflation rate**: quarterly log difference of the GDP deflator. **Nominal interest rate**: effective Federal Funds rate. **Per-capita hours**: number of hours worked in the total economy, divided by the civilian non-institutional population, 16 years and older. **Real per-capita GDP**: nominal GDP divided by population and the GDP deflator. **Real per-capita consumption**: sum of nominal expenditure on non-durables and services divided by population and the GDP deflator. **Real per-capita investments**: sum of nominal expenditure on consumer durables and total private investment divided by population and the GDP deflator. **Hourly wage inflation**: nominal compensation per hour in the total economy, from NIPA, and average hourly earnings of production and non-supervisory employees, computed by Bureau of Labor Statistics from the Establishment Survey. **Credit spread**: difference between the interest rate on BAA-rated corporate bonds and the ten-year U.S. government bond rate. **Real per-capita...**
**net worth**: Market Value of Equities Outstanding - Net Worth (Market Value) - Balance Sheet of Nonfarm Nonfinancial Corporate Business (MVEONWMVBSNCCB) divided by population and the GDP deflator. **Real per-capita credit**: sum of total Liabilities - Balance Sheet of Nonfarm Nonfinancial Corporate Business (TLBSNCCB) and total Credit Market Instruments - Liabilities - Balance Sheet of Nonfarm Nonfinancial Corporate Business (TCMILBSNCCB) divided by population and the GDP deflator.

### E Prior and posterior distributions and convergence

![Figure E.10: Prior (gray thin line) and posterior (dark thick line) distributions](image_url)

Figure E.10: Prior (gray thin line) and posterior (dark thick line) distributions
Figure E.11: Prior (gray thin line) and posterior (dark thick line) distributions
Figure E.12: Brooks and Gelman (1998) convergence diagnostics. The red and blue lines represent specific measures of the parameter vectors both within and between chains. First panel: constructed from an 80% confidence interval around the parameter mean. Second panel: a measure of the variance. Third panel: based on third moments. The overall convergence measures are constructed on an aggregate measure based on the eigenvalues of the variance-covariance matrix of each parameter.