After the Tide:

Commodity Currencies and Global Trade*

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October 24, 2016

Abstract

The decade prior to the Great Recession saw a boom in global trade and rising transportation costs. High-yielding commodity exporters' currencies appreciated, boosting carry trade profits. The Global Recession sharply reversed these trends. We interpret these facts with a two-country general equilibrium model that features specialization in production and endogenous fluctuations in trade costs. Slow adjustment in the shipping sector generates boom-bust cycles in freight rates and, as a consequence, in currency risk premia. We validate these predictions using global shipping data. Our calibrated model explains about 57 percent of the narrowing of interest rate differentials post-crisis.

Keywords: shipping, trade costs, carry trade, currency risk premia, exchange rates, international risk sharing, commodity trade
JEL codes: G15, G12, F31

*We benefitted from comments and suggestions by Andy Abel, George Alessandria (the editor), Mathieu Taschereau-Dumouchel, Doireann Fitzgerald (the discussant), Ivan Shaliastovich, and conference participants in the Carnegie-Rochester-NYU Conference on Public Policy, for which this paper was prepared.

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1 Introduction

The decade prior to the Great Recession saw a boom in global trade, including a rapid rise in commodity prices, trade volumes, and, consequently, in the cost of transporting goods around the world. At the same time currencies of commodity-exporting currencies appreciated, boosting the carry trade profits in foreign exchange markets (commodity currencies typically earn higher interest rates, making them attractive to investors).[1] The onset of the Global Recession led to a sharp reversal in all of these trends, with only a weak recovery subsequently. We interpret these facts through the lens of an international asset pricing model, focusing on two groups of countries whose currencies represent the two sides of a typical carry trade strategy. The first group consists of developed countries that are major exporters of basic commodities (Australia, Canada, New Zealand, and Norway) - the typical "investment" currencies.[2] The second group consists of developed economies that primarily export complex manufactured goods (the Euro zone, Japan, Sweden, and Switzerland), typically seen as providing "funding" currencies for exchange rate speculation due to their historically low interest rates.

Building on Ready, Roussanov, and Ward (2016), we develop a two-country general equilibrium model that features complete financial markets and specialization in trade. The key friction that we emphasize in this paper is the slow adjustment of capacity in the shipping sector, which results in highly variable costs of international trade (or at least their component that is attributed to shipping). The model can jointly account for the dynamic behavior of real exchange rates and interest rates (and therefore carry trade returns), commodity prices, and shipping costs. All of these series exhibit a sharp drop during the crisis, followed by very slow recovery, with shipping costs being the most sluggish. Our quantitative model implies that expected returns on the commodity currency carry trade fall by about

[1] Early literature on the uncovered interest rate parity puzzle focused on the fact that movements in bilateral exchange rates over time do not offset differences in interest rates - e.g. Hansen and Hodrick (1980), Fama (1984). In contrast, recent literature shows that the bulk of carry trade profitability stems from persistent differences in real interest rates across countries - Lustig, Roussanov, and Verdelhan (2011), Ready, Roussanov, and Ward (2016), Hassan and Mano (2014).

[2] These currencies have been of particular interest in the international economics literature that studies the connection of exchange rates with underlying macroeconomic fundamentals - e.g. Chen and Rogoff (2003), Chen, Rogoff, and Rossi (2010), and Ferraro, Rossi, and Rogoff (2011).

one half in the wake of the Great Recession.

The model is designed to capture the carry trade in currency markets. In the model, differences in average interest rates and risk exposures between countries that are net importers of basic commodities ("producer countries") and commodity-exporting countries ("commodity countries") are rationalized by appealing to a natural economic mechanism: trade costs.[3] We model trade costs by considering a simple model of time-varying trade frictions. At any time the cost of transporting a unit of good from one country to the other depends on the aggregate shipping capacity available. While the capacity of the shipping sector adjusts over time to match the demand for transporting goods between countries, it does so slowly, e.g. due to gestation lags in the shipbuilding industry. In order to capture this intuition we assume that the marginal cost of shipping an extra unit of good is increasing - i.e., trade costs in our model are convex, as in Ready, Roussanov, and Ward (2016).

Convex shipping costs imply that the sensitivity of the commodity country to world productivity shocks is lower than that of the country that specializes in producing the final consumption good, simply because it is costlier to deliver an extra unit of the consumption good to the commodity country in good times, but cheaper in bad times. Therefore, under complete financial markets, the commodity country's consumption is smoother than it would be in the absence of trade frictions, and, conversely, the producer country's consumption is riskier. Since the commodity country faces less consumption risk, it has a lower precautionary saving demand and, consequently, a higher interest rate on average, compared to the country producing manufactured goods. Since the commodity currency is risky - it depreciates in bad times from the perspective of the producer country's consumer - it commands a risk premium. Therefore, the interest rate differential is not offset on average by exchange rate movements, giving rise to a carry trade. The role of slowly-adjusting shipping capacity, which we emphasize here, is to amplify these fluctuations in trade costs, and, therefore, in exchange rates, especially during cyclical transitions. When global output expands due to rising productivity, trade costs increase sharply until shipping capacity catches up with output (and exports); a contraction that follows such a build-up produces a sharp drop in

[3]Trade costs have a long tradition in international finance: e.g., Dumas (1992), Hollifield and Uppal (1997). Obstfeld and Rogoff (2001) argue that trade costs hold the key to resolving several major puzzles in international economics.

transport costs as accumulated shipping capacity is large relative to the amount of goods being shipped.

In order to evaluate the model's ability to generate quantitatively reasonable magnitudes of currency risk premia and interest rates we calibrate it by allowing for the possibility of very large jumps in productivity - i.e., rare disasters, as in the literature on the equity premium puzzle (e.g., Longstaff and Piazzesi (2004), Barro (2006), Gabaix (2012), Wachter (2013)). The calibrated model is able to account for the observed interest rate differentials and average returns on the commodity currency carry trade strategies without overstating consumption growth volatility, even in samples that contain disasters, or implying an unreasonably high probability of a major disaster.[4]

We use our model as a laboratory for understanding the behavior of commodity currencies around the Great Recession. We feed in a series of productivity shocks observed in the data. Over the period 2002-2006 these shocks generate a boom in commodity prices (as commodity supply struggles to catch up quickly) and a rise in global shipping costs (as shipping capacity also lags behind). The commodity country exchange rate appreciates as well, yielding high carry trade profits, essentially matching those observed in the data. The latter result is not mechanical, as in the presence of complete financial markets terms of trade do not drive exchange rates; rather, this is due to the fact that increasing trade costs make markets more segmented, with marginal utility of producer country ("Japan") consumers fall faster than that of commodity country ("Australia"). The ensuing global crisis is represented in our model by a large negative productivity shock in the producer country (we abstract from demand shocks or financial frictions for simplicity). As a result, output and trade in the final good collapse, as do commodity prices and shipping costs. The commodity currency depreciates due to a sharp spike in the marginal utility in "Japan" relative to that of "Australia", generating large losses for the currency carry trade. Since shipping capacity is very slow to adjust, trade costs remain depressed even as output and trade recover. Interest rate differentials and expected carry trade returns also decrease as low trade costs imply a

[4]Farhi and Gabaix (2016) generate a carry trade risk premium in a rare disaster setting by assuming that both the disaster probability and countries' exposures are exogenously time-varying. In our setting, the probability of a disaster shock is constant over time, while the magnitude of its impact on the exchange rate, and therefore its contribution to the risk premium, vary endogenously over time with the state of the global economy and trade frictions.

greater degree of risk sharing (i.e., a closer alignment of marginal utilities, and consequently less scope for a currency risk premium). We show that all of these predictions are consistent with the empirical evidence for the countries that we consider.

Our analysis sheds a new light on the role of time-varying transport costs in international trade. In our model trade costs increase in (global) good times endogenously, since that is when exports tend to rise. This is consistent with arguments in Hummels (2007) and papers cited therein, emphasizing the effects of port congestion and delays in shipping and the role of fuel costs, which in the recent decades have behaved procyclically, as well as evidence on the value of speed of shipment analyzed in Hummels and Schaur (2013). It is also corroborated by the empirically observed behavior of the shipping cost indices that we consider as the most explicit (albeit narrow) measures of transportation costs. Other models in international finance instead assume that trade costs increase (exogenously) in bad times, potentially due to a tightening of trade credit - e.g. Maggiori (2012). We do not need to take a stand on the role of trade finance in the trade collapse during the Great Recession, which is a subject of an empirical debate. Indeed, while some studies, such as Amiti and Weinstein (2011) and Chor and Manova (2012) argue that there is some empirical support for the role of trade credit and financial frictions, Levchenko, Lewis, and Tesar (2011) find very little evidence, at least for the U.S. In addition, Eaton, Kortum, Neiman, and Romalis (2011) and Gopinath, Itskhoki, and Neiman (2012) find little evidence in prices of traded goods that would be consistent with a large role of trade costs in reducing trade volumes during the crisis. While our model of international trade is relatively stylized compared to the recent models that focus on understanding trade frictions at the micro level (e.g., Arkolakis (2010), Alessandria, Kaboski, and Midrigan (2013)), our contribution is to highlight the importance of shipping capital for the dynamics of global trade, international risk sharing, and real exchange rates.

2 Motivating Evidence

Our approach builds on the evidence in Ready, Roussanov, and Ward (2016) that countries whose primary exports are basic commodities exhibit qualitatively different behavior of macroeconomic aggregates than countries that concentrate in exporting complex manufactured goods. As argued by Lustig, Roussanov, and Verdelhan (2011), persistent differences in countries' exposures to global shocks drive the bulk of currency risk premia earned in foreign exchange markets. These differences lead to persistent interest rate differentials (or, equivalently, forward discounts), which, in turn, translate into expected excess returns earned on currency positions, as spot exchange rates on average systematically deviate from the forward rates.[5]

Table 1 displays the average interest rate differentials (proxied by one-month forward discounts) and the corresponding average one-month excess returns on eight currencies vis-a-vis the U.S. dollar, over the period 2000-2015 (since the introduction of the Euro). These currencies are among the 10 most actively traded currencies in the foreign exchange market (known as the G10 currencies). The economies representing the top four currencies - the Euro zone, the Japanese yen, the Swedish Krona, and the Swiss franc - all feature advanced manufacturing sectors and a high share of complex manufactured goods in their exports. The bottom four currencies are known as "commodity currencies": the Australian, Canadian, and New Zealand dollars, and the Norwegian Krone.[6] Alongside the exchange rate variables the table displays the average Import Ratio introduced by Ready, Roussanov, and Ward (2016), which reflects the extent to which a country is a net exporter of commodities vs. manufactured goods. This ratio introduced in is calculated as

$$\text{Import Ratio} = \frac{\text{Net Imports of Complex Goods} + \text{Net Exports of Basic Commodities}}{\text{Total Trade in All Goods}},$$

using U.N. COMTRADE data to construct good-specific trade series at annual frequency.

[5]Denoting log forward exchange rate one month ahead $f_t = \log(F_t)$ and log spot exchange rate $s_t = \log(S_t)$, both expressed in units of foreign currency per one U.S. dollar, the forward discount is equal to the interest rate differential: $f_t - s_t \approx i_t^\star - i_t$, where $i^\star$ and i denote the foreign and domestic nominal one month risk-free rates, by covered interest rate parity. The log excess return rx on buying a foreign currency in the forward market and then selling it in the spot market after one month is then given by

$$rx_{t+1} = f_t - s_{t+1},$$

while the arithmetic excess return is given by

$$Rx_{t+1} = \frac{F_t}{S_{t+1}} - 1.$$

[6]The other two countries making up the G10 group - the U.K. and the U.S. - have a relatively equal shares of complex and basic goods in their net exports, as analyzed in Ready, Roussanov, and Ward (2016).

Table 1: Average Currency Returns and Forward Discounts

Country	**Currency Excess Return**	**Forward Discount**	**Import Ratio**
Japan	-2.25	-2.28	-0.54
Switzerland	-1.54	0.40	-0.21
Germany (Euro)	-0.28	-0.58	-0.18
Sweden	-0.02	-0.19	-0.11
Canada	0.25	1.13	0.21
Norway	1.29	0.69	0.51
New Zealand	2.81	1.70	0.53
Australia	2.46	1.47	0.65

Table plots average annualized currency returns, forward discounts, and import ratios for the post-euro sample (1999 - 2015). Currency returns are the excess return to rolling over 1-month currency forwards. Forward discounts are the average annualized discount on 1-month forward contracts. Import ratio is calculated as in Ready, Roussanov, and Ward (2016), and reflects the extent to which a country is a net exporter of commodities. This ratio is calculated as:

$$\text{Import Ratio} = \frac{\text{Net Imports of Complex Goods} + \text{Net Exports of Basic Commodities}}{\text{Total Trade in All Goods}}$$

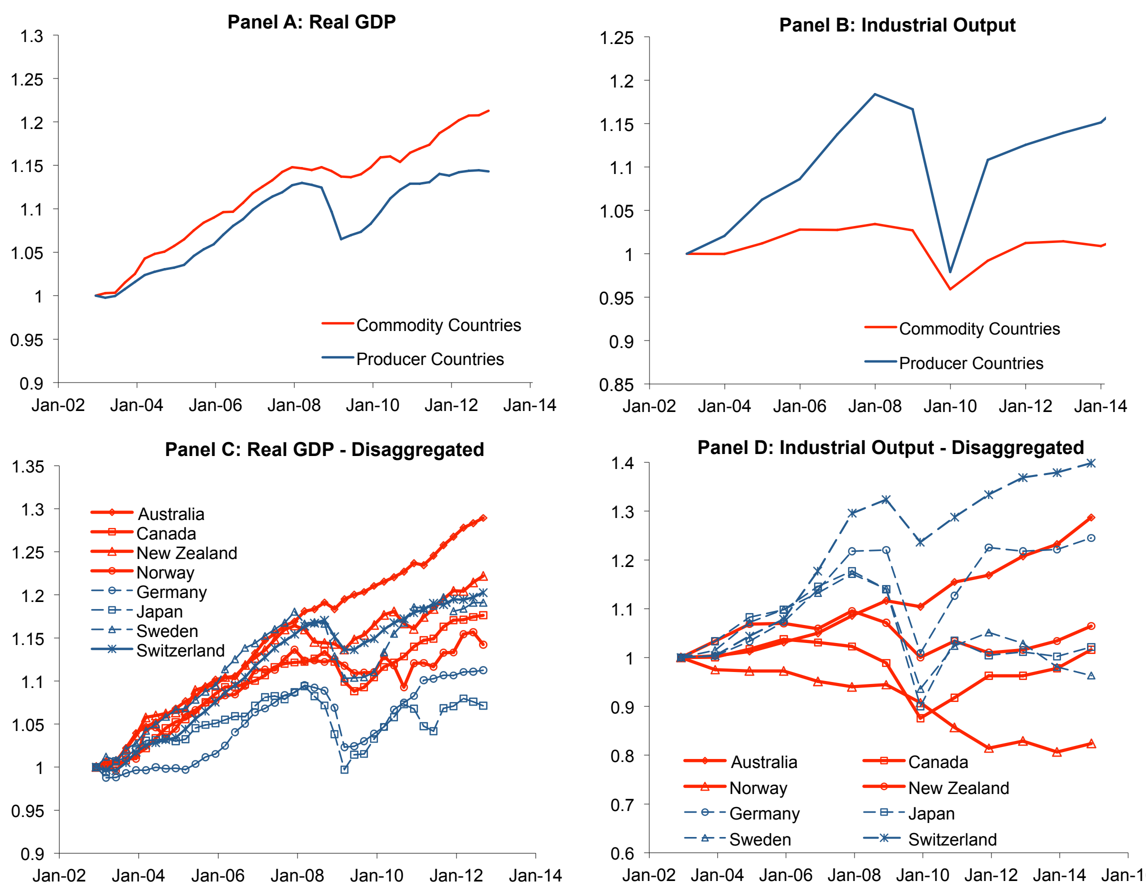
Currency data is from Bloomberg and trade data is from the U.N. Comtrade Database. Goods are classified as complex or basic as in Ready, Roussanov, and Ward (2016).

All of the countries in the latter group primarily export basic commodities, as indicated by the positive Import ratios (the other countries' Import ratios are negative). The table reveals that the top four currencies have persistently low interest rates relative to the U.S., which translate on average into negative excess returns. In contrast, the bottom four currencies exhibit higher interest rates (positive forward discounts for the U.S. dollar), which translate into positive average excess returns. Note that across countries both average forward discounts and average excess returns are roughly monotonic in the Import ratio. As argued by Ready, Roussanov, and Ward (2016), this systematic relationship captures essentially all of the cross-sectional variation in *unconditional* average currency returns, i.e. the currency carry trade.

While the currency carry trade is a highly profitable trading strategy on average, it performs poorly during global market downturns (see Lustig, Roussanov, and Verdelhan (2011) for detailed analysis). The carry trade suffered particularly large losses during the onset of the global financial crisis and the Great Recession. Figure 1 shows that two groups of countries indeed display very different behavior of macroeconomic aggregates during the crisis. In the "producer" countries (complex manufactured goods exporters) both GDP and industrial output grow rapidly between 2003 and 2007 and then plunge during the Great Recession, largely recovering afterwards, but still remaining below trend until at least 2014. In contrast, for the "commodity countries" we see on average a somewhat slower rate of productivity growth but only a slight downturn in GDP during the crisis and a much smaller drop in industrial output than in the producer countries.

The apparent resilience of the commodity countries during the Great Recession is particularly surprising given the dramatic decline in their terms of trade due to the collapse of commodity prices (e.g., see discussion in Eaton, Kortum, Neiman, and Romalis (2011)). Figure 2 Panel A plots the aggregate Commodity Price Index compiled by the Commodity Research Bureau (CRB) along the with the two measures of global shipping costs. The first measure is the widely used Baltic Dry Index (BDI), an exchange-traded composite of dry bulk freight rates, which captures the cost of shipping dry commodities (e.g., iron ore). The second measure (Harpex) is a composite index of freight rates for container shipping along several common routes (e.g., Hong Kong to Rotterdam), and therefore captures the

Figure 1: Commodity and Producer Country Output



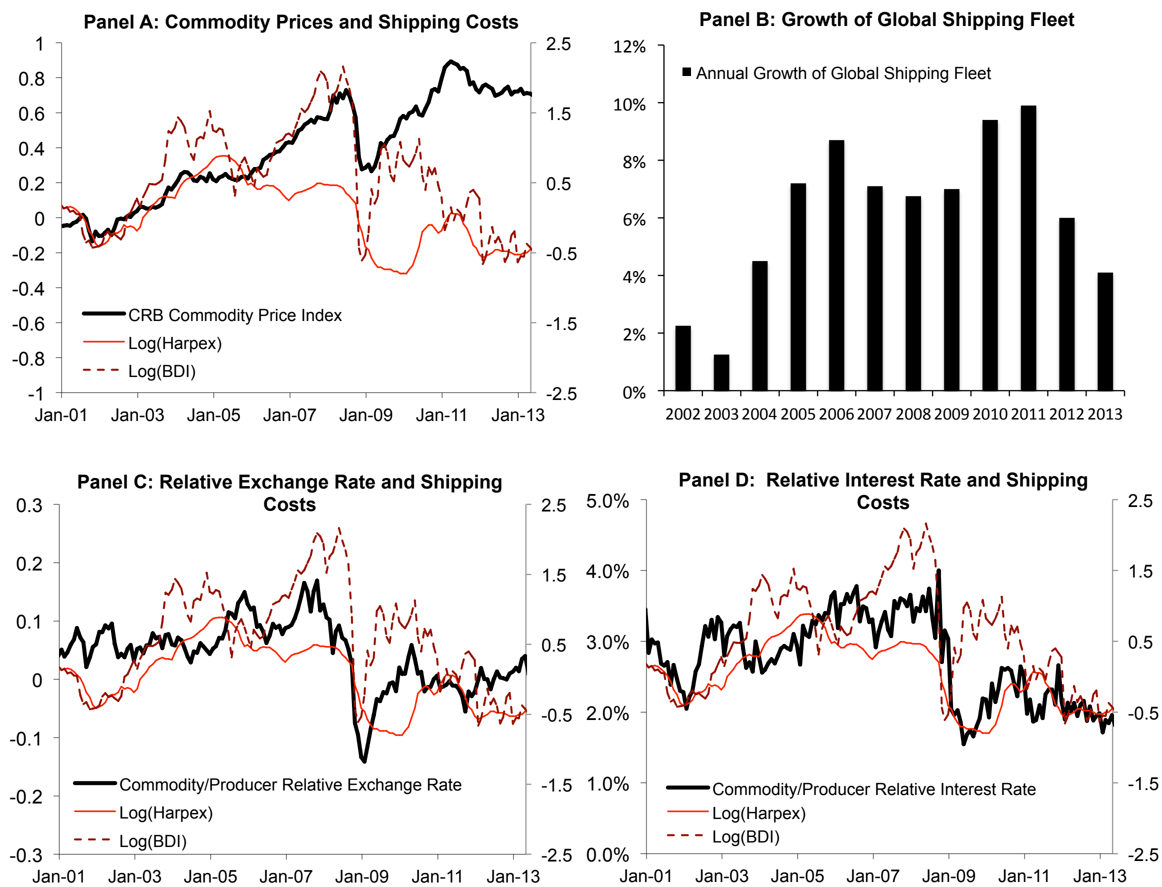
Panel A plots an equal weighted index of log real GDP for the four commodity countries (Australia, Canada, New Zealand, and Norway) and the four producer countries (Germany, Japan, Switzerland, and Sweden). Panel B repeats this plot using and index of industrial output which uses industrial output growth weighted by GDP. Panels C and D show data for the individual countries. Data are from the OECD. Switzerland's productivity data is omitted due to lack of availability.

cost of shipping more complex/finished goods. While container shipping rates are typically contracted forward and are therefore relatively smooth, bulk shipping rates are determined largely in a spot market and therefore fluctuate at fairly high frequencies. The figure shows that commodity prices indeed fell by about 20% on average during the global financial crisis, although they recovered between 2009 and 2010, reaching new highs in 2011. Interestingly, global shipping costs, which grew rapidly in the early 2000-s (especially the BDI) fell much more dramatically, both BDI and Harpex losing over 90% of their values between 2007 and 2009. Both indices recover roughly half-way by 2010 but remain depressed throughout the remainder of our sample.

What explains this discrepancy between the behavior of commodity prices and the price of shipping (both commodities and finished goods)? Panel B plots the plots the annual growth rate of freight shipping capacity. Both dry bulk and container ships are large and essentially irreversible investments: they take between 2 to 3 years to build but can be operated for decades. During the trade boom of the early 2000-s as the demand for shipping services drove up shipping costs, used ship prices increased rapidly, the largest of them (CapeSize) selling for over $100 million each (see Greenwood and Hanson (2015) for details). This increased demand drove shippers to increase investment in new ships, resulting in annual increases in shipping capacity between between 7 and 9% in 2007-2009. With the collapse of global trade during the Great Recession shipping costs (and prices of ships collapsed by between 70 and 80%), yet new ships ordered during the boom continued coming on-line, increasing capacity and driving down shipping costs even as the global economy began to recover.

The differing patterns in shipping costs and commodity prices is particularly interesting when examined in the context of commodity country currencies. Panel C plots the two measures of shipping costs along with the spot exchange between producer and commodity countries calculated as the relative price of gdp-weighted baskets of our producer and commodity country currencies. Panel D plots relative interest rates, calculated in a similar manner, again with the two measures of shipping costs. What is notable about these plots is that the patterns in commodity currencies, both in terms of their value and associated interest rates, quite closely follow the observed patterns in shipping costs. The relative value and relative interest rate are both high prior to the crisis, fall dramatically during the crisis,

Figure 2: Shipping Costs and Commodity Prices



Panel A plots the log of the CRB Spot Commodity index, as well as the logs of the Harpex container ship index and the Baltic Dry Index. Panel B plots the annual growth of the global merchant shipping fleet. Panel C plots the cumulative change the relative spot exchange rates between a gdp-weighted portfolio of the four commodity countries (Australia, Canada, New Zealand, and Norway) and the four producer countries (Germany, Japan, Sweden, and Switzerland). Panel D plots the difference between the gdp-weighted average of commodity country and producer country short-term (1-month) interest rates calculated form currency forwards. Currency forward data are from Barclays and Reuters. Price index data are from Datastream. Shipping fleet data are from United Nations Conference on Trade and Development.

and then remain subsequently low despite the recovery in commodity prices. The low relative interest rates are important, because they potentially indicate a substantial reduction in expected returns to the ongoing carry-trade strategy.

Our model aims to account for these features of the data relating to the period around the global financial crisis and the Great Recession that are summarized here. The first is the average interest rate differential between the "commodity" and the "producer" countries, and the associated carry trade risk premium. The second is the relative resilience of the commodity countries during the global downturn, despite a substantial decline in their terms of trade. The third key component is the joint behavior of shipping costs, commodity country currency values, and the relative interest rate between commodity country and producer currencies. In what follows we show that these features are inherently connected via a simple risk sharing mechanism that relies on complete financial markets and costly trade in goods.

3 Model

3.1 Setup

There are two countries each populated by a representative consumer endowed with CRRA preferences over the same consumption good with identical coefficients of relative risk aversion γ and rates of time preference ρ. The two countries specialize in the production of a single good and trade with each other through interactions with a shipping industry and commodity traders.

The "commodity" country produces a basic input good using a linear production technology

$$\max_{l_c} P^* z_c l_c - w_c l_c.$$

One unit of commodity country's non-traded input (l_c) (e.g., labor, land, etc.) is supplied inelastically, and the price of the commodity in the country is P^*. The wage rate in the commodity country, w_c, is pinned down by the first-order condition

$$w_c = P^* z_c.$$

The "producer" country only produces a final consumption good using basic commodity input b and labor:

$$\max_{l_p} \underbrace{z_p b^{1-\beta} l_p^{\beta}}_{\equiv y_p} - Pb - w_p l_p,$$

which is subject to a productivity shock z_p with one unit of producer country's non-traded input l_p also supplied inelastically. The wage paid to labor is w_p and P is the price of unit of basic commodity. From the first-order condition over labor and zero profits, the price of the basic commodity is given by

$$P = \frac{(1-\beta) y_p}{b}.$$

Commodity traders purchase the entire supply of the commodity, exchange it (at the exchange rate S) and then ship it to the producer country, receiving the payment for the value of the goods there, and paying the cost of shipping τ_c per unit. The traders are in a competitive industry and earn zero profits:

$$P z_c (1 - \tau_c) - \frac{P^*}{S} z_c = 0 \Rightarrow SP(1 - \tau_c) = P^*,$$

which pins down the price of the commodity in the commodity country.

The countries are spatially separated so that transporting goods from one country to the other incurs shipping costs. Our shipping industry comprises two sectors that specialize in the transportation of commodities and the transportation of finished goods; these can be thought of as the owners of dry bulk ships and container ships, respectively. Both industries feature per-unit costs of transporting good x

$$\kappa_0^i + \kappa_1^i \frac{x}{z_k},$$

which depend on the total amount of goods shipped in the same direction, x, and the shipping capacity available at time t, z_k. For simplicity we assume that this shipping capacity (or, equivalently, shipping sector productivity) is exogenous (although a model with investment in shipping capacity yields similar implications). Since the costs of shipping raw commodities and manufactured goods are likely to be different, we allow two sets of parameters ($i \in c, f$). This specification extends the variable iceberg cost of Backus, Kehoe, and Kydland (1992).

The commodity-shipping sector chooses the quantity to ship given the price of trade subject to the above cost function. The commodity-shipping sector earns zero profits, motivated by the fact that dry bulk ships are nearly perfect substitutes for each other and, as such, are essentially commoditized. This implies that the shipping cost function implied by the solution to this sector's problem is

$$z_c \tau_c - \kappa_0^c z_c - \kappa_1^c \frac{z_c^2}{z_k} = 0 \Rightarrow \tau_c = \kappa_0^c + \kappa_1^c \frac{z_c}{z_k}.$$

Given this price of trade, the total units of the basic commodity the producer country receives is

$$b = z_c(1 - \tau_c(z_c, z_k)) = z_c \left(1 - \kappa_0^c - \kappa_1^c \frac{z_c}{z_k}\right).$$

In the final-good-shipping industry, while we specify it as competitive, and therefore taking prices as given, we allow the sector itself to have positive profits (due to the decreasing returns). This is motivated by the fact that shipping of final goods is a more specialized industry. While it features many competing firms, entry into a particular market (shipping route, type of cargo, etc.) is limited in the short run. Thus, the final good shipping sector, which we assume belongs to the commodity country, maximizes profits $\Pi_f(X_s, z_k) = \tau_f X - \kappa_0^f X - \kappa_1^f \frac{X^2}{z_k}$, which are rebated to the commodity country as dividends and included in its consumption.[7]

The price of shipping services then satisfies the first-order condition with respect to X,

[7] This is the standard approach taken in the literature, e.g. see Kose and Yi (2006). Alternatively, we could assume that, as in the commodity shipping sector, free entry enforces a zero profit condition setting

$$\tau_f = \kappa_0^f + \kappa_1^f \frac{X}{z_k}.$$

We consider this case as an extension below. Unlike our benchmark specification, in this case the competitive equilibrium is not efficient, since the shipping sector does not internalize the congestion externality that arises from the convex transportation technology.

yielding

$$\tau_f = \kappa_0^f + 2\kappa_1^f \frac{X}{z_k}.$$

The production economy outlined here is very simple (e.g., it is essentially static, as there are no capital or other inter-temporal investment margins), intended to highlight the main mechanism based on the interplay of specialization and trade costs. Gourio, Siemer, and Verdelhan (2013) and Colacito, Croce, Ho, and Howard (2013) study currency risk premia in fully dynamic production economies that could potentially be generalized to incorporate the type of heterogeneity we consider.

Consumption allocations for the commodity country and the producer country, c_c and c_p, are determined by the output of the producer country y_p and the amount X of final consumption good exported to the commodity country. We will consider complete financial markets as our benchmark case, so that equilibrium consumption allocations to the two countries over time and across states of nature will be determined as a result of a risk-sharing arrangement, and the real exchange rate is pinned down by the absence of arbitrage in the financial markets (as well as the markets for the consumption good).

3.2 Dynamics

We assume that the shocks to productivity experienced by the final good producer are permanent, so that its evolution (in logs) follows a jump-diffusion process:

$$d \log z_{pt} = (\mu - \mu_Z \eta)\, dt + \sigma_p dB_{pt} + dQ_t.$$

Let $N(t)$ be a Poisson process with intensity η, and let $-Z_1, -Z_2, \ldots$ be a sequence of identically distributed random variables drawn from a truncated Pareto distribution with minimum jump Z_{min}, maximum jump Z_{max}, and shape parameter α. Denote this distribution's mean as μ_Z. Define the compound Poisson process:

$$Q(t) = \sum_{j=1}^{N(t)} Z_j = \int_0^t Z_s dN_s, \; t \geq 0.$$

$$\Rightarrow dQ(t) = Z_{N(t)} dN_t,$$

so that μ is the uncompensated drift of the jump-diffusion, and the growth rate of the productivity shock process can be written as

$$\begin{aligned}\frac{dz_{pt}}{z_{pt^-}} &= \left(\mu - \mu_Z \eta + \frac{1}{2}\sigma_p^2\right) dt + \sigma_p dB_{pt} + (e^{Z_{N(t)}} - 1) dN_t \\ &\mathrel{\hat{=}} \mu_p dt + \sigma_p dB_{pt} + (e^{Z_{N(t)}} - 1) dN_t,\end{aligned}$$

where $z_{pt^-} = \lim_{s\uparrow t} z_{ps}$ is the process's left-limit, a convention used throughout.

In order to ensure stationarity of the model economy, we further assume that commodity country productivity shock are cointegrated with the producer country shocks. Specifically, we assume that their cointegrating residual

$$q_t = \log z_{pt} - \beta \log z_{ct}$$

is stationary, following a mean-reverting jump-diffusion process

$$dq_t = \left[(1-\beta)(\mu - \mu_Z \eta) - \beta \psi q_t\right] dt + \sigma_p dB_{pt} - \beta \sigma_c dB_{ct} + dQ_t,$$

so that the commodity country productivity shock process (in logs) follows

$$d \log z_{ct} = (\mu + \psi q_t) dt + \sigma_c dB_{ct},$$

and therefore we can write

$$\begin{aligned}\frac{dz_{ct}}{z_{ct^-}} &= \left(\mu + \psi q_t + \frac{1}{2}\sigma_c^2\right) dt + \sigma_c dB_{ct} \\ &\mathrel{\hat{=}} \mu_{ct} dt + \sigma_c dB_{ct}.\end{aligned}$$

This cointegrated relationship can be interpreted as a reduced form representation of an economy where supply of the commodity is inelastic in the short run (based on the currently explored oil fields, say) but adjusts in the long run to meet the demand by the final good producers (e.g., as new fields are explored more aggressively when oil prices are high).

Similarly, we assume that shipping sector productivity is cointegrated with the commodity supply, with the cointegrating residual defined

$$q_{kt} = \log z_{ct} - \log z_{kt},$$

which follows a mean-reverting process

$$dq_{kt} = (\psi q_t - \psi_k q_{kt})dt + \sigma_c dB_{ct} - \sigma_k dB_{kt}$$

so that the shipping shock process follows

$$\begin{aligned} d\log z_{kt} &= (\mu + \psi_k q_{kt})dt + \sigma_k dB_{kt} \\ \Rightarrow \frac{dz_{kt}}{z_{kt^-}} &= \left(\mu + \psi q_{kt} + \frac{1}{2}\sigma_k^2\right) dt + \sigma_k dB_{kt} \\ &\mathrel{\hat{=}} \mu_{kt} dt + \sigma_k dB_{kt}, \end{aligned}$$

where the Brownian motions B_{pt}, B_{ct}, and B_{kt} are independent. The latter assumption captures the idea that shipping capacity cannot be adjusted quickly in response to shocks, which can lead to substantial volatility in costs of shipping over time, and therefore shipping costs that are very sensitive to demand shocks in the short run (e.g., Kalouptsidi (2014), Greenwood and Hanson (2015)). Our modeling of cointegrated jump-diffusion processes is similar to the model of cointegrated consumption and dividend dynamics in Longstaff and Piazzesi (2004). We can solve for output and commodity price dynamics by application of Ito's lemma (see Appendix).

3.3 Complete markets and consumption risk sharing

In order to emphasize that our mechanism does not rely on any financial market imperfections, we consider consumption allocations under complete markets. This is a standard benchmark in international finance, and is reasonable at least when applied to developed countries.[8] Under complete markets, each country's representative consumer chooses the

[8]For example, Fitzgerald (2012) estimates that risk-sharing via financial markets among developed countries is nearly optimal, while goods markets trade frictions are sizeable.

time- and state-contingent consumption allocations: the commodity country consumption is $c_{cs} = X_s(1-\tau_f(X_s, z_k))+\Pi_f(X_s, z_k)$, and the producer country consumption is $c_{ps} = y_{ps}-X_s$, where X_s is exports of final good to the commodity country.

The producer country's problem is

$$\max_{\{X_t\}_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\frac{c_{pt}^{1-\gamma}-1}{1-\gamma}dt\right], \text{ subject to} \tag{1}$$

$$c_{pt} = y_{pt} - X_t, \text{ for all } t \geq 0 \tag{2}$$

$$W_{p0} \geq \mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p}c_{pt}dt\right], \tag{3}$$

where π_t^p is the stochastic discount factor that prices in the units of producer country consumption. Its initial wealth W_{p0} is the present value of its income (i.e., the Cobb-Douglas labor share of total output βy_p), and is therefore given by

$$W_{p0} = \beta\mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p}y_{pt}dt\right].$$

The first-order condition for X_t here is simply

$$e^{-\rho t}c_{pt}^{-\gamma} = \lambda_p\frac{\pi_t^p}{\pi_0^p}, \tag{4}$$

where λ_p is the Lagrange multiplier on the commodity country's resource constraint where the country takes prices as given and is constant since data at different dates represent the evolution over time of a single plan (and is feasible because of complete markets).

The commodity country's problem is

$$\max_{\{X_t\}_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\frac{c_{ct}^{1-\gamma}-1}{1-\gamma}dt\right], \text{ subject to} \tag{5}$$

$$c_{ct} = X_t\left(1-\tau_f(X_t, z_{kt})\right) + \Pi_f(X_s, z_k), \text{ for all } t \geq 0 \tag{6}$$

$$W_{c0} \geq \mathbb{E}\left[\int_0^\infty X_t\frac{\pi_t^p}{\pi_0^p}dt\right], \tag{7}$$

where W_{c0} is the country's wealth expressed in the units of producer country's consumption good, which is the present-value of future income received as payment for the commodity delivered to the producer country and discounted by the stochastic discount factor π_t^p state-by-state:

$$W_{c0} = (1-\beta)\,\mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p} y_{pt} dt\right].$$

The latter follows from the fact that the production function for the final good is Cobb-Douglas and the share of output that is expended on the commodity input is equal $1-\beta$.

The first-order condition with respect to X_t (taking shipping costs and profits as given) is

$$e^{-\rho t} c_{ct}^{-\gamma}\left(1-\kappa_0^f - 2\kappa_1^f \frac{X_t}{z_{kt}}\right) = \lambda_c \frac{\pi_t^p}{\pi_0^p}, \tag{8}$$

where λ_c is the Lagrange multiplier.

Complete markets imply that the stochastic discount factor π_t^p is unique, so that putting together the ratio of first-order conditions (4) and (8) obtains

$$c_{ct}^{-\gamma}\left(1-\kappa_0^f - 2\kappa_1^f \frac{X_t}{z_{kt}}\right) = \frac{\lambda_c}{\lambda_p} c_{pt}^{-\gamma}. \tag{9}$$

This corresponds to the first-order condition of a central planner in the equivalent Pareto problem.[9] It is standard in the international asset pricing literature to consider the planner solution directly (e.g., Verdelhan (2010) and Ready, Roussanov, and Ward (2016)). The planner's Pareto weight for the producer country consumer relative to the commodity country consumer can be set to equal the ratio of Lagrange multipliers $\lambda = \frac{\lambda_c}{\lambda_p}$, depending on each country's initial wealth. We refer to this ratio in what follows, even though it is not a free parameter; the complete decentralized solution is detailed in the appendix.

Since the trade costs are increasing in the amount of goods shipped (holding shipping capacity fixed), the cost of transporting an extra unit of the final consumption good is increasing in total output y_{pt}. When output is high, exports to the commodity country

[9] Since the intratemporal budget constraint for the commodity country is $c_{ct} = X_t\left(1-\kappa_0^f - 2\kappa_1^f \frac{X_t}{z_{kt}}\right) + \kappa_1^f \frac{X_t^2}{z_{kt}} = X_t\left(1-\kappa_0^f - \kappa_1^f \frac{X_t}{z_{kt}}\right)$, equation (9) is equivalent to the first order condition of the planning problem, which internalizes the congestion arising in the final good shipping sector by taking shipping costs into account.

rise, while shipping becomes increasingly costly.The effects of individual state variables on the final good trade cost τ_f are intuitive: greater shipping capacity decreases the cost of shipping, while higher productivity of the final goods producer increases trade costs by raising output and, consequently, the amount of goods shipped to the commodity country (higher productivity in the commodity country has a similar effect, as it feeds into final good output).

3.4 Exchange rates

The spot exchange rate in the absence of arbitrage is proportional to the ratio of the marginal utilities of the two representative agents,

$$S_t = \lambda \frac{\pi_{pt}}{\pi_{ct}} = \lambda \left(\frac{c_{ct}}{c_{pt}} \right)^{\gamma} = \left(1 - \kappa_0^f - 2\ \kappa_1^f \frac{X_t}{z_{kt}} \right) \tag{10}$$

where the last equality follows from (9), implying that the real exchange rate is proportional to the marginal value to the commodity country consumer of a unit of the consumption good shipped from the country where it is produced (e.g., see Dumas (1992), Hollifield and Uppal (1997), Verdelhan (2010)).

The real exchange rate is monotonic in the ratio of the two countries' consumption levels, is linear in the quantity of final good output exported to the commodity country, X_t, and is therefore closely related to the trade costs. Following good productivity shocks in either final good or commodity producing countries, total output y_p and exports X both increase, and therefore the producer country exchange rate depreciates. This is due to the fact that shipping costs lower the value of a marginal unit of the consumption good exported by its producer to the commodity country consumer, and more so when more of the good is shipped. Consequently, as (10) shows, both consumption and its marginal utility declines more slowly for the commodity country consumer than for the producer country consumer in good times, and also rises more slowly in bad times.[10] Positive shocks to the shipping

[10]In autarky, the commodity currency appreciates following good shocks to the final good production technology as its good becomes more highly demanded - this is the terms-of-trade effect, which is present even in the absence of complete financial markets, as emphasized by Cole and Obstfeld (1991). The effects of the commodity country productivity differ, however: terms of trade logic implies that commodity currency appreciates when the commodity becomes scarce following a bad supply shock. This is not generally true in

capacity z_k reduce the cost of shipping and therefore act in the opposite direction, increasing the value of the unit of X to the commodity country and therefore lowering its exchange rate (X will increase endogenously in response to higher shipping capacity, however, partially offsetting the influence of shipping cost shocks on the exchange rate.).

3.5 Asset pricing

Stochastic discount factors for the two countries are given by

$$\begin{aligned} \pi_{pt} &= e^{-\rho t} c_{pt}^{-\gamma} \\ \Rightarrow \frac{d\pi_{pt}}{\pi_{pt^-}} &= -\left\{\rho + \gamma\mu_{cpt} - \frac{1}{2}\gamma(1+\gamma)\sigma_{cpt}^T\sigma_{cpt}\right\} dt - \gamma\sigma_{cpt}^T dB_t + \left(e^{-\gamma\mathcal{J}_p} - 1\right) dN_t \end{aligned}$$

for the final good producer and

$$\begin{aligned} \pi_{ct} &= e^{-\rho t} c_{ct}^{-\gamma} \\ \Rightarrow \frac{d\pi_{ct}}{\pi_{ct^-}} &= -\left\{\rho + \gamma\mu_{cct} - \frac{1}{2}\gamma(1+\gamma)\sigma_{cct}^T\sigma_{cct}\right\} dt - \gamma\sigma_{cct}^T dB_t + \left(e^{-\gamma\mathcal{J}_c} - 1\right) dN_t \end{aligned}$$

for the commodity producer, where $\mathcal{J}_p$ and $\mathcal{J}_c$ are log changes in the marginal utilities induced by jumps.

Risk-free rates are the (negative) drifts of the stochastic discount factors:

$$r_{pt}^f = \rho + \gamma\mu_{cpt} - \frac{1}{2}\gamma(1+\gamma)\sigma_{cpt}^T\sigma_{cpt} - \eta\mathbb{E}_Z\left[e^{-\gamma\mathcal{J}_p} - 1\right] \tag{11}$$

and

$$r_{ct}^f = \rho + \gamma\mu_{cct} - \frac{1}{2}\gamma(1+\gamma)\sigma_{cct}^T\sigma_{cct} - \eta\mathbb{E}_Z\left[e^{-\gamma\mathcal{J}_c} - 1\right], \tag{12}$$

for the final goods and commodity producer, respectively. The terms $\mathbb{E}_Z$ denote expectations taken over the distribution of jump sizes conditional on a jump occurring. The first two terms of the interest rate expressions above are equal between the two countries on average, as long-run consumption growth rates are equalized by the social planner. However, the last

our complete markets setup, as a decline in commodity supply leads to lower output of the final good, and higher value for the producer country currency.

terms – the precautionary saving demands – differ. Since the final goods producer absorbs the bulk of productivity shocks to output, consuming a greater share in good times and a lower share in bad times, it experiences greater consumption volatility. Consequently, it has a greater precautionary demand and a lower interest rate on average. Similarly, the conditional expectation of marginal utility growth upon a jump is greater for the producer country consumer due to the same effect.

Since trade costs are persistent as long as shipping capacity adjusts slowly in response to demand, the interest rate variation is driven in part by the expected convergence in consumption due to cointegration (captured by the drift terms) and by the dispersion in conditional risk exposures of the pricing kernels (captured by the precautionary and jump terms). In particular, when output outstrips shipping capacity, the dispersion between the risk terms in the two countries is high, where as when shipping capacity is abundant relative to output this dispersion is lower.

3.6 Expected excess returns: the carry trade

We can define the instantaneous excess return process for the currency trading strategy that is long the commodity currency (and short the producer currency) as

$$dRet_t = (r^f_{ct} - r^f_{pt})dt - \frac{dS_t}{S_{t^-}}.$$

This return can be earned by a final-good producing country investor directly, by shipping a unit of consumption good (borrowed at rate r^f_{pt}) and purchasing S_t units of the commodity-country risk free bonds, earning interest r^f_{ct} on these bonds, and converting it back into its own consumption good by shipping fewer units of the consumption good to the commodity country. It can also be obtained indirectly, by trading a state-contingent claim that replicates the payoff on this strategy, given complete financial markets. A commodity country investor can obtain a similar return, adjusted for the exchange rate.

The conditional expected excess return on this strategy (i.e., the currency risk premium) is given by the covariance of the exchange rate with the producer country pricing kernel (e.g., Bakshi and Chen (1997)):

$$\mu^{FX} = \mathbb{E}\left[dRet_t|\mathcal{F}_t\right] = \mathbb{E}\left[\frac{dS_t}{S_{t^-}}\frac{d\pi_{pt}}{\pi_{pt^-}}|\mathcal{F}_t\right],$$

since the returns are expressed in the producer country numeraire (an equivalent statement holds for the commodity country pricing kernel if the returns are expressed in the commodity currency units). In general, this risk premium is not equal to zero, so that the uncovered interest parity relation $\mathbb{E}\left[\frac{dS_t}{S_{t^-}}|\mathcal{F}_t\right] = (r^f_{ct} - r^f_{pt})dt$ need not hold.

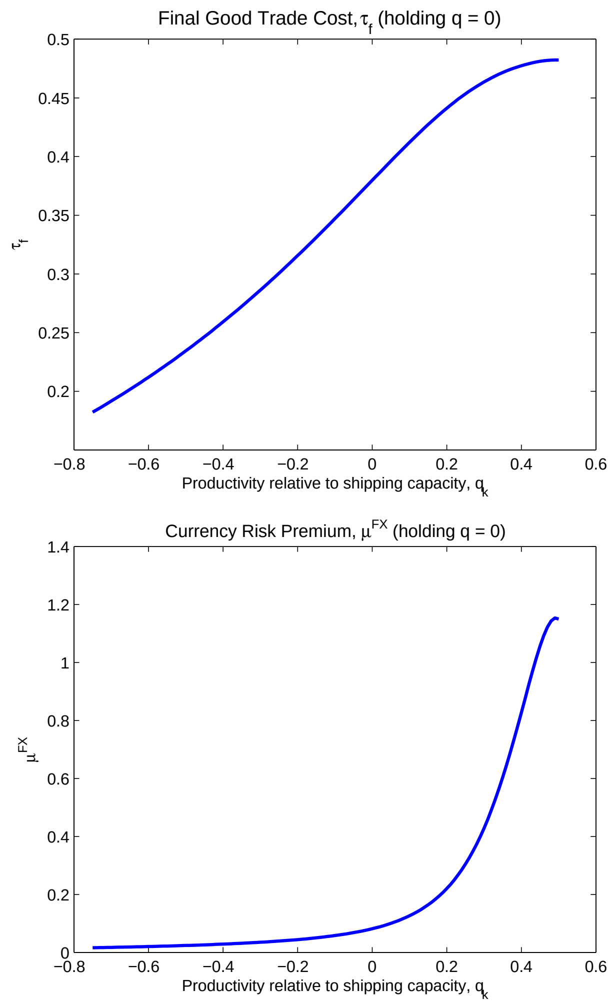
In fact, this commodity currency trading strategy is profitable, on average, since the commodity currency is risky: it tends to appreciate in good times (when final good output is high) and depreciate in bad times, so that $\mathbb{E}\left[\frac{dS_t}{S_{t^-}}\frac{d\pi_{pt}}{\pi_{pt^-}}\right] > 0$. As long as exchange rates are persistent and close to random walks, the bulk of average carry excess return comes from the interest rate differentials.

While both interest rates fluctuate, with the commodity country interest rate being more volatile, and sometimes falling below that of the final good producer, on average the latter is lower. Therefore, a long position in the commodity currency and a short position in the "safe haven" currency of the final good producer is indeed a carry trade strategy, at least unconditionally.

This strategy is a form of unconditional carry-trade strategy insofar as the commodity currency interest rate is on average higher than the producer country interest rate, i.e. as long as the precautionary terms are large enough.

However, the expected returns on this carry trade strategy do move stochastically over time as a function of the conditioning variables, since the quantity of risk embedded in the exchange rate depends on the degree of consumption risk-sharing at the given moment. Figure 3 displays the final good trade costs τ_f and the conditional currency risk premium μ^{FX} as functions of the two cointegrating residuals q_t and q^k_t, evaluated at $q_t = 0$, so that a higher q^k_t due to large output of the final good relative to the available shipping capacity translates into high shipping costs and high expected excess returns. We explore this mechanism quantitatively using the fully-specified model in Section 4 below when we calibrate our model to match the dynamics of the Great Recession.

Figure 3: Trade Costs and Currency Risk Premium



4 Quantitative analysis

So far we have only explored the qualitative implications of our model. We now turn to quantitative analysis. Ideally, we would like to calibrate the model parameters to closely match a set of key empirical moments. The fact that the model features only two countries (each completely specialized in producing one kind of good) makes such a moment-matching exercise challenging. In order to circumvent this challenge we treat the key manufacturing-good exporting countries as a group, assuming that they are representative of a final-good producer country in the model. Similarly, the key commodity-exporting (developed) countries are assumed as a group to be representative of the commodity country in the model. Our motivating empirical evidence above appears to corroborate this distinction, even though the difference between the two types of countries is much less stark in reality than our model assumes. We form two baskets using the set of G10 countries: one of the countries with the four highest import ratios (commodity countries) and the other of the four lowest (final good producer countries). We average macroeconomic and financial variables across countries within each basket and compare their properties to those implied by the model. Table 2 summarizes these Benchmark moments while Table 3 lists the parameter values used in the both calibrations.

The model is too streamlined to try to take to the data and minimize a metric of model fit, we instead discipline the model by quantitatively matching a few key macroeconomic *quantities* of the data along with some of its other qualitative features and, conditional on this, then see how consistent the model's generated asset *prices* are.

We first calibrate the distribution of jump sizes so that its tail approximately corresponds to the distribution of empirically observed consumption disasters compiled by Barro and Ursua (2008) (the largest disaster in their sample corresponds to a consumption drop of 70%, which is approximately the same as the upper bound of our jump distribution $Z_{max} = 1.25$). Disasters - large jumps that cause a 5% or greater drop in consumption - occur at least once over a 30-year period with probability of 16% in the simulated samples given that the jump intensity η is such that a jump occurs on average every 20 years, the smallest jump

Table 2: **Calibration moments**

This table reports summary statistics generated by the model solved using both the monopolistic benchmark case and the decentralized method described in the text, and compares them to data analogues from the G10 country set. All of the financial variables (real interest rates, commodity prices, exchange rates, currency returns) are sampled monthly (monthly carry trade returns are based on continuously rolled-over positions in the model and one-month forward contract returns in the data). The commodity country set includes Australia, Canada, New Zealand and Norway. The producer country set consists of Germany/Euro, Japan, Sweden, and Switzerland. All means and standard deviations are annualized, in percentage points. Output growth in the data is calculated as the gdp-weighted growth of industrial output from the OECD. Consumption growth in the data is the GDP-weighted growth of total household consumption expenditure. X_t is calculated in the data as the total net exports of complex goods by the producer countries. GDP-weighted GDP growth is also shown in the data for reference. τ in the data is proxied by growth in the Harpex, for which data begins in 1999. Data are shown for a quarterly sample from 1995-2012, and a longer annual sample from 1987-2012. The model moments are averages across 10,000 simulated paths of 30 year length, reported as unconditional means and standard deviations.

Panel A: Model			Panel B: Data				
	Benchmark			Quarterly (1995 - 2012)		Annual (1987 - 2012)	
	Mean	Std		Mean	Std	Mean	Std
			Output, Consumption, and Exports				
Δy_{pt}	2.18	2.81	Δy_{pt}	1.35	5.49	1.35	5.68
Δy_{ct}	2.09	1.24	Δy_{ct}	1.26	2.39	1.84	2.93
Δc_{pt}	2.18	2.67	Δc_{pt}	1.23	1.18	1.64	1.30
Δc_{ct}	2.04	1.83	Δc_{ct}	4.03	0.91	3.06	1.21
ΔX_t	2.16	2.96	ΔX_t	2.55	9.52	4.94	8.85
			ΔGDP_{pt}	1.04	1.81	1.72	2.25
			ΔGDP_{ct}	1.58	1.39	2.56	1.32
			Interest Rates and Carry Trade Returns				
r^f_{pt}	3.15	0.1	r^f_{pt}	2.08	1.61	2.12	2.16
r^f_{ct}	6.56	0.38	r^f_{ct}	4.62	1.14	5.16	1.74
$dRet_t$	4.15	5.96	$dRet_t$	3.80	11.44	2.45	11.05
			Trade Costs, Commodity Prices, and Exchange Rates				
dS_t	-0.76	5.94	dS_t	-1.26	11.55	0.60	10.75
dP_t	0.16	3.46	dP_t	5.52	15.13	-1.26	11.55
$d\tau$	0.183	3.75	$d\tau$	-3.44	21.20		

size is 2.5%, and the power law distribution of jump sizes has a tail exponent of 1.15.[11] Matching macroeconomic quantities and asset prices in a model with standard time-additive preferences is notoriously difficult. While the economic mechanism of our model does not rely on rare disasters, introducing jumps drawn from a fat-tailed distribution, as described above, helps to generate the carry trade risk premium while maintaining reasonably low risk-free rates via the precautionary channel, all the while keeping the volatilities of output and consumption series close to their empirical counterparts.

The next step is pinning down the movements in the exogenous state variables. We simply set μ to generate average consumption and output growth rates near 2 percent. Our data analog for the shipping shock is the global shipping fleet as shown in Table 2 . While our specification is two simple to accommodate the time-to-build aspect of this data, we set $\sigma_k = .01/360$ to match approximately the volatility of innovations to the growth rate of shipping, and we set to $\psi_k = 0.001/360$ to give a vaery slow speed of mean-reversion. Related, in reduced-form we introduce a slow mean-reversion of commodity productivity to producer productivity, setting $\psi = 0.04/360$, mimicking that commodity production reacts to an increase in demand with a lag. The parameters governing mean reversion of the commodity production and shipping prices are chosen so that the commodity production reverts more quickly than shipping capital. This is consistent with the behavior of commodity prices and shipping costs after the crisis, and also consistent with Bessembinder, Coughenor, Seguin, and Smoller (1995) who document relatively rapid mean reversion in commodity prices, and Kalouptsidi (2014) who emphasizes the long production lags in the shipping industry.

In the model, all output is consumed, so output volatility is necessarily roughly equivalent to consumption volatility. This is contrast to the data, in which volatility of output is substantially higher than consumption. We therefore choose shock volatilities that yield output and consumption volatilities roughly between the values observed in the data, and σ_c and σ_p to 1.5%/360 and 2.5%/360 respectively. The producer country's consumption and output inherits the greater variance, consistent with the data. This completes the specification of exogenous shocks.

[11] Backus, Chernov, and Martin (2011) argue that equity option prices imply lower probabilities of consumption disasters than the magnitude required to match the equity premium.

We set $\beta = 0.5$ to give each countries an equal share of total producer output. Theoretical models of Hassan (2013) and Martin (2011) relate currency risk premia to country size, and by setting "size" equal we help distinguish our mechanism. We set $\kappa_0^c = \kappa_0^f = 0$ to reduce the number of parameters and because we do not try to match the average level of trade costs.

The remaining parameters κ_1^c, κ_1^f, γ, and ρ are jointly chosen so as to match and means and standard deviations of both risk-free rates and the carry trade return, and volatility of exchange rate (the model's stationarity ensures the change in the exchange rate's mean will be zero in population). Because these parameters are set jointly we conduct a sensitivity analysis on the two key ones, κ_1^f and γ, in Table 4.

Equation (10) shows that the value of κ_1^f is related to exchange rate volatility, and we choose a value near 0.7 that helps generate a number comparable to the data. Note that the value measure the marginal sensitivity of an additional good shipped and does not directly translate into the average level of trade cost.

The value of κ_1^c is not directly important in generating variation in the exchange rate. Producer country output is effectively given by the three exogenous shocks and final good exports are chosen that determine the value of the exchange rate. Increasing κ_1^c does, however, separate commodity country's output dynamics from the producer's. Lowering the commodity trade cost increases the flow of goods to the producer and increasing the volatility of the final-good trade cost and consumptions.

The modest degree of relative risk aversion $\gamma = 5$ allows us to closely match the average carry trade return of just over 4% per annum, while keeping the consumption growth volatilities not much higher than 2% per annum. Nevertheless, the model overshoots the levels of the risk-free rates somewhat in the benchmark calibration. Because of this we set $\rho = 0.001$. This is a familiar tension form the literature on the equity premium puzzle (e.g. Mehra and Prescott (1985)): it is difficult to generate a sufficiently strong precautionary motive with the CRRA utility to overcome the first-order terms in (11) and (12) within the confines of power utility with the levels of risk aversion that allow for substantial risk premia.[12]

[12]There is some debate in the literature about the extent to which rare disasters and peso problems contribute to currency risk premia. Models such as Farhi and Gabaix (2016) and Gourio, Siemer, and Verdelhan (2013) rely on rare disasters for explanations of the forward premium puzzle. Empirical evidence in Farhi,

The model also struggles to match the exchange rate volatility observed in the data. The volatility of exchange rates (and therefore currency carry strategy returns) in the benchmark model is below the empirical volatility of carry returns for the currencies we consider, at about 6% per annum vs. approximately 11.5% in the data.

When considering output, we are interested in an empirical proxy for the output of physical goods from each economy, and we therefore we use an empirical measure of total industrial production from the OECD as our main measure rather than raw GDP growth. We do report the GDP growth statistics for comparison. Likewise, for our measure of X_t, the exports of complex goods from the producer country to the commodity country, we use a measure of net exports of our classified complex good from the four producer G-10 countries. For consumption we would ideally have a measure of complex good consumption, but such a measure is not readily available, so we therefore use the standard measure of total household consumption of goods and services again from the OECD. Using these measures, volatilities of consumption and output growth are close to those in the data, with output volatility being lower in the model, and consumption volatility somewhat higher.

As pointed out above, we did not target the average levels of trade costs. The variable trade cost coefficients combined with the shipping sector dynamics generated by the model imply that the fraction of total exports of the final good that is lost to transportation frictions is substantial, at 29% and 18%, for commodities, both well within the range of values estimated by Anderson and van Wincoop (2004). We also do not target the dynamics of exports, trade costs, and commodity prices in our calibration. The benchmark model produces reasonable properties for these time series, even though it cannot match the levels of volatility observed in the data.

Since the model has no capital and, therefore, only features exports of consumption goods, it cannot produce as high a volatility of exports as observed in the data (just over 3% in the model compared to 9.5% in the data). The volatility of commodity prices (which is the only relative price in the model) is similar in magnitude, at about 3.5%. The empirical analogue, the Commodity Research Bureau Industrial Commodities index has a much higher volatility,

Fraiberger, Gabaix, Ranciere, and Verdelhan (2009), Jurek (2009), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2008), and Chernov, Graveline, and Zviadadze (2012) points to the importance of crash risk in explaining jointly the carry trade risk premia and prices of currency options.

Table 3: **Parameter values**

Parameter	**Value** Benchmark Model	**Description**
β	0.5	Cobb-Douglas producer-country labor share
γ	5	Relative risk aversion
ρ	0.001	Rate of time preference (annualized)
κ_0^c	0.0	Fixed commodity trade cost
κ_1^c	0.3	Variable commodity trade cost
κ_0^f	0.0	Fixed final trade cost
κ_1^f	0.7	Variable final trade cost
σ_p	0.025	Productivity shock volatility (annualized)
σ_k	0.01	Shipping shock volatility (annualized)
σ_c	0.015	ommodity shock volatility (annualized)
μ	0.02	Uncompensated TFP growth rate (annualized)
ψ	0.04/360	Mean reversion of commodity supply (z_c to z_p)
ψ_k	0.001/3600	Mean reversion of shipping capacity (z_k to z_c)
η	20 years	Average jump frequency
α	1.15	Power tail of jump
Z_{min}	2.5%	Minimum jump size
Z_{max}	125%	Maximum jump size

at over 15%. However, our assumed dynamics of commodity supply as well as the structure of demand driven by the Cobb-Douglas production function are probably too simple to capture the empirical commodity price dynamics.

Our empirical measure of trade costs is the Harpex Shipping index, which is an aggregate of container freight rates and is thus the right measure for the cost of shipping finished goods. It is likely a rather imperfect proxy, given that it only applies to a subset of goods, and only captures a fairly narrow component of total cost of trading such goods internationally. However, to our knowledge this is the only relevant measure of transportation cost that is observable at relatively high frequency. The dynamics of the trade costs produced by the model are much less volatile than those exhibited by Harpex: our model implies trade cost volatility of around 4% per annum vs. the 21% volatility exhibited by Harpex. Some of the discrepancy could be due to the fact that other components of trade costs that we do not observe are much less volatile. It could also indicate that our quadratic trade cost specification does not allow for enough variation, compared to the data, which is corroborated by unreported sensitivity analysis using more steeply convex trade cost functions.

4.1 Sensitivity analysis

Table 4 reports results of sensitivity analysis, in which the model is simulated for alternative values of the key parameters.

"Externality" calibration refers to the case where final good shippers earn zero profits, thereby not internalizing the congestion externality. In contrast to the benchmark model, this alternative solution struggles to produce a reasonably high exchange rate volatility, while at the same time generating higher volatility of exports and thus a higher volatility of marginal trade costs. The reason is that in the competitive equilibrium the representative agents attempt to equalize their marginal utilities at every point in time and every state of nature, without taking into account the full cost of doing so. Consequently, the allocation under externality features a much smoother profile of real exchange rates than the benchmark allocation, in which the representative shipping firm (or the planner) internalizes the effect of exports on trade costs. Since consumption sharing occurs via exports, the amount of final good shipped to the commodity country responds more to output fluctuations in the com-

Table 4: Sensitivity Analysis

Table shows changes in the benchmark moments of the calibrated model described in Table 2 in response to changing values governing the trade costs function $\tau_f = \kappa_0^f + \delta\kappa_1^f \frac{X}{z_k}$ and risk aversion (γ).

	Benchmark ($\delta = 2$)		Externality ($\delta = 1$)		$\kappa_1^f = 0.6$		$\gamma = 4$	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Output, Consumption, and Exports								
Δy_{pt}	2.18	2.81	2.15	2.79	2.15	2.87	2.18	2.80
Δy_{ct}	2.09	1.24	2.05	1.22	2.07	1.23	2.05	1.23
Δc_{pt}	2.18	2.67	2.09	2.34	2.14	2.69	2.09	2.73
Δc_{ct}	2.04	1.83	2.06	2.12	2.05	2.10	2.06	1.85
ΔX_t	2.18	2.96	2.20	3.24	2.16	3.05	2.17	2.85
Interest Rates and Carry Trade Returns								
r_{pt}^f	3.15	0.08	4.29	0.37	2.92	0.10	5.25	0.06
r_{ct}^f	6.56	0.25	5.37	0.37	5.88	0.22	6.57	0.22
$dRet_t$	4.15	5.96	1.01	1.75	3.35	4.07	1.86	4.87
Trade Costs, Commodity Prices, and Exchange Rates								
dS_t	-0.76	5.95	-0.14	1.49	-0.54	4.05	-0.55	4.86
dP_t	0.15	3.46	0.13	3.45	0.13	3.52	0.14	3.43
$d\tau_f$	0.16	3.75	0.20	4.11	0.15	3.84	0.16	3.62

petitive equilibrium, resulting in a less volatile wedge between the marginal utilities of the two countries, and hence a lower, (though still positive) premium to the carry-trade. Qualitatively the results are largely the same, in both the competitive and benchmark solution methods, the carry trade risk premium falls in times of lower trade costs.

The effect of the externality is quite different from that of simply lowering the convex shipping cost in the benchmark model (the table displays the result of lowering κ_1^f from 0.7 to 0.6). Because the marginal cost of shipping an additional final good is now lower, the variability of the exchange rate is dampened, and with it producer consumption volatility. The ability to share risk has effectively increased and the carry trade investor now requires a lower risk premium. The volatility of commodity country consumption, however, increases, as the country bears relatively more risk than before. Lowering exchange rate volatility while raising commodity consumption volatility creates a tradeoff. The volatility of the commodity price is a function of the exogenous shocks and κ_1^c and is little affected by the change in κ_1^f.

The other key parameter is the coefficient of relative risk aversion γ. Lowering it to 4 from 5 unsurprisingly cuts the carry trade risk premium almost by a half, from just over 4% per annum to just under 2%. The reduction in the average interest rate differential comes from the increase in the average risk free rate for the producer country to 5.3% due to the smaller demand for precautionary savings. Volatility of the real exchange rate also falls, to slightly under 5%, because a smaller need of risk sharing brings marginal utilities closer. In consequence, risk-free rates increase while exchange rate volatility falls, and our preferred calibration balances this tradeoff.

5 Great Recession: Model and Data

In order to gauge the severity of the Great Recession on shipping capacity and globalization post-recession, we simulate a Great Recession in our model and later compare its predictions regarding exchange rates and currency returns to the observed patterns data.

5.1 Simulating a Great Recession in the Model

To test the model's ability to capture the dynamics of exchange rates, commodity prices, and trade costs around the Great recession, we perform a simulation exercise where we take our benchmark calibration as the structure of our economy and use empirical data on realized growth rates of productivity and shipping capital to proxy for the fundamental shocks. Specifically, we take industrial production growth rates for our portfolio of producer countries and commodity countries as proxies for Δz_c and Δz_p.[13] Our proxy for Δz_k is the growth rate in the global merchant fleet. We detrend this variable by subtracting the difference between its average growth from 2000 - 2013 and the average growth of our industrial production measures over this period to account for the increase in demand for ships arising from the development of emerging countries, which are outside the model. We initialize the shocks to be $z_p = 1$, $z_c = 1.2$, and $z_k = 1.1$, which reflect the average relative values in our benchmark simulations. Lastly, all variables, with the exception of interest rates and relative interest rates, are defined as having a value set to 1 at the onset of the Great Recession in 2008.

Figure 4 shows the results of this simulation. Panel A shows the sequences of the exogenous state variables (z_p, z_c, z_k) in levels. The productivity of the producer country rises prior to the Great Recession, and then falls suddenly, consistent with Figure 1. In contrast, the productivity of the commodity country is largely smooth over the period. Shipping capital grows slowly early in the period, more quickly prior to the recession, and then especially quickly following the recession, as new ships ordered prior to the recession are completed (see Figure 2).

The remaining panels show the resulting time-series of endogenous model variables plotted with their data analogues. Panel B shows the model-implied series of commodity prices plotted against the level of the CRB Commodity Index. Both the model and the data exhibit a drop in the crisis, and the a recovery subsequent to the crisis as producer country industrial output recovers as well. In the data, the recovery of commodity prices is stronger than in the model. In contrast, Panel C shows the time-series of model-implied marginal trade costs

[13] Using labor productivity growth rates generates similar qualitative patterns, although the magnitude of the drop in the exchange rate is smaller during the Great Recession. But because we do not have a model of labor supply, we believe that using industrial production betters matches the decline in global economic activity.

plotted with the level of the Harpex. While the model is unable to match the large volatility of this series, we see the same general pattern. These costs rise in the boom, drop in the crisis, rebound briefly only to fall again as shipping capital accumulates in the later stages of the recovery. Panel D shows that this low level of marginal costs in the model translates to a continued low value for the commodity currency, and that this pattern is both quantitatively and qualitatively mirrored by the observed commodity/producer exchange rate in the data.

Finally, Panel E plots the interest rates for the two countries in the model with the GDP-weighted average interest rates of the producer and commodity countries in the data, while Panel F the time series of differences between the two interest rates in the model and data. In the model, the low barriers to risk sharing created by the continued low trade costs lead to a reduction in the risk-premium associated with the carry-trade. This leads to a narrowing of the interest rate differential, a pattern that is again consistent with the data. Quantitatively, the interest differential in the model falls by about 1% from 2007 to 2009, while the corresponding differential in the data falls by roughly 1.75%; the expected excess returns on the commodity currency carry trade at the end of our sample in 2013 are 2%, about one-third lower than before the Great Recession.

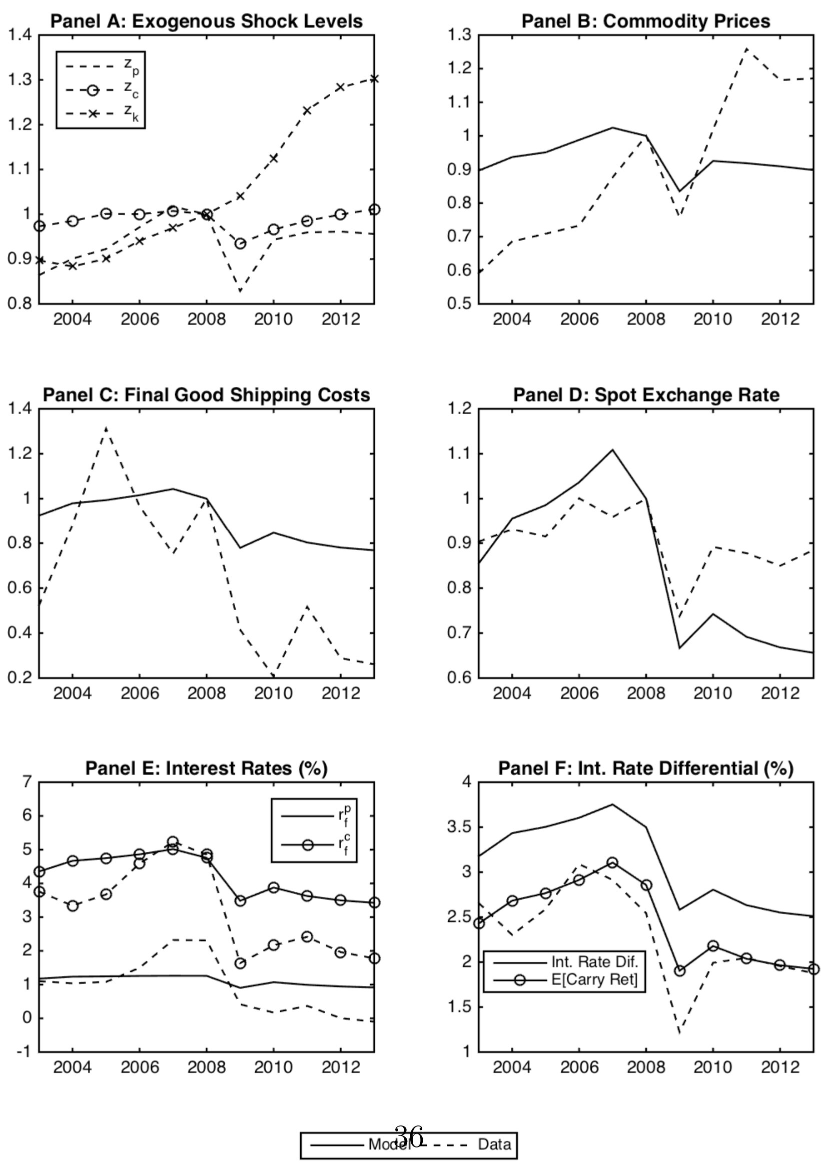
6 Conclusion

We introduce a simple two-country model featuring specialization in trade and a time-varying trade friction to study the behavior of exchange rates and currency carry trade returns around the Great Recession. In the model, persistent changes in trade costs arise from low frequency movements in shipping capacity, leading to endogenous time-varying dynamics in global market segmentation.

We use the calibrated model to account for the behavior of the relevant economic time series before and after the Great Recession. In our simulation, a run-up in global productivity prior to the recession leads to increases in commodity prices (as commodity supply struggles to catch up quickly) and in global shipping costs (as shipping capacity also lags behind). A widening wedge between the marginal utilities in the producer and the commodity countries leads to large carry trade profits. A large negative shock to productivity that follows a

Figure 4: Crisis Simulation: Model and Data

This figure shows the results of simulated global recession in the benchmark calibration of the model. The exercise simulates a single time series using realized annual growth rates for producer country and commodity country industrial production as well as growth rates of the global shipping fleet as proxies for shocks to the fundamental variables z_p,z_c, and z_k. The levels for the fundamentals, initialized at their steady-state value, are shown in Panel A. The remaining five panels show the model-implied time series for endogenous variables along with their data analog. Panel B plots model commodity costs with the level of the CRB all commodity index. Panel C plots marginal trade costs for the finished good in the model with the Harpex Index. Panel D plots the model value of the commodity currency with the commodity/producer country exchange rate from the data. Panel E plots the interest rates of the two countries in the model against the commodity and producer country interest rates in the data. Panel F plots the difference between the model and data interest rate series in Panel E, together with the expected carry trade returns implied by the model. All variables, with the exception of interest rate variables, are indexed to a value of one at the onset of the Great Recession in 2008. Data are described in Figures 1 and 2.



boom (i.e., the Great Recession) causes a sharp decline in commodity prices and trade costs, and a currency appreciation in the country which produces complex manufactured goods relative to the country which produces basic commodities. Subsequently, shipping costs remain depressed, due to overcapacity in the shipping sector.

Low shipping costs lead to low expected returns on the currency carry trade strategy as they imply a high level of global integration. Expected carry trade returns going forward are reduced by one third relative to their boom-year level. We verify the predictions of the model in the data, and show that the period since the recession has featured persistently low shipping costs, and lower interest rate differentials leading to lower returns to the carry trade strategy.

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Appendix

Model derivations

Output dynamics

Commodity output y_{ct} equals the level of z_{ct}, so that the final good output dynamics are given by

$$
\begin{aligned}
y_{pt} &= z_{pt}[z_{ct}(1-\tau_c(z_{ct}, z_{kt}))]^{1-\beta} \\
&= z_{pt} I(z_{ct}, z_{kt})^{1-\beta} \\
dy_{pt} &= dz^c_{pt} I_t^{1-\beta} \\
&+ z_{pt}(1-\beta) I_t^{-\beta} I_c dz^c_{ct} + \frac{1}{2} z_{pt}(1-\beta)\left(I_t^{-\beta} I_{cc} - \beta I_t^{-\beta-1} I_c^2\right) dz^c_{ct} dz^c_{ct} \\
&+ z_{pt}(1-\beta) I_t^{-\beta} I_k dz^c_{kt} + \frac{1}{2} z_{pt}(1-\beta)\left(I_t^{-\beta} I_{kk} - \beta I_t^{-\beta-1} I_k^2\right) dz^c_{kt} dz^c_{kt} \\
&+ d\left(\sum_{0<s\leq t}(y_{ps} - y_{ps^-})\right) \\
&= z_{pt}\mu_p I_t^{1-\beta} dt + z_{pt}\sigma_p I_t^{1-\beta} dB_{pt} \\
&+ \left(z_{pt}(1-\beta) I_t^{-\beta} I_c z_{ct}\mu_{ct} + \frac{1}{2} z_{pt}(1-\beta)\left(I_t^{-\beta} I_{cc} - \beta I_t^{-\beta-1} I_c^2\right) z_{ct}^2 \sigma_c^2\right) dt \\
&+ \left(z_{pt}(1-\beta) I_t^{-\beta} I_k z_{kt}\mu_{kt} + \frac{1}{2} z_{pt}(1-\beta)\left(I_t^{-\beta} I_{kk} - \beta I_t^{-\beta-1} I_k^2\right) z_{kt}^2 \sigma_k^2\right) dt \\
&+ z_{pt}(1-\beta) I_t^{-\beta} I_c z_{ct}\sigma_c dB_{ct} + z_{pt}(1-\beta) I_t^{-\beta} I_k z_{kt}\sigma_k dB_{kt} + z_{pt} I_t^{1-\beta}(e^{Z_{N(t)}} - 1) dN_t \\
\Rightarrow \frac{dy_{pt}}{y_{pt^-}} &= \mu_p dt + \sigma_p dB_{pt} \\
&+ (1-\beta)\left[\frac{I_c}{I_t} z_{ct}\mu_{ct} + \frac{1}{2}\left(\frac{I_{cc}}{I_t} - \beta\frac{I_c^2}{I_t^2}\right) z_{ct}^2\sigma_c^2\right] dt + (1-\beta)\frac{I_c}{I_t} z_{ct}\sigma_c dB_{ct} \\
&+ (1-\beta)\left[\frac{I_k}{I_t} z_{kt}\mu_{kt} + \frac{1}{2}\left(\frac{I_{kk}}{I_t} - \beta\frac{I_k^2}{I_t^2}\right) z_{kt}^2\sigma_k^2\right] dt + (1-\beta)\frac{I_k}{I_t} z_{kt}\sigma_k dB_{kt} \\
&+ \left(e^{Z_{N(t)}} - 1\right) dN_t \\
&\mathrel{\hat{=}} \mu_{yt} dt + \sigma_{yt}^T dB_t + (e^{Z_{N(t)}} - 1) dN_t,
\end{aligned}
$$

where $I(z_{ct}, z_{kt})$ and its derivatives are defined as follows:

$$
\begin{aligned}
I_t = I(z_{ct}, z_{kt}) &= z_{ct}(1 - \tau_c(z_{ct}, z_{kt})) \\
I_c &= (1 - \kappa_0^c) - 2\kappa_1^c \frac{z_{ct}}{z_{kt}} \\
I_{cc} &= -2\kappa_1^c / z_{kt} \\
I_k &= \kappa_1^c \frac{z_{ct}^2}{z_{kt}^2} \\
I_{kk} &= -2\kappa_1^c \frac{z_{ct}^2}{z_{kt}^3}
\end{aligned}
$$

Commodity price dynamics are given by

$$
\begin{aligned}
P_t &= (1 - \beta) z_{pt} \left[z_{ct}(1 - \tau_c(z_{ct}, z_{kt})) \right]^{-\beta} \\
&= \frac{(1 - \beta) y_{pt}}{(1 - \tau_c(z_{ct}, z_{kt})) z_{ct}}
\end{aligned}
$$

Competitive equilibrium allocation

We begin by generalizing (9):

$$
c_{ct}^{-\gamma} \left(1 - \kappa_0^f - \delta \kappa_1^f \frac{X_t}{z_{kt}} \right) = \frac{\lambda_c}{\lambda_p} c_{pt}^{-\gamma}. \tag{A-1}
$$

Here δ takes two possible values, depending on the type of competitive equilibrium we consider: in the case $\delta = 1$ the final good shipping industry earns zero profit, whereas in our benchmark case $\delta = 2$, profits are positive.

Solving for the consumption allocations, we have

$$
c_{ct} = c_{pt} \left[\frac{\lambda_p}{\lambda_c} \left(1 - \kappa_0^f - \delta \kappa_1^f \frac{X_t}{z_{kt}} \right) \right]^{\frac{1}{\gamma}}. \tag{A-2}
$$

Plugging this into the present value budget constraints in (3) and (7) and requiring that they are satisfied with equality for both countries obtains

$$
\begin{aligned}
0 &= \mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p} c_{pt} dt\right] - W_{p0} = \mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p} c_{pt} dt\right] - W_{c0}\frac{\beta}{1-\beta} \\
&= \mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p}\left(c_{pt} - \frac{\beta}{1-\beta}X_t\right) dt\right] = \mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p}\left(c_{pt} - c_{ct}\frac{\beta}{1-\beta}\left(1-\kappa_0^f - \delta\kappa_1^f\frac{X_t}{z_{kt}}\right)^{-1}\right) dt\right] \\
&= \mathbb{E}\left[\int_0^\infty \frac{\pi_t^p}{\pi_0^p}\left(c_{pt} - c_{ct}\frac{\beta}{1-\beta}\left(1-\kappa_0^f - \delta\kappa_1^f\frac{X_t}{z_{kt}}\right)^{-1}\right) dt\right].
\end{aligned}
$$

This equality will be satisfied if

$$\left[\frac{\lambda_p}{\lambda_c}\left(1-\kappa_0^f - \kappa_1^f\frac{X_t}{z_{kt}}\right)\right]^{\frac{1}{\gamma}} = \frac{1-\beta}{\beta}\left(1-\kappa_0^f - \delta\kappa_1^f\frac{X_t}{z_{kt}}\right).$$

Under competitive equilibrium ($\delta = 1$) in the special case of log utility ($\gamma = 1$) this yields

$$\frac{\lambda_p}{\lambda_c} = \frac{1-\beta}{\beta}.$$

In general the ratio of the Lagrange multipliers that satisfies the resource constraint is found numerically by iterating on the Monte Carlo simulation.

Exports of final consumption good

The first-order condition (9) implies that

$$G(X_t, z_{ct}, z_{pt}, z_{kt}) \equiv \left[X_t(1-\kappa_0^f - \kappa_1^f\frac{X_t}{z_{kt}})\right]^{-\gamma}\left(1-\kappa_0^f - \delta\kappa_1^f\frac{X_t}{z_{kt}}\right) - \lambda(y_{pt} - X_t)^{-\gamma} = 0 \quad \text{(A-3)}$$

must hold state by state for all t. In general, this nonlinear equation must be solved numerically, except for the special case of log utility ($\gamma = 1$), when it simplifies to

$$\kappa_1^f(\delta + \lambda)X_t^2 - [z_{kt}(1-\kappa_0^f)(1+\lambda) + \delta\kappa_1^f y_{pt}]X_t + (1-\kappa_0^f)y_{pt}z_{kt} = 0.$$

Solving this equation yields

$$X_t = \frac{z_{kt}(1-\kappa_0^f)(1+\lambda) + \ \delta\kappa_1^f y_{pt} - \sqrt{[z_{kt}(1-\kappa_0^f)(1+\lambda) + \ \delta\kappa_1^f y_{pt}]^2 - 4(1-\kappa_0^f) y_{pt} z_{kt} \kappa_1^f (\delta+\lambda)}}{2\kappa_1^f(\delta+\lambda)}$$

which is the only root that allows positive producer-country consumption. We can write

$$X_t = \frac{h(z_{ct}, z_{pt}, z_{kt}) - \sqrt{g(z_{ct}, z_{pt}, z_{kt})}}{2\kappa_1^f(\delta+\lambda)},$$

where

$$\begin{aligned} h(z_{ct}, z_{pt}, z_{kt}) &= z_{kt}(1-\kappa_0)(1+\lambda) + \ \delta\kappa_1 z_{pt} I_t^{1-\beta}, \\ g(z_{ct}, z_{pt}, z_{kt}) &= h(z_{ct}, z_{pt}, z_{kt})^2 - 4(1-\kappa_0)\kappa_1(\delta+\lambda) z_{pt} I_t^{1-\beta} z_{kt}. \end{aligned}$$

The derivatives of the export function and its components follow:

$$\begin{aligned} X_i &= \frac{h_i - \frac{1}{2} g^{-1/2} g_i}{\kappa_1(\delta+\lambda)}, \ \forall\, i = \{c, p, k\} \\ X_{ii} &= \frac{h_{ii} + \frac{1}{4} g^{-3/2} g_i^2 - \frac{1}{2} g^{-1/2} g_{ii}}{\kappa_1(\delta+\lambda)}. \end{aligned}$$

In the general CRRA case the derivatives of the export function can be found by implicit differentiation:

$$\begin{aligned} \frac{dX}{dz_i} &= -\frac{G_{z_i}}{G_X} \ \text{ for } i \in c, p, k \\ \frac{d^2 X}{(dz_i)^2} &= -\left(\frac{G_X \left(G_{z_i, X} \frac{dX}{dz_i} + G_{z_i, z_i} \right) - G_{z_i} \left(G_{X,X} \frac{dX}{dz_i} + G_{X, z_i} \right)}{(G_X)^2} \right) \end{aligned}$$

By normalizing each partial differential by X_t and by Ito's lemma,

$$
\begin{aligned}
dX_t(z_{ct}, z_{pt}, z_{kt}) &= X_{ct}X_t dz^c_{ct} + X_{pt}X_t dz^c_{pt} + X_{kt}X_t dz^c_{kt} \\
&\quad + \frac{1}{2}X_{cct}X_t dz^c_{ct}dz^c_{ct} + \frac{1}{2}X_{ppt}X_t dz^c_{pt}dz^c_{pt} + \frac{1}{2}X_{kkt}X_t dz^c_{kt}dz^c_{kt} \\
&\quad + d\left(\sum_{0<s\leq t}(X_s - X_{s^-})\right) \\
\Rightarrow \frac{dX_t}{X_{t^-}} &= \left\{X_c\mu_{ct}z_{ct} + X_{pt}\mu_p z_{pt} + X_{kt}\mu_{kt}z_{kt} + \frac{1}{2}X_{cct}\sigma_c^2 z_{ct}^2 + \frac{1}{2}X_{ppt}\sigma_p^2 z_{pt}^2 + \frac{1}{2}X_{kkt}\sigma_k^2 z_{kt}^2\right\}dt \\
&\quad + X_{ct}\sigma_c z_{ct}dB_{ct} + X_{pt}\sigma_p z_{pt}dB_{pt} + X_{kt}\sigma_k z_{kt}dB_{kt} + d\left(\sum_{0<s\leq t}\frac{X_s - X_{s^-}}{X_{t^-}}\right) \\
&\doteq \mu_{Xt}dt + \sigma_{Xt}^T dB_t + \left(e^{\mathcal{J}_X} - 1\right)dN_t,
\end{aligned}
$$

where $\mathcal{J}_X = \log\left(\xi(q_{t^-} + Z_{N(t)}, q_{kt^-})\right) - \log\left(\xi(q_{t^-}, q_{kt^-})\right)$, the log change in final goods exported, and where $\xi_t = \frac{X_t}{z_{kt}} \doteq \xi\left(q_t, q_{kt}\right)$ is exports of final good per unit of shipping capacity as a function of the two stationary state variables.

Since in the general case the export function must be found numerically, it is convenient to restate equation (9) as

$$
\left[\xi_t(1 - \kappa_0^f - \kappa_1^f\xi_t)\right]^{-\gamma}\left(1 - \kappa_0^f - \delta\kappa_1^f\xi_t\right) - \lambda\left[\exp\left(q_t + q_{kt}\right)\left(1 - \kappa_0^c - \kappa_1^c\exp\left(q_{kt}\right)\right)^{1-\beta} - \xi_t\right]^{-\gamma} = 0
$$

Then the numerical solution for ξ_t can be interpolated for use in simulations.

Consumption

For the consumption allocations we have

$$\begin{aligned}
c_{pt} &= y_{pt} - X_t \\
\Rightarrow dc_{pt} &= dy_{pt}^c - dX_t^c + d\left(\sum_{0<s\leq t}(c_{ps} - c_{ps^-})\right) \\
\Rightarrow \frac{dc_{pt}}{c_{pt^-}} &= \frac{1}{c_{pt^-}}\left(\mu_{yt} - \mu_{Xt}\right)dt + \frac{1}{c_{pt^-}}\left(\sigma_{yt}^T - \sigma_{Xt}^T\right)dB_t + d\left(\sum_{0<s\leq t}\frac{c_{ps} - c_{ps^-}}{c_{pt^-}}\right) \\
&\mathrel{\mathring{=}} \mu_{cpt}dt + \sigma_{cpt}^T dB_t + \left(e^{\mathcal{J}_p} - 1\right)dN_t
\end{aligned}$$

for the final good producer, and

$$\begin{aligned}
c_{ct} &= X_t\left(1 - \kappa_0^f - \kappa_1^f\frac{X_t}{z_{kt}}\right) \\
dc_{ct} &= (1 - \kappa_0^f)dX_t^c - \kappa_1^f d\left(\frac{(X_t^2)^c}{z_{kt}^c}\right) + d\left(\sum_{0<s\leq t}(c_{cs} - c_{cs^-})\right) \\
\Rightarrow \frac{dc_{ct}}{c_{ct^-}} &= \frac{1}{c_{ct^-}}\left\{\mu_{Xt}(1 - \kappa_0^f) - \kappa_1^f\left[\frac{1}{z_{kt}}(2X_t\mu_{Xt} + \sigma_{Xt}^T\sigma_{Xt}) - \frac{X_t^2}{z_{kt}}(\mu_{kt} - \sigma_k^2) - 2X_tX_{kt}\sigma_k^2\right]\right\}dt \\
&+ \frac{1}{c_{ct^-}}(1 - \kappa_0^f)\sigma_{Xt}^T dB_t - \frac{1}{c_{ct^-}}\kappa_1^f\frac{2X_t}{z_{kt}}\sigma_{Xt}^T dB_t - \frac{1}{c_{ct^-}}\kappa_1^f\frac{X_t^2}{z_{kt}}\sigma_k dB_{kt} \\
&+ d\left(\sum_{0<s\leq t}\frac{c_{cs} - c_{cs^-}}{c_{ct^-}}\right) \\
&\mathrel{\mathring{=}} \mu_{cct}dt + \sigma_{cct}^T dB_t + \left(e^{\mathcal{J}_c} - 1\right)dN_t
\end{aligned}$$

for the commodity producer.

Risk-free rates

In order to compute risk-free rates the expected growth rate of marginal utility conditional on a jump occurring must be computed as a function of the state variables. Let

$$\begin{aligned}\mathbb{E}_Z\left[e^{-\gamma\mathcal{J}_c}\right] &= \mathbb{E}_Z\left(\frac{\xi\left(q_{t^-}+Z,q_{kt^-}\right)\left(1-\kappa_0^f-\kappa_1^f\xi\left(q_{t^-}+Z,q_{kt^-}\right)\right)}{\xi\left(q_{t^-},q_{kt^-}\right)\left(1-\kappa_0^f-\kappa_1^f\xi\left(q_{t^-},q_{kt^-}\right)\right)}\right)^{-\gamma}\\ &\mathrel{\mathring{=}} \zeta_c\left(q_{t^-},q_{kt^-}\right),\end{aligned}$$

since the distribution of jump sizes is time invariant. Similarly, let

$$\begin{aligned}\mathbb{E}_Z\left[e^{-\gamma\mathcal{J}_p}\right] &= \mathbb{E}_Z\left(\frac{\exp\left(q_{t^-}+Z+q_{kt^-}\right)\left(1-\kappa_0^c-\kappa_1^c\exp\left(q_{kt^-}\right)\right)^{1-\beta}-\xi\left(q_{t^-}+Z,q_{kt^-}\right)}{\exp\left(q_{t^-}+q_{kt^-}\right)\left(1-\kappa_0^c-\kappa_1^c\exp\left(q_{kt^-}\right)\right)^{1-\beta}-\xi\left(q_{t^-},q_{kt^-}\right)}\right)^{-\gamma}\\ &\mathrel{\mathring{=}} \zeta_p\left(q_{t^-},q_{kt^-}\right).\end{aligned}$$

These functions can be evaluated by integrating over the distribution of jump sizes Z given by the pdf $\varphi\left(Z\right)=\frac{\alpha Z_{min}^{\alpha}x^{-\alpha-1}}{1-\left(\frac{Z_{min}}{Z_{max}}\right)^{\alpha}}$; this is done numerically using Gaussian quadrature.

Exchange rate

Since the spot exchange rate is defined as

$$S_t=\lambda\left(\frac{c_{pt}}{c_{ct}}\right)^{-\gamma}=\left(1-\kappa_0^f-\delta\kappa_1^f\frac{X_t}{z_{kt}}\right),$$

we can derive the dynamic evolution of exchange rate changes as

$$\begin{aligned}dS_t=-\delta\kappa_1^f&\left[\left(\frac{1}{z_{kt}}dX_t-\frac{X_t}{z_{kt}}(\mu_{kt}-\sigma_k^2)dt-\frac{X_t}{z_{kt}}\sigma_k dB_{kt}-X_{kt}\sigma_k^2dt\right)+\frac{1}{z_{kt}}d\left(\sum_{0<s\le t}\left(X_s-X_{s^-}\right)\right)\right]\\ =-\delta\kappa_1^f&\left[\frac{X_t}{z_{kt}}\left\{X_{ct}\mu_{ct}z_{ct}+X_{pt}\mu_p z_{pt}+X_{kt}\mu_{kt}z_{kt}+\frac{1}{2}X_{cct}\sigma_c^2z_{ct}^2+\frac{1}{2}X_{ppt}\sigma_p^2z_{pt}^2+\frac{1}{2}X_{kkt}\sigma_k^2z_{kt}^2\right\}dt\right.\\ &-\frac{X_t}{z_{kt}}(\mu_{kt}-\sigma_k^2)dt-X_tX_{kt}\sigma_k^2dt+\frac{X_t}{z_{kt}}X_{ct}\sigma_c z_{ct}dB_{ct}+\frac{X_t}{z_{kt}}X_{pt}\sigma_p z_{pt}dB_{pt}+\frac{X_t}{z_{kt}}\left(X_{kt}-1\right)\sigma_k dB_{kt}\\ &\left.+\left(\xi\left(q_{t^-}+Z_{N(t)},q_{kt^-}\right)-\ \xi\left(q_{t^-},q_{kt^-}\right)\right)dN(t)\right]\\ \Rightarrow\frac{dS_t}{S_{t^-}}&\mathrel{\mathring{=}}\mu_{St}dt+\sigma_{St}^TdB_t+\left(e^{\mathcal{J}_S}-1\right)dN(t),\end{aligned}$$

where $\mathcal{J}_S = \log\left(1 - \kappa_0^f - \delta\kappa_1^f \xi\left(q_{t^-} + Z_{N(t)}, q_{kt^-}\right)\right) - \log\left(1 - \kappa_0^f - \delta\kappa_1^f \xi\left(q_{t^-}, q_{kt^-}\right)\right)$

Expected Returns

Let

$$\mathbb{E}\left[dRet_t|\mathcal{F}_t\right] = \mathbb{E}\left[\frac{dS_t}{S_{t^-}}\frac{d\pi_{pt}}{\pi_{pt^-}}|\mathcal{F}_t\right] \mathrel{\mathring{=}} \mu_t^{FX}dt,$$

where μ_t^{FX} is the instantaneous conditional currency risk premium, which can be calculated as

$$\mu_t^{FX} = -\gamma\sigma_{St}^T\sigma_{cpt} + \eta\mathbb{E}_Z\left[\left(e^{\mathcal{J}_S} - 1\right)\left(e^{-\gamma\mathcal{J}_p} - 1\right)\right].$$