Endogenous Regime Shifts in a New Keynesian Model with a Time-varying Natural Rate of Interest

Kevin J. Lansing
Federal Reserve Bank of San Francisco

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1 Any opinions expressed here do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.
Numerous ZLB (or ELB) episodes in global data
U.S. data: ZLB binding 2008.Q4 to 2015.Q4

“Promising to remain at zero for a long time is a double-edged sword.” (Bullard 2010).
Standard NK model has multiple RE equilibria

- Taylor rule + Fisher Eqn. + ZLB \( \Rightarrow \) Two steady states. (Benhabib, Schmitt-Grohé & Uribe *AER, JET* 2001a,b).

- \( r^* \) = “natural rate of interest” (also called “equilibrium” or “neutral” rate). The real rate consistent with full utilization of resources and steady inflation at central bank’s target \( \pi^* \).
  
  **Evidence**: \( r^* \) shifts over time (Laubach & Williams 2003, 2015).

- Two long-run endpoints (steady states): (1) targeted where \( i = r^* + \pi^* \) and (2) deflation where \( i = 0 \) and \( \pi = -r^* \).

- Two local RE solutions: (1) targeted equilibrium is locally unique, and (2) deflation equilibrium allows for sunspot shocks (focus on MSV solution here; no sunspots).
Standard NK model has multiple RE equilibria

- **This paper**: NK model with shifting $r_t^*$. Agent employs weighted-average of the two local forecast rules. Weights depend on past forecast performance, i.e., $RMSFE$.

- Forecast rules from deflation equilibrium induce more volatility in $\pi_t$ and $y_t$ in response to $r_t^*$ shocks.

- **Results**: Negative $r_t - r_t^* \implies$ more weight on deflation forecast rules $\implies$ deflation can become self-fulfilling. Episode accompanied by severe recession (highly negative output gap) with nominal rate at ZLB. Similar to 2007-09 Great Recession.

- But even in normal times, agent may place nontrivial weight on deflation forecast rules, causing central bank to consistently undershoot $\pi^*$ (like now: $\pi_t^{U.S.} < 0.02$ since mid-2012).
Related literature (partial list)

- **Infrequent but long-lived ZLB episodes in global data**
  Dordal-i-Carreras, Coibion, Gorodnichenko & Wieland (2016)

- **Transition between regimes driven by sunpots**
  Aruoba, Cuba-Borda, & Schorfheide (2014, WP)
  Aruoba & Schorfheide (2015, WP)

- **Adaptive learning to select among multiple equilibria**
  Evans & Honkapohja (2005, RED),
  Eusepi (2007, JME)
  Evans, Guse, & Honkapohja (2008, EER)
  Benhabib, Evans & Honkapohja (2014, JEDC)

- **Optimal monetary policy with shifting natural rate**
  Eggertsson and Woodford (2003, BPEA)
  Evans, Fisher, Gourio & Krane (2015, BPEA)
  Hamilton, Harris, Hatzius, & West (2016. IMF Econ. Rev.)
  Gust, Johannsen, López-Salido (2015, WP)
  Basu & Bundick (2015, NBER WP 21838)
New Keynesian model with zero lower bound (ZLB)

\[ y_t = E_t y_{t+1} - \alpha \left[ i_t - E_t \pi_{t+1} - r_t \right] + \nu_t, \quad \nu_t \sim N(0, \sigma^2_\nu) \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t, \quad u_t \sim N(0, \sigma^2_u) \]
\[ i_t^* = \rho i^*_{t-1} + (1 - \rho) \left[ E_t r_t^* + \pi^* + g_\pi (\bar{\pi}_t - \pi^*) + g_y (y_t - y^*) \right] \]
\[ \bar{\pi}_t = \omega \pi_t + (1 - \omega) \bar{\pi}_{t-1}, \quad \bar{\pi}_t \approx \frac{1}{4} (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}) \]
\[ i_t = \max \{0, i_t^*\} \]

Natural rate of interest (exogenous):
\[ r_t \equiv - \log \left[ \beta \exp(\nu_t) \right] + E_t \Delta \bar{y}_{t+1} \]

Discount factor + Expected potential output growth

\[ r_t = \rho_r r_{t-1} + (1 - \rho_r) r_t^* + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \]
\[ r_t^* = r_{t-1}^* + \eta_t, \quad \eta_t \sim N(0, \sigma^2_\eta) \]
Two long-run endpoints (steady states)

Targeted Endpoint
\[ \pi_t = \pi^* \]
\[ y_t = y^* \equiv \pi^* (1 - \beta) / \kappa \]
\[ i^*_t = r^*_t + \pi^* \]
\[ i_t = i^*_t \]

Deflation Endpoint
\[ \pi_t = -r^*_t \]
\[ y_t = -r^*_t (1 - \beta) / \kappa \]
\[ i^*_t = (r^*_t + \pi^*) \left[ 1 - g_\pi - \frac{g_y (1 - \beta)}{\kappa} \right] \]
\[ i_t = 0 \]

Shifting Endpoint Time Series Model (Kozick-Tinsley, JMCB 2012)

\[ E_t r^*_t = \lambda \left[ \frac{r_t - \rho_r r_{t-1}}{1 - \rho_r} \right] + (1 - \lambda) E_{t-1} r^*_{t-1} \]

Kalman gain
\[ \lambda = \frac{-(1 - \rho_r)^2 \phi + (1 - \rho_r) \sqrt{(1 - \rho_r)^2 \phi^2 + 4 \phi}}{2} \]
\[ \phi \equiv \frac{\sigma^2_\eta}{\sigma^2_\varepsilon} \]
\[ E_t (r_{t+k} - r^*_{t+k}) = (\rho_r)^k (r_t - E_t r^*_t), \quad \rho_r = 0.857 \]
Two local RE equilibria

Targeted Equilibrium (Unique) assumes $i_t^* = i_t > 0$

$$
\pi_t = \ldots + A_{11}(r_t - E_tr_t^*) + A_{12}(\bar{\pi}_{t-1} - \pi^*) + A_{13}u_t + A_{14}v_t
$$
$$
y_t = \ldots + A_{21}(r_t - E_tr_t^*) + A_{22}(\bar{\pi}_{t-1} - \pi^*) + A_{23}u_t + A_{24}v_t
$$
$$
i_t^* = \ldots + A_{31}(r_t - E_tr_t^*) + A_{32}(\bar{\pi}_{t-1} - \pi^*) + A_{33}u_t + A_{34}v_t
$$

Deflation Equilibrium (MSV) assumes $i_t^* \leq 0$, $i_t = 0$

$$
\pi_t = \ldots + B_{11}(r_t - E_tr_t^*) + u_t + \kappa v_t
$$
$$
y_t = \ldots + B_{21}(r_t - E_tr_t^*) + v_t
$$
$$
i_t^* = \ldots + B_{31}(r_t - E_tr_t^*) + B_{32}(\bar{\pi}_{t-1} - \pi^*) + B_{33}u_t + B_{34}v_t
$$

Solution coefficients when $\beta, \omega \to 1$ and $g_y \to 0$:

$$
\frac{B_{11}}{A_{11}} = \frac{B_{21}}{A_{21}} = \frac{B_{31}}{A_{31}} = 1 + \frac{(1-\rho)g_\pi}{(\rho_r-\rho)} \frac{\rho_r\kappa}{[(1-\rho_r)^2-\rho_r\kappa]} \gg 1
$$

$\Rightarrow$ Deflation equilibrium exhibits much more volatility.
**Model parameter values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.2</td>
<td>Interest rate coefficient in Euler equation.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.995</td>
<td>Discount factor in Phillips curve.</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.025</td>
<td>Output gap coefficient in Phillips curve.</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>0.02</td>
<td>Central bank inflation target.</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.684</td>
<td>$\pi_t \sim 4$-quarter inflation rate.</td>
</tr>
<tr>
<td>$g_\pi$</td>
<td>1.5</td>
<td>Policy rule response to inflation.</td>
</tr>
<tr>
<td>$g_y$</td>
<td>0.5</td>
<td>Policy rule response to output gap.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.80</td>
<td>Interest rate smoothing parameter.</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.857</td>
<td>Persistence parameter for natural rate.</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.0099</td>
<td>Std. dev. temporary shock to natural rate.</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.0016</td>
<td>Std. dev. permanent shock to natural rate.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0226</td>
<td>Optimal Kalman gain for $E_t r_t^*$.</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.008</td>
<td>Std. dev. of aggregate demand shock.</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.016</td>
<td>Std. dev. of cost push shock.</td>
</tr>
</tbody>
</table>

RE solution coefficients: $B_{11}/A_{11} \sim B_{21}/A_{21} \sim B_{31}/A_{31} \sim 5.1$
Natural rate process approximates Laubach-Williams r-star

Bounds for simulations: $0.002 \leq r_t^* \leq 0.0298$ (1988.Q1 to 2015.Q4).

U.S. Real Interest Rates

- $r_t$, Real interest rate
- $r_t^*$, Two-sided estimate from Laubach and Williams
- $E_{t \to t}^*$, Shifting Endpoint Model, $\rho=0.857$, $\lambda=0.0226$
Model simulation: Targeted Equilibrium

- **Real Interest Rate**
  - Graph showing fluctuations over time.
  - Two lines: $r_t$ and $r^*_t$.

- **4-Quarter Inflation Rate**
  - Graph showing stability over time.

- **Desired Nominal Interest Rate**
  - Graph showing fluctuations over time.

- **Output Gap**
  - Graph showing stability over time.
Model simulation: Deflation Equilibrium

- **Real Interest Rate**
- **4-Quarter Inflation Rate**
- ** Desired Nominal Interest Rate**
- **Output Gap**

The graphs illustrate the dynamics of various economic indicators over time, with a focus on real interest rates and inflation rates, as well as nominal interest rates and output gaps.
Endogenous forecast rule switching
Discrete choice framework along the lines of Brock and Hommes (1997, 1998)

\[
\hat{E}_t y_{t+1} = \mu_t E_t^{\text{targ}} y_{t+1} + (1 - \mu_t) E_t^{\text{defl}} y_{t+1}
\]

\[
\hat{E}_t \pi_{t+1} = \mu_t E_t^{\text{targ}} \pi_{t+1} + (1 - \mu_t) E_t^{\text{defl}} \pi_{t+1}
\]

\[
\hat{E}_t i^*_{t+1} = \mu_t E_t^{\text{targ}} i^*_{t+1} + (1 - \mu_t) E_t^{\text{defl}} i^*_{t+1}
\]

\[
\mu_t = \frac{\exp \left[ \psi \left( \text{RMSFE}_t^{\text{defl}} - \text{RMSFE}_t^{\text{targ}} \right) \right]}{1 + \exp \left[ \psi \left( \text{RMSFE}_t^{\text{defl}} - \text{RMSFE}_t^{\text{targ}} \right) \right]} \quad \psi = 75
\]

“Intensity of choice”

Forecast fitness measure for \( i = \text{targ}, \text{defl} \):

\[
\text{RMSE}_t^i = \frac{1}{8} \sum_{i=1}^{8} \left[ \left( y_{t-j} - E_t^{i}_{t-j-1} y_{t-j} \right)^2 + \left( \pi_{t-j} - E_t^{i}_{t-j-1} \pi_{t-j} \right)^2 
+ \left( i^*_{t-j} + E_t^{i}_{t-j-1} i^*_{t-j} \right)^2 \right]^{0.5}
\]
Given current forecasts, solve for equilibrium variables

\[
i_t^* = \frac{1}{\rho} \left\{ \hat{E}_t i_{t+1}^* - (1 - \rho) \left[ E_t r_{t+1}^* + \pi^* + g_\pi \omega \left( \hat{E}_t \pi_{t+1}^* - \pi^* \right) \right. \right.
\]
\[
\left. \left. + (1 - \omega) g_\pi (\bar{\pi}_t - \pi^*) + g_y \left( \hat{E}_t y_{t+1} - y^* \right) \right] \right\}
\]

\[
i_t = \max \{0, i_t^*\}
\]

\[
y_t = \hat{E}_t y_{t+1} - \alpha \left[ i_t - \hat{E}_t \pi_{t+1} - r_t \right] + \nu_t
\]

\[
\pi_t = \beta \hat{E}_t \pi_{t+1} + \kappa y_t + u_t
\]

\[
\bar{\pi}_t = \omega \pi_t + (1 - \omega) \bar{\pi}_{t-1}
\]
Overlapping distributions induce endogenous regime shifts

- Distribution of Desired Nominal Interest Rate
- Distribution of Quarterly Inflation Rate
- Distribution of Nominal Interest Rate
- Distribution of Output Gap
Weight on targeted forecast rules can decline rapidly.
Switching model: Severe recession, deflation, ZLB binding
U.S. data: Severe recession, deflation, ZLB binding
Comparing simulations: Targeted, Deflation, Switching
Switching model: Inflation distribution shifts left

- Distribution of Desired Nominal Interest Rate
- Distribution of Quarterly Inflation Rate
- Distribution of Nominal Interest Rate
- Distribution of Output Gap
Switching model: Infrequent but long-lived ZLB episodes
## Quantitative Comparison

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td><strong>Mean $\pi_{t-3 \rightarrow t}$</strong></td>
<td>2.20%</td>
<td>1.99%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.09%</td>
<td>0.81%</td>
</tr>
<tr>
<td>Corr. Lag 1</td>
<td>0.89</td>
<td>0.75</td>
</tr>
</tbody>
</table>

| **Mean $y_t$** | | | | |
|----------------| | | | |
| Mean | $-1.51\%$ | 0.40% | $-0.32\%$ | 0.24% |
| Std. Dev. | 2.02% | 0.97% | 2.83% | 1.34% |
| Corr. Lag 1 | 0.96 | 0.27 | 0.78 | 0.55 |

| **Mean $i_t^*$** | | | | |
|----------------| | | | |
| Mean | 3.45% | 3.59% | $-2.15\%$ | 2.42% |
| Std. Dev. | 2.84% | 1.84% | 6.35% | 3.46% |
| Corr. Lag 1 | 0.99 | 0.88 | 0.85 | 0.89 |

| % periods $i_t = 0$ | | | | |
|----------------| | | | |
| Mean ZLB duration | 25.9% | 2.59% | 63.3% | 17.5% |
| Max. ZLB duration | 29 qtrs. | 20 qtrs. | 96 qtrs. | 67 qtrs. |

## Effect of Raising the Inflation Target

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\pi^* = 0.02$</th>
<th>$\pi^* = 0.03$</th>
<th>$\pi^* = 0.04$</th>
<th>$\pi^* = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev. $\pi_{t-3\rightarrow t}$</td>
<td>1.08%</td>
<td>1.04%</td>
<td>0.91%</td>
<td>0.83%</td>
</tr>
<tr>
<td>Std. Dev. $y_t$</td>
<td>1.34%</td>
<td>1.12%</td>
<td>1.01%</td>
<td>0.98%</td>
</tr>
<tr>
<td>Std. Dev. $i_t^*$</td>
<td>3.46%</td>
<td>2.72%</td>
<td>2.14%</td>
<td>1.92%</td>
</tr>
<tr>
<td>% periods $i_t = 0$</td>
<td>17.5%</td>
<td>5.72%</td>
<td>0.99%</td>
<td>0.11%</td>
</tr>
<tr>
<td>Mean ZLB duration</td>
<td>4.0 qtrs.</td>
<td>3.3 qtrs.</td>
<td>2.9 qtrs.</td>
<td>3.1 qtrs.</td>
</tr>
<tr>
<td>Max. ZLB duration</td>
<td>67 qtrs.</td>
<td>55 qtrs.</td>
<td>38 qtrs.</td>
<td>32 qtrs.</td>
</tr>
</tbody>
</table>

Note: Model results computed from a 300,000 period simulation.

- Higher $\pi^*$ can prevent switching to volatile deflation equilibrium where recessions are more severe.
- Numerous papers examine benefits of higher $\pi^*$ using models that ignore deflation equilibrium. This methodology likely understates the benefits of a higher $\pi^*$. 
Conclusion

- Standard NK model with shifting $r_t^*$ and occasionally binding ZLB. Two RE equilibria. Endogenous forecast rule switching based on past RMSFE performance.

- Model can produce Great Recessions when $r_t - E_t r_t^*$ is negative, causing agent to place significant weight on deflation forecast rules. Escape from ZLB occurs endogenously when $r_t - E_t r_t^*$ eventually starts rising.

- In normal times, non-trivial weight on deflation forecast rules may cause central bank to undershoot $\pi^*$ (like today?).

- When $\pi^* = 0.04$, probability of ZLB episode is small $\simeq 1\%$ and average duration of ZLB episode is only 3 quarters.