

Demography, Capital Flows and International Portfolio Choice over the Life-cycle *

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Abstract

In an aging world, how does a country's demographic structure impact external positions in safe and risky assets and their respective prices? We answer these questions combining endogenous portfolio choice over the life-cycle with a two-region, general equilibrium model. We show that when one region is aging faster than the other, its demand for both safe and risky assets increases, whereas a greater portfolio share is allocated into safe assets. Absent perfectly elastic supply, this results in a change in autarky rates and, in an open economy, in international asset trades. Calibrating the model to the U.S. and the EU, a negative net external position emerges in safe assets in the U.S. vis-à-vis the EU, explaining a significant portion of observed bilateral positions. Further, we predict persistent bilateral positions throughout the demographic transition. The model allows to quantitatively assess the impact of demographic change on trade in different types of assets, whereas previously, the focus has been on aggregate capital flows.

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1 Introduction

Many societies are currently experiencing major changes to their demographic structure. But while all world regions are aging, the trend and level of this transformation across regions differ over time. This holds true not only across stages of development, but also when comparing developed economies. Figure 1 shows, the EU-15¹ is aging faster than the US: The old age dependency ratio of the EU 15 is projected to rise from 0.33 today to 0.65 in 2100, whereas in the US, the old age dependency ratio is slightly lower today (at 0.25) and is likely to diverge even more from the EU in 2100 (0.53). What is the impact of these differential aging trends on external asset positions, portfolio compositions and asset prices? That is the question that this paper seeks to answer.

It is well known that if aging trends differ significantly across regions, demography can be a powerful determinant of international capital flows (Backus et al., 2014; Krueger and Ludwig, 2007; Börsch-Supan et al., 2006; Barany et al., 2015, among many others). Population aging influences the aggregate level of savings in an economy via two main channels. First, increased life expectancy causes individuals to save more during working life in order to provide for a longer retirement period. Second, the combination of falling birth rates and increased longevity influences the relative population share of each cohort. Older age groups that have high dis-saving will make up an increasing share in the total population. The overall effect on the aggregate savings rate will be positive as the population ages, diminishing as a larger share of the population dis-saves during retirement (Backus et al., 2014; Börsch-Supan et al., 2006). So all aging regions will experience a rise in aggregate savings, but the degree of which depends on the extent of demographic change. Different regional demand for assets will generate different autarky rates of return and in an open economy, absent capital market imperfections, international capital flows will result.

The life-cycle effects of savings will also affect an individual's portfolio allocation

¹This term includes all members of the European Union that joined prior to 2004: Austria, Belgium, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain and Sweden.

of savings into safe and risky assets. A large literature has documented a life-cycle shape to an individual's risky share of assets: the portfolio share of risky assets over the course of their working life increases, but shifts towards safe assets when retired (e.g. Poterba and Samwick, 2001; Guiso et al., 2002, Ameriks and Zeldes, 2004; Chang et al., 2014).² The intuition is that when labor income is relatively risk-free (or uncorrelated with risky asset returns), the present discounted value of future streams of labor income acts as a substitute for a safe asset. As the value of this safe asset is reduced towards retirement, so too, should the risky share Merton, 1971; Samuelson, 1969; Cocco et al., 2005.

Less well studied is how this life-cycle portfolio share of risky assets and demographic change may impact the composition of country portfolios. One would expect a larger share of retirees in the economy to imply a stronger relative demand for safe assets. Declines in fertility will also shift the balance between savers (young cohorts) and dis-savers (old cohorts), generating additional effects on autarky asset prices and returns. In two regions with different demographic composition, different relative prices for safe and risky assets will result. Were these regions financially integrated, there would be international asset trades that differ by riskiness of the asset. The country aging faster would have a higher demand for safe assets (lower autarky return) and thus, import safe assets from the relatively younger region. For risky assets, the direction of cross-border flows depends on the two regions' relative speed of demographic change.

Within this context, we consider the external positions of the EU-15 and US as two highly financially integrated world regions with varying aging trends. Since the early 1990s, diverging bilateral asset positions have emerged in the EU-15 vis-à-vis the US, with the EU-15 having a net asset position in debt instruments and net liability position in equity instruments (Figure 2). The majority of the debt position can be represented by safe debt instruments, such as government and agency secured bonds (see Section 1.2). An emergence of a safe net external asset position in EU-15 and a risky net external position in the US is indeed supportive of a changing asset demand structure driven by demographic transitions in the two regions: As the faster aging re-

²The hump-shaped form of risky asset shares is less pronounced when conditioning on participation in asset markets, while participation itself is also hump-shaped over the life-cycle.

gion, individuals in the EU-15 should have stronger preference for holding safe assets than US individuals.

The global imbalances manifesting themselves over the recent years, on aggregate and by asset type, have already been the subject of a large strand of literature (see e.g. Obstfeld and Rogoff, 2005, Blanchard et al., 2005, Gourinchas and Rey, 2007b, Caballero et al., 2008, Mendoza et al., 2009, among many others). However, we are not aware of a paper that considers demographics as an explanatory factor for the composition of country external portfolios. Our paper does so by building an OLG general equilibrium model, that is able to explain how demographic change affects savings, the allocation of savings into risky and safe assets, and ultimately financial transactions between two world regions.

Our approach allows us to decompose the effects into those on optimal household behavior (*life-cycle channel*), aggregate population shares (*distribution channel*) and on asset prices and returns (*valuation channel*). In an EU-15, US calibration, we can explain a significant share of the existing bilateral portfolio imbalances and are able to make predictions about future asset trades. We therefore add to the discussion about the persistence and sustainability of the currently observed large external positions in national balance sheets, and their asymmetric risk content.

Quantifying the role of demographic factors is important because portfolio imbalances that arise as an optimal consumption-smoothing and risk-sharing response to demographic change require a different policy response than, for example, imbalances due to fragilities in the international financial system or misled national policies. What is more, the general equilibrium nature of our model allows us to assess the contribution of demographic change on returns to both safe and risky assets. In this way, we add to the literature on persistently low interest rates that has offered various demand side explanations (Bernanke, 2005, Eggertsson and Mehrotra, 2014) and supply side explanations (Gourinchas and Jeanne, 2012). In contrast to those explanations, demographic change is a long-term phenomenon and one which is not easily corrected by policy measures. Knowing that the strong demand for safe assets from aging economies will persist and increase further implies that new responses need to be found. Our paper thus provides a starting point for re-thinking the policy response

to low interest rates and global imbalances.

In our framework, we capture a rich demographic structure of each region by stochastic survival probabilities and birth rates, reflecting both dimensions of demographic change. Agents in the model earn a stochastic labor income during working age and a fixed pension income during retirement; and agents smooth consumption by saving into two types of assets: a risk-free bond and a stock with stochastic returns, which earns a risk premium. In choosing their optimal portfolio, the agents take into account their financial wealth, the net-present value of their remaining lifetime income and their life expectancy as well as asset returns. Aggregate demand for both types of assets thus reflects the demographic structure of the region.

In our two region model world, we consider heterogeneity across regions solely in demographic variables and perfect capital integration. Asset supply in each region is modelled by Lucas tree endowment. External asset positions will reflect excess demand in each region for safe and risky assets at equilibrium interest rates that clear global markets. Thus, also asset positions and equilibrium asset returns will entirely reflect the cross-country differences in demographics.

We find that differences in relative demographic transitions between the EU-15 and US are powerful predictors of composition of the regions' bilateral portfolios. Our model can explain around two-thirds of the bilateral net safe asset position in our stylized facts. Positions are slightly reduced through out the transition as a larger share of population is dis-saving during retirement. We find that the *distribution channel*, or changing age distribution of the population drives our results, but that demographic change's impact on individual behavior becomes more important for older cohorts and as the demographic transition progresses.

In the following sections, we will first present the model. Section 3 describes how we parameterize the benchmark version of the model to match the data. In Section 4, we present simulation results for the effects of demographic change on key parameters of the benchmark model. Section 5 concludes.

1.1 Relation to the literature

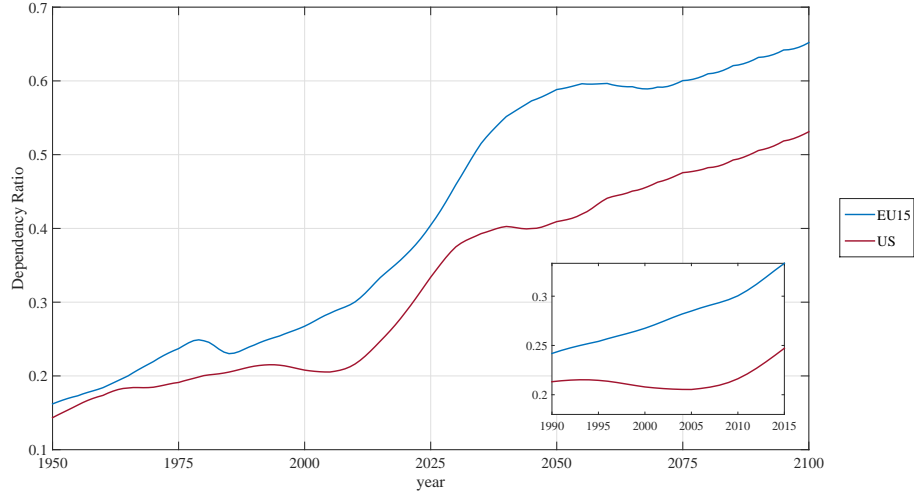
The impact of demographic change on saving and asset prices is well-researched in a closed economy setting. Life-cycle theory (going back to Modigliani and Brumberg, 1954, 1990 and Ando and Modigliani, 1963) analyzes how savings behavior of individual agents changes over the course of their lives, and across cohorts. Building on these models, Mason (1981, 1987) and Fry and Mason (1982) study the effect of increased population growth on life-cycle savings. Lee et al. (2000) and Bloom et al. (2003) explore the role of a higher life expectancy.

The optimal mix between safe and risky assets at various stages of life is the subject of portfolio choice theory. Merton (1971) shows that when labor income substitutes for safe asset holdings, agents with CRRA preferences should invest a constant fraction of their wealth in risky assets. Cocco et al. (2005) characterize optimal portfolio choice when labor income is risky and positively correlated with the returns to the risky asset. Papers on the effect of demographic change on asset prices often focus on the U.S. “baby boomer” generation, e.g. Brooks (2000, 2006), Abel (2001, 2003), Poterba (2001), Ang and Maddaloni (2005).

The literature studying the effect of demographic change in an open economy setting has so far focused on aggregate capital flows. Structural models are for example presented by Brooks (2003), Domeij and Floden (2006), Krueger and Ludwig (2007), Backus et al. (2014) and most recently by Barany et al. (2015), who also consider differences in pension systems and financial development. Coeurdacier et al. (2015) explore the role of financial frictions for the effect of cross-country differences of age profiles on international capital flows. Börsch-Supan et al. (2006) analyze the effect of pension reform in an open economy. Empirical studies include Taylor and Williamson (1994), Higgins and Williamson (1997), Higgins (1998), Lane and Milesi-Ferretti (2002) and Chinn and Prasad (2003).

To our knowledge, our paper is the first to explore international portfolio allocation and demographic change in an open economy life-cycle model. We contribute to the literature by making predictions for the direction and magnitude of flows separately for safe and risky assets - which leads to different policy implications than the study of

Figure 1: Demographic trends EU-15 vs. US



Old age dependency ratios for the medium variant for fertility and normal migration. Source: United Nations Population Division.

aggregate flows. Additionally, our model can explain a significant portion of the valuation effects present in international foreign asset positions through general equilibrium price effects. There is one empirical paper, De Santis and Lüthmann (2009), which considers - among other things - the link between demographics and various types of capital flows. They find that a higher old-age dependency induces net equity inflows and net outflows of debt instruments. This is in line with our results.

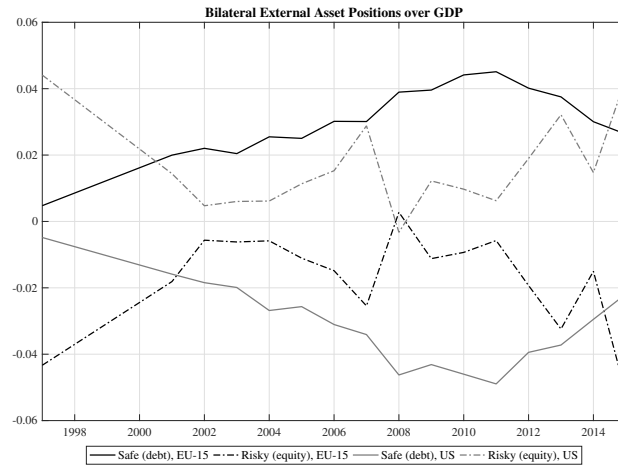
1.2 Stylized Facts

Our research is motivated by several stylized facts. First, there are large variations in demographic trends not only across levels of development, but also within developed regions. Specifically, if one examines the dependency ratio data and projections from the UN Population Division, there are sizeable variables in the speed of aging.

Figure 1 depicts the dependency ratios from the UN Population Prospects for the US and EU-15 from 1950 to 2100.³ Both regions are aging over this period, due to a

³The projections shown for 2016 onwards are the medium variant for fertility and rely on normal

Figure 2: Bilateral debt and equity positions EU-15 vs. US



Bilateral net debt (safe) and equity (risky) positions relative to GDP. Source: IMF Coordinated Portfolio Investment Survey.

combination of an increase in life expectancy and a decrease in fertility. Starting at an old-age dependency ratio of around 0.15 in 1950, the ratio will rise to 0.50 at the end of the projection. The speed of aging is projected to increase over the next decades, among other things due to the retiring of the “baby boomer” cohorts.

But there are also differences in how the US and the EU-15 are aging: The old-age dependency ratio is constantly higher in the EU-15, and there have been and are projected to be several periods of acceleration of the demographic transition of the EU-15, further increasing the difference between the two regions. In 2050, the old-age dependency ratio of the EU-15 will be around 18 percentage points stronger than that of the US.

Second, bilateral positions in the two regions diverge by risk properties. This can be shown by correcting bilateral debt and equity positions, as reported by the CPIS, for risky corporate debt. The exact definitions can be found in the Appendix. The US has a negative net bilateral safe asset position and a positive net bilateral risky asset

assumptions for migration (United Nations, 2015).

position, which means that the US is a net exporter of safe assets and a net importer of risky assets vis-à-vis the EU-15 (Figure 2). This “hedge fund” like bilateral portfolio of the US mirrors its international asset position as for example explored by Gourinchas and Rey (2007a) and Gourinchas et al. (2010). However, we are not aware of a paper that has presented this finding for bilateral asset positions between these regions.

In our model world, there exist only the US and the EU-15, whereas in reality, many other countries are participating in international asset trades. What is the share of bilateral asset positions in the total safe and risky positions of the two regions? This question is not easy to answer, because not all US,EU-15 investments are accounted for as bilateral investments. Offshore financial centers (OFCs) play a role as intermediary of bilateral investments for the purpose of tax evasion and money laundering. Asset positions vis-à-vis an offshore financial center might therefore in reality represent bilateral US, EU-15 investments (see e.g. Milesi-Ferretti and Lane, 2010). While the extent of this is impossible to verify, in Table 1, we present an upper and a lower bound for the share of bilateral debt and equity in total external debt and equity assets⁴: The lower bound attributes all OFC-US and OFC, EU-15 investments to the rest of the world (RoW), the upper bound attributes them to the respective other region. As can be seen, all bilateral shares lie in an interval of around 0.4 and 0.65. These numbers are quite sizeable.

⁴For the sake of comparability, we only use CPIS data here, therefore distinguishing between debt and equity instead of safe and risky assets. Since countries do not report liabilities to the IMF, the numbers in CPIS are inferred. Because these are less precise than data on assets, we restrict our focus on assets. The numbers necessarily exclude any investment that is unaccounted for because it is not reported, a lot of which could also be bilateral (see Zucman, 2013).

Table 1: Share of bilateral positions in total external positions

	EU-15		US	
	debt	equity	debt	equity
OFCs as RoW	0.45	0.44	0.45	0.42
OFCs as bilateral	0.57	0.64	0.59	0.62

Source: CPIS, using a definition of offshore financial centers (OFCs) by the IMF. Numbers are averages over 1997 - 2015.

2 A Multi-Period OLG Model

We augment the workhorse quantitative overlapping generations model of Auerbach and Kotlikoff (1987) along the following dimensions. First in order to highlight the role of demography, we consider a two region world, where regions differ only by their demographic transitions. Second, we allow for cohort specific survival probabilities and birth rates so that we can model the impact of a full demographic transition. Third, we allow for a portfolio allocation problem of individuals, by introducing a safe asset, or bond, and a risky asset, or stock, with endogenous prices. This model environment will allow us to track different channels of demographic change on external positions: effects over the life-cycle, effects of changing population shares, and effects on prices.

In general, we distinguish between individual choice variables and aggregate, or variables taken as given at the individual level, by capitalizing the latter.

2.1 Individuals

Within each region j and in each period t , a continuum of individuals, indexed by i , is born and forms a new cohort. The measure of each cohort reflects demographic change as discussed in Section 2.2.

Individuals are born at age N^b , endowed with a Lucas tree, work until age N^r over

which time, they receive an age-dependent labor income, and live to a maximum age of N^d . Prior to reaching age N^d , individuals survive to age $n + 1$ with probability $\delta_{n,t}$ conditional on being alive at age n . These survival probabilities effect longevity and are cohort specific due to demographic transition. The individual objective function is sum of expected discounted lifetime utility:

$$U_{i,t}^j = \mathbb{E} \sum_{n=N^b}^{N^d} \left(\prod_{l=0}^{n-1} \delta_{l,t+l}^j \right) \beta^{n-N^b} u(c_{i,n,t+n-N^b}^j), \quad (1)$$

where $c_{i,n,t}^j$ is consumption of individual i in region j , who is n years old at time t , $\beta\delta_{n,t}^j$ is the subjective discount factor that takes the region-specific stochastic survival probability into account. Preferences are of the CRRA type, with parameter ϑ .

2.2 Demographics

We model a rich demographic structure, accounting for differences in birth rates (fertility) and survival probabilities (longevity). Agents take into account a cohort-specific series of age-dependent survival probabilities when making decisions. However, demographics also impact the aggregate size of the population, and the distribution of population across cohorts.

In each period, a continuum of individuals is born, forming a cohort. The birth rate, γ_t will impact the measure of each cohort. The size or measure of the youngest cohort $L_{N^b,t}$ evolves according to

$$L_{N^b,t+1} = (1 + \gamma_{t+1})L_{N^b,t}. \quad (2)$$

In addition, the survival probabilities will impact the size of cohorts over time. In each period, there is a probability of survival to the next period conditional on reaching age n of $\delta_{n,t}$. The size of a cohort of age n in period t is $L_{n,t}$. This cohort will reach size

in period t of

$$L_{n,t} = \left(\prod_{l=0}^{n-1} \delta_{N^{b+l}, t-n+l} \right) L_{N^b, t-n}. \quad (3)$$

Total population size is the sum over all cohorts alive at time t : $L_t = \sum_{n=N^b}^{N^d} L_{n,t}$.

2.3 Market Structure

2.3.1 Financial Assets

Financial markets are incomplete and Individuals smooth consumption by allocating their savings into a safe asset or bond and risky asset or stock. Holdings of bonds and stocks by individuals represent tradable claims on the dividends provided by two tranches of each Lucas (1978) tree with which the individuals are endowed. Thus, the total number of trees in the economy corresponds to the population.

A safe tranche pays a safe dividend of d in each period and trades at price Q_t , whereas the risky tranche pays a stochastic dividend, \tilde{d}_t and trades at price \tilde{Q}_t . From the asset pricing equations of households, gross returns of each asset are

$$R_{t+1} = \frac{Q_{t+1} + d_{t+1}}{Q_t} \quad (4)$$

$$\tilde{R}_{t+1} = \frac{\tilde{Q}_{t+1} + \tilde{d}_{t+1}}{\tilde{Q}_t} \quad (5)$$

where $\tilde{d}_t = d_t + r_{p_t} + \epsilon_t$. Where r_{p_t} is a risk premium and ϵ_t is an *i.i.d.* shock, which follows a normal distribution with mean zero and variance σ_ϵ^2 . As is standard, each asset's return is comprised of a capital gain and dividend price ratio. Returns therefore depend both on the (stochastic) dividends and on the aggregate demand and supply of assets, which determine the price and are influenced by demographics. Asset returns are perfectly correlated across regions.⁵

⁵In the perfectly financially integrated model world, this means that asset returns equalize across regions. This assumption reduces the total number of assets in the model world to two: one safe and

The stochastic component of the risky returns introduces aggregate risk. The shock to the risk premium is independent of shocks to labor income. This assumption is motivated in keeping the mechanism of our model clear, however, it is also empirically relevant case. Several papers have found a very small correlation between labor income and stock returns (Cocco et al., 2005).

We model the aggregate supply of both safe and risky assets in each region as proportional to the the sum of agents at time t L_t^j , or the sum of all living cohorts:

$$\mathbb{B}_t^j = \lambda^j \sum_{n=N^b}^{N^d} L_{n,t}^j \quad \mathbb{S}_t^j = \tilde{\lambda}^j \sum_{n=N^b}^{N^d} L_{n,t}^j, \quad (6)$$

with scaling parameters $\lambda, \tilde{\lambda}$, for safe and risky assets, respectively. In this formulation, we keep the supply side of assets simple, but responsive to demographic change. Some elasticity of supply to aging should be expected, given that demographic change is a long-term and predictable development. Conversely, it is unlikely that the supply is fully elastic: For the supply of government bonds, the prototype safe asset, considerations other than demand matter, for example the sustainability of the fiscal position of the government. For the supply of equity, a firm considers the shape of the production function and the supply of input factors other than capital when deciding on the amount of capital to employ.

Because of constant scaling parameters, safe assets are always supplied in a fixed ratio to risky assets. This specification is not unreasonable. Gorton et al. (2012) show that the percentage of all U.S. assets that can be considered safe has remained stable between 1952 and 2010.

A discussion of our calibration of these parameters follows in Section 3.1.3.

one risky. With this assumption, we abstract from important issues such as motivating the equity and bond home bias, which we believe are beyond the scope of this paper.

2.3.2 Market for Goods

Agents choose consumption level of a composite good, $c_{i,n,t}$. For simplicity, we abstract from modelling (real) exchange rates and assume that the law of one price always holds. In our model world, there is only one good, the price of which is normalized to 1 in both regions. Therefore, trade in goods does not play a role. We choose this approach to highlight more clearly the impact of demographic change on international trade in assets.

2.4 Individual's Problem

In each period, working individuals start the period with a certain amount of cash on hand $x_{i,n,t}$, and choose consumption $c_{i,n,t}$ and an allocation of savings into bonds $b_{i,n,t}$ and stocks $s_{i,n,t}$ in order to maximize expected lifetime utility Eq. (1) subject to budget constraints, Eqs. (7) and (8). Each working individual faces the budget constraint,

$$c_{i,n,t} + Q_t b_{i,n,t} + \tilde{Q}_t s_{i,n,t} = \overbrace{(Q_t + d_t)b_{i,n-1,t-1} + (\tilde{Q}_t + \tilde{d}_t)s_{i,n-1,t-1} + y_{i,n,t}}^{x_{i,n,t}} \quad \text{if } n \leq N^r, \quad (7)$$

where, cash on hand is the right hand side of Eq. (7), $y_{i,j,t}$ is a stochastic, age-dependent labor income process.

Retired individuals are subject to a nearly identical set of budget constraints as working individuals, apart from the pension income, $\tilde{y}_{i,j,t}$, discussed in more detail in the following Section 6.2.

$$c_{i,n,t} + Q_t b_{i,n,t} + \tilde{Q}_t s_{i,n,t} = \overbrace{(Q_t + d_t)b_{i,n-1,t-1} + (\tilde{Q}_t + \tilde{d}_t)s_{i,n-1,t-1} + \tilde{y}_{i,n,t}}^{x_{i,n,t}} \quad \text{if } n > N^r, \quad (8)$$

A natural borrowing constraint arises due to a zero income shock discussed in

Section 2.4.1, such that

$$s_{i,n,t} \geq 0 \quad b_{i,n,t} \geq 0.$$

Individuals are not willing to short stocks or bonds.

We often refer to the risky share of assets at the individual level. From the left-hand-side of Eqs.(7) and (8), we define end of period assets $a_{i,n,t}$:

$$a_{i,n,t} \equiv Q_t b_{i,n,t} + \tilde{Q}_t s_{i,n,t}. \quad (9)$$

This implies a share of risky assets as:

$$\omega_{i,n,t} = \frac{Q_t b_{i,n,t}}{a_{i,n,t}}. \quad (10)$$

2.4.1 Income Process

Central to the life-cycle shape of asset accumulation and risky share is the labor and pension income process and its risk attributes. Labor income for individual i of age n is defined as

$$y_{i,n,t} = P_n \zeta_{i,n,t} \quad (11)$$

where

$$P_n = G_n P_{n-1}, \quad (12)$$

$P_{N^b} > 0$ given. P_n is a deterministic process exhibiting a hump shape over the life-cycle. This is homogeneous across individuals, cohorts and regions. This represents a premium on work experience, or that the amount of labor supplied by the agents or their productivity depends deterministically on their age.

The labor income process $y_{i,n,t}$ is a stochastic process, with transitory, idiosyncratic

shock $\zeta_{i,n,t}$. The transitory shock takes the form of zero income shock with

$$\zeta_{i,n,t} = \begin{cases} 0 & \text{with probability } p \\ 1 & \text{with probability } 1 - p. \end{cases}$$

A similar specification is frequently used in the life-cycle literature, going back to Zeldes (1989) and Carroll (1992).⁶ The shock can be thought of as an unemployment spell or an employment break due to medical reasons.

The shock to labor income is independent of the shock to stock returns. This assumption is motivated by keeping the mechanism of our model clear, however, it is also empirically relevant. The evidence in the literature is not clear: Papers have sometimes found a small positive correlation between labor income and stock returns or a small negative correlation (Heaton and Lucas, 2000).

We include the zero income shock to address an issue well known in the literature studying portfolio choice over the life-cycle: from the standard models arises the prediction that the risky share of assets is much higher than found in reality (Merton, 1971; Heaton and Lucas, 1997) and U-shaped, counter to empirical evidence of a hump-shaped risky share over the life-cycle (Guiso et al., 2002; Cocco et al., 2005; Fagereng et al., 2017). The intuition behind this result is that when human wealth, the present discounted value of future labor income, over financial wealth is high, and when human wealth is relatively safe and not correlated with risky returns, it mimics a large safe asset position. Relatively more endowed with this safe asset, the young hold a larger share of financial wealth in the risky asset, which declines with age and human wealth over the life-cycle. The old, on the other hand, have run down their financial assets so much that the financial assets become small enough to increase the risky share. This result is robust to stochastic income and even the low correlations between labor income and risky returns observed in the data (Cocco et al., 2005).

With the inclusion of the zero income shock, we generate a model counterpart of risky share more consistent with empirical evidence. In our model, agents will

⁶In contrast to those papers, our income process does not contain transitory or permanent income shocks. We abstract from those shocks in order to keep our model tractable.

accumulate a precautionary buffer in the safe asset when young (and maintain it when old) when the level of financial assets is low. The budget constraints in Eqs. (7) and (8) must hold in each period. In order to insure non-zero consumption in each period, individuals will accumulate safe financial assets in order to avoid zero consumption in the event of successive zero income draws. This is the same intuition behind the emergence of a natural borrowing constraint. A broader discussion on how our labor income process compares to that which is more standard in the literature is presented in the Appendix 2.4.1

In our benchmark specification, pension income is a fixed fraction $\phi \in [0, 1]$ of the deterministic labor income earned in the period directly preceding retirement, N^r :

$$\tilde{y}_{i,n,t} = \phi P_{i,N^r,t} \quad \forall n = N^r + 1, \dots, N^d. \quad (13)$$

Since the pension income is a share of deterministic labor income, individuals are prevented from receiving no pension income should they draw a zero income shock in the period preceding retirement. We choose the simple constant replacement rate pension scheme because we do not want to impose a specific pension system. There are several different pension systems around the world, among them fully funded plans and pay-as-you-go (PAYGo) systems. (Private retirement schemes are in our model part of the individual savings.) Imposing e.g. a PAYGo system would require specifying how the system is adjusted to demographic change: Through an increase in the contribution rate, a decrease the replacement rate, or through an increase in the pension age. We will consider alternative pension schemes in a future iteration of the paper.

Pension income is also subject to zero shocks of the same nature as the zero labor income shocks. In general, zero income periods should not be encountered by retirees that receive a public pension, but we consider these shocks rather to reflect periods of high expenditure, e.g. due to health shocks. On the role of medical expenses of the elderly, see (De Nardi et al., 2010; De Nardi et al., 2016; Ameriks et al., 2015). We show that zero income shocks can be recast as expenditure shock, taking a special form in the Appendix 6.2.2.

2.5 Aggregates and the Open Economy

We consider two world regions, which differ only in terms of demographic characteristics. Financial markets are perfectly integrated, so agents of both regions are free to buy bonds and stocks anywhere in the the two regions. Furthermore, our assumption of a real exchange rate equal to unity and fixed relative prices for goods, renders the consumption of a single identical world good. Thus, agents in region j can consume a domestic good and foreign good, save in the form of domestic bond and stocks or foreign bonds and stocks. Foreign goods and assets are denoted with an asterisk.

In each region, individual consumption $c_{i,n,t}$, stock holdings, $s_{i,n,t}$ and bond holdings $b_{i,n,t}$ are averaged within each cohort, $c_{n,t}^j = \frac{1}{L_{n,t}} \int_0^{L_{n,t}} c_{i,n,t}^j di$; $s_{n,t}^j = \frac{1}{L_{n,t}} \int_0^{L_{n,t}} s_{i,n,t}^j di$; and $b_{n,t}^j = \frac{1}{L_{n,t}} \int_0^{L_{n,t}} b_{i,n,t}^j di$.

Aggregate demand for goods, bonds and stocks will be the sum over cohorts weighted by the cohort size:

$$\begin{aligned}
 \text{consumption :} & & C_t^j &= \sum_{n=N^b}^{N^d} L_{n,t}^j (c_{n,t}^j + c_{n,t}^{j*}) \\
 \text{bonds :} & & B_t^j &= \sum_{n=N^b}^{N^d} L_{n,t}^j (b_{n,t}^j + b_{n,t}^{j*}) \\
 \text{stocks :} & & S_t^j &= \sum_{n=N^b}^{N^d} L_{n,t}^j (s_{n,t}^j + s_{n,t}^{j*}).
 \end{aligned}$$

2.5.1 Market Clearing

In this section, we derive relevant market clearing conditions, building slowly from an economy under autarky to a two region world.

Autarky

Under autarky cross-border asset trades are shut down and the local demand for assets will be cleared by local supply of assets at equilibrium asset prices:

$$\begin{aligned}
 S_t^j &= \sum_{n=N^b}^{N^d} L_{n,t}^j s_{n,t}^j = \mathbf{S}_t = \tilde{\lambda}^j \sum_{n=N^b}^{N^d} L_{n,t}^j \\
 B_t^j &= \sum_{n=N^b}^{N^d} L_{n,t}^j b_{n,t}^j = \mathbf{B}_t = \tilde{\lambda}^j \sum_{n=N^b}^{N^d} L_{n,t}^j.
 \end{aligned} \tag{14}$$

Averaging the individual flow individual budget constraints Eqs. (7) and (8) over individuals and aggregating over working cohorts and retired cohorts, respectively, yields the aggregate resource constraint under autarky:

$$C_t + Q_t(B_t - B_{t-1}) + \tilde{Q}_t(S_t - S_{t-1}) = Y_t + dB_{t-1} + \tilde{d}_t S_{t-1}. \tag{15}$$

where Y_t is the aggregate labor and pension income in the economy.

If the market clearing conditions under autarky, Eqs. (14) hold, the market for goods clears by Walras's Law and Eq. (15) becomes,

$$C_t = \overbrace{Y_t + (d\lambda + \tilde{d}_t \tilde{\lambda})L_{t-1}}^{\text{Aggregate Endowment}} - Q_t \lambda (L_t - L_{t-1}) - \tilde{Q}_t \tilde{\lambda} (L_t - L_{t-1}). \tag{16}$$

Asset prices under autarky, Q_t and \tilde{Q}_t , will be as such that aggregate consumption will be equal to the aggregate endowment minus the additional savings required to offset the change in the supply of assets due to population change.

Two Region World

In a world consisting of two perfectly integrated regions $j = e, u$, the asset prices will adjust such that the market for assets and goods clear globally. The relevant market clearing conditions for stocks and bonds are:

$$\begin{aligned}
 S_t &\equiv S_t^e + S_t^u = \sum_{n=N^b}^{N^d} L_{n,t}^e s_{n,t}^e + \sum_{n=N^b}^{N^d} L_{n,t}^u s_{n,t}^u = \mathbf{S}_t \equiv \mathbf{S}_t^e + \mathbf{S}_t^u = \sum_{n=N^b}^{N^d} \tilde{\lambda}^e L_{n,t}^e + \sum_{n=N^b}^{N^d} \tilde{\lambda}^u L_{n,t}^u \\
 B_t &\equiv B_t^e + B_t^u = \sum_{n=N^b}^{N^d} L_{n,t}^e b_{n,t}^e + \sum_{n=N^b}^{N^d} L_{n,t}^u b_{n,t}^u = \mathbf{B}_t \equiv \mathbf{B}_t^e + \mathbf{B}_t^u = \sum_{n=N^b}^{N^d} \lambda^e L_{n,t}^e + \sum_{n=N^b}^{N^d} \lambda^u L_{n,t}^u.
 \end{aligned} \tag{17}$$

Due to our assumption of perfectly correlated returns across regions, agents of both regions are indifferent between holding domestic and foreign assets. Consequently, hedging behavior, which is an important feature of many portfolio choice models, does not play a role. We abstract from hedging because, while it certainly motivates international asset purchases, it is not central for explaining the effect of demographic change on international asset trades. We are interested in how aging affects the *net* positions in risky and safe assets, not the *gross* positions. As long as demographic change does not alter the international correlation of risky returns or the correlation between labor income and asset returns, the role of hedging does not change between the different demographic set-ups that we consider. We believe that this is a credible assumption.

In addition, the hedging motive seems to be less important in the data than it is in most international portfolio choice models. We observe a strong equity home bias, which an important literature seeks to explain. (For surveys of the literature on this so called *international diversification puzzle*, see Lewis, 1999 and Gourinchas and Rey, 2013.)

In the two region case, total asset demand in region u will consist of local assets and cross-border asset holdings from region e . A region's net foreign asset position in

each instrument is the difference between asset demand and local asset supply,

$$\begin{aligned}
NFB_t^u &= \overbrace{B_t^{uu} + B_t^{ue}}^{\text{bond demand by } u} - \overbrace{B_t^{uu} - B_t^{eu}}^{\text{bond supply by } u} = B_t^{ue} - B_t^{eu} \\
NFS_t^u &= \overbrace{S_t^{uu} + S_t^{ue}}^{\text{stock demand by } u} - \overbrace{S_t^{uu} - S_t^{eu}}^{\text{stock supply by } u} = S_t^{ue} - S_t^{eu},
\end{aligned} \tag{18}$$

where the first superscripted letter denotes the region demanding/supplying the assets and the second superscripted letter denotes on which region the claims are. So S_t^{uu} are claims by agents in region u on region u 's risky dividends and S_t^{ue} are claims by agents in region u on region e 's risky dividends, and so on. Region e 's net foreign asset position is computed accordingly.

Considering the above definition, we can rewrite each region's aggregate resource constraint,

$$\begin{aligned}
Q_t(NFB_t^u - NFB_{t-1}^u) + \tilde{Q}_t(NFS_t^u - NFS_{t-1}^u) &= Y_t^u + d_t \mathbb{B}_{t-1}^u + \tilde{d}_t^u \mathbb{S}_{t-1}^u - C_t^u - \\
Q_t(\mathbb{B}_t^u - \mathbb{B}_{t-1}^u) - \tilde{Q}_t(\mathbb{S}_t^u - \mathbb{S}_{t-1}^u) + d_t NFB_{t-1}^u + \tilde{d}_t NFS_{t-1}^u. & \tag{19}
\end{aligned}$$

This is a standard balance of payments equation. The left hand side of Eq.(19) is the change in the net external asset position in bonds and stocks in region u . The right hand side is the standard definition for the current account: the aggregate endowment $Y_t^u + d_t \mathbb{B}_{t-1}^u + \tilde{d}_t^u \mathbb{S}_{t-1}^u$ minus consumption C_t^u and investment (here the change in supply induced by population growth), $Q_t(\mathbb{B}_t^u - \mathbb{B}_{t-1}^u) - \tilde{Q}_t(\mathbb{S}_t^u - \mathbb{S}_{t-1}^u)$, plus net income on foreign assets, $d_t NFB_{t-1}^u + \tilde{d}_t NFS_{t-1}^u$.

Adding the two aggregate resource constraints in regions e and u , and substituting the world market clearing conditions Eqs.(17), the net external positions cancel, resulting in the consumption aggregate clearing condition,

$$\begin{aligned}
C_t = Y_t + d(\lambda^e L_{t-1}^e + \lambda^u L_{t-1}^u) + \tilde{d}_t(\tilde{\lambda}^e L_{t-1}^e + \tilde{\lambda}^u L_{t-1}^u) - Q_t(\lambda^e L_t^e + \lambda^u L_t^u - \lambda^e L_{t-1}^e - \lambda^u L_{t-1}^u) - \\
\tilde{Q}_t(\tilde{\lambda}^e L_t^e + \tilde{\lambda}^u L_t^u - \tilde{\lambda}^e L_{t-1}^e - \tilde{\lambda}^u L_{t-1}^u). \tag{20}
\end{aligned}$$

Consumption in the two regions is equal to the aggregate endowment minus investment

across the two regions, which is directly related to the change in the population over the two world regions.

2.6 Equilibrium and Timing

At the beginning of each period t , the members of the youngest cohort $L_{N^b,t}$ are born with no initial wealth, but receive their labor income. All living agents receive interest income on existing savings in safe and risky assets, returning R_t and \tilde{R}_t , respectively according to Eqs. (4). Individuals of working age receive their labor income, $y_{n,t}$ and retired agents collect pension benefits, $\tilde{y}_{n,t}$. Together, these income streams constitute cash on hand, $x_{n,t}$, which is allocated optimally to consumption and asset purchases. Returns are allocated to end-of-period wealth of the previous period, so agents must form expectations of future returns when choosing portfolio allocation. After agents have made their consumption and saving decisions, a share of each cohort, $(1 - \delta_{n,t})$ dies. Their remaining assets are redistributed to the aggregate stock of assets in the economy.⁷

The equilibrium consists of paths for consumption $\{c_{i,n,t}\}$, bonds $\{b_{i,n,t}\}$, $\{b_{i,n,t}^*\}$ and stocks $\{s_{i,n,t}\}$, $\{s_{i,n,t}^*\}$ as well as the paths for prices, interest rates, and wages for each of the two regions e, u . The equilibrium paths are chosen such that: (i) agents maximize the expected life time utility, Eq.(1), subject to a sequence of budget constraints, Eqs. (7) and (8), taking the paths for prices as given in the economy; and (iii) such that markets clear globally for stocks, bonds, Eq. (17) and goods markets in each period.

3 Simulation Exercise [preliminary]

In order to test our model's ability to explain international external positions, we perform two simulation exercises of the demographic transition.

⁷Up until now, we have ignored the accidental bequests that occur with stochastic lifetimes. We assume that in each period a portion of the endowment allocated to the return on assets held by agents that die expires and is not reallocated to living agents. We assume no annuity insurance against accidental bequests. We may reallocate bequests at a later stage.

Demographic data from UN Population Prospects spans 1950 to 2095 in five year increments. We assume an initial steady state population distribution that reflects demographic data in 1950. In order to generate the initial steady state, we simulate birth cohorts starting from 1870 to 1950 resulting in a full age distribution across age cohorts in 1950.⁸

Starting in 1950 the demographic transition begins. Agents born in 1950 take into account the full demographic transition directly through survival probabilities and indirectly through birth rates that influence aggregate variables. Because demographic data is available every 5 years and we still simulate a full life-cycle, we assume that individuals take demographic structure as given for five year periods. The simulation works as follows: individuals born in 1950 take survival probabilities from age 20 to 24 from 1950 demographics data, at age 25, forward looking individuals will take survival probabilities from age 25 to 29 from 1955 demographic data. This constitutes through the projections up to 2095. We use an analogous approach for birth rates in the simulation of aggregate variables.

Post-transition in 2095 (the last projection year from the UN data), we simulate a steady state by taking demographic data as fixed at 2095 projections. This generates a full age distribution the first cohort of which is born in 2095. This approach allows us to examine both the pre- and post-transition steady state variables as well as the full transition.

As a first exercise, we assume a small open economy, that starts as a closed economy in 1950 and opens to the rest of the world in 1990. Prior to opening, the economy's age demographic transition is identical to that of the rest of the world. After opening, the small open economy experiences an accelerated demographic transition, against a rest of the world with a stationary age distribution. Thus, autarky rates prior to opening will be identical to the world interest rates after opening. Autarky and

⁸The population structure observed in 1950 will have arisen out of demographic structures starting in 1870 and changing over time until 1950. Clearly over this time between Europe and the US there are large immigration swings as well as changes in birth rates. However, we are only interested in generating a model counterpart to the observed population structure in 1950 so that we may focus on the transition starting thereafter. Although interesting, We are less concerned with the transition leading up to 1950 and we accept this abstraction given a lack of available data.

world returns after opening are taken as the average real return on 3 month US treasury bill and derived from risk premium estimated from S& P500 returns from (Fama and French, 2002). This exercise abstracts from the general equilibrium effects on asset prices, highlighting the impact of demographic change on asset demand over the entire transition.

In a second simulation exercise, we simulate a two-region version of the model with a US, EU-15 parameterization. A future iteration of our paper will include fully endogenous world asset price determination, determined by global asset market clearing. For now, we assume opening in the two regions in 1990. We proxy world equilibrium rates of return by observed real rates of return in the US from 1990 to 2015. Solving the two region model for these observed interest rates, we generate our model predictions for external asset positions in the two regions. In our two-region world, regions differ only in their demographic transitions. This exercise thus assumes that observed changes in safe and risky returns are entirely due to differences in relative demographic transitions.

3.1 Parameterization

The parameters used in our simulation exercise are presented in Table 2. The first set of parameters are taken from standard values in the literature and the remaining parameters are estimated from data.

3.1.1 Demographics

Agents are born at age 20 and the maximum lifetime is set to age 100. Over the life-cycle agents survive with age-dependent survival probabilities taken from data and projections from the United Nations World Population Prospects (United Nations, 2015).⁹

The UN provides survival probabilities to a maximum age of 85. The survival

⁹The UN data provides several projections. We use the survival probabilities that take into account migration flows, and a medium variant for fertility projections United Nations, 2015.

Table 2: Parameters in Simulation Exercises

Age at birth	N^b	20	(Cocco et al., 2005)
Maximum Age	N^d	100	(Cocco et al., 2005)
Subjective discount factor	β	0.96	(Cocco et al., 2005)
CRRA parameter	ϑ	8	(Mehra and Prescott, 1985)
Risk premium	r_p	0.04	(Fama and French, 2002)
Shock to risk premium	σ_ϵ^2	0.21	(Fama and French, 2002)
Probability of zero income	p	0.01	(Carroll, 1992)
Age-dependent income growth	G_n, P_n		(Cocco et al., 2005)
Pension income replacement rate	ϕ	0.68	(Cocco et al., 2005)
Survival probabilities	$\delta_{n,t}$		UN Population Prospects, medium variant, 1950 to 2095
Birth rate	γ_t		UN Population Prospects, projected changes in 20 year old cohort, 1950 to 2095
Retirement Age	N^r	65	Legal retirement age, United States
Safe dividend	d	1	normalization
Safe asset supply parameter, soe	λ^e	148.03	Calibrated in Section 3.1.3
Risky asset supply parameter, soe	$\tilde{\lambda}^e$	46.72	Calibrated as in Section 3.1.3
Safe asset supply parameter, two region	λ		Calibrated as in Section 3.1.3
Risky asset supply parameter, two region	$\tilde{\lambda}$		Calibrated as in Section 3.1.3

probabilities are interpolated using cubic spline interpolation, and then extrapolated for remaining ages up to 100 years. The maximum age of 100 removes a growing number of centenarians observed in the age distribution from our model. This will understate the total population share of the older cohorts in our results.

3.1.2 Non-Financial Income

Labor income plays a key role asset accumulation and allocation over the life-cycle. In a first step, we use parameters describing the deterministic shape of labor income over the life-cycle as estimated in Cocco et al. (2005), which has become the standard

approach in the literature.¹⁰ These estimates determine the shape of the deterministic component of labor income, or G_n in Eq. (12). The labor income process over the life-cycle is estimated for three different educational groups as a third order polynomial of age and other control variables using data from the Panel Study of Income Dynamics (PSID).¹¹ A more detailed account of the estimation equations as well as estimates are given in Section 6.2 in the Appendix. Labor and pension income is subject to a zero income shock in each period. Agents have labor or pension income of zero with probability of p . We set this equal to a value of 1 percent, common value used in the literature and taken from Carroll (1992).

Agents retire at the end of age 65, receiving the first pensions at age 66. This number is slightly lower than the normal retirement age in the US and many European countries, but it roughly matches the average effective retirement age currently prevailing as well as projections from the OECD until 2060 (OECD, 2015). Considering the discourse on pension reforms that are currently under discussion on both sides of the Atlantic, it seems reasonable not to expect large increases in retirement age over the course of our simulation period. Our baseline model keeps the retirement age fixed throughout the demographic transition, however, in a later extension we may also consider the impact on asset accumulation and allocation on changing retirement age.

As for the determination of pension income, we consider a constant replacement rate of final last period's labor income. For the constant replacement rate variant, we use the replacement rate estimated by Cocco et al. (2005) from the PSID for high school graduates or, $\phi = 0.68212$. Undoubtedly private pension schemes play an important role in the U.S., with 47% of working-aged citizens enrolled in a private pension plan (OECD, 2015). We abstract from such pension schemes, because these reflect savings decisions of the agents, which we model as part of the portfolio choice problem.

¹⁰In a later iteration, we will estimate the process for the EU-15.

¹¹Their definition of labor income includes unemployment compensation and other social assistance to correctly assess the risk connected to the labor income process.

3.1.3 Parameters on Asset Supply

The parameters that scale asset supply to a proportion of population are estimated as to allow asset markets to clear. Cross-border financial flows between the US and EU-15 are very small (especially by today's standards) until the 1990s. Thus, we calibrate the parameters λ and $\tilde{\lambda}$ as those which clear the asset markets under autarky with demand generated by our model and supply to the population size generated by demographic data over the period 1950 to 1990.

In the small open economy simulation, until 1990 observed asset returns over that period correspond to autarky returns. After opening, we maintain fixed parameters and allow the prices to reflect differing asset demand as a result of demographic transition.

The parameters that result are per capita asset demands as of financial account opening:

$$\lambda^j = \frac{B_t^j}{L_t^j} \quad \tilde{\lambda}^j = \frac{S_t^j}{L_t^j},$$

where B_t^j and S_t^j are the aggregate asset demands simulated by our model and L_t^j are regional aggregate population sizes that result directly from the demographics input into our simulation. The scaling parameters essentially fix the level of per capita assets to the level in 1990.

In the two region simulation, asset markets clear globally. Our interim approach of taking observed rates of return in the US as equilibrium interest rates, leaves the possibility that markets will not clear at observed interest rates. In order to ensure that markets clear, we allow the scaling parameters on a global level to adjust.

From the market clearing condition Eq.(17), we calculate the global scaling parameters that will clear the global asset markets after opening,

$$\lambda_t = \frac{B_t}{L_t}$$

$$\tilde{\lambda}_t = \frac{S_t}{L_t},$$

where B_t and S_t are asset demands over the two regions at observed interest rates and L_t is population over the two regions.

4 Results

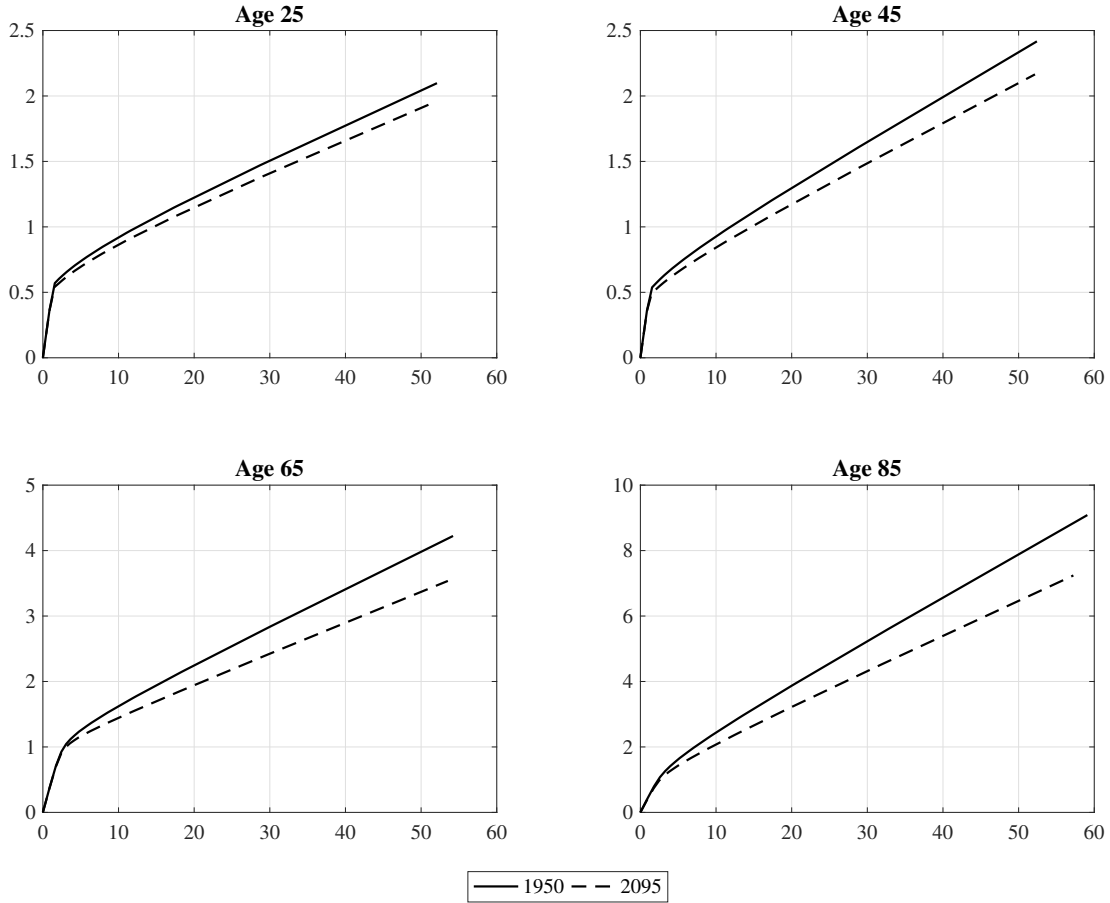
We present the solution to our model for the demographic transition, by showing the solutions for pre- and post- transition steady states. Then, we move on to present the results of our simulation exercises. We highlight, in particular, the effects of demographic transition on individual behavior, a *life-cycle channel*, and through changing relative size of age cohorts over time, a *distribution channel*. In a further iteration of the paper, we will also highlight the role of the *valuation channel* that operates via changing asset prices.

4.1 Policy Functions

We solve the model for policy functions of consumption $c_{i,n,t}$ and $b_{i,n,t}$ and $s_{i,n,t}$ for all ages N^b to N^d for the state variable: cash on hand, $x_{i,n,t}$. Figure 3 depicts consumption and the risky share for a specific age for 1950 and 2095 steady states for varying cash on hand.

For a given financial wealth, or cash on hand, individuals reduce their consumption post demographic transition. This change is influenced by increased expected longevity of agents. Absent changes in the retirement age and pension provision, aging will increase asset accumulation over the life-cycle to provide for a longer retirement period. We also assume a time-invariant deterministic labor income shape. The effect will be reduced if agents expect to earn more over the life-cycle.

Figure 3: Policy functions, consumption

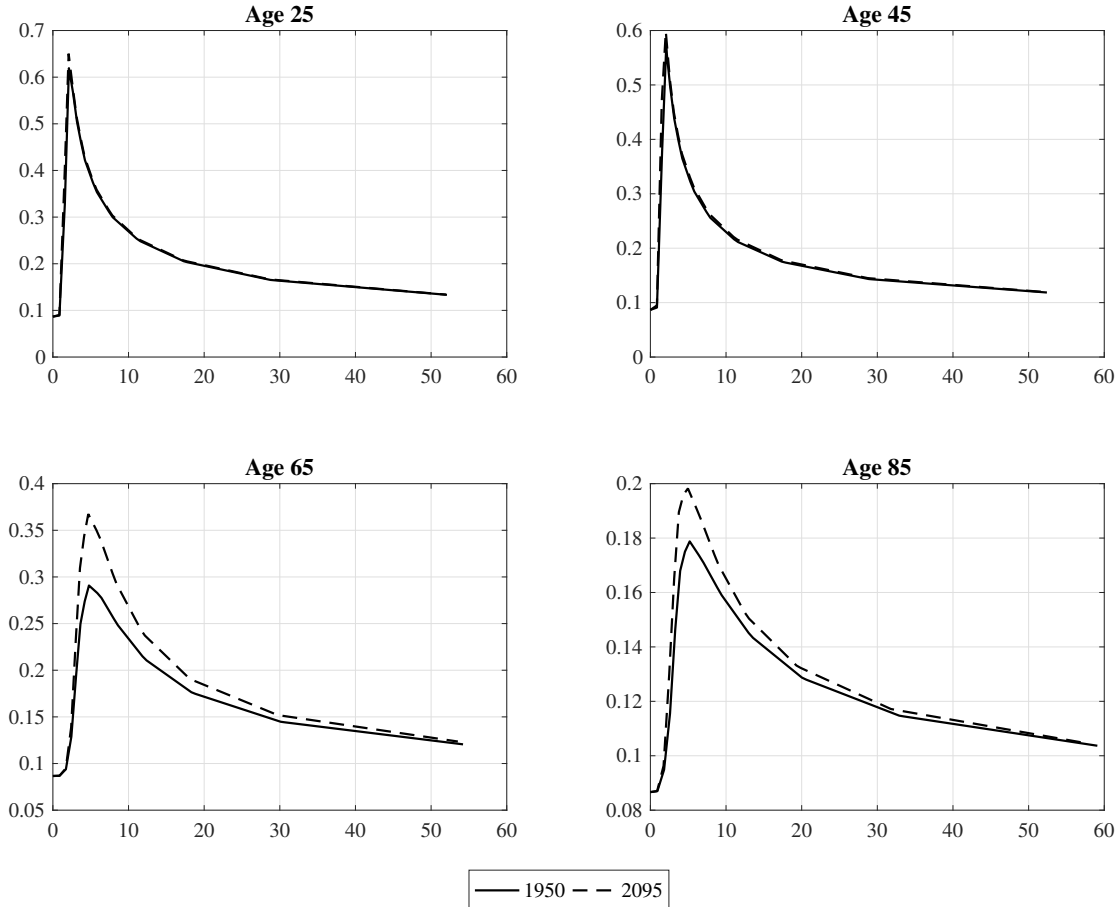


The four panels show the level of normalized consumption for varying levels of normalized financial wealth or cash on hand for ages 25, 45, 65, and 85, respectively. The solid black line depicts the solution of the model for the 1950 demographic data. The dashed black line shows the model solutions for 2095 demographic projections. In order to reduce the number of state variables, we solve the model for normalized variables by P_n , the deterministic component of labor income.

The effect of life-cycle savings from demographic change will also impact the share of savings allocated to stocks. Figure 4 depicts our model's solution for the share of savings in each period allocated to risky assets.

As is well known in the portfolio choice literature, the risky share of assets over the life-cycle is largely driven by the ratio of human wealth to total financial wealth

Figure 4: Policy functions, risky share



The four panels show the risky share of savings for varying levels of normalized financial wealth or cash on hand for ages 25, 45, 65, and 85, respectively. The solid black line depicts the solution of the model for the 1950 demographic data. The dashed black line shows the model solution for 2095 demographic projections.

(Merton, 1971; Guiso et al., 2002; Cocco et al., 2005; Fagereng et al., 2017). The intuition is that human wealth, or the present discounted value of labor income over the life-cycle, acts as a substitute for the safe asset holdings as long as the variance of innovations to labor income are not too high and its innovations are not highly correlated with innovations to excess return on risky assets.¹²

¹²Cocco et al. (2005) show that at empirically relevant correlations between the innovations to risk

The introduction of a zero income shock reduce the risky share for lower levels of wealth. Agents, however, with a relatively high level of prudence, will accumulate safe assets fairly quickly in order to avoid a very low level of consumption in the case of successive zero income draws.

There is a noticeable difference between the risky share of financial wealth with demographic change, particularly for retirees. As will become clear in the simulation results in the following section, this is due to a larger wealth accumulation after retirement with increased longevity. As human wealth for retirees is small, changes in total financial wealth will have large impacts on the risky share.

4.2 A Small Open Economy

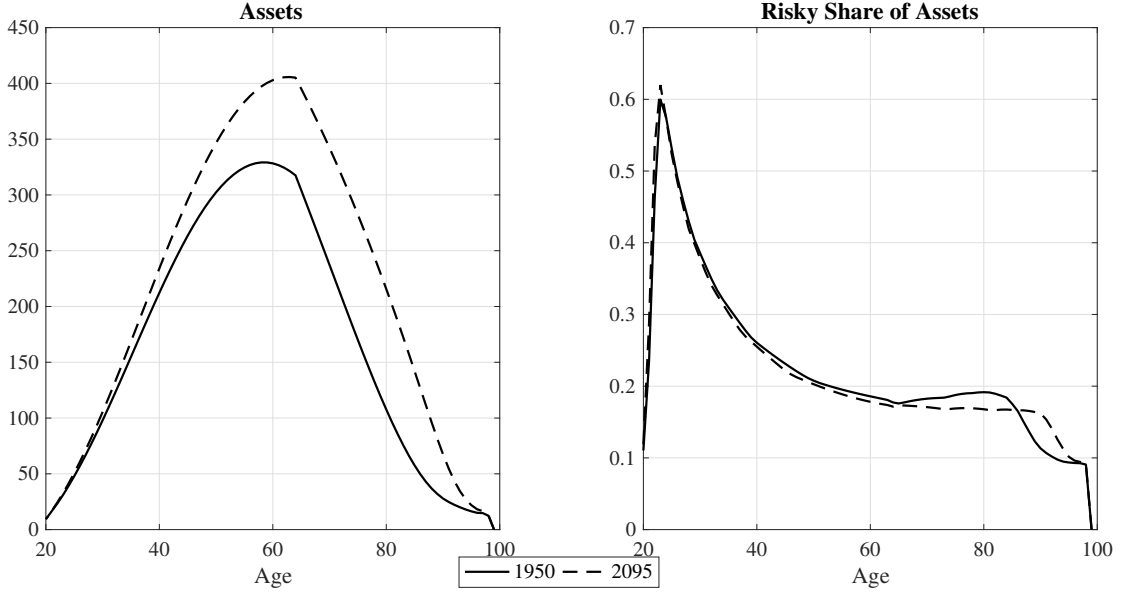
Our small open economy is closed at the beginning at the pre-transition steady state. The demographic transition begins and the economy is aging as a closed economy until 1990, at which point it opens to the rest of the world with a stationary demographic structure. We follow an agent born in the pre-transition steady state and one born in the post-transition steady state over her life-cycle.

Figure 5, shows financial assets and the share of assets invested in risky asset over the life-cycle for pre- and post-demographic transition. Asset accumulation over the life-cycle increases by 18 percent from 1950 to 2095 at the peak, just before retirement. Individuals in 1950 run down assets less quickly as probabilities of survival increase in old age.

The risky share is largely unchanged for the first 10 years of life. Then, the risky share is slightly lower for ages 30 to around 90, increasing above the 1950 share around age 90. This pattern is driven by demographic induced changes to the human wealth to financial wealth ratio.

premium and the persistent innovation to labor income, the shape the policy function remains high (essentially 1) at low levels of wealth. Only with a correlation of the innovation of around 0.4, does the risky share over the life-cycle begin to have a hump-shape.

Figure 5: Assets and Risky Share over the Life-cycle



The left panel shows financial assets from Eq. (9) for an individual over the life-cycle in 1950 in the solid black line and in 2095 in the dashed black line. The right panel shows the risky share of financial assets $\omega_{i,n,t}$ from Eq. (10) for an individual over the life-cycle in 1950 in the solid black line and in 2095 in the dashed black line.

Total human wealth of an individual at the beginning of life is:

$$H_{i,N^b,t} = \mathbb{E} \sum_{n=N^b}^{N^r} \frac{y_{i,n,t+n-N^b}}{R^{n-N^b}} + \sum_{n=N^r+1}^{N^d} \frac{\tilde{y}_{i,n,t+n-N^b}}{R^{n-N^b}}, \quad (21)$$

where \mathbb{E} is mathematical expectations operator and R is the safe asset gross return. Human wealth includes labor income and pension income and evolves dynamically according to,

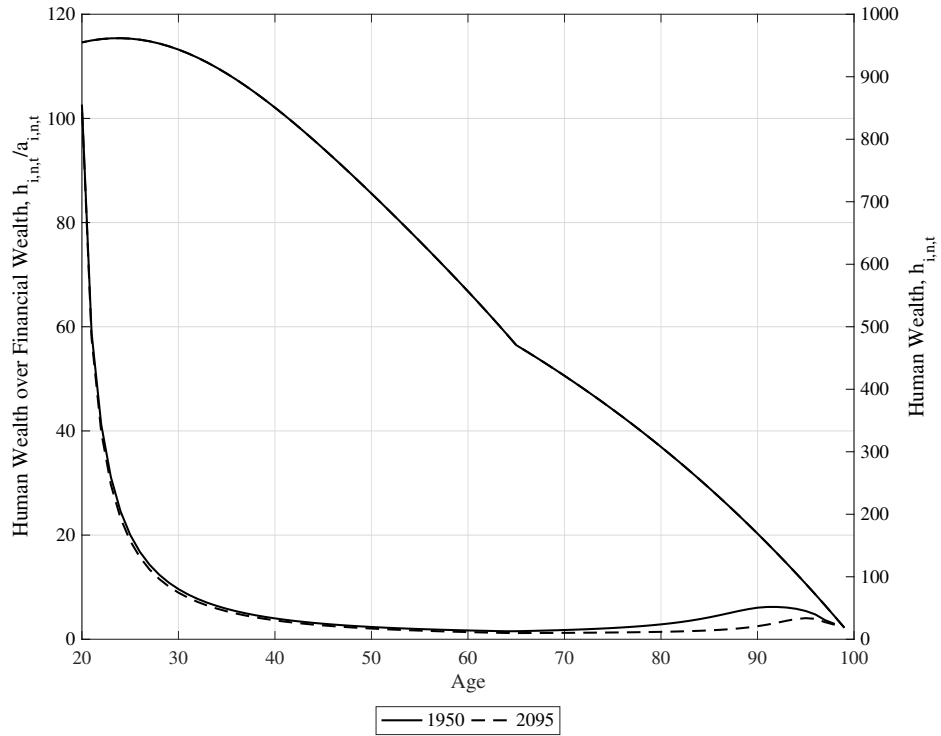
$$H_{i,n,t} = R(H_{i,n-1,t-1} - y_{i,n,t}) \quad \text{for } N^b < n \leq N^r \quad (22)$$

$$H_{i,n,t} = R(H_{i,n-1,t-1} - \tilde{y}_{i,n,t}) \quad \text{for } N^r < n < N^d. \quad (23)$$

Total financial wealth is defined at the individual level in Eq. (9). When the ratio

$H_{i,n,t}/a_{i,n,t}$ is large, it acts as a large stock of safe assets. There should be a strong positive correlation between the human wealth to financial wealth ratio and the risky share over the life-cycle.

Figure 6: Human Wealth over Financial Wealth over the Life-cycle



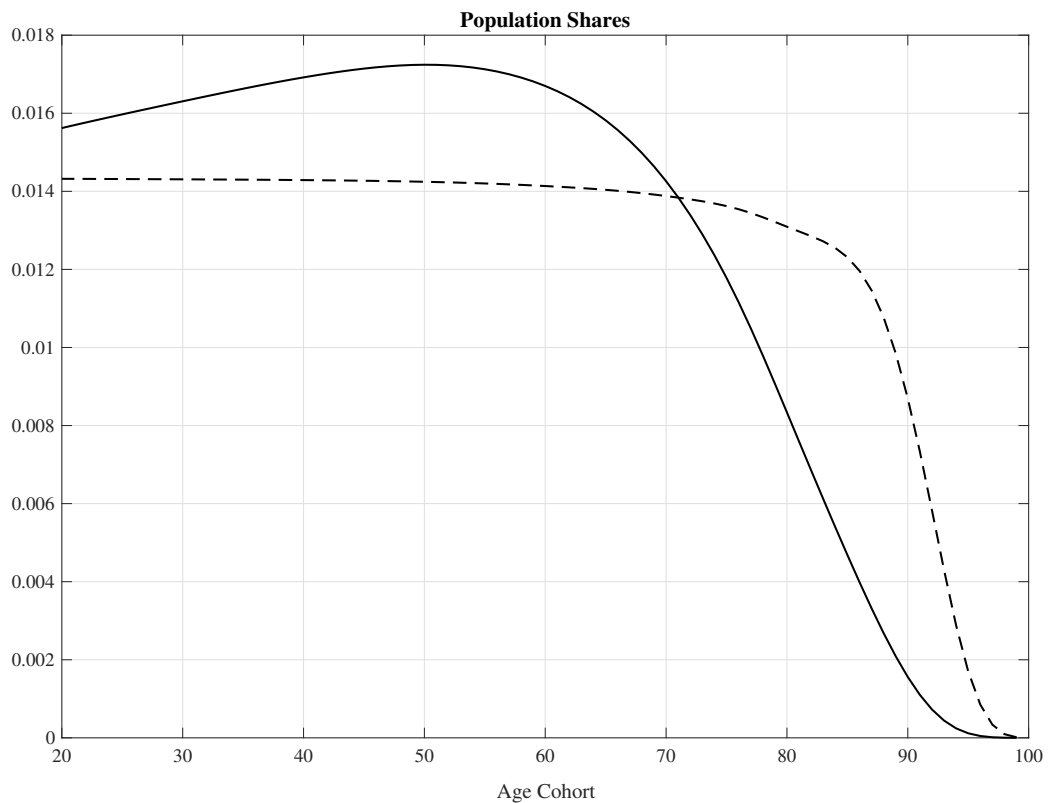
The solid black line depicts human wealth in Eq. (22) using the right axis for both 1950 and 2095. The solid black line using the left axis depicts human wealth to financial asset ratio in 1950. The dashed black line shows the ratio for 2095 demographic projections.

Figure 6 shows the ratio $H_{i,n,t}/a_{i,n,t}$ and human wealth over the life-cycle for pre- and post-demographic transition. It is clear that there is a strong, positive correlation between the ratio and risky share over the life-cycle. There is a relatively larger difference in the ratio between 1950 and 2095 towards the end of the life-cycle, when human wealth becomes very low. As the ratio falls more after age 85 for 2095 demography, this pushes up the risky share towards end of life, explaining the behavior of the risky

share towards the end of life in Figure 5.

The demographic transition not only effects individuals over the life-cycle, but will influence the relative size of cohorts. Figure 7 shows the age distribution of the population for the pre- and post- demographic transition. There is a clear shift of population mass after the demographic transition to the over 70 age cohorts. This will have separate effects on the small open economy's demand for safe and risky assets.

Figure 7: Age Distribution of Population



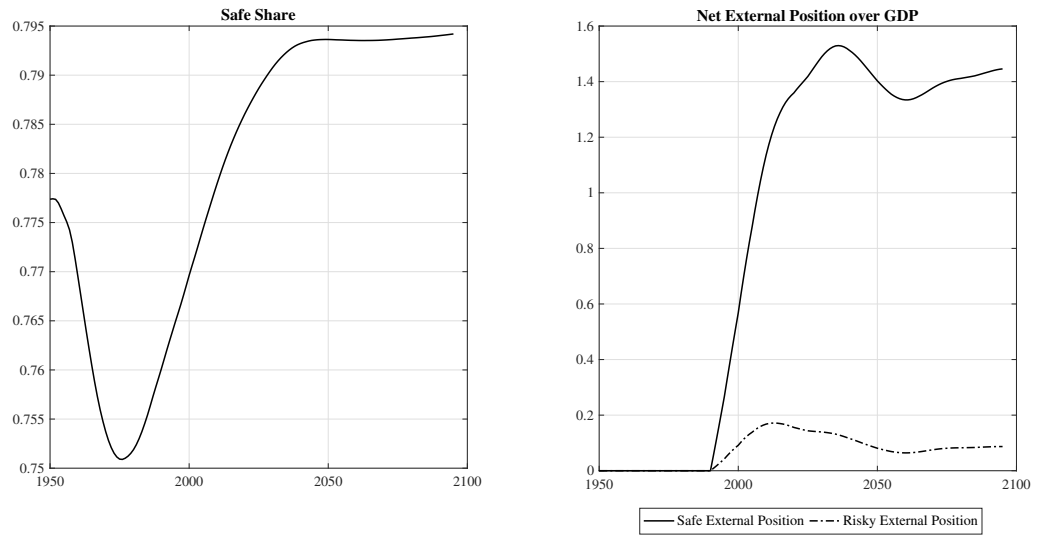
The solid black line shows age distribution for 1950 demographic data. The dashed black line shows age distribution for 2095 demographic projections.

The combined life-cycle and age distribution effects will lead to interesting dynamics over the demographic transition. Increasing population shares of older cohorts should increase the share of safe assets in the economy. However, older cohorts will

be dis-saving more relative to younger cohorts. Furthermore, increasing asset accumulation over the life-cycle will increase asset holdings in the economy.

We examine these combined effects by running a simulation of the demographic transition and calculating the implied safe share of total asset demand and external positions resulting from asset demand in excess of local supply. These are asset demands that prevail at given world interest rates.

Figure 8: Net External Asset Position over GDP, 1950-2095



The left panel shows the aggregate safe share of assets demanded in the small open economy calculated as $\sum_{n=N^b}^{N^d} \frac{b_{n,t}}{a_{n,t}}$. The right panel shows the net external positions in safe and risky assets over the aggregate endowment. The solid black line shows safe external positions and the dashed black line shows the risky external positions.

Figure 8 shows the aggregate share of safe assets in the small open economy and the net external positions over the aggregate endowment. The aggregate safe share of assets is:

$$\Omega_t = \sum_{n=N^b}^{N^d} \frac{b_{n,t}}{a_{n,t}}.$$

In our simulation, the safe share initially decreases until around 1980 and increases thereafter.¹³ The share of safe assets is very high, rising from around 0.75 in 1980 to just under 0.795 after 2050. The safe share of assets stabilized thereafter towards the end of demographic transition.

Visible in the right panel of Figure 8, net external positions over GDP are calculated as:

$$NFB_t = \frac{B_t^e - \mathbb{B}_t^e}{X_t^e}$$

$$NFS_t = \frac{S_t^e - \mathbb{S}_t^e}{X_t^e},$$

where $X_t^e = (R - 1)B_t^e + (\tilde{R} - 1)S_t^e + Y_t^e$ is the aggregate endowment in region e .

Upon opening in 1990, large external positions emerge in safe assets, projecting to rise to over 150 percent of GDP around 2030 and settling to around 140 percent after 2050. Smaller net external asset positions in risky assets also emerge from rising to 20 percent of GDP around 2010 and falling to 10 percent throughout the transition. Aging increases the demand for safe and risky assets in excess of local asset supply. As aging progresses, a settling occurs in external positions that reflects a larger share of the population dis-saving during retirement.

So far, we have identified two separate channels from demographic transition to net external positions in the small open economy. In a next step we measure the relative contribution to observed aggregate asset accumulation of each channel.

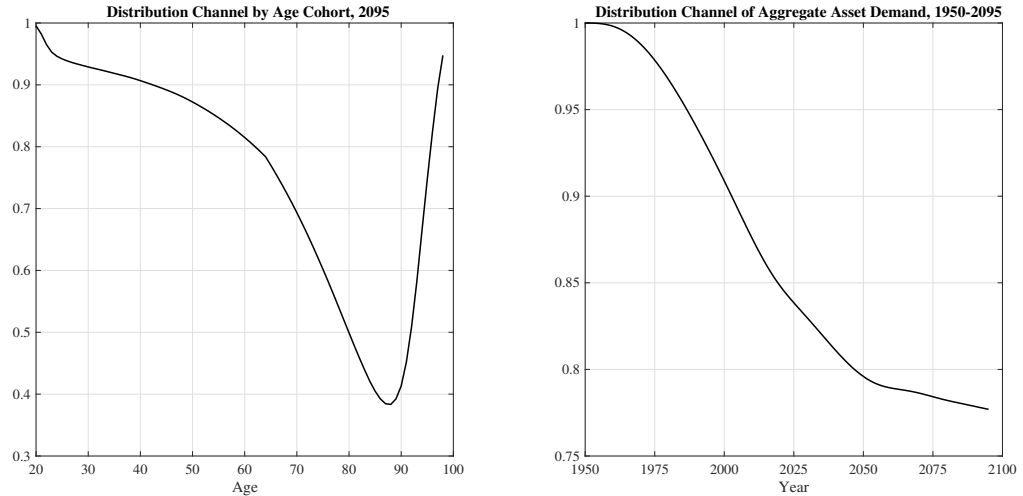
The *life-cycle channel* operates via individual behavior and takes into account time varying expected survival probabilities. The *distribution channel* operates via time varying birth rates and survival probabilities that impact the age distribution of the population over time.

In order to isolate the effects of the channels, we perform a counterfactual experiment. We solve the model, holding demographic structure constant at the 1950

¹³The safe share initially declines as a result of a drop in the dependency ratio after increased birth cohorts in Europe after after the end of WWII.

pre-transition steady state and allow the age distribution to reflect time varying birth rates and survival probabilities. We calculate the ratio of aggregate asset demand in this experiment to our baseline model aggregate asset demand.

Figure 9: The Distribution Channel



The left panel shows the distribution channel across cohorts in the post-demographic steady state. The right panel shows size of the distribution channel over time.

Figure 9 shows the ratio of asset demand without the life-cycle channel to our baseline model asset demand for the cross-sectional asset holdings by age cohort in 2095 and on an aggregate basis for the demographic transition. It is clear that the majority of our results are explained by the *distribution channel*. In the developed regions we study, the majority of demographic change on longevity takes place in older cohorts. Therefore, the distribution channel becomes less important for older portions of the age distribution. In other words, the life-cycle channel explains 50 percent of aggregate asset demand of the elderly, cohorts over age 80.

Over the demographic transition, the distribution channel explains the majority of aggregate asset demand, falling to just under 80 percent after 2050. However, as the demographic transition progresses, survival probabilities increase more and more for the elderly, and the life-cycle channel gains influence.

In this small open economy, with rich demographic structure, we are able to replicate the emergence of very large safe asset external positions of around 120 percent of GDP. This is much larger than our stylized facts for external position for the EU-15 over GDP. However, in a two-region world, it will be *relative* demographic trends that determine cross-border asset trades and resulting external positions.

4.3 Two Region World

We perform a two region simulation, parameterizing our model to demographic data in US and EU-15. We assume the two regions are closed until 1990: up until this point local asset demand is satiated by local asset supply.

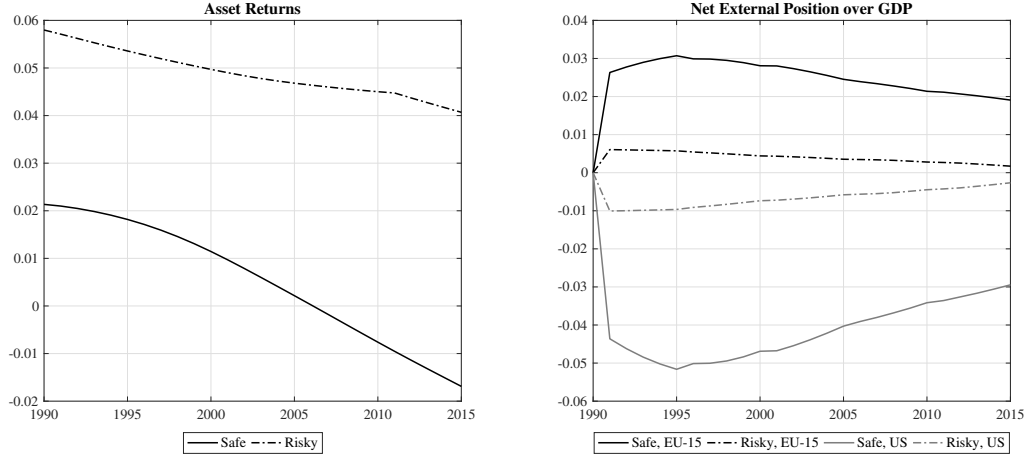
We take observed return rates in the US from 1990 to 2015 as a measure of world return rates for safe and risky assets. We solve the model for two regions taking these world interest rates as given and evaluate the net external positions that arise over this time period.

Observed rates of return for safe and risky assets and external positions over GDP in both regions are shown in Figure (10).

Safe asset returns are falling over the period 1990 to 2015, a fact well documented in the literature (Bernanke, 2005; Caballero et al., 2008). The excess return on risky asset taken here is slightly increasing over the period. This pattern of returns is consistent with increasing relative demand for safe assets due to demographic change in our model.

Given world returns, considering the different demographic transitions over the period 1990 to 2015, our model predicts the emergence of a safe external asset position over GDP rising to around 3 percent of GDP in EU-15 vis-à-vis the US. This reflects an economy's increased demand relative of safe assets due to its more advanced aging. Autarky returns are depressed in the region and successive capital flows to the younger region. The net external asset position is much smaller in magnitude, rising to just 0.5 percent of GDP, reflecting a lower relative demand for risky assets. The transition over this period is interesting. Note in Figure 1, the US dependency ratio flattens out

Figure 10: The Distribution Channel



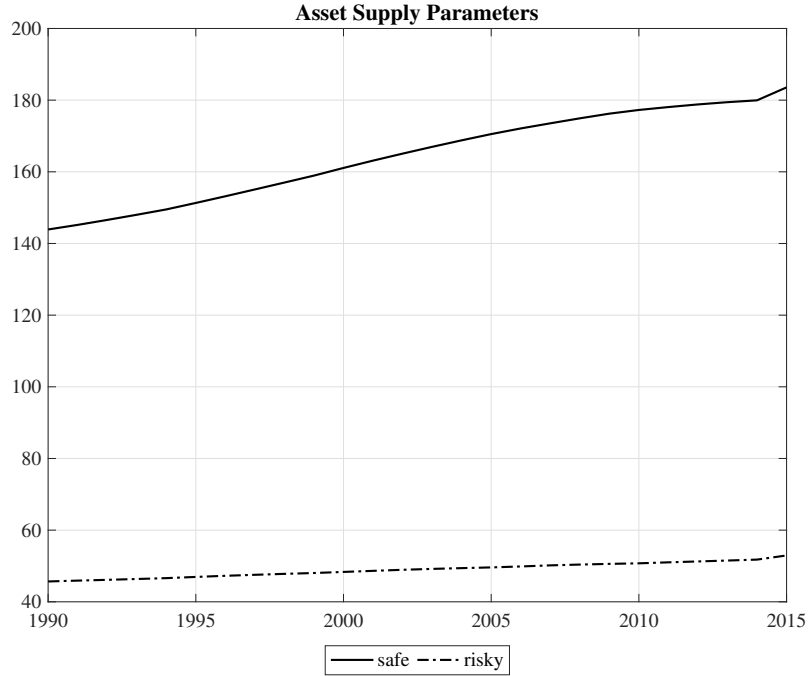
The left panel depicts real rates of returns for safe (3 month treasury bills) and risky assets (S&P500 real returns) from 1990 to 2015. The right panel depicts the safe and risky bilateral external asset positions over GDP from Eq. (18). The grey and black solid lines show the safe external positions over GDP for the US and EU-15, respectively. The dashed lines show the risky external positions over GDP.

starting in 1990 and begins to rise around 2005. We observe larger initial external positions in EU-15 early over this simulation period, that decline to around 2 percent of GDP in 2015.

We noted before, that empirical interest rates over the simulation period may not coincide to market clearing rates of return in a model where regions differ only by demographics. We show that the observed rates of return are falling faster than our model would imply.

Figure 11 shows the scaling parameters for safe assets λ_t , and risky assets $\tilde{\lambda}$ that clear global asset markets for given rates of return in Figure fig:tworegion`results. Particularly for safe assets, the scaling parameter is rises from 140 to over 180 in 2015, indicating that the global supply of safe assets would need to increase by almost 30 percent over this period in order to keep up with increased safe asset demand. The scaling parameter for risky asset rises much less from around 45 in 1990 to 53 in 2015.

Figure 11: Supply Scaling Parameters, 1990-2015



The solid line depicts the scaling parameter for world safe asset supply, λ_r . The dashed black line shows the scaling parameter for world risky assets, $\tilde{\lambda}$.

There are very important factors influencing global safe asset demand returns over this period, which we do not capture in our stylized model of the world; not the least of which is the financial crisis starting in 2008. An important next step in our research will be to fully endogenize asset prices, in order to determine how much relative demographic transition can explain dynamics in rates of returns.

5 Conclusion

We build a two region, general equilibrium model with OLG structure and endogenous portfolio choice in order to ascertain the impact of the demographic transition

(increasing longevity and falling birth rates) on the composition of international portfolios. When a country ages faster than the rest of the world, large external positions emerge in both safe and risky assets. The magnitude of the safe position is over 10 times larger than the risky position. As the demographic transition progresses, the positions are slightly reduced as a larger share of population begins dis-saving during retirement.

These results are driven by an increasing aggregate share of safe asset demanded in the economy, which stabilizes towards the end of the demographic transition. We identify and quantify two channels that contribute: a *life-cycle channel*, which operates via increased longevity and impact the individuals saving and portfolio allocation behavior; and *distribution channel* that operates via birth rates and longevity rates changing the age distribution of cohorts, which are at different life-cycle stages.

We find that cross-sectionally and longitudinally, the *distribution channel* is relatively more important in explaining our model's predictions for aggregate asset demand, falling in relative important for cohorts most impacted by changing longevity and as the demographic transition progresses.

In a two region simulation, using for observed interest rates over a period 1990-2015, we show that differences in relative demographic transitions between the EU-15 and US are powerful predictors of composition of the regions' bilateral portfolios. Our model can explain around two-thirds of the bilateral net safe asset position in our stylized facts. Our model predicts the emergence of a net risky asset position in the region with a faster demographic transition. This is inconsistent with observed net risky liability position for the EU-15, for example. We note several points that may contribute to this finding.

First, bilateral positions will reflect behavior of all sectors of the economy, not just on the households sector, which we explain in our model. It could be that household sector is holding a net external asset positions in risky (equity) assets in EU-15 as well, but these flows are dominated by financial sector or corporate sector positions. Indeed, our model consistently predicts a much smaller magnitude of cross-border holdings of equity. In a next step, we will quantify the relative size of household sector

contributing to these bilateral positions by using financial account data, which is more disaggregated by sector.

Second, the bilateral portfolio data is only disaggregated by type of instrument, debt or equity, not by risk profiles of these instruments. In our stylized facts we have addressed this partially by reclassifying cross-border corporate debt holdings as risky assets. This adjustment reduces the net asset position in equity for the EU-15 slightly, but does not reverse the sign.

This project is very much a work in progress. So far, we have not quantified the importance of a third channel: *the valuation channel*. This operates via world rates of return influenced by changing international demand for safe and risky assets. This channel will be vital in order to provide quantitative predictions of the total importance of demographic trends on the composition of portfolios. Furthermore, with endogenous prices, we will also be able to quantify the valuation effects of demographic transition on external portfolios, which have been gaining importance in recent years.

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6 Appendix

6.1 Data

Table 3: Definition of safe and risky assets from CPIS

Safe assets	Risky assets
short- and long-term debt instruments, e.g. bonds, debentures, treasury bills, negotiable certificates of deposit, commercial papers, bankers' acceptances (CPIS), excluding corporate bonds (Flow of Funds)	equity and investment fund shares, e.g. shares, stocks, participations or similar documents (CPIS), plus corporate bonds (Flow of Funds)

The share of corporate bonds in the total external bond positions is calculated using Flow of Funds holdings of US corporate bonds by the RoW as share of total RoW holdings of US bonds. This relies on the assumption that the RoW share of corporate bonds is highly correlated with the EU-15 share of corporate bonds. Given that the EU-15 makes for the majority of total US bond holdings, this assumption is straightforward. The CPIS reports summary data for all sectors of the economy. This also includes reserve holdings by the central bank. Since both the US Dollar and the Euro are reserve currencies, holdings of the respective other reserve currency are, however, small: less than 1 percent in the US and around 2-3 percent in Eurozone Germany and France.

6.2 Labor Income Process

The log of the deterministic component of labor income in Eq. (12) is estimated as by regressing individual labor income for high school graduates on dummies for age and other individual characteristics and, as follows,

$$\ln(G_n) = 0.6004 + 0.1682 \times age - 0.0323 \times \frac{age^2}{10} + 0.0020 \times \frac{age^3}{100}. \quad (24)$$

From Cocco et al. (2005), the deterministic labor income component is estimated using PSID data and controlling for age and other characteristics, such as education, marital status and household size.

Table 4: Definition of safe and risky assets for various data sources

Source	Risky assets	Safe assets
SCF, HFCS	Stocks; stock brokerage accounts; mortgage-backed bonds; foreign and corporate bonds; mutual funds, trusts and annuities invested in stocks	checking and savings accounts; money market accounts; certificates of deposit; cash value of life insurance; government or state bonds; mutual funds, trusts and annuities invested in bonds
Financial Accounts (<i>Federal Reserve</i>)	corporate equities, mutual funds shares, corporate bonds, (proprietor's equity)	currency and deposits, commercial paper; money-market funds; treasury and agency debt; state bonds; bond funds; cash value of life insurance
Financial Accounts (<i>Eurostat</i>)	total equity, mutual funds, (unlisted equity)	money and deposits; debt securities; life-insurance

Sources: CPIS from the IMF, US flow of funds, Federal Reserve, Eurostat, country financial accounts and Survey of consumer finances and Household Finance and Consumption Survey.

6.2.1 Labor income moments

The inclusion of the unemployment shock will embody our labor income process with different stochastic properties than one including a transitory and permanent shock. We examine this below.

Our single *i.i.d.* innovation to labor income $\zeta_{i,n,t}$ has mean $(1 - p)$ and variance $p(1 - p)$. This process implies a mean income:

$$\mathbb{E}y_{i,n,t} = P_n(1 - p),$$

The deterministic component of labor income is reduced by the probability of the zero income shock.

The variance of the income process is:

$$\mathbb{V}(y_{i,n,t}^2) = \mathbf{G}^2 P_{N^b}^2 (1 - p)p,$$

where $\mathbf{G} = \prod_{j=N^b}^{n-1} G_j$ is the history of deterministic component of labor income. The variance follows this term and is hump shaped over the life-cycle.

We consider an income process more standard in the literature, with transitory *i.i.d.* innovation $\ln(\theta_{i,n,t}) \sim \mathcal{N}(0, \sigma_\theta^2)$ and persistent innovation $\ln(\eta_{i,n,t}) \sim \mathcal{N}(0, \sigma_\eta^2)$:

$$\begin{aligned} y_{i,n,t} &= P_n \theta_{i,n,t} \\ P_n &= P_{n-1} G_n \eta_{i,n,t}. \end{aligned}$$

The mean of the income process is:

$$\mathbb{E}(y_{i,n,t}) = \mathbf{G} P_{N^b},$$

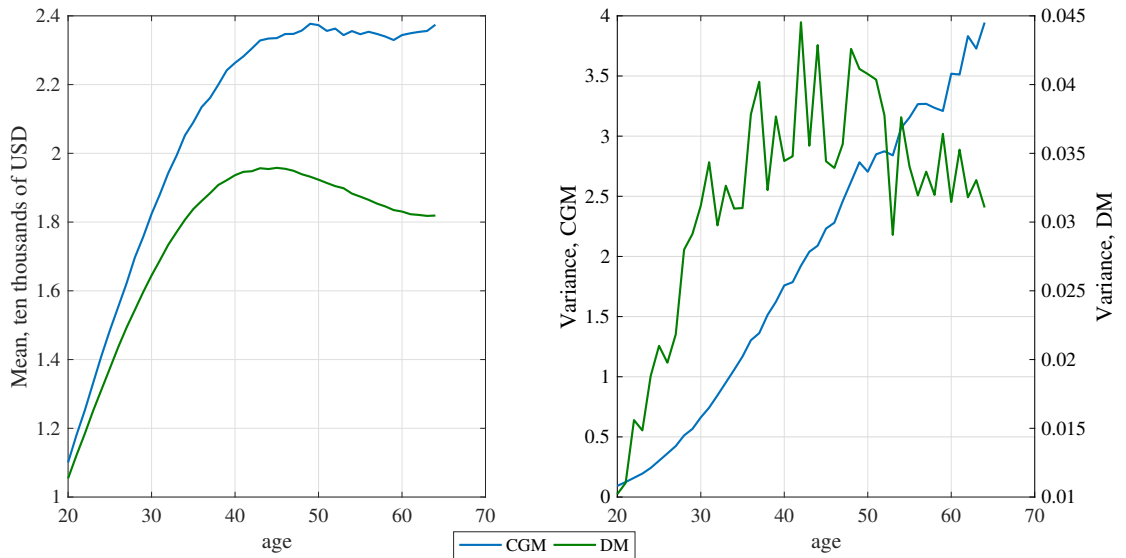
The expectation of the income process with no zero income shock is higher.

The variance of the income process is,

$$\mathbb{V}(y_{i,n,t}) = \mathbb{G}_n^2 P_{N^b}^2 ((\sigma_\eta^2 + 1)^t) \sigma_\theta^2.$$

Over the life-cycle, the variance is increasing much more than with our zero income shock due to the presence of highly persistent innovations. This is an important factor to account for within cohort dispersion of income. Furthermore, this will result in a much larger unconditional variance of the standard labor income process.

Figure 12: Moments of Labor Income Processes



The left panel depicts the unconditional mean of the zero income shock process *DM* in green and in Cocco et al. (2005) *CGM* in blue. The right panel shows the unconditional variance of both processes over working years of the life-cycle.

Figure 12 shows results of a Monte-carlo simulation exercise of 10,000 draws without replacement. We simulate the mean and variance of our labor income process *DM* and that in Cocco et al. (2005) *CGM*. These results reiterate the analytical derivations of the first two moments above. On the one hand, agents in our model world are receiving lower expected value of labor income, which will reduce savings over the life-cycle. On the other, we are making this labor income unconditionally less risky.

In addition to the moment differences, intuition from permanent income hypothesis tells us that individuals will react more to permanent shocks than transitory income shocks. Individuals will save more if negative income shocks can be very persistent and will consume more (save less) out of a positive persistent shock. Therefore, we would expect there to be differences in the level of savings over the life-cycle in our model relative to those models standard in the life-cycle literature. As a robustness check, in a further iteration of this paper, we will test our results for inclusion of the labor income process with permanent (persistent) and transitory innovations, in order to determine the impact on the level of savings, and thus on the level of aggregate asset demands. We note that since there are no differences between regions in the labor income processes, we expect there to be level effects on external positions, but not effects on the most important qualitative predictions of this paper.

6.2.2 Expenditure Shocks in Retirement

We recast the retirees budget constraint Eq. (8) with the inclusion of expenditure shocks. We reinterpret the zero pension shock as an *i.i.d.* medical expenditure shock, $m_{i,n,t}$ of the form

$$m_{i,n,t} = \begin{cases} -\phi P_{N^r} & \text{with probability } p \\ 0 & \text{with probability } 1 - p. \end{cases} .$$

This generates the budget constraint for retirees:

$$c_{i,n,t} + Q_t b_{i,n,t} + \tilde{Q}_t s_{i,n,t} = \overbrace{(Q_t + d_t)b_{i,n-1,t-1} + (\tilde{Q}_t + \tilde{d}_t)s_{i,n-1,t-1} + \tilde{y}_{i,n,t} - m_{i,n,t}}^{x_{i,n,t}} \quad \text{if } n > N^r, (25)$$

and pension income is zero when the medical expenditure shocks are non-zero with probability p .