Multiple Equilibria in Open Economy Models with Collateral Constraints: Overborrowing Revisited[*]

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Abstract

This paper establishes the existence of multiple equilibria in infinite-horizon open-economy models in which the value of tradable and nontradable endowments serves as collateral. In this environment, the economy displays self-fulfilling financial crises in which pessimistic views about the value of collateral induces agents to deleverage. The paper shows that under plausible calibrations, there exist equilibria with underborrowing. This result stands in contrast to the overborrowing result stressed in the related literature. Underborrowing emerges in the present context because in economies that are prone to self-fulfilling financial crises, individual agents engage in excessive precautionary savings as a way to self-insure.

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1 Introduction

Open-economy models with collateral constraints display a pecuniary externality originating in the fact that the price of pledgable objects is endogenous to the model but exogenous to individual agents. A result stressed in the literature is that these economies overborrow, that is, they borrow more than they would if agents internalized the externality (Bianchi, 2011; Korinek, 2011; Jeanne and Korinek, 2010).

A second type of instability caused by the aforementioned pecuniary externality, which has been given less attention in the literature, is the emergence of nonconvexities whereby in the aggregate an increase in borrowing relaxes the collateral constraint. This perverse relationship arises for plausible calibrations and can give rise to multiple equilibria, as shown by Jeanne and Korinek (2010) in the context of an economy with a stock collateral constraint. The first contribution of this paper is to extend this result by showing analytically that nonconvexities can give rise to multiple equilibria in the context of open economy models with flow collateral constraints in which borrowing is limited by the value of tradable and nontradable endowments. This result is of interest because this type of collateral constraint is widely used in the open-economy literature on pecuniary externalities (e.g., Bianchi, 2011; Benigno et al. 2013 and 2014; Ottonello, 2015). We show that in this environment, self-fulfilling financial crises can emerge as a result of pessimistic views about the value of collateral that induce agents to deleverage.

The second contribution of this paper is to show that in these equilibria agents borrow less than they would if they could internalize the pecuniary externality. Thus, multiplicity of equilibrium gives rise to underborrowing, in the sense that if the government applies capital control taxes optimally, the level of external debt is higher than in the unregulated competitive equilibrium. Underborrowing is the result of excessive self-insurance on the part of the private sector as a means to cope with an environment prone to self-fulfilling collapses in the value of collateral.

As is well known, Ramsey-optimal policy is mute with respect to implementation. In the context of the present analysis, this means that the Ramsey-optimal capital control policy is consistent with the Ramsey optimal allocation, but can also be consistent with other nonoptimal allocations. A natural question is therefore what kind of capital control policy can implement the Ramsey optimal allocation. The third contribution of the paper is to explicitly address the issue of implementation. In particular, we show that capital control policies that are triggered by sudden and discrete bursts in capital outflows can avoid self-fulfilling financial crises and implement the Ramsey optimal allocation. According to this class of capital control policies, the government threatens to tax capital flight if a panic

attack induces agents to collectively deleverage. This threat discourages nonfundamental runs on the country's debt, leaving as the sole possible equilibrium the Ramsey-optimal one.

Existing quantitative studies avoid the multiplicity problem by choosing calibrations for which nonconvexities are absent. This concern in choosing model parameterizations is explicitly mentioned, for instance, in Jeanne and Korinek (2010) in the context of a stock-collateral-constraint model and in Benigno et al. (2014) in the context of a flow-collateral-constraint model, and is implicit in the parameterizations adopted in Bianchi (2011) and Ottonello (2015), among others. A central quantitative contribution of the present paper is to depart from this practice by solving for equilibrium dynamics in the presence of nonconvexities. We show that under plausible calibrations, the presence of nonconvexities can give rise to equilibria exhibiting underborrowing. This result stands in contrast to the overborrowing result stressed in the related literature. In an economy calibrated with parameters typically used in the emerging-market business-cycle literature and fed with shocks estimated on quarterly Argentine data, we find equilibria in which the unregulated economy underborrows.

An exception to the standard overborrowing result stressed in the quantitative literature on pecuniary externalities due to collateral constraints is Benigno, Chen, Otrok, Rebucci, and Young (2013). The main departure of this paper from the related literature is that it considers a production economy. In an environment with production, the social planner sustains more debt than in the competitive economy by engineering sectoral employment allocations conducive to elevated values of the collateral in terms of tradable goods. The result of this paper is complementary but different from the one presented here. In the present study, underborrowing arises even in the context of an endowment economy and is due to the multiplicity of equilibrium caused by the dependence of the value of collateral on the aggregate level of external debt.

The remainder of the paper is organized as follows. Section 2 presents an open economy with a flow collateral constraint in which tradable and nontradable output has collateral value. Section 3 characterizes analytically multiplicity of equilibrium. It shows the existence of up to two equilibria with self-fulfilling crashes in the value of collateral. Section 4 studies Ramsey optimal capital control policy. It shows that the unregulated economy underborrows relative to the economy with Ramsey optimal capital controls. Section 5 presents a capital-control policy rule that can implement the Ramsey optimal allocation. Section 6 quantitatively characterizes debt dynamics in a stochastic economy in which agents coordinate on equilibria driven by pessimistic beliefs and establishes that underborrowing occurs under plausible calibrations. Section 7 concludes.

2 An Economy With A Flow Collateral Constraint

A large number of studies of open economies with collateral constraints assume that the object that serves as collateral is a flow. We will focus on the case in which tradable and nontradable output have collateral value, which is the type of flow collateral constraint most frequently studied in the related literature. Under this formulation, the source of pecuniary externalities is the relative price of nontradable goods in terms of tradables, or the real exchange rate. The collateral constraint gives rise to a pecuniary externality because individual households fail to internalize the effect of their borrowing decision on the relative price of nontradables and hence the value of their own collateral. As a result, the equilibrium features inefficient credit booms and contractions. This type of flow collateral constraint was introduced in open economy models by Mendoza (2002). The externality that emerges when debt is denominated in tradables goods but leveraged on nontradable income and the consequent room for macroprudential policy was emphasized by Korinek (2007) in the context of a three-period model. Bianchi (2011) extends the Korinek model to an infinite-horizon framework and derives quantitative predictions for optimal prudential policy.

Consider a small open endowment economy in which households have preferences of the form

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where c_t denotes consumption in period t, $U(\cdot)$ denotes an increasing and concave period utility function, $\beta \in (0,1)$ denotes a subjective discount factor, and $\mathbb{E}_t$ denotes the expectations operator conditional on information available in period t. The period utility function takes the form $U(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ with $\sigma > 0$. We assume that consumption is a composite of tradable and nontradable goods of the form

$$c_t = A(c_t^T, c_t^N) \equiv \left[a c_t^{T\,1-1/\xi} + (1-a) c_t^{N\,1-1/\xi} \right]^{1/(1-1/\xi)}, \tag{2}$$

where c_t^T denotes consumption of tradables in period t and c_t^N denotes consumption of nontradables in period t. Households are assumed to have access to a single, one-period, risk-free, internationally-traded bond denominated in terms of tradable goods that pays the interest rate r_t when held from periods t to $t+1$. The household's sequential budget constraint is given by

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + \frac{d_{t+1}}{1 + r_t}, \tag{3}$$

where d_t denotes the amount of debt due in period t and d_{t+1} denotes the amount of debt assumed in period t and maturing in $t+1$. The variable p_t denotes the relative price of

nontradables in terms of tradables, and y_t^T and y_t^N denote the endowments of tradables and nontradables, respectively. Both endowments are assumed to be exogenously given. The collateral constraint takes the form

$$d_{t+1} \leq \kappa(y_t^T + p_t y_t^N), \tag{4}$$

where $\kappa > 0$ is a parameter. Households internalize this borrowing limit. However, this borrowing constraint introduces an externality, because each individual household takes the real exchange rate, p_t, as exogenously determined, even though their collective absorptions of nontradable goods is a key determinant of this relative price. From the perspective of the individual household, the collateral constraint is well behaved, in the sense that the higher the debt level the tighter the collateral constraint becomes.

Households choose a set of processes $\{c_t^T, c_t^N, c_t, d_{t+1}\}$ to maximize (1) subject to (2)-(4), given the processes $\{r_t, p_t, y_t^T, y_t^N\}$ and the initial debt position d_0. The first-order conditions of this problem are (2)-(4) and

$$U'(A(c_t^T, c_t^N))A_1(c_t^T, c_t^N) = \lambda_t, \tag{5}$$

$$p_t = \frac{1-a}{a}\left(\frac{c_t^T}{c_t^N}\right)^{1/\xi}, \tag{6}$$

$$\left(\frac{1}{1+r_t} - \mu_t\right)\lambda_t = \beta\mathbb{E}_t\lambda_{t+1}, \tag{7}$$

$$\mu_t \geq 0, \tag{8}$$

and

$$\mu_t\left[d_{t+1} - \kappa(y_t^T + p_t y_t^N)\right] = 0, \tag{9}$$

where $\beta^t\lambda_t$ and $\beta^t\lambda_t\mu_t$ denote the Lagrange multipliers on the sequential budget constraint (3) and the collateral constraint (4), respectively. As usual, the Euler equation (7) equates the marginal benefit of assuming more debt with its marginal cost. During tranquil times, when the collateral constraint does not bind, one unit of debt payable in $t+1$ increases tradable consumption by $1/(1+r_t)$ units in period t, which increases utility by $\lambda_t/(1+r_t)$. The marginal cost of an extra unit of debt assumed in period t and payable in $t+1$ is the marginal utility of consumption in period $t+1$ discounted at the subjective discount factor, $\beta\mathbb{E}_t\lambda_{t+1}$. During financial crises, when the collateral constraint binds, the marginal utility of increasing debt falls to $[1/(1+r_t) - \mu_t]\lambda_t$, reflecting a shadow penalty for trying to increase debt when the collateral constraint is binding.

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In equilibrium, the market for nontradables must clear. That is,

$$c_t^N = y_t^N.$$

Then, a competitive equilibrium is a set of processes $\{c_t^T, d_{t+1}, \mu_t\}$ satisfying

$$\left(\frac{1}{1+r_t} - \mu_t\right) U'(A(c_t^T, y_t^N))A_1(c_t^T, y_t^N) = \beta\mathbb{E}_t U'(A(c_{t+1}^T, y_{t+1}^N))A_1(c_{t+1}^T, y_{t+1}^N), \qquad (10)$$

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r_t}, \qquad (11)$$

$$d_{t+1} \leq \kappa\left[y_t^T + \left(\frac{1-a}{a}\right)c_t^{T 1/\xi} y_t^{N 1-1/\xi}\right], \qquad (12)$$

$$\mu_t\left[\kappa y_t^T + \kappa\left(\frac{1-a}{a}\right)c_t^{T 1/\xi} y_t^{N 1-1/\xi} - d_{t+1}\right] = 0, \qquad (13)$$

$$\mu_t \geq 0, \qquad (14)$$

given processes $\{r_t, y_t^T, y_t^N\}$ and the initial condition d_0.

The fact that c_t^T appears on the right-hand side of the equilibrium version of the collateral constraint (12) means that during contractions in which the absorption of tradables falls, the collateral constraint endogenously tightens. Individual agents do not take this effect into account in choosing their consumption plans. This is the nature of the pecuniary externality in this model.

From the perspective of the individual household, equations (3) and (4) define a convex set of feasible debt choices, d_{t+1}. That is, if two debt levels d^1 and d^2 satisfy (3) and (4), then any weighted average $\alpha d^1 + (1-\alpha)d^2$ for $\alpha \in [0,1]$ also satisfies these two conditions. From an equilibrium perspective, however, this ceases to be true in general. The reason is that the relative price of nontradables, p_t, which appears on the right-hand side of the collateral constraint (4) is increasing in consumption of tradables by equation (6), which, in turn, is increasing in d_{t+1} by the resource constraint (11). To see this, use equilibrium condition (11) to eliminate c_t^T from equilibrium condition (12) to obtain

$$d_{t+1} \leq \kappa\left[y_t^T + \left(\frac{1-a}{a}\right)\left(y_t^T + \frac{d_{t+1}}{1+r_t} - d_t\right)^{1/\xi} y^{N 1-1/\xi}\right].$$

It is clear from this expression that the right-hand side is increasing in the equilibrium level of external debt, d_{t+1}. Moreover, depending on the values assumed by the parameters κ, a, and ξ, the equilibrium value of collateral may increase more than one for one with d_{t+1}. In

other words, an increase in debt, instead of tightening the collateral constraint may relax it. In this case, the more indebted the economy becomes, the less leveraged it is. As we will see shortly, this possibility can give rise to multiple equilibria and self-fulfilling drops in the value of collateral. Furthermore, if the intratemporal elasticity of substitution ξ is less than unity, which is the case of greatest empirical relevance for many countries (Akinci, 2011), the equilibrium value of collateral is convex in the level of debt. This property may cause the emergence of two distinct values of d_{t+1} for which the collateral constraint binds and two disjoint intervals of debt levels for which the collateral constraint is slack.

3   Self-Fulfilling Financial Crises

The focus of this section is to characterize self-fulfilling financial crises under flow collateral constraints. For analytical convenience, assume that the CRRA period utility function and the CES aggregator function introduced above satisfy $\sigma = 1/\xi = 2$, which is an empirically plausible case. We simplify the economy by assuming that the tradable and nontradable endowments and the interest rate are constant and equal to $y_t^T = y^T$, $y_t^N = 1$, and $r_t = r$, for all t. Further, assume that $a = 0.5$, and $\beta(1 + r) = 1$. Given these assumptions, the equilibrium conditions (10)-(13) can be written as

$$c_{t+1}^T \sqrt{1 - (1+r)\mu_t} = c_t^T, \tag{15}$$

$$c_t^T + d_t = y^T + \frac{d_{t+1}}{1+r}, \tag{16}$$

$$d_{t+1} \leq \kappa[y^T + (y^T + d_{t+1}/(1+r) - d_t)^2], \tag{17}$$

$$\mu_t \left\{ \kappa[y^T + (y^T + d_{t+1}/(1+r) - d_t)^2] - d_{t+1} \right\} = 0, \tag{18}$$

with $c_t^T > 0$ and $\mu_t \geq 0$ and d_0 given.

Let us first characterize conditions under which an equilibrium exists in which traded consumption and debt are constant for all $t \geq 0$, that is, an equilibrium in which $c_t^T = c_0^T$ and $d_t = d_0$ for all $t \geq 0$, where d_0 is given. We refer to this equilibrium as a steady-state equilibrium. By (15), in a steady-state equilibrium $\mu_t = 0$ for all t. This means that in a steady-state equilibrium the slackness condition (18) is also satisfied for all t.

When $d_{t+1} = d_t = d$, the collateral constraint (17) becomes

$$d \leq \kappa \left[y^T + \left(y^T - \frac{rd}{1+r} \right)^2 \right]. \tag{19}$$

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Figure 1: Feasible Debt Levels in the Steady State Under A Flow Collateral Constraint
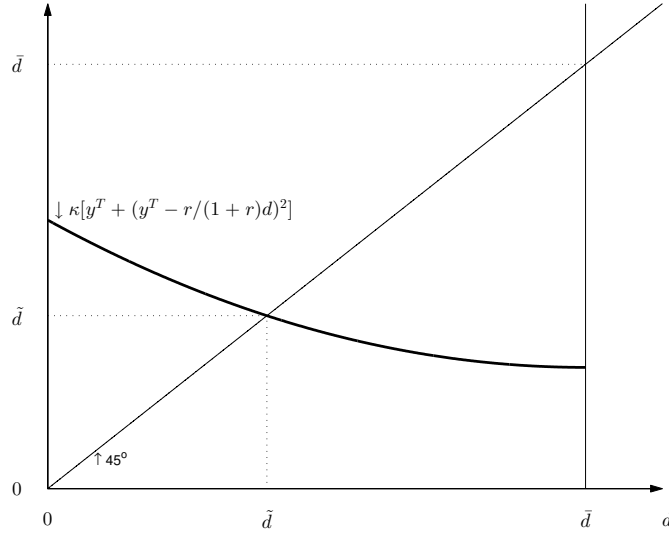

We refer to this expression as the steady-state collateral constraint. Figure 1 displays the left- and right-hand sides of the steady-state collateral constraint (19) as a function of d. The left-hand side is the 45° line. The right-hand side, shown with the thick solid line, is a quadratic expression with a minimum at the natural debt limit $\bar{d} \equiv y^T(1+r)/r$. It follows that the steady-state collateral constraint is well behaved, in the sense that the higher is the steady-state level of debt the tighter is the steady-state collateral constraint.

At the natural debt limit, consumption of tradables is zero. This means that a steady-state equilibrium can exist only for initial values of debt less than $\bar{d}$. At $\bar{d}$, the right-hand side of the collateral constraint equals κy^T and the left-hand side equals $y^T(1+r)/r$. We assume that $\kappa < (1+r)/r$, so that at $\bar{d}$ the left-hand side is larger than the right-hand side, and the steady-state collateral constraint is violated. Let $\tilde{d} < \bar{d}$ be the value of d at which the steady-state collateral constraint (19) holds with equality, that is, the value of d at which the right-hand side of the steady-state collateral constraint crosses the 45° line as indicated in the figure. Any value of initial debt, d_0, less than or equal to $\tilde{d}$ satisfies the steady-state collateral constraint (19). Since we have already shown that a constant value of debt also satisfies all other equilibrium conditions, we have demonstrated that any initial value of debt less than or equal to $\tilde{d}$ can be supported as a steady state equilibrium.

Do there exist other equilibria? The answer is yes. Consider an economy with an initial debt level $d_0 < \tilde{d}$ as shown in figure 2. The figure reproduces from figure 1 the right-hand side of the steady-state collateral constraint (19) shown with a thick solid line. Because in the graph the initial level of debt, d_0, satisfies $d_0 < \tilde{d}$, we have from the previous analysis that $d_t = d_0$ for all t can be supported as an equilibrium. Now consider the collateral constraint

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Figure 2: Multiple Equilibria With Flow Collateral Constraints
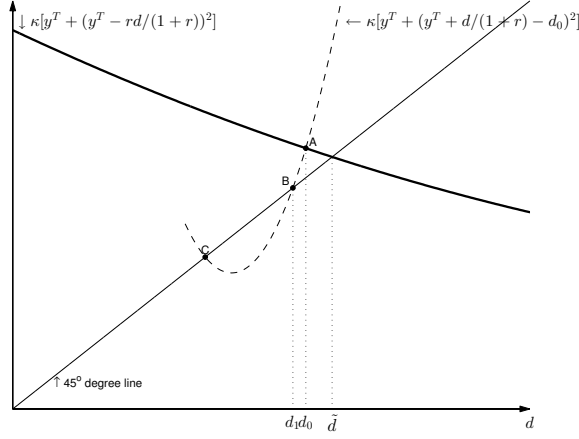
in period 0, given by

$$d \leq \kappa \left[y^T + \left(y^T + \frac{d}{1+r} - d_0 \right)^2 \right], \tag{20}$$

expressed as a function of the level of debt in period 1, denoted by d. We refer to expression (20) as the period-0 collateral constraint. The figure plots the right-hand side of the period-0 collateral constraint with a broken line. The right-hand sides of the period-0 and steady-state collateral constraints intersect when $d = d_0$, point A in the figure. At point A, the right-hand side of the period-0 collateral constraint (the broken line) is upward sloping, with a slope equal to $2\kappa/(1+r)(y^T - rd_0/(1+r)) > 2\kappa/(1+r)(y^T - r\bar{d}/(1+r)) = 0$.

The right-hand side of the period-0 collateral constraint can cross the $45°$ line to the left of d_0 either zero or two times. Suppose that it crosses the $45°$ line twice, as shown in figure 2. This is possible for some parameter configurations.[1] At the crossing with the higher debt level, indicated by point B in the figure, the slope of the right-hand side of the period-0 collateral constraint must be positive. This means that at point B, c_0^T is positive (recall that the slope of the period-0 collateral constraint is $2\kappa/(1+r)c_0^T$. We wish to show that point B can be supported as an equilibrium. In this equilibrium $d_t = d_1 < d_0$ for all $t > 0$. To establish this result, we must show that equilibrium conditions (15)-(18) are satisfied for all $t \geq 0$, with $c_t^T > 0$ and $\mu_t \geq 0$. We have already shown that $c_0^T > 0$ at point B. Now note that at point B the collateral constraint is binding in period 0, since the right-hand side of the period-0 collateral constraint crosses the $45°$ line, which is the left-hand side of

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[1]A sufficient condition for the existence of two crossings of this type for some range of $d_0 < \tilde{d}$ is that the slope of the right-hand side of the period-0 collateral constraint be larger than unity at $d_0 = d_1 = \tilde{d}$. This condition is satisfied as long as $\kappa r/(1+r)(1 - \kappa r/(1+r))y^T > a/(1-a)((1+r)^2 - 1)/4$.

the period-0 collateral constraint. Thus, equilibrium conditions (17) and (18) are satisfied in period 0. Also, the facts that $d_1 < d_0$ and $d_1 = d_2$ imply that $c_0^T < c_1^T$, which can be verified by comparing the resource constraint (16) evaluated at $t = 0$ and $t = 1$. In turn, $c_0^T < c_1^T$ implies, by the Euler equation (15), that a strictly positive value of the Lagrange multiplier μ_0 makes the Euler equation hold with equality in period 0. This establishes that the debt level associated with point B satisfies all equilibrium conditions in period 0. Since $d_1 < \tilde{d}$, we have, from the preceding analysis of steady-state equilibria, that $d_t = d_1$ for all $t \geq 1$ can be supported as an equilibrium. This completes the proof of the existence of multiple equilibria, one with $d_t = d_0$ for all $t \geq 0$ and the collateral constraint never binding, and another one with $d_1 < d_0$ and the collateral constraint binding in period 0 and never binding thereafter. The latter equilibrium takes place at a level of period-1 debt at which, from an aggregate point of view, the period-0 collateral constraint behaves perversely in the sense that more borrowing would loosen rather than tighten the borrowing restriction.

The intuition behind the existence of the second equilibrium is as follows. Imagine the economy being originally in a steady state with debt constant and equal to d_0. Unexpectedly, the public becomes pessimistic and aggregate demand contracts. The contraction in aggregate demand means that households want to consume less of both types of good, tradable and nontradable. Because nontradables are in fixed supply, their relative price, p_0, must fall to bring about market clearing. As a result, the value of collateral, given by $\kappa(y^T + p_0 y^N)$, also falls. This reduction in collateral is so large that it forces households to deleverage. This generalized decline in the value of collateral represents the quintessential element of a financial crisis. To reduce their net debt positions, households must cut spending, validating the initial pessimistic sentiments, and making the financial crisis self-fulfilling. The contraction in the debt position and the fall in the relative price of nontradables imply that the self-fulfilling financial crisis occurs in the context of a current account surplus and a depreciation of the real exchange rate.

Figure 2 displays a period-0 collateral constraint that crosses the 45° line once with a positive slope (point B) and once with a negative slope (point C). Point C cannot be an equilibrium because it is associated with negative period-0 consumption, $c_0^T < 0$. To see this, recall that the slope of the period-0 collateral constraint in period 0 is given by $2\kappa/(1+r)c_0^T$. So if the slope is negative, so is c_0^T.

Debt levels between points A and B cannot be equilibria of the type discussed here, even though these debt levels are feasible in the sense that they satisfy the resource and collateral constraints. They cannot represent equilibria because they fail to satisfy the Euler equation (15) in period 0. To see this, note that at such points the period-0 collateral constraint is slack, implying that $\mu_0 = 0$. We also have that at any point between A and B

Figure 3: Third Equilibrium Under Flow Collateral Constraints
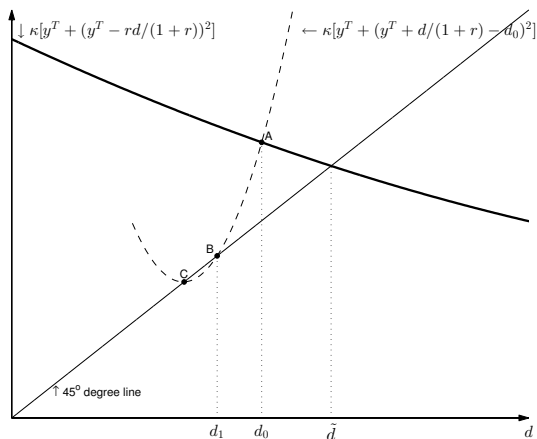
c_1^T is strictly higher than c_0^T, since the economy deleverages in period 0 and is in a steady-state equilibrium in period 1. This means that at any point between A and B, the left-hand side of the Euler equation (15) is larger than its right-hand side.

However, if the period-0 collateral constraint crosses the 45° line twice with a positive slope and before $\tilde{d}$, as shown in figure 3, then a third equilibrium emerges (point C). The proof of this claim is identical to the one establishing that point B is an equilibrium. A third equilibrium of this type entails a larger drop in the value of collateral and more deleveraging than in the equilibrium associated with point B. This suggests that in the current environment self-fulfilling financial crisis can come in different sizes.

4 Underborrowing

The pecuniary externality created by the presence of the relative price of nontradables in the collateral constraint induces an allocation that is in general suboptimal, not only when compared to the allocation that would result in the absence of a collateral constraint, but also relative to the best allocation possible among all of the ones that satisfy the collateral constraint. As a result, the collateral constraint opens the door to welfare improving policy intervention.

The standard result in the literature on pecuniary externalities due to a flow collateral constraint of the type studied here is that the unregulated economy overborrows. That is, external debt is higher than it would be if households internalized the pecuniary externality. The focus of this section is to establish whether overborrowing continues to obtain in economies exhibiting multiple equilibria.

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The policy intervention we consider here is capital controls. This instrument is of interest for two reasons. First, the optimal capital control policy fully internalizes the pecuniary externality, in the sense that it induces the representative household to behave as if it understood that its own borrowing choices influence the relative price of nontradables and therefore the value of collateral. Second, capital controls are of interest because they represent a tax on external borrowing, which is the variable most directly affected by the pecuniary externality.

We assume that the government is benevolent in the sense that it seeks to maximize the well being of the representative household. Further, we assume that the government has the ability to commit to policy promises. That is, we characterize the Ramsey optimal capital control policy in the context of an open economy with a flow collateral constraint.

Let τ_t be a proportional tax on debt acquired in period t. If τ_t is positive, it represents a proper capital control tax, whereas if it is negative it has the interpretation of a borrowing subsidy. The revenue from capital control taxes is given by $\tau_t d_{t+1}/(1 + r_t)$. We assume that the government consumes no goods and that it rebates all revenues from capital controls to the public in the form of lump-sum transfers (lump-sum taxes if $\tau_t < 0$), denoted ℓ_t.[2] The budget constraint of the government is then given by

$$\tau_t \frac{d_{t+1}}{1 + r_t} = \ell_t. \tag{21}$$

The household's sequential budget constraint now becomes

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y^N + (1 - \tau_t) \frac{d_{t+1}}{1 + r_t} + \ell_t.$$

It is apparent from this expression that the capital control tax distorts the borrowing decision of the household. In particular, the gross interest rate on foreign borrowing perceived by the private household is no longer $1 + r_t$, but $(1 + r_t)/(1 - \tau_t)$. All other things equal, the higher is τ_t, the higher is the interest rate perceived by households. Thus, by changing τ_t the government can encourage or discourage borrowing. All optimality conditions associated with the household's optimization problem (equations (5)-(9)) are unchanged, except for the debt Euler equation (7), which now takes the form

$$\left(\frac{1 - \tau_t}{1 + r_t} - \mu_t \right) \lambda_t = \beta \mathbb{E}_t \lambda_{t+1}.$$

A competitive equilibrium in the economy with capital control taxes is then a set of

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[2]Alternatively, one could assume that revenues from capital control taxes are rebated by means of a proportional income transfer. Since tradable and nontradable income is exogenous to the household, this transfer would be nondistorting and therefore equivalent to a lump-sum transfer.

processes c_t^T, d_{t+1}, λ_t, μ_t, and p_t satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \tag{22}$$

$$d_{t+1} \leq \kappa \left[y_t^T + p_t y^N \right], \tag{23}$$

$$\lambda_t = U'(A(c_t^T, y^N)) A_1(c_t^T, y^N), \tag{24}$$

$$\left(\frac{1 - \tau_t}{1 + r_t} - \mu_t \right) \lambda_t = \beta \mathbb{E}_t \lambda_{t+1}, \tag{25}$$

$$p_t = \frac{A_2(c_t^T, y^N)}{A_1(c_t^T, y^N)}, \tag{26}$$

$$\mu_t[\kappa(y_t^T + p_t y^N) - d_{t+1}] = 0, \tag{27}$$

$$\mu_t \geq 0, \tag{28}$$

given a policy process τ_t, exogenous driving forces y_t^T and r_t, and the initial condition d_0.

The benevolent government sets capital control taxes to maximize the household's lifetime utility subject to the restriction that the optimal allocation be supportable as a competitive equilibrium. It follows that all of the above competitive equilibrium conditions are constraints of the Ramsey government's optimization problem. Formally, the Ramsey-optimal competitive equilibrium is a set of processes τ_t, c_t^T, d_{t+1}, λ_t, μ_t, and p_t that solve the problem of maximizing

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, y^N)) \tag{29}$$

subject to (22)-(28), given processes y_t^T and r_t and the initial condition d_0. In the welfare function above, we have replaced consumption of nontradables, c_t^N, with the endowment of nontradables, y^N, because the Ramsey planner takes into account that in a competitive equilibrium the market for nontradables clears at all times.

The above set of equilibrium conditions can be reduced to two expressions. Specifically, processes c_t^T and d_{t+1} satisfy equilibrium conditions (22)-(28) if and only if they satisfy (22) and

$$d_{t+1} \leq \kappa \left[y_t^T + \frac{1 - a}{a} \left(\frac{c_t^T}{y^N} \right)^{\frac{1}{\xi}} y^N \right]. \tag{30}$$

To see this, suppose c_t^T and d_{t+1} satisfy (22) and (30). We must establish that (22)-(28) are also satisfied. Obviously, the resource constraint (22) holds. Now pick p_t to satisfy (26). This is possible, because the process c_t^T is given. Use this expression to eliminate p_t from (23). The resulting expression is (30), establishing that (23) holds. Next, pick λ_t to satisfy (24).

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Now, set $\mu_t = 0$ for all t. It follows immediately that the slackness condition (27) and the non-negativity condition (28) are satisfied. Finally, pick τ_t to ensure that (25) holds, that is,

$$\tau_t = 1 - \beta(1 + r_t)\mathbb{E}_t \frac{U'(A(c_{t+1}^T, y^N))A_1(c_{t+1}^T, y^N)}{U'(A(c_t^T, y^N))A_1(c_t^T, y^N)}. \tag{31}$$

Next, we need to show the reverse statement, that is, that processes c_t^T and d_{t+1} that satisfy (22)-(28) also satisfy (22) and (30). Obviously, (22) is satisfied, and combining (23) with (26) yields (30). This completes the proof of the equivalence of the constraint set (22)-(28) and the constraint set (22) and (30).

We can then state the Ramsey problem as

$$\max_{\{c_t^T, d_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, y^N)) \tag{29}$$

subject to

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \tag{22}$$

$$d_{t+1} \leq \kappa \left[y_t^T + \frac{1-a}{a} \left(\frac{c_t^T}{y^N} \right)^{\frac{1}{\xi}} y^N \right]. \tag{30}$$

Comparing the levels of debt in the Ramsey equilibrium and in the unregulated equilibrium (i.e., the equilibrium without government intervention), we can determine whether the lack of optimal government intervention results in overborrowing or underborrowing.

Consider the Ramsey optimal allocation in the perfect-foresight economy analyzed in section 3. Suppose that the initial value of debt, d_0, satisfies $d_0 < \tilde{d}$, as shown in figure 2. Since one possible competitive equilibrium is $d_t = d_0$ and $c_t^T = y^T - rd_0/(1+r)$ for all $t \geq 0$, and since this equilibrium is the first best equilibrium (i.e., the equilibrium that would result in the absence of the collateral constraint), it also has to be the Ramsey optimal equilibrium. The capital control tax associated with the Ramsey optimal equilibrium can be deduced from inspection of equation (31). Because consumption of tradables is constant over time and because in this analytical example $\beta(1 + r) = 1$, it follows that $\tau_t = 0$ for all $t \geq 0$.

Compare now the level of debt in the Ramsey optimal allocation with the one associated with the unregulated competitive equilibrium. Here, we must take into account that there are multiple unregulated competitive equilibria. Consider first the case of two unregulated equilibria, depicted as points A and B in figure 2. Suppose the unregulated competitive equilibrium happens to be the one in which the collateral constraint binds in period 0, point

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B in figure 2. In this case the unregulated economy underborrows at all times, since the level of debt at point B is less than the Ramsey optimal level of debt, d_0. If, on the other hand, the unregulated competitive equilibrium happens to be the unconstrained equilibrium (point A in the figure), then there is neither underborrowing nor overborrowing, since its associated level of debt coincides with the Ramsey optimal level, d_0. Thus, in this economy, there is either underborrowing or optimal borrowing, depending on whether the competitive equilibrium happens to be the constrained or the unconstrained one.

Similarly, in the economy depicted in figure 3, which has three unregulated equilibria, given by points A, B, and C, the Ramsey optimal equilibrium is at point A, with constant consumption and capital control taxes equal to zero at all times. If the economy coordinates on equilibria B or C, it underborrows, and if it coordinates on equilibrium A, it neither underborrows nor overborrows.

5 Implementation

The Ramsey optimal policy is mute with regard to equilibrium implementation. In the context of the economy studied in the previous section, this means that the Ramsey optimal policy $\tau_t = 0$ does not guarantee that the competitive equilibrium will be the Ramsey optimal one (e.g., point A in figures 2 and 3). In particular, the policy rule $\tau_t = 0$ for all t may result in an unintended competitive equilibrium, like point B in figure 2 or points B or C in figure 3. Thus a policy of setting $\tau_t = 0$ at all times may fail to implement the Ramsey optimal allocation. However, any capital-control policy that succeeds in implementing the Ramsey optimal allocation must deliver $\tau_t = 0$ for all t in equilibrium. The difference between a policy that sets $\tau_t = 0$ under all circumstances and a policy capable of implementing the Ramsey-optimal allocation is not the capital control tax that results in equilibrium, but the tax rates that would be imposed off equilibrium.

To shed light on the issue of implementation, here we study a capital-control feedback rule capable of bringing about the Ramsey-optimal equilibrium in the model economy of section 3. Specifically, consider the capital control policy

$$\tau_t = \tau(d_{t+1}, d_t)$$

satisfying $\tau(d, d) = 0$. According to this rule, capital controls depend, not on the level of debt, but on the magnitude of capital flows. Under this tax-policy rule, the Euler equation

in period 0 (e.g., the combination of equations (24) and (25) evaluated at $t = 0$) becomes

$$\frac{c_1^T}{c_0^T} = \frac{1}{\sqrt{1 - \tau(d_1, d_0) - (1 + r)\mu_0}}. \tag{32}$$

In the intended (Ramsey) equilibrium, we have that $c_1^T/c_0^T = 1$, that $d_1 = d_0$, and that $\mu_0 = 0$, so the Euler equation holds and $\tau(d_1, d_0) = 0$. This means that the proposed policy is consistent with the Ramsey optimal allocation.

But the same capital control policy can rule out the unintended equilibria. Recalling that c_0^T and c_1^T satisfy $c_0^T = y^T + d_1/(1+r) - d_0$ and $c_1^T = y^T - rd_1/(1+r)$, we can write the Euler equation (32) as

$$\frac{y^T - rd_1/(1+r)}{y^T + d_1/(1+r) - d_0} = \frac{1}{\sqrt{1 - \tau(d_1, d_0) - (1 + r)\mu_0}}. \tag{33}$$

Now pick the function $\tau(\cdot, \cdot)$ in such a way that if a self-fulfilling crisis occurs and the economy deleverages, then the Euler equation holds only if μ_0 is negative. Specifically, set $\tau(d_1, d_0)$ to satisfy

$$\frac{y^T - rd_1/(1+r)}{y^T + d_1/(1+r) - d_0} < \frac{1}{\sqrt{1 - \tau(d_1, d_0)}}.$$

for all $d_1 < d_0$.[3] Clearly, this policy requires $\tau(d_1, d_0) > 0$ if $d_1 < d_0$. Under this capital control policy, the Euler equation would not hold for any value of d_1 less than d_0, since it would require $\mu_0 < 0$, which violates the nonnegativity constraint (28). This means that any equilibrium in which the economy deleverages is ruled out.

The capital control policy that rules out self-fulfilling crises and ensures that only the Ramsey optimal (and first-best) equilibrium emerges is one in which the policy maker is committed to imposing capital control taxes in the case of capital outflows, that is, if $d_1 < d_0$. This type of capital control policy serves as a metaphor for a variety of policies that are often contemplated in emerging countries during financial panics and that aim at temporarily restricting capital outflows, including restrictions on foreign exchange markets and profit and dividend repatriations. In the present perfect-foresight economy, the mere threat of the imposition of capital control taxes in the case of capital outflows suffices to fend off self-fulfilling crises and thus such taxes never need to be deployed in equilibrium.

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[3]An example of a policy that satisfies this restriction is $\tau(d_1, d_0) = 1 - \left(\frac{y^T - rd_1/(1+r)}{y^T + d_1/(1+r) - d_0} + \alpha\right)^{-2}$, for any $\alpha > 0$.

6   Underborrowing In A Stochastic Economy

We now characterize numerically the debt dynamics in a stochastic version of the flow collateral-constrained economy presented in section 2. To this end, we assume a joint stochastic process for the tradable endowment and the country interest rate and calibrate the structural parameters of the model to match certain features of the Argentine economy.

We calibrate the model at a quarterly frequency. Table 1 summarizes the calibration. We set κ so that the upper limit of net external debt is 30 percent of annual output. This value is in line with those used in the quantitative literature on output-based flow collateral constraints (e.g., Bianchi, 2011). Because the time unit in the model is a quarter, this calibration restriction implies a value of κ of 1.2 $(= 0.3 \times 4)$. The calibration of the remaining parameters follows Schmitt-Grohé and Uribe (2016). We set $\beta = 0.9635$, $\sigma = 1/\xi = 2$, $a = 0.26$, and $y^N = 1$. The exogenous variables y_t^T and r_t are assumed to follow a bivariate AR(1) process of the form

$$
\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t}{1+r} \end{bmatrix} = A \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}}{1+r} \end{bmatrix} + \epsilon_t, \tag{34}
$$

where $\epsilon_t \sim N(\emptyset, \Sigma_\epsilon)$. Schmitt-Grohé and Uribe (2016) estimate this process on Argentine quarterly data over the period 1983:Q1 to 2001:Q4. The estimated parameters are

$$
A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \qquad \Sigma_\epsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \qquad r = 0.0316.
$$

6.1   Equilibrium Approximation

To approximate the equilibrium, we develop an Euler equation iteration procedure over a discretized state space. The appendix describes the numerical algorithm. The economy possesses two exogenous states, y_t^T and r_t, and one endogenous state, d_t. We discretize $\ln y_t^T$ using 21 evenly spaced points centered at 0 and discretize $\ln(1+r_t)/(1+r)$ using 11 evenly spaced points centered at 0. Thus, both grids are symmetric. The upper bound of the grids of $\ln y_t^T$ and $\ln((1+r_t)/(1+r))$ are taken to be the $\sqrt{10}$ times the corresponding unconditional standard deviations implied by the estimated VAR system (34). The resulting intervals are $[\ln \underline{y}^T, \ln \overline{y}^T] = [-0.3858, 0.3858]$ and $\left[\ln\left(\frac{1+r}{1+r}\right), \ln\left(\frac{1+\overline{r}}{1+r}\right)\right] = [-0.0539, 0.0539]$. We compute the transition probability matrix using the simulation approach of Schmitt-Grohé and Uribe (2009). For the endogenous state variable, d_t, we use 501 equally spaced points in the interval

Table 1: Calibration of the Stochastic Economy

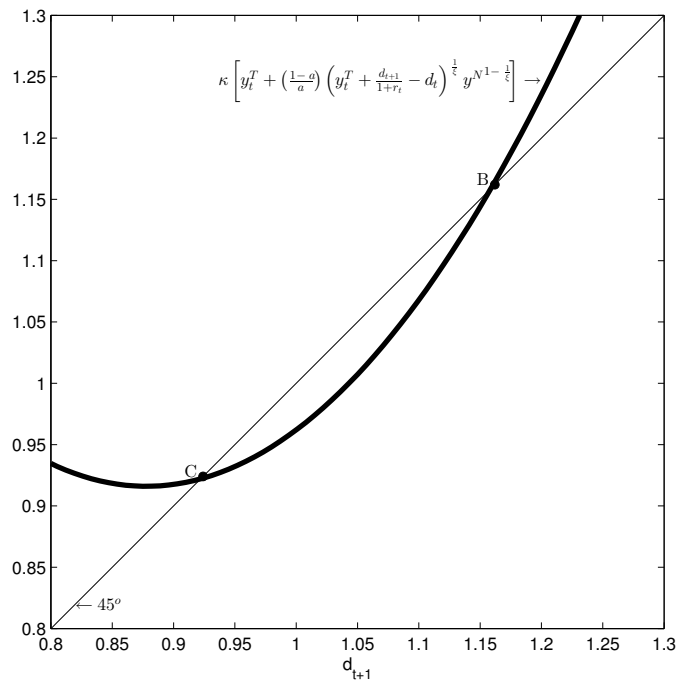| Parameter	Value	Description
κ	1.2	Parameter of collateral constraint
σ	2	Inverse of intertemporal elasticity of consumption
β	0.9635	Quarterly subjective discount factor
r	0.0316	Steady state quarterly country interest rate
ξ	0.5	Elasticity of substitution between tradables and nontradables
a	0.26	Parameter of CES aggregator
y^N	1	Nontradable output
y^T	1	Steady-state tradable output
Discretization of State Space		
n_{y^T}	21	Number of grid points for $\ln y_t^T$, equally spaced
n_r	11	Number of grid points for $\ln[(1+r_t)/(1+r)]$, equally spaced
n_d	501	Number of grid points for d_t, equally spaced
$\left[\ln \underline{y}^T, \ln \overline{y}^T\right]$	[-0.3858,0.3858]	Range for tradable output
$\left[\ln\left(\frac{1+\underline{r}}{1+r}\right), \ln\left(\frac{1+\overline{r}}{1+r}\right)\right]$	[-0.0539,0.0589]	Range for interest rate
$[\underline{d}, \overline{d}]$	[0,3.5]	Debt range

Note. The time unit is one quarter.

$[\underline{d}, \overline{d}] = [0, 3.5]$.

As in the analytical example of section 3, the present economy features a collateral constraint that in equilibrium may intersect the 45-degree line twice with a positive slope, implying that the set of values d_{t+1} that satisfy both the period-t resource constraint and the period-t collateral constraint may not be convex. For example, figure 4 displays the value of collateral as a function of d_{t+1} for the state $(y_t^T, r_t, d_t) = (0.7633, 0.0541, 1.5960)$. In this state, there are two disjoint sets of d_{t+1} for which the collateral constraint is satisfied. In between these two sets, the price of nontradables is too low to guarantee the satisfaction of the borrowing limit. More than one third of all possible states (y_t^T, r_t, d_t) display nonconvexities of the type shown in figure 4.

The numerical solution must take a stance on how to handle the possibility of indeterminacy of the rational expectations equilibrium of the type identified in section 3. Failing to address this issue may result in nonconvergence of numerical algorithms. Specifically, we focus on two canonical equilibrium selection mechanisms suggested by the preceding theoretical analysis.
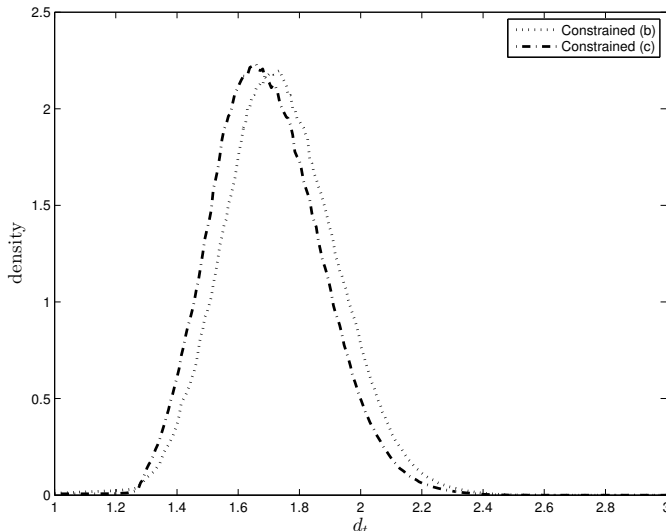
(b) If for a given current state (y_t^T, r_t, d_t) there are one or more values of d_{t+1} for which all equilibrium conditions are satisfied pick the largest one for which the collateral constraint is binding.

Figure 4: Multiple Binding Debt Levels In the Stochastic Economy

$$\kappa \left[y_t^T + \left(\tfrac{1-a}{a} \right) \left(y_t^T + \tfrac{d_{t+1}}{1+r_t} - d_t \right)^{\frac{1}{\xi}} y^{N 1 - \frac{1}{\xi}} \right] \rightarrow$$

B

C

← 45°

d_{t+1}

Note. The value of collateral is evaluated at the state $(y_t^T, r_t, d_t) =$ $(0.7633, 0.0541, 1.5960)$. All parameters take the values indicated in table 1.

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Figure 5: External Debt Densities

Note. Replication program plotdu.m.

(c) If for a given current state (y_t^T, r_t, d_t) there are one or more values of d_{t+1} for which all equilibrium conditions are satisfied pick the smallest one for which the collateral constraint is binding.

Criteria (b) and (c) favor self-fulfilling equilibria, as in points B and C in figure 3 with (c) favoring larger crises. One could in principle design algorithms to identify other possible equilibria. For instance, one could introduce a sunspot variable that randomizes across all debt choices, or a subset of debt choices, for which all equilibrium conditions are satisfied. Such equilibria may or may not exist. This type of search is beyond the scope of the present study, for our objective here is to find equilibria displaying underborrowing of the type identified in the analytical characterization presented in section 4.

A second aspect of the proposed solution algorithm is a forward-looking mechanism, we call path-finder, that avoids debt choices that lead with positive probability to areas of the state space for which either consumption is non-positive or the aggregate collateral constraint is violated in the future. This refinement of the solution algorithm facilitates convergence in the presence of nonconvexities in the aggregate feasible debt set.

6.2   Model Predictions

Figure 5 displays the unconditional distribution of external debt, d_t.[4] The different equilibrium selection criteria give rise to different debt distributions, revealing the presence of

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[4]To avoid clutter, the densities are smoothed out. Specifically, for each grid point d_i the associated smoothed density is the average of the densities associated with points d_{i-19} to d_i for $i = 20, \ldots, 501$.

Figure 6: External Debt Densities With And Without The Collateral Constraint
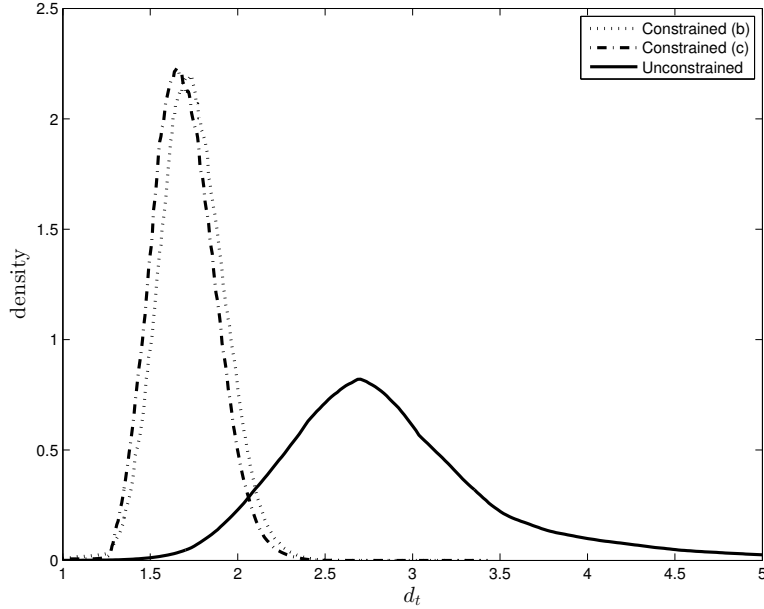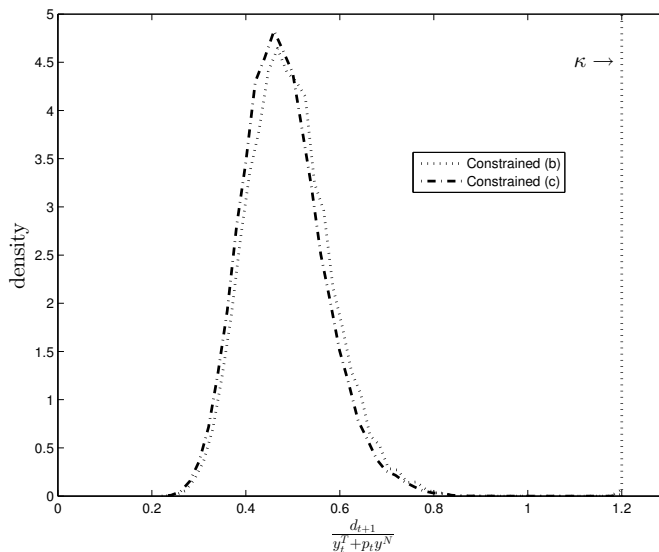

Note. Replication program plotdu.m.

multiple equilibria. The more pessimistic equilibrium selection criterion (c) (dash-dotted line in the figure), which favors larger self-fulfilling debt crises, yields a debt distribution with a mean of 12.0 percent of annual output. The distribution of debt associated with selection criterion (b) is located to the right of the one associated with criterion (c), although the difference is not large, only 0.4 percentage points of output on average. However, if one were to attempt to compute the equilibrium assuming uniqueness, standard Euler-equation iteration procedures will in general not converge.

To highlight the borrowing restrictions imposed by the collateral constraint, figure 6 displays the unconditional distribution of external debt, d_t, in a version of the present economy without a collateral constraint (solid line). For comparison, the figure reproduces the debt distributions in the economy with the collateral constraint. The presence of the collateral constraint significantly limits the ability of households to borrow. In the economy without the collateral constraint, the mean debt to output ratio 46 percent, almost four times larger than in the economy with the collateral constraint. Furthermore, the collateral constraint compresses the debt distribution around its mean. The unconditional standard deviation of the debt-to-output ratio is six times smaller in the collateral-constrained economy than in the unconstrained economy (2 versus 12 percentage points of output). This does not mean, however, that the collateral constraint is hit frequently. It actually turns out that the contrary is the case. The collateral constraint almost never binds. Figure 7 plots the distribution of leverage, defined as $d_{t+1}/\left(y_t^T + p_t y^N\right)$, as well as the upper bound on leverage

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Figure 7: The Equilibrium Distributions of Leverage

Note. (b) and (c) refer to the equilibrium selection criteria. Replication program plot_leverage.m.

given by κ. It is apparent from this figure that the probability that the collateral constraint binds is virtually nil. In fact, in a simulation of one million quarters, the constraint binds only 287 times under equilibrium selection criterion (c) and 1,113 times under criterion (b). This means that households choose to stay clear of the endogenous debt limit virtually all of the time. They manage to avoid being caught with a binding constraint by engaging in precautionary savings. They save because being up against the constraint forces them to deleverage. This collective deleveraging causes the price of collateral to collapse, which reinforces the need to deleverage, which in turn leads to even larger declines in consumption.

In section 4 we showed analytically that when the collateral constraint introduces nonconvexities, the unregulated competitive equilibrium can display underborrowing, in the sense that the level of external debt is below the Ramsey-optimal level. Here, we show that this result also obtains in a stochastic economy under a plausible calibration.

The Ramsey optimal allocation is relatively easy to compute because the Ramsey problem can be cast in the form of a Bellman equation problem. Specifically, the recursive version of the Ramsey problem of maximizing (29) subject to (22) and (30) is given by

$$v(y^T, r, d) = \max_{c^T, d'} \left\{ U(A(c^T, y^N)) + \beta \mathbb{E}\left[v(y^{T'}, r', d') \, \big| \, y^T, r \right] \right\}$$

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subject to

$$c^T + d = y^T + \frac{d'}{1+r}$$

$$d' \leq \kappa \left[y^T + \frac{1-a}{a} \left(\frac{c^T}{y^N} \right)^{\frac{1}{\xi}} y^N \right],$$

where a prime superscript denotes next-period values. Although the constraints of this control problem may not represent a convex set in tradable consumption and debt, the fact that the Ramsey allocation is the result of a utility maximization problem, implies that its solution is generically unique. The calibration of the economy is the same as that used for the unregulated economy, summarized in table 1.

Figure 8: Equilibrium Underborrowing
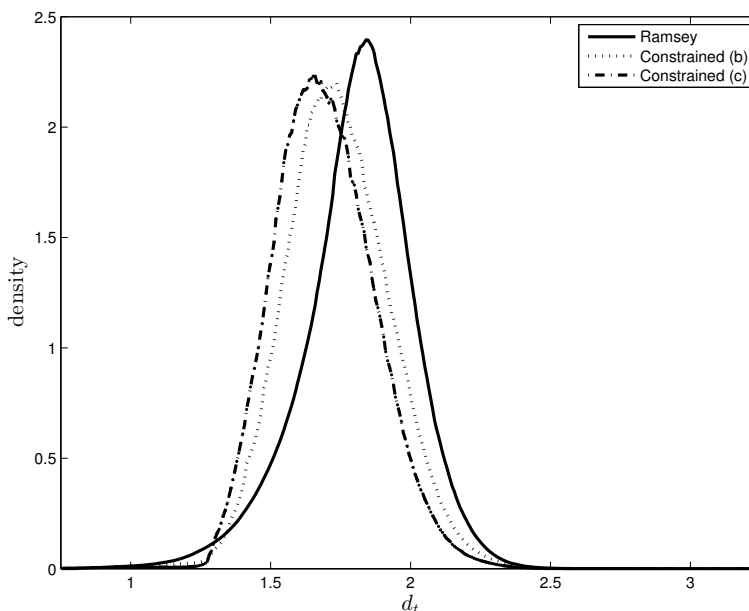
Note. Replication program plotd_ramsey.m.

Figure 8 displays with a solid line the unconditional distribution of net external debt, d_t, under Ramsey optimal capital control policy. For comparison, it reproduces from figure 5 the unconditional distributions of debt in the unregulated economy. The figure shows that the unregulated economy displays underborrowing, in the sense that its debt distribution lies to the left of the one associated with the Ramsey optimal capital control policy. The average annual debt-to-output ratio in the Ramsey economy is 13.1 percentage points of output compared with 12.4 and 12.0 percentage points of output in the unregulated economies (b) and (c), respectively. In the unregulated economy, households have an incentive to over self

insure. This is due to the fact that the unregulated economy is highly fragile as it is more prone to financial crises caused by a binding collateral constraint.

7   Conclusion

A peculiar aspect of open economy models in which borrowing is limited by the value of tradable and nontradable output is that the equilibrium value of collateral is increasing in the level of external debt. For plausible calibrations, this relationship can become perverse, in the sense that an increase in debt increases collateral by more than one for one. That is, as the economy becomes more indebted it becomes less leveraged. This problem can give rise to a nonconvexity whereby two disjoint ranges of external debt for which the collateral constraint is satisfied are separated by a range for which the collateral constraint is violated.

This paper shows that in this environment, the economy displays self-fulfilling financial crises in which pessimistic views about the value of collateral induces agents to deleverage. In the context of a stochastic economy and under plausible calibrations, the paper shows that there exist equilibria with underborrowing, in the sense that the equilibrium level of debt is lower than what is optimal for a Ramsey planner with access to capital control taxes.

The underborrowing result stands in contrast to the overborrowing result stressed in the related literature. Underborrowing emerges in the present context because in economies that are prone to self-fulfilling financial crises, individual agents engage in excessive precautionary savings as a way to self insure.

The paper addresses the issue of implementation of the Ramsey optimal equilibrium. This is a nontrivial problem, because the Ramsey optimal policy only specifies what taxes are levied in the Ramsey equilibrium, but not what taxes would be levied off equilibrium. As a result, the Ramsey-optimal capital control policy does not ensure implementation of the Ramsey-optimal allocation. In particular, other, possibly welfare inferior, equilibria may be consistent with the Ramsey-optimal capital control policy. This paper shows that a capital control policy that threatens to tax capital outflows in the event of a self-fulfilling financial crisis can make such events incompatible with a rational expectations equilibrium and therefore eliminate them as possible outcomes, ensuring the emergence of the desired equilibrium.

Appendix: Numerical Solution Algorithm

This appendix describes the numerical algorithm used to approximate the equilibrium of the stochastic economy with a flow collateral constraint studied in section 6. The algorithm is a modified Euler-equation iteration procedure. To handle the possibility of indeterminacy of the rational expectations equilibrium of the type identified in section 3, we impose that the algorithm either favors equilibria like point C or like point B in figure 4. In addition, the algorithm does not allow choosing values of debt in period t for which there may exist no choice of debt in period $t+1$ for which consumption of tradables is positive and the collateral constraint is satisfied. This refinement of the solution algorithm, which we call pathfinder, facilitates convergence in the presence of nonconvexities in the aggregate feasible debt set.

1. Let y^T, r, and d denote the state of the economy in the current period and d' denote debt next period. Note that d' is in the information set of the current period.

2. Use the resource constraint (11) to compute consumption of tradables,

$$C^T(y^T, r, d, d') \equiv y^T + d'/(1+r) - d$$

for all current states (y^T, r, d) and for all $d' \in \{\underline{d}, \ldots, \overline{d}\}$. Consumption is then given by

$$C(y^T, r, d, d') \equiv \left[aC^T(y^T, r, d, d')^{1-1/\xi} + (1-a)y^{N1-1/\xi} \right]^{1/(1-1/\xi)}.$$

Compute the marginal utility of tradable consumption as

$$\Lambda(y^T, r, d, d') \equiv aC(y^T, r, d, d')^{-\sigma} \left(\frac{C^T(y^T, r, d, d')}{C(y^T, r, d, d'))} \right)^{-1/\xi}.$$

Compute the relative price of nontradables as

$$P(y^T, r, d, d') \equiv \frac{1-a}{a} \left(\frac{C^T(y^T, r, d, d')}{y^N} \right)^{1/\xi}.$$

Compute the value of collateral as

$$M(y^T, r, d, d') \equiv \kappa[y^T + P(y^T, r, d, d')y^N].$$

3. There may exist states (y^T, r, d) for which no choice of debt d' exists such that consumption of tradables is positive and the collateral constraint is satisfied. Let $Z(y^T, r, d)$ be an indicator function that takes the value 1 if $C^T(y^T, r, d, d') < 0$ or $d' > M(y^T, r, d, d')$

or both for all $d' \in \{\underline{d}, \ldots, \overline{d}\}$ and let $Z(y^T, r, d)$ take the value 0 otherwise. We will use this indicator below to avoid debt choices that lead with positive probability to areas of the state space for which $Z(y^{T'}, r', d') = 1$. We refer to this aspect of the solution algorithm as a path-finder.

4. Iteration $n = 1, 2, \ldots$ starts with a guess for the policy function of net external debt $d' = D_n(y^T, r, d)$.

5. Compute the expected value of the marginal utility of tradable consumption conditional on information available in the current period,

$$\Lambda_n^e(y^T, r, d, d') \equiv \mathbb{E}\Lambda(y^{T'}, r', d', D_n(y^{T'}, r', d')|y^T, r, d, d').$$

Define
$$\mu_n(y^T, r, d, d') = \frac{\Lambda(y^T, r, d, d')}{1 + r} - \beta \Lambda_n^e(y^T, r, d, d').$$

6. Pick d' as the solution to

$$D_{n+1}(y^T, r, d) = \operatorname*{arg\,min}_{d' \in \{\underline{d}, \ldots, \overline{d}\}} |\mu_n(y^T, r, d, d')|$$

subject to

$$C^T(y^T, r, d, d') > 0,$$

$$M(y^T, r, d, d') \geq d',$$

$$\mathbb{E}Z(y^{T'}, r', d'|y^T, r, d') = 0.$$

The first constraint says that consumption of tradables must be strictly positive. The second constraint says that the collateral constraint must be respected. And the third constraint rules out next-period debt choices that will place the economy with positive probability into areas of the state space next period for which $Z(y^{T'}, r', d') = 1$. This constraint is the path-finder aspect of the solution algorithm.

7. For a given current state (y^T, r, d), define the smallest possible choice of next-period debt at which the collateral constraint is binding as

$$d^c = \min_{d_j \in \{d_2, \ldots, \overline{d}\}} d_j$$

subject to

$$[M(y^T, r, d, d_j) - d_j][M(y^T, r, d, d_{j-1}) - d_{j-1}] \leq 0$$

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$$\mathbb{E}Z(y^{T'}, r', d_j | y^T, r, d_j) = 0$$

and

$$C^T(y^T, r, d, d_j) > 0,$$

where d_j is the jth element of the debt grid. If d^c exists for a given current state (y^T, r, d), check whether $\mu_n(y^T, r, d, d^c) \geq 0$. If so, set $D_{n+1}(y^T, r, d) = d^c$. If two such values exist pick the smaller one for equilibrium selection criterion (c) and the larger one for equilibrium selection criterion (b). This step ensures that the algorithm favors equilibria like points B or C, respectively, in figure 4.

8. If

$$\max_{\{y^T, r, d\}} |D_{n+1}(y^T, r, d) - D_n(y^T, r, d)| < \text{Tol}$$

for some tolerance level Tol, the procedure is completed. We set $\text{Tol} = 0$. Else, go to item 4. for a new iteration.

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