Comparing auction designs where suppliers have uncertain costs and uncertain pivotal status*

Pär Holmberg[†]and Frank A. Wolak[‡]

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Abstract

We analyze how market design influences bidding in multi-unit procurement auctions where suppliers have uncertain costs. Similar to wholesale electricity markets there is a risk that a supplier is pivotal, i.e. that realized demand is larger than the realized total production capacity of the competitors. In our setting with flat marginal costs, we show that welfare improves if the auctioneer restricts offers to be flat. We solve for a unique Bayesian NE and find that the competitiveness of market outcomes improves with increased market transparency. We identify circumstances where the auctioneer prefers uniform to discriminatory pricing, and vice versa.

Key words: cost uncertainty, asymmetric information, uniform-price auction, discriminatory pricing, market transparency, wholesale electricity market, treasury auction, bidding format, Bayesian Nash equilibria

JEL codes: C72, D43, D44, L13, L94

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[†]Research Institute of Industrial Economics (IFN), Stockholm. Associate Researcher of the Electricity Policy Research Group, University of Cambridge.

[‡]Program on Energy and Sustainable Development (PESD) and Department of Economics, Stanford University, 579 Serra Mall, Stanford, CA 94305-6072, e-mail:wolak@zia.stanford.edu.

1 Introduction

Multi-unit auctions are used to trade commodities, securities, emission permits and other divisible goods. Our discussion focuses on electricity markets, where producers submit offers before the demand and production capacities are fully known. Due to demand shocks, unexpected outages and intermittent output from renewable energy sources, there is a risk that a producer is pivotal, i.e. that realized demand is larger than the realized total production capacity of the competitors. We are interested in how such markets are influenced by the auction design. Most electricity markets use uniform-pricing where the highest accepted offer sets the transaction price for all accepted production. A few markets, such as the British real-time market, use discriminatory pricing, where each accepted offer is instead paid its own offer price.[1] Bilateral and continuous trading in forward markets often have similarities with discriminatory pricing. We also discuss how bid constraints, which give producers less flexibility when making their offers, influence welfare and payoffs among participants in electricity markets.

Our model accounts for asymmetric information in suppliers' production costs. Our analysis is, for example, of relevance for European wholesale electricity markets, where the European Commission has introduced regulations that increase the market transparency, so that uncertainties and information asymmetries are reduced. According to EU No. 543/2013, the hourly production in every single plant should be published. EU No. 1227/2011 (REMIT) mandates all electricity market participants to disclose insider information, such as the scheduled availability of plants.

In electricity markets, marginal costs can be estimated from engineering data on plant characteristics and input fuel price indexes. Long before delivery, in forward markets, the uncertainty about future fuel prices is to a large extent a common uncertainty among producers. The relative size of the common uncertainty typically decreases closer to the delivery. In the spot market, an owner of a thermal plant has private information about the actual price paid for its input fuel and how the plant is maintained and operated. We believe that the cost uncertainty and the information asymmetry are greatest in hydro-dominated markets. Such markets are also special in that they have a significant common uncertainty component also in the spot market. The opportunity cost of using water stored in the reservoir behind a specific generation unit is typically estimated by solving a stochastic dynamic program based on estimates of the probability distribution of future water inflows and future offer prices of thermal generation units, which can leave a significant scope for differences across market participants in their estimates of the generation unit-specific opportunity cost of water. The uncertain opportunity cost is exacerbated by political risks such as the possibility of regulatory intervention and each producer's subjective beliefs about the probability of these events occurring during the planning period. The influence of these polit-

[1] In addition, some special auctions in the electricity market, such as counter-trading in the balancing market and/or the procurement of power reserves, sometimes use discriminatory pricing (Holmberg and Lazarzcyk, 2015; Anderson et al., 2013).

ical risks on cost uncertainty is likely to be the greatest during extreme system conditions when water is scarce and the probability of regulatory intervention is high.

Our simplified model of the electricity market considers a multi-unit auction with two capacity-constrained producers facing an inelastic demand. Demand is uncertain and realized after offers have been submitted. Similar to the model of the electricity market by von der Fehr and Harbord (1993), each firm has a flat marginal cost (independent of output) and must make a flat offer. We generalize von der Fehr and Harbord (1993) by introducing uncertain interdependent costs. Analogous to Milgrom and Weber's (1982) auction for single objects as well as Ausubel et al.'s (2014) and Vives' (2011) models of multi-unit auctions, each firm makes its own estimate of production costs based on private imperfect information that it receives, and then makes an offer.[2] As is customary in game theory, we refer to this private information as a private signal. We solve for a unique Bayesian NE when signals are drawn from a bivariate distribution that is known to the suppliers.

In our setting with flat marginal costs, the bid constraint that offers must also be flat improves welfare and ensures that there are no welfare losses in equilibrium. A comparison of our results to Vives (2011) suggests that the bid constraint is particularly beneficial for uniform-price auctions where producers have large common uncertainties in their costs. This is relevant for uniform-price auctions of forward contracts and hydro-dominated electricity markets, where the opportunity cost has a significant common uncertainty and is approximately flat for a wide range of outputs.

We show that the expected payoffs for uniform and discriminatory pricing are equal when signals are independent. An auctioneer tends to favour discriminatory pricing when a higher signal of a producer is more informative of the competitor's signal. The opposite is true when a higher signal is less informative of the competitor's signal. Advantages and disadvantages with uniform pricing tend to be amplified if producers are pivotal with a higher probability. Equilibrium offers in a discriminatory auction are determined by the expected sales of the highest and lowest bidder, respectively. In our setting, the variance in these sales after offers have been submitted – due to demand shocks, outages and intermittent renewable production – will not influence the bidding behaviour of producers or their expected payoffs in the discriminatory auction. Offers and payoffs in the uniform-price auction are also insensitive to this variance in sales, as long as the probability is negligible that a market shock would change the pivotal status of at least one producer. For independent signals, expected payoffs (but not offers) in the uniform-price auction are independent of the variance in sales, even if the pivotal status of producers changes with a positive probability.

Independent of the payment scheme, we find that mark-ups decrease if producers' are more likely to receive similar information. This is related to Vives (2011) who finds that mark-ups decrease when producers receive less noisy cost

[2] Milgrom and Weber (1982) and Ausubel et al. (2014) analyse sales auctions, so in their settings each agent estimates the value of the good that the auctioneer is selling.

information before competing in a uniform-price auction. In single object auctions, disclosure of information is beneficial for the auctioneer under more general circumstances, a result which is often referred to as the linkage principle or the publicity effect (Milgrom and Weber, 1982). It is known from Perry and Reny (1999) that there are exceptions from the linkage principle for multi-unit auctions. Still, taken together, these results suggest that publicly available information of relevance for production costs – such as weather conditions, fuel prices, prices of emission permits – is likely to improve the competitiveness of market outcomes in electricity markets. It is also easier for a producer to estimate the marginal cost of its competitors if the market operator discloses detailed historical bid data and/or detailed production data. Thus, our results support the argument that the transparency increasing measures of the European Commission should improve the performance of European electricity markets. In addition, information provision about outcomes from financial markets just ahead of the operation of related physical markets should lower the market uncertainty. Similarly, trading of long-term contracts, which help producers predict future electricity prices, should reduce the extent of informational asymmetries among suppliers about the opportunity cost of water.

Extending this logic further, our results suggest that regulatory risks are particularly harmful for competition in hydro-dominated wholesale electricity markets, especially when water is scarce, because of the potential informational asymmetries about the likelihood of regulatory interventions. Thus, we recommend clearly defined contingency plans for intervention by the regulator in case of extreme system conditions. This could potentially mitigate the extraordinarily high-priced periods that typically accompany low-water conditions in hydro-dominated markets such as California, Colombia, and New Zealand.

Because increased transparency reduces the payoff of producers in our model, we would not expect producers to agree to voluntarily disclose production cost-relevant information. This has similarities to Gal-Or (1986) who shows that producers that play a Bertrand equilibrium would try to conceal their private costs from each other. Moreover, increased transparency would only be helpful up to a point, because there is a lower bound on equilibrium mark-ups when producers are pivotal. Another caveat is that we only consider a single shot game. As argued by von der Fehr (2013), there is a risk that increased transparency in European electricity markets can facilitate tacit collusion in a repeated game.

Our study focuses on procurement auctions, but the results are analogous for multi-unit sales auctions. Purchase constraints in sales auctions correspond to production capacities in our setting.[3] As an example, U.S. treasury auctions have the 35% rule, which prevents a single bidder from buying more than 35% of the securities sold. Similar rules are used in spectrum auctions by the Federal Communications Commission (FCC) and in California's auctions of Greenhouse Gas emission allowances. Purchase constraints are used to avoid the outcome

[3] To some extent, bidders' financial constraints would also correspond to production capacities. Financial constraints of bidders partly explain the bidding behaviour in security auctions (Che and Gale, 1998).

where a single bidder purchases the vast majority of the good sold, which would give it significant market power in secondary markets. On the other hand, such constraints increase the probability that a bidder will be pivotal and/or make bidders pivotal with a larger margin in the auction. The latter would make bidding less competitive and the auctioneer's revenues would go down.

Analogous to the demand uncertainty in our model, the auctioneer's supply of treasury bills is typically uncertain when bids are submitted due to an uncertain amount of non-competitive bids (Wang and Zender, 2002) or because the auctioneer wants to wait for the latest market news before finally announcing its supply of treasury bills.

Most treasury auctions around the world use discriminatory pricing (Bartolini and Cottarelli, 1997). An important exception is the U.S. Treasury, which switched from the discriminatory format to the uniform-price format during the 1990s. Analogous to our model, bidders' marginal valuation of securities is fairly insensitive to the purchased volume. Moreover, securities do often have a large common value component. This indicates that bid constraints should increase welfare and auction sales revenues in uniform-price security auctions. Finally, our results show that it is beneficial for auctioneers of securities to disclose market relevant information before the auction starts, so that bidders have access to similar information.

The remainder of the paper is organized as follows. Section 2 compares details in our model with the previous literature. Section 3 formally introduces our model, which is analysed for auctions with discriminatory and uniform-pricing in Section 4. The paper is concluded in Section 5. All proofs are in the Appendix.

2 Comparison with related studies

Divisible-good auctions do often have restrictions on how many offer prices each producer can submit or, equivalently, how many steps a producer is allowed to have in its supply function. Similar to models of electricity markets by von der Fehr and Harbord (1993) and Fabra et al. (2006), we make the assumption that offers must be flat; a producer must offer its whole production capacity at the same unit price. We generalize their setting to cases where costs are uncertain. Our model also generalizes Parisio and Bosco (2003), which is restricted to producers with independent private costs in uniform-price auctions. Our bid constraint makes the discriminatory auction identical to a Bertrand game with uncertain costs and uncertain demand. Thus, we generalise the Bertrand models by Gal-Or (1986) and Spulber (1995), which consider producers with independent private costs. Another consequence of the bid constraint is that uniform and discriminatory pricing are equivalent when firms are non-pivotal with certainty, i.e. when the capacity of each producer is always larger than realized demand. Independent of the auction format, the payoff is then zero for the producer with the highest offer price and the other producer is paid its own offer price. This corresponds to the first-price single-object auction that is studied by Milgrom and Weber (1982). We generalize their model to the case where producers are pivotal with a positive probability. Ausubel et al. (2014) and Vives (2011) consider multi-unit auctions for producers that are non-pivotal with certainty. Unlike them, we allow producers to be pivotal with a positive probability, as is often the case in electricity markets. Moreover, we contribute relative to them by studying the effect of a bid constraint. The bid constraint and the assumption that the pivotal status of producers is uncertain are useful when we prove uniqueness of equilibria. Unlike Ausubel et al. (2014) and Fabra et al. (2006), we compare auction designs for settings with unique equilibria. Vives (2011) focuses on linear SFE in uniform-price auctions.

It follows from Ausubel et al. (2014) that auctions where producers have asymmetric information about flat marginal costs can only be efficient if offers are also flat. In our model, the bid constraint ensures that offers are flat and that welfare losses can be avoided. Ausubel et al. (2014) provide a few examples where equilibrium offers are flat and allocations efficient without a bid constraint, but that is not true in general. Equilibrium offers in discriminatory auctions tend to be flatter (more elastic with respect to the price) than in uniform-price auctions (Genc, 2009; Anderson et al., 2013; Ausubel et al., 2014). Therefore, we conjecture that our bid constraint will have a greater positive influence on market performance in uniform-price auctions.

Related to the above, the results in Vives (2011) illustrate that the lack of bid constraints can have anti-competitive consequences in uniform-price auctions. In an auction where the costs are positively interdependent, a high clearing price is bad news for a firm's costs, because this increases the probability that the competitor has received a high-cost signal. Ausubel et al. (2014) refer to this as a generalized winner's curse. As illustrated by Vives (2011), a producer therefore has an incentive to reduce its output when the price is unexpectedly high and increase its output when the price is unexpectedly low. This will make supply functions steeper or even downward sloping in auctions with nonrestrictive bidding formats, and this will significantly harm competition. If costs have a large common uncertainty, then mark-ups in a uniform-price auction can be as high as for the monopoly case (Vives, 2011). Our restrictive bidding format avoids this problem. The bid constraint gives a producer less flexibility to indirectly condition its output on the competitor's signal. It does not matter how sensitive a producer's cost is to the competitor's signal, our results are the same irrespective of whether the costs are private, common or anything in between those two extremes.

Our reading of previous theoretical comparisons of auction formats by for example Holmberg (2009), Hästö and Holmberg (2006), Pycia and Woodward (2015) and Ausubel et al. (2014) is that they tend to conclude that discriminatory pricing is weakly preferable to uniform-pricing from the auctioneer's perspective. We think that the bid constraint makes payoffs in the two auction formats more similar and that details in the bidding format can influence the ranking of auction designs. Empirical studies by Armantier and Sbaï (2006;2009) and Hortaçsu and McAdams (2010) find that the treasury would prefer uniform pricing in France and Turkey, respectively, while Kang and Puller (2008) find that discriminatory pricing would be best for the treasury in South Korea. Wolak (2007) and Kastl (2012) have developed structural econometric models that account for further details in the bidding format, which can be useful in future empirical assessments of multi-unit auction designs.

As for example illustrated by Wilson (1979), Klemperer and Meyer (1989), Green and Newbery (1992) and Ausubel et al. (2014), there are normally multiple NE in divisible-good auctions when some offers are never price-setting. The bid constraint mitigates this problem. In our setting, there is a unique equilibrium in the discriminatory auction also for a given demand level and given production capacities. Some uncertainty in demand or production capacities, so that the pivotal status of producers is uncertain, is required to get uniqueness in our uniform-price auction. Uniqueness of equilibria is another reason why highly anti-competitive equilibria in uniform-price auctions can be avoided. In the special case where producers are pivotal with certainty, there is, in addition to the symmetric Bayesian equilibrium that we calculate, also an asymmetric high-price equilibrium (von der Fehr and Harbord, 1993) in the uniform-price auction. This equilibrium is very unattractive for consumers of electricity, because the highest offer, which sets the clearing price, is always at the reservation price.[4] Thus, for circumstances when the high-price equilibrium exists and is selected by producers, the uniform-price auction is significantly worse than the discriminatory auction for an auctioneer (Fabra et al., 2006).

In practice, the number of pivotal producers in wholesale electricity markets depends on the season and the time-of-day (Genc and Reynolds, 2011), but also on market shocks. Pivotal status indicators as measures of the ability to exercise unilateral market power have been evaluated by Bushnell et al. (1999) and Twomey et al. (2005) and have been applied by the Federal Energy Regulator Commission (FERC) in its surveillance of electricity markets in U.S. Such binary indicators are supported by von der Fehr and Harbord's (1993) high-price equilibrium in uniform-price auctions, where the market price is either at the marginal cost of the most expensive supplier or the reservation price, depending on whether producers are non-pivotal or pivotal with certainty. Our equilibrium is more subtle, the pivotal status can be uncertain before offers are submitted and the expected market price increases continuously when producers are expected to be pivotal with a larger margin.

In order to facilitate comparisons with previous studies, we are interested in results for the limit where the cost uncertainty decreases until the costs are common knowledge. In this limit, our model of the discriminatory auction corresponds to the classical Bertrand game. For producers that are non-pivotal with certainty, we get the competitive outcome with zero mark-ups, both for uniform and discriminatory pricing. This result agrees with the competitive outcomes for non-pivotal producers in von der Fehr and Harbord (1993) and in Fabra et al. (2006). If signals are independent and producers pivotal, it follows from Harsanyi's (1973) purification theorem that in the limit when costs are common knowledge, our Bayesian Nash equilibria for uniform-price and discriminatory auctions correspond to the mixed-strategy NE analysed by Anderson et al. (2013), Anwar (2006), Fabra et al. (2006), Genc (2009), Son et al. (2004) and von der Fehr and Harbord (1993). Anal-

[4] The equilibrium offer from the low-price bidder must be sufficiently low to ensure that the high-price bidder would not find it profitable to deviate and undercut the low-price bidder.

ogous mixed strategy NE also occur in the Bertrand-Edgeworth game, where producers are pivotal, (Edgeworth, 1925; Allen and Hellwig, 1986; Beckmann, 1967; Levitan and Shubik, 1972; Maskin, 1986; Vives, 1986; Deneckere and Kovenock, 1996; Osborne and Pitchik, 1986).

The costs are not influenced by private information in the limit where they are common knowledge. In our model, the results are the same for the more general case where the costs are insensitive to common variations in signals. Hence, non-pivotal producers do not get any informational rent even if they have private information about the outcomes where producers have different signals.

3 Model

There are two risk-neutral producers in the market. Each producer $i \in \{1, 2\}$ receives a private signal s_i with imperfect cost information. The joint probability density $\chi(s_i, s_j)$ is continuously differentiable and symmetric, so that $\chi(s_i, s_j) \equiv \chi(s_j, s_i)$. Moreover, $\chi(s_i, s_j) > 0$ for $(s_i, s_j) \in (\underline{s}, \overline{s}) \times (\underline{s}, \overline{s})$.[5] Signals are affiliated when

$$\frac{\chi(u, v')}{\chi(u, v)} \leq \frac{\chi(u', v')}{\chi(u', v)}, \tag{1}$$

where $v' \geq v$ and $u' \geq u$. Thus, if the signal of one player increases, then it (weakly) increases the probability that its competitor has a high signal relative to the probability that its competitor has a low signal. Signals are strictly affiliated when the inequality in (1) is strict. We say that signals are negatively affiliated when the opposite is true, i.e.

$$\frac{\chi(u, v')}{\chi(u, v)} \geq \frac{\chi(u', v')}{\chi(u', v)}, \tag{2}$$

where $v' \geq v$ and $u' \geq u$. Note that independent signals are both affiliated and negatively affiliated. We let

$$F(s_i) = \int_{-\infty}^{s_i} \int_{-\infty}^{\infty} \chi(u, v)\, dv du$$

denote the marginal distribution, i.e. the unconditional probability that supplier i receives a signal below s_i. Moreover,

$$f(s_i) = F'(s_i).$$

As in von der Fehr and Harbord (1993), we consider the case when each firm's marginal cost is flat up to its production capacity constraint $\widetilde{q}_i$.[6] But in our setting, marginal costs and possibly also $\widetilde{q}_i$ are uncertain when offers are submitted.

[5] We do not require $\chi(s_i, s_j) > 0$ at the boundary, but $\frac{\chi_1(u,\overline{s})}{\chi(u,\overline{s})} = \frac{\chi_2(\overline{s},u)}{\chi(\overline{s},u)}$ is assumed to be bounded for $u \in [\underline{s}, \overline{s}]$.

[6] This corresponds to flat demand in the sales auction of Ausubel et al. (2014).

The production capacities of the two producers could be correlated, but they are symmetric information and we assume that they are independent of production costs and signals. In Europe, this assumption could be justified by the fact that any insider information on production capacities must be disclosed to the market according to EU No. 1227/2011 (REMIT). Capacities are symmetric ex-ante, so that $\mathbb{E}[\widetilde{q}_i] = \mathbb{E}[\widetilde{q}_j]$. Realized production capacities are assumed to be observed by the auctioneer when the market is cleared.[7]

We refer to $c_i(s_i, s_j)$ as the marginal cost of producer i, but actually costs are actually not necessarily deterministic for given s_i and s_j. More generally, $c_i(s_i, s_j)$ is the expected marginal cost conditional on all information available among producers in the market. We use the convention that a firm's own signal is placed first in its list of signals. Firms' marginal costs are symmetric ex-ante, i.e. $c_i(s_a, s_b) = c_j(s_a, s_b)$. But costs and information about costs are normally asymmetric ex-post, after private signals have been observed. We assume that

$$\frac{\partial c_i(s_i, s_j)}{\partial s_i} > 0, \tag{3}$$

so that a firm's marginal cost increases with respect to its own signal. We also require that a firm's cost is weakly increasing with respect to the competitor's signal:

$$\frac{\partial c_i(s_i, s_j)}{\partial s_j} \geq 0. \tag{4}$$

A firm's private signal has more influence on its own cost than on the competitor's cost:

$$\frac{\partial c_i(s_i, s_j)}{\partial s_i} > \frac{\partial c_j(s_j, s_i)}{\partial s_i}. \tag{5}$$

Taken together, (3) and (4) imply that:

$$\frac{dc_i(s, s)}{ds} > 0. \tag{6}$$

The special case with independent signals and $\frac{\partial c_i(s_i,s_j)}{\partial s_j} = 0$ corresponds to the private independent cost assumption, which is, for example, used in the analysis by Parisio and Bosco (2003) and Spulber (1995). The common cost/value assumption that is used by Wilson (1979) and others corresponds to $c_i(s_i, s_j) \equiv c_j(s_j, s_i)$. Our model approaches the latter case in the limit where $\frac{\partial c_i(s_i,s_j)}{\partial s_i} - \frac{\partial c_j(s_j,s_i)}{\partial s_i} \searrow 0$.

Costs are insensitive to private information about both producers in the limit where costs are common knowledge. For our Bayesian NE, it turns out that bidding behaviour is determined by properties of the cost function along its diagonal where producers receive identical private information. Thus, for us it is sufficient to define a weaker form of common knowledge about costs.

[7] Alternatively, similar to the market design of the Australian wholesale market, producers could first choose bid prices and later adjust production capacities at those prices just before the market is cleared. Anyway, we assume that the reported production capacities are publicly verifiable, so that bidders cannot choose them strategically.

Definition 1 *Production costs are insensitive to common variations in signals in the limit where* $\frac{dc_i(s,s)}{ds} \searrow 0$ *for* $s \in [\underline{s}, \overline{s})$.

As in von der Fehr and Harbord (1993), demand can be uncertain $D \in [\underline{D}, \overline{D}]$. It could be correlated with the production capacities, but demand is assumed to be independent of the production costs and signals. In addition, it is assumed that all outcomes are such that $0 \leq D \leq \widetilde{q}_i + \widetilde{q}_j$, so that there is always enough production capacity to meet the realized demand. As in von der Fehr and Harbord (1993), demand is inelastic up to a reservation price $\overline{p}$. Analogous to Milgrom and Weber (1982), we assume that the reservation price is set at the highest relevant marginal cost realization, i.e. $\overline{p} = c_i(\overline{s}, \overline{s})$ for $i \in \{1, 2\}$. This assumption can be motivated by the fact that an auctioneer would lower its procurement cost by lowering the reservation price whenever $\overline{p} > c_i(\overline{s}, \overline{s})$.

A firm $i \in \{1, 2\}$ submits its offer after it has received its private signal s_i. We assume that the bidding format constrains offers such that each firm must offer its entire production capacity at one unit price $p_i(s_i)$. The auctioneer accepts offers in order to minimize its procurement cost. Thus, output from the losing producer, which has the highest offer price, is only accepted when that firm is pivotal and in that case the auctioneer first accepts the entire production capacity from the winning producer, which has the lowest offer price. Ex-post, we denote the winning producer, which gets a high output, by subscript H. The losing producer, which gets a low output, is denoted by the subscript L. Winning and losing producers have the following expected outputs:

$$q_H = \mathbb{E}[\min(\widetilde{q}_H, D)] \tag{7}$$

and

$$q_L = \mathbb{E}[\max(0, D - \widetilde{q}_H)]. \tag{8}$$

A rationing rule is used when producers submit offers at the same price and realized demand is strictly less than the realized market capacity. We assume that the rationing rule is such that, whenever rationing is needed, any producer would get a significantly larger output, an increment bounded away from zero, if it reduced its offer price by any positive amount.

In a uniform-price auction, the highest accepted offer price sets the market price for all accepted offers. In a discriminatory auction, each accepted offer is paid its individual offer price. The payoff of each producer is given by its revenue minus its realized production cost. We solve for Bayesian Nash equilibria in a one-shot game, where $p_i(s_i)$ is weakly monotonic and piece-wise differentiable.

4 Analysis

4.1 Discriminatory pricing

We start the section on discriminatory pricing by deriving the best response of a firm $i \in \{1, 2\}$. We denote its competitor by $j \neq i$. We use the best response to explicitly solve for the unique Bayesian NE.

Each firm is paid as bid under discriminatory pricing. The demand uncertainty and the production capacity uncertainties are independent of the cost uncertainties. Thus, the expected profit of firm i when receiving signal s_i is:

$$\begin{aligned}\pi_i(s_i) = {} & (p_i(s_i) - \mathbb{E}[c_i(s_i,s_j)|\, p_j \geq p_i]) \Pr(p_j \geq p_i|\, s_i)\, q_H \\ & + (p_i(s_i) - \mathbb{E}[c_i(s_i,s_j)|\, p_j \leq p_i]) (1 - \Pr(p_j \geq p_i|\, s_i))\, q_L.\end{aligned} \tag{9}$$

In the Appendix, we show that:

Lemma 1 *In markets with discriminatory pricing:*

$$\begin{aligned}\tfrac{\partial \pi_i(s_i)}{\partial p_i} = {} & \Pr(p_j \geq p_i|\, s_i)\, q_H + (1 - \Pr(p_j \geq p_i|\, s_i))\, q_L \\ & + \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right) \tfrac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} (q_H - q_L),\end{aligned} \tag{10}$$

whenever $p_i(s_i)$ and $p_j(s_j)$ are locally differentiable and locally invertible for signals that have offer prices around $p_i(s_i)$.

The first two terms on the right-hand side of (10) correspond to the price effect. This is what the producer would gain in expectation from increasing its offer price by one unit if the acceptance probabilities were to remain unchanged. However, on the margin, a higher offer price lowers the probability of being the winning producer by $\frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i}$. Switching from being the winning to the losing bidder reduces the accepted quantity by $q_H - q_L$. We refer to $\frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i}(q_H - q_L)$ as the quantity effect, i.e. the quantity that is lost on the margin from a marginal price increase. The mark-up for lost sales, $p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)$, times the quantity effect gives the lost value of the quantity effect. This is the last term on the right-hand side of (10). Note that marginal changes in $p_i(s_i)$ only result in changes in output for cases where the competitor, producer j, is bidding really close to p_i, which corresponds to the competitor receiving the signal $p_j^{-1}(p_i)$. This explains why $c_i\left(s_i, p_j^{-1}(p_i)\right)$ is the relevant cost in the mark-up for lost sales in the quantity effect.

We find it useful to introduce the function $H^*(s)$, which is proportional to the quantity effect and inversely proportional to the price effect for a given signal s.

Definition 2

$$H^*(s) := \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\bar{s}} \chi(s,s_j)\, ds_j q_H + \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j q_L}. \tag{11}$$

$H^*(s)$ depends on exogenous variables/parameters and captures the essential aspects of the information structure, the auction format and the essential properties of demand and the production capacities. The unique equilibrium is symmetric, so in the following we sometimes find it convenient to drop subscripts.

Proposition 1 *If*

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}}\chi(s,s_j)\,ds_j q_H + q_L\int_{\underline{s}}^x \chi(s,s_j)\,ds_j}{\chi(s,x)}\right)\geq 0, \tag{12}$$

for all $s,x\in(\underline{s},\overline{s})$*, then there is a unique Bayesian NE in the discriminatory auction. There is a unique equilibrium for more general probability distributions when* $\frac{dc(v,v)}{dv}$ *is large for* $v<\overline{s}$*. The unique equilibrium is symmetric, without welfare losses and has the property that* $p'(s)>0$*, where*

$$p(s)=c(s,s)+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^v H^*(u)du}dv \tag{13}$$

for $s\in[\underline{s},\overline{s})$*. The above expression can be simplified for the following circumstances:*

1. In the limit when production costs are insensitive to common variations in signals, (13) can be simplified to:

$$p(s)=c(\underline{s},\underline{s})+e^{-\int_s^{\overline{s}}H^*(u)du}\left(\overline{p}-c(\underline{s},\underline{s})\right). \tag{14}$$

2. The condition in (12) is satisfied if signals are independent, and (13) can then be simplified to:

$$p(s)=c(s,s)+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}\left(\frac{(1-F(v))\,q_H+F(v)\,q_L}{(1-F(s))\,q_H+F(s)\,q_L}\right)dv. \tag{15}$$

   In the limit when production costs are insensitive to common variations in signals, then (15) can be further simplified to:

$$p(s)=c(\underline{s},\underline{s})+\left(\frac{q_L}{((1-F(s))\,q_H+F(s)\,q_L)}\right)\left(\overline{p}-c(\underline{s},\underline{s})\right). \tag{16}$$

3. The condition in (12) is satisfied if signals are affiliated and producers are non-pivotal with certainty, in which case (11) can be simplified to:

$$H^*(s):=\frac{\chi(s,s)}{\int_s^{\overline{s}}\chi(s,s_j)\,ds_j}. \tag{17}$$

   If, in addition, production costs are in the limit where they are insensitive to common variations in signals, the equilibrium offer is perfectly competitive, i.e. $p(s)=c(\underline{s},\underline{s})$ for $s\in[\underline{s},\overline{s})$.

Demand is inelastic so the total output is efficient. The marginal costs are flat and in our unique equilibrium, the firm with the highest cost and signal makes the highest offer. Thus, the bid constraint ensures that the allocated output is efficient and that welfare losses are avoided.

The term $\int_{\underline{s}}^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_{\underline{s}}^{v} H^*(u)du} dv$ in (13) corresponds to a mark-up. Given that $H^*(s)$ is proportional to the quantity effect and inversely proportional to the price effect, it makes sense that a high $H^*(s)$ results in more competitive offers with lower mark-ups. For example, higher production capacities so that $q_H - q_L$ increases, and less restrictive purchase constraints in analogous sales auctions, will make bidding more competitive. We also note from Definition 2 that $H^*(s)$ and $p(s)$ are determined by the expected sales of the high price bidder and the low price bidder, but $H^*(s)$ and $p(s)$ are independent of the variances of those sales.

Another conclusion that we can draw from Proposition 1 is that bidding behaviour is only influenced by properties of $c_i(s_i, s_j)$ at points where $s_i = s_j$. Thus, for given properties along the diagonal of the cost function where signals are identical, it does not matter for our analysis whether the costs are private, so that $\frac{\partial c_i(s_i,s_j)}{\partial s_j} = 0$, or whether the costs have a common uncertainty component, such that $\frac{\partial c_i(s_i,s_j)}{\partial s_j} > 0$. As noted above, the reason is that when solving for the locally optimal offer price, a producer is only interested in cases where the competitor is bidding really close to p_i. In a symmetric equilibrium, this occurs when the competitor receives a similar signal. The properties of $c(\cdot)$ for signals where $s_i \neq s_j$ could influence the expected production cost of a firm, but not its bidding behaviour. This would be different if each producer submitted an offer with multiple offer prices or even a continuous supply function as in Vives (2011), so that a producer could indirectly condition its output on the competitor's information.

Costs that are common knowledge constitute a special case of the limit where firms' marginal costs are insensitive to common variations in signals, as in (14). If costs are common knowledge, the signals only serve the purpose of coordinating producers' actions as in a correlated equilibrium (Osborne and Rubinstein, 1994). If, in addition, signals are independent as in (16), signals effectively become randomization devices of a mixed-strategy NE. To illustrate this, signals could be transformed from s to $P = p(s)$, i.e. a signal that directly gives the offer price that a firm should choose. The price signal has the probability distribution $G(P) = F(p^{-1}(P))$. If we rewrite (16), we get that

$$G(P) = \frac{q_H}{q_H - q_L} - \frac{\overline{p} - c}{P - c} \frac{q_L}{q_H - q_L}. \tag{18}$$

This probability distribution of offer prices corresponds to the mixed-strategy NE that is calculated for discriminatory auctions by Fabra et al. (2006). This confirms Harsanyi's (1973) purification theorem that a mixed-strategy NE is equivalent to a pure-strategy Bayesian NE, where costs are common knowledge and signals are independent.

Only the lowest offer price is accepted when $\widetilde{q}_i > D$ for both producers and all outcomes, so that producers are non-pivotal with certainty as in Case 3. In this special case, there is no difference between our discriminatory and uniform-price auctions, because the winning offer sets its own price also in the uniform-price auction. This case also corresponds to the first-price single-object auction, which is analysed by Milgrom and Weber (1982). As in Milgrom and Weber (1982), private information normally gives an informational rent, so if costs are asymmetric information, then non-pivotal bidders also have a positive mark-up. However, mark-ups are zero in the limit when production costs are insensitive to common variations in signals, even if non-pivotal producers have private information for outcomes where signals differ. For the special case where costs are common knowledge, this result concurs with von der Fehr and Harbord (1993) and Fabra et al. (2006), where mark-ups are zero in auctions with both uniform and discriminatory pricing. Recall that our discriminatory auction is identical to the Bertrand model, so our results also apply to the Bertrand game.

4.2 Uniform-pricing

As mentioned earlier, Case 3 in Proposition 1 also applies to producers that are non-pivotal with certainty in a uniform-price auction. Now, we consider the other extreme where producers are pivotal with certainty in a uniform-price auction. As in the discriminatory auction, we solve for a symmetric equilibrium. Later, we will consider the general case where the pivotal status of producers is uncertain, in which case the unique equilibrium is symmetric.

The highest offer sets the market price in a uniform-price auction when producers are pivotal with certainty. The demand and production capacity uncertainties are independent of the signals and cost uncertainties. Thus, when producers are pivotal with certainty, the expected profit of firm i when receiving signal s_i is:

$$\begin{array}{c}\pi_i(s_i) = \mathbb{E}\left[p_j - c_i(s_i,s_j)\right| p_j \geq p_i] \Pr\left(p_j \geq p_i\right| s_i)\, q_H \\ + \left(p_i(s_i) - \mathbb{E}\left[c_i(s_i,s_j)\right| p_j \leq p_i]\right)\left(1 - \Pr\left(p_j \geq p_i\right| s_i)\right) q_L.\end{array} \tag{19}$$

Lemma 2 *In a uniform-price auction with producers that are pivotal with certainty, we have:*

$$\begin{array}{c}\frac{\partial \pi_i(s_i)}{\partial p_i} = \left(1 - \Pr\left(p_j \geq p_i\right| s_i)\right) q_L \\ + \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)(q_H - q_L)\end{array} \tag{20}$$

whenever $p_i(s_i)$ and $p_j(s_j)$ are locally differentiable and locally invertible for signals that have offer prices around $p_i(s_i)$.

The first-order condition for the uniform-price auction is similar to the first-order condition for the discriminatory auction in Lemma 1, but there is one difference. In contrast to the discriminatory auction, the lowest bidder does not gain anything from increasing its offer price in a uniform-price auction when producers are pivotal with certainty. Thus, the price effect has one term less in the uniform-price auction, which reduces the price effect. There is a corresponding change in the H function which is proportional to the quantity effect and inversely proportional to the price effect.

$$\hat{H}(s) = \frac{(q_H - q_L)\,\chi(s,s)}{q_L \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j}. \tag{21}$$

Proposition 2 *The symmetric Bayesian Nash equilibrium offer in a uniform-price auction where producers are pivotal with certainty is given by*

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \hat{H}(u)du} dv \tag{22}$$

for $s \in [\underline{s}, \overline{s})$ if signals are negatively affiliated. The symmetric equilibrium exists for more general probability distributions when $\frac{dc(v,v)}{dv}$ is large for $v < \overline{s}$. The equilibrium is without welfare losses and has the property that $p'(s) > 0$. The expression can be simplified for the following circumstances:

1. In the limit when production costs are insensitive to common variations in signals, (22) can be simplified to:

$$p(s) = c(\underline{s}, \underline{s}) + e^{-\int_s^{\overline{s}} \hat{H}(u)du} (\overline{p} - c(\underline{s}, \underline{s})). \tag{23}$$

2. Independent signals are negatively affiliated. In this case, (22) simplifies to:

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} \left(\frac{F(s)}{F(v)} \right)^{\frac{(q_H - q_L)}{q_L}} dv. \tag{24}$$

In the limit when production costs are insensitive to common variations in signals, then (24) can be further simplified to

$$p(s) = c(\underline{s}, \underline{s}) + (F(s))^{\frac{(q_H - q_L)}{q_L}} (\overline{p} - c(\underline{s}, \underline{s})). \tag{25}$$

Equation (22) has properties similar to the corresponding expressions for the discriminatory auction in Proposition 1. But the ratio of the quantity and price effects differs. It follows from Definition 2 and (21) that $\hat{H}(s) > H^*(s)$ or, equivalently, that the price effect is relatively smaller in the uniform price auction as compared to a discriminatory auction. Thus, producers make offers with lower mark-ups in uniform-price auctions. On the other hand, in a uniform-price auction, the losing producer with the highest offer price sets the transaction price for both accepted offers, so in the end it is not self-evident that a uniform-price auction would lower the procurement cost of an auctioneer. We will analyse this further in Section 4.4.

We can use an argument similar to the one we used for the discriminatory auction to show that the limit result in (25) corresponds to the mixed-strategy NE that is derived for uniform-price auctions by von der Fehr and Harbord (1993). (25) can also be used to calculate the expected clearing price.

Proposition 3 *If the signals are independent, the production costs are insensitive to common variations in signals, and producers are pivotal with certainty, then the expected market price in the uniform-price auction is given by:*

$$\overline{p} - \frac{(\overline{p} - c)(q_H - q_L)}{q_H + q_L},$$

where $c = c(\underline{s}, \underline{s})$.

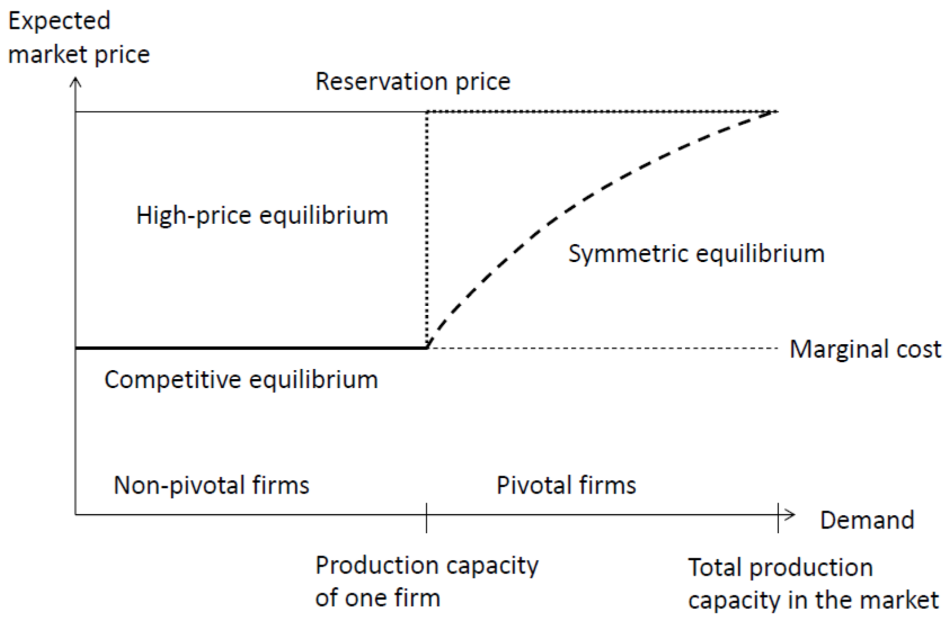

Figure 1: Comparative statics analysis for our symmetric equilibrium and von der Fehr and Harbord's (1993) asymmetric high-price equilibrium in a uniform-price auction where producers have a certain pivotal status, costs are common knowledge and signals are indepedent.

In the special case with certain demand and certain production capacities that are pivotal, we have $q_H = \widetilde{q}$ and $q_L = D - \widetilde{q} > 0$, so that the expected market price is given by

$$\overline{p} - \frac{(\overline{p} - c)(2\widetilde{q} - D)}{D}. \qquad (26)$$

Figure 1 plots this relationship, which gives a comparative statics analysis of the expected transaction price with respect to a certain demand level. The expected market price increases continuously as demand increases and it does not reach the reservation price until demand equals the total production capacity in the market. With more firms in the market, the expected price in our model would stay near the marginal cost until demand is near the total production capacity in the market, where the expected price will take off towards the reservation price. This would be reminiscent of what is often called "hockey-stick pricing" that is typical for wholesale electricity markets (Hurlbut et al., 2004; Holmberg and Newbery, 2010). In Figure 1, we also plot the high-price equilibrium in von der Fehr and Harbord (1993). In this equilibrium, the market price jumps directly from the competitive price with zero mark-ups up to the reservation price when demand increases at the critical point where producers switch from being non-pivotal to being pivotal with certainty in a uniform-price auction.

In the comparative statics analysis in Figure 1, where costs common knowledge, the expected transaction price is continuous at the point where producers switch from being non-pivotal to pivotal. This is not the case for uncertain costs. In the special case where producers are just pivotal with certainty, so that $q_L \searrow 0$, then it follows from Proposition 2 that $p(s) = c(s, s)$. This corresponds to Milgrom and Weber's (1982) results for second-price sales auctions, because the lowest bidder gets to produce (almost) the whole demand while the highest bidder sets the uniform market price. When comparing this to Case 3 in Proposition 1, which also applies to uniform-price auctions with non-pivotal producers, we note that the comparative statics analysis of our symmetric equilibrium has a discontinuity at the critical point where producers' capacities switch from being nonpivotal with certainty to being pivotal with certainty. Somewhat counter-intuitively, offer prices decrease at this critical point, even if demand increases. The reason for this is that the offer that sets the market price also switches at this point, which drastically changes the bidding behaviour. Just pivotal firms bid truthfully as in a second-price auction while non-pivotal firms set their own price and use similar bidding strategies as in a first-price procurement auction, i.e. firms' mark-ups are strictly positive for uncertain costs. The following proves that the producer's revenues can also shift downwards at the critical point where producers' capacities switch from being nonpivotal with certainty to being pivotal with certainty.

Proposition 4 *If producers' signals are strictly affiliated, then the expected payoff of the auctioneer is strictly larger for just pivotal producers than for producers that are just non-pivotal with certainty in markets with uniform pricing and a symmetric equilibrium.*

4.2.1 Uncertain pivotal status

In the general case, the pivotal status of producers is uncertain when offers are submitted. In this case, all offers are price-setting with some probability, which will ensure a unique equilibrium. In particular, the high-price equilibrium does not exist when the pivotal status of producers is uncertain.[8] Unlike the discriminatory auction, allowing for uncertain pivotal status makes the analysis of the uniform-price auction more complicated. The problem is that the lowest bidder, which has the highest output, would set its own transaction price, as in a discriminatory auction, for outcomes when the highest bidder is non-pivotal, while the highest bidder would set the transaction price of the lowest bidder when the highest bidder is pivotal. Thus, unlike the discriminatory auction, the payoff of the winning producer generally depends on the probability that the highest bidder is non-pivotal. We denote this probability by Π^{NP}. Demand and production capacities are independent of the signals, so the pivotal status of producers is also independent of signals.

[8] Uncertain pivotal status implies that the lowest bidder will set its transaction price with a positive probability. As shown by von der Fehr and Harbord (1993), this implies that the lowest bidder would find it optimal to choose an offer just below the high-price offer at the reservation price. But this means that the high-price bidder, in its turn, would find it optimal to deviate and slightly undercut the low-price bidder.

Lemma 3 *In a uniform-price auction, where $p_i(s_i)$ and $p_j(s_j)$ are locally differentiable and locally invertible, we have:*

$$\begin{array}{c} \frac{\partial \pi_i(s_i)}{\partial p_i} = \Pr\left(p_j \geq p_i|\, s_i\right) q_H^{NP} \Pi^{NP} + \left(1 - \Pr\left(p_j \geq p_i|\, s_i\right)\right) q_L \\ + \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)(q_H - q_L), \end{array} \quad (27)$$

where

$$q_H^{NP} = \mathbb{E}\left[\widetilde{q}_H |\, \widetilde{q}_H \geq D\right].$$

Thus, the quantity effect is similar, as when producers are pivotal with certainty. But the price effect depends on the probability that the highest bidder is non-pivotal. Increasing an offer price contributes to the price effect when a producer's offer is price-setting, i.e. when the producer is pivotal and has the highest offer price or when the producer has the lowest offer price and the highest bidder is non-pivotal. The function $\hat{H}(s)$ generalizes as follows:

Definition 3

$$\hat{H}(s) = \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\overline{s}} \chi(s,s_j)\, ds_j q_H^{NP} \Pi^{NP} + \int_{\underline{s}}^s \chi(s,s_j)\, ds_j q_L}.$$

Proposition 5 *If the pivotal status is uncertain and*

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s,s_j)\, ds_j q_H^{NP} \Pi^{NP} + q_L \int_{\underline{s}}^x \chi(s,s_j)\, ds_j}{\chi(s,x)}\right) \geq 0 \quad (28)$$

for all $s, x \in (\underline{s}, \overline{s})$, then there is a unique Bayesian NE in the uniform-price auction. There is a unique equilibrium for more general probability distributions when $\frac{dc(v,v)}{dv}$ is large for $v < \overline{s}$. The unique equilibrium is symmetric, without welfare losses, and has the property that $p'(s) > 0$, where:

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \hat{H}(u)du} dv, \quad (29)$$

for $s \in [\underline{s}, \overline{s})$. The expression can be simplified for the following circumstances:

1. In the limit when production costs are insensitive to common variations in signals, (29) can be simplified to:

$$p(s) = c(\underline{s}, \underline{s}) + e^{-\int_s^{\overline{s}} \hat{H}(u)du} \left(\overline{p} - c(\underline{s}, \underline{s})\right). \quad (30)$$

2. Independent signals satisfy the condition in (28), in which case (29) simplifies to:

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} \left(\frac{(1 - F(v))\, q_H^{NP} \Pi^{NP} + F(v)\, q_L}{(1 - F(s))\, q_H^{NP} \Pi^{NP} + F(s)\, q_L}\right)^{\frac{(q_H - q_L)}{q_H^{NP} \Pi^{NP} - q_L}} dv. \quad (31)$$

If, in addition to independent signals, production costs are insensitive to common variations in signals, then (31) can be simplified to

$$p(s) = c(\underline{s}, \underline{s}) + \left(\frac{q_L}{((1 - F(s)) q_H^{NP} \Pi^{NP} + F(s) q_L)} \right)^{\frac{(q_H - q_L)}{q_H^{NP} \Pi^{NP} - q_L}} (\overline{p} - c(\underline{s}, \underline{s})) . \tag{32}$$

As in the pivotal case, the price effect is smaller in the uniform-price auction as compared to the discriminatory auction. Thus, offers are lower, but transaction prices are set differently and could still be higher on average. Similar to the discriminatory auction, the mark-ups are lower when $\widetilde{q}$ increases, as this increases the quantity effect. We note that as Π^{NP} increases towards 1, so that producers are more likely to be non-pivotal, the bidding behaviour in the uniform-price auction gets closer to offers in the discriminatory auction, which concurs with our discussion in Section 4.1. At the other extreme, when Π^{NP} decreases towards 0, bidding gets closer to the uniform-price auction with producers that are pivotal with certainty. For a given q_H^{NP}, producers will increase their offer prices when Π^{NP} increases. This may seem counterintuitive, but this is to compensate for the fact that there is a higher risk that the market price is set by the lowest offer price rather than the highest offer price.

4.3 Transparency improves auction performance

We will now use Proposition 1 and Proposition 5 to draw conclusions about the influence from the information structure. It is useful to normalize the signals and cost functions when comparing different information structures.

Definition 4 *We say that signals and cost functions have been normalized if* $c(s, s) = s$.

We have by assumption that $\frac{dc_i(s,s)}{ds} > 0$, so any signal s and cost function $c(s, s)$ can be normalized by the transformations $\widetilde{s}_i = c(s_i, s_i)$, $\widetilde{s}_j = c(s_j, s_j)$ and $\widetilde{c}(\widetilde{s}_i, \widetilde{s}_j) = c(s_i, s_j)$.

Definition 5 *For normalized signals and cost functions, we say that two pairs of probability density functions and marginal cost functions,* $\left\{\chi^A(s_i, s_j), c^A(s_i, s_j)\right\}$ *and* $\left\{\chi^B(s_i, s_j), c^B(s_i, s_j)\right\}$*, are equivalent if the two pairs have the same marginal distribution of normalized signals and the same joint distribution of marginal costs.*

It follows from Definitions 2 and 3 that $H^*(u)$ and $\hat{H}(s)$ both increase when the density at $\chi(s, s)$ increases relative to both $\int_{\underline{s}}^{s} \chi(s, s_j) ds_j$ and $\int_{s}^{\overline{s}} \chi(s, s_j) ds_j$. The reason is simply that the quantity effect of increasing one's offer price increases if, conditional on the reception of a signal s, it becomes more likely that the competitor receives a similar signal s and chooses a similar offer price. Thus, we can conclude from Propositions 1 and 5 that

Corollary 1 *Consider two auctions A and B with identical payment schemes (uniform or discriminatory pricing). Producers have lower mark-ups in auction A, characterized by the normalized pair* $\left\{\chi^A(s_i, s_j), c^A(s_i, s_j)\right\}$*, in comparison to auction B, characterized by the normalized pair* $\left\{\chi^B(s_i, s_j), c^B(s_i, s_j)\right\}$*, if the two pairs are equivalent and if the signals are more likely to be similar in auction A in the sense that normalized signals in auction A have a relatively higher probability density for identical signals, so that*

$$\frac{\chi^A(s,s)}{\int_{\underline{s}}^{s} \chi^A(s,s_j)\,ds_j} > \frac{\chi^B(s,s)}{\int_{\underline{s}}^{s} \chi^B(s,s_j)\,ds_j}$$

and

$$\frac{\chi^A(s,s)}{\int_{s}^{\overline{s}} \chi^A(s,s_j)\,ds_j} > \frac{\chi^B(s,s)}{\int_{s}^{\overline{s}} \chi^B(s,s_j)\,ds_j}$$

for $s \in (\underline{s}, \overline{s})$.

In particular, if increased transparency makes signals more similar without changing the relevant properties of costs, then this will lower the mark-ups. Similarly, for hydro-dominated markets, mark-ups would also decrease for increased political and regulatory transparency if the result is that producers are more likely to observe similar signals.

4.4 Ranking of auction formats

We already know from Section 4.1 that the two auction formats are equivalent in the non-pivotal case. Below we show that there are cases where the two auction formats are equivalent also when producers are pivotal with a positive probability, so that $q_L > 0$.

Lemma 4 *If the signals are independent and the costs are common knowledge, the expected profit for a producer is given by*

$$\pi(s) = q_L\left(\overline{p} - c(\underline{s}, \underline{s})\right), \tag{33}$$

for both the uniform-price and the discriminatory auction and irrespective of the probability that the highest bidder is pivotal.

The equivalence result implies that the comparative statics analysis of the symmetric equilibrium in Figure 1 also applies to the expected transaction price in the discriminatory auction. There is a simple intuition for this equivalence result. If the signals are independent and the costs are common knowledge, then our Bayesian NE corresponds to a mixed-strategy NE. In a mixed-strategy NE, a producer gets the same expected payoff for any price that is chosen with a positive probability in equilibrium. Thus, irrespective of the auction format, the expected payoff can be calculated from the case when a producer chooses a price near the reservation price and is almost surely undercut by the competitor, which gives the payoff in (33). The proposition below generalizes the equivalence result to uncertain costs.

Proposition 6 *If the signals are independent, then the expected profit for a producer is the same for the uniform-price and the discriminatory auction and independent of the probability that the highest bidder is pivotal.*

As compared to independent signals, it follows from Corollary 1 that the markups in both auctions will decrease if producers are more likely to receive similar information. If a producer's signal s becomes more informative of the competitor's signal as s increases, so that $\int_s^{\overline{s}} \chi(s, s_j)\, ds_j$ decreases relative to $\int_{\underline{s}}^{s} \chi(s, s_j)\, ds_j$, then it follows from Definitions 2 and 3 that $H^*(s)$ will tend to increase relative to $\hat{H}(s)$, and it follows from Propositions 1 and 5 that this would make discriminatory pricing relatively more attractive for an auctioneer. It would be the other way around if a higher signal was instead less informative of the competitor's signal. It also follows from Definitions 2 and 3 that advantages and disadvantages of uniform-pricing tend to increase if producers are pivotal with a higher probability, i.e. Π^{NP} decreases, for fixed q_L and q_H.

5 Concluding discussion

We consider a duopoly model of a divisible-good procurement auction with production uncertainty, such as a wholesale electricity market. Each producer receives a private signal with imperfect cost information from a bivariate probability distribution (known to each producer) and then chooses one offer price for its whole production capacity. The demand and production capacities could also be uncertain. A producer is pivotal when the realized capacity of the competitor is smaller than realized demand. Marginal costs are flat (independent of output). We assume that the bidding format has the constraint that offers must also be flat.

The bid constraint facilitates uniqueness of equilibria. There is a unique Bayesian NE, which is symmetric, in the discriminatory auction. The uniform-price auction has a unique equilibrium, which is symmetric, when the pivotal status of producers is uncertain. The bid constraint also reduces production inefficiencies and ensures that welfare losses can be avoided. The bid constraint mitigates Vives (2011) highly anti-competitive outcomes for uniform-price auctions where costs have large common uncertainties. Costs do often have relatively large common uncertainties in forward markets and in hydro-dominated electricity markets. In such markets, the marginal costs are also approximately flat for a wide range of outputs. This indicates that it should be optimal for welfare and from the auctioneer's perspective to limit the number of allowed steps in producers' supply schedules in uniform-price auctions of forward contracts and hydro-dominated electricity markets. Alternatively, as is often the case in practice, the auctioneer could also avoid uniform-price auctions when trading forward contracts long before delivery.

In expectation, uniform and discriminatory pricing are equivalent when the signals are independent. Consumers of electricity tend to favour discriminatory pricing when a higher signal of a producer is more informative of the competitor's signal. The opposite is true when a higher signal of a producer is less informative of the competitor's signal. Advantages and disadvantages of uniform pricing tend to be amplified if producers are pivotal with a higher probability. We show that equilibrium offers in a discriminatory auction are determined by the expected sales of the producer with the highest and lowest offer price, respectively. The variance of these sales – due to demand shocks, production outages and volatile renewable production – will not influence the bidding behaviour of producers. Bidding in the uniform-price auction is also insensitive to this variance, as long as it is not sufficiently large to occasionally change the pivotal status of at least one producer. Moreover, for given expected sales and independent signals, the probability that a producer is pivotal in a uniform-price auction does not influence the expected payoffs. For strictly affiliated signals and certain demand in a uniform-price auction, a comparative statics analysis of our equilibrium has, somewhat counter-intuitively, a discontinuous decrease in producers' payoffs if there is a small increase in demand, such that producers switch from being non-pivotal to pivotal with certainty.

The markups fall in both auction formats if producers are more likely to receive similar information. This concurs with a related result in Vives (2011), which shows that less informational noise makes uniform-price auctions more competitive, and with the linkage principle for single object auctions in Milgrom and Weber (1982). Taken together, these results support the measures taken by the European Commission to increase the transparency in European wholesale electricity markets. However, disclosure of information is only beneficial up to a point. A pivotal producer can deviate to the reservation price, which ensures it a minimum profit. Moreover, in a repeated game, there is a risk that increased transparency will facilitate tacit collusion as argued by von der Fehr (2013).

We are concerned that cost uncertainty and asymmetric information could result in significant mark-ups in hydro dominated electricity markets with scarce water. This could help explain the extraordinarily high price-periods that typically accompany scarcity of water in such markets. One measure that could mitigate this is to clearly define contingency plans for intervention by the market operator and the regulator under extreme system conditions. In hydro-dominated markets, improved political transparency has similar pro-competitive effects as improved market transparency.

The results are analogous for multi-unit sales auctions, such as security auctions. In particular, given that bidders' marginal valuation of financial instruments should be approximately flat and bidders' valuations of securities typically have large common uncertainties, we believe that it would be beneficial for welfare and the auctioneer that uniform-price auctions of securities or emission permits use a bidding format that significantly restricts the number of steps in the bid-schedules. Purchase constraints in sales auctions increase the probability that bidders are pivotal and make them pivotal by a wider margin. This results in less competitive outcomes, at least in a one-shot game. On the other hand, purchase constraints may improve the competitiveness of secondary markets.

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Appendix

We start the Appendix by proving equilibrium properties that will be useful when proving uniqueness and symmetry of Bayesian NE in auctions with discriminatory or uniform-pricing. In Appendix B, we prove some relationships for conditional probabilities and conditional expected values that will be used when solving for equilibria in the two auctions. In Appendix C, we prove results for the discriminatory auction. Results for the uniform-price auction are proven in Appendix D and payoffs in the two auctions are compared in Appendix E.

Appendix A: Uniqueness and symmetry of the equilibrium

We first introduce the following definitions:

Definition 6

1. We say that $p_i(s_i)$ is sometimes price-setting if, conditional on that producer i receiving the signal $s_i \in [\underline{s}, \overline{s}]$, there is a strictly positive probability that

producer i has a strictly positive output and is paid the transaction price $p_i(s_i)$.

2. We say that firm i has an accumulation of offers at p if there is a range of signals (s_1, s_2), such that $p_i(s_i) = p$ for $s_i \in (s_1, s_2)$.

Lemma 5 *Consider a Bayesian NE in a uniform-price or discriminatory auction where producer i has the strategy $p_i(s_i)$ for $s_i \in [\underline{s}, \overline{s}]$. The following equilibrium properties can be proven:*

1. Firm i cannot have a sometimes price-setting offer $p_i(s_i) \in (p_0, p_1)$ if the competitor j does not have any offer in the range (p_0, p_1) for any signal $s_j \in [\underline{s}, \overline{s}]$. Similarly, firm i cannot have a sometimes price-setting offer $p_i(s_i) \in [p_0, p_1)$ if firm j does not have any offer in the range (p_0, p_1) for any signal $s_j \in [\underline{s}, \overline{s}]$ and firm j does not have an accumulation of offers at p_0.

2. If firm j has an accumulation of offers at p_0 for signals $s_j \in (s_1, s_2)$, then there is no signal $s_i \in [\underline{s}, \overline{s}]$ such that: $p_i(s_i) = p_0 > c_i(s_i, s_2)$.

3. If the lowest offer that can occur for any producer in equilibrium is sometimes price-setting, then firms must have the same strategy when receiving the lowest signal, i.e. $p_i(\underline{s}) = p_j(\underline{s})$.

4. Assume that firm i has an offer $p_i(s)$ for signal s which is sometimes price-setting and such that: $p_i(s) = p_j(s)$, $p_i(\check{s}) < p_i(s)$ and $p_j(\check{s}) < p_j(s)$ for any existing $\check{s} < s$, then there is no accumulation of offers at the price $p_i(s)$.

Proof. 1) Make the contradictory assumption that the statement is true. Firm i can then increase the offer for signal s_i up to a price $p \in (p_i(s_i), p_1)$. Such a change will never change the output of producer i for the stated circumstances, but it will sometimes increase the revenue of firm i (whenever $p_i(s_i)$ is price-setting), so the deviation is strictly profitable.

2) Make the contradictory assumption that the statement is true. Firm i can then reduce its offer price $p_i(s_i)$ by an arbitrarily small amount $\varepsilon > 0$. Due to properties of the assumed rationing rule, such a deviation will for the signal s_i increase the output of firm i by an amount that is bounded away from zero whenever the competitor receives a signal s_j in the range (s_1, s_2). The condition $p_0 > c_i(s_i, s_2) \geq c_i(s_i, s_j)$ for $s_j \leq s_2$ ensures that it is profitable for firm i to increase its output for those circumstances. Thus, the deviation is profitable for sufficiently small ε, so that any resulting reductions in the transaction price of firm i become sufficiently small.

3) Weak monotonicity of $p_i(s_i)$ and $p_j(s_j)$ imply that firm i has no offer below $p_i(\underline{s})$ and that firm j has no offer below $p_j(\underline{s})$. Make the contradictory assumption that $p_j(\underline{s}) \neq p_i(\underline{s})$. Without loss of generality, we assume that $p_i(\underline{s}) < p_j(\underline{s})$, where $p_i(\underline{s})$ is sometimes price-setting. However, it follows directly from 1) that this is not possible in equilibrium.

4) Make the contradictory assumption that at least one firm j has an accumulation of offers at the price $p_j(s)$ for signals $s_j \in [s, s_2]$. Without loss of generality,

we assume that firm j has the (weakly) largest accumulation of offers at $p_j(s)$, i.e. there is no signal $\hat{s} > s_2$ such that $p_i(\hat{s}) = p_j(s)$. It follows from 2) that $p_i(s) = p_j(s) = p_j(s_2) \leq c_i(s, s_2) = c_j(s, s_2)$, where the latter equality follows from symmetry of costs.[9] It also follows that $p_j(s_2) \leq c_j(s, s_2) < c_j(s_2, s)$, because as assumed in (5), a firm is strictly more sensitive to changes in its own signal as compared to changes in the competitor's signal. Thus, we have from (3) that $p_j(s_2) < c_j(s_2, s_i)$ for $s_i \in [s, \overline{s}]$. But this would imply that when receiving signal s_2, firm j would have a strictly negative payoff whenever $p_j(s_2)$ is price-setting. Thus, firm j can increase its payoff by increasing $p_j(s_2)$. ■

We can use the technical results above to prove the following, which will be useful when proving uniqueness and symmetry of equilibria for both auctions.

Lemma 6 *Consider an auction with uniform-pricing where producers are non-pivotal with a positive probability or a discriminatory auction. Assume that the necessary first-order conditions of offers from producers i and j have the symmetry property that $p_i'(s) = p_j'(s)$ whenever $p_i(s) = p_j(s) = p$ and there is no accumulation of offers at p. For such a first-order condition, any existing Bayesian NE in the considered auction must be unique and symmetric. Moreover, the unique symmetric equilibrium offer $p_i(s)$ must be invertible.*

Proof. Assume that the considered auction has an equilibrium. The lowest offer that can occur in the equilibrium is at least partly accepted with a positive probability. We consider an auction with either discriminatory or uniform pricing. In the latter case, producers are non-pivotal with a positive probability. Thus, the lowest offer is sometimes price-setting in the considered auction. Hence, it follows from 3) in Lemma 5 that $p_i(\underline{s}) = p_j(\underline{s})$. 1) and 4) ensure that there are no discontinuities in $p_i(s_i)$ at $\underline{s}$ and no accumulation of offers at $p_i(\underline{s})$. Let s^* be the highest signal in the range $[\underline{s}, \overline{s}]$, such that no producer has an accumulation of offers or a discontinuity in its offer function for $s < s^*$. Thus, the assumed symmetry property of the first-order condition and piece-wise differentiability of $p_i(s)$ and $p_j(s)$ ensure that $p_i'(s) = p_j'(s)$ for the range of signals $(\underline{s}, s^*)$. The symmetry of the initial condition $p_i(\underline{s}) = p_j(\underline{s})$ and the symmetry of slopes $p_i'(s)$ imply that $p_i(s^*) = p_j(s^*)$. Moreover, offers $p_i(s^*) = p_j(s^*)$ are not undercut with certainty by the other firm and are therefore sometimes price-setting if $s^* < \overline{s}$. Thus, we can use 1) and 4) to rule out cases where $s^* < \overline{s}$. Uniqueness follows from the assumption that $\overline{p} = c_i(\overline{s}, \overline{s})$, which ensures that $p_i(\overline{s}) = \overline{p}$ for a symmetric equilibrium, even if producers are non-pivotal. Finally, we note that weak-monotonicity of $p(s)$ combined with no accumulation of offers implies that $p(s)$ must be piece-wise strictly monotonic, and therefore invertible for any Bayesian NE. ■

Compared to discriminatory pricing, we need a stricter sufficient condition to ensure uniqueness in the uniform-price auction: producers need to be non-pivotal with a positive probability. The reason is that an offer may sometimes be

[9] Recall that we use the convention that a firm's own costs are always first in the list of signals.

accepted in a uniform-price auction, even if it is never price-setting, as in the high-price equilibrium by von der Fehr and Harbord (1993). In the above uniqueness argument, we use Milgrom and Weber's (1982) assumption that $\overline{p} = c_i(\overline{s}, \overline{s})$. This assumption is crucial when ensuring uniqueness in an auction where suppliers are non-pivotal with certainty, as in a single object auction. However, if the pivotal status of suppliers is uncertain, then the uniqueness result would also hold for $\overline{p} > c_i(\overline{s}, \overline{s})$.

Appendix B: Relationships for conditional probabilities

Before proving the lemmas and propositions that have been presented in the main text, we will derive some results that will be used throughout these proofs. By assumption, $p_j(s_j)$ is monotonic and invertible. Thus, we get

$$\begin{aligned} \Pr\left(p_j \geq p_i \middle| s_i\right) &= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j}{\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j} \\ \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} &= \frac{-p_j^{-1\prime}(p_i)\chi\left(s_i,p_j^{-1}(p_i)\right)}{\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j}, \end{aligned} \tag{34}$$

where the last result follows from Leibniz' rule. The above results and Leibniz' rule are used in the following derivations.

$$\begin{aligned} \mathbb{E}\left[c_i(s_i,s_j)\middle| p_j \geq p_i\right] &= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j} = \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\Pr(p_j \geq p_i|s_i)\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j} \\ \frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i} &= \frac{p_j^{-1\prime}(p_i)\chi\left(s_i,p_j^{-1}(p_i)\right)\int_{p_j^{-1}(p_i)}^{\overline{s}}\left(c_i(s_i,s_j)-c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\chi(s_i,s_j)ds_j}{\left(\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j\right)^2} \\ &= \frac{-\frac{\partial \Pr\left(p_j \geq p_i\middle|s_i\right)}{\partial p_i}\int_{p_j^{-1}(p_i)}^{\overline{s}}\left(c_i(s_i,s_j)-c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\chi(s_i,s_j)ds_j}{(\Pr(p_j \geq p_i|s_i))^2\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j}. \end{aligned} \tag{35}$$

From (34) and (35), we have that:

$$\begin{aligned} &-\frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i}\Pr\left(p_j \geq p_i\middle| s_i\right) - \mathbb{E}\left[c_i(s_i,s_j)\middle| p_j \geq p_i\right]\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ &= \left(\frac{\int_{p_j^{-1}(p_i)}^{\overline{s}}\left(c_i(s_i,s_j)-c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j} - \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j}\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ &= -\frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i\left(s_i,p_j^{-1}(p_i)\right)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j}\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ &= -c_i\left(s_i,p_j^{-1}(p_i)\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}. \end{aligned} \tag{36}$$

Using the above equation, we can derive the following result:

$$\begin{aligned} &\left(1 - \frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i}\right)\Pr\left(p_j \geq p_i\middle| s_i\right) + \left(p_i - \mathbb{E}\left[c_i(s_i,s_j)\middle| p_j \geq p_i\right]\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ &= \Pr\left(p_j \geq p_i\middle| s_i\right) + \left(p_i - c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}. \end{aligned} \tag{37}$$

Similarly, from (34), we have that

$$\begin{aligned} 1-\Pr\left(p_j\geq p_i|\,s_i\right) &= \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)}\chi(s_i,s_j)ds_j}{\int_{\underline{s}}^{\overline{s}}\chi(s_i,s_j)ds_j} \\ \mathbb{E}\left[c_i\left(s_i,s_j\right)|\,p_j\leq p_i\right] &= \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)}c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\int_{\underline{s}}^{p_j^{-1}(p_i)}\chi(s_i,s_j)ds_j} = \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)}c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{(1-\Pr(p_j\geq p_i|s_i))\int_{\underline{s}}^{\overline{s}}\chi(s_i,s_j)ds_j} \\ \frac{\partial\mathbb{E}[c_i(s_i,s_j)|p_j\leq p_i]}{\partial p_i} &= \frac{p_j^{-1\prime}(p_i)\chi\left(s_i,p_j^{-1}(p_i)\right)\int_{\underline{s}}^{p_j^{-1}(p_i)}\left(c_i\left(s_i,p_j^{-1}(p_i)\right)-c_i(s_i,s_j)\right)\chi(s_i,s_j)ds_j}{\left(\int_{\underline{s}}^{p_j^{-1}(p_i)}\chi(s_i,s_j)ds_j\right)^2} \\ &= \frac{-\frac{\partial\Pr\left(p_j\geq p_i\left|s_i\right.\right)}{\partial p_i}\int_{\underline{s}}^{p_j^{-1}(p_i)}\left(c_i\left(s_i,p_j^{-1}(p_i)\right)-c_i(s_i,s_j)\right)\chi(s_i,s_j)ds_j}{(1-\Pr(p_j\geq p_i|s_i))^2\int_{\underline{s}}^{\overline{s}}\chi(s_i,s_j)ds_j}. \end{aligned} \tag{38}$$

It now follows from (38) that:

$$\begin{aligned} -\tfrac{\partial\mathbb{E}[c_i(s_i,s_j)|p_j\leq p_i]}{\partial p_i}\left(1-\Pr\left(p_j\geq p_i|\,s_i\right)\right)+\mathbb{E}\left[c_i\left(s_i,s_j\right)|\,p_j\leq p_i\right]\tfrac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i} \\ = \tfrac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}c_i\left(s_i,p_j^{-1}\left(p_i\right)\right). \end{aligned} \tag{39}$$

Appendix C: Discriminatory auction

Proof. (Lemma 1) It follows from (9) that

$$\begin{aligned} \tfrac{\partial\pi_i(s_i)}{\partial p_i} &= \left(1-\tfrac{\partial\mathbb{E}[c_i(s_i,s_j)|p_j\geq p_i]}{\partial p_i}\right)\Pr\left(p_j\geq p_i|\,s_i\right)q_H \\ &+\left(p_i-\mathbb{E}\left[c_i\left(s_i,s_j\right)|\,p_j\geq p_i\right]\right)\tfrac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}q_H \\ &+\left(1-\tfrac{\partial\mathbb{E}[c_i(s_i,s_j)|p_j\leq p_i]}{\partial p_i}\right)\left(1-\Pr\left(p_j\geq p_i|\,s_i\right)\right)q_L \\ &-\left(p_i-\mathbb{E}\left[c_i\left(s_i,s_j\right)|\,p_j\leq p_i\right]\right)\tfrac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}q_L. \end{aligned} \tag{40}$$

Using (37) and the relation in (39) yields:

$$\begin{aligned} \tfrac{\partial\pi_i(s_i)}{\partial p_i} &= \Pr\left(p_j\geq p_i|\,s_i\right)q_H+\left(p_i-c_i\left(s_i,p_j^{-1}\left(p_i\right)\right)\right)\tfrac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}q_H \\ &+c_i\left(s_i,p_j^{-1}\left(p_i\right)\right)\tfrac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}q_L \\ &+\left(1-\Pr\left(p_j\geq p_i|\,s_i\right)\right)q_L-p_i\tfrac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}q_L, \end{aligned}$$

which gives (10). ■

The following lemma is useful when deriving results for the non-pivotal case.

Lemma 7 *$e^{-\int_s^v H(u)du}>0$ for $\underline{s}\leq s<v<\overline{s}$ and $e^{-\int_s^{\overline{s}} H(u)du}=0$ for $\underline{s}\leq s<\overline{s}$.*

Proof. It follows from (17) that

$$H\left(u\right)=\frac{\chi\left(u,u\right)}{\int_u^{\overline{s}}\chi\left(u,s_j\right)ds_j}=-\frac{d}{du}\ln\left(\int_u^{\overline{s}}\chi\left(u,s_j\right)ds_j\right)+\frac{\int_u^{\overline{s}}\chi_1\left(u,s_j\right)ds_j}{\int_u^{\overline{s}}\chi\left(u,s_j\right)ds_j}. \tag{41}$$

The assumptions that we make for the joint probability density imply that $\frac{\int_u^{\overline{s}} \chi_1(u,s_j)ds_j}{\int_u^{\overline{s}} \chi(u,s_j)ds_j}$ is bounded. Thus, $e^{-\int_s^v H(u)du}$ is strictly positive, unless

$$
\begin{aligned}
e^{\left[\ln\left(\int_u^{\overline{s}} \chi(u,s_j)ds_j\right)\right]_s^v} &= e^{\ln\left(\int_v^{\overline{s}} \chi(v,s_j)ds_j\right)-\ln\left(\int_s^{\overline{s}} \chi(s,s_j)ds_j\right)} \\
&= \frac{\int_v^{\overline{s}} \chi(v,s_j)\,ds_j}{\int_s^{\overline{s}} \chi(s,s_j)\,ds_j}
\end{aligned}
$$

is equal to zero. This is the case if and only if $\int_v^{\overline{s}} \chi(v,s_j)\,ds_j = 0$. It follows from the assumptions that we make on the joint probability distribution that this is the case if and only if $v = \overline{s}$. ■

Proof. (Proposition 1) Consider a signal $s \in (\underline{s}, \overline{s})$. Assume that $p_i(s) = p_j(s) = p(s)$ and that there is no accumulation of offers at $p(s)$. Piece-wise differentiability, weak-monotonicity of $p_j(s)$ and no accumulation of offers at $p_j(s)$ implies that $p_j(s)$ must also be piece-wise strictly monotonic in some neighbourhood around s, and therefore invertible in that range. Thus, $p_j^{-1}(p_i) = s$. Hence, we get the following first-order condition from (10).

$$
\begin{aligned}
\tfrac{\partial \pi_i(s_i)}{\partial p_i} &= \Pr(p_j \geq p|\, s)\, q_H + (1 - \Pr(p \geq p|\, s))\, q_L \\
&+ (p - c_i(s,s))\, \tfrac{\partial \Pr(p_j \geq p|s)}{\partial p}\,(q_H - q_L) = 0.
\end{aligned}
$$

Using (34) and that $p_j^{-1\prime}(p_i) = \frac{1}{p_j'(s)}$, the condition can be written as follows:

$$
\int_s^{\overline{s}} \chi(s,s_j)\,ds_j q_H + \int_{\underline{s}}^{s} \chi(s,s_j)\,ds_j q_L - \frac{(p - c(s,s))}{p_j'(s)} \chi(s,s)(q_H - q_L) = 0.
$$

The condition is similar for both firms. Symmetry of the underlying parameters together with $\int_s^{\overline{s}} \chi(s,s_j)\,ds_j q_H > 0$ and $\int_{\underline{s}}^{s} \chi(s,s_j)\,ds_j q_L \geq 0$, ensures that $p_j'(s) = p_i'(s)$. Thus, it follows from Lemma 6 that any existing Bayesian NE must be symmetric and unique. Below we solve for this equilibrium.

We can use the definition in (11) to write the first-order condition on the following form:

$$
p'(s) - (p - c(s,s))\, H^*(s) = 0. \tag{42}
$$

Multiplication by the integrating factor $e^{\int_s^{\overline{s}} H^*(u)du}$ yields:

$$
\begin{aligned}
&p'(s)\, e^{\int_s^{\overline{s}} H^*(u)du} - pH^*(s)\, e^{\int_s^{\overline{s}} H^*(u)du} \\
&= -c(s,s)\, H^*(s)\, e^{\int_s^{\overline{s}} H^*(u)du},
\end{aligned}
$$

so that

$$
\frac{d}{ds}\left(p(s)\, e^{\int_s^{\overline{s}} H^*(u)du}\right) = -c(s,s)\, H^*(s)\, e^{\int_s^{\overline{s}} H^*(u)du}.
$$

Next we integrate both sides from s to $\overline{s}$.

$$
\begin{aligned}
\overline{p} - p(s)\, e^{\int_s^{\overline{s}} H^*(u)du} &= -\int_s^{\overline{s}} c(v,v)\, H^*(v)\, e^{\int_v^{\overline{s}} H^*(u)du}\, dv \\
p(s) &= \overline{p} e^{-\int_s^{\overline{s}} H^*(u)du} + \int_s^{\overline{s}} c(v,v)\, H^*(v)\, e^{-\int_s^{v} H^*(u)du} dv.
\end{aligned}
$$

We use integration by parts to rewrite the above expression as follows:

$$p(s) = \overline{p}e^{-\int_s^{\overline{s}} H^*(u)du} + \left[-c(v,v)\,e^{-\int_s^v H^*(u)du}\right]_s^{\overline{s}} + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v H^*(u)du} dv,$$

which gives (13), because $c(\overline{s},\overline{s}) = \overline{p}$. It is clear from (13) that $p > c(s,s)$ for $s \in [\underline{s},\overline{s})$. Hence, it follows from (42) that $p'(s) > 0$ for $s \in [\underline{s},\overline{s})$.

It remains to show that $p(s)$ is an equilibrium. It follows from (10) and (34) that

$$\begin{aligned}
\frac{\partial \pi_i(s)}{\partial p} &= \frac{\int_{p_j^{-1}(p)}^{\overline{s}} \chi(s,s_j)\,ds_j}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\,ds_j} q_H + \frac{\int_{\underline{s}}^{p_j^{-1}(p)} \chi(s,s_j)\,ds_j}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\,ds_j} q_L \\
&\quad - \frac{p_j^{-1\prime}(p)\,\chi\left(s,p_j^{-1}(p)\right)}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\,ds_j}\left(p - c_i\left(s,p_j^{-1}(p)\right)\right)(q_H - q_L). \\
\frac{\partial \pi_i(s)}{\partial p} &= \frac{\chi\left(s,p_j^{-1}(p)\right)}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\,ds_j}\left(\frac{\int_{p_j^{-1}(p)}^{\overline{s}} \chi(s,s_j)\,ds_j}{\chi\left(s,p_j^{-1}(p)\right)} q_H + \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi(s,s_j)\,ds_j}{\chi\left(s,p_j^{-1}(p)\right)} q_L \right. \\
&\quad \left. - p_j^{-1\prime}(p)\left(p - c_i\left(s,p_j^{-1}(p)\right)\right)(q_H - q_L)\right).
\end{aligned}$$

We know that $\frac{\partial \pi_i(s)}{\partial p} = 0$ for $s = p_j^{-1}(p)$. Thus, whenever $\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s,s_j)ds_j q_H + q_L \int_{\underline{s}}^x \chi(s,s_j)ds_j}{\chi(s,x)}\right) \geq 0$, it follows from the above and (3) that $\frac{\partial \pi_i(s)}{\partial p} > 0$ when $s > p_j^{-1}(p) \iff p < p_j(s)$ and that $\frac{\partial \pi_i(s)}{\partial p} < 0$ when $s < p_j^{-1}(p) \iff p > p_j(s)$. Thus, $p(s)$ globally maximizes the profit of firm i for any signal s when the inequality in (12) is satisfied.

In case 1) when costs are common knowledge, we have $\frac{dc(v,v)}{dv} \searrow 0$ for $v < \overline{s}$, so it follows from (13) that

$$p(s) \to c(\underline{s},\underline{s}) + e^{-\int_s^{\overline{s}} H^*(u)du} \int_s^{\overline{s}} \frac{dc(v,v)}{dv} dv,$$

which gives (14).

For independent signals in case 2, we have $\chi(s,s_j) = f(s)\,f(s_j)$, so the inequality

$$\begin{aligned}
&\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s,s_j)\,ds_j q_H + q_L \int_{\underline{s}}^x \chi(s,s_j)\,ds_j}{\chi(s,x)}\right) \\
&= \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s)\,f(s_j)\,ds_j q_H + q_L \int_{\underline{s}}^x f(s)\,f(s_j)\,ds_j}{f(s)\,f(x)}\right) = \\
&= \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s_j)\,ds_j q_H + q_L \int_{\underline{s}}^x f(s_j)\,ds_j}{f(x)}\right) = 0 \geq 0
\end{aligned}$$

is satisfied. Moreover, we have from Definition 2 that

$$
\begin{aligned}
H^*(s) &= \frac{f(s)(q_H - q_L)}{\int_s^{\overline{s}} f(s_j)\, ds_j q_H + \int_{\underline{s}}^{s} f(s_j)\, ds_j q_L} \\
&= -\frac{d}{ds} \ln \left(\int_s^{\overline{s}} f(s_j)\, ds_j q_H + \int_{\underline{s}}^{s} f(s_j)\, ds_j q_L \right).
\end{aligned}
$$

Thus, (13) can be written as in (15). If, in addition, the costs are insensitive to common variations in signals, so that $\frac{dc(v,v)}{dv} \searrow 0$ for $v < \overline{s}$, then (15) can be simplified to (16) as follows:

$$
\begin{aligned}
p(s) &= c(\underline{s}, \underline{s}) + \left(\frac{q_L}{((1 - F(s)) q_H + F(s) q_L)} \right) \int_s^{\overline{s}} \frac{dc(v,v)}{dv} dv \\
&= c(\underline{s}, \underline{s}) + \left(\frac{q_L}{((1 - F(s)) q_H + F(s) q_L)} \right) (\overline{p} - c(\underline{s}, \underline{s})).
\end{aligned}
$$

Producers are non-pivotal with certainty and $q_L = 0$ in case 3. Thus $H^*(s)$, simplifies to (17). For affiliated signals, we have $\frac{d}{ds}\left(\frac{\chi(s,s_j)}{\chi(s,x)}\right) \geq 0$ if $s_j \geq x$, which ensures that the global second-order condition in (12) is satisfied when $q_L = 0$. If, in addition, we have that the costs are insensitive to common variations of signals, then it follows from (14) and Lemma 7 that equilibrium offers are perfectly competitive for $s < \overline{s}$. ■

Appendix D: Uniform-price auction

The following derivations will be useful when analysing uniform-price auctions. It follows from (34) and Leibniz' rule that:

$$
\begin{aligned}
\mathbb{E}[p_j - c_i(s_i, s_j) | p_j \geq p_i] &= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} (p_j(s_j) - c_i(s_i,s_j)) \chi(s_i,s_j) ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j) ds_j} \\
&= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} (p_j(s_j) - c_i(s_i,s_j)) \chi(s_i,s_j) ds_j}{\Pr(p_j \geq p_i | s_i) \int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j) ds_j}
\end{aligned}
$$

$$
\frac{\partial \mathbb{E}[p_j - c_i(s_i,s_j) | p_j \geq p_i]}{\partial p_i} = \frac{-\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \int_{p_j^{-1}(p_i)}^{\overline{s}} \left(p_j(s_i) - c_i(s_i,s_j) - \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\right) \chi(s_i,s_j) ds_j}{(\Pr(p_j \geq p_i | s_i))^2 \int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j) ds_j}. \tag{43}
$$

Similar to (36), it can be shown that:

$$
\begin{aligned}
&\frac{\partial \mathbb{E}[p_j - c_i(s_i,s_j) | p_j \geq p_i]}{\partial p_i} \Pr(p_j \geq p_i | s_i) + \mathbb{E}[p_j - c_i(s_i, s_j) | p_j \geq p_i] \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} \\
&\quad = \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right).
\end{aligned} \tag{44}
$$

Proof. (Lemma 2) We have from (19) that

$$
\begin{aligned}
\frac{\partial \pi_i(s_i)}{\partial p_i} &= \frac{\partial \mathbb{E}[p_j - c_i(s_i,s_j) | p_j \geq p_i]}{\partial p_i} \Pr(p_j \geq p_i | s_i) q_H \\
&+ \mathbb{E}[p_j - c_i(s_i, s_j) | p_j \geq p_i] \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} q_H \\
&+ \left(1 - \frac{\partial \mathbb{E}[c_i(s_i,s_j) | p_j \leq p_i]}{\partial p_i}\right) (1 - \Pr(p_j \geq p_i | s_i)) q_L \\
&- (p_i - \mathbb{E}[c_i(s_i, s_j) | p_j \leq p_i]) \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} q_L.
\end{aligned} \tag{45}
$$

Next we use (39) and (44) to simplify this expression to (20). ■

Proof. (Proposition 2) Note that (20) is very similar to (10) and the statements can be proven in a very similar way to the proof of Proposition 1. In particular, it can be shown that the first-order condition is given by:

$$\int_{\underline{s}}^{s} \chi(s, s_j)\, ds_j q_L - \frac{(p - c(s,s))}{p'(s)} \chi(s,s)(q_H - q_L) = 0$$

$$p'(s) - p\hat{H}(s) = -c(s,s)\hat{H}(s).$$

The property of negatively affiliated signals in (2) implies that $\frac{d}{ds}\left(\frac{\int_{\underline{s}}^{x} \chi(s,s_j)ds_j}{\chi(s,x)}\right) \geq 0$ for $x > s_j$, which is sufficient to ensure global optimality. ■

Proof. (Proposition 4) In the non-pivotal case, the lowest offer price sets the market price and the winning producer (with the lowest offer price) gets to produce the entire demand, which corresponds to a first-price procurement auction. In the just pivotal case, the highest offer price sets the market price and the winning producer gets to produce the entire demand, which corresponds to a second-price auction. Thus, the statement follows from Milgrom and Weber (1982). ■

Proof. (Proposition 3) We let $G(P)$ be the probability that a producer's offer price is below P. This is the same as the probability that s is below $p^{-1}(P)$. Hence, it follows from (25) that

$$G(P) = \left(\frac{P - c}{\overline{p} - c}\right)^{\frac{q_L}{q_H - q_L}}.$$

From the theory of order statistics, we know that

$$G^2(P) = \left(\frac{P - c}{\overline{p} - c}\right)^{\frac{2q_L}{q_H - q_L}}$$

is the probability distribution of the highest offer price, which sets the price. Hence, the probability density of the market price is given by $2G(p)G'(p)$. Thus, the expected market price is given by:

$$\int_c^{\overline{p}} 2G(p)G'(p)\, p dp = \left[G^2(p)\, p\right]_c^{\overline{p}} - \int_c^{\overline{p}} G^2(p)\, dp$$

$$= \overline{p} - \left[\frac{(p - c)^{\frac{2q_L}{q_H - q_L}+1}}{\left(\frac{2q_L}{q_H - q_L} + 1\right)(\overline{p} - c)^{\frac{2q_L}{q_H - q_L}}}\right]_c^{\overline{p}} = \overline{p} - \frac{(\overline{p} - c)(q_H - q_L)}{q_H + q_L}.$$

■

Proof. (Lemma 3) The demand and production capacity uncertainties are independent of the signals and the cost uncertainties. Thus, the expected profit of firm i when receiving signal s_i is:

$$\begin{aligned}\pi_i(s_i) = {} & \mathbb{E}\left[p_j - c_i(s_i, s_j) \mid p_j \geq p_i\right] \Pr(p_j \geq p_i \mid s_i)\, q_H^P \left(1 - \Pi^{NP}\right) \\ & + \mathbb{E}\left[p_i(s_i) - c_i(s_i, s_j) \mid p_j \geq p_i\right] \Pr(p_j \geq p_i \mid s_i)\, q_H^{NP} \Pi^{NP} \\ & + \left(p_i(s_i) - \mathbb{E}\left[c_i(s_i, s_j) \mid p_j \leq p_i\right]\right)\left(1 - \Pr(p_j \geq p_i \mid s_i)\right) q_L,\end{aligned} \tag{46}$$

where

$$q_H^P = \mathbb{E}\left[\widetilde{q}_H \middle| \widetilde{q}_H < D\right].$$

It follows from differentiation of (46) and the relations in (37), (39) and (44) that:

$$\begin{array}{c}\frac{\partial \pi_i(s_i)}{\partial p_i} = \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right) q_H^P\left(1 - \Pi^{NP}\right) \\ + \left(\Pr\left(p_j \geq p_i\right| s_i) + \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\right) q_H^{NP}\Pi^{NP} \\ + \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\left(c_i\left(s_i, p_j^{-1}(p_i)\right) - p_i\right) q_L \\ + \left(1 - \Pr\left(p_j \geq p_i\right| s_i)\right) q_L,\end{array} \tag{47}$$

so

$$\begin{array}{c}\frac{\partial \pi_i(s_i)}{\partial p_i} = \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)\left(q_H^P\left(1 - \Pi^{NP}\right) + q_H^{NP}\Pi^{NP} - q_L\right) \\ + \Pr\left(p_j \geq p_i\right| s_i) q_H^{NP}\Pi^{NP} + \left(1 - \Pr\left(p_j \geq p_i\right| s_i)\right) q_L,\end{array}$$

which can be simplified to (27), because $q^H = q_H^P\left(1 - \Pi^{NP}\right) + q_H^{NP}\Pi^{NP}$. ■

Proof. (Proposition 5) The proof is similar to the proof of Proposition 1.

■

Appendix E: Ranking of auction formats

Proof. (Lemma 4)

For the discriminatory auction, it follows directly from (9) and (16) that

$$\begin{aligned}\pi(s) &= (p(s) - c(\underline{s}, \underline{s}))(1 - F(s)) q_H + (p(s) - c(\underline{s}, \underline{s})) F(s) q_L \\ &= q_L(\overline{p} - c(\underline{s}, \underline{s})),\end{aligned}$$

when costs are common knowledge and signals are independent. Going through the same calculations for the uniform-price auction is rather tedious, because the winning producer is sometimes paid the offer price of the losing producer, so the expected transaction price is less straightforward. Thus, we use a different approach for the uniform-price auction. It follows from (47) that

$$\begin{array}{c}\frac{\partial \pi_i}{\partial p_i} = \frac{\partial \Pr(p_j \geq p_i)}{\partial p_i}\left(p_i - c_i(\underline{s}, \underline{s})\right) q_H^P\left(1 - \Pi^{NP}\right) \\ + \left(\Pr\left(p_j \geq p_i\right) + \left(p_i - c_i(\underline{s}, \underline{s})\right)\frac{\partial \Pr(p_j \geq p_i)}{\partial p_i}\right) q_H^{NP}\Pi^{NP} \\ + \frac{\partial \Pr(p_j \geq p_i)}{\partial p_i}\left(c_i(\underline{s}, \underline{s}) - p_i\right) q_L \\ + \left(1 - \Pr\left(p_j \geq p_i\right)\right) q_L = 0\end{array} \tag{48}$$

whenever signals are independent, $s_i < \overline{s}$ and the costs are common knowledge. Hence, in equilibrium the expected payoff of a producer will not change if it changes the offer price in the range $[c, \overline{p}]$ for a given signal. This is expected as this special case corresponds to a mixed-strategy NE in accordance with Harsanyi's purification theorem. Thus, to calculate the expected equilibrium payoff for producer i, we can assume that it makes an offer at $\overline{p}$. The competitor plays the equilibrium strategy, so it will almost surely undercut $\overline{p}$, i.e. $\Pr(p_j \geq p_i) = 0$. The expected profit of producer i can now be calculated from (46):

$$\pi_i(s) = (\overline{p} - c(\underline{s}, \underline{s}))\, q_L.$$

■

Proof. (Proposition 6) It follows from (46) and (29) that the expected revenue of a producer in a uniform price auction after observing the signal s is:

$$\begin{aligned} R(s) &= \frac{\int_s^{\overline{s}} p(s_j)\chi(s,s_j)ds_j q_H^P\left(1-\Pi^{NP}\right)+\int_s^{\overline{s}} p(s)\chi(s,s_j)ds_j q_H^{NP}\Pi^{NP}+\int_{\underline{s}}^{s} p(s)\chi(s,s_j)ds_j q_L}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j} \\ &= \frac{\int_s^{\overline{s}}\left(c(s_j,s_j)+\int_{s_j}^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_{s_j}^{v}\hat{H}(u)du}dv\right)\chi(s,s_j)ds_j q_H^P\left(1-\Pi^{NP}\right)}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j} \\ &\quad+\frac{\int_s^{\overline{s}}\left(c(s,s)+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\hat{H}(u)du}dv\right)\chi(s,s_j)ds_j q_H^{NP}\Pi^{NP}}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j} \\ &\quad+\frac{\int_{\underline{s}}^{s}\left(c(s,s)+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\hat{H}(u)du}dv\right)\chi(s,s_j)ds_j q_L}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j}. \end{aligned} \tag{49}$$

We can also use the above expression to calculate the expected revenue in the discriminatory auction by setting $\Pi^{NP} = 1$. $R(s)$ can be rewritten as follows:

$$R(s) = \frac{\int_s^{\overline{s}} c(s,s)\,\chi(s,s_j)\,ds_j q_H}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)\,ds_j} + \frac{\int_{\underline{s}}^{s} c(s,s)\,\chi(s,s_j)\,ds_j q_L}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)\,ds_j} + \frac{\Theta(s)}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)\,ds_j}. \tag{50}$$

$\Theta(s)$ is defined below. It captures how differences in the auction format and the probability that producers are pivotal influence the expected revenue.

$$\begin{aligned} \Theta(s) &= \int_s^{\overline{s}}\int_{s_j}^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_{s_j}^{v}\hat{H}(u)du}dv\chi(s,s_j)\,ds_j q_H^P\left(1-\Pi^{NP}\right) \\ &\quad+\int_s^{\overline{s}}\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\hat{H}(u)du}dv\;\chi(s,s_j)\,ds_j q_H^{NP}\Pi^{NP} \\ &\quad+\int_{\underline{s}}^{s}\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\hat{H}(u)du}dv\;\chi(s,s_j)\,ds_j q_L \\ &\quad+\int_s^{\overline{s}}\left(c(s_j,s_j)-c(s,s)\right)\;\chi(s,s_j)\,ds_j q_H^P\left(1-\Pi^{NP}\right). \end{aligned}$$

Next, we change the order of integration for the double integral and adjust limits, so that the integrals describe the same domain of integration.

$$\begin{aligned} \Theta(s) &= \int_s^{\overline{s}}\frac{dc(v,v)}{dv}\int_s^{v}e^{-\int_{s_j}^{v}\hat{H}(u)du}\chi(s,s_j)\,ds_j dv q_H^P\left(1-\Pi^{NP}\right) \\ &\quad+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\hat{H}(u)du}\int_s^{\overline{s}}\chi(s,s_j)\,ds_j dv\; q_H^{NP}\Pi^{NP} \\ &\quad+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\hat{H}(u)du}\int_{\underline{s}}^{s}\chi(s,s_j)\,ds_j dv q_L \\ &\quad+\int_s^{\overline{s}}\left(c(s_j,s_j)-c(s,s)\right)\;\chi(s,s_j)\,ds_j q_H^P\left(1-\Pi^{NP}\right). \end{aligned}$$

Assume now that $\frac{dc(v,v)}{dv}$ is zero for v below $w \geq s$. In this case, we have:

$$\begin{aligned}
\Theta(s) &= \int_w^{\overline{s}} \frac{dc(v,v)}{dv} \int_s^v e^{-\int_{s_j}^v \hat{H}(u)du} \chi(s,s_j)\, ds_j dv q_H^P \left(1-\Pi^{NP}\right) \\
&+ \int_w^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \hat{H}(u)du} \int_s^{\overline{s}} \chi(s,s_j)\, ds_j dv\ q_H^{NP}\Pi^{NP} \\
&+ \int_w^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \hat{H}(u)du} \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j dv q_L \\
&+ \int_w^{\overline{s}} \left(c(s_j,s_j) - c(w,w)\right)\ \chi(s,s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right),
\end{aligned}$$

if $w \geq s$. We have $\frac{d\Theta(s)}{dw} = 0$ if $w < s$, otherwise

$$\begin{aligned}
\frac{d\Theta(s)}{dw} &= -\frac{dc(w,w)}{dw} \int_s^w e^{-\int_{s_j}^w \hat{H}(u)du} \chi(s,s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right) \\
&- \frac{dc(w,w)}{dw} e^{-\int_s^w \hat{H}(u)du} \left(\int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j q_L + \int_s^{\overline{s}} \chi(s,s_j)\, ds_j q_H^{NP}\Pi^{NP}\right) \\
&- \frac{dc(w,w)}{dw} \int_w^{\overline{s}} \chi(s,s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right). \qquad (51)
\end{aligned}$$

Next, we use that signals are independent, i.e. $\chi(s,s_j) = f(s) f(s_j)$. We have from Definition 3 that

$$\begin{aligned}
\hat{H}(s) &= \frac{f(s)(q_H - q_L)}{\int_s^{\overline{s}} f(s_j)\, ds_j q_H^{NP}\Pi^{NP} + \int_{\underline{s}}^{s} f(s_j)\, ds_j q_L} \\
&= -\frac{d}{ds} \ln\left(\left(1-F(s)\right) q_H^{NP}\Pi^{NP} + F(s) q_L\right) \frac{(q_H - q_L)}{q_H^{NP}\Pi^{NP} - q_L} \\
\int_s^w \hat{H}(u)\, du &= \left[\frac{(q_H - q_L)\ln\left(\left(1-F(s)\right) q_H^{NP}\Pi^{NP} + F(s) q_L\right)}{q_L - q_H^{NP}\Pi^{NP}}\right]_s^w \\
&= \frac{(q_H - q_L)\ln\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP} + F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP} + F(s)q_L}\right)}{q_L - q_H^{NP}\Pi^{NP}} \\
e^{-\int_s^w \hat{H}(u)du} &= \left(\frac{\left(1-F(w)\right) q_H^{NP}\Pi^{NP} + F(w) q_L}{\left(1-F(s)\right) q_H^{NP}\Pi^{NP} + F(s) q_L}\right)^{\frac{-(q_H - q_L)}{q_L - q_H^{NP}\Pi^{NP}}}. \qquad (52)
\end{aligned}$$

We substitute (52) and that $\chi(s,s_j) = f(s) f(s_j)$ into (51)

$$\begin{aligned}
\frac{d\Theta(s)}{dw} &= -\frac{dc(w,w)}{dw} \int_s^w \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP} + F(w)q_L}{(1-F(s_j))q_H^{NP}\Pi^{NP} + F(s_j)q_L}\right)^{\frac{-(q_H - q_L)}{q_L - q_H^{NP}\Pi^{NP}}} f(s) f(s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right) \\
&- \frac{dc(w,w)}{dw} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP} + F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP} + F(s)q_L}\right)^{\frac{-(q_H - q_L)}{q_L - q_H^{NP}\Pi^{NP}}} \int_s^{\overline{s}} f(s) f(s_j)\, ds_j q_H^{NP}\Pi^{NP} \\
&- \frac{dc(w,w)}{dw} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP} + F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP} + F(s)q_L}\right)^{\frac{-(q_H - q_L)}{q_L - q_H^{NP}\Pi^{NP}}} \int_{\underline{s}}^{s} f(s) f(s_j)\, ds_j q_L \\
&- \frac{dc(w,w)}{dw} f(s) \int_w^{\overline{s}} f(s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right) \qquad (53)
\end{aligned}$$

Next, we use the substitution $F = F(s_j)$, so that $dF = f(s_j)\, ds_j$.

$$\begin{aligned}\frac{d\Theta(s)}{dw} = & -\frac{dc(w,w)}{dw} \int_{F(s)}^{F(w)} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F)q_H^{NP}\Pi^{NP}+Fq_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s)\, dF q_H^P \left(1-\Pi^{NP}\right) \\ & -\frac{dc(w,w)}{dw} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s)(1-F(s))\, q_H^{NP}\Pi^{NP} \\ & -\frac{dc(w,w)}{dw} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s) F(s)\, q_L \\ & -\frac{dc(w,w)}{dw} f(s)(1-F(w))\, q_H^P \left(1-\Pi^{NP}\right).\end{aligned} \tag{54}$$

The first integral can be solved as follows:

$$\begin{aligned}& \int_{F(s)}^{F(w)} \left(\frac{(1-F(w))\, q_H^{NP}\Pi^{NP}+F(w)\, q_L}{(1-F)\, q_H^{NP}\Pi^{NP}+Fq_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} dF \\ & = \left((1-F(w))\, q_H^{NP}\Pi^{NP}+F(w)\, q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \int_{F(s)}^{F(w)} \left((1-F)\, q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} dF \\ & = \left((1-F(w))\, q_H^{NP}\Pi^{NP}+F(w)\, q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \left[\frac{\left((1-F)\, q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}+1}}{\left(q_H-q_H^{NP}\Pi^{NP}\right)}\right]_{F(s)}^{F(w)} \\ & = \left((1-F(w))\, q_H^{NP}\Pi^{NP}+F(w)\, q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \left[\frac{\left((1-F)\, q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}+1}}{q_H^P\left(1-\Pi^{NP}\right)}\right]_{F(s)}^{F(w)},\end{aligned}$$

because $q_H = q_H^{NP}\Pi^{NP} + q_H^P\left(1-\Pi^{NP}\right)$. Using this result, we can rewrite (54) as follows:

$$\begin{aligned}\frac{d\Theta(s)}{dw} = & -\frac{dc(w,w)}{dw} \left((1-F(w))\, q_H^{NP}\Pi^{NP}+F(w)\, q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s) \\ & \left[\left((1-F)\, q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}+1}\right]_{F(s)}^{F(w)} \\ & -\frac{dc(w,w)}{dw} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s)(1-F(s))\, q_H^{NP}\Pi^{NP} \\ & -\frac{dc(w,w)}{dw} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s) F(s)\, q_L \\ = & -\frac{dc(w,w)}{dw} \left((1-F(w))\, q_H^{NP}\Pi^{NP}+F(w)\, q_L\right) f(s) \\ & -\frac{dc(w,w)}{dw} f(s)(1-F(w))\, q_H^P\left(1-\Pi^{NP}\right) \\ = & -\frac{dc(w,w)}{dw} \left((1-F(w))\, q_H+F(w)\, q_L\right) f(s)\end{aligned} \tag{55}$$

Hence, it follows that $\frac{d\Theta(s)}{dw}$ is independent of Π^{NP} and thus also independent of whether the auction has a uniform or a discriminatory format. The same type of independency applies to $\frac{dR(s)}{dw}$. We have from Lemma 4 that the expected revenue $R(s)$ is the same, independent of Π^{NP} and independent of the auction format, if $w \nearrow \overline{s}$ and the signals are independent. From the above reasoning, it follows that the result in Lemma 4 can be generalized to any $w \in (\underline{s}, \overline{s})$. ■