General Equilibrium Dampened
(i) from Micro to Macro   (ii) Forward Guidance

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ESEM @ Edinburgh
December 31, 2016
Motivation

- **GE effects key to macroeconomics (and elsewhere)**
  - upend partial-equilibrium (PE) intuitions
  - limit usefulness of micro-based evidence a la Mian-Sufi
  - drive interpretations of phenomena + policy implications

- **But:** GE effects hinge on
  - common knowledge (CK) of structure and state of economy
  - immense coordination in beliefs and behavior
  - all hardwired in solution concept + info assumptions
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This talk, Part I: Dampening GE

- Formalize notion
  
  “GE Adjustment Takes Times”

- Framework: abstract but flexible “supply and demand”

- Main result: Equivalence between
  
  ▶ relax solution concept → Tattonment (“off equilibrium”)
  ▶ relax info / CK → imperfect coordination (“on equilibrium”)

- Broader lessons/implications:
  
  ▶ lack of CK = relaxation of RE solution concept = dampen GE
  ▶ resuscitate PE intuitions in GE settings
  ▶ enhance value of empirical work a la Mian-Sufi
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This talk, Part II: Application to Forward Guidance

- **Context:** A New-Keynesian economy during a liquidity trap

- **Question:** Ability to stimulate economy my promising low interest rates after ZLB has ceased to bind

- **Puzzling prediction:** Ability is large and increases with horizon at which forward guidance operates

- **Our contribution:**
  - puzzle driven solely by GE effects
  - lack of CK \(\rightarrow\) anchors expectations of income and inflation
    \(\rightarrow\) attenuates relevant GE effects \(\rightarrow\) reduces power of forward guidance
  - additional results: paradox of flexibility, discounted Euler/NKPC...
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Related Literature

- **Part I: Higher-order uncertainty in macroeconomics**
  - Angeletos and Lian (2016): chapter in *Handbook of Macroeconomics*

- **Part II: Forward guidance**
  - Wiederholt (2016), Farhi and Werning (2016)
  - McKay, Nakamura and Steinsson (2016a,b), Gabaix (2016)
  - Garcia-Schmidt and Woodford (2015)
First paper: Angeletos and Lian (2016a)

Dampening General Equilibrium:
Macro is Micro in the Short Run
Framework

- Minimal framework for studying PE vs GE, and micro vs macro
  - many locations, competitive firms and households
  - idiosyncratic and aggregate shocks
  - two (relative) prices $\rightarrow$ three goods
    - = numeraire + another tradable + one non-tradable per location

- What’s next?
  - micro-foundations, demand and supply
  - review standard predictions
  - two variants: (i) Tatonnment and (ii) Incomplete Info/Lack of CK
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Supply

- Representative competitive firm at each location $i \in [0, 1]$.
- Technology (production possibilities set):

  $$F(q_i, q_i^*, q_i^Z; a_i) \leq 0$$

  - $q_i, q_i^*, q_i^Z =$ production of non-tradables, tradables, numeraire
  - $a_i =$ technology shock = “supply shock”
Supply

- Firm’s problem

\[
\max \left\{ p_i q_i + p^* q^*_i + q^Z_i \right\}
\]

s.t. \( F(q_i, q^*_i, q^Z_i; a_i) \leq 0 \)

- \( p_i, p^* = \) price of non-tradables, tradables

- Supply of local non-tradable

\[ q_i = S(p_i, p^*, a_i) \]

- Supply of tradable and numeraire:

\[ q^*_i = S^*(...) \quad q^Z_i = S^Z(...) \]
Demand

- Representative competitive household at each location $i \in [0, 1]$.
- Preferences:
  \[ u_i = U(c_i, c_i^*, c_i^Z; \xi_i) \]
  - $\xi_i =$ preference shock = “demand shock”
- Budget constraint:
  \[ p_i c_i + p^* c_i^* + c_i^Z = y_i \]
  - $y_i =$ income $= p_i q_i + p^* q_i^* + q_i^Z$
Demand

- Demand for local non-tradable:
  \[ c_i = D(p_i, p^*, \xi_i, y_i) \]

- Demand for tradable and numeraire
  \[ c_i^* = D^*(...) \quad c_i^Z = D^Z(...) \]
Partial Equilibrium

- Partial equilibrium ≡
  - market clearing for non-tradable, but arbitrary $p^*$
  - i.e., momentarily allow market for tradable to be off equilibrium

- Let $\theta_i = (a_i, \xi_i)$. Quantity and price of non-tradable

  \[ c_i = q_i = Q(p^*, \theta_i) \quad \text{and} \quad p_i = P(p^*, \theta_i) \]

- Net (excess) demand for tradable

  \[ n_i \equiv c_i^* - q_i^* = N^*(p^*, \theta_i) \]
General Equilibrium

- Market clearing for tradable: \( p^* \) solves
  \[
  \int N^* (p^*, \theta_i) \, di = 0
  \]

- GE imposes
  \[
  p^* = P^* (\bar{\theta})
  \]

- Assumptions
  - \( \frac{\partial}{\partial p^*} \int N^* \, di < 0 \) (stable equilibrium)
  - \( \frac{\partial}{\partial \theta} \int N^* \, di \neq 0 \), or equivalently \( \frac{\partial}{\partial \theta} P^* \neq 0 \) (non-zero GE effects)
Macro Effect of an Aggregate Shock

- How does the economy respond to a shock that moves $\theta_i$ for all $i$?
  - demand shock: housing wealth, consumer deleveraging...
  - supply shock: productivity, payroll taxes...

- To simplify, work with log-linearized conditions
  - all variables in log-deviations from “steady state”

- To be concrete, focus on expenditure on non-tradable
  - local expenditure on non-tradable:
    \[ x_i = q_i + p_i = X(p^*, \theta_i) \]
  - corresponding aggregate:
    \[ \bar{x} = \int x_i di = X(p^*, \bar{\theta}) \]
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Micro vs Macro

- **Micro elasticity, or PE effect**
  - at local level
    \[
    \varepsilon_{micro}^i \equiv \left. \frac{dx_i}{d\theta_i} \right|_{p^* \text{ constant}}
    \]
  - aggregate counterpart
    \[
    \varepsilon^{micro} \equiv \int \varepsilon_{micro}^i di = \left. \frac{d\tilde{x}}{d\bar{\theta}} \right|_{p^* \text{ constant}} = \frac{\partial X}{\partial \theta}
    \]

- **Macro effect**
  \[
  \varepsilon^{Macro} \equiv \left. \frac{d\tilde{x}}{d\theta} \right|_{p^* \text{ adjusts in GE}} = \frac{\partial X}{\partial \theta}_{PE} + \frac{\partial X}{\partial p^*} \frac{\partial p^*}{\partial \theta}_{GE} \neq \varepsilon^{micro}
  \]
Micro vs Macro

- **Micro elasticity, or PE effect**
  - at local level
    \[ \varepsilon_{i}^{\text{micro}} \equiv \left. \frac{dx_i}{d\theta_i} \right|_{p^* \text{ constant}} \]
  - aggregate counterpart
    \[ \varepsilon^{\text{micro}} \equiv \int \varepsilon_{i}^{\text{micro}} \, di = \left. \frac{d\bar{x}}{d\bar{\theta}} \right|_{p^* \text{ constant}} = \frac{\partial X}{\partial \theta} \]

- **Macro effect**
  \[ \varepsilon^{\text{Macro}} \equiv \left. \frac{d\bar{x}}{d\theta} \right|_{p^* \text{ adjusts in GE}} = \underbrace{\frac{\partial X}{\partial \theta}}_{PE} + \underbrace{\frac{\partial X}{\partial p^*} \frac{\partial p^*}{\partial \theta}}_{GE} \neq \varepsilon^{\text{micro}} \]
PE effect

\[ S(\cdot, p^*) \]
\[ D(\cdot, p^*, \xi_0) \]
\[ D(\cdot, p^*, \xi_1) \]

PE effects
GE amplifies PE

- PE effects: $S(\cdot, p^*_0)$, $D(\cdot, p^*_0, \xi_0)$
- GE effects: $S(\cdot, p^*_1)$, $D(\cdot, p^*_1, \xi_1)$
- PE: $O$
GE attenuates PE

\[ D(\cdot, p^*_0, \xi_0) \]

\[ D(\cdot, p^*_1, \xi_1) \]

\[ O \]

\[ PE \]

\[ GE \]

\[ S(\cdot, p^*_0) \]

\[ S(\cdot, p^*_1) \]

\[ \text{GE effects} \]

\[ \text{PE effects} \]
Connection to Empirical Work

- Recent empirical macro:
  - exploits cross-sectional variation
  - provides estimate of $\epsilon^{micro}$
  - Mian-Sufi, Nakamura-Steinsson, etc

- Tension between
  - what is estimated ($\epsilon^{micro}$)
  - what is of interest ($\epsilon^{Macro}$)

- Key problem:
  - GE effect partialed out as time fixed effect in regressions

- Our contribution: lessens the problem (at least in the short run)
Standard paradigm:
  ▶ adjustment in $p^*$ is instantaneous
  ▶ perfect coordination

What we are after:
  ▶ slow adjustment in $p^*$

How?
  ▶ relax solution concept: Tâtonnement
  ▶ relax info assumption: remove common knowledge
Tâtonnement

• Let $t$ index round of iteration in Tatonnement process
  ▶ soon to reinterpret $t$ as time

• $\forall i, t$, local market for non-tradables clears with given perception $\hat{p}_t^*$
  ▶ gives PE outcomes with $p^* = \hat{p}_t^*$ to reinterpret $t$ as time

• “Walrasian auctioneer” adjusts $\hat{p}^*$ slowly from old GE level to new one
  ▶ $\hat{p}^*$ solves the following ODE
    \[
    \frac{d\hat{p}_t^*}{dt} = -b_t \cdot \left[ N^* (\hat{p}_t^*, \bar{\theta}_{new}) \right]
    \]
  ▶ with initial condition
    \[
    \hat{p}_0^* = P^*(\bar{\theta}_{old})
    \]
  ▶ and for some exogenous $\{b_t\}$ with $b_t \geq b > 0 \ \forall t$. 
Tâtonnement

- Let \( t \) index round of iteration in Tâtonnement process
  - soon to reinterpret \( t \) as time

- \( \forall i, t \), local market for non-tradables clears with given perception \( \hat{p}_t^* \)
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    \[
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    \]
    - with initial condition
      \[
      \hat{p}_0^* = P^*(\bar{\theta}_{old})
      \]
    - and for some exogenous \( \{b_t\} \) with \( b_t \geq b > 0 \ \forall t \).
Tâtonnement: micro vs macro

- **Macro effect at** $t$:

  \[
  \varepsilon_{\text{Tât}}(t) = \varepsilon^{\text{micro}} + w(t) \cdot (\varepsilon^{\text{Macro}} - \varepsilon^{\text{micro}})
  \]

  where $w(t)$ is increasing in $t$, with $w(0) = 0$ and $w(\infty) = 1$

- PE same as in benchmark
  - because of local market clearing

- GE is dampened by factor $w$
  - because of erroneous perceptions of $p^*$
Tâtonnement: micro vs macro
Corollary. $\varepsilon_{\hat{\text{T}}\hat{\text{a}}t}$ is monotone and continuous in $t$, with

$$\varepsilon_{\hat{\text{T}}\hat{\text{a}}t}(0) = \varepsilon^{\text{micro}}$$

and

$$\varepsilon_{\hat{\text{T}}\hat{\text{a}}t}(\infty) = \varepsilon^{\text{Macro}}$$

That is, we can span the gap between the micro and the macro by varying the round $t$ in Tâtonnement.
Incomplete Information

- Goal: translate from “off equilibrium” to “on equilibrium”
- Same payoff environment
- Non-tradable decisions in the “morning” under incomplete information
  - perfect knowledge of local conditions \((\theta_i, q_i, p_i)\)
  - lack common knowledge (CK) of global conditions \((\bar{\theta}, p^*)\)
  - private signal about the latter: \(s_i = \bar{\theta} + v_i\)
- Tradable decisions in the “afternoon”
  - global conditions \((\bar{\theta}, p^*)\) become common knowledge
Equilibrium

- **Rational-Expectations Equil** with inco info (similar to PBE)

- Morning: local markets for non-tradable clear, giving
  \[
  q_i = Q(E_i[p^*], \theta_i) \quad p_i = P(E_i[p^*], \theta_i)
  \]
  where \( E_i[p^*] \) is the rational expectation of \( p^* \) conditional on \( s_i \)

- Afternoon: \( p^* \) clears global market for tradable, giving
  \[
  p^* = P^q(\bar{q}, \bar{\theta})
  \]
  where \( \bar{q} \) is the realized agg quantity of non-tradable
**Characterization**

- **Lemma.** Equilibrium outcomes satisfy

\[
q_i = \alpha E_i[\tilde{q}] + \eta E_i[\tilde{\theta}] + \zeta \theta_i
\]

- Isomorphic to a “beauty contest”
  - GE effect akin to strategic interaction in games
  - \( \alpha < 1 \): degree of strategic complementarity/substitutability

- **Corollary.** Rational expectation of \( p^* = \) hierarchy of beliefs about \( \tilde{\theta} \)

\[
\bar{E}[p^*] = \Omega \cdot \sum_{h=1}^{\infty} \alpha^{h-1} \bar{E}^h[\tilde{\theta}]
\]

  - GE effects = HOB
Characterization

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  - GE effects = HOB
Lack of CK = Anchored Expectations

- Beliefs
  - first-order beliefs:
    \[ \bar{E}^1 \bar{\theta} \equiv \int E_i \bar{\theta} di = \lambda \bar{\theta} \]
    where \( \lambda \equiv \frac{\kappa}{\kappa + \sigma^2_{\theta}} \in (0, 1) \) captures deviation of common knowledge
  - higher-order beliefs (HOB):
    \[ \bar{E}^h \bar{\theta} \equiv \bar{E} \left[ \bar{E}^{h-1}[\bar{\theta}] \right] = \lambda^h \bar{\theta} \]

- HOB vary less than lower-order beliefs
  \( \Rightarrow \) expectations of \( p^* \) are anchored

- GE is stronger \( \Rightarrow \) HOB more important
  \( \Rightarrow \) the stronger the GE effect, the stronger its own attenuation
Micro vs Macro

- PE as in benchmark
  - due to perfect knowledge of local conditions

- GE dampened
  - due to lack of common knowledge of global conditions

Macro effect revisited:

\[ \varepsilon_{Inc}(\lambda) \equiv \frac{d\bar{x}}{d\theta} = \varepsilon^{micro} + g(\lambda) \cdot \left( \varepsilon^{Macro} - \varepsilon^{micro} \right) \]

- \( g(\lambda) \) is monotone in \( \lambda \), with \( g(0) = 0 \) and \( g(1) = 1 \)
- \( g(\lambda) \) decreases with \( \alpha \)
  - tends to be closer to zero when GE effect is larger
Proposition. For any $\{b_t\}$ and any $t$, there exists a $\lambda$ such that

1. rational expectations $\bar{E}[p^*]$ in inco-info economy same as ad hoc perceptions $\hat{p}_t^*$ in Tâtonnement economy
2. outcomes in inco-info economy same as in Tâtonnement economy
3. equal GE attenuation

$$\varepsilon_{\hat{T}ât}(t) = \varepsilon_{Inc}(\lambda)$$

The converse is also true.
Complementary Results and Take-home Lesson

Similar equivalence results for

- adaptive expectations
- reflective equilibrium (Garcia-Schmidt & Woodford, 2015)
- limited-depth reasoning

Take-home lesson:

lack of CK = relaxation of solution concept = GE dampened
Extension: GE Takes Time

- A dynamic extension
- Essentially repeated version of static economy

\[ F(q_i,t, q^*_i,t, q^Z_i,t; a_i) \leq 0 \]

\[ u_i = \sum_{t=0}^{\infty} e^{-\rho t} U(c_i,t, c^*_i,t, z_i,t; \xi_i) \]

- Slow learning about \( \bar{\theta} \) (or, equivalently, about global response)
Learning and GE Adjustment

- To avoid perfect aggregation of information:
  - idiosyncratic “iceberg costs” for tradable good
  - noisy private learning through realized prices

- **Lemma.** There exists an increasing sequence \( \{\lambda_t\} \) such that, \( \forall t, \)
  \[
  \bar{E}_t^h [\bar{\theta}] = \lambda_t^h \bar{\theta}
  \]

  \[
  \bar{E}_t [p^*] = \Omega \sum_{h=1}^{\infty} \alpha^{h-1} \bar{E}_t^h [\bar{\theta}] = f(\lambda_t) \bar{\theta}
  \]

- **Proposition.** Macro elasticity at \( t \)
  \[
  \varepsilon_t = \varepsilon_{Inc}(\lambda_t) = \varepsilon^{micro} + w_t \cdot (\varepsilon^{Macro} - \varepsilon^{micro}) \]
  \underbrace{GE_t}_t

  where \( w_t = g(\lambda_t) \) is increasing in \( t \), with \( w_0 < 1 = w_\infty \)

- Similar to static model, except that now \( \lambda_t \) increases with time
Slow GE Adjustment

- Formalization of notion that GE adjustment takes time
  - in short run, macro effect is close to micro/PE effect
  - but as time passes, it converges to what predicted by standard model

- Speed of convergence?
  - not surprisingly, it depends on quality of learning
  - more interestingly, it depends on magnitude of GE effect

- Prop. For any given $\{\lambda_t\}$, the sequence $\{w_t\}$ converges to 1 at a rate that is decreasing in $\alpha$. In this sense,

  stronger GE effect $\rightarrow$ slower GE adjustment!
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  stronger GE effect → slower GE adjustment!
Second paper: Angeletos and Lian (2016b)

Forward Guidance without Common Knowledge
Forward Guidance Puzzle

- Context: an economy during a liquidity trap
  - zero-lower bound (ZLB) binds for $t \leq T - 1$

- Forward Guidance = promise at $t$ to keep interest rates low at $t \geq T$

- The Puzzle: standard NK model predicts that the stimulating effect is quantitatively large, increases with $T$, and explodes as $T \to \infty$
Our contribution

- Prelim result: beauty-contest representation of NK model
  - disentangle PE and GE effects
  - recast GE effects as HOB

- Main result: remove CK
  - anchor expectations of income and inflation
  - attenuate relevant GE effects
  - reduce power of forward guidance

- Complementary results: paradox of flexibility, discounted Euler/NKPC
Differences from Earlier Abstract Framework

- Concrete context, precise micro-foundations, policy focus
- Truly dynamic environment
  - forward-looking expectations
  - dynamic beauty contest
- Specific novel insights
  - GE effects tied to expectations of future income and future inflation
  - longer horizons map to beliefs of higher order
  - dampening increases with $T \rightarrow$ lessen forward-guidance puzzle
  - dampening increases with price flexibility $\rightarrow$ lessen paradox of flexibility
Textbook NK Model

- **demand block:** Euler condition (aka IS curve)

\[ y_t = \mathbb{E}_t [y_{t+1} - (R_t - \pi_{t+1})] \]

- **supply block:** NK Philips Curve

\[ \pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa y_t \]

- Monetary Policy: ZLB and forward guidance
  
  - \( R_t = 0 \ \forall t \leq T - 1 \)
  
  - \( R_T \) free \( \rightarrow \) forward guidance moves \( \mathbb{E}_0[R_T] \)
  
  - \( y_t = \pi_t = 0 \ \forall t \geq T + 1 \) \( \rightarrow \) ex post optimal

- The puzzle: \( \left| \frac{\partial y_0}{\partial \mathbb{E}_0[R_T]} \right| \) increases with \( T \) and explodes as \( T \to \infty \)
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- **The puzzle**: \( \left| \frac{\partial y_0}{\partial \mathbb{E}_0[R_T]} \right| \) increases with \( T \) and explodes as \( T \to \infty \)
NK as Multi-Layer Beauty Contest

- Remove CK (of policy and/or of responses of others)

- Euler condition → dynamic BC among consumers
  - feedback from future spending to future income to current spending

- NKPC → dynamic BC among firms
  - feedback from future inflation to future MCs to current inflation

- Equilibrium: higher-layer BC between consumers and firms
  - feedback from future inflation to current spending
NK as Multi-Layer Beauty Contest

- Euler condition → dynamic BC among consumers
  \[ y_t = \int c_{it} di \]
  \[ c_{it} = f(\text{expected PV of income}) = f(\mathbb{E}_{it}[y_{t+k}]) \]
  \[ y_t = -\sum_{k=1}^{+\infty} \beta^{k-1} \left\{ \bar{E}_t[R_{t+k-1}] - \bar{E}_t[\pi_{t+k}] \right\} + (1-\beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[y_{t+k}] \right\} \]

- NKPC → dynamic BC among firms
  \[ p_t = \theta p_{t-1} + (1-\theta) \int p_{it}^* di \]
  \[ p_{it}^* = f(\text{expected PV of nominal MC}) = f(\mathbb{E}_{it}[p_{t+k}]) \]
  \[ \pi_{t}^* = \kappa y_t + \kappa \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t^f[y_{t+k}] + \frac{1-\theta}{\theta} \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t^f[\pi_{t+k}] + \kappa \mu_t \]
NK as Multi-Layer Beauty Contest

- Euler condition $\rightarrow$ dynamic BC among consumers

$$y_t = \int c_{it} di$$
$$c_{it} = f(\text{expected PV of income}) = f(\mathbb{E}_{it}[y_{t+k}]) \quad \implies$$
$$y_t = -\sum_{k=1}^{+\infty} \beta^{k-1} \left\{ \bar{E}_t[R_{t+k-1}] - \bar{E}_t[\pi_{t+k}] \right\} + (1-\beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[y_{t+k}] \right\}$$

- NKPC $\rightarrow$ dynamic BC among firms

$$p_t = \theta p_{t-1} + (1-\theta) \int p_{it}^* di$$
$$p_{it}^* = f(\text{expected PV of nominal MC}) = f(\mathbb{E}_{it}[p_{t+k}]) \quad \implies$$
$$\pi_t^* = \kappa y_t + \kappa \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t[f[y_{t+k}]] + \frac{1-\theta}{\theta} \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t[f[\pi_{t+k}]] + \kappa \mu_t$$
GE Attenuation

- **three GE mechanisms** = three types of strategic complementarity
  - within demand block: *income multiplier*
  - within supply block: *pricing complementarity*
  - between two blocks: *inflationary/deflationary spiral*

- **key insight:** lack of CK attenuates all three at once!
Forward Guidance Dampened

**Proposition.** With non-vanishing lack of CK,

\[
\frac{\partial y_0}{\partial \bar{E}_0 R_T} \bigg|_{\text{variant}} \to 0 \quad \text{as } T \to \infty
\]

\[
\frac{\partial y_0}{\partial \bar{E}_0 R_T} \bigg|_{\text{standard}} \to 0 \quad \text{as } T \to \infty.
\]

**Proposition.** When lack of CK is sufficiently large,

\[
\frac{\partial y_0}{\partial \bar{E}_0 R_T} \bigg|_{\text{variant}} \to 0 \quad \text{as } T \to \infty,
\]

whereas \[
\frac{\partial y_0}{\partial \bar{E}_0 R_T} \bigg|_{\text{standard}} \to \infty.
\]
Forward Guidance Dampened

**Proposition.** With non-vanishing lack of CK,

\[
\left. \frac{\partial y_0}{\partial E_0 R_T} \right|_{\text{variant}} \to 0 \quad \text{as } T \to \infty
\]

\[
\left. \frac{\partial y_0}{\partial E_0 R_T} \right|_{\text{standard}} \to \infty.
\]

**Proposition.** When lack of CK is sufficiently large,

\[
\left. \frac{\partial y_0}{\partial E_0 R_T} \right|_{\text{variant}} \to 0 \quad \text{as } T \to \infty,
\]

whereas \( \left. \frac{\partial y_0}{\partial E_0 R_T} \right|_{\text{standard}} \to \infty. \)
Paradox of Flexibility, Discounting, and More

- Dampening is stronger when prices are more flexible

![Graph showing the relationship between dampening and price flexibility.](image)

- Lack of CK manifests as discounting of future expectations in Euler and NKPC of isomorphomorphic representative-agent model

- Insights relevant also for
  - shocks at ZLB, deflationary spirals, paradox of flexibility, eq. selection, neo-Fisherian predictions...
Conclusion

- Worth revisiting solution concept and GE effects in macro
  - even if we maintain individual rationality and PE effects

- Lack of CK = relaxation of solution concept = GE dampened
  - formalization of “GE takes time”
  - in short run, “Macro is (close) to Micro”

- Topical application: Forward Guidance

- Other applications...
  - aggregate demand and Keynesian multipliers
  - Ricardian equivalence, fiscal stimuli