SEARCH RELATIVITY

Ying Tung Chan and Chi Man Yip*

Abstract

This paper proposes a unified theory that explains three sets of seemingly unrelated features of the unemployment rate by educational attainment (EUR). First, the higher the educational group, the lower is the EUR. Any economy with a higher proportion of highly-educated workers is expected to have a lower overall unemployment rate (OUR). Why are the OUR and the distribution of educational attainment uncorrelated? Second, it seems natural that the EUR increases with OUR. But why is such increase more pronounced for a lower educational group? Third, the lowest possible EUR is about half the OUR in each of the 50 states. What explains the magic number of one-half? In our model, unemployed workers with heterogeneous productivity compete with one another during a job search process. The job finding rate increases with the relative position in the distribution of search intensity, not its level. The increase in the share of the highly-educated improves the relative position of highly-educated unemployed workers; meanwhile, it creates the negative externality of the same size on the medium- and the low-educated unemployed, leaving the average transition rate and thus the OUR unchanged. Our model derives a novel formula of the EUR. The null hypotheses that the actual and the predicted values are from the same distribution cannot be rejected for high-school graduates and Bachelor's degree holders in 46 and 43 out of 50 states.

Keywords: Educational Unemployment Rate; Fundamental Frictional Unemployment Rate; Relative Search Intensity; Unemployment Distribution.

JEL Classification Numbers: E24, J64.

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*Chan: Department of Economics, McGill University, Room 414, Leacock Building, 855 Sherbrooke Street West, Montreal, Quebec H3A 2T7 (email: yingtung.chan@mail.mcgill.ca); Yip: Department of Economics, University of Calgary, 2500 University Dr. N.W., Calgary, AB, T2N 1N4 (email: cman.yip@gmail.com) We would like to thank Scott Taylor and Trevor Tombe for their patience, guidance, and encouragement support. We thank Meng Sun and Qiongda Zhao for their comments and our many insightful conversations. We thank Nicole Fortin, Kim Huynh, Lucija Muehlenbachs, Stefan Staubli, Atsuko Tanaka, and Jean-Francois Wen and seminar participants at the Canadian Economics Association-Annual Conference for their useful comments. We owe our classmates Younes Ahmadi, Yutaro Sakai, and Qian Sun for their suggestions.

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1 Introduction

Back to the days of Marx (1923) and Keynes (1937), economists were concerned with wages and unemployment rates. Since then, these two variables have been at the heart of economic literature, especially in the field of Labor Economics, Macroeconomics, and Public Economics. For the theoretical convenience, plenty of earlier works study an average wage level and an unemployment rate of an economy, abstracting the discussion on their distributions. Since the past decades, wage distribution have received lots of attention from both theoretical works (Galor and Zeira, 1993; Burdett and Mortensen, 1998; Krusell and Smith, 1998; Postel-Vinay and Robin, 2002; Moscarini and Postel-Vinay, 2013) and empirical studies (Juhn et al., 1993; DiNardo et al., 1996; Katz et al., 1999; Lemieux, 2006; Autor et al., 2008). Nevertheless, works, especially theoretical ones, that study an unemployment distribution across various demographic groups and its properties are surprisingly rare.[1]

In particular, this paper studies the relationship between the unemployment rate by educational attainment (EUR) and the overall unemployment rate (OUR). We motivate this study by three sets of observations in the United States: (i) the statistical puzzle of an unemployment rate identity, (ii) the underlying relationship between EUR and OUR, and (iii) the magic number of "one-half".[2]

The Statistical Puzzle of an Unemployment Rate Identity. First, Figure 1 shows that the EUR decreases with educational level. This negative relationship holds for more than twenty consecutive years. Given an unemployment rate identity, any economy with a higher proportion of highly-educated workers is expected to have a lower OUR.[3] However, Figure 2 illustrates that the OUR and the distribution of educational attainment are uncorrelated. *So, why does the EUR decrease with educational level? And what makes the OUR and the distribution of educational attainment uncorrelated?*

The Underlying Relationship Between EUR and OUR. Figure 3 illustrates the relationship between the EUR and the OUR by state and educational level. Clearly, each EUR and OUR are positively correlated regardless of educational level. Moreover, there seems to exist a mechanism that governs the slopes in the figure, except high-school dropouts. When regressing the EURs on the OURs, it is straightforward to see that the higher the

[1] To the best of our knowledge, this paper is the first theoretical paper that primarily investigates the unemployment distribution across educational groups.

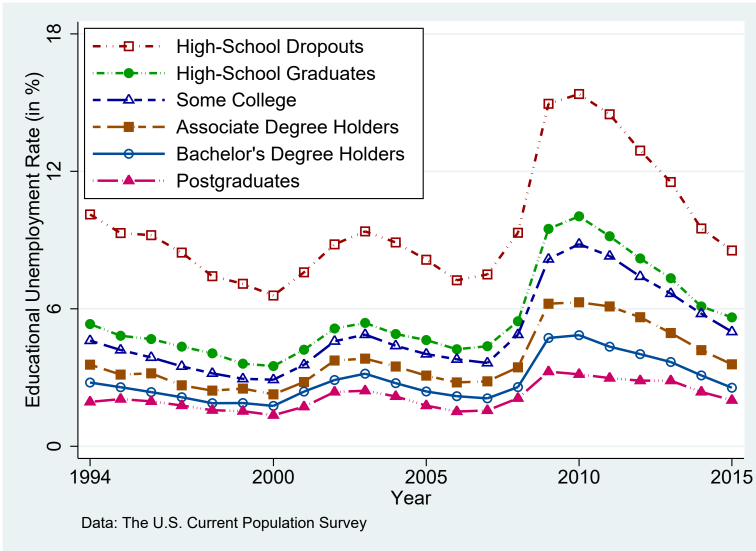
[2] More rigorous empirical examinations on the properties of the EUR are given in section 2.1. Four salient facts about the EUR and the OUR in the United States are documented and examples that show the incapability of the existing search and matching model in explaining the observed trends in unemployment are demonstrated.

[3] The unemployment rate identity is given by

$$OUR \equiv \sum_j h_j u_j, \text{ where } \sum_j h_j = 1$$

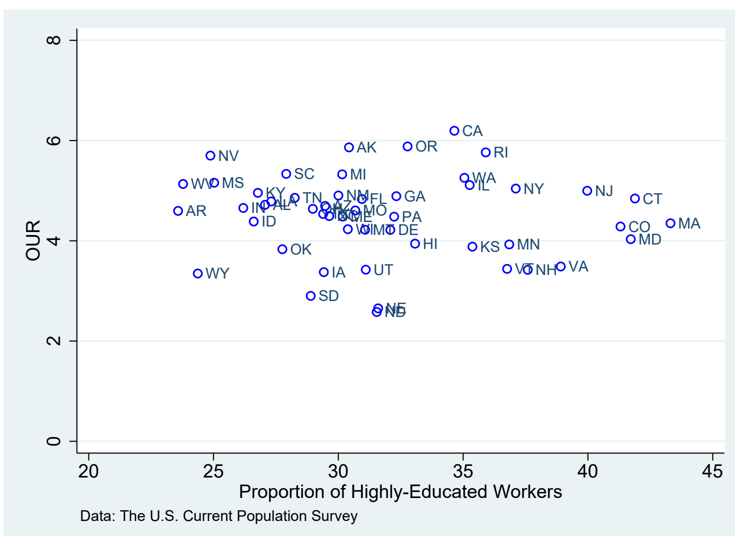
where h_j and u_j are the share and the EUR of educational group j. Undoubtedly, the equation holds by identity.

Figure 1: Yearly EURs in the United States, 1994-2015



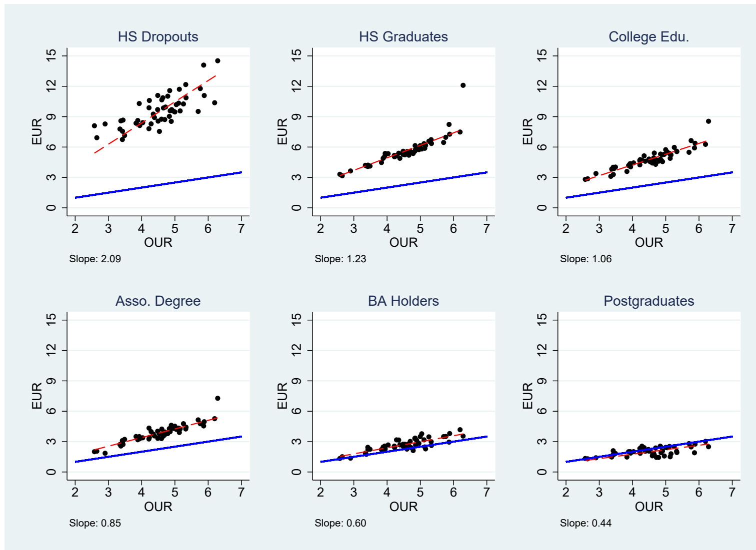
Notes: This figure displays the unemployment rate by educational attainment. Data are collected from the US CPS. Samples are restricted to labor force participants aged 25-60.

Figure 2: The Association between OUR & the Proportion of Highly-Educated Workers in the United States, 1994-2015



Notes: This figure displays the relationship between the OUR and the proportion of the highly-educated by state during 1994-2015. Data are collected from the US CPS. Samples are restricted to labor force participants aged 25-60. This figure excludes DC because it is an outlier. Each dot illustrates the average OUR and the average fraction of highly-educated in a state during 1994-2015. The highly-educated are defined as those who are Bachelor's degree holders and/or the postgraduates.

Figure 3: The Correlation between the EUR and the OUR by State, 1994-2015



Notes: This figure displays the relationship between the unemployment rate by educational attainment and the overall unemployment rate by state and educational level. Data are collected from the US CPS. Samples are restricted to labor force participants aged 25-60. The dash line is the fitted value of the EUR. The solid line represents $EUR = OUR/2$.

educational level, the flatter is the slope. *So, what economic mechanism governs the responsiveness of the EUR to the OUR? And why are the responsiveness of the EUR to the change in the OUR more pronounced for a lower educational group?*

The Magic Number of "One-Half". Figure 1 suggests that the higher the educational level, the lower is the EUR. While the post-graduate is the highest educational level in all states, their EUR is supposed to be the lowest. But what determines the level of the lowest possible EUR in any economy? In Figure 3, the solid line represents the mathematical relationship in which EUR is half the OUR. Clearly, points lie around this solid line for the post-graduate, suggesting that the EUR of the postgraduate is about half the OUR. *So, why is the lowest possible EUR related to the OUR? What mechanism creates the magic number of one-half?*

While these three sets of observations seem irrelevant to one another, this paper proposes a unified theory to explain these *three seemingly unrelated* features of EURs. Despite having an extensive literature on the OUR (Ljungqvist and Sargent, 2008; Elsby et al., 2009; Davis et al., 2010; Shimer, 2012; Sahin et al., 2014), the study on the EUR is rare. The OUR is indeed a weighted average of all EURs, and the weight is the fraction of each educational group in the population, which is accessible and rather stable along its increasing trend. Once the properties of the EURs are well understood, the properties of the OUR would be mastered well, not vice versa. Undoubtedly, to understand these three sets of observations not only opens the “black box” of EUR but it also complements the existing literature on the OUR.

To explain a series of research questions regarding the EUR, we construct a search equilibrium model that is widely applicable in explaining the distribution of unemployment across educational groups. The key innovation of this paper is that a matching technology and a relative position of one’s search intensity in an economy jointly determine a job finding rate. The former factor is standard in the search and matching literature (Rogerson et al., 2005). The latter feature is the main departure of this paper from the existing literature. An unemployed worker submits his own search intensity to maximize his value function. A higher rank of search intensity will lead to a higher transition rate; therefore, the job finding rate of any worker depends largely on others’ strategy. We can also interpret the job seeking game as an auction for a higher transition rate: an unemployed worker (a bidder) with the highest bid of intensity is rewarded with the highest transition rate, a bidder with the second highest bid is rewarded with the second highest rate, and so on. In pursuit of a higher transition rate, the unemployed worker needs to have a slightly better curriculum vitae, perform slightly better in a job interview, and/or devote slightly more time to seek jobs than the candidates slightly above the worker on the intensity ladder. To increase one’s search intensity without climbing up the intensity ladder, therefore, does not enhance his job finding rate. The determination of the optimal search intensity thus requires unemployed workers to consider both the marginal search cost and whether an additional search effort allows them to climb up the intensity ladder.

The EUR of a higher educational group is lower because of their higher relative position in the intensity ladder, not because of the higher level of search intensity. Workers with a higher educational group tend to have a higher productivity level and thus wage; therefore, the higher search benefit incents them to search more intensively. This higher search intensity rewards them a higher job finding rate because of their higher relative position in the search intensity distribution. Although workers with a higher educational level search more intensively, a higher fraction of highly-educated workers does not imply a lower OUR. The increase in one’ search intensity does enhance his relative position in the search intensity ladder. Meanwhile, it creates a negative externality of the same size on others: there exist workers whose’ ranking declines. Consequently, the two forces cancel and the OUR is independent of the fraction of highly-educated workers.

The magic number of one-half is attributed to the result, in which the job finding rate of the post-graduate is always twice the average rate of the economy as a whole. Recall that the job finding rate is proportional to the relative position of the search intensity distribution in the unemployment. Since the post-graduates possess the highest search benefit, they tend to search the most intensively and thus rank top in the search intensity ladder. Therefore, the cumulative distribution function of the search intensity for the post-graduate is one. We also show that the expectation of any cumulative distribution function is always equal to one-half. Consequently, the post-graduates always search twice as fast as the average of the economy, explaining why the lowest possible EUR is about half the OUR.

Our theory departs from the conventional thought on the determinants of search technology. The existing literature tends to suggest that the level of search intensity determines one's transition rate to employment. Our theory, on the contrary, proposes that the difference in a transition rate arises (in large part) from the relative position of search intensity, not their levels. For example, Kroft and Pope (2014) find that an increasing popularity of a website like Craigslist largely reduces search cost in a job search process but has no impact on the unemployment rate. Our proposed theory provides one of the potential explanation: the OUR is uncorrelated with the average level of search intensity. Theoretically, the reduction in search cost might increase the level of search intensity. The increase in one' search intensity does enhance his relative position in the search intensity ladder. Meanwhile, it creates a negative externality of the same size on others: there exist workers whose' ranking declines. Overall, a rise in the job seeking rate for a certain group of unemployed has no effect on the OUR.

Our model is not only theoretically appealing, but it also explains the three sets of observations and leads to several striking results. First, this model derives a novel formula of the EURs. We show that our proposed model succeeds in explaining the features of unemployment. Beyond the trends, we evaluate the derived EURs using the US data during 1994-2015. For each educational group in each state, most the null hypotheses that the actual and the derived EURs are from the same distribution cannot be rejected at any conventional significance level. For example, the null hypotheses cannot be rejected for high-school graduates and Bachelor's degree holders in 46 and 43 out of 50 states at five percent significance level. This implies that our model does well predict not only the trends in EURs, but also their magnitudes over the past two decades. Moreover, the formula requires only two input variables: the OUR and the distribution of educational level, both of which are easily accessible. Using the least number of input variables (two), our simple formula allows economists, policymakers, and the public to accurately map the OUR to a more relevant information to the public — the EUR. In particular, our formula indicates that the EUR of the postgraduates is as simple as $u/(2-u)$, where u is the OUR. The derived formula, though simple, is so accurate that the statistical tests cannot distinguish the distribution between the actual and the predicted values. The striking results suggest that the search intensity relativity, thought not necessarily the only factor, is the fundamental mechanism that governs the underlying relationship between EUR and the OUR.

Second, this model derives the formula of the "fundamental frictional unemployment rate", which is defined as the unemployment rate at which the associated unemployment spell cannot be further shortened simply by increasing search effort or productivity of workers. In other words, this paper computes the unemployment rate that is driven sheerly by a search friction (a matching technology), abstracting other factors like search effort or ability. While the difference in the OUR across countries could be attributed to the difference in search friction, workers' search intensity, and their productivity, it is difficult to learn about the search friction simply by comparing the OURs across countries. Similarly, whether a rise in the OUR in recession results from the increase in search friction or the decline in the incentive to seek job remains an empirical question, but is difficult to investigate. The fundamental frictional unemployment rate allows economists to quantify the frictional unemployment rate so as to study the properties of search friction. Furthermore, this measure allows us to disentangle the overall unemployment that is attributable to other factors such as rationing unemployment (Michaillat, 2012), mismatch unemployment (Sahin et al., 2014) and ambiguous unemployment (Chan and Yip, 2016) from search frictional unemployment.

Our work is related to the search-and-matching literature with a continuum of worker types. Acemoglu (2001), Albrecht and Vroman (2002), Wong (2003), and Dolado et al. (2009) are works with a finite number of worker types in the random search literature. With a special interest in the wage distribution, most of these works perform well in modeling between and within educational group wage inequality. While an unemployment distribution is out of the scope, most of these works unsurprisingly predict that the unemployment rates for different types of workers are identical. Nevertheless, this implication might be inconsistent with the data. For example, Ashenfelter and Ham (1979) document that the unemployment rate and the educational attainment are inversely related in the United States. Also, Topel (1993) finds that men of lower wages have a higher risk of unemployment. We will revisit the educational unemployment in the United States in the next section.

In the directed search literature, works like Eeckhout and Kircher (2010) and Peters (2010) model an economy with a continuum of worker types.[4] In Peters (2010), a continuum of workers of different types directs their applications to different jobs. Firms value workers' characteristics differently and give a job offer to the job seeker they most value. His model, unlike the other directed search literature, succeeds in getting rid of the tradeoff between higher wages and lower unemployment rates, resolving the common problem in this literature. As pointed out by the author, the setting that firms could not condition wages on workers' characteristics is awkward and might slightly depart from the reality.[5]

Section 2 illustrates further evidence on the first two sets of observations and provides examples to show that the existing search and matching models fail to capture at least one

[4] Eeckhout and Kircher (2010) also allows for a continuum of firm types.

[5] The model could predict that wages and skill levels are positively related as the high-skilled unemployed tend to direct their search to the positions of higher wages.

of the observations. The basic model environment is described in section 3, followed by the construction of a steady state Nash equilibrium in section 4. Our model is shown to capture the features of unemployment in section 4. Beyond the trends, section 5 evaluates the magnitude of the unemployment distribution and the EURs derived by the model. The derived formulas are shown to perform well (if not considered as nearly perfectly) using the United States Current Population Survey 1994-2015. In addition, the section derives the formula of the Fundamental Frictional Unemployment Rate. Section 6 concludes the paper.

2 Historical Facts and Motivating Examples

This section documents several salient features of unemployment corresponding to the first two sets of observations.[6] Second, we provide two motivating examples to demonstrate the incapability of the existing search and matching model in explaining the observed features of the unemployment rates.

2.1 Salient Features of Unemployment

This subsection illustrates the features of unemployment rates, which correspond to the first two sets of observations. Data are obtained from the United States Current Population Survey 1994-2015 throughout this subsection.[7] Samples are restricted to the labor force participants aged 25-60.

Feature 1: The EUR decreases in educational attainment. Figure 1 displays the EUR of six different educational groups.[8] The figure indicates that the higher the educational level, the lower is the corresponding EUR. Moreover, the six lines are completely segregated during 1994-2015, suggesting that such negative association between an educational level and their EUR holds during the entire period of examination. Of course, such relationship might be driven by a change in the composition of the sample. Hence, we estimate the association between the EUR and the educational level using the following model.

$$Unemployed_{ijt} = \alpha + X_{ijt}^{T}\beta + \varepsilon_{ijt}$$

where $Unemployed_{ijt}$ takes the value of one if a respondent i with educational level j is unemployed at time t, and zero otherwise. X_{ijt} is a vector of control variable including age

[6] Since Figure 3 clearly shows that the lowest EUR is about half the OUR in almost all the states, we do not conduct any further analysis on the third sets of observations.

[7] Analyses start from 1994 because the US CPS was redesigned in 1994.

[8] Similar features are seen in Canada and the United Kingdom.

and dummies for female, white, and marital status. *Female* is one if a respondent is female, and zero otherwise. *White* is one if a respondent is white, and zero otherwise. *Married* is one if a respondent is married, and zero otherwise. $\hat{\alpha}$ measures the average EUR of the corresponding educational level over the period of examination, conditioning on the demographic characteristics. In the absence of the controlling variables, $\hat{\alpha}$ is the average EUR over 1994-2015.

Results are reported in Table 1. Each column reports estimates from a different model specification, which is described at the bottom of the table. According to the estimates in column (1), the average EURs are 9.6%, 5.6%, 4.9%, 3.8%, 2.9%, and 2.2% for for high-school dropouts, high-school graduates, workers with some college education, Associate degree holders, Bachelor's degree holders, and postgraduates. The results reflect that an EUR decreases with educational level. Moreover, this relationship is robust to model specification.

Feature 2: The OUR is uncorrelated with the distribution of educational level. The OUR and the distribution of educational level are uncorrelated both in the short- and the long-term. First, we show the long-term relationship between the mean OUR and the mean share of the highly-educated by state during 1994-2015 in Figure 2.[9] The figure suggests that the association between the OUR and the proportion of the high-educated is at best weak. One might worry that the relationship between the OUR and the distribution of educational level is hidden because we average out the association between the two variables over the examination period. To see the short-term relationship, we plot the relationship between the yearly OUR and the yearly share of highly-educated workers by state in Figure A.1. The figure suggests neither positive nor negative association between the OUR and the share of highly-educated workers. To further confirm that the OUR is not associated with the distribution of educational level, we compute the cyclical components of the OUR and the proportion of the low- and the highly-educated by state using Hodrick-Prescott filter (Hodrick and Prescott, 1997), with the multiplier equal to 6.25. The yearly cyclical components are plotted in Figure A.2, which shows that the association between the cyclical components of the OUR and the proportion of the low- and the highly-educated are weak over 1994-2015. Hence, we can conclude that the OUR and the distribution of educational level are uncorrelated both in the short- and the long-term.

In fact, an OUR is a weighted sum of EURs by the unemployment rate identity: $u \equiv \sum_{j=1}^{N} u_j h_j$, where j denotes educational group, h_j is the share of the educational group in the population, u_j is the EUR of the group j, and u is the OUR. The first feature of unemployment suggests that the groups with a higher educational attainment are associated

[9]Highly-educated workers are defined as those who are Bachelor's degree holders and/or the postgraduates, and low-educated workers are defined as those who are high-school dropouts and/or high-school graduates. The rest are defined as Medium-educated worker. These definitions are used throughout the article. The observation of DC is dropped because its share of highly-educated is at least 15 percent more than anyone of other states.

Table 1: The Educational Unemployment Rates in the United States

Dependent Variable: Unemployed				
	(1)	**(2)**	**(3)**	**(4)**
Sample: High-School Dropouts				
Constant	0.096***	0.138***	0.277***	0.287***
	(0.003)	(0.008)	(0.010)	(0.010)
Adjusted R-squared	0.000	0.011	0.021	0.026
Sample: High-School Graduates				
Constant	0.056***	0.066***	0.158***	0.166***
	(0.002)	(0.003)	(0.004)	(0.004)
Adjusted R-squared	0.000	0.010	0.016	0.021
Sample: Workers with Some College Education				
Constant	0.049***	0.049***	0.109***	0.116***
	(0.002)	(0.003)	(0.004)	(0.004)
Adjusted R-squared	0.000	0.009	0.013	0.017
Sample: Associate Degree Holders				
Constant	0.038***	0.031***	0.067***	0.076***
	(0.001)	(0.002)	(0.004)	(0.004)
Adjusted R-squared	0.000	0.007	0.009	0.012
Sample: Bachelor's Degree Holders				
Constant	0.029***	0.027***	0.038***	0.043***
	(0.001)	(0.002)	(0.003)	(0.003)
Adjusted R-squared	0.000	0.005	0.005	0.008
Sample: Postgraduates				
Constant	0.022***	0.020***	0.027***	0.034***
	(0.001)	(0.002)	(0.002)	(0.002)
Adjusted R-squared	0.000	0.003	0.003	0.005
Year×Month Fixed Effect	No	Yes	Yes	Yes
State Fixed Effect	No	Yes	Yes	Yes
Age	No	No	Yes	Yes
Female Dummy	No	No	Yes	Yes
White Dummy	No	No	Yes	Yes
Married Dummy	No	No	No	Yes

Notes: Robust standard errors in parentheses are clustered at the state level. Data are collected from the US CPS. Samples are restricted to labor force participants aged 25-60. The number of observation is 1,179,338, 4,188,690, 2,491,002, 1,437,020, 2,996,926, and 1,563,744 for high-school dropouts, high-school graduates, workers with some college education, Associate degree holders, Bachelor's degree holders, and postgraduates. Unemployed is one if the respondent is unemployed, and zero otherwise. Female is one if the respondent is female, and zero otherwise. White is one if the respondent is white, and zero otherwise. Married is one if the respondent is married, and zero otherwise. Significance levels: ***=1%, **=5%, *=10%.

with a lower unemployment rate. Hence, any economy with a higher fraction of highly-educated workers are expected to have a lower OUR. Nevertheless, the second documented feature suggest that the correlation between the OUR and the distribution of educational attainment is at best weak. Hence, the first two features creates the statistical puzzle of the unemployment rate identity.[10]

Feature 3: All EURs increase in slumps. As known, there was a financial crisis during 2008-2010. According to Figure 1, the EURs started to rise in 2008 and reached its peak in 2010 regardless of educational attainment, providing a support that all the EURs are higher in slumps. To control for other characteristics of the sample, we estimate the association between the EURs and the real GDP as follows.

$$EUR_{jt} = \alpha + \gamma GDP_t + X_{jt}^T \beta + \varepsilon_{jt}$$

where EUR_{jt} is the state EUR of the educational group j in a quarter t, GDP_t is a quarterly real GDP divided by 1000, and X_{jt} is the state share of female, white, married respondents and the state share of the respondents aged 40-60. Coefficients are estimated separately using samples of six educational groups. Results are reported in Table 2. Each column shows estimates from a different model specification, which is described at the bottom of the table. The estimates suggest that the EURs and the real GDP are negatively associated at one percent significance level regardless of educational groups. The result is robust to model specification. For example, an 1000 dollar increase in a real GDP is associated with a reduction of the EUR of the high-school dropouts by 4.0 percentage points.

Feature 4: The negative employment effect is more pronounced for a lower educational group in slumps. Figure 1 shows that the distances between lines are rather steady before the 2008 financial crisis, but increase during 2008-2010. This reflects that the lower the educational level, the larger is the negative employment effect in slumps. In fact, according to Table 2, the magnitudes of the estimates decrease with educational level. Controlling the year and quarter effect and other characteristics of the samples, results suggest that the responsiveness to a negative shock fall with educational groups. For example, an 1000 dollar increase in a real GDP is associated with a reduction of the EUR by 4.0, 2.5, 2.3, 1.6, and 1.2 percentage points for high-school dropouts, high-school graduates, workers with some college education, associate degree holders, and Bachelor's degree holders. Obviously, the magnitude of the employment effect unambiguously decreases with edu-

[10]Moreover, if the proportion of the highly-educated h_j increases over time, it is expected that u falls with time. According to the CPS data, the proportion of the highly-educated increases from 28% to 38% and the fraction of their low-educated counterpart falls from 44% to 34% during 1994-2015. These trends, together with the first feature of unemployment, infer that the fraction of the educational group with a lower EUR rises over time. Hence, the US OUR is expected to decline over time. Surprisingly, recent literature debates on the rise in the US natural rate of unemployment after the 2008 financial crisis (Daly et al., 2012; Daly and Hobijn, 2015).

Table 2: The State Educational Unemployment Rates & Real GDP in the United States

Dependent Variable: State EUR (in %)

	(1)	**(2)**	**(3)**
Sample: High-School Dropouts			
Real GDP/1000	-3.934***	-4.026***	-4.016***
	(0.609)	(0.598)	(0.605)
Adjusted R-squared	0.455	0.461	0.462
Sample: High-School Graduates			
Real GDP/1000	-2.505***	-2.503***	-2.505***
	(0.252)	(0.252)	(0.252)
Adjusted R-squared	0.694	0.695	0.695
Sample: Workers with Some College Education			
Real GDP/1000	-2.266***	-2.278***	-2.278***
	(0.373)	(0.368)	(0.368)
Adjusted R-squared	0.629	0.629	0.629
Sample: Associate Degree Holders			
Real GDP/1000	-1.568***	-1.556***	-1.558***
	(0.339)	(0.337)	(0.337)
Adjusted R-squared	0.439	0.442	0.442
Sample: Bachelor's Degree Holders			
Real GDP/1000	-1.236***	-1.243***	-1.243***
	(0.176)	(0.178)	(0.178)
Adjusted R-squared	0.498	0.498	0.498
Sample: Postgraduates			
Real GDP/1000	-1.195***	-1.211***	-1.210***
	(0.211)	(0.214)	(0.214)
Adjusted R-squared	0.244	0.247	0.247
Observations	4,488	4,488	4,488
Year Fixed Effect	Yes	Yes	Yes
Quarter Fixed Effect	Yes	Yes	Yes
Age40 Share	No	Yes	Yes
Female Share	No	Yes	Yes
White Share	No	Yes	Yes
Married Share	No	No	Yes

Notes: Robust standard errors in parentheses are clustered at the state level. The national quarterly real GDP is from the U.S. Bureau of Economic Analysis. Other variables are collected from the US CPS. Samples are restricted to labor force participants aged 25-60. Age40 is the fraction of respondents aged 40-60 in the corresponding state. Female is the fraction of female respondents by state. White (Married) is the fraction of white (married) respondents by state. Significance levels: ***=1%, **=5%, *=10%.

cational level. Although the responsiveness of the EUR for Bachelor's degree holders and postgraduates are close, the employment effect on the graduates is sightly lower. Generally, both Figure 1 and Table 2 point to the feature that the negative employment effect is more pronounced for a lower educational group in slumps.

Here summarizes the documentation of the four features of unemployment: (i) the EUR decreases with educational attainment, (ii) the OUR is uncorrelated with the distribution of educational attainment, (iii) the EURs increase in an economic downturn, and (iv) the negative employment effect is more pronounced for a lower educational group in slumps. While the first two features speak directly to the statistical puzzle of the unemployment rate identity, the feature 3 and 4 raise another sets of questions: why do all the EURs increase in slumps? why is the responsiveness of the EUR smaller for a higher educational group? what governs the responsiveness of the EUR?

2.2 Motivating Examples

This subsection demonstrates the incapability of a standard search and matching model in capturing some of the documented features of unemployment. Two cases are considered, with the first one neglecting the level of search intensity, and the second one considering it. Without loss of generality, we normalize the measure of workers to unity. Workers differ in their productivity levels δ. Denote $H(\delta)$ as the cumulative distribution function of the productivity level in a population, with the support $\mathcal{R}_{++}$ and the corresponding productivity density function $h(\delta)$. Denote $G(\delta)$ as the cumulative distribution function of the productivity level in unemployment and the corresponding productivity density function $g(\delta)$.

Without Considering the Level of Search Intensity. We first consider the simplest search and matching model, in which search intensity is not incorporated. In a steady state equilibrium, for each δ, the flows in and out of unemployment are equal as follows.

$$\lambda(h(\delta) - g(\delta)u) = pg(\delta)u$$

where λ, p, and u are a separation rate from a position, a transition rate from unemployment to employment and the measure of the unemployment respectively. Rearranging terms, the steady state unemployment rate amongst workers of type δ is given by

$$\frac{g(\delta)u}{h(\delta)} = \frac{\lambda}{\lambda + p}$$

This gives an unemployment rate that is widely applied in the existing literature. It shows that the unemployment rates of workers with productivity δ strictly increases with a separation rate λ and decreases with a job seeking rate p. Also, the rates are identical regardless of workers' productivity level. As known, the productivity level, on average, increases with educational level. Therefore, the model suggests that the unemployment rates are the same for all workers, regardless of educational attainment. It predicts that the EURs for various educational levels increase by the same amount in slumps. This class of model fails in capturing the heterogeneity in the unemployment rate across educational group and the heterogeneous response of the EURs to the real GDP. Hence, without taking the search intensity level into account, the implied EURs cannot capture some of the features of unemployment.

Considering the Level of Search Intensity. Now, we investigate another model in which unemployed workers of type δ are allowed to choose the corresponding optimal search intensity $s^*(\delta)$ to maximize his outside option values. The higher the search intensity level, the more likely the worker will find a job. The corresponding job finding rate is given by $s^*(\delta)p$. With no doubt, workers with a higher productivity will choose to search job more intensively because of a higher search benefit.[11] To equate the flows in and out of unemployment, we obtain the following equation in a steady-state.

$$\begin{aligned}\lambda(h(\delta) - g(\delta)u) &= s^*(\delta)pg(\delta)u \\ \frac{g(\delta)u}{h(\delta)} &= \frac{\lambda}{\lambda + s^*(\delta)p}\end{aligned}$$

With a higher transition rate from unemployment to employment, workers with a higher productivity level (a higher educational level) are associated with a lower unemployment rate, in line with the first documented feature of unemployment: the EUR decreases with educational level. Nevertheless, integrating both sides over δ, the steady-state OUR is given by

$$u = \frac{\lambda}{\lambda + p\int_0^\infty s^*(\delta)dG(\delta)}$$

Without making further assumption, the OUR largely depends on the productivity (education attainment) distribution amongst unemployment. Consider that the proportion of the highly-educated worker increases. Since highly-educated workers optimally choose a higher search intensity level, the OUR falls with the proportion of the groups with higher educational level. This is inconsistent with our observation in that the OUR is not associated with the distribution of educational attainment. In fact, this model illustrates the statistical puzzle of the unemployment rate identity. While the highly-educated workers search more intensively, their job seeking rate is higher and thus their EUR is lower. Hence, any economy with a higher proportion of the highly-educated will have a lower OUR in the steady state. However, this is inconsistent with one of the documented features: the OUR and the

[11] Interested readers are referred to Pissarides (2000).

distribution of the educational attainment are uncorrelated.

The two examples, though simple, uncover the problem in the search-and-matching literature; the existing search equilibrium model fails to capture some of the features of unemployment.[12] This failure calls for a model that could explain the three sets of seemingly unrelated observations and ideally captures both the trend and the magnitude of the EURs, allowing economists to thoroughly understand not only the OUR but also the EURs.

3 The Basic Model

Consider an economy with a continuum of utility maximizing workers and a continuum of profit-maximizing firms. Without loss of generality, the mass of workers is fixed and normalized to unity. Workers and firms are risk neutral and share an identical real interest rate r.[13] Workers are either employed or unemployed.[14] Workers are employed if they hold a position in a firm, and unemployed otherwise. A firm could be filled by at most one worker, and any worker can take up at most one position. Since one firm is basically equivalent to one vacancy, we follow the tradition in this literature to address a firm as a vacancy. Hence, vacancies are either filled or unfilled.

Workers differ in their productivity level δ. Denote $H(\delta)$ as the cumulative distribution function of δ, and the corresponding density function as $h(\delta)$. We assume that the productivity is finite; otherwise, its likelihood is zero. Hence, the only assumption on the distribution function of δ is $\lim_{\delta \to \infty} \delta h(\delta) = 0$.[15] To simplify the analysis, we assume that the lower support of $H(\delta)$ exceeds unemployment benefits z so that workers are willing to sign a contract during a job interview.[16]

[12] Undoubtedly, none of these models could explain the magic number of "one-half" : the lowest EUR is always half the OUR.

[13] This simplification is common in this literature. (Mortensen and Pissarides, 1994; Moen, 1997; Moscarini, 2005; Rogerson et al., 2005; Gonzalez and Shi, 2010; Fujita and Ramey, 2012; Michaillat, 2012) Some applications of the search and matching model assume agents to be risk-averse. The literature that investigates the optimal unemployment benefits with search frictions is one of the examples. (Fredriksson and Holmlund, 2006; Guerrieri et al., 2010) Recent literature also investigates job search behaviors with the preference of ambiguity aversion. (Chan and Yip, 2016)

[14] The model assumes that no decision on labor supply, either the number of working hours or labor force participation, is made. This simplification is standard (Mortensen and Pissarides, 1994; Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Fujita and Ramey, 2012; Michaillat, 2012), and is in line with empirical regularities: cyclical variations in total working hours (unemployment) basically arise from changes in the number of employment but not changes in working hours per worker (labor force participation) (Shimer, 2010).

[15] The assumption is a sufficient condition for the existence of the equilibrium in a steady state, which will be shown later. In fact, it is natural to assume the productivity distribution to be either a log-normal distribution or a Pareto distribution, both of which satisfy the assumption on $H(\delta)$. Indeed, a normal distribution and an exponential distribution also satisfy this assumption. We do not strict the distribution of δ for a broader set of readers who are interested in the steady state equilibrium of similar framework with other productivity distributions.

[16] This assumption rules out the possibility of not participating in a labor market. As mentioned, the decision

Denote $J^E(\delta)$ and $J^U(\delta)$ as value functions of employment and unemployment respectively. An employed worker with a production value δ receives a wage w and faces a separation shock at a Poisson rate λ. When the shock arrives, the employed worker becomes unemployed. Hence, the Hamilton-Jacobi-Bellman equation can be written as follows.

$$rJ^E(\delta) = w(\delta) + \lambda(J^U(\delta) - J^E(\delta)) \tag{1}$$

An unemployed worker receives an unemployment benefit z. He pays a search cost $C : \mathbb{R}_+ \mapsto \mathbb{R}_+$ and selects the optimal level of search intensity s. Assume that the search cost is zero if $s = 0$ and the function is strictly concave. Hence, $C(0) = 0$, $C'(s) > 0$ and $C''(s) > 0$ for all $s \in \mathbb{R}_+$. An unemployed worker transits from unemployment to employment at a rate of $F(s)p$, where $p \in (0,1)$ is a matching rate of the economy.[17] In contrast to the existing literature, the transition rate also depends on $F(s)$, which describes the relative position of search intensity in unemployment. To capture the properties of a transition rate, we assume that $F(s)$ is a cumulative distribution function of search intensity in unemployment for several reasons. First, $F(s)$ increases in s so that the transition rate increases with the rank in the search intensity ladder. Second, $F(s)$ is lower bounded by zero so that the transition rate is zero if no search effort is made in a job search process. Third, $F(s)$ is upper bounded by one to guarantee that a transition rate $F(s)p$ less than one. $F(s) = 1$ does not lead to a transition rate equal to one due to another search friction that is applied generally to the entire economy.[18] It is noteworthy that the distribution of search intensity $F(s)$ is endogenized. The selection of the optimal search intensity largely depends on the relative position in the ladder. Similar to an auction, each unemployed worker submits his own search intensity to bid for a higher transition rate $F(s)p$, and is rewarded with a transition rate $F(s)p$. In the Nash steady state equilibrium, each unemployed worker chooses s to maximize the value of unemployment given the optimal search intensity s of other

on labor force participation is beyond the scope of this paper. Our model matches well with the data under this simplification probably because cyclical variations in unemployment basically arise from changes in the number of employment but not changes in labor force participation (Shimer, 2010).

[17] Since endogenizing market tightness does not provide additional economic insight of this paper but significantly complicates the analysis, we assume that the transition rate does not depend on market tightness. We will show that endogenizing market tightness does not alter the decomposition formula. Moreover, the steady state Nash equilibrium will be shown to exist when market tightness is endogenized in our model.

[18] One might be concerned that workers of different productivity levels might not compete with one another in a job search process. In reality, jobs are not perfectly segregated by productivity level. Therefore, it is reasonable that the high school dropouts and the high school graduates might compete for a similar job type, and the Bachelor's degree holders and the postgraduates might also perform a similar task in their positions. Meanwhile, it is rarely to see that the high school dropouts and the postgraduates compete for the same job. We will show that the steady state Nash equilibrium is in line with these observations. To decide whether to increase one's search intensity, a worker has to consider whether such increase improves his ranking in the ladder. Also, we will show that workers with similar productivity level are in the similar rank; therefore, workers with significant difference in productivity will not compete with one another in the equilibrium, in line with the reality.

unemployed workers.

Given $J^E(\delta)$ and others' best response function s^*, an unemployed worker of type δ chooses his action s to maximize his value of unemployment as follows.

$$rJ^U(\delta) = \max_s \left\{z - C(s) + F(s)p(J^E(\delta) - J^U(\delta))\right\} \tag{2}$$

Regarding vacancies, we assume that their numbers are fixed. When a vacancy is filled by a worker with a production value δ, it generates a production value δ and pays a wage w to the worker. A filled vacancy faces a separation shock at a rate of λ. When the shock arrives, a filled vacancy becomes unfilled, and receives zero profits. Hence, an asset value function of a filled vacancy is written as follows.

$$rJ^F(\delta) = \delta - w(\delta) - \lambda J^F(\delta) \tag{3}$$

Following the existing literature, we assume that wages are determined by maximizing the generalized Nash product. Consequently, expected gains from search are split according to the generalized Nash bargaining solution as follows.

$$J^E(\delta) - J^U(\delta) = \beta\left(J^E(\delta) - J^U(\delta) + J^F(\delta)\right) \tag{4}$$

where $\beta \in (0,1)$ is a bargaining power of workers. The higher the value of β, the greater is the workers' bargaining power. Equating flows in and flow out of unemployment, a steady state unemployment rate is given by

$$\lambda(1-u) = \underbrace{\int F(s^*(\delta))pdG(\delta)}_{\text{Average Job Finding Rate}} \times u \tag{5}$$

where $G(\delta)$ is a cumulative distribution function of the unemployed workers of type δ, with $g(\delta)$ the corresponding probability density function. Hence, $G(\delta)$ measures the unemployment distribution across productivity. The LHS and the RHS of the equation (5) is the flow in and flow out of unemployment, where $p\int F(s^*(\delta))dG(\delta)$ is the average transition rate from unemployment to employment, and $s^*(\delta)$ is the optimal search intensity submitted by the unemployed worker of type δ. Similarly, a steady state δ-specific unemployment is given by

$$\lambda(h(\delta) - g(\delta)u) = F(s^*(\delta))pg(\delta)u \tag{6}$$

The LHS captures the number of employed workers of type δ flowing into unemployment, where $h(\delta) - g(\delta)u$ is the measure of employed workers of type δ. $F(s^*(\delta))p$ is a transition rate into employment of the unemployed workers of type δ. So, the RHS captures their flows out of unemployment, where $g(\delta)u$ is the measure of unemployed workers of type δ.

4 Characterization of Steady-State Nash Equilibrium

Definition 1. *A steady state Nash equilibrium is defined as* $\{s(\delta), J^E(\delta), J^U(\delta), J^F(\delta), u, g(\delta)\}$, *for all* δ,

1. *(Optimal Search Intensity):* $s(\delta)$ *maximizes* $J^U(\delta)$ *given* $s^*(\delta)$ *of other unemployed workers;*
2. *(Value Functions):* $J^E(\delta)$, $J^U(\delta)$, *and* $J^F(\delta)$ *satisfy equations (1)-(3);*
3. *(Rent-Sharing):* $w(\delta)$ *maximizes the generalized Nash product, satisfying the sharing rule (4);*
4. *(Steady-State Accounting):* u *and* $g(\delta)$ *satisfy equations (5) and (6).*

Using equations (1), (3), and (4), the wage equation can be written by

$$w(\delta) = rJ^U(\delta) + \beta(\delta - rJ^U(\delta)) \tag{7}$$

A wage is equal to an outside option value plus a fraction of economic rent. Using equation (2) and (7), the outside option value is given by

$$rJ^U(\delta) = \max_s \left\{ z - C(s) + F(s)\frac{\beta p}{r+\lambda}(\delta - rJ^U(\delta)) \right\} \tag{8}$$

Lemma 1. *If* $\delta_1 > \delta_2$, $s^*(\delta_1) \geq s^*(\delta_2)$ *in a steady state Nash equilibrium.*

Proof. Denote $\Phi(\delta)$ as $\frac{\beta p}{r+\lambda}(\delta - rJ^U(\delta))$. If $\partial\Phi(\delta)/\partial\delta \leq 0$, then $\partial rJ^U(\delta)/\partial\delta \geq 1$. Applying the envelope theorem to equation (8), $\partial\Phi(\delta)/\partial\delta \leq 0$ implies that $\partial rJ^U(\delta)/\partial\delta \leq 0$. A contradiction results. Hence, we can conclude that $\partial\Phi(\delta)/\partial\delta > 0$. We now prove that $s^*(\delta_1) \geq s^*(\delta_2)$ by contradiction. Suppose it is not the case. Given $\delta_1 > \delta_2$, there exists a steady state Nash equilibrium such that $s_1 < s_2$, where s_i is denoted as the optimal search intensity of the worker of type δ_i. Workers of type δ_2 picks s_2 because $F(s_2)\Phi(\delta_2) - C(s_2) \geq F(s_1)\Phi(\delta_2) - C(s_1)$. Therefore, we have

$$\begin{aligned}
(F(s_2) - F(s_1))\Phi(\delta_2) &\geq C(s_2) - C(s_1) \\
(F(s_2) - F(s_1))\Phi(\delta_1) &> C(s_2) - C(s_1) \\
F(s_2)\Phi(\delta_1) - C(s_2) &> F(s_1)\Phi(\delta_1) - C(s_1)
\end{aligned}$$

The second inequality arises because $\Phi(\delta)$ is strictly increasing in δ and $\delta_1 > \delta_2$. According to the last inequality, it is strictly better off for workers of type δ_1 to choose s_2 instead of

s_1. Therefore, we can conclude that $s^*(\delta_1) \geq s^*(\delta_2)$ if $\delta_1 > \delta_2$ in a steady state Nash equilibrium. □

Intuitively, the higher the productivity, the higher is the bargaining wage due to rent-sharing. Hence, an unemployed worker with a higher δ has a higher net search benefit. Consequently, they are willing to pay a search cost at least as high as those with lower δ. Lemma 1 shows that if the net search benefit is sufficiently large for workers of type δ_2 to search at s_2, it is also high enough for a worker of type $\delta \geq \delta_2$ to search at the same intensity s_2. We will show that $ds^*(\delta)/d\delta > 0$ is the rule to construct the strategy in a steady state Nash equilibrium. Furthermore, Lemma 1 implies that all the unemployed workers of type $x \leq \delta$ will search with $s^*(x)$ not higher than $s^*(\delta)$, meaning that $G(\delta) = F(s^*(\delta))$. With the least production value $\delta = z$, $s^*(z)$ is always the lowest search intensity. Therefore, $0 = F(s^*(z)) = G(z)$. With zero transition rate, unemployed workers of type z will search with no effort. That is, $s^*(z) = 0$.

Differentiating both sides of $G(\delta) = F(s^*(\delta))$ with respect to δ yields $g(\delta) = f(s^*(\delta))ds^*(\delta)/d\delta$. Using $F(s^*(\delta)) = G(\delta)$, $G(z) = 0$ and equation (5), a steady state unemployment rate is given by

$$u = \frac{\lambda}{\lambda + \int_z^\infty F(s^*(x))pdG(x)} = \frac{\lambda}{\lambda + p\int_z^\infty G(x)dG(x)} = \frac{\lambda}{\lambda + \frac{1}{2}p} \tag{9}$$

Ostensibly, the OUR (9) is as usual as the one derived from the existing search-and-matching literature: u strictly increases with λ but decreases with p. Basically, equation (9) ensures that the derived OUR is unchanged even though the assumption on heterogeneity in workers' productivity is relaxed. In other words, the assumption on homogeneity in workers' productivity in a search-and-matching model does not lose any predictive power on the OUR.

Using $F(s^*(\delta)) = G(\delta)$, the δ-specific unemployment rate (6), and the OUR (9), Appendix 7.1 gives the proof that $G(\delta)$ can be written as follows.

$$G(\delta) = \frac{u}{2(1-u)}\left(\sqrt{1 + \frac{4(1-u)H(\delta)}{u^2}} - 1\right) \tag{10}$$

Differentiating $G(\delta)$ with respect to δ and rearranging terms, the δ-specific unemployment rate u_δ is given by

$$u_\delta \equiv \frac{g(\delta)u}{h(\delta)} = \left(1 + \frac{4(1-u)H(\delta)}{u^2}\right)^{\frac{-1}{2}} \tag{11}$$

which is a function of the OUR and the distribution of productivity. When δ tends to infinity, it is straightforward to show that u_δ approaches $\frac{u}{2-u}$. Thus far, the steady state OUR (9) and δ-specific unemployment rate (11) have been derived.

Differentiating equation (2) with respect to s and using equations (1) and (7) give the best response function of workers of δ as follows.

$$C'(s^*(\delta)) = f(s^*(\delta))\frac{\beta p}{r+\lambda}(\delta - rJ^U(\delta)) \tag{12}$$

The optimal search intensity equates a marginal search cost to a marginal search benefit. Using $f(s^*(\delta))ds^*(\delta)/d\delta = g(\delta)$ and equation (12), one could verify that search effort strictly increases with δ.

$$\frac{ds^*(\delta)}{d\delta} = \frac{\beta p g(\delta)(\delta - rJ^U(\delta))}{(r+\lambda)C'(s^*(\delta))} > 0 \tag{13}$$

Using $f(s^*(\delta))ds^*(\delta)/d\delta = g(\delta)$ and equation (2), (10) and (11), the optimal $s^*(\delta)$ in equation (12) can be found by solving $C \equiv C(s^*(\delta))$ in the following first order linear differential equation.[19]

$$\frac{dC}{d\delta} = T(\delta)(\delta - z) + T(\delta)C \tag{14}$$

where $T(\delta) \equiv \frac{\beta p h(\delta)}{(r+\lambda)u\Phi_1(\delta)(1+\Phi_2(\delta))}$, $\Phi_1(\delta) \equiv \sqrt{1 + \frac{4(1-u)H(\delta)}{u^2}}$, $\Phi_2(\delta) \equiv \frac{\beta\lambda}{r+\lambda}(\Phi_1(\delta)-1)$. The initial condition is given by $C(s^*(z)) = 0$. The unique solution to this initial value problem is given by

$$C(s^*(\delta)) = \int_z^\delta T(x')(x'-z)e^{\int_{x'}^\infty T(y)dy}dx' \tag{15}$$

Denote $\Omega(\delta,p)$ as the RHS of the above equation. Appendix 7.3 shows that $\lim_{p\to 0}\Omega(\delta,p)$ is zero, $\lim_{p\to\infty}\Omega(\delta,p)$ is positive finite, and $\lim_{p\to\infty}\frac{\partial\Omega(\delta,p)}{\partial p}$ is zero. The assumption $\lim_{\delta\to\infty}\delta h(\delta) = 0$ ensures that the solution is finite for all $\delta \geq z$. Since $C(s^*(\delta))$ strictly increases with s^*, the unique s^* is given by

$$s^*(\delta) = C^{-1}\left(\Omega(\delta,p)\right) \tag{16}$$

$\partial\Omega(\delta,p)/\partial p \geq 0$ implies that $s^*(\delta)$ is increasing in p for all $\delta \geq z$. With a higher p, an increase in a transition rate and thus a marginal search benefit from an increase in the relative ranking in intensity is amplified. Unemployed workers would have sufficient incentive to deviate to the new Nash equilibrium with higher $s^*(\delta)$. Interestingly, in response to the rise in p, all unemployed workers but the ones with a production value equal to z increase their search intensity but the ranking of no one gets improved.

[19] The derivation is given in the Appendix.

Now, we show that the marginal search cost (MSC) intersects the marginal search benefit (MSB) at the $s^*(\delta)$ where no worker of type δ has any incentive to deviate from this strategy. Suppose that the MSB curve cuts the MSC curve from below. Since $f(s)$ is upper bounded, given $rJ^U(\delta)$ the marginal search benefit is upper bounded. However, $\lim_{s\to\infty} C'(s) = \infty$. Therefore, if there exists an intersection such that the MSB curve cuts the MSC curve from below, there exists another intersection at $s > s^*(\delta)$. A contradiction results because the unique $s^*(\delta)$, given by equation (16), implies that the MSC and the MSB curve intersect once in the interval $\mathbb{R}_{++}$. Now, we can conclude that $s^*(\delta)$ happens at the intersection where the MSC curve cuts the MSB curve from below. Workers of δ have no incentive to cut search intensity because the marginal search benefit is greater than the marginal search cost for all $s \in (0, s^*(\delta))$. Also, they are reluctant to increase search intensity because the marginal search benefit is less than the marginal search cost for all $s > s^*(\delta)$. Given that other workers follow the strategy $s^*(\delta)$ from equation (16), any worker of type δ has no incentive to deviate from the search effort $s^*(\delta)$ that satisfies equation (12) and is given by (16).

Proposition 1. ***(Existence of the Equilibrium)*** *There exists a steady state Nash equilibrium defined in Definition 1. A steady state OUR and the δ-specific unemployment rate are given by equation (9) and (11) respectively.*

Proposition 1 states the existence of the equilibrium. Nevertheless, it does not guarantee the uniqueness of the equilibrium. In fact, for all $\delta \geq z$, $s^*(\delta) = 0$ will yield another steady state Nash equilibrium that satisfies Definition 1. Given that all other unemployed workers do not search, the best response function of workers of any type is $s^*(\delta) = 0$, and the corresponding transition rate $F(s)p$ is equal to p, which is the highest. If a worker deviates from the equilibrium and search at any positive intensity $s^*(\delta) > 0$, his transition rate remains p. However, he has to pay a positive search cost $C(s^*(\delta)) > 0$. Hence, given that no unemployed workers search at positive intensity, deviating from $s^*(\delta) = 0$ lowers one's payoff. In this equilibrium, $s^*(\delta) = 0$ and $C(s^*(\delta)) = 0$. The value function of unemployment of type δ reduces to

$$rJ^U(\delta) = z + p(J^E(\delta) - J^U(\delta))$$

which is equivalent to the value function of unemployment in a standard search-and-matching model without search intensity. In other words, our model nets the standard model. As discussed in section 2.2, this type of model fails to capture some of the recent features of unemployment. Hereafter, we will focus on the steady state Nash equilibrium established so far. Theorem 1 will summarize the four properties of these derived unemployment rates in the steady state Nash equilibrium.

Theorem 1. ***(Properties of Unemployment Rates)*** *In a steady state Nash equilibrium,*

1. *the δ-specific unemployment rate u_δ unambiguously decreases with δ;*
2. *u is independent of the distribution of productivity $H(\delta)$;*
3. *a rise in λ and a fall in p increase u_δ and u;*
4. *if $\frac{2H(\delta)}{u}(\frac{1}{u}-1) > 1$, the δ-specific unemployment rate with a lower δ increases more in slumps; and*
5. *the lowest δ-specific unemployment rate u_δ is equal to $\frac{u}{2-u}$.*

Proof. See the Appendix 7.4. □

The First Property. The first statement indicates that the higher the productivity of a group, the lower will be their unemployment rate. This implication is seemingly no difference from the standard one in the literature in that an unemployed worker with a higher search intensity level is more likely to transit into employment. However, the mechanisms that drive them to get rid of unemployment faster are different. In the literature, the unemployed worker is more likely to get hired because of the higher level of search intensity. Here, the level of search intensity essentially plays no role in determining the transition rate in our model. Instead, a higher search intensity places an unemployed worker at a higher position in the search ladder, which in turn rewards him a higher transition rate.

The Second Property. It is noteworthy that u is a function of a separation rate λ and a matching rate that is generally applied to an economy p, not the average level of search intensity or the distribution of δ. This property is important for three reasons. First, the independence of u from the productivity distribution explains at least two phenomena. It explains not only why the correlation between the OUR and the distribution of educational attainment is close to zero, as shown in Figure 2, but also why the natural rate of unemployment remains steady for quite a long time (at least 50 years in the U.S.) even though the proportion of highly-productive workers increases over time (which in turn increases the human capital). Second, one should be aware that it is rather difficult to obtain the information on workers' search intensity and is hard to measure search intensity in practice. The independence guarantees that the derived OUR is free from any function of search intensity, making practitioners easier to uncover the OUR simply with the two transition rates. Third, the independence property is unconditional on the stage of the development of an economy, meaning that the independence in principle holds in economies regardless of the distribution of productivity.

Mathematically, the OUR is negatively associated with the average transition rate, given by $p\int_z^\infty F(s^*(x))dG(x)$. Since the higher the productive, the higher is the search intensity and thus the relative position in the search intensity distribution. The search intensity distribution coincides with the distribution of productivity in the unemployment. That is,

$F(s^*(\delta)) = G(\delta)$. The average transition rate reduces to $p\int_z^\infty G(x)dG(x)$. Since the mean value of any cumulative distribution function is $1/2$, the average transition rate further reduces to $p/2$, independent of the distribution of search intensity or productivity. Intuitively, consider a worker increases his search effort such that his ranking slightly improves. On the one hand, such increase in the search effort shrinks his average unemployment spell. On the other hand, there exist other unemployed workers whose rankings decline in the search effort ladder, lengthening the average unemployment spell of those with the decline in the rankings. More importantly, the negative externality, created by the increase in one's search effort, is of the same magnitude as the improvement in the one's ranking. Overall, the average transition rate of an economy is unaffected by the reallocation of the ranking, remaining $p/2$. This explains why a rise in one's search effort could shorten his own average unemployment duration but not the unemployment spell of the economy as a whole, leaving the OUR independent of the distribution of productivity or search effort.

The Third Property. The third statement indicates that a higher λ and a lower p increase the δ-specific unemployment rates. Intuitively, the higher separation rate increases the flow into unemployment and the lower job finding rate reduces the flow out out of unemployment, both of which lengthen the unemployment duration and increase the δ-specific unemployment rate. We can also infer that it is the increase in λ and/or the decrease in p that leads to a rise in the δ-specific unemployment rates, which in turn increases the OUR in slumps.

The Fourth Property. This statement illustrates that the increase in δ-specific unemployment rate is more pronounced for workers with a lower production value under a condition. Since one of the common component of a transition rate $F(s^*(\delta))p$ declines in slumps, it takes longer time for an unemployed worker to get rid of unemployment regardless of δ. Hence, u_δ increases in slumps. Indeed, a fall in p has two forces on a δ-specific unemployment rate. In a steady state, the flow out of unemployment of type δ amounts to $F(s^*(\delta))pg(\delta)u$. A decline in a matching rate p reduces the difference in the transition rate $F(s^*(\delta))p$ between the workers of various search effort because $F(s^*(\delta))$ is strictly increasing in δ. Consequently, a decline in a matching rate p reduces the difference in the δ-specific unemployment rates between workers of different production values. However, in response to a lower p in slumps, the decrease in the flow out of unemployment will also be smaller amongst workers with a higher unemployment stock $g(\delta)u$. Notice that the unemployment rate is higher amongst workers with lower productivity, increasing the volatility of their unemployment rates in the economic downturn. If the second effect is dominant (i.e. $\frac{2H(\delta)}{u}(\frac{1}{u} - 1) > 1$), the δ-specific unemployment rate increases more for workers with a lower δ in slumps. The intuition is similar for the rise in λ.

The Fifth Property. This property relates the lowest possible δ-specific unemployment rate to the OUR. Recall from the first property that the higher the productivity level, the lower is the δ-specific unemployment rate. Hence, δ-specific unemployment rate reaches its minimum when δ tends to infinity. Using equation (11), it is straightforward to show that the lowest possible unemployment rate is $\frac{u}{2-u}$, which is close to half of the OUR. Notice that the unemployment rate of one group is negatively associated with the average job finding rate in the group. For example, the higher the transition rate $F(s^*(\delta))p$, the smaller is the corresponding δ-specific unemployment rate. As shown in Lemma 1, the higher the productivity level, the higher is the search intensity level. Workers with the highest productivity level will search the most intensively in an economy so that their unemployed workers rank top in the intensity ladder. Hence, their transition rate becomes p. Regarding the economy as a whole, the average transition rate is equal to $\int_0^1 F(s^*(\delta))pG(\delta)$. Again, Lemma 1 implies that the search intensity distribution $F(s^*(\delta))$ coincides with the productivity distribution in the unemployment $G(\delta)$. Therefore, the rate reduces to $p\int_0^1 G(\delta)dG(\delta)$, which is equal to $p/2$.[20] Consequently, the job finding rate of the entire population is about half the rate of the workers with the highest productivity level; therefore, the unemployment rate for the workers with the highest productivity about twice the OUR. This striking result holds regardless of the distribution of the productivity.

Clearly, these properties of unemployment in Theorem 1 correspond to the three sets of observations. But the fourth property is valid only if $\frac{2H(\delta)}{u}(\frac{1}{u}-1)$ holds. How likely does the inequality hold? Notice that $\frac{2H(\delta)}{u}(\frac{1}{u}-1)$ strictly increases with $H(\delta)$ and decreases with u. According to US CPS data, the proportions of high-school graduates or below followed a decreasing trend and reached its lowest point at 34% in 2015. According to Bureau of Labor Statistics Data, the highest recorded monthly unemployment rate (since 1948) was 10.8% in November and December in 1982. With $H(\delta) = 34\%$ and $u = 10.8\%$, $\frac{2H(\delta)}{u}(\frac{1}{u}-1)$ should exceed 52, far above one. In other words, the smallest value of $\frac{2H(\delta)}{u}(\frac{1}{u}-1)$ exceeds 52 during 1948-2015. In fact, the inequality still holds even if the share of high school graduate or below $H(\delta)$ is as low as 5% and the unemployment rate is as high as 25%. Following the current decreasing trend in the share of high school graduate or below (minus 1% each year), we expect that the condition holds not only during 1948-2015 but also the coming thirty years or even more. In other words, the fourth property in Theorem 1, together with the other properties, is expected to hold in the United States during 1948-2045.

5 Model Implications

Recall that the first objective of this paper is to construct a model that explains the three features of unemployment. As stated in Theorem 1, the OUR and the δ-specific unemploy-

[20] $p\int_0^1 G(\delta)dG(\delta) = p\int_0^1 x dx = p/2$.

ment rates derived from our model do capture the features. We should highlight that our modification is indeed widely applicable but this paper purposely presents a simple search equilibrium model to shed light on the crucial role of the relativity of search intensity during a job search process. In section 2.2, we show that the existing search and matching model, in which the search intensity level matters in determining a job finding rate, fails in capturing more than one unemployment features, still less in predicting its magnitudes.

The objective of this section is twofold: first, to evaluate the predictive power of our model, where only the relativity of search intensity matters in determining a job finding rate; and second, to discuss the implications of this model. In particular, this section derives and evaluates the formulas of the unemployment distribution (10), the EURs (11), and the fundamental frictional unemployment rate. Throughout the section, the formulas are evaluated using the United States Current Population Survey 1994-2015, and the samples of examination are restricted to a labor force aged 25-60.

In practice, it is difficult to observe the exact production value of a worker. We therefore make two assumptions:

Assumption 1. *Productivity strictly increases with educational attainment.*

Assumption 2. *A productivity distribution is continuous over its support.*

We use j to denote an educational group: the higher the number of j the higher is the educational level. Denote $\bar{\delta}_j$ and $\underline{\delta}_j$ as the highest and the lowest productivity level in an educational group j. Denote H_j as the cumulative distribution function of a worker of type $\bar{\delta}_j$ in educational group j. Assumption 1 implies that $\underline{\delta}_{j+1} > \bar{\delta}_j$, and Assumption 2 ensures that given any j, $|H(\underline{\delta}_{j+1}) - H(\bar{\delta}_j)| < \varepsilon$ for all $\varepsilon > 0$.

5.1 Unemployment Distribution

This subsection discusses the unemployment distribution derived from the model. According to equation (10), $G(\delta)$ is the cumulative distribution function of productivity in the unemployment, which is indeed the unemployment distribution. Using equations (10) and (11), simple algebra gives the following theorem.

Theorem 2. ***(Unemployment Distribution)*** *Suppose the assumption 1 and 2 are satisfied. In a steady state Nash equilibrium, the cumulative distribution function of the educational* *group j in the unemployment G_j is given by*

$$G_j = \frac{1}{2}\frac{\Psi(u)}{\Psi(u_j)}$$

where $\frac{\Psi(u)}{\Psi(u_j)}$ *is the unemployment odds ratio;*

$\Psi(u) \equiv \frac{u}{1-u}$ *is the overall unemployment odds;*

$\Psi(u_j) \equiv \frac{u_j}{1-u_j}$ *is the educatioal unemployment odds j;*

and $u_j = \left(1 + \frac{4(1-u)H_j}{u^2}\right)^{-\frac{1}{2}}$.

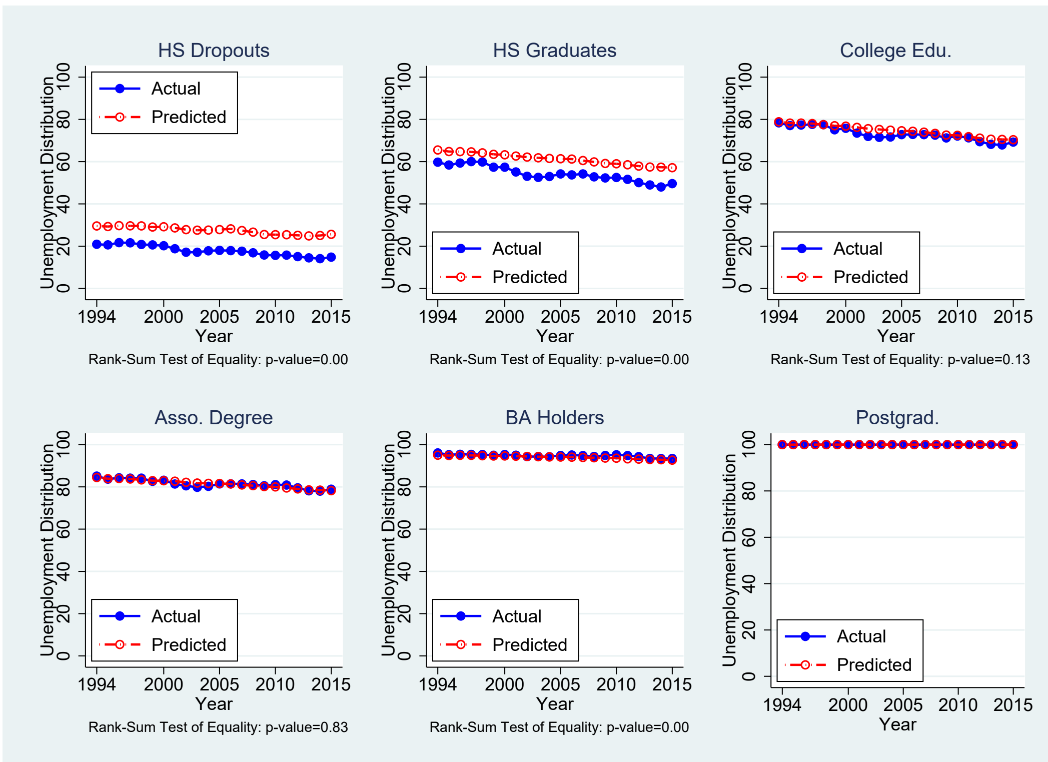
Proof. See the Appendix 7.5. □

Our model derives a novel formula of the unemployment distribution across educational levels, which is equal to half the unemployment odds ratio. It is noteworthy that $G(\delta)$ is endogenized in our model; no restriction is imposed on the function $G(\delta)$. To be a valid cumulative distribution function, $G(\delta)$ has to satisfy three properties: (i) its increases with productivity, (ii) it equals zero at its lower support, and (iii) it equals one at its upper support. First, differentiating $G(\delta)$ with respect to δ, simple algebra shows that $G(\delta)$ strictly increases with productivity δ. Second, it is straightforward to verify that $G(\delta)$ approaches zero (one) when $H(\delta)$ approaches zero (one). We can therefore conclude that $G(\delta)$ is a valid cumulative distribution function.

Next, we proceed to verify the predictive power of G_j. According to Theorem 2, the unemployment odds ratio is a function of two variables: the OUR u and the distribution function of educational level H_j, which are easily accessible. Using the US CPS, we compute the yearly u and H_j for each educational group. Both the actual (the solid lines) and the predicted (the dash lines) annual G_j are plotted in Figure 4. Indeed, the predicted G_t are about 10 percent and five percent more than the actual ones for high-school dropouts and high-school graduates. One of the possibility, that accounts for the difference, is that low-educated unemployed workers decide not only on their search intensity, but also on whether to participate to a labor force. Nevertheless, our model did not explicitly endogenize the labor force participation decision, and we leave it for future research avenue.[21] We perform the Wilcoxon rank-sum test of equality in each educational group. Surprisingly, the null hypothesis that the actual and the predicted G_j are from an identical distribution cannot be rejected at any conventional significance level for workers with some college education and associate degree holders. Although the rank-sum test rejects the equality between the actual and the predicted G_t for Bachelor's degree holders, they are so close that the two lines overlap with each other during the entire period of examination. Regarding postgrad-

[21] For example, incorporating the search relativity in the framework of Alvarez and Shimer (2011) could be a fruitful and interesting area of future research.

Figure 4: Evaluation of the Unemployment Distribution



Notes: Data are from the US CPS. Samples are restricted to the labor force aged 25-60. The predicted data (the dotted line) are generated from Theorem 2.

Table 3: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted Unemployment Distribution

State	**Unemployment Distribution**			
	(1)	**(2)**	**(3)**	**(4)**
	HS Grad.	Some College	AS Degree	BA Holders
AK	0.00**	0.01**	0.00**	0.00**
AL	0.00**	0.17	0.00**	0.00**
AR	0.03**	0.06	0.00**	0.00**
AZ	0.00**	0.01**	0.23	0.81
CA	0.00**	0.00**	0.05**	0.21
CO	0.00**	0.00**	0.00**	0.04**
CT	0.00**	0.00**	0.04**	0.89
DE	0.00**	0.44	0.93	0.04**
FL	0.00**	0.00**	0.03**	0.98
GA	0.00**	0.21	0.42	0.00**
HI	0.00**	0.06	0.72	0.31
IA	0.00**	0.93	0.02**	0.17
ID	0.00**	0.03**	0.44	0.36
IL	0.00**	0.36	0.87	0.09
IN	0.00**	0.42	0.01**	0.00**
KS	0.00**	0.12	0.41	0.96
KY	0.00**	0.66	0.00**	0.00**
LA	0.17	0.01**	0.00**	0.00**
MA	0.00**	0.00**	0.01**	0.96
MD	0.00**	0.37	0.69	0.09
ME	0.00**	0.51	0.72	0.57
MI	0.00**	0.16	0.00**	0.00**
MN	0.00**	0.00**	0.03**	0.64
MO	0.00**	0.45	0.31	0.00**

Notes: p-values are reported. Significance levels: **=5%. HS Grad., Some College, AS Degree, and BA Holders denote high-school graduates, workers with some college education, associate degree holders, and Bachelor's degree holders.

Table 4: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted Unemployment Distribution (Cont.)

State	**Unemployment Distribution**			
	(1)	**(2)**	**(3)**	**(4)**
	HS Grad.	Some College	AS Degree	BA Holders
MS	0.00**	0.24	0.00**	0.00**
MT	0.00**	0.00**	0.96	0.96
NC	0.00**	0.83	0.1	0.04**
ND	0.00**	0.61	0.14	0.53
NE	0.00**	0.31	0.71	0.61
NH	0.00**	0.00**	0.00**	0.01**
NJ	0.00**	0.08	0.27	0.09
NM	0.01**	0.57	0.17	0.02**
NV	0.00**	0.01**	0.62	0.67
NY	0.00**	0.01**	0.05**	0.00**
OH	0.00**	1	0.03**	0.00**
OK	0.00**	0.53	0.05**	0.00**
OR	0.00**	0.12	0.57	0.06
PA	0.00**	0.02**	0.72	0.19
RI	0.00**	0.41	0.27	0.03**
SC	0.00**	0.48	0.00**	0.00**
SD	0.00**	0.02**	0.01**	0.78
TN	0.01**	0.5	0.02**	0.00**
TX	0.00**	0.13	0.89	0.02**
UT	0.00**	0.00**	0.17	0.74
VA	0.00**	0.01**	0.2	0.59
VT	0.00**	0.00**	0.04**	0.37
WA	0.00**	0.00**	0.12	0.2
WI	0.00**	0.67	0.19	0.00**
WV	0.07	0.15	0.00**	0.00**
WY	0.00**	0.03**	0.76	0.15

Notes: p-values are reported. Significance levels: **=5%. HS Grad., Some College, AS Degree, and BA Holders denote high-school graduates, workers with some college education, associate degree holders, and Bachelor's degree holders.

uates, the predicted G_t is always equal to one because their productivity is assumed to be the highest in the economy due to Assumption 1. With certainty, the two lines are identical in this educational group. We conduct the same exercise in each of the 50 states in the United States, and plot the actual and the predicted values in Figure B.1. The corresponding p-values are reported in Table 3. Surprisingly, the predictability of Theorem 2 in each state is as good as its performance in an economy as a whole. Over 50 percent of the states, the null hypothesis on the equality between the two sets of values cannot be rejected for some college education, associate degree holders, and Bachelor's degree holders. We conclude that Theorem 2 succeeds in predicting the unemployment distribution in magnitude for workers with some college education, associate degree holders, and Bachelor's degree holders and performs fairly well for high-school graduates.

5.2 Educational Unemployment Rate

Recall that another objective of this paper is to derive a formula that allows economists, policymakers, and the public to disaggregate the OUR into various δ-specific unemployment rates (the EURs). Indeed, equation (11) is the disaggregation formula that maps an OUR to the unemployment rate of the xth percentile of a productivity distribution. Although x is unobservable, equation (11) can be computed under Assumption 1 and 2 as follows.

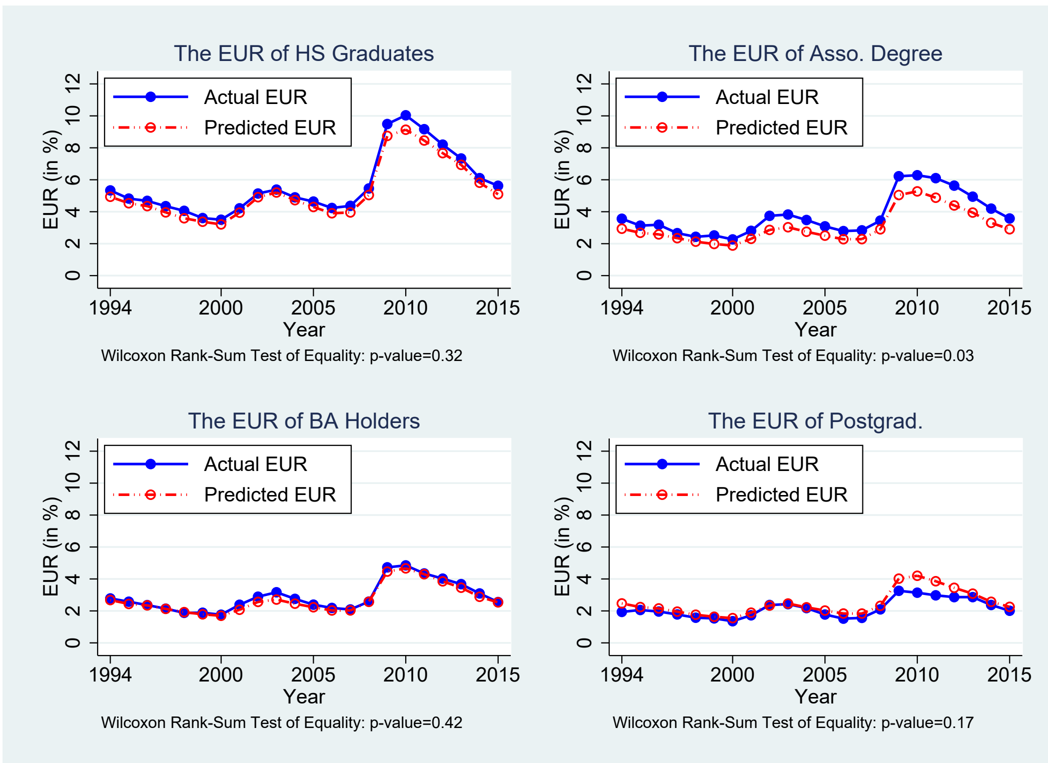
Theorem 3. ***(Disaggregation Theory)*** *Suppose the assumption 1 and 2 are satisfied. In a steady state Nash equilibrium, the EUR of the educational group j is given by*

$$\begin{aligned} u_j &= \frac{2u}{(B_j + B_{j-1})} \\ \textit{where} \quad & B_j \equiv (u^2 + 4(1-u)H_j)^{\frac{1}{2}} \end{aligned} \tag{17}$$

Proof. See the Appendix 7.6. □

Using the US CPS, the actual and the predicted EURs of high-school graduates, associated degree holders, Bachelor's degree holders, and postgraduates are computed. Both the actual (the solid lines) and the predicted (the dash lines) annual unemployment rates of 1994-2015 are plotted in Figure 5. As expected due to Theorem 1, the two sets of values display a similar trend regardless educational level. More strikingly, the magnitude of the actual and the predicted EURs are so close that the null hypothesis that they are drawn from an identical distribution cannot be rejected by the Wilcoxon rank-sum test for the high-school graduates, the Bachelor's degree holders, and the postgraduates. With no doubt, the disaggregation theory does a superb performance in mapping the OUR into the EURs in the United States.

Figure 5: Evaluation of the Disaggregation Theory



Notes: Data are from the US CPS. Samples are restricted to the labor force aged 25-60. The predicted data (the dotted line) are generated from Theorem 3.

Table 5: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted EURs

State	**Disaggregation**				**Approximation**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
	HS Grad.	AS Degree	BA Holders	Postgrad.	Postgrad.
AK	0.02**	0.00**	0.01**	0.00**	0.00**
AL	0.29	0.03**	0.23	0.00**	0.00**
AR	0.04**	0.71	0.13	0.00**	0.00**
AZ	0.47	0.05	0.32	0.72	0.85
CA	0.16	0.04**	0.21	0.53	0.69
CO	0.76	0.15	0.09	0.21	0.15
CT	0.94	0.28	0.26	1.00	0.81
DE	0.34	0.32	0.85	0.27	0.37
FL	0.66	0.05**	0.26	0.94	0.81
GA	0.30	0.02**	0.66	0.05**	0.10
HI	0.27	0.20	0.98	0.45	0.51
IA	0.74	0.32	0.41	0.24	0.28
ID	0.85	0.03**	0.89	0.25	0.29
IL	0.32	0.06	0.83	0.21	0.37
IN	0.69	0.24	0.74	0.03**	0.03**
KS	0.57	0.08	0.34	0.56	0.89
KY	0.22	0.01**	0.27	0.00**	0.00**
LA	0.08	0.00**	0.00**	0.00**	0.00**
MA	0.89	0.16	0.13	0.93	0.64
MD	0.36	0.32	0.67	0.32	0.53
ME	0.54	0.28	0.89	0.22	0.34
MI	0.67	0.29	0.64	0.01**	0.01**
MN	0.71	0.04**	0.25	0.61	0.40
MO	0.89	0.08	0.93	0.00**	0.01**

Notes: p-values are reported. Significance levels: **=5%. HS Grad., AS Degree, BA Holders, Postgrad. denote high-school graduates, associate degree holders, Bachelor's degree holders, and postgraduates. Column (1)-(4) uses Theorem 3, and column (5) uses Theorem 4.

Table 6: Wilcoxon Rank-Sum Test of Equality between the Actual and the Predicted EURs (cont.)

State	**Disaggregation**				**Approximation**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
	HS Grad.	AS Degree	BA Holders	Postgrad.	Postgrad.
MS	0.04**	0.10	0.02**	0.00**	0.00**
MT	0.94	0.00**	0.81	0.76	0.93
NC	0.37	0.57	0.64	0.27	0.35
ND	0.93	0.02**	0.26	0.94	0.93
NE	0.98	0.11	0.94	0.72	1.00
NH	0.51	0.01**	0.02**	0.20	0.10
NJ	0.62	0.08	0.26	0.32	0.43
NM	0.18	0.02**	0.78	0.02**	0.04**
NV	0.32	0.05**	0.53	0.80	0.89
NY	0.59	0.04**	0.03**	0.19	0.39
OH	0.56	0.21	0.98	0.03**	0.05**
OK	0.34	0.01**	0.91	0.01**	0.01**
OR	0.64	0.04**	0.61	0.14	0.56
PA	0.32	0.00**	0.72	0.53	0.64
RI	0.42	0.08	0.71	0.15	0.21
SC	0.37	0.49	0.22	0.00**	0.00**
SD	0.51	0.64	0.04**	0.51	0.62
TN	0.15	0.17	0.50	0.03**	0.05**
TX	0.02	0.01**	0.81	0.20	0.29
UT	0.96	0.29	0.27	0.79	0.67
VA	0.85	0.04**	0.34	0.83	0.57
VT	0.98	0.32	0.11	0.80	0.89
WA	0.94	0.01**	0.23	0.20	0.36
WI	0.91	0.21	0.78	0.09	0.11
WV	0.13	0.01**	0.04**	0.00**	0.00**
WY	0.76	0.02**	0.87	0.21	0.24

Notes: p-values are reported. Significance levels: **=5%. HS Grad., AS Degree, BA Holders, Postgrad. denote high-school graduates, associate degree holders, Bachelor's degree holders, and postgraduates. Column (1)-(4) uses Theorem 3, and column (5) uses Theorem 4.

Table 7: Summary of the Wilcoxon Rank-Sum Test of Equality

Significance	Disaggregation				Approximation
Level	(1)	(2)	(3)	(4)	(5)
	HS Grad.	AS Degree	BA Holders	Postgrad.	Postgrad.
1%	50 (100)	40 (80)	48 (96)	39 (78)	39 (78)
5%	46 (92)	26 (52)	43 (86)	34 (68)	35 (70)
10%	45 (90)	20 (40)	42 (84)	33 (66)	33 (66)
25%	40 (80)	12 (24)	34 (68)	21 (42)	29 (58)
50%	27 (54)	3 (6)	23 (46)	16 (32)	18 (36)

Notes: There are 50 states in the United States. DC is excluded. The number reported in this table is the number of states that fails to reject the null hypothesis that the observed and the predicted EURs are from an identical distribution at the corresponding significance level. The number in parentheses is the percent of the total number of the states, in which the null hypothesis that the observed and the predicted EURs are from an identical distribution cannot be rejected at the corresponding significance level. HS Grad., AS Degree, BA Holders, Postgrad. are high-school graduates, associate degree holders, Bachelor's degree holders, and postgraduates. Column (1)-(4) uses the disaggregation theory (3), and column (5) uses the disaggregation approximation theory (4).

To further evaluate the predictability of Theorem (3), we apply the disaggregation theory to predict the yearly EURs of high-school graduates, associated degree holders, Bachelor's degree holders, and postgraduates by state during 1994-2015. Both the actual (the solid lines) and the predicted (the dash lines) annual unemployment rates of 1994-2015 are plotted in Figure C.1. According to the figure, the observed and the predicted EURs are close in its trend and magnitude for most the educational groups and states. We perform the Wilcoxon rank-sum test for each of the educational groups in each state, and report the corresponding p-values in Table 5. The table suggests that most the tests cannot reject the null hypothesis that the actual and the predicted EURs are from an identical distribution. Table 7 summarizes the results of the Wilcoxon rank-sum test. According to 7, the null hypothesis cannot be rejected at one percent level in all of the 50 states for the high-school graduates. Meanwhile, we cannot reject the null hypothesis in 40, 48, and 39 out of 50 states for associate degree holders, Bachelor's degree holders, and postgraduates at one percent significance level. At the conventional significance level (five percent), we cannot reject the null hypothesis in over half the United States regardless of educational level. Surprisingly, the null hypothesis cannot be rejected over 50 percent of the states for high-school graduates (90 percent), Bachelor's degree holders (84 percent), and postgraduates (66 percent) at 10 percent significance level. More strikingly, in the group of high-school graduates and Bachelor's degree holders, the null hypothesis cannot be rejected in about half the United States at 50 percent significance level. These results suggest that the derived formula (3) performs so well (if not considered as nearly perfectly) in predicting both the trends and the magnitudes of the EURs that most the statistical tests could not reject the null hypothesis that the derived formula (3) is identical to the underlying data generating process of the EURs.

Remarks. Thus far, we have demonstrated the superb performance of Theorem 2 and 3. Before closing this subsection, several points deserve attention. First, our theories utilize the least number of input variables. The unemployment distribution and the EUR are functions of only two variables: an OUR u and a cumulative productivity distribution function H_j, free from other parameters. The degree of freedom is indeed zero. Consider two formulas with different numbers of input variables. If both the formulas could predict any one of the unemployment distribution and the EUR at the same accuracy level, it is likely that the formula with less number of input variables is preferred.

Of course, the accessibility of input variables matters in the selection of any two formulas. Our theory requires the fraction of each educational groups and the overall unemployment rate. Needless to mention, it is straightforward to obtain an OUR and the distribution of the educational groups in most (if not all) countries. On the contrary, it is rather challenging to obtain the information about individual search intensity level, the functional form of a matching function, and a separation rate λ. Therefore, given the same accuracy, the formulas have its advantage over others (if others exist) in the accessibility of its inputs.

Third, the two formulas are not computer-intensive. Unlike nowadays fancy estimation methods, our formulas involve no recursive structure, systems of equations or samples of an enormous number of observations. Indeed, the computation takes almost no time and is easy to implement for economists, policymakers, and the public.

Of course, with only two input variables, the predictability of the formulas is our concern. As demonstrated, the Wilcoxon rank-sum test of equality cannot reject most the null hypotheses that the actual and the observed values are drawn from the same distribution at any conventional significance level in most the states. With no doubt, we can conclude that our model describes well the macroeconomic relationship between an OUR and an unemployment distribution and between an OUR and EURs, and the underlying theory succeeds in mapping the OUR into the unemployment distribution and the EURs in at least four aspects: its predictability, its least number of input variables, its accessibility of the required inputs, and its easiness of implementation.

5.3 Fundamental Frictional Unemployment Rate

In this section, we discuss a measure of the fundamental frictional unemployment rate (FFUR). To begin with, recall that the unemployment rate of workers falls with their average search intensity. This subsection asks if workers keep increasing their search intensity and/or productivity, does their unemployment rate approach zero? In other words, if search cost is sufficiently low and productivity is sufficiently high, does search frictional unemployment vanish? If not, what is the frictional unemployment rate, abstracting search effort and productivity?

Here, we define the FFUR as the unemployment rate that associates with the unemployment spell that cannot be shortened by further increasing one's search effort or productivity. Therefore, there exists no EUR that can be lower than the FFUR of an economy. Lemma 1 implies that those with the highest productivity should search the most intensively. It is therefore expected that the search intensity of the highest productivity workers ranks top in the search ladder. That is, $F(s) = 1$, leaving their transition rate equal to p. With the highest productivity in the economy, their unemployment rate in principle sheerly arises from search friction, net of search effort and productivity. Workers with higher productivity experience a shorter unemployment spell because they choose to search more intensively. Those, who possess the highest productivity, place themselves on the top of the search ladder. An extra search intensity does not allow the workers to surpass others in the search game. Their unemployment rate remains at the FFUR even though their productivity and search intensity further increase.

Using equation 11, when $H(\delta)$ approaches one, the FFUR is equivalent to $u/(2-u)$. The FFUR can be interpreted as the efficiency of the matching technology in one country's labor market. The lower is the rate, the more efficient is the matching technology. Truly, to conclude whether a higher natural rate of unemployment is attributable to the inefficient matching technology or the workers' lower search effect is rather hasty. The derived FFUR plays its role in establishing a positive relation between an overall unemployment rate and the efficiency of a matching technology. As the fundamental frictional unemployment rate strictly increases with u, the derived FFUR confirms that a lower natural rate of unemployment does reflect a more efficient matching technology in a labor market. More importantly, using $u/(2-u)$, the magnitude of FFUR, though unobservable, can be quantified.

In fact, if the share of the educational group j with the highest productivity is small enough, H_j approaches one. If Assumption 1 is satisfied, the postgraduate workers are the ones with the highest productivity. With the following assumption, the EUR of postgraduates is approximately equal to the FFUR.

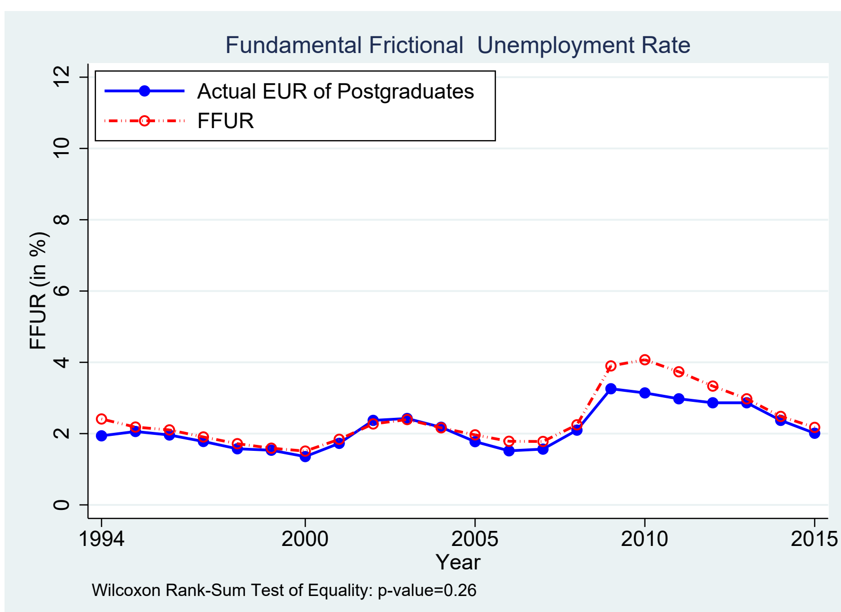
Assumption 3. *The share of the postgraduate workers is sufficiently small.*

Theorem 4. ***(Disaggregation Approximation Theory)*** *Suppose the assumption 1 and 3 are satisfied. In a steady state Nash equilibrium, the unemployment rate of the postgraduates is given by*

$$u_{Postgraduates} \approx \lim_{H(\delta)\to 1} \left(1 + \frac{4(1-u)H(\delta)}{u^2}\right)^{\frac{-1}{2}} = \frac{u}{2-u} \tag{18}$$

The disaggregation approximation theory shows that the unemployment rate of the postgraduates is a function of an OUR only. We evaluate this disaggregation approximation theory by comparing the FFUR with the EUR of postgraduates using the US CPS. If the two set of values are different, we should cast doubt on our argument in this subsection. Figure 6 shows that the actual EUR of postgraduates and the FFUR exhibit a similar trend both in recession and expansion. The two sets of values are so close that they overlap for ten consecutive years during 1994-2004. However, the FFUR are slightly higher than the actual EUR in slumps during 2008-2010. With the exception of the fair performance in slumps, the approximation method basically performs well in capturing the EUR of the postgraduates. We again simulate the FFUR and the EUR of postgraduates by state, and perform the Wilcoxon rank-sum test of equality. The corresponding p-values are reported in Table 5 and 7 summarizes the results of the Wilcoxon rank-sum test. This exercise is astonishing because the null hypothesis that the two sets of values are from an identical distribution cannot be rejected in 70 percent of the States at five percent significance level. We cannot reject the hypothesis in over half the United States at 25 significance level. With only one input variable, the predictive power of the disaggregation approximation theory is exceptionally high. This approximation theory preserves all the advantages of the disaggregation theory: its predictability, its least number of input variables, its accessibility of the required input, and its easiness of implementation. Nevertheless, the drawback of this theory is that it can only be applied to the postgraduates.

Figure 6: Evaluation of Fundamental Frictional Unemployment Rate



Notes: Data are from the US CPS. Samples are restricted to a labor force aged 25-60. The predicted data (the dotted line) are generated from Theorem 4.

This FFUR is derived by assuming that only the relativity of search intensity matters in determining a transition rate. Suppose both the level and the relativity of search intensity are also the determinants of the transition rate. The unemployment spell for the postgraduates is expected to be shorter than the one associated with FFUR; the EUR of the postgraduates in principle should be far below the FFUR. If an unemployment rate is solely driven by search friction, the distance between the FFUR and the EUR of postgraduates should be attributable to the level of search intensity. According to Figure 6, the two sets of values are so close that the statistical test could not reject the null hypothesis that the derived formula is identical to the underlying data generating process of the EURs at any conventional significance level. It basically leaves almost no room that can be explained by the level of search intensity. But the actual EURs are indeed lower than the FFUR in slumps. Of course, one could argue that parts of the gap is attributed to the level of search intensity. Consider that the existing search equilibrium model, where the level of search intensity is one of the determinants of the transition rate, could not capture more than one documented unemployment trend, as shown in section 2.2. We cast doubt on the possibility that the gap between the FFUR and the EUR is explainable by the level of search intensity. In fact, one of the possibility is that the unemployment rate of this period is not only generated by search friction, but also by rationing (Michaillat, 2012) and ambiguity (Chan and Yip, 2016), which are shown to play a crucial role in unemployment in economic downturns. We therefore conclude that while the relativity of search intensity is the key determinants of the transition rate, the possibility that the level of search intensity matters in determining the transition rate is at best low.

6 Conclusion

This article documents three sets of seemingly unrelated features of unemployment, in which the existing search and matching equilibrium model cannot capture if the search intensity level determines a job finding rate. We construct a model that only the relativity position of search intensity matters in determining the transition rate. We show that our model could explain all the documented features of unemployment.

Our model derive three novel formulas for the unemployment distribution, the EURs, and the fundamental frictional unemployment rate. Our numerical exercises suggest that the formulas predict the magnitude of the corresponding variables so well that most of the null hypothesis in which the actual and the predicted values of the variables are identical cannot be rejected at any conventional significance level.

This paper brings a new research direction in several aspects. First, this paper succeeds in decomposing the OUR into EURs of various education groups. It will be fruitful and interesting to decompose the overall unemployment duration into the unemployment duration by educational attainment. Second, with the decomposition theory, it is interesting to also compute and explore the properties of the elasticity of EURs with respect to OUR. Of course, it would be interesting to estimate such elasticity and verify its properties. Third, the proposed theory embeds the search relativity in the undirected search model. Incorporating the relativity in a directed search model with on-the-job search allows the model to also predict the on-the-job transition rate. Such generalization completes our understanding in the job-seeking behaviours of not only the unemployed but also the employed.

7 Appendix: Proof

7.1 The Derivation of Equation (10)

Using equation (6), $G(\delta)$ is the solution of the following differential equation.

$$G'(\delta)u = \frac{\lambda h(\delta)}{\lambda + G(\delta)p} \tag{19}$$

Notice that u is shown to be independent of δ. Rearranging terms, we have

$$(\lambda + G(\delta)p)G'(\delta)u = \lambda h(\delta)$$

Integrating both side from z to x, we have

$$\begin{aligned} u\int_z^x \lambda + G(\delta)pdG(\delta) &= \int_z^x \lambda h(\delta)d\delta \\ u\lambda G(\delta) + up\int_z^x G(\delta)dG(\delta) &= \lambda H(x) \\ up\frac{G^2(\delta)}{2} &= \lambda(H(\delta) - G(\delta)u) \end{aligned}$$

Solving the quadratic equation gives the solution of $G(\delta)$ as in equation (10).

7.2 The Derivation of Equation (14)

Using $F(s^*(\delta)) = G(\delta)$ and equation (10),

$$F(s^*(\delta))\frac{\beta p}{r+\lambda} = \frac{\beta\lambda}{r+\lambda}(\Phi_1(\delta) - 1) \tag{20}$$

where $\Phi_1(\delta) = \sqrt{1 + \frac{2pH(\delta)}{\lambda u}}$.

Using equation (8) and (20), we have

$$\delta - rJ^U(\delta) = \frac{\delta + C(s^*(\delta)) - z}{1 + \Phi_2(\delta)}$$

where $\Phi_2(\delta) = \frac{\beta\lambda(\Phi_1(\delta)-1)}{r+\lambda}$. Substituting the above equation, $f(s^*(\delta))ds^*(\delta)/d\delta = g(\delta)$ and equation (11) into equation (12), we have

$$C'(s^*(\delta))\frac{ds^*(\delta)}{d\delta} = \frac{h(\delta)}{u\Phi_1(\delta)}\frac{\beta p}{r+\lambda}\frac{\delta + C(s^*(\delta)) - z}{1 + \Phi_2(\delta)}$$

which can be written as equation (14).

7.3 Proofs of the Properties of $\Omega(\delta, p)$

$$T(\delta) = \frac{\lambda\beta h(\delta)A}{\beta\lambda(1 + 2AH(\delta)) + (r + \lambda - \beta\lambda)\sqrt{1 + 2AH(\delta)}}$$

where $A = 2(1-u)/u^2$. It is straightforward to show that $\lim_{p\to 0} T(\delta) = 0$ and $\lim_{p\to\infty} T(\delta)$ is finite. Hence, $\lim_{p\to 0} \Omega(\delta, p) = 0$ and $\lim_{p\to\infty} \Omega(\delta, p)$ is positive finite. Differentiating $T(\delta)$ with respect to p, we have

$$\frac{\partial T(\delta)}{\partial p} = \frac{2-u}{u^2}\frac{\partial T(\delta)}{\partial A}$$

One could easily verify that $\partial T(\delta)/\partial p > 0$, $\lim_{p\to\infty} \frac{\partial T(\delta)}{\partial p} = 0$. Differentiating $\Omega(\delta, p)$ with respect to p, we have

$$\frac{\partial \Omega(\delta, p)}{\partial p} = \int_z^\delta \frac{\partial T(x')}{\partial p}(x' - z)e^{\int_{x'}^\infty T(x)dx}dx' + \int_z^\infty \left(\int_{x'}^\infty \frac{\partial T(x)}{\partial p}dx\right) T(x')(x' - z)e^{\int_{x'}^\infty T(x)dx}dx'$$

Hence, we have $\partial\Omega(\delta, p)/\partial p \geq 0$ and $\lim_{p\to\infty} \partial\Omega(\delta, p)/\partial p = 0$.

7.4 Proof of Theorem 1

Differentiating u_δ in equation (11) with respect to δ, we have

$$\frac{du_\delta}{d\delta} = -\frac{2(1-u)h(\delta)}{u^2}\left(1 + \frac{4(1-u)H(\delta)}{u^2}\right)^{\frac{-3}{2}} < 0$$

Differentiating the above derivative with respect to p, we have

$$\frac{d^2u_\delta}{dud\delta} = -\frac{u + 2(1-u)}{u(1-u)}\frac{2H(\delta)(1-u) - u^2}{u^2 + 4(1-u)H(u)}$$

$d^2u_\delta/dud\delta > 0$ iff $2H(\delta)(1-u) < u^2$.

7.5 Proof of Theorem 2

Using equations (10) and (11), we have

$$\begin{aligned} G(\delta) &= \frac{u}{2(1-u)}\left(\sqrt{1+\frac{4(1-u)H(\delta)}{u^2}}-1\right) \\ &= \frac{1}{2}\frac{u}{1-u}\left(\frac{1}{u_\delta}-1\right) \\ &= \frac{1}{2}\Psi\frac{1-u_\delta}{u_\delta} \\ &= \frac{1}{2}\frac{\Psi}{\Psi_\delta} \\ &= \frac{1}{2}\Theta_\delta \end{aligned}$$

Assumption 1 and 2 imply that $\underline{\delta}_j > \bar{\delta}_{j-1}$. Hence, we have

$$G_j = \frac{1}{2}\Theta_j$$

7.6 Proof of Theorem 3

$$\begin{aligned} u_j &= \frac{\int_{\underline{\delta}_j}^{\bar{\delta}_j}\left(1+\frac{4(1-u)H(\delta)}{u^2}\right)^{\frac{-1}{2}} h(\delta)d\delta}{H_j - H_{j-1}} \\ &= \frac{\int_{\underline{\delta}_j}^{\bar{\delta}_j}\left(1+\frac{4(1-u)H(\delta)}{u^2}\right)^{\frac{-1}{2}} dH(\delta)}{H_j - H_{j-1}} \\ &= \frac{\int_{H_{j-1}}^{H_j}\left(1+\frac{4(1-u)x}{u^2}\right)^{\frac{-1}{2}} dx}{H_j - H_{j-1}} \\ &= \frac{u}{H_j - H_{j-1}}\int_{H_{j-1}}^{H_j}\left(u^2+4(1-u)x\right)^{\frac{-1}{2}} dx \\ &= \frac{u}{H_j - H_{j-1}}\frac{1}{4(1-u)}\int_{B_{j-1}}^{B_j} y^{\frac{-1}{2}}dy \\ &= \frac{u}{H_j - H_{j-1}}\frac{1}{2(1-u)}y^{\frac{1}{2}}\Big|_{B_{j-1}}^{B_j} \\ &= \frac{2u}{(B_j^{\frac{1}{2}} + B_{j-1}^{\frac{1}{2}})} \end{aligned}$$

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