Interactions of Financial and Real Frictions Along the Business Cycle

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Abstract

We model the interactions of financial frictions and real frictions along the business cycle, using a DSGE model calibrated for the US economy, with heterogeneous households, banks, firms, a housing market, and wage bargaining. The model features labor and investment frictions, in the form of convex costs, and financial frictions, in the form of credit constraints and the risk of banks diversion of funds. In addition, there are price frictions and habits in consumption.

We examine technology, monetary policy, and credit shocks. We look at the response to these shocks of real aggregate variables, financial market variables, housing market variables, and labor market variables.

We find that the interactions of real frictions and financial frictions have important implications for the effects of financial shocks on the macroeconomy.

Key Words: real frictions, financial frictions, business cycles, monetary policy, macroprudential policy.

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1 Introduction

The Global Financial Crisis (GFC), its repercussions and ramifications have seen both policymakers and academic researchers become acutely aware of the importance of financial markets, including the shocks and frictions inherent in them, for business cycles. At the same time policymakers keep using labor market indicators, such as wage inflation, the rate of unemployment, and more, to be key inputs in their policy decisions. In this paper we ask what are the connections between the two. In particular we ask how do labor market frictions and shocks relate to financial frictions and shocks over the business cycle, seeking to identify channels of effect running from financial markets to labor markets and vice versa. While looking at the labor market, it makes sense to examine the market for capital in similar vein.

The emerging literature on DSGE modelling with financial frictions since the onset of the crisis, surveyed below, either spells out a financial sector or adds financial frictions and/or shocks to the modelling of the firm to determine the intermediation process between firms and households. As suggested by Gertler, Kiyotaki, and Prestipino (2016), these recent models represent an improvement on existing models by adding features that are important in understanding the recent crisis. But the interaction of financial frictions with frictions in the real economy, such as labor and capital markets frictions, has hardly been investigated by the recent literature. In this paper we use a DSGE model that links financial markets and financial frictions with real markets and real frictions, to enhance our understanding of how shocks are transmitted through the real economy and explore the linkages between financial markets and the real economy.

We use a DSGE model calibrated to the US economy, with heterogeneous households, banks, firms, a housing market, and wage bargaining. The model features labor and investment frictions, in the form of convex costs, and financial frictions, in the form of credit constraints and the risk of banks diversion of funds. In addition there are price frictions and habits in consumption. Essentially this a standard DSGE model in the spirit of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007),

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to which we add labor frictions as in Merz and Yashiv (2007) and Yashiv (2016), and financial frictions as in Iacoviello (2015) and Gertler and Kiyotaki (2011, 2015). This approach enables us to obtain a comprehensive model to investigate the behaviour of real aggregate variables (GDP, capital and investment, employment, and hiring), financial market variables (interest rate spreads, volumes of lending and deposits, bank net worth), housing market variables (prices and sales) and labor market variables (wages, employment, unemployment, and hiring). We examine technology and monetary policy shocks, as well as credit shocks, aiming to determine the consequences of the interactions of real and financial frictions.

We identify two main financial market issues at the focus of our analysis: firm borrowing from banks and the leverage and credit spreads characterizing the latter. We link these to gross hiring costs and gross investment costs in a number of channels. We then attempt to disentangle the relative roles played by the various frictions and shocks in this system.

The paper proceeds as follows: Section 2 presents the literature. Section 3 discusses the model, highlighting the key features in our model that distinguish it from previous models. Section 4 presents the methodology, including the calibration of the model. Section 5 presents the results. Section 6 discusses the results and presents robustness checks. Section 7 offers concluding remarks.

2 Literature

Business cycle research in Macroeconomics has been facing new challenges following the 2007-2009 GFC; see Linde, Smets and Wouters (2016) and Ramey (2016) for broad discussions. Prior to the crisis, macroeconomic researchers and policymakers relied on the benchmark DSGE model, as formulated in Christiano, Eichenbaum, and Evans (2005) and in Smets and Wouters (2007) and described in detail in the Gali (2015) textbook. Basically this model is a New-Keynesian model with price frictions, modelled after Rotemberg (1982) or Calvo (1983). But the important events in financial markets and housing markets, and their substantial effects on the overall economy, were missing from these standard models. Much of the ensuing work has been an attempt to embed various concepts of frictions, and in particular financial frictions, in existing business cycle DSGE models to account for such developments.

The literature, emerging over the past decade, incorporating financial frictions in macroeconomic models is already voluminous. Surveys and discussions may be found in Gertler and Kiyotaki (2011), Brunnermeier, Eisenbach, and Sannikov (2013), Ramey
(2016), and Gertler, Kiyotaki and Prestipino (2016). In what follows we discuss the specific papers relevant for the current one.

In terms of DSGE modelling we draw upon Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). For investment frictions we follow the approach used by Chirstiano, Eichenbaum and Evans (2005) and Christiano, Eichenbaum and Trabandt (2016). For labor frictions we postulate gross hiring costs, following Merz and Yashiv (2007) and Yashiv (2016), which use an approach akin to the one used for investment costs by Lucas and Prescott (1971) and by Tobin (1969) and Hayashi (1982). More recently, King and Thomas (2006), Khan and Thomas (2008), Alexopoulos (2011), and Alexopoulos and Tombe (2012) provide justifications for the hiring costs formulation used here.

For financial shocks and frictions we draw upon papers that study two important sources of shocks within the economy: the housing market and the banking sector. One is Iacoviello (2015), which adds the housing market to a DSGE model with financial frictions to capture the role of housing losses in triggering and amplifying the 2007 crisis and the effects to the real economy. His model includes heterogeneous households, bankers, which intermediate funds between savers (patient households) and borrowers, who use loans for consumption, secured on residential housing (impatient households) and firms (entrepreneurs). In equilibrium both bankers and entrepreneurs are credit constrained and as such, deleveraging by banks, due to a housing market shock, results in a credit crunch, that spills overs to corporate loans, amplifying and propagating the shock to the real economy. The second is Gertler and Kiyotaki (2011, 2015), who model agency issues in financial intermediation by banks. These agency issues lead to the emergence of a spread between banks’ lending and funding rates. Because agents in the economy need to borrow from banks, movements in the spread – caused by shocks within the banking sector – will have real effects. Gertler and Karadi (2011) have implemented the Gertler and Kiyotaki (2011) model in a DSGE framework. Gersbach and Rochet (2017) have discussed this framework in the context of an analysis of credit cycle stabilization and counter-cyclical capital requirements on banks. Important related contributions have been made by Jermann and Quadrini (2012) and by Curdia and Woodford (2016).
3 The Model

3.1 The Set-Up

The basic set-up is a standard New Keynesian DSGE model. There are two types of households, featuring habit formation, disutility of work, utility from housing, and borrowing à la Iacoviello (2015). There are two types of firms, with the monopolistically-competitive firms facing Rotemberg (1982) price fictitious, investment adjustment costs as in Christiano, Eichenbaum and Evans (2005), and hiring costs as in Merz and Yashiv (2007). Labor markets are frictional with Nash wage bargaining in the DMP tradition; see, for example, Merz (1995). There is a banking sector with frictions following Gertler and Kiyotaki (2011, 2015).

To allow for simultaneous lending and borrowing within the household sector we follow Iacoviello (2015) and assume that there are two types of households: patient households, to be denoted with index $H$ – forming a fraction $(1 - \sigma)$ of the labor force – who hold deposits with banks and own housing, and impatient households, to be denoted with index $S$ – forming a fraction $\sigma$ of the labor force – who borrow against their housing wealth. Each household type is of measure 1. We assume that frictions exist such that households can not borrow from or lend to each other. This creates a need for a banking sector that can channel funds from the saving households to the borrowing households. Savers make deposits at these banks, who then lend the money to borrowers. The borrowers face a collateral constraint, only able to borrow up to a given ratio of the value of their housing. The saving households are also assumed to own the firms and the banks and receive dividends from them.

Firms produce output using labor and capital. We assume that frictions exist such that firms are unable to fund all of their investment out of retained earnings; rather, they have to borrow from banks to finance a fraction of their investment. In addition, we assume that they have to borrow ‘working capital’ from banks. This working capital is used to pay a fraction of their wage bill as well as a fraction of the costs associated with hiring new workers.

Finally, banks channel funds from the patient households (depositors) through to impatient households and firms. As in Gertler and Kiyotaki (2011, 2015), they face an endogenous leverage constraint arising out of the friction that they can divert a fraction of their assets should that be more profitable than continuing as an ongoing entity. In addition, there is turnover in the banking sector with dying banks’ net worth returning to the patient households (as the owners of the banks) and new banks being set up.
using initial net worth supplied to them by the patient households.

Figure 1 shows graphically the agents in the model and the transactions between them.

Figure 1

3.2 The Labor Market

We normalise the labor force to 1. There are two types of households: patient households, to be denoted with index $H$, forming a fraction $(1 - \sigma)$ of the labor force, and impatient households, to be denoted with index $S$, forming a fraction $\sigma$ of the labor force. Each household type is of measure 1. The stocks of employment are denoted $N$ and of unemployment denoted $U$.

The labor market is frictional and workers who are unemployed at the beginning of each period $t$ are denoted by $U_{0,t}$. It is assumed that these unemployed workers can start working in the same period if they find a job. Given the same matching technology facing all workers and one pool of unemployment for all households, this happens with probability $f_t = \frac{h_t}{U_{0,t}}$, where $h_t$ denotes the total number of worker matches. Workers separate from employment at rate $\delta_N$.

The stocks are given by:

1. $1 = N_{H,t} + U_{H,t}$
2. $1 = N_{S,t} + U_{S,t}$
3. $U_{t} = (1 - \sigma)U_{H,t} + \sigma U_{S,t}$
4. $N_{t} = (1 - \sigma)N_{H,t} + \sigma N_{S,t}$

The matching probability is:

$$f_t = \frac{h_t}{U_{0,t}}$$

where

$$U_{0,t} = U_{t-1} + \delta_N N_{t-1}$$

$$U_{t} = U_{0,t} - h_t$$
This implies the following relations:

\[ h_t = f_t U_t^0 = \frac{f_t}{1 - f_t} U_t = \frac{f_t}{1 - f_t} (1 - N_t) \]  

(5)

The hiring flows are given by:

\[ h_t = (1 - \sigma) h_{H,t} + \sigma h_{S,t} \]  

(6)

So the stocks evolve as follows:

\[ U_t = (1 - f_t) U_t^0 \]  

(7)

\[ N_{H,t} = (1 - \delta_N) N_{H,t-1} + h_{H,t} \]

\[ N_{S,t} = (1 - \delta_N) N_{S,t-1} + h_{S,t} \]

\[ N_t = (1 - \delta_N) N_{t-1} + h_t \]

### 3.3 Households

As noted above, there are two types of households: patient households, to be denoted with index \( H \) – forming a fraction \((1 - \sigma)\) of the labor force – who hold deposits with banks and own housing, and impatient households, to be denoted with index \( S \) – forming a fraction \( \sigma \) of the labor force – who borrow against their housing wealth. Each household type is of measure 1. In what follows we present the optimisation problem for the two types of households. Identical households within each sector, each indexed by \( j \) or \( i \), will decide on the same consumption so:

\[ C_{H,t} = c_{H,j,t} \]  

(8)

\[ C_{S,t} = c_{S,i,t} \]

\[ C_t = (1 - \sigma) C_{H,t} + \sigma C_{S,t} \]

We assume that the total housing stock is constant and normalised to 1. Identical households within each sector, each indexed by \( j \) or \( i \), will decide on the same housing so:
\[ H_{H,t} = H_{H,j,t} \]
\[ H_{S,t} = H_{S,j,t} \]
\[ H_t = (1 - \sigma) H_{H,t} + \sigma H_{S,t} = 1 \forall t \]

3.3.1 Patient Households

The problem for patient households is to maximise their utility subject to a budget constraint and the evolution of employment. A typical patient household \( H \), using subscript \( j \), obtains utility from consumption, \( c_{H,j,t} \), from housing, \( H_{H,j,t} \), and from leisure (i.e., suffer disutility from working, \( N_{H,j,t} \)). They accumulate housing and bank deposits, \( D_{H,j,t} \), which pay the (gross) risk-free nominal rate of interest, \( R_t \). We also assume that they own the firms and the banks, receive profits from the firms, and get net transfers from the banks; we denote these by \( \Pi_{H,j,t} \).

Hence, we can write the problem for patient household \( j \) as follows:

Maximise

\[
\max_{c_{H,j,t}, H_{H,j,t}} E_0 \sum_{t=0}^{\infty} p_t^H [(1 - \eta) \ln(c_{H,j,t} - \eta C_{H,j,t-1}) + J A_{H,t} \ln H_{H,j,t} - \frac{\tau}{1 + \xi} N_{H,j,t}^{1+\xi}] 
\]

subject to:

(i) the budget constraint in nominal terms is given by:

\[
\left[ \frac{P_t c_{H,j,t} + D_{H,j,t}}{D} + \frac{\phi_t (1 - \rho)}{2} \left( \frac{D_{H,j,t} - D_{H,j,t-1}}{D} \right)^2 \right] = R_{t-1} D_{H,j,t-1} + P_t w_t N_{H,j,t} + \Pi_{H,j,t} \quad (11)
\]

(ii) employment evolution is given by (using (5) for this type of household):

\[
N_{H,j,t} = (1 - \delta_N) N_{H,j,t-1} + h_{H,t} \quad (12)
\]

\[
= (1 - \delta_N) N_{H,j,t-1} + \frac{f_{H,t} H_t}{1 - f_{H,t}} (1 - N_{H,t})
\]
where \( f \) and \( \tau \) are preference parameters, \( \beta_H \) is the discount factor for patient households, \( A_{H,t} \) is a housing demand shock, \( q_t \) is the real price of housing, \( w_t \) is the real wage, \( \Pi_{H,j,t} \) denotes the sum of dividend payments received from the firms and the banks less the capital that they put in to newly-created banks, \( P_t \) is the aggregate price level and \( T_t \) denotes lump-sum taxes paid to the government. Notice that we have ‘external’ habits in consumption. That is, the utility of household \( H \) depends on their consumption vis-à-vis the previous period’s average consumption of patient households, \( \bar{C}_{H,t-1} \). There are quadratic adjustment costs on deposits (and where \( D \) with no time subscript denotes steady state deposits).

Assuming all patient households are identical with measure 1, the first-order conditions for this problem for the aggregate patient household sector are given by:

(i) the intertemporal Euler equation for consumption:

\[
\frac{1}{C_{H,t} - \eta C_{H,t-1}} \left[ 1 + \phi_D \left( \frac{D_t - D_{t-1}}{D} \right) \right] = \beta_H \frac{P_t}{P_{t+1} C_{H,t+1} - \eta C_{H,t}} \left[ R_t + \phi_D \left( \frac{D_{t+1} - D_t}{D} \right) \right]
\]

where we have used the fact that total deposits, \( D_t \) will be given by:

\[
D_t = (1 - \sigma)D_{H,j,t}
\]

(ii) the housing demand equation:

\[-\frac{IA_{H,t}}{H_{H,t}} + \frac{(1 - \eta) q_t}{C_{H,t} - \eta C_{H,t-1}} = \beta_H \frac{P_t}{P_{t+1} C_{H,t+1} - \eta C_{H,t}} \left[ R_t + \phi_D \left( \frac{D_{t+1} - D_t}{D} \right) \right] \]

(iii) the value of employment \( V_{1,t}^{NH} \), to be used in wage bargaining below:

\[
\frac{V_{1,t}^{NH}}{1 - f_t} = w_t - \tau \bar{N}_{H,t} \frac{C_{H,t} - \eta C_{H,t-1}}{(1 - \eta)} + \beta_H (1 - \delta_N) E_t \left[ V_{1,t+1}^{NH} \frac{C_{H,t} - \eta C_{H,t+1}}{C_{H,t+1} - \eta C_{H,t}} \right]
\]

where \( f_t \) is the job-finding rate.

### 3.3.2 Impatient Households

Impatient households, indexed by \( S \), discount the future more heavily than patient households, such that \( \beta_S < \beta_H \). As with patient households, a typical impatient household obtains utility from consumption, \( c_{S,j,t} \), from housing, \( H_{S,j,t} \), and from leisure (i.e.,
suffer disutility from working, $N_{S,i,t}$). They accumulate housing and borrow from banks (denoted $L_M$) against their housing wealth. Note that, although we call it mortgage borrowing throughout the paper, such borrowing may represent total household secured borrowing, where the security is provided by the value of their housing wealth. As such, we impose a constraint on this borrowing. Specifically, borrowing adjusts slowly towards a target loan-to-value ratio (representing the extent to which their borrowing is ‘secured’). This target loan-to-value ratio will be given by $m_M A_{M,i,t}$, where $A_{M,i,t}$ is a shock to the target. This captures changes in the borrowing capacity of impatient households due to, for example, tighter screening practices by the banks and/or restrictions imposed on this type of lending by the macroprudential regulatory authority. Again, we assume that impatient households have ‘external’ habits in consumption with their utility today depending on their consumption relative to the previous period’s average consumption of impatient households, $C_{S,t-1}$.

Hence, we can write the problem of the typical impatient household $S$ (with subscript $i$ the representative household index) as follows.

Maximise

$$\max_{\varepsilon_{S,i,t}, H_{S,i,t}} E_0 \sum_{t=0}^{\infty} \beta^t \left( (1 - \eta) \ln(\varepsilon_{S,i,t} - \eta C_{S,t-1}) + JA_{H,t} \ln H_{S,i,t} - \frac{\tau}{1 + \delta} N_{S,i,t}^{1+\tau} \right)$$

subject to:

(i) the budget constraint in nominal terms:

$$P_t c_{S,i,t} + P_t q_t (H_{S,i,t} - H_{S,i,t-1}) + R_{L,t-1} L_{M,i,t-1} + \frac{\phi_s \sigma}{2} \left( \frac{(L_{M,i,t} - L_{M,i,t-1})^2}{L_{M,i,t}} \right) + T_{S,i,t} = L_{M,i,t} + P_t w_t N_{S,i,t}$$

(ii) the loan constraint in nominal terms:

$$L_{M,i,t} = \rho_s L_{M,i,t-1} + (1 - \rho_s) m_M A_{M,i,t} q_t H_{S,i,t} P_t$$

(iii) employment evolution (using (5) for this type of household):
\[ N_{S,t} = (1 - \delta_N) N_{S,t-1} + h_{S,t} \]
\[ = (1 - \delta_N) N_{S,t-1} + \frac{f_t}{1 - f_t} (1 - N_{S,t}) \]  

where \( R_{L,t} \) denotes the banks' gross lending rate, \( \frac{\phi_s \sigma}{2} \left( \frac{(L_{M,t} - L_{M,t-1})^2}{L_M} \right) \) represents costs of adjusting mortgage borrowing for impatient household \( i \) and \( L_M \) denotes the steady-state level of lending to impatient households ('mortgage' lending).

Assuming all impatient households are identical and with measure 1, the first-order conditions for this problem for the aggregate impatient household sector are given by:

(i) the inter-temporal Euler equation for consumption.

\[ \frac{1}{(C_{S,t} - \bar{C}_{S,t-1})} \left[ 1 - \frac{\phi_s (L_{M,t} - L_{M,t-1})}{L_M} - \mu_{S,t} \right] = \frac{1}{C_{S,t+1} - \bar{C}_{S,t+1}} P_t \left[ R_{L,t} - \phi_s \frac{(L_{M,t+1} - L_{M,t})}{L_M} - \mu_{S,t+1} \right] \]  

\[ (21) \]

Here we have used:

\[ L_{M,t} = \sigma L_{M,i,t} \]  

(ii) the housing demand equation:

\[ \frac{-J A_{H,t}}{H_{S,t}} + \frac{(1 - \eta)}{C_{S,t} - \bar{C}_{S,t-1}} q_t \left[ 1 - \mu_{S,t}(1 - \rho_s) m_{M,i,t} A_{m,i,t} \right] = \beta_S E_t \left[ \frac{(1 - \eta)}{C_{S,t+1} - \bar{C}_{S,t+1}} q_{t+1} \right] \]  

\[ (22) \]

Notice here the presence of an additional term \(- \mu_{S,t}(1 - \rho_s) m_{M,i,t} q_t\) relative to the housing demand equation for patient households. This reflects the fact that the impatient households not only desire housing for its own (utility) sake, but also because an increase in their housing wealth loosens their borrowing constraint. The value to them of such a ‘marginal loosening’ will be given by \( \mu_s \).

(iii) the value of employment, to be used in wage bargaining below.

\[ \frac{V_{N_S}}{1 - f_t} = w_t - \tau N_{S,t} \frac{C_{S,t} - \eta C_{S,t-1}}{A_{p,t}(1 - \eta)} + \beta_S (1 - \delta_N) E_t \left[ \frac{V_{N_S}}{C_{S,t+1} - \eta C_{S,t+1}} \right] \]  

\[ (23) \]
3.4 Firms

There is a unit measure of monopolistically-competitive firms indexed by \( l \in [0,1] \) and of final goods aggregator firms. We assume price stickiness à la Rotemberg (1982), meaning firms maximise the present discounted value of current and expected future profits subject to quadratic price adjustment costs, hiring frictions and investment frictions, to be elaborated below. In what follows we present the optimisation problem for the two types of firms.

3.4.1 Final Goods Firms

Final good aggregator firms operate in a competitive market and produce \( y_t \) using goods \( y_{l,t} \) as inputs. The price of the final good used for consumption, investment and government purchases is given by \( P_t \). Final firms maximise

\[
\max P_t y_t - \int_0^1 P_{l,t} y_{l,t} \, dl
\]

subject to

\[
Y_t = \left( \int_0^1 y_{l,t}^{(e-1)/e} \, dl \right)^{e/(e-1)}.
\]

where \( e > 0 \) is the elasticity of demand for an individual firm’s good.

Taking first order conditions with respect to \( y_t \) and \( y_{l,t} \) and merging we solve for the demand function

\[
y_{l,t} = \left( \frac{P_t}{P_{l,t}} \right)^e y_t
\]

(25)

3.4.2 Intermediate Goods Firms

Intermediate goods firms produce output using capital, \( k \), and labor, \( N \). We assume they can vary the extent to which they utilise their capital and denote this by \( z \). The gross output of a representative firm \( l \) at time \( t \) is:

\[
y_{l,t} = A_t N_{l,t}^{1-a} (z_{l,t} k_{l,t-1})^a
\]

(26)

where \( A_t \) is an aggregate technology shock and the firm faces the demand function derived above (25).

where \( e > 0 \) is the elasticity of demand for an individual firm’s good.
In order to produce this output, the firm has to hire $h_{l,t}$ workers:

$$N_{l,t} = (1 - \delta_N)N_{l,t-1} + h_{l,t}, \quad 0 < \delta_N < 1. \quad (27)$$

In order to hire these workers, the firm has to pay a hiring cost given by:

$$g(h_{l,t}, N_{l,t}) = \frac{\phi_h}{2} \left( \frac{h_{l,t}}{N_{l,t}} \right)^2 P_t y_t \quad (28)$$

We interpret hiring costs as training costs and other costs that are related to the hiring rate. The modelling of these costs follows previous work by Merz and Yashiv (2007) and Yashiv (2016), whereby the cost function is quadratic in the hiring rate, where $\phi_h$ is a positive parameter governing the degree of hiring frictions.

In every period $t$, the existing capital stock depreciates at the rate $\delta_{K,l,t}$ and is augmented by new investment subject to investment costs:

$$k_{l,t} = (1 - \delta_{K,l,t})k_{l,t-1} + A_{k,l}I_{l,t} \left[ 1 - S \left( \frac{j_{l,t}}{I_{l,t-1}} \right) \right] \quad (29)$$

where $A_{k,l}$ is an investment-specific technology shock and, following Christiano, Eichenbaum and Evans (2005), we assume that the cost function $S$ satisfies $S(1) = S'(1) = 0$ and $S''(1)$ is a positive constant. We assume that the greater the extent to which capital is utilised, the faster it depreciates:

$$\delta_{K,l,t} = \delta_K + r_k \left( \omega \frac{z_{l,t}^2}{2} + (1 - \omega) z_{l,t} + \omega - 1 \right) \quad (30)$$

where $\omega$ is a technological parameter, $\delta_K$ is the steady-state capital depreciation rate, $r_k$ is the steady-state return on capital and steady-state capital utilisation is assumed to equal 1.

The firm borrows from banks in order to pay a fraction $0 \leq \Omega_1 \leq 1$ of their investment costs, a fraction $0 \leq \Omega_2 \leq 1$ of their wage bill, and a fraction $0 \leq \Omega_3 \leq 1$ of their hiring costs. Thus firm loans $L_{E,t}$ are given by:

$$L_{E,t} = \Omega_1 P_t I_{l,t} + \Omega_2 W_t N_{l,t} + \Omega_3 P_t \left( \frac{\phi_h}{2} \left( \frac{h_{l,t}}{N_{l,t}} \right)^2 y_t \right) \quad (31)$$

where $L_{E,z,t}$ is the stock of loans with gross nominal lending rate $R_{l,t} = 1 + r_{L,t}$.

We assume that the intermediate firms are owned by the patient households. So, they will maximise the present discounted value of the current and future expected
streams of profits they send to their owners where the stochastic discount factor will be that given by the patient households’ problem. Following Rotemberg (1982), we assume that firms face quadratic costs of adjusting their prices. The maximisation problem for firm \( l \) is thus:

\[
\max_{h_{t,j}, h_{t,y}, I_t} E_t \sum_{t=1}^{\infty} \frac{\beta_t(1-\eta)}{C_{H,t} - \eta C_{H,t-1}} P_t \left( \frac{P_{t,y_{t,j}}}{P_{t-1,y_{t,j}}} - P_{t,I_{t,j}} - W_t N_{t,I_{t,j}} + L_{E,t,J} - R_{L,t,J-1} L_{E,t,J-1} \right) - \left( \frac{\phi_t h_{t,y_{t,j}}}{N_{t,y}} \right)^2 + \chi \left( \frac{P_{t,I_{t,j}}}{P_{t-1,I_{t,j}}} - 1 \right)^2 P_t y_t \right) 
\]

s.t. (25), (26), (27), (29), (30) and (31).

Assuming all firms are symmetric and so set the same price, hiring rates and investment, the first-order conditions for this problem imply the following, where \( Q_t^N \) and \( Q_t^K \) are the real values of an additional employee and an additional unit of the capital good, respectively.

(i) Prices.

\[
\frac{1 - \epsilon}{\chi} + \frac{ermc_t}{\chi} - \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} + \frac{E_t \left[ \beta_H (C_{H,t} - \eta C_{H,t-1}) \left( \frac{P_{t+1}}{P_t} - 1 \right) \left( \frac{P_{t+1}}{P_t} \right)^2 \frac{y_{t+1}}{y_t} \right]}{\chi} = 0 
\]

where \( mc_t \) is the Lagrange multiplier on (26) and is spelled out below.

Log-linearising this equation around a zero-inflation steady state produces the familiar New Keynesian Phillips curve linking inflation this period with expected inflation next period and real marginal cost:

\[
\left( \frac{P_t}{P_{t-1}} - 1 \right) = \pi_t = \beta_H E_t [\pi_{t+1}] + \frac{(\epsilon - 1) \ln \left( \frac{mc_t}{mc} \right)}{\chi} 
\]

(ii) Hiring.

Marginal hiring costs are given by:

\[
Q_t^N = (1 - \Omega_Z) h_{t,y_{t,j}} y_t N_{t,y} + E_t \left[ \beta_H (C_{H,t} - \eta C_{H,t-1}) \left( \frac{P_t}{P_{t+1}} \right) \right] \left[ R_{L,t} \Omega_Z h_{t,y_{t,j}} y_t N_{t,y} \right] 
\]
Marginal hiring revenues are given by:

\[
Q^N_t = \left(1 - \alpha \right) rmc_t \frac{y_t}{N_t} - w_t (1 - \Omega_2) + \phi_h \left( \frac{h_t}{N_t} \right)^2 \frac{y_t}{N_t} (1 - \Omega_3) + \frac{E_t}{Q^N_{t+1}} \left[ \beta_H \left( C_{H,t} - \eta C_{H,t-1} \right) \right] - R_{L,t} \frac{P_t}{P_{t+1}} \left( \Omega_2 w_t - \Omega_3 \phi_h \left( \frac{h_t}{N_t} \right)^2 \frac{y_t}{N_t} \right) + \frac{\Omega_2 w_t N_t}{y_t} - \frac{\Omega_3 \phi_h h_t N_t}{N_t} + \frac{\Omega_2 w_t N_t}{y_t}.
\]  

(36)

We can combine these equations to obtain the following expression for real marginal cost:

\[
m_t = \frac{N_t}{y_t (1 - \alpha)} + \frac{(1 - \Omega_3) \phi_h N_t}{(1 - \alpha) y_t} E_t \left[ \frac{h_t}{N_t} \frac{y_t}{N_{t+1}} \right] + \frac{1}{(1 - \alpha) y_t} \frac{N_t}{y_t} E_t \left[ \frac{R_{L,t+1}}{R_{t+1}} \right] \phi_h h_t \frac{N_t}{N_{t+1}} \frac{y_t}{N_{t+1}} + \frac{1}{(1 - \alpha) y_t} \frac{N_t}{y_t} E_t \left[ \frac{R_{L,t+1}}{R_{t+1}} \right] \phi_h h_t \frac{N_t}{N_{t+1}} \frac{y_t}{N_{t+1}} + \frac{1}{(1 - \alpha) y_t} \frac{N_t}{y_t} E_t \left[ \frac{R_{L,t+1}}{R_{t+1}} \right] \phi_h h_t \frac{N_t}{N_{t+1}} \frac{y_t}{N_{t+1}}
\]

where, to aid intuition, we have assumed zero deposit adjustment costs.

If we set \( \Omega_2 = \Omega_3 = \phi_h = 0 \), then real marginal costs are given by the familiar expression

\[
m_t = \frac{w_t N_t}{y_t (1 - \alpha)} = \frac{w_t N_t}{y_t (1 - \alpha)}
\]

which underlies the use of the labor share in empirical estimates of the New Keynesian Phillips curve, eg, Gali, Gertler, and Lopez-Salido (2005). The introduction of hiring frictions, i.e., setting \( \phi_h > 0 \), changes the expression for real marginal cost to

\[
m_t = \frac{w_t N_t}{y_t (1 - \alpha)} + \frac{\phi_h}{(1 - \alpha)} \left( 1 - \frac{h_t}{N_t} \right) \frac{h_t}{N_t} + \frac{(1 - \delta_N) \phi_h N_t}{(1 - \alpha) y_t} E_t \left[ \frac{h_t+1}{N_{t+1}} \frac{y_{t+1}}{N_{t+1}} \right] \frac{h_t+1}{N_{t+1}} \frac{y_{t+1}}{N_{t+1}}
\]

The intuition here is that to increase output, in addition to paying wages (the first term on the right-hand side of this equation), firms must pay the costs of hiring additional workers (the second term on the right-hand side of this equation) and, next period, will have to pay the costs of hiring workers to replace those who became unemployed at the end of this period (the third term on the right-hand side of this equation).

Comparing this with the expression for real marginal cost in the presence of finan-
cial frictions, i.e., with $\Omega_2 \neq 0, \Omega_3 \neq 0$, one sees three additional terms that reflect the fact that firms need to borrow to pay wages and to pay hiring costs (both this period and the next). If firms could finance these costs out of retained earnings, the cost would be $R$. But because they are having to borrow from banks, they have to pay an interest rate of $R_{Lt}$. Thus, the opportunity cost of this borrowing will be given by the spread of the lending rate over the deposit rate. As we will show below, the frictions within the banking sector determine this spread. Thus there is a channel through which financial frictions, by determining the spread, will affect hiring and real marginal costs, due to hiring frictions. Hence they will affect inflation via the New Keynesian Phillips curve.

(iii) Investment.

Marginal investment costs are given by:

$$1 = \Omega_1 + A_{k,t} Q^K_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) - \left( \frac{I_t}{I_{t-1}} \right) S' \left( \frac{I_t}{I_{t-1}} \right) \right) + E_t \left[ \frac{\beta H (C_{H,t} - \eta C_{H,t+1})}{(C_{H,t+1} - \eta C_{H,t})} \left[ -R_{Lt} \Omega_t \frac{P_t}{P_{t+1}} + A_{k,t+1} Q^K_{t+1} \left( \frac{I_{t+1}}{I_t} \right) S' \left( \frac{I_{t+1}}{I_t} \right) \right] \right]$$

This equation implies that investment depends negatively on the spread, $R_{Lt} - R_t$ as the firm has to borrow from banks to finance investment and the spread is measures the opportunity cost of this borrowing. This is another channel through which a worsening of financial frictions, leading to a rise in the spread, will have a negative effect on the real economy.

Marginal investment revenues are given by:

$$Q^K_t = E_t \left[ \frac{\beta H (C_{H,t} - \eta C_{H,t+1})}{(C_{H,t+1} - \eta C_{H,t})} \left[ \frac{\alpha \cdot rmc_{t+1} y_{z,t+1}}{k_t} + Q^K_{t+1} (1 - \delta_{k,t}) \right] \right]$$

Firms set the marginal cost of investing in capital today, $Q^K_t$, equal to the discounted value of the expected benefit accruing to them tomorrow. This benefit, in turn, has two parts: the marginal product of capital, $\frac{\alpha \cdot rmc_{t+1} y_{z,t+1}}{k_t}$, and the value of the undepreciated capital left in the firm at the end of the period, $Q^K_{t+1} (1 - \delta_{k,t})$.

Finally, optimization of capacity utilization equates the marginal benefit from more capacity with the marginal cost, which is linear in utilization (see equation (30)).

$$\frac{\alpha \cdot rmc_{t+1} y_{z,t}}{k_{t-1}} = Q^K_t r_k (\omega z_t + 1 - \omega)$$

Hence the marginal product of capital depends on the extent to which it is utilised.
3.5 Wage Determination

We assume that wages are negotiated on behalf of all employed workers by a representative union, without distinction between workers from different households, and that the solution is the Nash solution.

Wages are assumed to maximise a geometric average of the household’s and the firm’s surplus weighted by the parameter $\gamma$, which denotes the bargaining power of the households:

$$ W_t = \arg \max \left\{ \left( \sigma V_{t}^{NS} + (1 - \sigma) V_{t}^{NH} \right)^\gamma \left( Q_t^N \right)^{1-\gamma} \right\}. \quad (41) $$

The first order condition to this problem leads to the Nash sharing rule:

$$ (1 - \gamma) V_t^N = \gamma Q_t^N \quad (42) $$

where

$$ V_t^N = \sigma V_t^{NS} + (1 - \sigma) V_t^{NH} \quad (43) $$

For incentive compatibility the representative union has to deliver a present discounted value of being employed to a worker that is at least as good as they could obtain on their own:

$$ V_t^{NS} \leq V_t^N \quad (44) $$

$$ V_t^{NH} \leq V_t^N \quad (45) $$

Hence:

$$ V_t^N = V_t^{NS} = V_t^{NH} \quad (45) $$

We reproduce the relevant expressions:

$$ \frac{V_t^{NH}}{1 - f_t} = w_t - \tau N_{H,t}^g \left( \frac{C_{H,t} - \eta C_{H,t-1}}{1 - \eta} \right) + \beta_H (1 - \delta_N) N_{H,t} \left[ \frac{(C_{H,t} - \eta C_{H,t})}{(C_{H,t+1} - \eta C_{H,t})} V_{t+1}^{NH} \right] \quad (46) $$

$$ \frac{V_t^{NS}}{1 - f_t} = w_t - \tau N_{S,t}^g \left( \frac{C_{S,t} - \eta C_{S,t-1}}{1 - \eta} \right) + \beta_S (1 - \delta_N) N_{S,t} \left[ \frac{(C_{S,t} - \eta C_{S,t})}{(C_{S,t+1} - \eta C_{S,t})} V_{t+1}^{NS} \right] \quad (47) $$
\[ Q_t^N = (1 - \Omega_3)\phi_h \frac{h_t}{N_t} \frac{y_t}{N_t} + E_t \left[ \frac{\beta_h (C_{H,t} - \eta C_{H,t-1}) P_t}{(C_{H,t+1} - \eta C_{H,t}) P_{t+1}} \left[ R_{L,t} \Omega_3 \phi_h \frac{h_t}{N_t} \frac{y_t}{N_t} \right] \right] \quad (48) \]

\[ Q_t^N = \frac{(1 - \alpha) r m c_t y_{z,t}}{N_t} - w_t (1 - \Omega_2) + \phi_h \left( \frac{h_t}{N_t} \right)^2 \frac{y_t}{N_t} (1 - \Omega_3) + E_t \left[ \frac{\beta_h (C_{H,t} - \eta C_{H,t-1})}{(C_{H,t+1} - \eta C_{H,t})} \left[ -R_{L,t} \frac{P_t}{P_{t+1}} \left( \Omega_2 w_t - \Omega_3 \phi_h \left( \frac{h_t}{N_t} \right)^2 \frac{y_t}{N_t} \right) + \frac{Q_t^N (1 - \delta_N)}{N_t} \right] \right] \quad (49) \]

Using equations (46) to (49) and the sharing rule (42) to eliminate the terms in \( Q_t^N \) and \( V_{t+1}^N \) one gets the following expression for the real wage:

\[
w_t = \frac{1}{1 - \Omega_2 \gamma \left( 1 - R_{L,t} \beta_h E_t \frac{c_{H,t}}{c_{H,t+1}} \right)} \left[ \begin{array}{c}
\left( 1 - \gamma \right) \tau N_{H,t} \frac{r m c_t \gamma}{\phi_h \frac{h_t}{N_t} \frac{y_t}{N_t}} \\
\left( 1 - \gamma \right) \tau N_{z,t} \frac{\left( C_{H,t} - \eta C_{H,t-1} \right)}{(1 - \eta)^2} \\
\left[ \phi_h \frac{h_t}{N_t} \frac{y_t}{N_t} \right] \\
\left( 1 - \gamma \right) \tau N_{H,t} \frac{r m c_t \gamma}{\phi_h \frac{h_t}{N_t} \frac{y_t}{N_t}} \\
\left( 1 - \gamma \right) \tau N_{z,t} \frac{\left( C_{H,t} - \eta C_{H,t-1} \right)}{(1 - \eta)} \\
\end{array} \right] \quad (50) \]

where:

\[
\frac{c_{H,t}}{c_{H,t+1}} = \frac{(c_{H,t} - \eta C_{H,t-1}) P_t}{(C_{H,t+1} - \eta C_{H,t}) P_{t+1}}
\]

To obtain some intuition for this equation, note that with zero hiring costs (\( \phi_h = 0 \)) and no financial frictions (\( \Omega_2 = \Omega_3 = 0 \)), it becomes:

\[
w_t = \left( 1 - \gamma \right) \tau N_{H,t} \frac{C_{H,t} - \eta C_{H,t-1}}{(1 - \eta)} + \gamma \cdot r m c_t (1 - \alpha) \frac{y_{z,t}}{N_t} \quad (51) \]

Wages are a weighted average of the worker’s reservation value, which takes into account utility from consumption and disutility from work, and the flow productivity value to the firm generated by the worker, which equals their marginal product.

If we now add hiring costs, then wages will be higher, as the firm has to partly compensate the worker for the hiring cost savings generated by a match having been formed:
Comparing this with equation (50), we can see that the effect of financial frictions is to lower the wage, as the need for firms to borrow to pay wages and/or hiring costs will lower the surplus value of any job match.

Equation (50) enables us to examine the effect of a rise in the spread on wages. Since firms have to borrow to pay wages, a rise in the spread will lead to a fall in wages. This can be seen in the denominator of equation (50). Against this, however, the rise in the spread will lead to a rise in hiring costs – since firms have to borrow to pay the hiring costs – and, in turn, this will lead to a rise in the surplus of an existing match and, hence, wages. This works through the numerator of equation (50). The net effect of the financial shock will depend on the extent to which firms have to borrow to pay hiring costs relative to wages. This is the key channel through which financial shocks, leading to movements in the spread, will affect real variables, operating through the real frictions in the economy.

3.6 Banks

Our modelling of the banking sector follows Gertler and Kiyotaki (2011, 2015) since we wish to be able to generate an interest rate spread, with banks having an endogenous leverage ratio.

We assume that banks issue loans to firms and to impatient households (mortgages) and finance these out of household deposits and their own net worth, n. As a result of financial market frictions, banks are constrained in their ability to raise deposits from households. Given this, they would attempt to save their way out of these constraints by accumulating retained earnings in order to move towards 100% equity finance. Following Gertler and Kiyotaki (2011, 2015), we limit this possibility by assuming that each period banks have an iid probability $1 - \zeta$ of exiting. Hence, the expected lifetime of a bank is $\frac{1}{1 - \zeta}$. When banks exit, their accumulated net worth is distributed as dividends to the patient households.

Each period, exiting banks are replaced with an equal number of new banks which initially start with a net worth $\nu$, provided by the patient households. A bank that survived from the previous period – bank $b$, say – will have net worth, $n_{b,t}$, given by:
\[ n_{t,t} = R_{t,t-1} (L_{E,b,t-1} + L_{M,b,t-1}) - R_{t-1} D_{b,t-1} \]  

(53)

where \( L_{M,b} \) is the total mortgage lending of bank \( b \), \( L_{E,b} \) is the total lending of bank \( b \) to firms and \( D_{b} \) are bank \( b \)'s deposits.

So, total net worth, \( n \), of the banking sector will be given by:

\[ n_t = \zeta (R_{t,t-1} (L_{E,t-1} + L_{M,t-1}) - R_{t-1} D_{t-1}) + (1 - \zeta) \nu \]  

(54)

Each period banks (whether new or existing) finance their loan book with newly issued deposits and net worth:

\[ L_{b,t} = D_{b,t} + n_{b,t} \]  

(55)

where \( L_{b} \) is total lending (to both mortgages and corporates) of bank \( b \).

Following Gertler and Kiyotaki (2011, 2015), we introduce the following friction into the banks’ ability to issue deposits. After accepting deposits and issuing loans, banks have the ability to divert some of their assets for the personal use of their owners. Specifically, they can sell up to a fraction \( \theta_t \) of their loans in period \( t \) and spend the proceeds during period \( t \). But, if they do, their depositors will force them into bankruptcy at the beginning of period \( t + 1 \). We model this as a parameter \( \tilde{\theta} \) with a multiplicative AR1 shock to the ease of diversion, as follows:\footnote{Gersbach and Rochet (2017, p. 121) show how similar results w.r.t. leverage can emerge from alternative forms of financial frictions, including moral hazard, the Inalienability of human capital, and haircuts and limits to arbitrage.}

\[
\theta_t = \tilde{\theta}^{1-\rho_\theta} \theta_{t-1} e^{\epsilon_{\theta,t}} \\
\ln \theta_t - \ln \tilde{\theta} = \rho_\theta (\ln \theta_{t-1} - \ln \tilde{\theta}) + \epsilon_{\theta,t}
\]

When deciding whether or not to divert funds, bank \( b \), will compare the franchise value of the bank, \( V_{b,t} \), against the gain from diverting funds, \( \theta_t (L_{E,b,t} + L_{M,b,t}) \). Hence, depositors will ensure that banks satisfy the following incentive constraint:

\[
\theta_t (L_{E,b,t} + L_{M,b,t}) \leq V_{b,t} \]  

(56)

We can write bank \( b \)'s problem as the choice of \( L_{E,b}, L_{M,b} \), and \( D_b \) each period to maximise its franchise value:
\[
V_{b,t} = \max_{L_{b,t}} E_t \left[ \sum_{j=1}^{\infty} \zeta^{j-1} (1 - \zeta) \frac{\beta_H^j (1 - \eta)}{(c_{H,t+j} - \eta C_{H,t+j}) P_{t+j}} n_{b,t+j} \right] \tag{57}
\]

subject to the incentive constraint (56) and the balance sheet constraints. Here we have assumed that the patient households own the banks.

The Bellman equation for bank \( b \)'s franchise value will be given by:

\[
V_{b,t} = E_t \left[ \beta_H^{(c_{H,t} - \eta C_{H,t-1})} P_t \left[ (1 - \zeta) n_{b,t+1} + \zeta V_{b,t+1} \right] \right] \tag{58}
\]

The balance sheet constraints imply:

\[
E_t \left( \frac{n_{b,t+1}}{n_{b,t}} \right) = \frac{R_{L,t} (L_{b,E,t} + L_{b,M,t}) - R_t D_{b,t}}{n_{b,t}}
\]

\[
= R_{L,t} \varphi_{b,t} - R_t (n_{b,t} - 1)
\]

where \( \varphi_{b,t} = \frac{(L_{b,e,t} + L_{b,m,t})}{n_{b,t}} = \frac{L_{b,t}}{n_{b,t}} \) is bank \( b \)'s leverage ratio, i.e., the ratio of assets to net worth.

As the banks set their loan rates higher than the deposit rate, then the expected growth rate of net worth will be an increasing function of the leverage ratio. Given that both the objective and constraints of the bank are constant returns to scale, we can rewrite the optimisation problem for bank \( b \) in terms of choosing the leverage ratio, and the lending split between mortgages and corporate loans, to maximise the ratio of its franchise value to net worth, \( \psi_t = \frac{V_t}{n_t} \).

Formally, maximise

\[
\psi_{b,t} = \max_{\varphi_{b,t}} E_t \left[ \sum_{j=1}^{\infty} \zeta^{j-1} (1 - \zeta) \frac{\beta_H^j (1 - \eta)}{(c_{H,t+j} - \eta C_{H,t+j}) P_{t+j}} n_{b,t+j} \right] \tag{60}
\]

subject to

\[
\theta_t \varphi_{b,t} = \psi_{b,t} \tag{61}
\]

where we have assumed parameter values such that the constraint binds in equilibrium.
Given constant returns to scale, we can aggregate up across all banks to the aggregate Bellman equation with franchise value $\Psi_t$:

$$\Psi_t = \max_{\varphi_t} E_t \left\{ \beta_H \left( \frac{C_{H,t} - \eta C_{H,t-1}}{C_{H,t+1} - \eta C_{H,t}} \right) P_t \left[ \left( 1 - \zeta + \zeta \Psi_{t+1} \right) \left( (R_{L,t} - R_t) \varphi_t + R_t \right) \right] \right\}$$

subject to:

$$\theta_t \varphi_t = \Psi_t$$

The solution implies:

$$E_t \left[ \beta_{t+1} \left( 1 - \zeta + \zeta \varphi_{t+1} \right) \left( R_{L,t} - R_t \right) \varphi_t \right] = \theta_t \varphi_t - E_t \beta_{t+1} \left( 1 - \zeta + \zeta \varphi_{t+1} \right) \frac{\left[ 1 + d_t \right] - E_t \left[ \beta_{t+1} d_{t+1} \right]}{\beta_{t+1}}$$

where $d_t \equiv \frac{D_t - D_{t-1}}{D_t}$.

Equation (64) includes the interest rate spread $(R_{L,t} - R_t)$. Ceteris paribus, the spread will be higher the tighter is the constraint (the higher is $\theta_t$).

### 3.7 Monetary and Fiscal Authorities

The government is assumed to run a balanced budget:

$$P_t G_t = T_t$$

Government spending is assumed to follow the stochastic process:

$$\ln (G_t) = \rho_G \ln (G_{t-1}) + (1 - \rho_G) \ln (\bar{G}) + \varepsilon_{G,t}$$

where $\bar{G}$ denotes the steady-state level of government spending and $\varepsilon_G$ is a white noise shock.

The central bank operates a Taylor Rule of the form:

$$\ln (R_t) = \rho_R \ln (R_{t-1}) + (1 - \rho_R) \ln \left( \frac{1}{\beta_H} \right) + (1 - \rho_R) \left( v_\pi \left( \frac{P_t}{P_{t-1}} - 1 \right) + v_y \ln \left( \frac{y_t}{\bar{y}} \right) \right) + \varepsilon_{R,t}$$

where $\bar{y}$ denotes the steady-state level of output and $\varepsilon_R$ is a white-noise shock. In line with most of the empirical and theoretical literature, eg, Altig et al. (2011) and Chris-
tiano et al. (2011), we assume that households and firms are unable to respond within period to the monetary policy shock. But we allow the banks to adjust their lending rates in response to the shock.

3.8 Market Clearing

Aggregating the budget constraints for each sector implies the goods market clearing condition:

\[ y_t = \frac{C_t + I_t + G_t + \frac{\phi_D}{2} \left( \frac{D_{t+1} - D_t}{D_t} \right)^2 + \frac{\phi_S}{2} \left( \frac{L_M - L_{M,t-1}}{L_M} \right)^2}{1 - \frac{\chi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 - \frac{\phi_h}{2} \left( \frac{h_t}{N_t} \right)^2} \] (68)

4 Empirical Implementation

In the next section we examine Impulse Response Functions (IRF) generated by the model. In this section we briefly present the empirical methodology we follow.

4.1 Rationale and Format

The key aim is to see the interactions of real and financial frictions. To do so we examine the effects of real, financial, and policy shocks under different model configurations. We shut down key elements pertaining to real and financial frictions in order to determine the relative role played by different parts of the model.

The model embeds the following shocks. First, we look at the more standard shocks – technology \(A_t\) and monetary policy \(\varepsilon_{r,t}\). Then we look at shocks related to financial markets – shocks to the LTV ratio \(A_{M,t}\), to housing preferences \(A_{H,t}\), and to bank diversion \(\theta_t\). For all shocks except the technology shock, we make them comparable as follows. We present a 25 bp increase in \(R_t\) for the monetary policy shock. This leads to an endogenous rise of \(R_L\) of almost the same magnitude upon impact. We then do all shocks – LTV \(A_{M,t}\), housing preferences \(A_{H,t}\), and bank diversion \(\theta_t\) – so that upon impact they will generate an endogenous rise of \(R_L\) of about the same magnitude.

We do so for four different configurations, which appear in the figures in different colors as follows:
The idea behind this set up is to shut down or open up two key dimensions of the real-financial interaction: one is the existence of real frictions, manifested in the hiring costs scale parameter $\phi_h$ and the investment costs function $S$; the other is the existence of links between firms and banks via borrowing to finance investment, wages and hiring costs, parameterized by $\Omega_1, \Omega_2, \Omega_3$. Model 1 (blue lines in the figures below) shuts down the interaction completely, while model 4 (red) has the interactions in full. Model 3 (green) connects firms to banks so there is a connection between investment and hiring and the banking system with its diversion issue, but there are no real frictions. Model 2 (black) features real frictions but does not connect firms to banks. Note that even when no direct interaction exists, there could still be connections in the GE set up. For example, the rate of interest $R_t$ reacts to output and inflation, exerting an effect on both banks (directly) and on firms (via the product demand they face).

We present the results in three panels for each shock. Each panel contains four lines according to the configurations of Table 1. Panel a in each case shows the response of real variables: GDP, consumption, investment, employment, the hiring rate, and real wages. Panel b in each case shows the financial variables – the rates (the interest rate, the lending rate, and the spread) and the volumes (deposits, lending, and net worth). Panel c in each case shows prices – inflation and real housing prices.

### 4.2 Calibration

We calibrate and simulate the model. Our calibration, in general, follows the existing literatures on DSGE models with a housing market, real frictions, and financial frictions.

Table 2A lists the parameters governing the household sector. In the main, we take our values from Iacoviello (2015). Specifically, we use his calibrated values for the discount factors of the patient and impatient households, the inverse Frisch elasticity of...
labor supply, the target loan-to-value rate on mortgage lending, and the share of impatient households in the population. We also use his estimated values for consumption habits and the parameters governing the adjustment costs for deposits and mortgage borrowing. We set the scale parameter on housing in the utility function, $J$, to 0.189, which implies a steady-state ratio of housing wealth to GDP equal to 11.6, and we set the scale parameter on leisure in the utility function, $\tau$, to 1.1784, which together with our assumed value for the steady-state job finding rate, $f$, of 0.66 implies a steady-state unemployment rate of 6%. Finally, our values for the steady-state job finding rate and the steady-state unemployment rate imply a value of 0.126 for the exogenous rate of job destruction, $\delta_N$.

Table 2A: Parameter Calibration Values

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>habit in utility</td>
</tr>
<tr>
<td>$\beta_H$</td>
<td>discounting, patient</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>discounting, impatient</td>
</tr>
<tr>
<td>$J$</td>
<td>scale, housing in utility</td>
</tr>
<tr>
<td>$\tau$</td>
<td>scale, work in utility</td>
</tr>
<tr>
<td>$\xi$</td>
<td>inverse Frisch elasticity</td>
</tr>
<tr>
<td>$m_M$</td>
<td>target loan-to-value ratio</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>share of impatient</td>
</tr>
<tr>
<td>$\delta_N$</td>
<td>worker separation rate</td>
</tr>
<tr>
<td>$f$</td>
<td>steady-state job finding rate</td>
</tr>
<tr>
<td>$\phi_D$</td>
<td>scale deposit AC</td>
</tr>
<tr>
<td>$\phi_S$</td>
<td>mortgage borrowing AC</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>AR1 of mortgage loans</td>
</tr>
</tbody>
</table>

Table 2B lists the parameters governing the firms. We use standard values for (i) the capital share, $\alpha$; (ii) the capital depreciation rate, $\delta_K$; and (iii) the elasticity of demand for individual goods, $\epsilon$. We set the scaling parameter for the price adjustment costs, $\chi$, equal to 117. Together with our calibration for the elasticity of demand, this implies a 0.085 slope of the New Keynesian Phillips curve that would be obtained with Calvo
(1983) price frictions with an average duration for prices of 4 quarters. The scaling parameter on the investment adjustment costs is set equal to the value estimated by Smets and Wouters (2007) for the US economy and the scaling parameter on the hiring costs is set equal to the value used by Faccini and Yashiv (2017). In the baseline we assume that firms have to finance their entire investment and wage bills by bank borrowing as well as all hiring costs. That is, we set $\Omega_1 = \Omega_2 = \Omega_3 = 1$. When studying the effects of financial frictions we set these parameters to 0. Finally, we set the workers’ bargaining power $\gamma$ to 0.29.

Table 2B: Parameter Calibration Values

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>capital share in Cobb Douglas</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>elasticity of demand</td>
</tr>
<tr>
<td>$\Omega_1$</td>
<td>proportion of investment financed by borrowing</td>
</tr>
<tr>
<td>$\Omega_2$</td>
<td>proportion of wage bill financed by borrowing</td>
</tr>
<tr>
<td>$\Omega_3$</td>
<td>proportion of hiring costs financed by borrowing</td>
</tr>
<tr>
<td>$\delta_K$</td>
<td>capital depreciation</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>scaling parameter, hiring costs</td>
</tr>
<tr>
<td>$S''(1)$</td>
<td>scaling parameter, investment adjustment costs</td>
</tr>
<tr>
<td>$\chi$</td>
<td>scaling parameter, price frictions</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>worker bargaining parameter</td>
</tr>
</tbody>
</table>

Finally, Table 2C lists the parameters governing the financial and public sectors. We calibrate the parameters governing the banks following Gertler and Kiyotaki (2015). Specifically, we set the survival rate for banks, $\zeta$, to 0.95, implying an average bank life expectancy of five years, and the proportion of bank assets that can be diverted, $\theta$, to 0.1939, implying an annualised steady-state spread of loan rates over deposit rates of one percentage point.

The coefficients on the Taylor rule take the standard values of 1.5 on inflation and 0.125 on quarterly output. We set the inertia coefficient, $\rho$, to 0.81, the value estimated by Smets and Wouters (2007). We set the steady-state share of government spending in GDP to 18%, the calibrated value used by Smets and Wouters.
Table 2C: Parameter Calibration Values

<table>
<thead>
<tr>
<th>symbol</th>
<th>Banks and the Public Sector</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_G)</td>
<td>govt. expenditure AR1</td>
<td>0.95</td>
</tr>
<tr>
<td>(\rho_R)</td>
<td>Taylor rule AR1</td>
<td>0.81</td>
</tr>
<tr>
<td>(\upsilon_\pi)</td>
<td>Taylor rule inflation coefficient</td>
<td>1.5</td>
</tr>
<tr>
<td>(\upsilon_y)</td>
<td>Taylor rule output coefficient</td>
<td>0.125</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>A bank’s probability of staying active</td>
<td>0.95</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Diversion rate</td>
<td>0.1939</td>
</tr>
</tbody>
</table>

Finally, all of our shocks – with the exception of the monetary policy shock, assumed to be white noise – are assumed to follow AR(1) processes with an autocorrelation coefficient of 0.95.

5 Real-Financial Interactions in the Effects of Shocks

In what follows we report the IRFs for five shocks according to the methodology outlined above. We first show the IRFs graphs and offer an explanation of the patterns seen. We then present in a table the differences across the four model configurations and discuss them. The next section provides for an integrative discussion of our findings.

5.1 Technology Shocks

Figure 2 shows the IRFs of a 1% reduction in \(A_t\), namely a negative technology shock.

Upon impact, the real variables (panel a) decline followed by a gradual return to steady state. There is quite a difference across models. The model with no bank credit to firms and no real frictions – depicted in the blue lines – has a rise upon impact in GDP, employment, hiring rates, investment and wages, and a relatively small fall in consumption. In the model with firm borrowing and real frictions, depicted in the red lines, while the labor market variables – employment, wages, and hiring rates – rise upon impact, GDP, consumption and investment fall. The intermediate cases, either no firm borrowing depicted in the black lines, or no real frictions in the green lines, are similar to the full model, the red lines; indeed the black lines overlap the red lines.
These results, but the blue case, are explained by the standard New Keynesian mechanism, whereby a contractionary productivity shock is indeed contractionary except for the labor market variables which at first expand, due to a rise in \( rmc_t \) (see Gali (2015)). The blue case of no borrowing and no real frictions is different; the rise in \( rmc_t \) is stronger (not plotted) and the positive reactions of the labor market variables (employment, wages and hiring rates) are stronger, as there are no hiring frictions. Investment rises rather than falls due to the strong rise in \( rmc_t \) with no investment frictions; GDP follows and rises as employment and capital both rise.

In terms of the financial variables (panel b) and prices (panel c), the responses across the four models are similar: inflation shoots up and then falls; house prices rise; interest rates and the spread rise and then fall back; deposits and lending fall then rise slowly back up; and net worth first rises and then falls. For the latter variables, as well as housing prices, it takes more than the 20 quarters depicted in the graphs for a return to steady state.

These results can be explained as follows: the contractionary productivity shock raises real marginal costs and hence inflation; the nominal rate \( R_t \) responds with an increase via the Taylor rule; consequently lending rates rise too. Consumption and lending contract.

Table 3 summarizes the results across models.
### Table 3
The Effects of a Negative Technology Shock

<table>
<thead>
<tr>
<th></th>
<th>no borrowing</th>
<th>borrowing</th>
<th>differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_1 = \Omega_2 = \Omega_3 \simeq 0$</td>
<td>$\Omega_1 = \Omega_2 = \Omega_3 = 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**no real frictions**

- Blue: $\phi_h = S \simeq 0$
  - $y, i, n, \frac{h}{n}, w \uparrow; c \downarrow$
  - $R, R_L, \text{spread } \uparrow$
  - $D, L \downarrow n \uparrow$
  - $\pi, q \uparrow$

- Green: $\phi_h = S \simeq 0$
  - $y, i, c \downarrow; n, \frac{h}{n}, w \uparrow$
  - $R, R_L, \text{spread } \uparrow$
  - $D, L \downarrow n \uparrow$
  - $\pi, q \uparrow$

**real frictions**

- Black: $\phi_h, S > 0$
  - $y, i, c \uparrow; n, \frac{h}{n}, w \uparrow$
  - $R, R_L, \text{spread } \uparrow$
  - $D, L \downarrow n \uparrow$
  - $\pi, q \uparrow$

- Red: $\phi_h, S > 0$
  - $y, i, c \downarrow; n, \frac{h}{n}, w \uparrow$
  - $R, R_L, \text{spread } \uparrow$
  - $D, L \downarrow n \uparrow$
  - $\pi, q \uparrow$

**differences**

- Substantial, some and changing especially for real vars
- Blue responds more
The table shows five sets of differences. First, we see relatively big differences between the blue and the red lines, i.e., with no firm borrowing and real frictions and with both. Hence these features of the economy are of importance. Second, shutting down real frictions, we see differences between the blue and the green lines, i.e., without and with firm borrowing. Thus firm borrowing makes a difference given no real frictions, operating to mitigate responses, especially of the real variables. Third, with real frictions, we see an overlap between the black and the red lines, i.e., without and with firm borrowing. Hence borrowing ceases to matter in the presence of real frictions and it is the real frictions that play the dominant role. Fourth we see substantial differences in the case of no firm borrowing: most variables respond more in the blue lines, when there are no real frictions, than in the black lines, when there are real frictions, again showing the importance of real frictions. Fifth, we see some differences between the red and green lines – when there is firm borrowing, the differences between no real frictions and real frictions models are not consistent across the different variables (e.g., wages react more strongly in the red case but employment and hiring rates do so in the green case).

5.2 Monetary Policy Shocks

Figure 3 shows the IRFs of a contractionary monetary policy shock $\varepsilon_{R,t}$ of 25 basis points.

Figure 3

Upon impact, the rise in $R$ leads to a rise in $R_L$. These lead to the following reactions: Households decrease consumption and demand for housing and for deposits; hence mortgage borrowing falls as do housing prices. Firms decrease investment and hiring with the rise in the interest rates and they borrow less. In terms of wage bargaining, the rise in the interest rate lowers the value of the match surplus and so wages fall. The aggregate economy contracts, with a fall in GDP; inflation falls in line with NKPC.

As interest rates fall back to steady state levels, the above processes are reversed. Table 4 summarizes the results across models.
Table 4
The Effects of a Contractionary Monetary Policy Shock Upon Impact

<table>
<thead>
<tr>
<th>no borrowing</th>
<th>borrowing</th>
<th>differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_1 = \Omega_2 = \Omega_3 \simeq 0$</td>
<td>$\Omega_1 = \Omega_2 = \Omega_3 = 1$</td>
<td></td>
</tr>
<tr>
<td><strong>no real frictions</strong></td>
<td><strong>blue</strong></td>
<td><strong>green</strong></td>
</tr>
<tr>
<td>$\phi_h = S \simeq 0$</td>
<td>$y, c, i, n, \frac{h}{n}, w \downarrow$</td>
<td>$y, i, n, \frac{h}{n}, w \downarrow$</td>
</tr>
<tr>
<td></td>
<td>$c \downarrow$ blue</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R, R_L \uparrow$; spread $\downarrow$ a little</td>
<td>$R \uparrow; R_L$, spread $\downarrow$</td>
</tr>
<tr>
<td></td>
<td>$D, L, n$ unchanged</td>
<td>$D, L, n$ unchanged</td>
</tr>
<tr>
<td></td>
<td>$\pi, q \downarrow$</td>
<td>$\pi, q \downarrow$ $\succ$ blue</td>
</tr>
<tr>
<td><strong>real frictions</strong></td>
<td><strong>black</strong></td>
<td><strong>red</strong></td>
</tr>
<tr>
<td>$\phi_h, S &gt; 0$</td>
<td>$y, i, n, \frac{h}{n}, w \downarrow$ very slightly;</td>
<td>$y, c, i, n, \frac{h}{n}, w \simeq$ black</td>
</tr>
<tr>
<td></td>
<td>$c \downarrow$ a lot</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R, R_L$, spread $\uparrow$</td>
<td>$R, R_L$, spread $\simeq$ black</td>
</tr>
<tr>
<td></td>
<td>$D, L, n$ unchanged</td>
<td>$D, L \downarrow, n \uparrow$</td>
</tr>
<tr>
<td></td>
<td>$\pi, q \downarrow$ a lot</td>
<td>$\pi, q \simeq$ black</td>
</tr>
<tr>
<td><strong>differences</strong></td>
<td><strong>substantial</strong></td>
<td><strong>some,</strong></td>
</tr>
<tr>
<td></td>
<td>blue responds more for real</td>
<td>green responds $\succ$ red</td>
</tr>
<tr>
<td></td>
<td>and less for financial and prices</td>
<td>except for $c, \pi, q$</td>
</tr>
</tbody>
</table>
The table exhibits only two similar cases – red and black, when real frictions are present, and firm borrowing does not matter much. All other pairwise comparisons exhibit big differences. This entails the following: first, in the case of no firm borrowing there are lower responses of real variables in the presence of real frictions (black) than without them (blue), but bigger responses for the financial variables and prices. Second, in the case of firm borrowing there are lower responses of all variables in the presence of real frictions (red) than without them (green), except for consumption, inflation and real house prices. Third, with no real frictions, there are lower responses of all variables in the presence of firm borrowing (green relative to blue), except for consumption, inflation and real house prices.

This can be explained as follows: Investment and hiring frictions mean that investment, hiring, employment and capital will be less volatile than in an otherwise identical model without these frictions (black vs. blue). Similarly, firm borrowing will mean that investment, capital and employment will be less volatile than in an otherwise identical model without firm borrowing (green vs. blue). With employment and capital less volatile, then output will be less volatile. But the reduction in volatility will not be as large as for investment (as the frictions only affect output ‘indirectly’). So, with the volatility of investment reduced relative to that of output, then the volatility of consumption (the rest of output) must be raised relative to output. Any reduction in the volatility of a quantity can be expected to increase the volatility of its associated price. That explains the increase in inflation volatility.

Overall, firm borrowing takes the economy in the same direction as real frictions, i.e., mitigates responses to shocks. Real frictions are dominant.

5.3 Housing Demand Shocks

Figure 4 shows the IRFs of a negative change in $A_{H,t}$, namely a negative housing demand shock.

Upon impact, house prices fall with lower housing demand. Hence there is less borrowing by impatient households, so loan volume declines. With the decline in housing value, which serves as collateral for impatient households, there is a rise in $R_L$ and the spread. Moving to the real variables, the rise in $R_L$ depresses investment and hiring, leading to a fall in capital, employment, and output. The central bank lowers $R$ using the Taylor rule.
Adjustment back to steady state: after about 3 quarters bank net worth drops and after 10 quarters lending picks up. Hence leverage rises and so $R_L$ and the spread now fall, reversing the effects on the real variables and on $R$. Inflation falls and then rises in accordance with the NKPC.

For the real variables – models 1 (blue) and 3 (green) with no real frictions display clear effects; models 2 (black) and 4 (red) with real frictions move less. For inflation, the relative magnitudes are as for the real variables, i.e., strongest responses of the blue model with no real frictions and no borrowing, then the green model with borrowing, and the lowest volatility for the two models with real frictions, the black and the red lines. The same pattern is true also for the interest rate $R_t$, which responds to GDP and inflation via the Taylor rule. The behavior of financial volumes (deposits, total lending) and rates (the lending rate $R_{L,t}$ and the spread) follows a different pattern; the black and blue with no firm borrowing react more strongly than the green and the red with firm borrowing. In the former case rates rise and volumes fall more than in the latter case. This is so as without firm borrowing there is only household borrowing which behavior reflects the housing demand shock. In terms of house prices, responses are similar across models.

Table 5 summarizes the results across models.
Table 5
The Effects of a Negative Housing Demand Shock Upon Impact

<table>
<thead>
<tr>
<th>no borrowing</th>
<th>borrowing</th>
<th>differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>no real frictions</td>
<td>blue</td>
<td>green</td>
</tr>
<tr>
<td>$\phi_h = S \simeq 0$</td>
<td>$y, c, i, n, h_n, w \downarrow$</td>
<td>$y, c, i, n, h_n, w \downarrow, &lt;$ blue</td>
</tr>
<tr>
<td></td>
<td>$R \downarrow, R_L \uparrow$</td>
<td>$R \downarrow, R_L \uparrow$</td>
</tr>
<tr>
<td></td>
<td>$D, L \downarrow$ a lot, $n \uparrow$</td>
<td>$D, L \downarrow, n \uparrow &lt;$ blue</td>
</tr>
<tr>
<td></td>
<td>$\pi, q \downarrow$</td>
<td>$\pi, q \downarrow &lt;$ blue</td>
</tr>
<tr>
<td>real frictions</td>
<td>black</td>
<td>red</td>
</tr>
<tr>
<td>$\phi_h, S &gt; 0$</td>
<td>$y, c, i, n, h_n, w \downarrow$ very slightly</td>
<td>$y, c, i, n, h_n, w \downarrow$ very slightly</td>
</tr>
<tr>
<td></td>
<td>$R \downarrow$ a little, $R_L \uparrow$</td>
<td>$R \downarrow$ a little, $R_L \uparrow$</td>
</tr>
<tr>
<td></td>
<td>$D, L, n \simeq$ blue</td>
<td>$D, L \downarrow, n \uparrow &gt;$ green</td>
</tr>
<tr>
<td></td>
<td>$\pi \downarrow$ somewhat, $q \downarrow$</td>
<td>$\pi \downarrow$ a little, $q \downarrow &lt;$ green</td>
</tr>
<tr>
<td>differences</td>
<td>substantial,</td>
<td>some</td>
</tr>
</tbody>
</table>
| | especially for real vars | | }
The table shows sets of differences across both real and financial variables. First, we see relatively big differences for almost all variables, between the blue and the red lines, i.e., with no borrowing and real frictions and with both. Thus, we see again that these features of the economy are of importance. Second, shutting down real frictions, we do see differences, sometimes substantial, between the blue and the green lines, i.e., without and with firm borrowing. Firm borrowing makes a difference given no real frictions, operating to mitigate responses. Third, with real frictions present, does firm borrowing make a difference? For the real variables we see an overlap between the black and the red lines, i.e., without and with firm borrowing. But we do see big differences between them for the financial variables, with borrowing mitigating responses. Firm borrowing takes the economy in the same direction as real frictions. Fourth, with no borrowing there is a substantial difference between blue (no real frictions) and black (with real frictions), the latter mitigating responses. This shows real frictions matter a lot. Fifth, doing the same comparison with borrowing, i.e., between green and red, there is some difference but not as strong as with borrowing. Thus, with borrowing already mitigating some responses, adding real frictions matters, but less so.

5.4 LTV Shocks

Figure 5 shows the IRFs of a rise in $A_{M,t}$ namely a shock tightening the LTV ratio.

Figure 5

This follows very much the same patterns as the negative housing demand shock. House prices fall with lower mortgage borrowing due to the increased LTV ratio. With less borrowing by impatient households loan volume declines. With the fall in housing value, there is a rise in $R_L$ and the spread. The rise in $R_L$ depresses investment and hiring, leading to a fall in capital, employment, and output. The central bank lowers $R$ using the Taylor rule. After about 3 quarters bank net worth drops and after 10 quarters lending picks up. Hence leverage rises and so $R_L$ and the spread now fall, reversing the effects on the real variables and on $R$. Inflation falls and then rises in accordance with the NKPC.

Table 6 summarizes the results and differences across models.
<table>
<thead>
<tr>
<th>no real frictions</th>
<th>blue</th>
<th>green</th>
<th>substantial;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_h = S \approx 0 )</td>
<td>( y, c, i, n, \frac{h}{\Omega}, w \downarrow )</td>
<td>( y, c, i, n, \frac{h}{\Omega}, w \downarrow \prec \text{blue} )</td>
<td>blue responds more</td>
</tr>
<tr>
<td>( \Omega_1 = \Omega_2 = \Omega_3 \approx 0 )</td>
<td>( \Omega_1 = \Omega_2 = \Omega_3 = 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>real frictions</td>
<td>black</td>
<td>red</td>
<td>very similar;</td>
</tr>
<tr>
<td>( \phi_h, S &gt; 0 )</td>
<td>( y, c, i, n, \frac{h}{\Omega}, w \downarrow \text{very slightly} )</td>
<td>( y, c, i, n, \frac{h}{\Omega}, w \downarrow \text{very slightly} )</td>
<td>except for ( D, L, n )</td>
</tr>
<tr>
<td>( \pi, q \downarrow )</td>
<td>( \pi, q \downarrow )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D, L, n \approx \text{blue} )</td>
<td>( D, L \downarrow, n \downarrow \prec \text{blue} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>differences</td>
<td>substantial;</td>
<td>some</td>
<td>especially for real vars</td>
</tr>
</tbody>
</table>
The table shows that the differences are similar to the ones found for the negative housing demand shock.

5.5 Credit Supply Shocks

One key aim of this paper is to assess the impact of linking financial and real frictions. The constraint on firms, whereby they have to borrow to finance the costs associated with investment, the wage bill, and hiring costs, introduces a direct relationship between financial frictions and firms’ output. As shown in equation (37), the opportunity cost of firms borrowing from banks to finance this working capital is the spread of the lending rate over the deposit rate. Movements in the spread, as a result of financial shocks, affect the real marginal cost of firms, and in turn, influence firms’ decisions on hiring, investment, and prices. This channel is missing when firms do this finance out of retained earnings instead of bank credit, thus reducing the impact of financial shocks on the real economy. In this sub-section we look at the credit supply shock. Figure 6 shows the IRFs of a rise in $\theta_t$, namely an increase in the ease of diversion by the banks.

Figure 6

Starting from panel b we see that banks cut back on lending, as required by depositors, given the greater risk of banks diverting their funds. This leads to a rise in the lending rate and the spread. The real variables react negatively upon impact as does inflation. The central bank lowers the interest rate as a result and so deposits decline. Housing prices drop with the ensuing lower demand for housing.

Table 7 summarizes the results and differences across models.
### Table 7
The Effects of a Rise in the Diversion Rate

<table>
<thead>
<tr>
<th>no borrowing</th>
<th>borrowing</th>
<th>differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_1 = \Omega_2 = \Omega_3 \simeq 0$</td>
<td>$\Omega_1 = \Omega_2 = \Omega_3 = 1$</td>
<td></td>
</tr>
</tbody>
</table>

**no real frictions**

<table>
<thead>
<tr>
<th>blue</th>
<th>green</th>
<th>substantial;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_h = S \simeq 0$</td>
<td>$y, c, i, n, \frac{b}{\pi}, w \downarrow$</td>
<td>$y, c, i, n, \frac{b}{\pi}, w \downarrow$</td>
</tr>
<tr>
<td>$R \downarrow, R_L \uparrow$</td>
<td>$R \downarrow, R_L \uparrow$</td>
<td></td>
</tr>
<tr>
<td>$D, L \downarrow$ a lot, $n \uparrow$</td>
<td>$D, L \downarrow$ a lot, $n \uparrow$</td>
<td></td>
</tr>
<tr>
<td>$\pi, q \downarrow$</td>
<td>$\pi, q \downarrow$</td>
<td></td>
</tr>
</tbody>
</table>

**real frictions**

<table>
<thead>
<tr>
<th>black</th>
<th>red</th>
<th>very similar;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_h, S &gt; 0$</td>
<td>$y, c, i, n, \frac{b}{\pi}, w \downarrow$ very slightly</td>
<td>$y, c, i, n, \frac{b}{\pi}, w \downarrow$ very slightly</td>
</tr>
<tr>
<td>$R \downarrow$ a little, $R_L \uparrow$</td>
<td>$R \downarrow$ a little, $R_L \uparrow$</td>
<td>black responds more</td>
</tr>
<tr>
<td>$D, L, n \simeq$ blue</td>
<td>$D, L \downarrow &lt;$ green, $n \uparrow &gt;$ green</td>
<td></td>
</tr>
<tr>
<td>$\pi \downarrow$ somewhat, $q \downarrow$</td>
<td>$\pi \downarrow$ a little, $q \downarrow &lt;$ green</td>
<td></td>
</tr>
</tbody>
</table>

**differences**

<table>
<thead>
<tr>
<th>substantial,</th>
<th>substantial,</th>
</tr>
</thead>
<tbody>
<tr>
<td>especially for real vars</td>
<td>usually red &lt; green</td>
</tr>
</tbody>
</table>

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While overall the patterns are similar to the shocks discussed around Tables 5 and 6, what stands out here is the difference of the green line w.r.t the other three configurations. In particular, there are bigger differences relative to the red lines, the full model. The green lines represent the configuration of no real frictions with firm borrowing.

The mechanism is as follows. The credit supply shock, resulting from an increase in frictions in the financial sector, leads to a rise in the lending rate and the spread. In the green case the response is mitigated. This leads to a fall in mortgage borrowing and in total lending. With less demand, house prices decline.

The real variables decline upon impact, except for consumption in the green model. With real frictions, the responses of the black and red models are minor. Real marginal costs decline (not shown) and so inflation declines, again with lower reaction of the black and red cases.

With inflation and output dropping, the central bank lowers the interest rate $R_t$. It does so more in the blue and green cases, whereby with no real frictions, the responses of inflation and output are stronger.

6 Discussion

6.1 Implications of the Results

Across all shocks, there are big differences between the model with no borrowing and real frictions and with both. In particular, real frictions matter a lot. Shutting down real frictions, there are differences, sometimes substantial, without and with firm borrowing. Firm borrowing makes a difference given no real frictions, operating to mitigate responses.

With real frictions present, does firm borrowing make a difference? For real variables, no. But we do see big differences between them for the financial variables, with borrowing mitigating responses. Firm borrowing takes the economy in the same direction as real frictions.

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6.2 Robustness

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7 Conclusions

We find that financial shocks and frictions have implications for the real economy and vice versa. The interactions between financial sector frictions and real frictions matter; both real frictions and firm borrowing operate to mitigate responses to shocks. We showed how monetary policy matters for variables of interest to macroprudential policy makers and vice versa.
References


8 Figures

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Figure 1: Set Up
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Figure 6a: Real Variables

Figure 6b: Financial Variables
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