Unemployment and Credit Risk

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Abstract

This paper studies the credit risk implications of labor market fluctuations, by incorporating defaultable debt into a textbook search model of unemployment. In the model, the present value of cash flows that firms extract from workers simultaneously drives unemployment dynamics and credit risk variation. The model generates fat right tails in both unemployment and credit spreads, and their strong comovement over the business cycle, in line with the historical U.S. data from 1929 to 2015. Quantitatively, the model reasonably replicates the level, volatility and cyclicality of credit spreads. Overall, the paper highlights labor market fluctuations as an important macroeconomic driver of credit risk variation.

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1 Introduction

For the period from 1929 to 2015, the U.S. corporate bond market exhibits strong co-movement with the labor market. Figure 1 plots the monthly percentage yield spread between Moody’s Baa- and Aaa-rated corporate bond together with the U.S. unemployment rate from April 1929 through March 2015. The figure shows a tight relation between the Baa-Aaa credit spread and the unemployment rate, with spreads generally widening as unemployment rises and vice versa. The correlation between the two series is 0.81.\(^1\) Perhaps most prominent is the extraordinarily high level of unemployment during the Great Depression, which is accompanied by unusually high credit spreads. Intriguingly, both series exhibit similar “double dip” dynamics over this period. In short, Figure 1 suggests that labor market conditions might be important for understanding credit risk in the corporate bond market.

\(^1\)Notably, the correlation is higher—and surprisingly so—than traditional determinants of credit spreads posited by structural credit risk models. In particular, the correlations of the quarterly series over the period 1929Q2-2015Q1 are, respectively: unemployment (0.81), aggregate stock market volatility (0.71), idiosyncratic stock volatility (0.37), and market leverage (0.61). Refer to Appendix A for variable construction.
This paper explores the credit risk implications of labor market fluctuations. Empirically, this paper documents that credit spreads are sensitive to labor market conditions in historical U.S. data. In particular, regressing Moody’s Baa-Aaa credit spread on the U.S. unemployment rate from 1929 through 2015 suggests that a one percentage point rise in the unemployment rate is associated with a surge of the Baa-Aaa credit spread by around 13.4 basis points. The magnitude remains sizable at 8.7 basis points, after controlling for traditional credit risk determinants and macroeconomic conditions. Importantly, the unemployment rate itself explains as much as 66% of the spread variation. Furthermore, the results are robust at first differences.

Motivated by the findings, this paper develops a model by incorporating defaultable debt into an otherwise standard Diamond-Mortensen-Pissarides (DMP) model of equilibrium unemployment. The model has three key features. First, firms own the production technology and hire workers to produce output. Firms have to search for unemployed workers via posting vacancies. Frictions in matching unemployed workers to vacant jobs create rents to be divided between firms and workers through Nash bargained wages. Second, equityholders run the firms but partially finance the activities with defaultable debt. The tax benefits of debt and default losses shape optimal financing decisions in a dynamic trade-off framework. Third, default is endogenous in that equityholders choose to optimally default on their debt obligations whenever the option to default is more valuable than paying back creditors.

The model offers some intuition about how variation in credit spreads is linked to unemployment fluctuations. As in Merton (1974), corporate debt in the model is economically equivalent to risk-free debt minus a put option written on the underlying assets of the firm. As the model is parsimonious, movements in the asset value of the firm is driven by movements in the asset value of employment relationships, as measured by the present value of current and future cash flows that workers bring to the firm. Unemployment fluctuations reveal the variation in the asset value of employment relationships, which drives firms’ default decisions—the decision to exercise the put option—and credit spread variation.

The model delivers two key results on credit risk. First, the model reasonably replicates salient features of credit spreads in the data. Owing to strong nonlinear dynamics, the economy occasionally runs into economic disasters per Rietz (1988) and Barro (2006). Default rates are also countercyclical in the model, typically rising in recessions with low productivity and high unemployment, when investors experience disastrously low consumption and high marginal utilities. The coincidence generates a substantial credit risk premium, giving rise to sizable, volatile, and countercyclical credit spreads. Finally, credit spreads in the model feature a fat right tail as in the data.

Second, the model is consistent with the relation between credit spreads and unemployment in historical U.S. data. In model simulations, credit spreads and unemployment closely track each other, with a correlation of 0.85. More important, economic disasters induce occasional coincident spikes in both series to usually high levels, resembling the Great Depression episode (Figure 1), which is a novel prediction of the model. Quantitatively, the model accounts for the strong re-
response of credit spreads to unemployment. In model regressions a one percentage point rise in
unemployment increases credit spreads by around 16.7 basis points.

What drives the strong response of credit spreads to labor market conditions? This paper
approaches the question through the lens of asset volatility, which captures the amount of busi-
ness risk that firms face. The model, which is reasonably calibrated with realistic unemployment
volatilities, generates sizable and countercyclical asset volatility. Comparative statics further show
that a sizable asset volatility is essential for the strong response of credit spreads to unemploy-
ment. Taken together, the paper points to the labor market as a significant source of business risk
for firms. While the credit risk literature has largely treated asset volatility as exogenous, this
paper sheds light on macroeconomic drivers of asset volatility.

This paper makes two contributions to the credit risk literature. First, it adds to the large
empirical literature on the determinants of credit spread variation (Duffee 1998; Collin-Dufresne,
Goldstein, and Martin 2001; Campbell and Taksler 2003; Chen, Lesmond, and Wei 2007; Cremers,
Driessen, and Maenhout 2008; Zhang, Zhou, and Zhu 2009; Ericsson, Jacobs, and Oviedo 2009;
Giesecke, Longstaff, Schaefer, and Streibulaev 2011; Krishnamurthy and Vissing-Jorgensen 2012;
Kang and Pflueger 2015; Bao, Chen, Hou, and Lu 2015). In particular, based on long historical
time series, this paper uncovers a new link between labor market conditions and credit spread
variation.\(^2\)

Second, this paper adds to a recent strand of literature on the macroeconomic determinants of
credit risk. Several studies (Hackbarth, Miao, and Morellec 2006; Chen, Collin-Dufresne, and
Goldstein 2009; Bhamra, Kuehn, and Streibulaev 2010; Chen 2010) propose that exposures to
macroeconomic risks give rise to sizable and volatile credit spreads in endowment economies.
These studies all assume that firms’ asset value evolves exogenously, and is delinked from firms’
real decisions. Motivated by the historical relation between credit spreads and unemployment,
this paper relates default to firms’ job creation decisions in a general equilibrium production
economy. This paper shows that labor market fluctuations are important for generating strong
dependent comovement between default and marginal utilities in a production setting.

Relatedly, a number of papers (Philippon 2009, Kuehn and Schmid 2014, Gomes and Schmid
2014) link endogenous movements in firms’ asset value to their capital investment decisions.
Those papers feature rich cross-sectional heterogeneity and derive interesting implications be-
tween credit risk and capital investment at the firm level. However, they have entirely abstracted

\(^2\) Practitioners have long recognized the importance of labor market conditions for corporate default. For instance,
Moody’s proposed a default forecasting model in 2007, called Credit Transition Model, in which the impact of
macroeconomic conditions on default is parsimoniously summarized with only two drivers: the unemployment
rate and the high yield spread over Treasuries. In response to why these macroeconomic factors are selected,
Moody’s wrote “We chose to use the U.S. unemployment rate as a measure of macroeconomic health over other, well
received measures (GDP or IP growth, for instance) for a couple of reasons. First, the contemporaneous correlation
between the aggregate default rate and changes in unemployment is about as good as that of any other conventional
measure. Second, the level of unemployment helps summarize recent economic history.” For more details, refer to
from frictions in the labor market, the focus of this paper.

This paper also contributes to the rare disasters literature (Rietz 1988; Barro 2006; Gabaix 2012; Gourio 2012; Wachter 2013), which has so far focused on equity prices. A notable exception is Gourio (2013), who embeds disasters into a standard real business cycle model to jointly explain the behavior of credit spreads, business cycles, and disasters. However, disasters are exogenously imposed in his model. The novelty of this paper is that it draws on strong nonlinear dynamics in the search economy to generate endogenous disasters. The model’s success to jointly explain the behavior of credit spreads and unemployment lends support to the model’s disaster mechanism.

Petrosky-Nadeau, Zhang, and Kuehn (2015) show that search and matching frictions in the labor market give rise to endogenous disasters, potentially explaining aggregate asset prices including the first and second moments of the equity premium and risk-free rate. My paper complements their work, but differs in two important aspects. First, my paper features defaultable debt, focuses on the credit risk implications of labor market conditions, and provides empirical evidence in support of the model’s key predictions. Second, the richer structure of this model also shows how financial frictions interact with labor search frictions in explaining aggregate asset prices and labor market volatility.

Apart from search frictions, several articles (Danthine and Donaldson 2002; Uhlig 2007; Favilukis and Lin 2015) explore how rigid wages affect the equity premium through operating leverage. Labor market frictions have also been shown to have important implications for the cross-section of stock returns (Belo, Lin, and Bazdresch 2014; Donangelo 2014; Donangelo, Gourio, and Palacios, 2015). Finally, Favilukis, Lin, and Zhao (2015) examine how rigid wages impact credit risk in a model featuring labor adjustment costs and long-run risks, but no unemployment.

The remainder of this paper proceeds as follows. Section 2 presents the stylized facts. Section 3 lays down the model, characterizes its equilibrium conditions, and briefly discusses the solution method. Section 4 presents the quantitative results. Section 5 examines the implications for labor market volatility. Section 6 concludes.

2 Stylized Facts

To formally examine the relation between labor market conditions and credit risk, I focus on the unemployment rate as the indicator of labor market conditions. The reason is that the U.S. unemployment rate is available for a long historical period (back to 1929). The long historical perspective distinguishes my analysis from most studies on credit spread variation that use postwar data.

2.1 Labor Market Conditions and Credit Spreads

The baseline regression is specified as in Campbell and Taksler (2003):

\[
CS_t = \beta_0 + \beta_1 U_t + \gamma Z_t + \epsilon_t
\]
This table presents the results of the regression

\[ CS_t = \beta_0 + \beta_1 U_t + \gamma Z_t + \epsilon_t \]

The dependent variables are levels of corporate bond spreads (Moody’s Baa-Aaa or Aaa-Treasury spreads). \( U_t \) denotes the unemployment rate. \( Z_t \) represents a vector of control variables: Market leverage is total liabilities divided by the sum of total liabilities and the market value of corporate equity in the non-financial corporate sector; Aggregate stock volatility is the 6-month moving average of monthly realized market volatility estimated from daily returns; Idiosyncratic stock volatility is the 6-month moving average of the cross-sectional dispersion of monthly stock returns; Treasury slope is the 10-year minus 3-month Treasury yields; Price-earning ratio is the price-earning ratio of the S&P 500 index; Industrial production is the growth rate of the industrial production index. For each regression, the table reports OLS coefficient estimates and Newey-West corrected \( t \)-statistics (in brackets) with the automatic lag selection method of Newey and West (1994). Data are quarterly and span 1929Q2 to 2015Q1.

<table>
<thead>
<tr>
<th></th>
<th>Baa-Aaa</th>
<th>Aaa-Treasury</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.134</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>[8.76]</td>
<td>[6.00]</td>
</tr>
<tr>
<td>Market leverage</td>
<td>2.444</td>
<td>1.393</td>
</tr>
<tr>
<td></td>
<td>[2.97]</td>
<td>[1.98]</td>
</tr>
<tr>
<td>Aggregate stock volatility</td>
<td>0.186</td>
<td>0.114</td>
</tr>
<tr>
<td></td>
<td>[6.61]</td>
<td>[4.78]</td>
</tr>
<tr>
<td>Idiosyncratic stock volatility</td>
<td>−0.300</td>
<td>−0.623</td>
</tr>
<tr>
<td></td>
<td>[−0.23]</td>
<td>[−0.70]</td>
</tr>
<tr>
<td>3-month Treasury yield</td>
<td>0.025</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>[1.15]</td>
<td>[1.93]</td>
</tr>
<tr>
<td>Treasury slope</td>
<td>0.103</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>[2.35]</td>
<td>[0.91]</td>
</tr>
<tr>
<td>Price-earning ratio</td>
<td>−0.003</td>
<td>−0.001</td>
</tr>
<tr>
<td></td>
<td>[−0.32]</td>
<td>[−0.21]</td>
</tr>
<tr>
<td>Industrial production</td>
<td>−0.008</td>
<td>−0.019</td>
</tr>
<tr>
<td></td>
<td>[−0.85]</td>
<td>[−2.39]</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.210</td>
<td>−1.053</td>
</tr>
<tr>
<td></td>
<td>[2.04]</td>
<td>[−1.95]</td>
</tr>
<tr>
<td>Observations</td>
<td>344</td>
<td>344</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.66</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td>0.48</td>
</tr>
</tbody>
</table>
in which $CS_t$ is the Baa-Aaa credit spread, $U_t$ is the U.S. unemployment rate, and $Z_t$ is a vector of controls. These controls are chosen based on previous research on credit spread variation.\(^3\) These variables include market leverage, aggregate stock market volatility, idiosyncratic stock volatility, the 3-month Treasury yield, the slope of the Treasury term structure (measured by the 10-year minus 3-month Treasury yield), the price-earning ratio for the S&P 500, and the growth rate of the industrial production index.\(^4\) Further details of the variables and their construction are relegated to Appendix A.

Table 1 reports the regression results, estimated using ordinary least squares (OLS).\(^5\) The coefficient of 0.134 in Column (1) implies that a one percentage point increase in the unemployment rate is associated with an increase of the Baa-Aaa spread by 13.4 basis points. Put differently, a one standard deviation increase in the unemployment rate (4.38\%) corresponds to a 59 ($= 4.38 \times 13.4$) basis point increase in the Baa-Aaa spread. The coefficient is statistically significant ($t = 8.76$). Importantly, the unemployment rate itself explains as much as 66\% of the spread variation.

Column (3) indicates that the coefficient on the unemployment rate remains highly statistically significant ($t = 6$), after controlling for variables suggested by the literature. The coefficient of 0.087 is smaller in magnitude relative to that in Column (1), likely driven by the collinearity between the unemployment rate and aggregate stock market volatility (correlation = 0.59). Nonetheless, the impact remains economically sizable. A one standard deviation increase in the unemployment rate leads to a widening of credit spreads by 38 basis points. For comparison, a one standard deviation increase in aggregate stock market volatility increases credit spreads by only 26 basis points.

The remaining columns in Table 1 report similar regressions of the Aaa-Treasury spread (the yield spread between Moody’s Aaa-rated debt and U.S. long maturity government debt). Notably, the coefficients on the unemployment rate are small. With controls, the unemployment rate enters insignificantly ($t = 1.35$ in Column 6). Also, the unemployment rate does not track much variation in the Aaa-Treasury spread. As such, to the extent that the Baa-Aaa spread represents compensation for default, the unemployment rate explains movements in credit risk, rather than non-default factors such as liquidity risk or tax differentials between corporate bonds and Treasury bonds, potentially captured by the Aaa-Treasury spread.

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\(^3\)See Duffee (1998); Collin-Dufresne, Goldstein, and Martin (2001); Campbell and Takasler (2003).

\(^4\)I choose industrial production rather than GDP to capture macroeconomic conditions, because GDP is not available at the quarterly frequency for the pre-1947 period.

\(^5\)Statistical inference is based on a heteroskedasticity- and autocorrelation-consistent asymptotic covariance matrix computed according to Newey and West (1987), with the automatic lag selection method of Newey and West (1994). As a robustness check, I also follow Krishnamurthy and Vissing-Jorgensen (2012) in adjusting the standard errors assuming an AR(1) error structure, motivated by a standard Box-Jenkins analysis of the autocorrelation function and partial autocorrelation function of the error terms. The results are similar.
Table 2: Explaining Changes in Corporate Bond Yield Spreads

This table presents the results of the regression

\[ \Delta CS_t = \beta_0 + \beta_1 \Delta U_t + \gamma \Delta Z_t + \epsilon_t \]

The dependent variables are changes in corporate bond spreads (Moody’s Baa-Aaa or Aaa-Treasury spreads). \( U_t \) denotes the unemployment rate. \( Z_t \) represents a vector of control variables: Market leverage is total liabilities divided by the sum of total liabilities and the market value of corporate equity in the non-financial corporate sector; Aggregate stock volatility is the 6-month moving average of monthly realized market volatility estimated from daily returns; Idiosyncratic stock volatility is the 6-month moving average of the cross-sectional dispersion of monthly stock returns; Treasury slope is the 10-year minus 3-month Treasury yields; Price-earning ratio is the price-earning ratio of the S&P 500 index; Industrial production is the growth rate of the industrial production index. For each regression, the table reports OLS coefficient estimates and Newey-West corrected \( t \)-statistics (in brackets) with the automatic lag selection method of Newey and West (1994). Data are quarterly and span 1929Q2 to 2015Q1.

<table>
<thead>
<tr>
<th>Baa-Aaa</th>
<th>Aaa-Treasury</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>( \Delta U_{unemployment} )</td>
<td>0.128</td>
</tr>
<tr>
<td>( \Delta Market leverage )</td>
<td>3.439</td>
</tr>
<tr>
<td>( \Delta Aggregate stock volatility )</td>
<td>0.037</td>
</tr>
<tr>
<td>( \Delta Idiosyncratic stock volatility )</td>
<td>-1.515</td>
</tr>
<tr>
<td>( \Delta 3\text{-month } Treasury yield )</td>
<td>-0.107</td>
</tr>
<tr>
<td>( \Delta Treasury slope )</td>
<td>-0.086</td>
</tr>
<tr>
<td>( \Delta Price-earning ratio )</td>
<td>-0.010</td>
</tr>
<tr>
<td>( \Delta Industrial production )</td>
<td>-0.012</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.002</td>
</tr>
<tr>
<td>Observations</td>
<td>343</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.11</td>
</tr>
</tbody>
</table>
2.2 Robustness

For robustness, Table 2 estimates the baseline regression in first differences, testing whether changes in the unemployment rate explain credit spread changes:

\[ \Delta CS_t = \beta_0 + \beta_1 \Delta U_t + \gamma \Delta Z_t + \epsilon_t \]

The coefficients on changes in the unemployment rate are 0.128 and 0.092, respectively, without (Column 1) and with controls (Column 3). Importantly, the coefficients are of similar magnitudes to those in the level regressions, suggesting that the impact of the unemployment rate on credit spreads is robust with respect to specifications. Consistent with Collin-Dufresne, Goldstein, and Martin (2001), the amount of explained variation in credit spread changes is much lower, compared with that in the level regressions. In particular, changes in the unemployment rate alone explain around 11% of the variation in credit spread changes. Adding changes in the unemployment rate raises the explanatory power of traditional variables from 31% to 36%.

Taken as a whole, this section documents a robust empirical fact in historical U.S. data. Corporate bond spreads are sensitive to labor market conditions, as captured by unemployment fluctuations. The evidence suggests that labor market fluctuations have potentially important implications for firms’ costs of borrowing.

3 The Model

The model embeds defaultable debt into an otherwise standard DMP model of equilibrium unemployment. Firms own the productive technology of the economy, and hire workers to produce output, subject to search and matching frictions. Equityholders operate the firms, but partially finance the activities with defaultable debt. Optimal financing decisions are shaped by the tax benefits of debt and default losses, in a dynamic trade-off framework. Finally, equityholders choose to optimally default whenever the option to default is more valuable than paying back creditors.

3.1 The Environment

There exists a continuum of measure one of firms indexed by \( i \in [0, 1] \), which operate the same constant returns to scale production technology, and produce output, \( Y_{it} \), with labor, \( N_{it} \):

\[ Y_{it} = X_t Z_{it} N_{it}, \]

in which \( X_t \) and \( Z_{it} \) denote aggregate productivity and firm-specific productivity, respectively. The log aggregate productivity, \( x_t \equiv \log(X_t) \), follows:

\[ x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \]
in which $\rho$ is the persistence, $\sigma$ denotes the conditional volatility, and $\epsilon_{t+1}$ is an independently and identically distributed (i.i.d.) standard normal shock.

The firm-specific productivity, $Z_{it}$, is i.i.d. across firms and over time, and follows a lognormal distribution with the cumulative distribution function denoted $\Phi(Z_{it})$. For the purpose of normalization, $E[Z_{it}] = 1$.

For parsimony, the model abstracts from capital in the production function. The aim is to focus on the impact of labor market conditions.

**Unemployment, Vacancies, and Matching**

The DMP model views the labor market as a trading place, where unemployed workers and firms with job vacancies meet to trade labor services. The trading process is characterized by a matching function, which relates the flow of new hires to the two key inputs in the matching process: the number of unemployed workers and the number of job vacancies. Matching frictions create rents to be divided between firms and workers through Nash bargained wages.

Specifically, each firm employs $N_{it}$ workers in the current period. Meanwhile, it posts vacancies, $V_{it}$, to attract unemployed workers for next period’s operation. The total numbers of employed workers, $N_t$, and vacant jobs, $V_t$, are, respectively:

$$N_t \equiv \int N_{it} \, di, \quad V_t \equiv \int V_{it} \, di.$$  

The size of the labor force is normalized to one, therefore aggregate unemployment is $U_t = 1 - N_t$. The extent to which the labor market is slack is characterized by labor market tightness, defined as $\theta_t \equiv V_t/U_t$.

The total number of new matches, $G$, are formed via a constant returns to scale matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}},$$

in which $\iota$ is the matching elasticity. The matching function is a market level relationship that characterizes the outcome of the process by which agents meet and match. The probability that a firm fills a vacancy (the vacancy filling rate), $q(\theta_t)$, is

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^{1/\iota})^{1/\iota}}.$$

The probability that an unemployed worker finds a job (the job finding rate), $f(\theta_t)$, is

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{(1 + \theta_t^{-1/\iota})^{1/\iota}}.$$

It follows that $f(\theta_t) = \theta_t q(\theta_t)$, $f'(\theta_t) > 0$, and $q'(\theta_t) < 0$. The tighter the labor market, the easier it is for workers to find a job, and the more difficult for firms to fill a vacancy.
Jobs are destroyed at a constant rate $s$ per period. Taken together, each firm’s employment, $N_{it}$, evolves according to:

$$N_{it+1} = (1 - s)N_{it} + q_t V_{it},$$

(1)

in which $q_t V_{it}$ represents the number of new hires.

Firms incur costs in posting vacancies. The unit cost per vacancy, $\kappa_t$, takes the form:

$$\kappa_t \equiv \kappa_0 + \kappa_1 q(\theta_t),$$

in which $\kappa_0$ is the flow cost of maintaining vacancies, and $\kappa_1$ is the fixed cost. The fixed cost, $\kappa_1$, captures the costs that are paid after the worker who is eventually hired arrives, such as, the costs of training, negotiating, and one-off administrative costs associated with adding the worker to the payroll.

**Financing**

Firms finance hiring activities and wage bills by issuing defaultable debt and equity. Debt finance takes the form of one-period zero-coupon bonds. The debt contract specifies the par value of the issuance, $B_{it+1}$, and the price, $Q_{it}$.

Firms balance the tax benefits of debt and expected default losses. Following Gourio (2013), a firm receives a tax subsidy of $\tau$ dollars for each dollar that the firm raises in the bond market. Specifically, a firm that issues debt, $B_{it+1}$, at the price, $Q_{it}$, receives $(\tau + 1)Q_{it}B_{it+1}$. Creditors recover a fraction $\xi \in (0, 1)$ of the firm value upon default, as in Leland (1994). While creditors bear the default losses ex post, equityholders ultimately bear the costs of default, because debt prices reflect the expected default losses ex ante.

Figure 2 depicts the timeline of events within period $t$. Firm $i$ enters period $t$ with workers $N_{it}$ and debt $B_{it}$. Upon observing the aggregate productivity, $X_{it}$, and firm-specific productivity, $Z_{it}$, firm $i$ makes the default decision. Following Hennessy and Whited (2007), I assume that equityholders will choose to default on their debt obligations whenever the equity value of the firm falls below zero. If the firm decides not to default, it produces and sells output, pays back debt, $B_{it}$, to creditors, issues new debt, $B_{it+1}$, makes wage payments to workers, and posts vacancies, $V_{it}$, to attract workers for the next period. At the end of the period $t$, matching takes place in the labor market. The number of new hires, $q_t V_{it}$, is added to the firm’s workforce at the beginning of period $t + 1$.

Implicitly, a firm receives the tax shields in the period in which it issues debt. This modeling strategy has the appealing property that it delinks current period profits from last period’s shock, thereby reducing the state space of the model by one dimension and greatly simplifying the determination of the bond price schedule. To see how, recall that if a firm gets the tax shields one period after debt issuance, the tax shields and hence current period profits would then depend on last period’s shock. As a result, last period’s shock would show up in the state space of the model. For a similar approach, see Strebulaev and Whited (2012).
Figure 2: Timeline of Events

<table>
<thead>
<tr>
<th>t−1</th>
<th>t+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (N_{it}, B_{it}) ) given</td>
<td>( (N_{it+1}, B_{it+1}) ) matching</td>
</tr>
<tr>
<td>( X_t, Z_{it} ) realized</td>
<td>pay workers;</td>
</tr>
<tr>
<td></td>
<td>pay back debt;</td>
</tr>
<tr>
<td></td>
<td>post vacancies ( V_{it} );</td>
</tr>
<tr>
<td></td>
<td>borrow ( B_{it+1} );</td>
</tr>
<tr>
<td></td>
<td>produce and sell output</td>
</tr>
</tbody>
</table>

If the firm decides to default, it exits the economy. A new firm enters the economy immediately to replace the exiting firm. Without loss of generality, I assume the new firm has the same number of workers and the same firm-specific productivity as the exiting firm, but with an initial debt of zero. The new firm then produces and sells output, issues debt, makes wage payments to workers, and posts vacancies.

**Equity Valuation**

Equityholders maximize the equity value of the firm, defined as the present value of future equity distributions. Let \( P(N_{it}, B_{it}, Z_{it}) \) denote firm \( i \)'s cum-dividend equity value in period \( t \):

\[
P(N_{it}, B_{it}, Z_{it}) \equiv \max \left( 0, S(N_{it}, B_{it}, Z_{it}) \right),
\]

in which \( S(N_{it}, B_{it}, Z_{it}) \) is the cum-dividend equity value prior to default decisions. The maximum captures the possibility of default at the beginning of the period, in which case the equityholders get nothing. The cum-dividend equity value prior to default, \( S(N_{it}, B_{it}, Z_{it}) \), obeys:

\[
S(N_{it}, B_{it}, Z_{it}) \equiv \max_{V_{it}, B_{it+1}} X_t Z_{it} N_{it} - W_t N_{it} - \kappa_t V_{it} + (\tau + 1) Q_{it} B_{it+1} - B_{it}
\]

\[
+ E_{t+1} \left[ \int_0^\infty P(N_{it+1}, B_{it+1}, Z_{it+1}) \, d\Phi(Z_{it+1}) \right],
\]

subject to \( N_{it+1} = (1 - s) N_{it} + q_t V_{it} \), and \( V_t \geq 0 \),

in which \( W_t \) is the wage rate, and \( M_{t+1} \) is the pricing kernel, which is determined in general equilibrium, consistent with the household behavior.

Default is triggered whenever the firm-specific productivity, \( Z_{it} \), is below the default threshold, \( Z_{it}^* \), determined by:

\[
S(N_{it}, B_{it}, Z_{it}^*) = 0.
\]
Debt Valuation

Creditors’ valuation of corporate debt equals next period’s expected discounted payoff:

\[
Q_{it}B_{it+1} = \mathbb{E}_t \left[ \left( 1 - \Phi(Z_{it+1}) \right) B_{it+1} \right]
\]

Payoff in the non-default states

\[
+ \xi \int_0^{Z_{it+1}} \left[ S(N_{it+1}, B_{it+1}, Z_{it+1}) + B_{it+1} \right] d\Phi(Z_{it+1})
\]

Payoff in the default states

(5)

In the non-default states (i.e. \( Z_{it+1} \geq Z_{it+1}^* \)), creditors collect the par value of debt, \( B_{it+1} \). In the default states (i.e. \( Z_{it+1} < Z_{it+1}^* \)), creditors collect a fraction \( \xi \) of the firm value, which comprises equity, \( S(N_{it+1}, B_{it+1}, Z_{it+1}) \), and debt, \( B_{it+1} \). This formulation implies that creditors not only collect the defaulting firm’s current period cash flows, but also extract the “going-concern value” of the firm. See a similar formulation in Hennessy and Whited (2007).

Households

As in Merz (1995), the representative household consists of a continuum of employed workers and unemployed workers, who provide perfect consumption insurance for one another. The household pools their incomes before choosing consumption plans and asset holdings. The representative household has recursive preferences (Epstein and Zin 1989) defined over aggregate consumption, \( C_t \):

\[
J_t = \left[ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta \left( \mathbb{E}_t \left[ J_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\psi}},
\]

in which \( J_t \) is the recursive utility, \( \beta \) is the subjective discount factor, \( \psi \) is the intertemporal elasticity of substitution, and \( \gamma \) is the relative risk aversion. The pricing kernel is:

\[
M_{t+1} \equiv \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{J_{t+1}^{1-\gamma}}{\mathbb{E}_t[J_{t+1}^{1-\gamma}]]} \right)^{\frac{\frac{1-\gamma}{1-\gamma}}{\frac{1-\gamma}{1-\gamma}}}
\]

Wages

Workers and firms bargain collectively over the wage rate via a Nash bargaining process. The worker can threaten to become unemployed, in which case the worker receives the flow value of unemployment activities, \( b \). The firm can threaten to end the job. In the end, the marginal surplus from a firm-worker match is divided by the Nash sharing rule, such that the worker keeps a share \( \eta \in (0, 1) \) of the surplus, in which \( \eta \) is the worker’s bargaining power. As in Cooper, Haltiwanger, and Willis (2007) and Petrosky-Nadeau (2014), wages are bargained after observing the aggregate productivity, but before the firm draws the firm-specific productivity. The assumption implies
that wages do not depend on firm-specific productivity.

The Nash sharing rule is:

\[
\frac{1}{\eta} H_t = \Lambda_t = \frac{1}{1-\eta} \Omega_t, \tag{8}
\]

in which \( \Lambda_t \) is the total marginal surplus from a firm-worker match, \( H_t \) is the worker’s marginal surplus, and \( \Omega_t \) is the firm’s marginal surplus. It follows that \( \Lambda_t = H_t + \Omega_t \).

The worker’s marginal surplus is (see Appendix C for the detailed derivations)

\[
H_t = W_t + (1-s)E_t[M_{t+1}H_{t+1}] - [b + f(\theta_t)E_t[M_{t+1}H_{t+1}]]. \tag{9}
\]

Upon a successful match, a worker gets the wage payment, \( W_t \), plus the expected discounted future surplus, \( E_tM_{t+1}H_{t+1} \), net of separation. If the worker chooses to stay unemployed, the worker would get the flow value of unemployment, \( b \). Moreover, with probability \( f(\theta_t) \), the worker would find a job, and get the expected discounted future surplus. As such, the last term represents the worker’s opportunity cost of employment.

The firm’s marginal surplus is (see Appendix C)

\[
\Omega_t = X_t \int_{Z_t^*}^{\infty} Z_t d\Phi(Z_t) - [1 - \Phi(Z_t^*)]W_t + (1-s)[1 - \Phi(Z_t^*)]\left[\frac{\kappa_t}{q(\theta_t)} - \lambda_t\right]. \tag{10}
\]

The first term represents the worker’s marginal contribution to output in the non-default states (i.e. \( Z_t \geq Z_t^* \)). The second term is the firm’s wage payment to the worker, which occurs when the firm does not default. The last term stands for the continuation value of the employment relationship, accounting for both the possibility of separation and the possibility of default. In equilibrium, with free entry to job creation, the continuation value equals the hiring costs that the worker saves for the firm, taking into account the non-negative vacancy constraint—\( \lambda_t \) denotes the multiplier associated with the vacancy constraint in (3).

Substituting (9) and (10) into (8) yields the wage determination equation:

\[
\frac{1}{1-\eta} \Omega_t = \frac{1}{\eta} \left[ W_t - b + \frac{\eta}{1-\eta} \left[ 1 - s - f(\theta_t) \right] E_t[M_{t+1}\Omega_{t+1}] \right]. \tag{11}
\]

Equation (11) simplifies in the absence of financial frictions, in which case firms do not take on leverage and no default would ever occur. As a result, the default threshold \( Z_t^* \) equals zero, and the firm’s marginal surplus becomes \( \Omega_t = X_t - W_t + (1-s)[\kappa_t/q(\theta_t) - \lambda_t] \). Substituting it into the wage determination equation (11), the wage rate, \( W_t \), collapses to the conventional form as in Pissarides (2000):

\[
W_t = (1 - \eta)b + \eta(X_t + \kappa_t \theta_t).
\]
Equilibrium and Aggregation

The i.i.d. nature of the firm-specific productivity together with the structure of the economy implies that all firms make identical hiring and borrowing decisions in all periods. Firms differ only in their default decisions. This feature significantly simplifies aggregation.

The market clears in the goods market:

\[ X_t N_t = C_t + \kappa_t V_t. \]  

(12)

As in Gourio (2013) and Jermann and Yue (2014), default losses are assumed to be transfers rather than real resource costs. The rationale for this assumption is, if default losses are due to legal fees or asset fire sales, losses to firms and creditors are recouped by lawyers and vulture investors (i.e. other members of the representative household).

A competitive equilibrium is defined as a set of functions for (i) firms’ vacancy policy, \( V_{it} \), and debt policy, \( B_{i,t+1} \); (ii) firms’ value functions, \( S_{it} \) and \( P_{it} \); (iii) the wage rate, \( W_t \), and the pricing kernel, \( M_{t+1} \), such that (i) firms’ policies are optimal and \( S_{it} \) and \( P_{it} \) satisfy the Bellman equations (4) and (3); (ii) the wage rate, \( W_t \), is given by the Nash bargaining solution (11); (iii) the pricing kernel, \( M_{t+1} \), satisfies (7); (iv) the goods market clears according to (12).

3.2 Equilibrium Characterization

Before turning to quantitative analysis, a useful step is to characterize the optimality conditions for firms’ decisions including hiring, financing, and default. The derivations of optimality conditions are relegated to Appendix B.

What Drives Default?

The default threshold, \( Z_{it}^{\star} \), fully characterizes the firm’s default decision. With free entry to job creation, the present value of future cash flows that a worker brings to the firm equals the hiring costs that the worker saves for the firm. Equation (4) which determines the default threshold can be written explicitly:

\[
X_t Z_{it}^{\star} N_{it} - W_t N_{it} + (1 - s)[\kappa_t / q(\theta_t) - \lambda_t] N_{it} = B_{it}
\]

(13)

The left-hand side is the value of the firm’s asset: current period cash flows (output minus wage bills) and present value of future cash flows (the continuation value of employment relationships). The right-hand side is the firm’s liability, \( B_{it} \). The default threshold, \( Z_{it}^{\star} \), is the cutoff level of firm-specific productivity such that the two are equal.

Equation (13) paints a clear picture on why firms default. Taking a perspective from the asset side, a firm may choose to default for two reasons. The current period cash flows are too
low, or the continuation value of employment relationships is too low. The former could occur when the firm has a very bad draw of firm-specific productivity (i.e. low $Z_{it}$), or when aggregate productivity is low (i.e. low $X_t$). The latter could occur when the labor market is slack, when unemployment is high, vacancy filling rate, $q(\theta_t)$, is high, and the vacancy duration, $1/q(\theta_t)$, is short. Intuitively, when hiring workers takes fewer resources, employment relationships are valued less, giving equityholders more incentives to default.

**Optimality Conditions for Hiring and Financing**

The job creation condition is:

$$
\frac{\kappa_t}{q(\theta_t)} - \lambda_t = \mathbb{E}_t M_{t+1} \left[ X_{t+1} (1 + L_1) + \left[ -W_{t+1} + (1 - \tau) \left( \frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda_{t+1} \right) \right] (1 + L_2) \right], \quad (14)
$$

in which

$$
L_1 \equiv \tau Z_{it+1}^* [1 - \Phi(Z_{it+1}^*)] - [1 - (\tau + 1) \xi] \int_0^{Z_{it+1}} Z_{it+1} d\Phi(Z_{it+1}), \quad (15)
$$

$$
L_2 \equiv \tau [1 - \Phi(Z_{it+1}^*)] - [1 - (\tau + 1) \xi] \Phi(Z_{it+1}^*). \quad (16)
$$

Equation (14) states that the marginal cost of hiring at time $t$ (the left-hand side) equals the discounted present value of marginal profit at time $t + 1$ from hiring an additional worker (the right-hand side). The marginal profit includes the marginal product of labor, $X_{t+1}$, net of the wage payment, $W_{t+1}$, plus the continuation value of an additional worker, which in equilibrium equals the hiring costs the worker saves for the firm $\kappa_{t+1}/q(\theta_{t+1}) - \lambda_{t+1}$, net of separation.

Importantly, all those components are adjusted to reflect the tax benefits and default losses associated with taking on leverage. In particular, the marginal product of labor, $X_{t+1}$, is augmented by a factor of $L_1$. The first term of $L_1$ in equation (15) captures the tax shields that firms exploit in the non-default states: One additional worker encourages an extra borrowing of $X_{t+1}^* Z_{it+1}^*$ through boosting the value of output (see equation 13), leading to tax shields in the amount of $\tau X_{t+1}^* Z_{it+1}^*$, which the firm is able to collect only in the non-default states (i.e. with probability $1 - \Phi(Z_{it+1}^*)$). The second term of $L_1$ reflects losses in the event of default. The worker’s marginal product is subject to losses by $[1 - (\tau + 1) \xi] X_{t+1} \int_0^{Z_{it+1}} Z_{it+1} d\Phi(Z_{it+1})$ in the default states (i.e. when $Z_{it+1} \leq Z_{it+1}^*$). Taken together, $L_1$ summarizes the impact of financial frictions on the firm’s marginal product of labor. Similarly, the wage payment and the continuation value components are augmented by a factor of $L_2$, to reflect the impact of tax shields and default losses.

Without financial frictions (the tax benefit $\tau = 0$ and the recovery rate $\xi = 1$), both $L_1$ and $L_2$ become zero. Accordingly, the job creation condition (14) reduces to the all-equity version as in
The first-order condition with respect to $B_{t+1}$ yields:

\[
\tau E_t M_{t+1} [1 - \Phi(Z_{it+1}^*)] = (1 - \xi)(1 + \tau) E_t M_{t+1} \left[ \frac{B_{it+1}}{X_{t+1} N_{it+1}} \frac{\partial \Phi(Z_{it+1}^*)}{\partial Z_{it+1}^*} \right].
\] (17)

Equation (17) determines the optimal financing choice of the economy. The left-hand side is the marginal benefit of debt. One additional dollar of debt brings the firm $\tau$ dollars of tax shields when the firm does not default, which occurs with probability $1 - \Phi(Z_{it+1}^*)$. The right-hand side is the marginal cost of debt. One additional dollar of debt increases the default threshold $Z_{it+1}^*$ by $\frac{1}{X_{t+1} N_{it+1}}$ (see equation 13), leading to an increase of the default probability by $\frac{1}{X_{t+1} N_{it+1}} \frac{\partial \Phi(Z_{it+1}^*)}{\partial Z_{it+1}^*}$. As a result, default losses increase by $(1 - \xi)(1 + \tau) \frac{B_{it+1}}{X_{t+1} N_{it+1}} \frac{\partial \Phi(Z_{it+1}^*)}{\partial Z_{it+1}^*}$. In equilibrium, firms lever up to the point where the benefit and cost of debt are balanced.

It is worth pointing out, both the benefit and cost of debt are discounted with the pricing kernel, $M_{t+1}$, indicating that equityholders weigh the benefit and cost of leverage with risk-neutral default probabilities (Almeida and Philippon 2007). In the absence of financial frictions (i.e. $\tau = 0$ and $\xi = 1$), equation (17) holds for any arbitrary level of debt, which essentially says that the optimal financial structure of the economy is indeterminate, as in Modigliani and Miller (1958).

### 3.3 Solution Method

A globally nonlinear solution method is crucial for accurately analyzing the model, owing to the focus of the paper on time-varying risk premium. In particular, the solution to the competitive equilibrium is obtained using projection methods. The details of the implementation are presented in Appendix D.

Rather than solving the multiplier function $\lambda_t$ in equation (14), I solve for the conditional expectation function in equation (14), denoted $E_t$. After obtaining $E_t$, I first calculate $\tilde{q}(\theta_t) = \kappa_0 / (E_t - \kappa_1)$. If $\tilde{q}(\theta_t) < 1$, it means the non-negativity vacancy constraint is not binding, therefore $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. If $\tilde{q}(\theta_t) \geq 1$, it means the non-negativity vacancy constraint is binding, and accordingly $V_t = 0$ and $q(\theta_t) = 1$.

The state space of the model consists of employment, $N_t$, debt, $B_t$, and aggregate productivity, $X_t$. Solving the model boils down to solving for four functions—the conditional expectation function $E_t$, the debt function $B_{t+1}$, the wage rate function $W_t$, and the indirect utility function $J_t—with four equilibrium conditions: the job creation condition (14), the optimality condition for financing (17), the wage determination equation (11), and the indirect utility equation (6).

---

7The reason is that the multiplier function $\lambda_t$ potentially has kinks (i.e. not smooth), due to occasionally binding non-negative vacancy constraint. A function with kinks imposes challenges for any numerical algorithm to approximate it. Working with $E_t$ gets around this issue, because $E_t$ shows up in equation (14) in the form of conditional expectations, meaning that it is by definition a sum of many functions, and tends to be smooth. The idea is in the spirit of the parameterized expectation approach in Christiano and Fisher (2000).
Table 3: Benchmark Quarterly Calibration

This table reports the parameters for the benchmark quarterly calibration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.991</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>10</td>
<td>Relative risk aversion</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.5</td>
<td>Elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>Financing</td>
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<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.01</td>
<td>Tax benefits of debt (dollar amount of tax shields)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.45</td>
<td>Recovery rates during default</td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.95</td>
<td>Aggregate productivity persistence</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0137</td>
<td>Aggregate productivity volatility</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.38</td>
<td>Conditional volatility of firm-specific productivity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.052</td>
<td>Workers’ bargaining weight</td>
</tr>
<tr>
<td>$b$</td>
<td>0.86</td>
<td>The value of unemployment activities</td>
</tr>
<tr>
<td>$s$</td>
<td>0.055</td>
<td>Job separation rate</td>
</tr>
<tr>
<td>$\iota$</td>
<td>1.27</td>
<td>Elasticity of the matching function</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>0.35</td>
<td>The proportional costs of vacancy posting</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>0.3</td>
<td>The fixed costs of vacancy posting</td>
</tr>
</tbody>
</table>

4 Quantitative Results

This section first discusses the calibration and aggregate moments, and then examines the model’s credit risk implications along two dimensions: key properties of credit spreads, and the relations between credit spreads and unemployment. Section 4.5 inspects the model’s mechanism through the lens of asset volatility. Finally, Section 4.6 presents comparative statics to further illustrate the intuition.

4.1 Calibration

Table 3 summarizes the parameter choices for the benchmark quarterly calibration. The first set of parameters concerns the preferences of the representative household. Following Bansal and Yaron (2004), the relative risk aversion, $\gamma$, is set to 10, the intertemporal elasticity of substitution, $\psi$, is set to 1.5, and the subject discount factor, $\beta$, is set to 0.991.

The second set of parameters pertains to labor market dynamics. The persistence of the productivity process, $\rho$, is set to 0.95, and its standard deviation, $\sigma$, is set to 0.0137, which are standard values in the labor search literature. The other six parameters are specific to the conventional search and matching framework: the job separation rate, $s$; the elasticity of the matching function, $\iota$; the workers’ bargaining weight, $\eta$; the flow value of unemployment activities, $b$; and the proportional and fixed costs of vacancy posting, $\kappa_0$ and $\kappa_1$. 

17
The job separation rate, $s$, is set to 0.055, consistent with the Survey of Income and Program Participation (SIPP) data (see Bils, Chang, and Kim 2011). The elasticity of the matching function, $\nu$, is set to 1.27, identical to the structural estimate of 1.27 in den Hann, Ramey, and Watson (2000). In the spirit of Hagedorn and Manovskii (2008), the worker’s bargaining power, $\eta$, is set to match the elasticity of wages with respect to labor productivity. Specifically, I set $\eta$ to be 0.052, which results in a wage elasticity of 0.47 in model simulations, close to the estimate of 0.45 in the data.

The flow value of unemployment activities, $b$, is in the spirit of Hagedorn and Manovskii (2008), who argue that $b$ should not deviate too much from the value of employment. In a perfectly competitive labor market, the two are equal. Specifically, $b$ is set to 0.86. It is close to the value of 0.85 used by Rudanko (2011) and Petrosky-Nadeau, Zhang, and Kuehn (2015). Because $b$ determines the size of the surplus from a match, it directly influences the average profit rate of firms. In Section 4.5, I further demonstrate that the choice of $b$ is empirically plausible, as judged from the profit-to-GDP ratio of the economy.

The proportional and fixed costs of vacancy posting $\kappa_0$ and $\kappa_1$ are pinned down jointly to target the mean and volatility of unemployment. This gives me $\kappa_0 = 0.35$ and $\kappa_1 = 0.3$, which implies a mean of 7.7% and a volatility of 13.2% for unemployment in simulations.

The last set of parameters governs the firms’ decisions to take on leverage. The tax benefit, $\tau$, is 0.01, consistent with the average interest rate of 7% on Baa-rated bond in the U.S. and a 15% effective tax advantage of debt (Leland 2004), which implies that the firm receives a tax subsidy of about one cent per dollar of debt raised in the bond market ($7\% \times 15\% = 0.105$). The recovery rate on defaulted bonds, $\xi$, is 45%, close to 42% in Chen (2010) for Baa-rated bonds. Finally, the volatility of firm-specific productivity, $\sigma_z$, is set to match the average default rate of 0.7% per year on Baa-rated bonds.

### 4.2 Aggregate Moments

Table 4 shows that the model captures aggregate business cycle dynamics (top panel) and aggregate asset prices (bottom panel) reasonably well. In particular, the model predicts an average consumption volatility of 2.90% per annum, lower than the predicted annual output volatility.

---

8In both the model and the data, the elasticity is measured as the coefficient by regressing HP-filtered log wages on HP filtered log labor productivity, with a smoothing parameter of 1,600. The data value of 0.45 indicates that a one percentage point increase in labor productivity is associated with a 0.45 percentage point increase in wages.

9The effective tax advantage of debt refers to the corporate tax rate offset by the personal tax rate advantage of equity. Graham (2000) estimates the corporate tax rate to be 35%, the personal tax rate on bond income 29.6%, and on dividends 12%. According to Miller (1977), the effective tax benefit of debt is $1 - (1 - 0.35) \times (1 - 0.12)/(1 - 0.296) = 0.188$, even larger than 15%.

10The default probability data are from Exhibit 32 of Moody’s annual report on corporate default and recovery rates (2013), which provides cumulative default probabilities across a variety of maturities. As in Gabaix (2012), a cumulative default probability is converted to an annual default probability by applying the formula $\frac{1}{N} \log(1 - x)$, where $x$ is the cumulative default probability and $N$ is the years to maturity. In particular, the 5-year, 10-year, and 20-year cumulative default probabilities for Baa-rated bonds are 3.096%, 7.112%, and 13.761%, respectively, over the 1920-2012 period. The implied annual default probabilities are 0.63%, 0.74%, and 0.74%, respectively. Their average provides an estimate of 0.7%. 18
Table 4: Aggregate Moments

This table presents annualized moments for aggregate output growth, $\Delta y$, aggregate consumption growth, $\Delta c$, aggregate excess stock market returns, $R - R_f$, and the risk-free rate, $R_f$. The data are real, sampled at an annual frequency, and cover the period from 1930 to 2014. The “Model” Panel presents the corresponding moments implied by the benchmark model, where $AR1\Delta y$ denotes the first-order autocorrelation. For each moment, I report the mean and the 5th, 50th, and 95th percentiles from 10,000 finite sample simulations of equivalent length to the data. The output and consumption growth rates are calculated by first aggregating quarterly output and consumption to yearly levels, then taking logs, then computing the first differences. For returns, the means are multiplied by four and standard deviation multiplied by two to annualize. All means and standard deviations are in percentage terms.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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<th></th>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>5%</td>
<td>50%</td>
<td>95%</td>
<td></td>
</tr>
<tr>
<td>$\sigma[\Delta y]$ (%)</td>
<td>4.87</td>
<td>3.45</td>
<td>2.32</td>
<td>3.02</td>
<td>6.21</td>
</tr>
<tr>
<td>$AR1[\Delta y]$</td>
<td>0.54</td>
<td>0.22</td>
<td>0.01</td>
<td>0.20</td>
<td>0.52</td>
</tr>
<tr>
<td>$\sigma[\Delta c]$ (%)</td>
<td>2.15</td>
<td>2.90</td>
<td>1.72</td>
<td>2.40</td>
<td>5.99</td>
</tr>
<tr>
<td>$AR1[\Delta c]$</td>
<td>0.47</td>
<td>0.23</td>
<td>0.02</td>
<td>0.21</td>
<td>0.53</td>
</tr>
<tr>
<td>$E[R - R_f]$ (%)</td>
<td>8.16</td>
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<td>$\sigma[R - R_f]$ (%)</td>
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<td>21.82</td>
<td>19.55</td>
<td>21.82</td>
<td>24.08</td>
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<tr>
<td>$E[R_f]$ (%)</td>
<td>2.90</td>
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</tr>
<tr>
<td>$\sigma[R_f]$ (%)</td>
<td>2.82</td>
<td>1.31</td>
<td>0.60</td>
<td>1.12</td>
<td>2.71</td>
</tr>
</tbody>
</table>

of 3.45%, owing to households’ desire to smooth consumption fluctuations. In addition, both the consumption and output growth volatilities in historical data fall comfortably within the 90% confidence bands of the bootstrapped model-implied distribution, indicating that the model economy is capable of generating the real-world data. In terms of persistence, the model predicts a first-order autocorrelation of 0.22 and 0.23, respectively, for output and consumption growth, somewhat lower than the data counterparts.

Turning to asset prices, the model generates an equity premium of 8.77% per annum, close to 8.16% in the data. The volatility of the equity premium is 21.82%, which compares fairly well with the empirical figure of 20.49%. Moreover, the mean risk-free rate in the model is 3.02%, slightly higher than the 2.9% real risk-free rate over the long U.S. sample (see Campbell and Cochrane 1999). Finally, risk-free rates are fairly stable in the model, with a volatility of 1.31% per annum, somewhat lower than the data (2.82%).

4.3 Credit Spreads

Table 5 evaluates the credit risk implications of the model. Panel A of Table 5 shows that the model reasonably replicates salient features of credit spreads. As a result of calibration, the default probability comes very close to the data. The average credit spread is 70 basis points, and
the 90% confidence interval of the model’s bootstrapped distribution ranges from 51 to 97 basis points. The data counterpart is from Duffee (1998), who shows that empirical estimates of the Baa-Aaa credit spread exhibit nontrivial variation across sectors and maturities, but nonetheless ranges largely between 70 to 105 basis points. In short, the model succeeds in accounting for a reasonably large fraction of observed Baa-Aaa credit spread, although it still understates somewhat the level of credit spreads. The quantitative performance of the model is comparable to several leading studies explaining the level of credit spreads, for example, Cremers, Driessen, and Maenhout (2008), and Bhamra, Kuehn, and Strebulaev (2010).

Table 5: Credit Risk Implications

This table presents properties of key credit market variables. Panel A reports the unconditional moments, where $AR(j)$ denotes the $j^{th}$ order autocorrelation. Panel B reports cyclicalities, measured as the correlation with output growth. For the data, the annual default probability is from Moody’s 2013 Annual Report on corporate default and recovery rates; credit spreads are the yield spreads between Baa- and Aaa-rated bonds with the callability feature adjusted, from Duffee (1998); market leverage is the book value of debt divided by the sum of the book value of debt and the market value of equity, from Huang and Huang (2012); and the cyclicalities are from Gomes and Schmid (2014). The “Model” Panel presents the corresponding statistics implied by the benchmark model. For each statistic, I report the mean and the 5th, 50th, and 95th percentiles across 10,000 finite sample simulations of equivalent length to the data.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Model Mean</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>5%</td>
<td>50%</td>
<td>95%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel A: Moments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Targeted</strong></td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Default probability (%)</td>
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<td>0.70</td>
<td>0.40</td>
<td>0.68</td>
<td>1.10</td>
<td></td>
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</tr>
<tr>
<td><strong>Untargeted</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit spread (%)</td>
<td>0.70-1.05</td>
<td>0.70</td>
<td>0.51</td>
<td>0.66</td>
<td>0.97</td>
<td></td>
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<tr>
<td>Credit spread volatility (%)</td>
<td>0.70</td>
<td>0.43</td>
<td>0.17</td>
<td>0.33</td>
<td>0.89</td>
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<tr>
<td>Credit spread skewness</td>
<td>2.28</td>
<td>2.20</td>
<td>0.86</td>
<td>1.92</td>
<td>4.60</td>
<td></td>
<td></td>
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<tr>
<td>Credit spread $AR(1)$</td>
<td>0.90</td>
<td>0.92</td>
<td>0.86</td>
<td>0.92</td>
<td>0.96</td>
<td></td>
<td></td>
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<tr>
<td>Credit spread $AR(4)$</td>
<td>0.71</td>
<td>0.70</td>
<td>0.54</td>
<td>0.71</td>
<td>0.84</td>
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<td></td>
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<tr>
<td>Credit spread $AR(10)$</td>
<td>0.45</td>
<td>0.41</td>
<td>0.18</td>
<td>0.41</td>
<td>0.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market leverage</td>
<td>0.40</td>
<td>0.37</td>
<td>0.32</td>
<td>0.37</td>
<td>0.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Cyclicalities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Credit spread</td>
<td>−0.36</td>
<td>−0.26</td>
<td>−0.36</td>
<td>−0.25</td>
<td>−0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default probability</td>
<td>−0.33</td>
<td>−0.56</td>
<td>−0.63</td>
<td>−0.56</td>
<td>−0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market leverage</td>
<td>−0.30</td>
<td>−0.34</td>
<td>−0.51</td>
<td>−0.35</td>
<td>−0.12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Following the credit spread puzzle literature, the empirical targets are taken from Duffee (1998), which handles the call option embedded in many corporate bonds, and thus allows a fair comparison between the model and the data. Specifically, Duffee (1998) estimates the Baa-Aaa spread among all business sectors (Panel D in his Table 1, p. 2231) to be 75 basis points for maturities of two to seven years, 70 basis points for maturities of seven to fifteen years, and 105 basis points for maturities of fifteen to thirty years.
Credit spreads in the model exhibit substantial volatility, averaging around 43 basis points. The 90% confidence interval ranges from 17 to 89 basis points, containing the volatility of the Baa-Aaa spread in the data (70 basis points). Interestingly, model-implied credit spreads feature a fat right tail, as evidenced by the skewness measure of 2.20 (versus 2.28 in the data), suggestive of episodes such as the Great Depression (see Figure 1). Furthermore, the model captures the persistence of credit spreads quite well, with the first, fourth, and tenth order autocorrelations at 0.92, 0.70, and 0.41, respectively, corresponding to the data values of 0.90, 0.71, and 0.45.

Turning to implications for financing, the model generates a market leverage of 37%, close to the average market leverage of 40% for Baa-rated firms. Given the seemingly large tax benefits of debt, it has remained a puzzle as to why firms appear to use debt conservatively, an observation commonly referred to as the under-leverage puzzle (Miller 1977; Graham 2000). The model’s success along this dimension shares the same insight as Almeida and Philippon (2007). Expected default losses, once evaluated with risk-neutral default probabilities implied from credit spreads, are sizable enough to offset equityholders’ incentives in taking on high leverage.

What drives the high levels of credit spreads in the model? Intuitively, investors care about the timing of default. If corporate bonds are more likely to default in bad times when investors’ marginal utility is high, they want to be compensated with high yields even with a small amount of default exposure. This intuition underlies the model’s predictions of high and volatile credit spreads. In what follows, I characterize the model’s implications for consumption dynamics and default, respectively.

Owing to strong nonlinear dynamics, the economy occasionally runs into economic disasters per Rietz (1988) and Barro (2006). To illustrate, Figure 3 depicts quantile-quantile plots of log consumption and output growth rates (based on one million periods simulated from the benchmark model) against the standard normal distribution. Panel A shows that log consumption growth rates (depicted by the blue line) exhibit sizable departures from the normal distribution (depicted by the dashed red line) at both ends of the distribution. More important, the left end highlights the presence of disasters, deep recessions whose likelihood is understated by the normal distribution. The right end indicates periods with abnormal growth, or recoveries, which usually occur when the economy climbs out of deep recessions. Output growth rates exhibit similar distributional patterns, as shown in Panel B.

Key properties of disaster episodes including the probability, size, and duration, are also measured, following Barro and Ursúa (2008). The model predicts that consumption disasters occur with a probability of 2.0% per year. The magnitude of consumption declines during disasters averages at 20%. Finally, an average consumption disaster takes around 5.7 years to unfold in the

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12Specifically, 10,000 artificial samples, each with equal length to the data, are simulated from the benchmark model. On each simulated sample, quarterly consumption and output are first time-aggregated to yearly levels. Disasters are then identified as cumulative fractional declines in consumption or output of at least 10%, by applying the peak-to-trough method. Finally, properties including the probability, size and duration, are measured and their cross-sample statistics are reported.
Figure 3: Disasters

This figure presents quantile-quantile plots of log consumption and output growth rates (from the model's simulated sample) versus the standard normal distribution, shown by the dashed red line. In each panel, the blue line plots sample quantiles of the simulated data (y-axis) against theoretical quantiles of the standard normal distribution (x-axis). If the simulated data are normally distributed, the blue line will coincide with the dashed red line. Both panels are based on the same simulated sample of one million periods from the benchmark model’s stationary distribution.

Panel A: Consumption Growth  Panel B: Output Growth

Countercyclical Default

Default rates are countercyclical in the model, as illustrated in Panel A of Figure 4, which plots the default probability against the aggregate productivity and employment. As the figure highlights, default probability rises sharply in bad economic times when both productivity and employment are low. Importantly, such times are exactly when investors experience disastrously low consumption and high marginal utilities. Following the intuition, one expects that investors will command compensation in the form of countercyclical credit spreads. Indeed, credit spreads widen dramatically when the economy slides into a recession typically featuring low productivity and high unemployment. This can be seen in Panel B of Figure 4, which graphs the model-implied credit spread against the aggregate productivity and employment. In simulations, the cyclical behavior of credit spreads and default rates is evident.

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13The model delivers similar disaster dynamics for output. In particular, the model predicts an output disaster occurs with a chance of 3.45% per year. Output declines average around 18% across disaster episodes. Finally, an output disaster lasts for 5.2 years on average.
Panels A and B plot default probability and credit spreads, respectively, against the aggregate productivity and employment, keeping debt fixed at the average level.

Panel A: Default Probability
Panel B: Credit Spreads (%)

of these variables is characterized by their correlations with output growth. Panel B of Table 5 shows that the cyclicalities of default probabilities, credit spreads, and market leverage are largely in line with their empirical counterparts.

The insight that large credit spreads arise due to coincidence of high default rates and high marginal utilities is not new. Motivated by the equity premium puzzle literature, Chen, Collin-Dufresne, and Goldstein (2009), Chen (2010), and Bhamra, Kuehn, and Streubäev (2010), among others, show that leading consumption-based asset pricing models featuring highly countercyclical pricing kernels, coupled with countercyclical default rates, can explain high levels of credit spreads relative to historical default and recovery rates. A common theme among those papers is that they assume firms’ asset value process or the cash flow process on which the asset values are derived, evolves exogenously, in a manner that they are delinked from firms’ real decisions. This paper differs substantially in that it ties default back to firms’ real decisions and the macroeconomy in general. Specifically, this paper relates firms’ default decisions to their job creation decisions, and demonstrates the link holds out the promise of endogenously yielding strong comovement between default rates and marginal utilities, and as a result, the insight from the endowment economy frameworks can be carried over.
4.4 Credit Spreads and Unemployment

The connection between credit spreads and unemployment can be elucidated by combining the insights of Merton (1974) with the search and matching theory of unemployment. The main insight of Merton (1974) is that the risky debt issued by a firm is economically equivalent to risk-free debt minus a put option written on the assets owned by the firm. In the context of this paper, movements in the asset value of the firm are solely driven by fluctuations in the asset value of employment relationships, as measured by the present value of current and future cash flows that workers bring to the firm.

On the other hand, the DMP model relates unemployment to job creation incentives, which are in turn driven by the asset value of employment relationships: In economically bad times, an employment relationship is valued less, employers put fewer resources into recruiting new workers, the labor market then slackens and unemployment rises. Apparently, the nexus is the asset value of employment relationships, which simultaneously drives firms’ job creation incentives, and the pricing of corporate debt.

A new insight that naturally emerges is, to the extent that unemployment reflects the asset value of employment relationships, it reveals information about the value of the embedded put option written on those assets, and credit risk. To flesh this insight out, I construct the asset value of employment relationships from the model, which is essentially 

\[ X_t - W_t + (1 - s)(\kappa_t/q(\theta_t) - \lambda_t) \]

It comprises two components: The first component represents the cash flow a worker generates in the current period, \( X_t - W_t \). The second component refers to the present value of future cash flows that the worker will bring to the firm, which in equilibrium equals the costs of hiring net of separation, \( (1 - s)(\kappa_t/q(\theta_t) - \lambda_t) \).

Panel A of Figure 5 illustrates the relation between credit spreads and the asset value of employment relationships as constructed above. Specifically, I simulate the model for one million periods and plot the model-implied credit spreads against the asset values. In line with Merton’s (1974) insight, Panel A resembles the familiar relation between the price of a put option against various values of the underlying asset. Notably, the relation is highly convex. When the asset value declines, the default option becomes nonlinearly more valuable.

Panel B of Figure 5 presents the scatterplot of unemployment against the asset value of employment relationships, from the same exact simulation. It highlights the central insight in the search and matching approach to unemployment. The incentives for creating jobs, namely the asset value of employment relationships, ultimately drive unemployment dynamics. Interestingly, the relation is similarly nonlinear. Unemployment rises at a precipitous pace as the asset value of employment relationships declines.

It follows immediately that credit spreads will strongly comove with unemployment in the model. Indeed, across model simulations, the correlation between credit spreads and unemployment averages around 0.85 (versus 0.81 in the data). To illustrate, Figure 6 displays an episode
Figure 5: Credit Spread, Unemployment, and Asset Value

Panel A plots credit spreads against the asset value of employment relationships (see the text for definition); Panel B plots unemployment against the asset value of employment relationships. Both panels are based on the same simulated sample of one million periods from the benchmark model’s stationary distribution.

from simulating the benchmark model, with blue denoting credit spreads (left axis) and red denoting the unemployment rate (right axis). Apart from the tight relation between the two series, most prominent are the occasional coincident spikes in both series to unusually high levels, reminiscent of historical episodes such as the Great Depression.

The Sensitivity of Credit Spreads to Unemployment

Table 6 evaluates how the quantitative predictions of the model compare with the empirical evidence presented in Section 2, by estimating similar regressions as in Section 2. Column (1) indicates that a one percentage point rise in unemployment leads to a widening of credit spreads by 16.7 basis points. The economic magnitude reduces slightly to 15.0 basis points, after including asset volatility and market leverage as controls, as displayed in Column (2). In comparison with Table 1, the model-implied sensitivity of credit spreads to unemployment comes fairly close to those in the data. In terms of explanatory power, the unemployment rate itself explains around 74% of credit spread variation. Adding control variables further raises the $R^2$ to 81%. Given the stylized nature of the model and arguably better measurement of variables in model regressions, perhaps not surprisingly, the model overstates somewhat the explanatory power.

The sensitivity of credit spreads to unemployment neatly captures the impact of labor mar-
Figure 6: Credit Spread and Unemployment in the Model

This figure plots an illustrative episode of credit spread (blue, left axis) against the unemployment rate (red, right axis) in the benchmark model.

ket conditions on credit risk. The reason is, to the extent that unemployment maps out the asset value of employment relationships, the sensitivity of credit spreads to unemployment roughly corresponds to the delta of the embedded put option written on the asset value of employment relationships. According to Merton (1974), the delta is essentially the risk-neutral default probability of corporate bonds. In other words, credit spreads will exhibit a higher sensitivity to unemployment only when fluctuations in labor market conditions render corporate bonds riskier.

Intuitively, bond prices react to the asset value of employment relationships exactly because a fraction of the asset value is lost due to default. Alternatively, if bonds do not default at all, bond prices and hence credit spreads will not contain any information about the asset value of employment relationships. It is important to point out that the model’s implications for the sensitivity of credit spreads to unemployment serve as a test for the model’s second-moment predictions, in a similar spirit as the hedge ratio test for the structural models of credit risk (e.g., Schaefer and Strebulaev 2008).
Table 6: Credit Spread Regressions in the Model

This table presents the results of the regression

\[ CS_t = \beta_0 + \beta_1 U_t + \gamma Z_t + \epsilon_t \]

based on simulated data out of the benchmark model. The table reports the average regression coefficients and average t-statistics (in brackets), across 10,000 simulated time series of equal length to the data. The table reports OLS coefficient estimates and Newey-West corrected t-statistics (in brackets) with the automatic lag selection method of Newey and West (1994). \( Z_t \) represents a vector of control variables: Asset volatility is the standard deviation of the growth rate of asset value; Market leverage is the book value of debt divided by the sum of the book value of debt and the market value of equity.

<table>
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<td>Unemployment</td>
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<td>[19.04]</td>
<td>[13.23]</td>
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<td>Asset volatility</td>
<td>0.116</td>
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<td></td>
<td>[7.23]</td>
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<tr>
<td>Market Leverage</td>
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<tr>
<td></td>
<td>[1.95]</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>−0.006</td>
<td>−0.012</td>
</tr>
<tr>
<td></td>
<td>[−8.78]</td>
<td>[−8.28]</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.74</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Predicting Changes in Unemployment

A robust feature of the data is that credit spreads contain information about future labor market conditions, and economic activity in general (Gilchrist and Zakrajsek 2012). Table 7 examines this feature by forecasting future changes in unemployment with credit spreads. Consistent with Gilchrist and Zakrajsek (2012), in the data a widening of the Baa-Aaa credit spread signals increases in future unemployment rates. The predictive coefficients are statistically significant up to two quarters and die off afterwards. Perhaps not surprisingly, \( R^2 \)’s decline with horizons as well. The model paints a similar picture. Widening credit spreads reliably precede a slack labor market featuring high unemployment. The model coefficients are largely comparable to those in the data, and \( R^2 \)’s decline with forecasting horizons.

4.5 Endogenous Asset Volatility

What drives the seemingly strong response of credit spreads to labor market fluctuations? This paper approaches the question through the lens of asset volatility, which captures the amount of business risk that firms face.

In the model, asset volatility is measured by the standard deviation of the growth rate of aggregate asset value of employment relationships. The constructed asset volatility thus measures the component of asset volatility that is common to all firms in the economy, or systematic asset
Table 7: Forecasting Changes in Unemployment

This table reports predictive regressions for changes in unemployment using credit spreads. The regression is

$$\sum_{h=1}^{H} [U_{t+h} - U_{t+h-1}] = \beta_0 + \beta_1 CS_t + \epsilon_{t+h}$$

The “Data” Panel displays predictive slopes, Newey-West corrected t-statistics, and $R^2$ in historical U.S. data (1948Q1-2015Q1). The “Model” Panel display average predictive slopes, Newey-West corrected t-statistics, and the mean, the 5th, and the 95th percentiles of $R^2$ across 10,000 finite sample simulations (of equivalent length to the data) of the benchmark model.

<table>
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<th>Model</th>
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<tr>
<td></td>
<td>$\beta_1$</td>
<td>$t(NW)$</td>
<td>$R^2$</td>
<td></td>
<td>$\beta_1$</td>
<td>$t(NW)$</td>
<td>$R^2$</td>
<td>$R^2(5%)$</td>
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<tr>
<td>1</td>
<td>0.26</td>
<td>3.95</td>
<td>0.09</td>
<td></td>
<td>0.44</td>
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<td>2.37</td>
<td>0.06</td>
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<td>0.45</td>
<td>1.43</td>
<td>0.05</td>
<td>0.01</td>
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<tr>
<td>3</td>
<td>0.40</td>
<td>1.59</td>
<td>0.03</td>
<td></td>
<td>0.26</td>
<td>0.44</td>
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<tr>
<td>4</td>
<td>0.34</td>
<td>1.03</td>
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<td></td>
<td>-0.02</td>
<td>-0.19</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Both the sizable and countercyclical natures of systematic asset volatility are crucial for the credit risk implications of the model. First of all, as noted before, the level of systematic asset volatility directly affects the extent of systematic nature of default, which determines the size of the credit risk premium, further impacting both the level of credit spreads and the sensitivity of credit spreads to unemployment. Second, countercyclical systematic asset volatility makes default more likely in bad economic times (Panel A, Figure 4), helping elevate the level of credit spreads.

The small surplus nature of the match between firms and workers plays a critical role in driving large and countercyclical asset volatility. Intuitively, the smaller the surplus from a match, the larger the proportional change of the surplus in response to exogenous shocks, and accordingly, the more responsively firms react to economic conditions in creating jobs. In the model, it is the asset value of employment relationships that drives the incentives for job creation. As a result, a small surplus translates into a sizable asset volatility. Furthermore, because job creation incentives are on average more responsive to shocks in bad times due to a even smaller surplus at such times, asset volatility tends to be countercyclical.

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14 See their Table 2. Since asset values are not directly observable, Chen, Yu, and Yang (2013) back out asset volatility from equity volatility based on Merton (1974), following Bharath and Shumway (2008).
More specifically, the flow value of unemployment activities, $b$, is the key parameter that pins down the size of the surplus. Intuitively, a high flow value of unemployment activities results in a small surplus from an employment relationship. A natural question to ask: Is the calibrated high $b$ value empirically plausible? As judged from the profit-to-GDP ratio in U.S. data, the answer is yes. To corroborate the claim, I obtain the profits and GDP data from NIPA Table 1.12 and 1.1.6, respectively, and estimate the annual profit-to-GDP ratio to be 9.15% for the period 1929 to 2014. In the model, I first time-aggregate the quarterly profit and GDP series to annual levels, and then calculate the profit-to-GDP ratio. Across model simulations, the profit-to-GDP ratio averages around 8.7%, close to the data estimate.

In short, this paper shows that a DMP model, which is reasonably calibrated with realistic unemployment volatilities, generates sizable and countercyclical asset volatility. Put differently, labor market fluctuations expose firms to substantial business risk, which renders credit risk sensitive to labor market conditions. Whereas the literature on corporate bond pricing has largely treated asset volatility—a primitive input to all structural models of credit risk—as exogenous, this paper suggests that the labor market represents a potentially important source of business risk for firms. In this respect, this paper sheds light on macroeconomic drivers of asset volatility.

### 4.6 Comparative Statics

Comparative statics, tabulated in table 8, further illustrate the intuition. Each column examines the key credit risk moments generated from models in which only one parameter changes relative to the benchmark calibration in Table 3.

Column (1) lowers the flow value of unemployment, $b$, to 0.6. A direct consequence of a lower flow value of unemployment is that the surplus from the match is much larger, meaning that firms’ job creation incentives are less sensitive to shocks.\(^{15}\) Indeed, asset volatility drops substantially from 13.78% per annum in the benchmark model to 5.13%. As a result, both the default probability and credit spreads fall dramatically—to 0.12% and 0.07%, respectively. Not surprisingly, the volatility of credit spreads also becomes nil. Because default becomes so rare while the tax advantage of debt stays highly lucrative, firms aggregatively lever up—market leverage rockets to an unrealistically high level at 0.76.

Interestingly, the sensitivity of credit spreads to unemployment even turns negative, meaning that credit spreads tighten as unemployment rises. To understand this counterfactual result, recall that firms are on average more profitable in good economic times and they want to lever up more to exploit the tax shields. The increase in leverage, however, makes firms more indebted, causing credit spreads to widen at such times. In the benchmark model, large asset volatility acts to offset firm’s incentives to lever up. In this case, however, the offsetting force greatly weakens due to a small asset volatility, resulting in procyclical credit spreads.

\(^{15}\)The profit-to-GDP ratio in this case is 20.2%, much higher than the data counterpart of 9.15%.
Table 8: Comparative Statics

This table presents the key credit risk moments from comparative statics exercises. Each column from (1) to (5) perturbs one parameter relative to the benchmark calibration in Table 3 while keeping all other parameters unchanged. For variable definitions and regression specifications, refer to Table 5 and Table 6.

<table>
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<th>(4)</th>
<th>(5)</th>
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<tr>
<td></td>
<td></td>
<td>low b</td>
<td>low ι</td>
<td>high σz</td>
<td>high τ</td>
<td>high ξ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b = 0.6</td>
<td>ι = 0.9</td>
<td>σz = 0.42</td>
<td>τ = 0.012</td>
<td>ξ = 0.5</td>
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</table>

<table>
<thead>
<tr>
<th>Unconditional Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default probability (%)</td>
</tr>
<tr>
<td>0.70</td>
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<tr>
<td>0.12</td>
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<tr>
<td>0.56</td>
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<tr>
<td>0.81</td>
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<tr>
<td>0.85</td>
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<tr>
<td>0.78</td>
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<tr>
<td>Credit spread (%)</td>
</tr>
<tr>
<td>0.70</td>
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<tr>
<td>0.07</td>
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<tr>
<td>0.60</td>
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<td>0.81</td>
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<tr>
<td>0.81</td>
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<tr>
<td>0.71</td>
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<tr>
<td>Credit spread volatility (%)</td>
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<td>0.43</td>
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<td>0.00</td>
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<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.61</td>
</tr>
<tr>
<td>0.47</td>
</tr>
<tr>
<td>0.43</td>
</tr>
<tr>
<td>Credit spread skewness</td>
</tr>
<tr>
<td>2.20</td>
</tr>
<tr>
<td>−0.87</td>
</tr>
<tr>
<td>2.18</td>
</tr>
<tr>
<td>2.58</td>
</tr>
<tr>
<td>2.11</td>
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<tr>
<td>2.21</td>
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<tr>
<td>Market leverage</td>
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<td>0.37</td>
</tr>
<tr>
<td>Asset volatility (%)</td>
</tr>
<tr>
<td>13.78</td>
</tr>
<tr>
<td>5.13</td>
</tr>
<tr>
<td>13.50</td>
</tr>
<tr>
<td>13.76</td>
</tr>
<tr>
<td>13.85</td>
</tr>
<tr>
<td>13.78</td>
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</table>

<table>
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<tr>
<th>Credit Spread Regressions</th>
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<tbody>
<tr>
<td>Univariate regression</td>
</tr>
<tr>
<td>0.167</td>
</tr>
<tr>
<td>−0.023</td>
</tr>
<tr>
<td>0.090</td>
</tr>
<tr>
<td>0.226</td>
</tr>
<tr>
<td>0.196</td>
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<tr>
<td>0.172</td>
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<tr>
<td>0.150</td>
</tr>
<tr>
<td>0.003</td>
</tr>
<tr>
<td>0.074</td>
</tr>
<tr>
<td>0.209</td>
</tr>
<tr>
<td>0.176</td>
</tr>
<tr>
<td>0.156</td>
</tr>
</tbody>
</table>

Figure 7 further demonstrates the role of $b$, by plotting various credit market statistics against different values of $b$. Panel A illustrates the importance of the small surplus between firms and workers in amplifying asset volatility, confirming the intuition articulated in Section 4.5. Panels B to D highlight how asset volatility further impacts the credit risk implications of the model. In particular, as asset volatility rises, the odds of default spike up (Panel B), and credit spreads widen accordingly (Panel C). Most importantly, as the labor market becomes more volatile, credit spreads exhibit a higher sensitivity to unemployment, consistent with the interpretation of the sensitivity as the delta of the embedded put option in Section 4.4.

Column (2) lowers the matching elasticity parameter, $ι$, from 1.25 to 0.9. A lower $ι$ means the labor market becomes more frictional in matching vacancies and unemployed workers. Hence, firms respond less actively to shocks in creating jobs, causing asset volatility to decline. It follows that default becomes less likely, credit spreads tighten and become less sensitive to labor market conditions.

Column (3) increases the firm-specific productivity, $σz$, from 0.38 to 0.42. $σz$ governs the variability of firm-level business conditions: More variability suggests a higher likelihood of default. Indeed, the annual default rate increases from 0.70% in the benchmark model to 0.81%, resulting in elevated and more volatile credit spreads and more sensitive credit risk to labor market conditions. Perhaps not surprisingly, systematic asset volatility barely changes, because firm-level shocks cancel out in the aggregate.

Columns (4) and (5) alter the two parameters governing financing: the tax shields, $τ$, and the
recovery rate, $\xi$. Both work through twisting firms’ incentives to lever up. As can be seen from the optimality condition for financing (17), either a larger tax benefit or a higher recovery rate entices firms to take on more leverage, which consequently increases the odds of default. With this observation in mind, the observed sensitivities of credit spreads to unemployment seem sensible.

4.7 Loose Ends

Given the central role of the asset value of employment relationships in this paper, a concern is whether it represents a significant component of firm value for U.S. firms. A notable example in the literature that has estimated this quantity is Merz and Yashiv (2007), who estimate Tobin’s $q$ for both capital and labor, by equating the market value of the firm to its capital stock and employment levels valued by their respective $qs$. With quarterly U.S. data from 1976Q1 through 2002Q4,
they estimate that the ratio of the marginal costs of hiring over the average output per worker—which can be interpreted as the Tobin’s $q$ for labor—is around 1.48, with a standard deviation of 0.57 (see their Table 4, p. 1427).

In my model, because the average output per worker is normalized to one, the corresponding quantity is $\kappa / q(\theta)$, which is the major component of the asset value of employment relationships (see equation 13). Across model simulations, $\kappa / q(\theta)$ averages around 1.22, somewhat below Merz and Yashiv’s point estimate, but well within the plausible ranges judged by the standard deviation. As such, my calibration is largely consistent with the magnitude of the asset value of employment relationships in U.S. data.

A related concern is whether adding capital will significantly alter the quantitative results. To start, recall that for most production functions capital has positive impact on the marginal product of labor. It follows that cyclical fluctuations in capital will amplify variation in firms’ job creation incentives, leading to more volatile employment relationships. In this respect, the inclusion of capital will tend to strengthen the credit risk implications of this paper.

5 Labor Market Volatility

Table 9 explores the model’s implications for labor market volatility. The data moments correspond to the period 1929Q2 to 2015Q1. Following the literature, monthly series of unemployment is first converted into quarterly series by taking within-quarter averages, which is then detrended as HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600.\footnote{For any general variable $X$, the proportional deviation from the mean is $(X - \bar{X})/\bar{X}$, where $\bar{X}$ is the mean of $X$.} For the model moments, the table presents the mean and the 5th, 50th, and 95th percentiles across 10,000 finite sample simulations of equivalent length to the data.

Panel A of Table 9 shows that the benchmark model predicts an unemployment volatility of 13%, and the 90% confidence interval of the bootstrapped distribution ranges from 5% to 28%, containing the data value of 20%. Unemployment in the model features a fat right tail, with a skewness of 2.7 (versus 2.0 in the data). In untabulated results, the model predicts a vacancy volatility of 15%, the labor market tightness volatility of 17%, and a downward-sloping Beveridge curve with a negative correlation of -0.28 between unemployment and vacancies.

How does credit risk affect unemployment volatility? To answer this question, I mute the impact of financial frictions by setting the tax benefits $\tau = 0$ and the recovery rate $\xi = 1$. It follows that firms in this model have no incentives to take on debt, and they are in effect all equity financed. I refer to this version of the model the “all-equity model.” Panel B shows that the all-equity model delivers a lower unemployment volatility. This is anticipated, because financial frictions serve as an amplification mechanism in the benchmark model. To see the point, consider a positive productivity shock. It not only enhances firms’ job creation incentives in terms of
Table 9: Labor Market Moments

This table presents moments of unemployment. The data moments are for the period 1929Q2 to 2015Q1. Monthly series of unemployment is converted into quarterly series by taking within-quarter averages, which is then detrended as HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. Panel A reports moments from the benchmark model. Panel B reports moments from the all-equity model, where the tax benefits $\tau = 0$ and the recovery rate $\xi = 1$. For each statistic, the table presents the mean and the 5th and 95th percentiles across 10,000 finite sample simulations of equivalent length to the data.

<table>
<thead>
<tr>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td><strong>Panel A: The Benchmark Model</strong></td>
<td></td>
</tr>
<tr>
<td>Unemployment volatility</td>
<td>0.20</td>
</tr>
<tr>
<td>Unemployment skewness</td>
<td>2.00</td>
</tr>
<tr>
<td><strong>Panel B: The All-equity Model</strong></td>
<td></td>
</tr>
<tr>
<td>Unemployment volatility</td>
<td>0.20</td>
</tr>
<tr>
<td>Unemployment skewness</td>
<td>2.00</td>
</tr>
</tbody>
</table>

cash flows, but also lowers the odds of default and the cost of borrowing, further encouraging job creation.

However, it appears that the amplifications effects are small quantitatively. Unemployment volatility increases from 12% in the all-equity model to only 13% in the benchmark model. Hence in the benchmark economy, the feedback effect from the credit market to the real economy is likely not sizable. To improve the model’s prediction along this dimension, one can introduce additional features (for example, multi-period debt) that might result in meaningful amplifications. This topic is left for future research.

6 Conclusion

Motivated by the tight historical relation between the Baa-Aaa credit spread and the unemployment rate in the U.S. economy, this paper explores how labor market conditions impact credit risk. To this end, this paper proposes a model that relates corporate default and credit risk to labor market conditions, within a standard DMP model of equilibrium unemployment that features defaultable debt.

A reasonably calibrated model replicates salient features of credit spreads, including the level, volatility, cyclicality, and skewness. More important, the model is consistent with the historical relation with credit spreads and unemployment in the U.S. economy. In the model, credit spreads and unemployment strongly comove. In response to endogenous disasters, both series spike up simultaneously to unusually high levels, reminiscent of historical episodes such as the Great De-
pression, which is a novel prediction of the model. Quantitatively, the model accounts for the strong response of credit spreads to unemployment in historical U.S. data. Further analysis points to the labor market as a significant source of business risk for firms.
References


Favilukis, Jack, Xiaoji Lin, and Xiaofei Zhao, 2015, The elephant in the room: the impact of labor obligations on credit risk, *Working paper*.


Moody’s Annual Default Study: Coroprate default and recovery rates, 1920-2012.


Appendix

A Data

This appendix describes the data sources and variable construction in Section 2. All variables are available at a monthly frequency except for market leverage, which is constructed at a quarterly frequency. To convert the monthly series to quarterly frequency (Table 1 and 2), I take the quarter-end observations for each calendar quarter (i.e. March, June, September, and December). CRSP refers to the Center for Research in Security Prices; BLS refers to the Bureau of Labor Statistics; FRED refers to the Federal Reserve Economic Data, maintained by the research division of the Federal Reserve Bank of St. Louis; GFD refers to the Global Financial Data database.

Yield on Baa-rated corporate bonds: The data are from FRED (series BAA) and available at a monthly frequency from 1919M1 to 2015M3.

Yield on Aaa-rated corporate bonds: The data are from FRED (series AAA) and available at a monthly frequency from 1919M1 to 2015M3.

Yield on long maturity Treasury bonds: The data are from FRED: series LTGOVTBD (the yield on long-term U.S. government securities) for 1929M4-1999M12; series GS20 (the yield on 20-year maturity Treasury bonds) for 2000M1-2015M3.

Yield on 10-year Treasury bonds: The data are from FRED: series m13033a for 1926M1-1941M12; series m13033b for 1942M1-1952M12; series GS10 for 1953M1-2015M3.

Yield on 3-month Treasury bonds: The data are from FRED: series m13029a for 1926M1-1933M12; series TB3MS for 1934M1-2015M3.

Unemployment rates: The data are from GFD and available at a monthly frequency from 1929M4 to 2015M3.

Aggregate stock market volatility: A monthly series of realized stock market volatility is estimated from daily returns on the CRSP index. Following Campbell and Taksler (2003), aggregate stock market volatility is constructed as a six-month moving average of the monthly series.

Idiosyncratic stock volatility: A monthly series of the cross-sectional dispersion of stock returns is estimated based on monthly returns from CRSP as in Goyal and Santa-Clara (2003). Following Campbell and Taksler (2003), idiosyncratic stock market volatility is constructed as a six-month moving average of the monthly series.

Market leverage: For 1952Q1-2015Q1, the data are from Table B102 of the Flow of Funds. Market leverage is calculated as total liabilities divided by the sum of total liabilities and the market value of corporate equity in the non-financial corporate sector. For 1929Q2-1951Q4, the liability data are from Larrain and Yogo (2008), available at Motohiro Yogo’s webpage (https://sites.google.com/site/motohiroyogo/home/research). Because the liability data are...
only available at an annual frequency, a quarterly series is obtained by linearly interpolating the annual series. The market value of corporate equity is from CRSP.


**Industrial production index**: The data are from FRED (series INDPRO) and available at a monthly frequency from 1919M1 to 2015M3.

### B Derivation of the First-Order Conditions

Rewrite the equityholder’s problem in (3) as

\[
S(N_{it}, B_{it}, Z_{it}) = X_t Z_{it} N_{it} - W_t N_{it} + F(N_{it}, B_{it}) - B_{it},
\]

\[
F(N_{it}, B_{it}) = \max_{V_{it}, B_{it+1}} -\kappa_t V_{it} + (\tau + 1) Q_{it} B_{it+1}
\]

\[
+ \mathbb{E}_t \int_{t+1}^{\infty} P(N_{it+1}, B_{it+1}, Z_{it+1}) d\Phi(Z_{it+1}),
\]

subject to \( N_{it+1} = (1 - s) N_{it} + q(\theta_t) V_{it} \), and \( V_{it} \geq 0 \).

Substituting the bond valuation equation (5) into (19) obtains

\[
F(N_{it}, B_{it}) = \max_{V_{it}, B_{it+1}} -\kappa_t V_{it}
\]

\[
+ \mathbb{E}_t \int_{t+1}^{\infty} X_{t+1} N_{it+1} \left[ 1 + \tau Z_{it+1}^* [1 - \Phi(Z_{it+1}^*)] - [1 - (\tau + 1) \xi] \int_{0}^{Z_{it+1}^*} Z_{it+1} d\Phi(Z_{it+1}) \right] + \lambda_t q(\theta_t) \hat{V}_{it},
\]

Since both (18) and (20) are linearly homogeneous in \( N_{it} \), it is more convenient to work with a scaled version of the problem. To this end, I define

\[
\hat{V}_{it} \equiv \frac{V_{it}}{N_{it}}, \quad \hat{F}_{it} \equiv \frac{F_{it}}{N_{it}}, \quad \hat{B}_{it} \equiv \frac{B_{it}}{N_{it}}, \quad \hat{S}_{it} \equiv \frac{S_{it}}{N_{it}},
\]

and rescale (18) and (20) by \( N_{it} \) to get

\[
\hat{S}_{it} = X_t Z_{it} - W_t + \hat{F}_{it} - \hat{B}_{it},
\]

\[
\hat{F}_{it} = \max_{V_{it}, B_{it+1}} -\kappa_t \hat{V}_{it} + [1 - s + q(\theta_t) \hat{V}_{it}]
\]

\[
\mathbb{E}_t \int_{t+1}^{\infty} X_{t+1} \left[ 1 + \tau Z_{it+1}^* [1 - \Phi(Z_{it+1}^*)] - [1 - (\tau + 1) \xi] \int_{0}^{Z_{it+1}^*} Z_{it+1} d\Phi(Z_{it+1}) \right] + \lambda_t q(\theta_t) \hat{V}_{it},
\]
where $\lambda_t$ is the multiplier associated with the nonnegative vacancy constraint.

Differentiating (22) with respect to $\widehat{V}_{it}$ and $\widehat{B}_{it+1}$, respectively, yields the optimality conditions for vacancy and debt:

$$
\frac{\kappa_t}{q(\theta_t)} - \lambda_t = \mathbb{E}_t M_{t+1} \left[ X_{t+1}[1 + L_1] + [-W_{t+1} + \widehat{F}_{it+1}][1 + L_2] \right],
$$

(23)

and

$$
\tau \mathbb{E}_t M_{t+1}[1 - \Phi(Z^{\ast}_{it+1})] = (1 - \xi)(1 + \tau) \mathbb{E}_t M_{t+1} \left[ \frac{B_{it+1}}{X_{t+1} N_{it+1}} \frac{\partial \Phi(Z^{\ast}_{it+1})}{\partial Z^{\ast}_{it+1}} \right],
$$

(24)

in which

$$
L_1 \equiv \tau Z^{\ast}_{it+1}[1 - \Phi(Z^{\ast}_{it+1})] - [1 - (\tau + 1)\xi] \int_0^{Z^{\ast}_{it+1}} Z_{it+1} d\Phi(Z_{it+1}),
$$

(25)

$$
L_2 \equiv \tau [1 - \Phi(Z^{\ast}_{it+1})] - [1 - (\tau + 1)\xi] \Phi(Z^{\ast}_{it+1}).
$$

(26)

Substituting the optimal vacancy condition (23) into the equityholder’s value function (22) yields:

$$
\widehat{F}_{it} = (1 - s) \left[ \frac{\kappa_t}{q(\theta_t)} - \lambda_t \right].
$$

(27)

Plugging (27) back to (23) yields the job creation condition:

$$
\frac{\kappa_t}{q(\theta_t)} - \lambda_t = \mathbb{E}_t M_{t+1} \left[ X_{t+1}[1 + L_1] + [-W_{t+1} + (1 - s) \left( \frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda_{t+1} \right)] [1 + L_2] \right].
$$

(28)

Plugging (27) back to (21) yields the equation determining the default threshold:

$$
X_t Z^{\ast}_{it} - W_t + (1 - s) \left[ \frac{\kappa_t}{q(\theta_t)} - \lambda_t \right] = \frac{B_{it}}{N_{it}}
$$

(29)

C Wages

This appendix derives the equilibrium wage rates under Nash bargaining between workers and firms. The derivation is adapted from Petrosky-Nadeau, Zhang, and Kuehn (2015).

Let $J_{N_t}$ denote the marginal value of an employed worker to the representative family, $J_{U_t}$ the marginal value of an unemployed worker to the representative family, $\phi_t$ the marginal utility of the representative family, $PP_{N_t}$ the marginal value of an employed worker to the representative firm, $PP_{V_t}$ the marginal value of an unfilled vacancy to the representative firm, The worker’s marginal surplus, $H_t$, is:

$$
H_t \equiv \frac{J_{N_t} - J_{U_t}}{\phi_t}.
$$

42
The firm’s marginal surplus, $\Omega_t$, is:

$$\Omega_t \equiv PP_N - PP_V.$$

The total surplus from the firm-worker match, $\Lambda_t$, is:

$$\Lambda_t \equiv H_t + \Omega_t.$$

The bargaining problem is to choose the wage rate, $W_t$, to maximize the Nash product:

$$\max_{W_t} H_t^{\eta} \Omega_t^{1-\eta}.$$  

The first-order condition for the Nash bargaining problem is the Nash sharing rule:

$$\frac{1}{\eta} H_t = \Lambda_t = \frac{1}{1 - \eta} \Omega_t. \quad (30)$$

**The Household**

Differentiating $J_t$ in equation (6) with respect to $C_t$ yields:

$$\phi_t = C_t^{-\frac{1}{p}} J_t^{\frac{1}{p}}.$$

The employment and unemployment evolution equations for the household are, respectively:

$$\begin{align*}
N_{t+1} &= (1 - s)N_t + f(\theta_t)U_t \\
U_{t+1} &= sN_t + (1 - f(\theta_t))U_t.
\end{align*}$$

Differentiating $J_t$ with respect to $N_t$ and then dividing by $\phi_t$ yields:

$$\frac{J_{N_t}}{\phi_t} = W_t + \mathbb{E}_t \left[ M_{t,t+1} \left( (1 - s) \frac{J_{N_{t+1}}}{\phi_{t+1}} + s \frac{J_{U_{t+1}}}{\phi_{t+1}} \right) \right].$$

Similarly, differentiating $J_t$ with respect to $U_t$ and then dividing by $\phi_t$ yields:

$$\frac{J_{U_t}}{\phi_t} = b + \mathbb{E}_t \left[ M_{t,t+1} \left( f(\theta_t) \frac{J_{N_{t+1}}}{\phi_{t+1}} + (1 - f(\theta_t)) \frac{J_{U_{t+1}}}{\phi_{t+1}} \right) \right].$$

Hence the worker’s marginal surplus is:

$$H_t \equiv \frac{J_{N_t} - J_{U_t}}{\phi_t} = W_t - b + [1 - s - f(\theta_t)]\mathbb{E}_t[M_{t+1}H_{t+1}] \quad (31)$$
The Firm

The aggregate cum-dividend market value of equity, $PP_t$, is:

$$PP_t = \int_0^\infty P(N_{it}, B_{it}, Z_{it}) \, d\Phi(Z_t) = X_t N_t \int_0^\infty Z_t \, d\Phi(Z_t) + [1 - \Phi(Z^*_t)] \left[ -W_t N_t + (1 - s) \left( \frac{\kappa_t}{q(\theta_t)} - \lambda_t \right) N_t - B_t \right].$$

Differentiating $PP_t$ with respect to $N_t$ and $V_t$ yields, respectively:

$$PP_{N_t} = X_t \int_0^\infty Z_t \, d\Phi(Z_t) + [1 - \Phi(Z^*_t)] \left[ -W_t + (1 - s) \left( \frac{\kappa_t}{q(\theta_t)} - \lambda_t \right) \right]$$

$$PP_{V_t} = 0.$$  

Hence the firm’s marginal surplus is:

$$\Omega_t = PP_{N_t} + PP_{V_t} = X_t \int_0^\infty Z_t \, d\Phi(Z_t) + [1 - \Phi(Z^*_t)] \left[ -W_t + (1 - s) \left( \frac{\kappa_t}{q(\theta_t)} - \lambda_t \right) \right].  \quad (32)$$

Plugging (31) and (32) back to (30) yields the wage determination equation:

$$\eta \Omega_t = (1 - \eta) \left[ W_t - b + \frac{\eta}{1 - \eta} [1 - s - f(\theta_t)] \frac{\kappa_t}{q(\theta_t)} - \lambda_t \right].  \quad (33)$$

Equation (33) simplifies in the absence of financial frictions, in which case firms are not taking on leverage and thus no default will ever occur. As a result, the default threshold $Z^*_t$ equals zero, and the firm’s marginal surplus becomes $\Omega_t = X_t - W_t + (1 - s) [\kappa_t/q(\theta_t) - \lambda_t]$. Substituting it into the wage determination equation (33), the wage rate, $W_t$, collapses to the conventional form as in Pissarides (2000):

$$W_t = (1 - \eta) b + \eta (X_t + \kappa_t \theta_t).$$

D Solution Method

The model is solved using a globally nonlinear solution that uses projection methods.

The state space of the model consists of employment, $N_t$, debt, $B_t$, and aggregate productivity, $X_t$. The task is to solve for four functions: the conditional expectation function $E_t$, the debt function $B_{t+1}$, the wage rate function $W_t$, and the indirect utility function $J_t$, with the following
four functional equations:

\[
E_t = E_t M_{t+1} \left[ X_{t+1} \left[ 1 + L_1 \right] + \left[ - W_{t+1} + (1 - s) E_{t+1} \right] \left[ 1 + L_2 \right] \right]
\]

\[
\tau E_t M_{t+1} \left[ 1 - \Phi(Z_{it+1}^*) \right] = (1 - \xi)(1 + \tau) E_t M_{t+1} \left[ B_{it+1} \frac{\partial \Phi(Z_{it+1}^*)}{\partial Z_{it+1}^*} \right]
\]

\[
\eta \Omega_t = (1 - \eta) \left[ W_t - b + \frac{\eta}{1 - \eta} [1 - s - f(\theta_t)] E_t [M_{t+1} \Omega_{t+1}] \right]
\]

\[
J_t = \left[ (1 - \beta) C_t^{1 - \frac{1}{\psi}} + \beta \left( E_t \left[ J_{t+1}^{1 - \gamma} \right] \right)^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right]^{\frac{1}{1 - \frac{1}{\psi}}}
\]

After obtaining \( E_t \), I first calculate \( \bar{q}(\theta_t) = \kappa_0 / (E_t - \kappa_1) \). If \( \bar{q}(\theta_t) < 1 \), it means the non-negativity vacancy constraint is not binding, therefore \( \lambda_t = 0 \) and \( q(\theta_t) = \bar{q}(\theta_t) \). If \( \bar{q}(\theta_t) \geq 1 \), it means the non-negativity vacancy constraint is binding, and accordingly \( V_t = 0 \), and \( q(\theta_t) = 1 \).

Specifically, log aggregate productivity, \( x_t \), is discretized into a Markov chain with 15 grid points via the Rouwenhorst (1995) method. On each grid point of \( x_t \), each of the four functions \( E_t \), \( B_{t+1} \), \( W_t \), and \( J_t \) is approximated by a two-dimensional cubic spline over the \((N_t, B_t)\) space with 40 grid points on the \( N_t \) space, [0.035, 0.99], and 10 grid points on the \( B_t \) space, [0, 1.38].