Nonlinear Persistence and Partial Insurance: Income and Consumption Dynamics in the PSID

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In this paper we highlight the important role the PSID has played in developing our understanding of income dynamics and partial insurance, see for example, Krueger and Perri (2005), Blundell, Pistaferri and Preston (BPP, 2008) and Attanasio and Pistaferri (2016). In the partial insurance approach, transmission parameters are specified that link ‘shocks’ to income with consumption growth. These transmission parameters can change across time and may differ across individuals reflecting the degree of ‘insurance’ available. They encompass self-insurance through simple credit markets as well as other mechanisms used to smooth consumption.

We explore the nonlinear nature of income shocks and describe a new quantile-based panel data framework for income dynamics, developed in Arellano, Blundell and Bonhomme (ABB, 2017). In this approach the persistence of past income shocks is allowed to vary according to the size and sign of the current shock. We find that the model provides a good match with data on family earnings and on individual wages from the PSID. We confirm the results on income dynamics using the extensive population register data from Norway.

Exploiting the enhanced consumption and asset data in recent waves of the PSID, we show that nonlinear persistence has key implications for consumption insurance. The approach is used to provide new empirical measures of partial insurance in which the transmission of income shocks to consumption varies systematically with assets, the level of the shock and the history of past shocks.

I. Earnings and consumption dynamics

A prototypical “canonical” panel data model of (log) family (earned) income $y_{it}$ is:

$$y_{it} = \eta_{it} + \epsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$ 

where $y_{it}$ is net of a systematic component, $\eta_{it}$ is a random walk with innovation $\nu_{it}$,

$$\eta_{it} = \eta_{i(t-1)} + \nu_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$ 

and $\epsilon_{it}$ is a transitory shock.

There is good economic reasoning behind this decomposition: persistent shocks to income are more difficult to insure, especially for young families with low assets. How families cope with persistent shocks is the main focus of this research. Short-run fluctuations will matter too, of course, especially for households with low assets (or low access to liquid assets).

In the partial insurance framework, consumption growth is related to income shocks:

$$\Delta c_{it} = \phi \psi v_{it} + \psi \epsilon_{it} + \nu_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$ 

where $c_{it}$ is log consumption net of a systematic component, $\phi$ is the transmission of persistent shocks $v_{it}$, and $\psi$ the transmission of transitory shocks; the $\nu_{it}$ are taste shocks, assumed to be independent across periods, see BPP.

This baseline panel data model specification can be summarised as:

$$\Delta c_{it} = \phi \psi v_{it} + \psi \epsilon_{it} + \nu_{it},$$

$$\Delta y_{it} = v_{it} + \Delta \epsilon_{it},$$

which implies covariance restrictions:

$$\text{var} (\Delta c_{it}) = \phi^2 \sigma^2_\phi + \psi^2 \sigma^2_\epsilon + \sigma^2_\nu.$$
The framework allows ‘unusual’ shocks to wipe out the memory of past shocks. Additionally the future persistence of a current shock will depend on future shocks. We will see that the presence of ‘unusual’ shocks matches the data well and has a key impact on consumption and saving decisions over the life cycle.

In this framework we maintain the permanent-transitory factor structure

$$y_{it} = \eta_{it} + \epsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$ 

but allow $\eta_{it}$ to follow a general first-order Markov process. Denoting the $r$th conditional quantile of $\eta_{it}$ given $\eta_{i,t-1}$ as $Q_r(\eta_{i,t-1}, \tau)$, we specify

$$\eta_{it} = Q_r(\eta_{i,t-1}, u_{it}),$$

where $(u_{it} | \eta_{i,t-1}, \eta_{i,t-2}, \ldots) \sim \text{Uniform}(0, 1)$, and $\epsilon_{it}$ has zero mean, independent over time.

The conditional quantile functions $Q_r(\eta_{i,t-1}, u_{it})$ and the marginal distributions $F_{\epsilon_{it}}$ can all be age specific.

This framework allows for quite general nonlinear dynamics of income, allowing a general form of conditional heteroscedasticity, skewness and kurtosis. To see this, consider the following measure of persistence

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_r(\eta_{i,t-1}, \tau)}{\partial \eta}.$$

which measures the persistence of $\eta_{i,t-1}$ when, at age $t$, it is hit by a shock $u_{it}$ that has rank $r$. This measures the persistence of histories. Below we show strong evidence for such nonlinearities in persistence.

### III. An Empirical Model for Consumption

To motivate the specification of consumption we use a standard life-cycle incomplete markets model (some arguments are latent). Let $c_{it}$ and $a_{it}$ denote log-consumption and assets (beginning of period) net of age dummies. We model consumption in levels and leave the nonlinear rule flexible. Our empirical specification is based on

$$c_{it} = g_t \left( a_{it}, \eta_{it}, \epsilon_{it}, v_{it} \right) \quad t = 1, \ldots, T,$$

2Blundell and Preston (1998) develop these covariance restrictions for repeated cross-sections.

3The first-order Markov assumption can be generalised to Markov ($p$), with any fixed $p$ (although this requires larger $T$).
FIGURE 1. QUANTILE AUTOREGRESSIONS OF LOG-earnings

(a) PSID data  
(b) Norwegian administrative data

Note: Residuals $y_{\tau,t}$ of log pre-tax household labor earnings, Age 25-60 1999-2009 (US), Age 25-60 2005-2006 (Norway). Estimates of the average derivative of the conditional quantile function of $y_{\tau,t}$ given $y_{\tau,t-1}$ with respect to $y_{\tau,t-1}$. Quantile functions are specified as third-order Hermite polynomials.

Source: Arellano, Blundell Bonhomme (2017).

where $v_{\tau,t}$ are independent across periods, and $g_{t}$ is a nonlinear, age-dependent function, monotone in $v_{\tau,t}$. $v_{\tau,t}$ may be interpreted a taste shifter that increases marginal utility. This consumption rule is consistent, in particular, with the standard life-cycle model, e.g. Kaplan and Violante (2010). ABB derive conditions under which $g$ is nonparametrically identified.

With consumption specification given by

$$c_{\tau t} = g_{t} \left( a_{\tau t}, \eta_{\tau t}, \epsilon_{\tau t}, v_{\tau t} \right), \quad t = 1, \ldots, T,$$

consumption responses to $\eta$ and $\epsilon$ are

$$\phi_{t}(a, \eta, \epsilon) = \mathbb{E} \left[ \frac{\partial g_{t}(a, \eta, \epsilon, v)}{\partial \eta} \right],$$

$$\psi_{t}(a, \eta, \epsilon) = \mathbb{E} \left[ \frac{\partial g_{t}(a, \eta, \epsilon, v)}{\partial \epsilon} \right].$$

where $\phi_{t}(a, \eta, \epsilon)$ and $\psi_{t}(a, \eta, \epsilon)$ reflect the transmission of the persistent and transitory earnings components, respectively. They generalise the partial insurance coefficients of BPP.

Similar techniques can be used in the presence of advance information, e.g.

$$c_{\tau t} = g_{t} \left( a_{\tau t}, \eta_{\tau t}, \eta_{\tau t+1}, \epsilon_{\tau t}, v_{\tau t} \right),$$

or consumption habits, e.g.

$$c_{\tau t} = g_{t} \left( c_{\tau t-1}, a_{\tau t}, \eta_{\tau t}, \epsilon_{\tau t}, v_{\tau t} \right).$$

also cases where the consumption rule depends on lagged $\eta$, or when $\eta$ follows a second-order Markov process, see Section 3 in ABB. The framework allow for additional, unobserved heterogeneity in earnings and consumption. Households will also differ in their initial productivity $\eta_1$ and initial assets.

IV. Data and Estimation

The PSID went through a redesign in the late 1990s, introducing new consumption and asset modules. Since 1999 it collects some 70% of consumption expenditures, and more than 90% since 2005. We use the sum of food at home, food away from home, gasoline, health, transportation, utilities, etc. We also make use of the more detailed asset data, see Blundell, Pistaferri and Saporta-Eksten (2016), BPS. For comparison we make use of family earnings data from administrative records from the Norwegian population registers see Blundell, Graber and Mogstad (2015).

The results we present on the PSID use data from the 1999 - 2009 surveys. Assets holdings are the sum of financial assets, real estate value, pension funds, and car value, net of mortgages and other debt. Income $y_{\tau t}$ are residuals of log

\(^4\text{The Norwegian results are part of the project on 'Labour Income Dynamics and the Insurance from Taxes, Transfers and the Family'. See ABB Appendix C.}\)
total pre-tax household labor earnings on a set of demographics - cohort and calendar time dummies, family size and composition, education, race, and state dummies. Log consumption \( c_{it} \) is also a residual, using the same set of demographics as for earnings. Following BPS, we select married male heads aged between 25 and 59. In this paper we focus on a balanced sub-sample of \( N = 792 \) households.

The conditional quantile function for the permanent income factor \( \eta_{it} \), given \( \eta_{i,t-1} \), is specified as

\[
Q_i(\eta_{i,t-1}, \tau) = Q(\eta_{i,t-1}, age_{i,t}, \tau)
= \sum_{k=0}^{K} a_k \varphi_k(\eta_{i,t-1}, age_{i,t}),
\]

where \( \varphi_k, k = 0, 1, ..., K \), are polynomials (Hermite). Similarly for \( \varepsilon_{it} \) etc. The consumption (log) function, \( g(a_i, \eta_i, \varepsilon_i, age_{i,t}) \), is specified as a flexible polynomial in assets, permanent income factor, the transitory shock and age.

Estimation takes place in two steps, see ABB for details. The first step recovers estimates of the income parameters. The second step recovers estimates of the consumption parameters, given an estimate of the income parameters. The estimation algorithm alternates between draws of latent variables from candidate posteriors and quantile regressions using those draws, see also Arellano and Bonhomme (2016).

V. Empirical Results

Figure 1 provides our initial evidence for non-linear income dynamics. It presents estimates of the average derivative of the conditional quantile function of \( y_{it} \) given \( y_{i,t-1} \) with respect to \( y_{i,t-1} \) for both the PSID in panel (a), and the Norwegian register data in panel (b). These are evaluated at percentiles of the shock \( \tau_{shock} \) and at a value of \( y_{i,t-1} \) that corresponds to the \( \tau_{init} \) percentile of the distribution of \( y_{i,t-1} \). The estimates in Figure 1 display distinct and systematic nonlinearity. The persistence of income shocks is much lower for large negative (positive) shocks for high (low) initial incomes. The results for the PSID are confirmed in panel (b) for the Norwegian data.

Turning to the income model, Figure 2, panel (a) provides estimates of the average derivative of the conditional quantile function of the persistent income factor \( \eta_{it} \) on \( \eta_{i,t-1} \) with respect to \( \eta_{i,t-1} \), evaluated at percentile \( \tau_{shock} \) and at a value of \( \eta_{i,t-1} \) that corresponds to the \( \tau_{init} \) percentile of the distribution of \( \eta_{i,t-1} \). The estimates are evaluated at mean age in the sample. Panel (b) in Figure 2 is based on data simulated according to our nonlinear earnings model with parameters set to their estimated values. It shows a close accordance with the persistence in the PSID income data, see panel (a) of Figure 1.

Moving to the estimated consumption model, Figure 3 displays the average derivative of the conditional mean of \( c_{it} \) given \( y_{it} \), \( a_{it} \) and \( age_{it} \) with respect to \( y_{it} \), evaluated at values of \( a_{it} \) and \( age_{it} \) corresponding to their \( \tau_{assets} \) and \( \tau_{age} \) percentiles, and averaged over the values of \( y_{it} \). It shows consumption responses vary systematically with age and assets and in a way that accords with stand life-cycle theory. It also shows a clear accordance between the consumption model and data.

Finally, we provide preliminary evidence that the nonlinear persistence we have uncovered in family earnings data is also evident in hourly wage data. Figure 4, panel (a), presents estimates of nonlinear persistence in the permanent component for PSID male hourly earnings. Panel (b) provides the implication from the simulated nonlinear model. These results show an important role for unusual shocks and nonlinear persistence in hourly wage data, suggesting nonlinear persistence maybe a key feature for life-cycle models of family labor supply.

VI. Summary and Conclusions

In this paper we have outlined a new framework to shed new light on income dynamics and nonlinear transmission of income shocks to consumption. We have exploited important new measurements for consumption and assets in the PSID. We have also shown the complementarities between ‘big’ administrative data, like the Norwegian registers, and purpose designed panel surveys, like the PSID.

A Markovian permanent-transitory model of household income, which reveals asymmetric persistence of unusual shocks, is shown to accord well with the persistence of income in the PISD and in Norwegian register data. An age-dependent nonlinear consumption rule as a function of assets, permanent income and transitory income, is also applied to the PSID and shown to generate new empirical measures of the de-
FIGURE 2. NONLINEAR PERSISTENCE

(a) Persistent component $\eta_{it}$  
(b) Earnings, nonlinear model

Note: PSID data. Graph (a) shows estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, based on estimates from the nonlinear earnings model. Graph (b) is based on data simulated according to our nonlinear earnings model with parameters set to their estimated values. Source: Arellano, Blundell and Bonhomme (2017).

FIGURE 3. CONSUMPTION RESPONSES TO $y_{it}$, BY ASSETS AND AGE

(a) PSID data  
(b) Nonlinear model

Note: Estimates of the average derivative of the conditional mean of $c_{it}$ given $y_{it}$, $a_{it}$ & $age_{it}$ with respect to $y_{it}$, evaluated at values of $a_{it}$ & $age_{it}$ corresponding to their $\tau_{assets}$ & $\tau_{age}$ percentiles, and averaged over the values of $y_{it}$. Source: Arellano, Blundell and Bonhomme (2017).
The results also point to nonlinearities in the dynamics of individual male wages. In future research we will explore the impact of such nonlinear persistence on family labour supply and consumption smoothing, building on Blundell, Saporta-Eksten and Pistaferri (2010) and Heathcote, Storesletten, and Violante (2014).

Future research could also usefully examine firm to firm transitions and lay-offs, it could focus on the role of housing equity and local labour markets. It could also look at health and other types of (partially insured) shocks.

A final word to the PSID. Congratulations at 50! Thanks for everything, for all those many micro-data innovations and looking forward to the next 50 years!

REFERENCES


