Search frictions and market power in negotiated price markets*

Jason Allen[a] Robert Clark[b] Jean-François Houde[c]

Abstract

This paper provides a framework for empirical analysis of negotiated-price markets in which buying is single-source. Negotiated-price markets feature search frictions, since consumers incur a cost to gather quotes, and long-term relationships between consumers and incumbent sellers, leading to the development of brand loyalty. Together, these characteristics imply that a firm with an extensive consumer base has an incumbency advantage. We use data from the Canadian mortgage market and a model of search and negotiation to characterize the impact of search frictions on consumer welfare and to quantify the role of search costs and brand loyalty for market power. Our results suggest that search frictions reduce consumer surplus by almost $12 per month per consumer, and that 28% of this reduction can be associated with discrimination, 22% with inefficient matching, and the remainder with the search cost. We also find that banks with large consumer bases have margins that are 70% higher than those with small consumer bases. The main source of this incumbency advantage is brand loyalty, however, the ability to price discriminate based on search frictions also accounts for almost a third of the advantage.

*This version: July 18, 2016. First version: February 2, 2012. Correspondence to [a]Jason Allen: Bank of Canada, Ottawa, Ontario, K1A 0G9; Email: jallen@bankofcanada.ca, [b]Robert Clark: Queen's University and HEC Montréal, Dunning Hall, 94 University Avenue, Kingston, Ontario, K7L 3N6; Email: robert.clark@hec.ca, [c]Jean-François Houde: Cornell University, 462 Uris Hall, Ithaca, NY 14850. E-mail: houde@cornell.edu. This research has benefited from the financial support of the NSF (SES-1024840). We thank the Canada Mortgage and Housing Corporation and Genworth Financial for providing us with the data. We also thank the Altus-Group. We thank the many seminar participants who have provided feedback on this paper. We have greatly benefited from discussions with Ken Hendricks, Ali Hortaçsu, Matt Lewis, Jack Porter, Dan Quint, Jacques Robert, Nicolas Sahuguet, Alan Sorensen, and Andrew Sweeting. The views in this paper are those of the authors and do not necessarily reflect those of the Bank of Canada. All errors are our own. This paper has previously circulated under the title: "Price negotiation in differentiated product markets: The case of insured mortgages in Canada".

1 Introduction

In a large number of markets, sellers post prices, but actual transaction prices are achieved via bilateral bargaining. This is the case for instance in the markets for new/used cars (Goldberg (1996), Scott-Morton et al. (2001), and Busse et al. (2006)), health insurance (Dafny (2010)), capital assets (Gavazza (2016)), financial products (Hall and Woodward (2012) and Allen et al. (2014a)), as well as for most business-to-business transactions (e.g. Joskow (1987), Town and Vistnes (2001), Grennan (2013), and Saiz (2015)).

In this paper, we are interested in two key features characterizing most of these markets. First, since buyers must incur a cost to gather price quotes, these markets are characterized by important search frictions. Second, the repeated relationship that develops between a buyer and a seller creates a *brand loyalty premium*, which increases the value of transacting with the same seller. This can be because of switching costs associated with changing suppliers, cost advantages of the incumbent sellers, or because of complementarities from the sale of related products (see Hannan and Adams (2011) for banks, Honka (2014) for insurance, and Chandra et al. (2015) for cars).

Search frictions and brand loyalty have implications for market power. Search costs open the door to price discrimination: the seller offering the first quote is in a quasi-monopoly position, and can make relatively high offers to consumers with poor outside options and/or high expected search costs. Brand loyalty reduces the bargaining leverage of consumers, because incumbent sellers provide higher value, which creates a form of lock-in. Together, these features imply that a firm with an extensive consumer base has an *incumbency advantage* over rival firms in the same market.

We consider one particular negotiated-price setting, the Canadian mortgage market, for which we have access to an administrative data-set on a large number of individually negotiated mortgage contracts, which we use to develop and estimate a model of search and price negotiation. In this market, national lenders post common interest rates, but in-branch loan officers have considerable freedom to negotiate directly with borrowers. Importantly, there is evidence of search frictions and brand loyalty in this setting. About 70% of consumers in this market combine day-to-day banking and mortgage services at their main financial institution and 80% get a rate quote from this lender. Moreover, despite the fact that approximately 60% of consumers search for additional quotes, only about 28% switch away from their main institution.

We make three contributions. First, we provide a framework designed to empirically analyze price negotiation in markets in which buying is single-source. This is the norm in consumer markets with negotiated prices, but also describes business-to-business negotiations where buyers transact with a single supplier. Second, we use the model to quantify the impact of search frictions on the welfare of consumers in these environments. Finally, we quantify the sources of market power, focusing especially on the incumbency advantage that stems from a large consumer base and decomposing it into two parts: (i) a first-mover advantage arising from price discrimination, and (ii) a loyalty premium originating from long-term relationships. This involves identifying the relative importance of search costs, and the loyalty premium associated with the *home* lender. Separately identifying these two sources of "state dependence" is an important contribution to both the industrial orgainzation and banking literatures, and has important policy implications. For instance, policies aimed at reducing search costs can have very different welfare implications if the small switching frequency that we observe is due to a high loyalty premium.

Despite their prevalence, markets with negotiated pricing have largely been ignored by empiricists. This is in part due to the fact that these markets pose important empirical challenges, the most serious of which is measuring the buyers' outside option. In posted-price markets this is straightforward. Rejected prices are those of competing products, and these are observed by the researcher. In contrast, in negotiated-price markets, the researcher cannot normally observe rejected prices. A number of approaches have been proposed to address this issue. The first is to impute the rejected price by using transaction prices from other buyers. Unfortunately, this approach comes with important selection problems. More recently, a new approach models the outside option as observed prices paid by a given buyer to alternative suppliers. This is justified by the simultaneous complete-information multi-lateral negotiation game proposed by Horn and Wolinsky (1988), and applied recently by Crawford and Yurukoglu (2012), Grennan (2013), Lewis and Pflum (2013), and Gowrisankaran et al. (2015). This method is suitable for the case of bargaining between buyers and their network of suppliers, but is not applicable when buyers transact with a single seller.

We propose a third approach based on equilibrium search, which is more appealing in markets where buying is single-source. Our approach is close in spirit to the literature in both labor economics and finance studying search and matching frictions in markets with bargaining. This literature, however, does not consider concentrated markets with differentiated firms, which characterize the situations listed above.[1] In contrast, although the existing models from the industrial organization literature studying search do take into account concentration and differentiation, they have mostly focused on cases where firms offer random posted-prices to consumers irrespective of their characteristics, thereby ignoring the presence of price discrimination.[2]

We build a two-period search model. Individuals are initially matched with their home bank for a quote, and can then decide, based on their expected net gain from searching, whether or not to gather additional quotes. The home bank uses this initial quote to price discriminate by screening high search-cost consumers. If it is rejected, consumers pay a search cost, and local lenders compete via an English auction for the contract.

This modeling strategy is related to search and bargaining models with asymmetric information developed by Wolinsky (1987), Chatterjee and Lee (1998), and Bester (1993), in which consumers

[1]The on-the-job search and over-the-counter markets literatures uses a similar price-setting mechanism. See for instance Postel-Vinay and Robin (2002), Dey and Flynn (2005) and Duffie et al. (2005).

[2]See for instance Hortaçsu and Syverson (2004) in the context of mutual funds, Alexandrov and Koulayev (2015) for mortgage rates, Hong and Shum (2006) for books, and Wildenbeest (2011) for grocery products. There is also a large literature in economics and marketing, devoted to measuring the magnitude of consumer search costs, using exogenous price distributions (see for instance Sorensen (2001), De Los Santos et al. (2012), and Honka (2014)).

negotiate with one firm, but can search across stores for better prices. In addition, the application of auction-like models to price negotiation settings has been used recently in the context of business-to-business transactions (e.g. Rosenbaum (2013), Beckert et al. (2016), Saiz (2015)), consumer markets (e.g. Hall and Woodward (2012) and Allen et al. (2014a)), and labor markets (e.g. Postel-Vinay and Robin (2002)). In our context, this modeling approach offers a tractable approximation to the multilateral negotiation that takes place when consumers decide to search. In addition, it allows us to discuss the identification of the model parameters in a transparent way.

The results can be summarized as follows. We find that firms face relatively homogeneous lending costs for the same borrower. In contrast, we find that borrowers face significant search costs and brand loyalty premia. On average, consumers in our sample face an upfront search cost of $1,150. In addition, the incumbent bank has an average cost advantage of $17.10/month (for a $100K loan) generating a sizeable loyalty premium.

We use the model estimates to characterize the impact of search frictions on consumer welfare and to measure market power. To quantify the welfare cost of search frictions, we perform a set of counter-factual experiments in which we eliminate the search costs of consumers. The surplus loss from search frictions originates from three sources: (i) misallocation of buyers and sellers, (ii) price discrimination, and (iii) the direct cost of gathering multiple quotes. Our results suggest that, overall, search frictions reduce average consumer surplus by almost $12 per month, over a five years period. Approximately 28% of the loss in consumer surplus comes from the ability of incumbent banks to price discriminate with their initial quote. A further 22% is associated with the misallocation of contracts, and 50% with the direct cost of searching. We also find that the presence of a posted-rate limits the ability of firms to price discriminate, and therefore reduces the welfare cost of search frictions. Competition also amplifies the adverse effects of search frictions on consumer welfare.

Our results also suggest that the market is fairly competitive. The average profit margin is estimated to be just over 20 basis points (bps), which corresponds to a Lerner index of 3.2%. However, margins vary considerably depending on whether consumers search and/or switch. On average, firms charge a markup that is 90% higher for consumers who are not searching. Banks' profits from switching consumers are $14.99/month (or 17.1 bps), compared to $20.22/month from loyal consumers (24.6 bps).

The increased profits earned from loyal consumers correspond to the incumbency advantage, and are directly related to the size of the bank's consumer base. To measure the source and magnitude of the advantage we use the simulated model to evaluate the correlation between consumer base and its profitability. We find that banks with the largest consumer bases earn, on average, 62% of the profits generated in their markets, compared to only 2% for those with the smallest. This difference is driven by the fact that large consumer-base lenders control a large share of the mortgage market, and earn significantly more profit per contracts than smaller banks.

We measure the incumbency advantage as the increased market power of banks with large consumers bases relative to those with the smallest. Our estimates suggest that banks with large consumer bases have margins that are 70% higher than those with small consumer bases. To identify the importance of the two sources of the incumbency advantage we simulate a series of counter-factual experiments aimed at varying the first-mover advantage and the differentiation component independently. Our results suggest that about 50% of the incumbency advantage can be directly attributed to brand loyalty, about 30% to search frictions and the remaining 20% to their interaction.

The paper is organized as follows. Section 2 presents details on the Canadian mortgage market and introduces our data sets. Section 3 presents the model, and Section 4 discusses conditions for non-parametric identification of the primitives. Section 5 discusses the estimation strategy and Section 6 describes the empirical results. Section 7 analyzes the impact of search friction and brand loyalty on consumer welfare and market power. Finally, section 8 concludes.

2 Institutional details and data

2.1 Institutional details

The Canadian mortgage market is dominated by six national banks (Bank of Montreal, Bank of Nova Scotia, Banque Nationale, Canadian Imperial Bank of Commerce, Royal Bank Financial Group, and TD Bank Financial Group), a regional cooperative network (Desjardins in Québec), and a provincially owned deposit-taking institution (Alberta's ATB Financial). Collectively, they control 90% of banking industry assets. For convenience we label these institutions the "Big 8."

Canada features two types of mortgage contracts – conventional, which are uninsured since they have a low loan-to-value ratio, and high loan-to-value, which require insurance (for the lifetime of the mortgage). Most new home-buyers require mortgage insurance. The primary insurer is the Canada Mortgage and Housing Corporation (CMHC), a crown corporation with an explicit guarantee from the federal government. During our sample period a private firm, Genworth Financial, also provided mortgage insurance, and had a 90% government guarantee. CMHC's market share during our sample period averages around 80%. Both insurers use the same insurance guidelines, and charge lenders an insurance premium, ranging from 1.75% to 3.75% of the value of the loan, which is passed on to borrowers.[3]

The large Canadian banks operate nationally and their head offices post prices that are common across the country on a weekly basis in both national and local newspapers, as well as online. Throughout our entire sample period the posted rate is nearly always common across lenders, and represents a ceiling in the negotiation with borrowers.[4]

[3] Appendix A describes the insurance rules, and defines all of the variables included in the data-set.

[4] In Canada pricing over the posted rate is illegal, and therefore this is a natural assumption. A similar setup is

According to the Ipsos-Reid survey, the majority of Canadians have a main financial institution where they combine their checking and mortgage accounts. Therefore, potential borrowers can accept to pay the rate posted by their home bank, or search for and negotiate over rates. Borrowers bargain directly with local branch managers or hire a broker to search on their behalf.[5] Our model excludes broker transactions and focuses only on branch-level transactions.

2.2 Mortgage data

Our main data set is a 10% random sample of insured contracts from the CMHC, from January 1999 to October 2002. The data-set contains information the terms of the contract (transaction rate, loan size, and house price), as well as financial and demographic characteristics of the borrower. In the empirical analysis we focus in particular on the income of the borrower, the FICO risk score, the loan-to-value ratio, and the 5-year bond-rate valid at the time of negotiation. In addition, we observe the closing date of the contract and the location of the purchased house up to the forward sortation area (FSA).[6,7]

The data set contains the lender information for twelve of the largest lenders during our sample period. For mortgage contracts where we do not have a lender name but only a lender type, these are coded as "Other credit union", and "Other trusts". The credit-union and trust categories are fragmented, and contain mostly regional financial institutions.[8] We therefore combine both into a single "Other Lender" category. Overall, therefore, consumers face 12 lending options.

We restrict our sample to contracts with homogenous terms. In particular, from the original sample we select contracts that have the following characteristics: (i) 25-year amortization period, (ii) 5-year fixed-rate term, (iii) newly issued mortgages (i.e. excluding refinancing), (iii) contracts that were negotiated individually (i.e. without a broker), (iv) contracts without missing values for key attributes (e.g. credit score, broker, and residential status).

The final sample includes around 26,000 observations, or about one-third of the initial sample. Approximately 18% of the initial sample contained missing characteristics; either risk type or business originator (i.e. branch or broker). This is because CMHC started collecting these transaction characteristics systematically only in the second half of 1999. We also drop broker transactions, (28%), as well as short-term, variable rate and refinanced contracts (40%).

implied in other retail markets featuring negotiation in the presence of *manufacturer's suggested retail prices.*

[5]Local branch managers compete against rival banks, but not against other branches of the same bank. Brokers are "hired" by borrowers to gather the best quotes from multiple lenders but compensated by lenders.

[6]The FSA is the first half of a postal code. We observe nearly 1,300 FSA in the sample. While the average forward sortation area (FSA) has a radius of 7.6 kilometers, the median is much lower at 2.6 kilometers.

[7]Based on the closing date we construct a posted price associated with each contract as the posted rate at closing if this yields a non-negative discount. If the generated discount is negative, the posted rate is taken to be the nearest-one that yields a positive discount.

[8]The "Other Bank" category includes mostly two institutions: Laurentian Bank and HSBC. The former is only present in Québec and Eastern Ontario, while the latter is present mostly in British Colombia and Ontario. We exploit this geographic segmentation and assign the "Other banks" customers to HSBC or Laurentian based on their relative presence in the local market around each home location.

We use the data to construct three main outcome variables: (i) monthly payment, (ii) negotiated discounts, and (iii) loyalty. The monthly payment, denoted by p_i, is constructed using the transaction interest rate, loan size, and the amortization period (60 months) specified in borrower i's contract. To construct negotiated discounts, we must first identify the posted rate valid at the time of negotiation. Since our contract data include only the closing date, to pin down the appropriate posted rate we infer the negotiation week that maximizes the aggregate fraction of consumers paying the posted rate (or 33 days prior to closing). Lastly, the loyalty variable is a dummy variable equal to one if a consumer has prior experience dealing with the chosen lender. Since 75% consumers are new home buyers, this most likely identifies the bank with which the borrower possess a savings or checking account. Note that this variable is not available for one lender, and we therefore treat the loyalty outcome as partly missing when constructing the likelihood function

Finally, since the main dataset does not provide direct information on the number of quotes gathered by borrowers, we supplement it with survey evidence from the Altus Group (FIRM survey). The survey asks 841 people who purchased a house during our sample period about their shopping habits. We use the aggregate results of this survey to construct auxiliary moments characterizing the fraction of consumers who report searching for more than one lender, by demographic groups. We focus in particular on city size, regions, and income groups.

2.2.1 Market-structure data

The market structure is described by the consumer base of each bank, and the number of lenders available in consumers' choice sets.

The consumer base of a lender is defined by its share of the market for day-to-day banking services. In the model, this is used to approximate the fraction of consumers in a given market that have prior experience with each potential lender. To construct this variable, we use micro-data from a representative survey conducted by Ipsos-Reid.[9] Each year, Ipsos-Reid surveys nearly $12,000$ households in all regions of the country. We group the data into by year, regions (10), and income categories (4). Within these sub-samples we estimate the probability of a consumer choosing one of the twelve largest lenders as their main financial institution, or home bank denoted by h. We use $\psi_h(x_i)$ to denote the probability that a consumer with characteristics x_i has prior experience with bank h.

The choice set of consumers is defined by the location of the house being purchased. In particular, we assume that consumers have access to lenders that have a branch located within 10 KM of the centroid of their FSAs. This choice is justified by the data: over 90% of loans are originated by a lender present within 10 KM of each FSA. In addition, the fact that rates are negotiated directly with loan officers limits the ability of consumers to perform the transaction online. Indeed, CMHC reports that less than 2% of mortgages are originated through the internet or phone.

[9] Source: Consumer finance monitor (CFM), Ipsos-Reid, 1999-2002.

Table 1: Descriptive statistics on mortgage contracts and loyalty in the selected sample

(a) Summary statistics

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Mean	Std-dev.	P25	P50	P75
Interest rate spread	120	59.3	81	115	161
Positive discounts	95.3	45.4	70	95	125
1(Discount=0)	.127	.333	0	0	0
Monthly payment	925	385	619	858	1169
Total loan ($/100K)	136	57.6	90.4	126	174
Income ($/100K)	68.4	27.9	48.5	64.1	82.1
FICO score	669	74	650	700	750
LTV	91	4.38	89.7	90	95
1(LTV=Max)	.385	.487	0	0	1
1(Previous owner)	.251	.433	0	0	1
1(Loyal)	.651	.477	0	1	1
Number of Lenders	8.65	1.44	8	9	10
Branch network	1.6	1.02	.989	1.37	1.93

(b) Reduced-form regression

	(1)	(2)
VARIABLES	Rate	1(Loyal)
1(Loyal)	0.097^a	
	(0.0079)	
Previous owner	0.025^a	0.11^a
	(0.0084)	(0.0072)
Branch network	0.023^a	0.026^a
	(0.0046)	(0.0045)
# Lenders (log)	-0.13^a	-0.076^a
	(0.022)	(0.019)
Observations	20,619	20,619
R-squared	0.612	0.095
Marg. effect: income	0.29	0.18
Marg. effect: loan	0.47	-0.19

Sample size = 26,218. Number of missing loyal observations = 5,599. The sample covers the period from January 1999 to October 2002. We trim the top and bottom 0.5% of observations in terms of income and loan size. Interest rates and discounts are expressed in percentage basis points (bps). The number of lenders is within 10KM of the borrowers new home (neighborhood). Relative branch is defined as the average network size of the chosen institution relative to the average size of others present in the same neighborhood. Each regression also includes markets and quarter/year fixed-effects, as well as other financial characteristics (i.e. posted-rate, bond-rate, FICO score, LTV, 1(LTV Max), loan size, income, loan/income.). Robust standard errors in parenthesis. Significance levels: ^{a} p<0.01, ^{b} p<0.05, ^{c} p<0.1.

The location of each financial institution's branches is available annually from Micromedia-ProQuest. We use this data set to match the new house location with branch locations, and construct the choice set of each consumer. Formally, a lender is part of consumer i's choice set if it has a branch located within less than 10 KM of the house location. We use $\mathcal{N}_i$ to denote the set of *rival* lenders available to consumer i (excluding the home bank), while n_i is the number of banks in $\mathcal{N}_i$.

2.3 Market features

Before introducing the model, we provides descriptive evidence outlining the key features of the Canadian mortgage market that we want to capture. Table 1a describes the main financial and demographic characteristics of the borrowers in our sample. Table 1b reports a subset of the coefficients of two reduced-form regressions describing the relationship between transaction characteristics and negotiation rates, as well as the probability of remaining loyal to the home-bank.

The estimation sample corresponds to a fairly symmetric distribution of income and loan-size. The average loan-size is about $136,000 which is twice the average annual household income. The loan-to-value (LTV) variable shows that many consumers are constrained by the minimum down-payment of 5% imposed by the government guidelines. Nearly 40% of households invest the minimum. Our focus in the paper will be on the monthly payment, L_i, made by borrower i, and so when we talk about quotes and rates, they will be based on a given monthly payment. The average monthly payment made by borrowers in our sample is $925.

In what follows we present five key *features* that characterize shopping behavior and outcomes in the Canadian mortgage market and most negotiated-price markets:

Feature 1: Mortgage transaction rates are dispersed. There is little within-week dispersion in posted prices, especially among the big banks, where the coefficient of variation on posted rates is very close to zero. In contrast, the coefficient of variation on transaction rates is 50%, and there is substantial residual dispersion as illustrated by the R^2 of 0.61 in Table 1b. See Allen et al. (2014b) for more details.

Feature 2: Consumers who are loyal and located in concentrated markets tend to pay higher rates. The rate regression shows that clients who remain loyal to their home bank receive discounts that are about 9.1 bps smaller than do new clients. It also shows that discounts are increasing in the number of local lenders and decreasing in relative network size.

Feature 3: Consumers search more than they switch. The search and negotiation process typically begins with the consumer's main financial institution—about 80% of consumers get a quote from their main institution (see Allen et al. (2014a)). A little over 60% of consumers search, but only about 28% switch away from their main institution.

Feature 4: Consumers tend to be more loyal in concentrated markets and to banks with larger branch networks. The loyalty regression shows that the likelihood of remaining loyal is decreasing in the number of lenders present in the market and increasing in relative network size.

Feature 5: Lenders with strong retail presence have larger market shares. On average consumers face 8.6 lenders within their neighborhood. Consumers tend to choose lenders with large branch networks; transacting with lenders that are nearly 60% larger than their competitors in terms of branches. Lenders with larger branch networks also tend to have a bigger share of the day-to-day banking market, generating a link between day-to-day market share and mortgage-market share that provides large banks with an incumbency advantage.

3 Model

Our modeling assumptions reflect the five key features listed in the previous section characterizing the mortgage industry, and that are also present in most negotiated-price markets. In addition, the model takes into account the fact that, during negotiation, loan officers can lower previously made offers in an effort to attract or retain potential clients. Furthermore, competition takes place locally between managers of competing banks, since consumers must contact loan officers directly to obtain discounts. We also suppose that branches that are part of the same network do not compete for the same borrowers, a feature of the Canadian mortgage market and of some, but not all, negotiated-price markets.

The next three subsections describe the model. First, we present preferences and cost functions, and the bargaining protocol. Then, we solve the model backwards, starting with the second stage of the game in which banks compete for consumers. Finally, we describe the consumer search decision, and the process generating the initial quote. To simplify notation we omit the borrower's index i, and will add it back in the next section for random variables and consumer characteristics.

3.1 Preferences and cost functions

Consumers solve a discrete-choice problem over which lender to use to finance their mortgage:

$$\max_{j \in \mathcal{J}} v_j - p_j, \tag{1}$$

where $\mathcal{J}$ is the set of lenders offering a quote, p_j denotes the monthly payment offered by lender j, and v_j denotes the maximum willingness-to-pay (or WTP) associated with bank j.

The choice set $\mathcal{J}$ is defined both by where consumers live, and by their search decision. In particular, consumers can obtain a quote from their home-bank (h) and from the n lenders in $\mathcal{N}$. We assume that the cost of obtaining a quote from the home-bank is zero, while the cost of getting additional quotes is $\kappa > 0$. This search cost does not depend on the number of quotes, and is distributed in the population according to CDF $H(\cdot)$.

The WTP of consumers is a combination of differentiation and mortgage valuation:

$$v_j = \begin{cases} \overline{v} + \lambda & \text{if } j = h, \\ \overline{v} & \text{else.} \end{cases}$$

The valuation for a mortgage, $\overline{v}$, is common across all lenders. Throughout we assume that it is large enough not to affect the set of consumers present in our sample. The parameter $\lambda \geq 0$ measures consumers' *willingness to pay for their home bank* relative to other lenders.

We also assume that banks have a constant borrower-specific marginal cost of lending. This measures the direct lending costs for the bank (i.e. default and pre-payment risks), net of the future benefits associated with selling complementary services to the borrower.[10] Since we do not observe the performance of the contract along the risk and complementarity dimensions, we use

[10] While lenders are fully insured against default risk, the event of default implies additional transaction costs to lenders that lower the value of lending to risky borrowers. Pre-payment risk is perhaps more relevant in our context, since consumers are allowed to reimburse up to 20% of their mortgage every year without penalty.

a reduced-form function to approximate the net present value of the contract. In particular, the monthly cost for bank j to lend to consumer i is:

$$c_j = \begin{cases} c - \Delta & \text{If } j = h, \\ c + \omega_j & \text{If } j \neq h, \end{cases} \tag{2}$$

where c is the common cost of lending to the consumer; ω_j the cost differential of lender j relative to the home bank (or its match value); and Δ is the *home bank's cost advantage*, arising from the fact it has an existing financial relationship with the consumer.[11] This cost advantage originates from the presence of switching costs, and/or complementarities between mortgage lending and other financial services (the home bank enjoys a cost advantage relative to rival lenders because it earns profits from other services).

As we will see below, the importance of brand loyalty in the market is driven by the sum of the cost and willingness-to-pay advantage of the home-bank: $\tilde{\Delta} = \Delta + \lambda$. We refer to $\tilde{\Delta}$ as the home-bank *loyalty premium*.

The idiosyncratic component, ω_j, is distributed according to $G(\cdot)$, with $E(\omega_j) = 0$. We use subscript (k) to denote the k^{th} lowest cost match value amongst the non home-bank lenders. The CDF of the k^{th} order statistic among the n lenders is given by $G_{(k)}(w|n) = \Pr(\omega_{(k)} < w|n)$.

Finally, lenders' quotes are constrained by a common posted price $\overline{p}$. The posted price determines both the reservation price of consumers (i.e. $\overline{v} > \overline{p}$), and whether or not consumers qualify for a loan at a given lender (i.e. $\overline{p} > c_j$).

3.2 Bargaining protocol and information

We model the negotiation process as a two-stage game. In the first-stage, the home-bank makes an initial offer p^0. At this point, the borrower can accept the offer, or search for additional quotes by paying the search cost κ. If the initial quote is rejected, the borrower organizes an English auction among the home-bank and the n other banks present in their neighborhood. The lender choice maximizes the utility of consumers, as in equation (1).

Information about costs and preferences is revealed sequentially as follows. At the initial stage, all parties observe the posted price $\overline{p}$, the number of rival banks n, the common component of the lending cost c, and the home-bank cost and WTP advantages (λ, Δ). These variables define the observed state vector: $s = (c, \lambda, \Delta, \overline{p}, n)$. The search cost is privately observed by consumers. The home bank knows only the distribution, which can vary across consumers based on observed de-

[11] Importantly, we rule out the possibility that the incumbent bank has more information than other lenders, since otherwise, the problem would involve adverse selection, and the initial quote would be much more complicated. For a discussion about competition when one firm has more information about a consumer learned from their past purchases see Fudenberg and Villas-Boas (2007). A subset of this literature has focused on credit markets and the extent to which lenders can learn about the ability of their borrowers to repay loans and use this information in their future credit-decisions and pricing. See for instance: Dell'Ariccia et al. (1999) and Dell'Ariccia and Marquez (2004).

mographic attributes. Finally, the idiosyncratic lending cost differences, ω_j, are privately observed by each lender in the second-stage of the game.

Before solving the game, two remarks are in order. First, note that consumers are price takers in the model, and therefore lenders have full bargaining power. This does not mean, however, that consumers have no bargaining leverage, since they have an informational advantage from knowing their own search cost. This prevents the home bank from extracting the entire surplus of consumers, as in Allen et al. (2014a).[12] Second, we assume that consumers pay the cost of generating offers at the auction stage (rather than firms). This implies that banks that are not competitive relative to the home bank are, in theory, indifferent between submitting and not submitting a quote. In these cases we assume that banks always submit a truthful offer that is consistent with their realized match values.

Next, we describe the solution of the negotiation by backward induction, starting with the competition stage.

3.3 Competition stage

Conditional on rejecting p^0, the home bank competes with lenders in the borrower's choice set. We model competition as an English auction with heterogeneous firms, and a cost advantage for the home bank. Since the initial quote can be recalled, firms face a reservation price: $p^0 \leq \overline{p}$.

We can distinguish between two cases leading to a transaction: (i) $\bar{p} < c - \Delta$, and (ii) $c < p^0 + \Delta \leq \overline{p} + \Delta$. In the first case the borrower does not qualify at the home bank. As such, the lowest cost bank wins by offering a price equal to the lending cost of the second most efficient qualifying lender:

$$p^* = \min\{c + \omega_{(2)}, \bar{p}\}. \tag{3}$$

This occurs if and only if, $0 < \bar{p} - c - \omega_{(1)}$.

If the borrower qualifies at the home bank, the highest surplus bank wins, and offer a quote that provides the same utility as the second best option. The equilibrium pricing function is:

$$p^* = \begin{cases} p^0 & \text{If } \bar{v} + \lambda - p^0 \geq \bar{v} - c - \omega_{(1)} \\ c + \omega_{(1)} + \lambda & \text{If } \bar{v} + \lambda - p^0 < \bar{v} - c - \omega_{(1)} < \bar{v} - c + \tilde{\Delta} \\ c - \tilde{\Delta} & \bar{v} - c - \omega_{(1)} > \bar{v} - c + \tilde{\Delta} > \bar{v} - c - \omega_{(2)} \\ c + \omega_{(2)} & \text{If } \bar{v} - c - \omega_{(2)} > \bar{v} - c + \tilde{\Delta}. \end{cases} \tag{4}$$

This equation highlights the fact that, at the competition stage, lenders directly competing with the home bank will on average have to offer a discount equal to the loyalty premium in order to

[12] Beckert et al. (2016) take a different approach, by assuming that consumers and firms split the *known* surplus from the auction using a Nash-Bargaining protocol. In this context, the relative bargaining power of consumers, instead of the search cost distribution, determines the split of the surplus.

attract new customers.[13]

3.4 Search decision and initial quote

The borrower chooses to search by weighing the value of accepting p^0, or paying a sunk cost κ in order to lower their monthly payment. The utility gain from search is:

$$\begin{aligned}\bar{\kappa}(p^0, s) &= \underbrace{\bar{v} + \lambda\left[1 - G_{(1)}(-\tilde{\Delta})\right] - E\left[p^*|p^0, s\right]}_{2^{nd} \text{ stage expected utility}} - \underbrace{\left[\bar{v} + \lambda - p^0\right]}_{1^{st} \text{ stage utility}} \\ &= p^0 - E\left[p^*|p^0, s\right] - \lambda G_{(1)}(-\tilde{\Delta}),\end{aligned}$$

where $1 - G_{(1)}(-\tilde{\Delta})$ is the retention probability of the home bank in the competition stage. A consumer will reject p^0 if and only if the gain from search is larger than the search cost. Therefore, the search probability is:

$$\Pr\left(\kappa < p^0 - E\left[p^*|p^0, s\right] - \lambda G_{(1)}(-\tilde{\Delta}|n)\right) \equiv H\left(\bar{\kappa}(p^0, s)\right). \tag{5}$$

Lenders do not commit to a fixed interest rate, and are open to haggling with consumers based on their outside options. This allows the home bank to discriminate by offering the same consumer up to two quotes: (i) an initial quote p^0, and (ii) a competitive quote p^* if the first is rejected.

The price discrimination problem is based on the expected value of shopping and the distribution of search costs. More specifically, anticipating the second-stage outcome, the home bank chooses p^0 to maximize its expected profit:

$$\max_{p^0 \leq \bar{p}} \quad (p^0 - c + \Delta)[1 - H(\bar{\kappa}(p^0, s))] + H(\bar{\kappa}(p^0, s))E(\pi_h^*|p^0, s),$$

where $E(\pi_h^*|p^0, s) = (p^0 - c + \Delta)(1 - G_{(1)}(p^0 - \lambda - c)) + \int_{-\tilde{\Delta}}^{p^0 - c - \lambda}(\omega_{(1)} + \tilde{\Delta})dG_{(1)}$. Importantly, the home bank will offer a quote only if it makes positive profit: $0 < \bar{p} - c$. The optimal initial quote first order condition is:

$$p^0 - c + \Delta = \underbrace{\frac{1 - H(\bar{\kappa}(p^0, s))}{H'(\bar{\kappa}(p^0, s))\bar{\kappa}_{p^0}(p^0, s)}}_{\substack{\text{Search cost} \\ \text{distribution}}} + \underbrace{E(\pi_h^*|p^0, s)}_{\substack{\text{Cost and quality} \\ \text{Differentiation}}} + \underbrace{\frac{H(\bar{\kappa}(p^0, s))}{H'(\bar{\kappa}(p^0, s))\bar{\kappa}_{p^0}(p^0, s)} \frac{\partial E(\pi_h^*|p^0, s)}{\partial p^0}}_{\text{Reserve price effect}}, \tag{6}$$

[13] Equations (3) and (4) also highlight the fact that the transaction price is determined by three lenders: the home bank, and the two most cost-efficient lenders. Therefore, while we assume that consumers search the entire choice-set, an implication of the model is that consumers need to obtain formal quotes from at most three lenders. This is in line with a Bertrand-Nash interpretation of the game, in which consumers learn the ranking of lenders' costs after paying the search cost, for instance through advertising, by calling banks directly, or indirectly through a real-estate agent.

where $\bar{\kappa}_{p^0}(p^0, s) = \frac{\partial \bar{\kappa}(p^0,s)}{\partial p^0}$. Equation (6) implicitly defines firms' profit margins from price discrimination. It highlights three sources of profits for the home bank: (i) positive average search costs, (ii) market power from differentiation in cost and quality (i.e. match value differences and home-bank cost advantage), and (iii) the reserve price effect. If firms are homogenous, the only source of profits will stem from the ability of the home bank to offer higher quotes to high search cost consumers.

Although the initial quote does not have a closed-form solution, in the following proposition (proven in Appendix B) we claim that, in the interior, it is additive in the common cost shock.

Proposition 1. *The optimal initial quote, p^0, is additive in c in the interior: $p^0 = c + \mu(\Delta, \lambda, n)$.*

Therefore we can characterize the initial quote by:

$$p^0 = \begin{cases} \bar{p} & \text{If } p^0(s) > \bar{p}, \\ \mu(n) + c & \text{Else.} \end{cases}$$

A number of predictions follow from Proposition 1, which we summarize in the following corollary.

Corollary 1. *The following predictions stem from Proposition 1:*
(i) The equilibrium search probability does not depend on c.
(ii) The equilibrium search probability is affected symmetrically by λ and Δ.
(iii) The distribution of p^* for switchers is only a function of $\tilde{\Delta}$.
(iv) In the interior, the average transaction price paid by loyal consumers is affected asymmetrically by λ and Δ, and the effect of λ is stronger.

To summarize, the model predicts three equilibrium functions: (i) the initial quote $p^0(s)$, (ii) the search-cost threshold $\bar{\kappa}(s)$, and (iii) the competitive price $p^*(\omega, s)$. In the interior solution, Proposition 1 and Corollary 1(i) imply that the initial quote's markup and the search-cost threshold are independent of c: $\mu(\Delta, \lambda, n)$ and $\bar{\kappa}(\Delta, \lambda, n)$.

4 Identification

In this section we provide conditions for non-parametric identification of the model. Our objective is to present a transparent argument that relies on observing the distribution of prices for loyal and switching consumers, as well as the conditional probability of remaining loyal. We propose a sequential approach that first identifies the lending-cost distribution and the loyalty-premium parameter from the distribution of prices for switchers. We then show that the distribution of the search cost and home-bank WTP advantage are separately identified from the conditional retention probability and average price of loyal borrowers.

Although the identification argument could naturally lead to a two-step estimator, our goal is not to estimate the model non-parametrically. Instead, the objective of this section is to clarify the link between the data and the primitives of the model, highlighting the role of the identifying assumptions. In the next section, we estimate a parametric version of the model, which allows us to more easily incorporate observable differences between consumers and firms.

There are four model primitives: (i) the distribution of the common lending cost (c_i), (ii) the distribution of idiosyncratic cost differences (ω_{ij}), (iii) the search-cost distribution (κ_i), and (iv) the loyalty premium parameters (λ_i, Δ_i). In addition, outcomes depend on the following observable state variables: (i) borrower financial characteristics x_i, (ii) the posted price $\bar{p}$, and (iii) the number of lenders n.

Crucial to our arguments presented below will be the existence of sufficient variation in $\bar{p}$ and n, conditional on x_i. In particular, our data set is a panel of locations and periods, over which we observe a (large) cross section of borrower characteristics x_i. For each consumer, the negotiation period $t(i)$ determines the posted price and other time-varying cost shifters, while the location determines the number of lenders available. Although we are not restricting the correlation between observable characteristics and $(\bar{p}_{t(i)}, n_i)$, it is essential that the supports of borrower characteristics be comparable across periods and markets.

Assumption 1 describes the relationship between the primitives of the model and observable characteristics of each transaction.

Assumption 1. *The primitive distributions of the model satisfy the following assumptions:*
(i) The distribution of the common lending cost is conditionally independent of $(n, \bar{p})$*:* $\Pr(c_i < c|x_i, n, \bar{p}) = F(c|x_i)$.
(ii) The idiosyncratic cost differences are IID across consumers: $\Pr(\omega_{ij} < \omega|x_i, n, \bar{p}) = G_j(\omega)$.
(iii) The search cost distribution is a function of borrower characteristics $z_i^1 \in x_i$ *and independent of* $(n, \bar{p})$*:* $\Pr(\kappa_i < \kappa|x_i, n, \bar{p}) = H(\kappa|z_i^1)$.
(iv) The loyalty premium parameters are deterministic functions of borrower characteristics $z_i^2 \in x_i$*:* $\lambda_i = \lambda(z_i^2)$ *and* $\Delta_i = \Delta(z_i^2)$.

These assumptions clarify that the model is identified by two key exclusion restrictions, as well as one important assumption regarding the distribution of the common cost and WTP advantages. On the first point, we assume that the lending and search costs are independent of the posted price and the structure of local markets. Similarly, Assumption 1(*iv*) restricts the loyalty premium to be a deterministic function of observable attributes. This is crucial since we do not separately observe search and loyalty. This restriction could in theory be relaxed with richer data on search and firm choices.

In addition, we use the following support assumptions. We do not impose these assumptions in the empirical application, since we use parametric distributions to estimate the model.

Assumption 2. *The data generating process is such that:*
(i) The common lending cost is distributed over a known support: $c_i \in [\underline{c}, \bar{c}]$.
(ii) The posted price, $\bar{p}$*, has a full support between a lower bound* $\bar{\bar{p}}$ *and* ∞.
(iii) The number of lenders has full support between 2 *and* $\bar{n} \geq 4$.

The rest of the section is organized as follows. First, we formally define the relationship between observed and predicted outcomes. Then, we discuss the identification of the model primitives in two steps: (i) identification of the lending cost distributions $\{F(c|x_i), G(\omega)\}$ and loyalty premium $\tilde{\Delta}$, and (ii) identification of the search cost distribution ($H(\kappa)$) and home-bank cost/WTP advantages $\{\lambda, \Delta\}$. In order to make the identification discussion more transparent, we focus on a special case of the model in which the loyalty premium parameters are constant across consumers (i.e. $\lambda(z_i^1) = \lambda$ and $\Delta(z_i^1) = \Delta$ for all i), and the search-cost distribution is independent of consumer observable characteristics (i.e. $H(\kappa|z_i^2) = H(\kappa)$). We also abstract from observable differences between lenders (i.e. $G_j(\omega) = G(\omega)$ for all j). Both assumptions can be relaxed without affecting the key results.

4.1 Identification problem and observed outcomes

The identification problem can be summarized as follows. First, we need to separately identify two sources of unobserved lending cost heterogeneity, the common component c_i and the lender-specific idiosyncratic component ω_{ij}, using only data on transaction prices. Second, we need to distinguish between two sources of brand loyalty—search costs and loyalty premia–using data on the conditional probability of remaining loyal to the home-bank. Finally, we need to demonstrate that the two sources of the loyalty premium—cost and WTP advantages—are separately identified.

Consider an *ideal* data set (i.e. one that satisfies Assumption 2), which allows us to measure three empirical distributions: (i) the conditional distribution of prices for switching consumers, (ii) the conditional distribution of prices for loyal consumers, and (iii) the conditional switching probability. These three outcomes are observed conditional on the vector of observed borrower characteristics, and over the full support of the distribution of the posted price and the number of lenders. The following three equations summarize the link between the model and the data:

$$\Phi^S\left(p\middle|x_i,\bar{p},n\right) = \frac{\Pr\left(p^*(\omega,s) \leq p, \omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s)|x_i,\bar{p},n\right)}{\Pr\left(\omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s)|x_i,\bar{p},n\right)} \tag{7}$$

$$\Phi^L\left(p\middle|x_i,\bar{p},n\right) = \frac{\Pr\left(p^0(s) \leq p, \bar{\kappa}_i > \bar{\kappa}(s)|x_i,\bar{p},n\right) + \Pr\left(p^*(\omega,s) \leq p, \omega_{(1)} > -\tilde{\Delta}, \bar{\kappa}_i > \bar{\kappa}(s)|x_i,\bar{p},n\right)}{1 - \Pr\left(\omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s)|x_i,\bar{p},n\right)} \tag{8}$$

$$S(x_i,\bar{p},n) = \Pr\left(\omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s)|x_i,\bar{p},n\right), \tag{9}$$

where $\Phi^S\left(p\middle|x_i,\bar{p},n\right)$ and $\Phi^L\left(p\middle|x_i,\bar{p},n\right)$ are the empirical distributions of prices for switching and loyal consumers, respectively, and $S(x_i,\bar{p},n)$ is the empirical switching probability. The first two equations highlight the fact that the distribution of prices for loyal consumers is a mixture of prices coming from searchers and non-searchers, while the distribution of prices for switching consumers is solely a function of the outcome of the auction. Similarly, the switching probability is a combination of two factors: consumers rejecting the initial offer, and the home bank losing at the competition stage.

4.2 Identification of the lending cost and loyalty premium

We first discuss identification of the distributions of c_i and ω_{ij}, as well as the loyalty premium ($\tilde{\Delta} = \lambda - \Delta$). To do so, we focus solely on the distribution of prices for switching consumers described in equation (7). This sub-sample is appealing, since prices are generated directly from the auction, and therefore are not directly a function of the search-cost distribution.

The challenge is to address a potential selection bias: consumers reaching the second stage of the game must first reject $p^0(s)$, which is a function of the unobserved common lending cost component. This endogenous selection implies that the lending cost distribution among switchers is different from the unconditional distribution $F(c|x_i)$.

To get around this problem we use the full-support assumption of $\bar{p}$ to eliminate the dependence of the selection probability on c. In particular, note that as $\bar{p} \to \infty$ the switching probability is independent of the common lending cost distribution:

$$\begin{aligned}\lim_{\bar{p}\to\infty} \Pr\left(\omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s)|x_i, \bar{p}, n\right) &= \lim_{\bar{p}\to\infty} \int G_{(1)}(-\tilde{\Delta}) H(\bar{\kappa}(s)) dF(c|x_i) \\ &= H(\bar{\kappa}(\lambda, \Delta, n)) G_{(1)}(-\tilde{\Delta}), \end{aligned} \tag{10}$$

where the last line follows from the fact that the search probability is independent of c when the initial quote is unconstrained (Corollary 1(i)).

This result implies that as $\bar{p} \to \infty$ the conditional distribution of prices for switching consumers is independent of the search-cost distribution:

$$\begin{aligned}\lim_{\bar{p}\to\infty} \Phi^S\left(p\middle|x_i, \bar{p}, n\right) &= \lim_{\bar{p}\to\infty} \frac{\Pr\left(p^*(\omega, s) \leq p, \omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s)|x_i, \bar{p}, n\right)}{\Pr\left(\omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s)|x_i, \bar{p}, n\right)} \\ &= \frac{H(\bar{\kappa}(\lambda, \Delta, n) \Pr(c_i + \min\{-\tilde{\Delta}, \omega_{(2)}\} < p, \omega_{(1)} < -\tilde{\Delta}|x_i, n)}{H(\bar{\kappa}(\lambda, \Delta, n)) G_{(1)}(-\tilde{\Delta})} \\ &= \Pr\left(c_i + \min\{-\tilde{\Delta}, \omega_{(2)}\} < p|\omega_{(1)} < -\tilde{\Delta}, x_i, n\right). \end{aligned} \tag{11}$$

The second equality follows from equation (10). Therefore, the distribution of c_i within the sample of *unconstrained* switching consumers is equal to the unconditional distribution of the common lending cost $F(c|x_i)$.[14]

Using this sub-sample, it is easy to show that the distributions of c_i and ω_{ij} and the loyalty premium $\tilde{\Delta}$ are separately identified. To see this, the distribution of prices in markets with two lenders is truncated from below by $\underline{c} - \tilde{\Delta}$. This is because the home bank faces only one rival ($n = 1$), and the price paid by switchers is equal to $p^* = c - \tilde{\Delta}$. Therefore, the minimum price paid by unconstrained switchers can be used to identify $\tilde{\Delta}$, while the remaining distribution of prices directly identifies the common lending cost:

$$F(c|x_i) = \Phi^S \left(p + \tilde{\Delta} \middle| x_i, \bar{p} = \infty, n = 1 \right).$$

The distribution of idiosyncratic cost differences ω_j can be identified using minimal variation in the number of bidders. In particular, the distribution of prices for each $n > 1$ is given by:

$$\Phi^S \left(p + \tilde{\Delta} \middle| x_i, \bar{p} = \infty, n \right) = F(p - \tilde{\Delta}|x_i) \frac{\left[G_{(1)}(-\tilde{\Delta}|n) - G_{(2)}(-\tilde{\Delta}|n) \right]}{G_{(1)}(-\tilde{\Delta}|n)} + \int_{\omega_{(2)} < -\tilde{\Delta}} F(p - \omega_{(2)}) \frac{dG_{(2)}(\omega_{(2)}|n)}{G_{(1)}(-\tilde{\Delta}|n)}. \quad (12)$$

Since $\tilde{\Delta}$ and $F(c|x_i)$ are known, equation (12) can be inverted under standard conditions to recover the distribution of idiosyncratic cost differences $G(\omega)$.

The previous argument depends on observing at least two market structures, including $n = 1$. However, observing duopoly markets is not necessary. For instance, Quint (2015) shows that observations of transaction prices and at least two different market structures of any size are sufficient to non-parametrically identify ascending auction models with additively-separable unobserved heterogeneity. Quint's identification argument relies on a condition that valuations be bounded below by zero. Quint's proof can be adapted to our procurement context with observations only for transaction prices from switchers. In this setting, identification requires that the common lending cost be bounded above as in Assumption 2(i) (see Appendix C).

If more than two sizes of auctions are observed, the model is over-identified which allows us to identify $\tilde{\Delta}$ without using the distribution of prices in $n = 1$. Intuitively, the loyalty premium is identified by observing how the distribution of prices for switchers changes with the number of lenders. In markets with a small number of lenders, the presence of a positive loyalty premium implies that the distribution of prices for switchers mostly reflects the common cost component, since the home bank is likely the next-best alternative for switching consumers. This is because $p^* = c_i - \tilde{\Delta}$ if $\omega_{(2)} > -\tilde{\Delta}$, and $\Pr\left(\omega_{(2)} > -\tilde{\Delta}|n\right)$ is close to one when n is small and $\tilde{\Delta}$ is large. In contrast, as the number of rival lenders increases, the probability that $\omega_{(2)} < -\tilde{\Delta}$ converges to one, which implies a stronger correlation between n and the price paid by switchers. Therefore, the

[14] This result is analogous to the *identification at infinity* arguments used in labor economics to identify Roy models (e.g. French and Taber (2011)).

loyalty premium is identified from the strength of the correlation between n and p^*, as the number of competitors becomes large.

4.3 Identification of the search-cost distribution and willingness-to-pay

In the previous subsection, we explained how to use variation in n and the full-support assumption to identify $F(c|x_i)$, $G(\omega)$ and $\tilde{\Delta}$. To see how the home-bank WTP advantage and the search-cost distribution are identified, consider first the switching probability conditional on a guess of λ:

$$\begin{aligned} S(x_i, \bar{p}, n) &= \Pr\left(\omega_{(1)} < -\tilde{\Delta}, \kappa_i < \bar{\kappa}(s) | x_i, \bar{p}, n\right) \\ &= \int G_{(1)}(-\tilde{\Delta}) H\left(\bar{\kappa}(s)\right) dF(c|x_i) = \int m(s) dF(c|x_i), \end{aligned} \tag{13}$$

where $\bar{\kappa}(s) = p^0(s) - E(p^*|p^0(s)) - \lambda G_{(1)}(-\tilde{\Delta})$ is the search-cost threshold.

Contrary to the argument used in the previous section, the search-cost distribution **cannot** be non-parametrically identified if all consumers are unconstrained by the posted price. To see this, note that when $\bar{p} \to \infty$ the predicted search probability is constant for all consumers facing the same market structure. At most, equation (13) would allow us to estimate the search probability for each $n = 1, 2, \ldots, \bar{n}$. Since the threshold function $\bar{\kappa}(s)$ is an implicit function of the entire search-cost distribution, this is clearly not enough to identify the entire distribution.

As a solution to this problem, we need to exploit variation in the probability of being constrained by the posted price. This can be done for instance by varying $\bar{p}$, or by varying elements of x_i that are positively correlated with the lending cost. For consumers receiving an initial quote of $\bar{p}$, the search threshold is a known function of $\{F(c|x), G(\omega), \Delta, \lambda\}$:

$$\bar{\kappa}(s) = \bar{p} - E(p^*|\bar{p}, s) - \lambda G_{(1)}(-\tilde{\Delta}), \quad \text{if } p^0(s) = \bar{p}.$$

It is easy to show that for these consumers, the search probability is an increasing function of $\bar{p}$, and a decreasing function of c_i. Therefore, exogenous variation in the posted-price and in the observable risk of consumers can be used to trace out the support of κ_i in equation (13) by varying continuously the search threshold of consumers.

More formally, conditional on knowing $\{F(c|x_i), G(\omega), \Delta, \lambda\}$, the identification of the search-cost distribution is analogous to the identification of non-parametric instrumental regression models (e.g. Newey and Powell (2003)). Under standard completeness conditions, one can show that there is a unique solution $m(s) \equiv G_{(1)}(-\tilde{\Delta})H\left(\bar{\kappa}(s)\right)$ to equation (13). With this non-parametric function in hand, we can use the solution of the model to compute the equilibrium search-cost thresholds and characterize the entire search-cost distribution.

Finally, to distinguish between the home-bank WTP and cost advantages, we must rely on the distribution of prices among loyal consumers. Recall from equation (8) that this distribution is a mixture of initial quote offers and auction prices. We know from Corollary 1(iv) that λ and Δ have different impacts on the average transaction price of loyal consumers. In contrast, λ and Δ affect symmetrically the equilibrium search probability (Corollary 1(ii)), and the distribution of prices for switchers is only a function of the sum $\tilde{\Delta}$ (Corollary 1(iii)). Therefore, while both parameters influence in the same way the observed retention probability, they have different effects on the average price difference between loyal and switching consumers. This moment can thus be used to identify λ separately from Δ.

4.4 Auxiliary data and additional moments

The previous discussion illustrates how the model parameters can be non-parametrically identified by looking separately at the distribution of prices for switchers, as well as the conditional retention probability and average price of loyal borrowers. While this strategy is transparent, it does not fully exploit the empirical predictions of the model. In particular, we have not used most of the variation present in the distribution of prices for loyal borrowers.

Furthermore, additional aggregate search moments can be used to improve the precision of the estimates. With this additional information, the separate identification of the search and switching parameters becomes even more transparent. We now have two measures of state-dependence: the average switching probability ($\bar{S}$) and the average search probability ($\bar{H}$). Using these measures, one can use the predicted switching probability in equation (9) to estimate the aggregate retention probability of the home-bank at the auction-stage:

$$\bar{S} = \bar{H} \times G_{(1)}(-\tilde{\Delta}), \qquad G_{(1)}(-\tilde{\Delta}) = \frac{\bar{S}}{\bar{H}}$$

For instance, in our sample the average switching probability is approximately 30%, while the aggregate search probability from the FIRM survey is 65%. On average, the home bank therefore wins the auction with probability 46%. Since the number of lenders per neighborhood is 8 on average, this implies that the loyalty premium is positive and large relative to the dispersion of idiosyncratic cost differences.

5 Estimation method

In this section we describe the steps taken to estimate the model parameters. We begin by describing the functional form assumptions imposed on consumers' and lenders' unobserved attributes. We then derive the likelihood function induced by the model, and discuss the sources of identification.

5.1 Distributional assumptions and functional forms

The lending cost function differs slightly from the model presentation. In particular, we account for loan size differences across borrowers, and we allow observed bank characteristics to affect the distribution of cost differences across lenders (i.e. ω_{ij} and Δ_i).

We model the monthly cost of lending $\$L_i$ over a 25 year amortization period using a linear function of borrower and lender characteristics:

$$c_{ij} = L_i \times (c_i + \omega_{ij}), \tag{14}$$

where the common cost component is normally distributed, $c_i \sim N(x_i\beta, \sigma_c^2)$, and the idiosyncratic cost differences are distributed according to a lender-specific type-1 extreme value distribution, $\omega_{ij} \sim \text{T1EV}(\xi_{ij} - \gamma\sigma_\omega, \sigma_\omega)$.[15]

The location parameter of the idiosyncratic cost difference distribution, ξ_{ij}, varies across lenders due to the presence of bank fixed-effects, and the size of the branch network in the neighborhood of the consumer (normalized by the average network size of rivals). The type-1 extreme-value distribution assumption leads to analytical expressions for the distribution functions of the first- and second-order statistics, and is often used to model asymmetric value distributions in auction settings (see for instance Brannan and Froeb (2000)).

The loan size is normalized so that the per-unit lending cost in equation (14) measures the monthly cost of a \$100,000 loan. The vector x_i controls for observed financial characteristics of the borrower (e.g. income, loan size, FICO score, LTV, etc), the bond-rate, as well as period and location fixed-effects. The location fixed-effects identify the region of the country where the house is located, defined using the first digit of the postal code (i.e. postal-code district). The period fixed-effects are defined at the quarter-year level.

The lending cost of the home bank is expressed slightly differently, because of the home-bank cost-advantage parameter:

$$c_{i,h(i)} = L_i \times \left(c_i + \Delta_{i,h(i)}\right),$$

where $h(i)$ is the home-bank index of borrower i, and $\Delta_{i,h(i)} = \xi_{i,h(i)} - \Delta(z_i^2)$ is consumer i's home-bank deterministic cost differential. In the application, we allow the cost-advantage parameter to depend on the borrower's income and home-ownership status:

$$\Delta(z_i^2) = L_i \times (\Delta_0 + \Delta_{inc}\text{Income}_i + \Delta_{owner}\text{Previous Owner}_i).$$

The WTP component of the loyalty premium is defined analogously as a linear function of

[15]The location parameter of the type-1 extreme-value distribution is adjusted by a factor $\gamma\sigma_\omega$ to guarantee that the error is mean zero (i.e γ is the euler constant).

income and home-ownership status:

$$\lambda(z_i^2) = L_i \times (\lambda_0 + \lambda_{inc}\text{Income}_i + \lambda_{owner}\text{Previous Owner}_i).$$

Finally, we assume that the search cost is exponentially distributed with a consumer-specific mean that depends on income and home-ownership status:

$$H(\kappa|z_i^1) = 1 - \exp\left(-\frac{1}{\alpha(z_i^1)}\kappa\right), \quad \log\alpha(z_i^1) = \alpha_0 + \alpha_{inc}\log\text{Income}_i + \alpha_{owner}\text{Previous Owner}_i.$$

5.2 Likelihood function

We estimate the model by maximum likelihood. The endogenous outcomes of the model are: the chosen lender and monthly payment $\{b(i), p_i\}$, as well as whether consumers remain loyal to their home-bank or switch. The observed prices are either generated from consumers accepting the initial quote (i.e. $p_i = p^0(s)$), or accepting the competitive offer (i.e. $p_i = p^*(\omega, s)$). Importantly, only the latter case is feasible if consumers switch financial institutions, while both cases have a positive likelihood for loyal consumers.

Moreover, the identity of the home bank is known for loyal consumers, while it unobserved for switching consumers. To construct the likelihood function, we first condition on the identity of the home bank for both types of transactions, and then integrate out h using the empirical distribution of h defined in Section 2.

In order to derive the likelihood contribution of each individual, we first condition on the choice-set $\mathcal{N}_i$,[16] the observed characteristics x_i, the identity of home bank h, the posted price valid at the time consumer (i) negotiated the contract $\bar{p}_{t(i)}$, and the model parameter vector $\theta = \{\beta, \xi, \sigma_\omega, \sigma_c, \alpha, \Delta, \lambda\}$. Let $\mathcal{I}_i = \{\mathcal{N}_i, x_i, \bar{p}_{t(i)}\}$ summarize the information known by the econometrician about consumer i.

In order to simplify notation, we use individual subscripts i for the borrower characteristics and random variables, with the understanding that all functions and variables are consumer-specific and depend on $\mathcal{I}_i$ and the parameter vector θ. For instance, $\Delta_{i,h} = \xi_{i,h} - \Delta(z_i^2)$ and $\lambda_i = \lambda(z_i^2)$ denote the home-bank cost and WTP advantages, and $\mu_i \equiv \mu(\mathcal{N}_i, \Delta_{i,h}, \lambda_i)$ is used to denote the initial quote markup (interior solution). In addition, we use c_i to summarize the state variable in the initial stage of the game, instead of $s_i = \{c_i, \bar{p}_{t(i)}, \mathcal{N}_i, \Delta_{i,h}, \lambda_i\}$. For instance, $\bar{\kappa}(c_i) \equiv \bar{\kappa}(s_i)$ and $p^0(c_i) \equiv p^0(s_i)$ correspond to the equilibrium search-cost threshold and initial quote, respectively.

Next we summarize the likelihood contribution for loyal and switching consumers. Appendix D describes in greater details the derivation of the likelihood function.

Likelihood contribution for loyal consumers The main obstacle in evaluating the likelihood

[16]We use $\mathcal{N}_i$ rather than n_i to characterize the choice set of consumers, since the identities of banks present in each neighborhood (not just the number) enter the distribution of lending costs.

function is that we do not observe whether or not consumers search. The unconditional likelihood contribution of loyal consumers is therefore:

$$\begin{aligned} &L(p_i, b(i) = h|\mathcal{I}_i, h, \theta) \\ &\quad = L\left(p_i = p^0(c_i), b(i) = h|\mathcal{I}_i, h, \theta\right) + L\left(p_i = p^*(\omega_i, c_i), b(i) = h|\mathcal{I}_i, h, \theta\right). \end{aligned} \tag{15}$$

The first term is a function of the solution to the optimal initial quote: $p^0(c_i) = \min\{\bar{p}_{t(i)}, c_i + \mu_i\}$. Since the markup is independent of c_i in the interior, the distribution of p_i takes the form of a truncated distribution:

$$L\left(p_i = p^0(c_i), b(i) = h|\mathcal{I}_i, h, \theta\right) = \begin{cases} f(p_i - \mu_i|x_i)\left[1 - H(\bar{\kappa}(p_i - \mu_i))\right] & \text{If } p_i < \bar{p}_{t(i)}, \\ \int_{\bar{p}_{t(i)}-\mu_i}^{\bar{p}_{t(i)}+\Delta_{i,h}} \left[1 - H(\bar{\kappa}(c_i))\right] dF(c_i|x_i) & \text{If } p_i = \bar{p}_{t(i)}. \end{cases} \tag{16}$$

The second element measures the probability of observing a constrained initial quote. This event occurs if $c_i > \bar{p}_{t(i)} - \mu_i$, and the consumer qualifies for a loan at its home bank (i.e. $c_i < \bar{p}_{t(i)} - \Delta_{i,h}$).

In addition to the search cost and the common lending cost, the likelihood contribution from searching consumers reflects the realization of the lowest cost differential in $\mathcal{N}_i$ (i.e. $\omega_{i,(1)}$). In particular, the transaction price is given by: $p_i = p^0(c_i)$ if $\omega_{i,(1)} > p^0(c_i) - c_i - \lambda_i$, or by $p_i = c_i + \omega_{i,(1)} + \lambda_i$ otherwise.

$$\begin{aligned} &L\left(p_i = p^*(\omega_i, c_i), b(i) = h|\mathcal{I}_i, h, \theta\right) \\ &\quad = \begin{cases} \left(1 - G_{(1)}(\mu_i - \lambda_i|\mathcal{N}_i)\right) H(\bar{\kappa}(p_i - \mu_i)) f(p_i - \mu_i|x_i) \\ \quad + \int_{p_i-\mu_i}^{p_i+\Delta_{i,h}} g_{(1)}(p_i - c_i - \lambda_i) H(\bar{\kappa}(c_i)) dF(c_i|x_i) & \text{If } p_i < \bar{p}_{t(i)}, \\ \int_{\bar{p}_{t(i)}-\mu_i}^{\bar{p}_{t(i)}+\Delta_{i,h}} \left(1 - G_{(1)}(\bar{p}_{t(i)} - c_i - \lambda_i|\mathcal{N}_i)\right) H(\bar{\kappa}(c_i)) dF(c_i|x_i) & \text{If } p_i = \bar{p}_{t(i)}. \end{cases} \end{aligned} \tag{17}$$

Likelihood contribution for switching consumers For switching consumers, the likelihood contribution depends on the relative position of the home bank in the surplus distribution of lenders belonging to $\mathcal{N}_i$. We use $g_b(\omega)$ to denote the density of the cost differential of the chosen lender, and $g_{-b}(\omega|\mathcal{N}_i)$ to denote the density of the most efficient lender in $\mathcal{N}_i$ *other* than b.[17]

If the observed price is unconstrained, the transaction price is equal to the minimum of $c_i - (\Delta_{i,h} + \lambda_i)$ and $c_i + \omega_{i,-b}$. If the consumer does not qualify for a loan at their home bank, the transaction price is the minimum of the posted price, and the second-lowest cost. This occurs if $c_i > \bar{p}_{t(i)} + \Delta_{i,h}$. Therefore, the transaction price for switching consumers is equal to $\bar{p}$ if and only

[17]The density $g_{-b}(\omega|\mathcal{N}_i)$ is $g_{(1)}(\omega|\mathcal{N}\backslash b)$.

if the chosen lender is the only qualifying bank. This leads to the following likelihood contribution:

$$
\begin{aligned}
&L\left(p_i, b(i) \neq h | \mathcal{I}_i, h, \theta\right) \\
&= \begin{cases} 1(\bar{p}_{t(i)} > p_i + \lambda_i) \left[\begin{array}{l} (1 - G_{-b}(-\Delta_{i,h} - \lambda_i | \mathcal{N}_i)) G_b(-\Delta_{i,h} - \lambda_i) \\ \times H(\bar{\kappa}(p_i + \Delta_{i,h} + \lambda_i)) f(p_i + \Delta_{i,h} + \lambda_i | x_i) \end{array} \right] & \text{If } p_i < \bar{p}_{t(i)}, \\ + \int_{p_i + \Delta_{i,h} + \lambda_i}^{\infty} G_b(p_i - c_i) H(\bar{\kappa}(c_i)) g_{-b}(p_i - c_i | \mathcal{N}_i) dF(c_i | x_i) & \\ \int_{\bar{p}_{t(i)} + \Delta_{i,h}}^{\infty} G_b(\bar{p} - c_i)(1 - G_{-b}(\bar{p}_{t(i)} - c_i | \mathcal{N}_i)) dF(c_i | x_i) & \text{If } p_i = \bar{p}_{t(i)}. \end{cases}
\end{aligned} \tag{18}
$$

Note that the first term is equal to zero if $\bar{p}_{t(i)} < p_i + \lambda_i$.[18] This condition ensures that the home bank's lending cost is below $\bar{p}_{t(i)}$ at the observed transaction price.

Integration of the home bank identity and selection The unconditional likelihood contribution of each individual is evaluated by integrating out the identity of the home bank. Recall, that h is missing for a sample of contracts, and is unobserved for switchers. In the former case we use the unconditional distribution of home banks, while in the latter case we condition on the fact that the chosen lender *is not* the home bank. This leads to the following unconditional likelihood:

$$
L\left(p_i, b(i) | \mathcal{I}_i, \theta\right) = \begin{cases} L(p_i, b(i) | \mathcal{I}_i, h = b(i), \theta), & \text{If } 1(\text{Loyal}_i) = 1, \\ \sum_{h \neq b(i)} \frac{\psi_h(x_i)}{\sum_{j \neq b(i)} \psi_j(x_i)} L(p_i, b(i) | \mathcal{I}_i, h, \theta) & \text{If } 1(\text{Loyal}_i) = 0, \\ \sum_h \psi_h(x_i) L(p_i, b(i) | \mathcal{I}_i, h, \theta) & \text{If } 1(\text{Loyal}_i) = \text{M/V}, \end{cases} \tag{19}
$$

where $\psi_h(x_i)$ is the unconditional probability distribution for the identity of the home bank.

In addition, the fact that we only observe *accepted* offers implies that the unconditional likelihood suffers from a sample selection problem. The probability that consumer i is in our sample is given by the probability of qualifying for a loan from at least one bank in i's choice set. This is given by the probability that the minimum of $c_i - \Delta_{i,h}$ and $c_i + \omega_{i,(1)}$.

$$
\Pr(\text{Qualify} | \mathcal{I}_i, \theta) = \sum_h \psi_h(x_i) \int F(\bar{p}_{t(i)} - \min\{\omega_{i,(1)}, -\Delta_{i,h}\} | x_i) dG_{(1)}(\omega_{i,(1)} | \mathcal{N}_i). \tag{20}
$$

Using this probability, we can evaluate the conditional likelihood contribution of individual i:

$$
L^c(p_i, b(i) | \mathcal{I}_i, \theta) = L\left(p_i, b(i) | \mathcal{I}_i, \theta\right) / \Pr(\text{Qualify} | \mathcal{I}_i, \theta). \tag{21}
$$

Aggregate likelihood function The aggregate likelihood function sums over the N observed contracts, and incorporates additional external survey information on search effort. We use the

[18] This reduces the smoothness of the likelihood, affecting primarily the parameters determining λ_i. To remedy this problem we smooth the likelihood by multiplying the second term in equation (18) by $(1+\exp((\lambda_i - \bar{p}_{t(i)} + p_i)/s))^{-1}$, where s is a smoothing parameter set to 0.01.

results of the annual FIRM survey (described in Section 2) to match the probability of gathering more than one quote along four dimensions: city-size, region, and income group.

Using the model and the observed new-home buyer characteristics we calculate the probability of rejecting the initial quote; integrating over the model shocks and the identity of the incumbent bank. Let $\bar{H}_g(\theta)$ denote this function for demographic group g. Similarly, let $\hat{H}_g$ denote the analog probability calculated from the survey.

We use the central-limit theorem to evaluate the likelihood of observing $\hat{H}_g$ under the null hypothesis that the model is correctly specified. That is, under the model specification, $\hat{H}_g - \bar{H}_g(\theta)$ is normally distributed with mean zero and variance σ_g^2/N_g, where σ_g^2 is the model predicted variance in the search probability across consumers in group g, and N_g is the number of households surveyed by the Altus Group.[19] The likelihood of the auxiliary data is therefore given by:

$$Q(\hat{H}|\theta) = \prod_g \phi\left(\sqrt{N_g}(\hat{H}_g - \bar{H}_g(\theta))/\sigma_g\right), \tag{22}$$

where $\phi(x)$ is the standard normal density.

Finally, we combine $Q(\hat{H}|\theta)$ and $L^c(p_i, b_i|X_i, \theta)$ to form the aggregate log-likelihood function that is maximized when estimating θ:[20]

$$\mathcal{L}(\mathbf{p}, \mathbf{b}|\mathbf{X}, \theta) = \sum_i \log L^c(p_i, b_i|\mathcal{I}_i, \theta) + \log Q(\hat{H}|\theta). \tag{23}$$

Notice that the two likelihood components are not on the same scale, since the FIRM survey contains fewer observations than the mortgage contract data-set. Therefore, we also test the robustness of our main estimates to the addition of an extra weight ρ that penalizes the likelihood for violating the aggregate search moments:

$$\mathcal{L}^\rho(\mathbf{p}, \mathbf{b}|\mathbf{X}, \theta) = \sum_i \log L^c(p_i, b(i)|\mathcal{I}_i, \theta) + \rho \log Q(\hat{H}|\theta). \tag{24}$$

Computation In order to evaluate the aggregate likelihood function, we must first solve the optimal initial offer defined implicitly by equation (6). This non-linear equation needs to be solved separately for every consumer/home-bank combination. We perform this operation numerically using a Newton algorithm that uses for the first and second derivatives of firms' expected profits. We also use starting values defined as the expected initial quote from the complete information

[19]We estimate σ_g by calculating the within group variance in search probability using the sample of individual contracts. Since this variance depends on the model parameter values, we follow a two-step approach: (i) calculate σ_g using an initial estimate of θ (e.g. starting with $\sigma_g = 1$), and (ii) hold σ_g fixed to estimate $\hat{\theta}$.

[20]The parameters are estimated by maximizing the aggregate log-likelihood function using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) numerical optimization algorithm within the Ox matrix programming language (Doornik 2007).

problem, for which we have an analytical expression. This procedure is very robust and converges in a small number of steps. Notice that since the interior solution is additive in c, this non-linear equation needs to be solved only once for each evaluation of the likelihood contribution of each household, $L(l_i, b(i)|\mathcal{I}_i, h, \theta)$. In addition, the integrals are evaluated numerically using a quadrature approximation.

6 Estimation results

6.1 Parameter estimates

Table 2 summarizes the maximum-likelihood estimates from three specifications, each one varying the source of the loyalty premium. In Specification (1), the loyalty premium takes the form of a WTP term, λ, for the home bank. In Specification (2), the home bank has a cost advantage, Δ, over competing lenders. Specification (3) nests both models.

Each specification implies that the home bank is more likely to "win" against rival banks at the competition stage, but have different implications for the price differences between loyal and switching borrowers. Holding fixed the magnitude of the idiosyncratic cost differences between lenders (σ_ω), the WTP model implies a larger average price difference between loyal and switching borrowers, relative to the cost advantage model. This difference is relatively small in the data: loyal borrowers pay about 10 bps more than switching borrowers, or about 10% of the standard-deviation of residual rates. In Specification (1), the model reconciles these two features with small estimates of σ_ω and λ_0. In contrast, the cost-advantage model leads to larger estimates of the differentiation parameters, Δ and σ_ω. Also, the cost-advantage model fits the data significantly better.

We formally assess the performance of the two modeling choices by estimating Specification (3). The last row reports the results of two likelihood-ratio tests testing the null-hypothesis that $\lambda_i = 0$ and $\Delta_i = 0$. We can easily reject the null hypothesis that the cost advantage parameters are zero; the test statistics is more than 40 times larger than the 1% critical value (i.e. 660.7 vs 16.3). In contrast, the null hypothesis of zero home-bank WTP parameters is much more modestly rejected (i.e. 45.7 vs 16.3).

A closer look at the estimates of λ in Specification (3) reveals that the intercept and *owner* parameters are not significantly different from zero (both statistically and economically), while the estimated cost advantage parameters are large and precisely estimated. The reverse is true for the interaction of income and loyalty. This suggests that the relationship between loyalty and income is better explained by the WTP model. Still, the effect of income on the loyalty premium is economically small and imprecise in all three specifications. Since the data do not support the WTP model, we use to the cost-advantage model as our **baseline** specification in our analysis of the empirical results.

Table 11 in the Appendix, evaluates the robustness of the results to the weight assigned to the

Table 2: Maximum likelihood estimation results

	Specification 1		Specification 2 Baseline		Specification 3	
	Estimate	(S.E.)	Estimate	(S.E.)	Estimate	(S.E.)
Heterogeneity and preferences						
Common shock (σ_c)	0.356	(0.003)	0.358	(0.003)	0.358	(0.003)
Idiosyncratic shock (σ_ω)	0.047	(0.002)	0.102	(0.002)	0.094	(0.002)
Avg. search cost (log)						
α_0	-1.539	(0.042)	-1.506	(0.026)	-1.592	(0.034)
α_{inc}	0.458	(0.052)	0.401	(0.038)	0.356	(0.045)
α_{owner}	0.184	(0.054)	0.086	(0.059)	0.143	(0.059)
Home-bank WTP						
λ_0	0.064	(0.003)			0.010	(0.007)
λ_{owner}	0.032	(0.002)			-0.016	(0.008)
λ_{inc}	0.002	(0.003)			0.023	(0.01)
Home-bank cost advantage						
Δ_0			0.146	(0.005)	0.126	(0.008)
Δ_{owner}			0.066	(0.004)	0.075	(0.008)
Δ_{inc}			0.012	(0.006)	-0.010	(0.01)
Cost function						
Intercept	5.332	(0.229)	5.495	(0.229)	5.479	(0.23)
Bond rate	0.307	(0.026)	0.306	(0.026)	0.306	(0.026)
Range posted-rate	-0.147	(0.017)	-0.145	(0.017)	-0.145	(0.017)
Total loan	-0.220	(0.073)	-0.208	(0.073)	-0.208	(0.073)
Income	-0.228	(0.026)	-0.214	(0.026)	-0.228	(0.027)
Loan/Income	-0.100	(0.01)	-0.102	(0.01)	-0.102	(0.01)
Previous owner	-0.003	(0.007)	0.047	(0.007)	0.051	(0.008)
House price	0.222	(0.066)	0.211	(0.066)	0.211	(0.066)
FICO	-0.662	(0.038)	-0.656	(0.038)	-0.660	(0.038)
LTV	1.111	(0.157)	1.092	(0.158)	1.093	(0.157)
$1(LTV = 95\%)$	0.029	(0.008)	0.029	(0.008)	0.029	(0.008)
Rel. network size	-0.019	(0.001)	-0.039	(0.002)	-0.036	(0.002)
Range of Bank FE	[-0.041 , 0.038]		[-0.088 , 0.063]		[-0.08 , 0.059]	
Quarter-year FE	Y		Y		Y	
Region FE	Y		Y		Y	
Sample size	26,218		26,218		26,218	
LLF/N	-2.059		-2.048		-2.047	
Search moments weight	1		1		1	
Likelihood ratio test ($\chi^2(3)$)	660.678		45.660			

Units: \$/100 per month. Average search cost function: $\log \alpha(z_i^1) = \alpha_0 + \alpha_{inc} \log(\text{Income}_i) + \alpha_{owner}\text{EHB}_i$. Home-bank willingness-to-pay: $\lambda(z_i^2) = L_i \times (\lambda_0 + \lambda_{inc}\text{Income}_i + \lambda_{owner}\text{Previous owner}_i)$. Home-bank cost advantage: $\Delta(z_i^2) = L_i \times (\Delta_0 + \Delta_{inc}\text{Income}_i + \Delta_{owner}\text{Previous owner}_i)$. Cost function: $c_{ij} = L_i \times (c_i + \omega_{ij})$, where $c_i \sim N(x_i\beta, \sigma_c^2)$ and $\omega_{ij} \sim \text{T1EV}\left(\bar{\xi}_j + \xi_{\text{branch}}\text{Rel. network size}_{ij} - \gamma\sigma_\omega, \sigma_\omega\right)$. The likelihood-ratio test reported in the last row test Model 1 and 2 against Model 3 (alternative hypothesis). The 1% significance level critical value is 16.266. Specification 2 is our **baseline** model.

auxiliary search moments. Specifically, we re-estimated the model with weights of 0 and 100 on the auxiliary search moments. A weight of 100 is analogous to increasing the sample size of the search survey to be roughly on par with the number of observations in the mortgage contract data. Doing so tends to increase the magnitude and heterogeneity of the loyalty-premium parameters (i.e λ and Δ), and changes the sign of the income coefficient in the search cost function. This allows the model to better match the observed heterogeneity in the search probability across market-size and income groups (see goodness of fit discussion below).

By setting a zero weight, the parameters are identified solely using the mortgage contract data. The results from Specifications (4) and (5) are similar to the results presented in Table 2, which is not surprising given the fact that the sample size in the contract data is much larger than in the search survey. The most noticeable differences between the two estimates are on the one hand that the average search cost is lower with a weight of zero (by about 15%-20%), and on the other hand that the dispersion of costs across lenders is larger (e.g. $\sigma_\omega = 0.12$ instead of $\sigma_\omega = 0.1$). Both features imply a larger predicted search probability in Specifications (4) and (5), relative to (2) and (3) (approximately 3 percentage points). The fact that these differences are fairly minor confirms that the model's key parameters can be identified without using direct information on search behavior.

Next, we discuss the economic magnitude of the parameter estimates, focusing in particular on the lending cost function and the search cost distribution. To better understand the magnitude of the estimates, recall that consumers choose a lender by minimizing their monthly payment net of the search cost. The monthly cost of supplying a $100,000 loan is a linear function of borrowers' observed and unobserved characteristics, and the parameters are expressed in $100 per month. For instance, in Table 2 the variance parameter of the common shock, $\sigma_c = 0.358$, implies that the common lending-cost standard-deviation for a $100,000 loan with fixed attributes is equal to $35.80/month.

Lending cost function The first two parameters, σ_c and σ_ω, measure the relative importance of consumer unobserved heterogeneity with respect to the cost of lending. The standard-deviation of the common component is 64% larger than the standard-deviation of idiosyncratic shock (i.e. 0.358 versus 0.128), suggesting that most of the residual price dispersion is due to consumer-level unobserved heterogeneity rather than to idiosyncratic differences across lenders.[21]

The estimate of σ_ω has key implications for our understanding of the importance of market power in this market. Abstracting from systematic differences across banks, the average cost difference between the first- and second-lowest cost lender, $c_{(1)}$ and $c_{(2)}$, is equal to $20 in duopoly markets, $17 with three lenders, and approaches $14 when N is equal 11.

In the model, market-power also arises because of systematic cost differences across banks: (i) bank fixed-effects, (ii) network size, and (iii) home-bank cost advantage. The estimates of the fixed-effects reveal relatively small differences across banks. Three of the eleven coefficients are not statistically different from zero (relative to the reference bank), and the range of fixed-effects is

[21] The standard deviation of an extreme-value random variable is equal to $\sigma_\omega \pi/\sqrt{6}$, or 0.102 in our case.

Table 3: Summary statistics on the home-bank cost advantage (Δ_i) by income and buyer status

VARIABLES	Mean	SD	P25	P50	P75
New home buyers					
Inc.$< 60K$	15.1	.111	15.1	15.1	15.2
Inc.$\geq 60K$	15.6	.273	15.4	15.5	15.7
Previous owners					
Inc.$< 60K$	21.7	.107	21.6	21.7	21.8
Inc.$\geq 60K$	22.3	.336	22	22.2	22.4
Total	17.1	2.94	15.2	15.5	21.5

Units: \$/Month. The cost advantage is measured for a \$100,000 loan.

equal to \$15/month in our baseline specification, or about the same scale as the standard-deviation of the idiosyncratic components.

We incorporate network size in the model by allowing the lending cost to depend on the relative branch network size of lenders in the same neighborhood. The estimates reveal that a lender with 3 times more branches than the average would experience a cost advantage of about \$12/month (compared to a single-branch institution). This is consistent with our interpretation of the lending cost function, as capturing elements of profits from complementary banking services that are increasing in branch-network size.

Turning to the estimate of Δ_i, we find that the presence of the loyalty premium corresponds to an average cost advantage of \$17.10/month (for a loan size of \$100,000). This cost advantage is substantial, given the fact that σ_ω is relatively small. At the estimated parameters, the probability that the home bank has a cost lower than the most efficient lender in $\mathcal{N}_i$ is equal to $G_{(1)}(\omega_h) = 51\%$; substantially more than the uniform probability of choosing a lender at random in the average choice set (i.e. $1/8 = 12\%$).

As mentioned, this cost advantage arises from the presence of switching costs, and/or complementarities between mortgage lending and other financial services, since the home-bank enjoys a cost advantage relative to rival lenders due to its profits from other services. To capture these gains, rival lenders must offer (costly) discounts on other products to get consumers to switch institutions.

Table 3 summarizes the distribution of Δ_i across borrowers. Recall that the loyalty premium is a deterministic function of income and prior-ownership status. We find that the home-bank cost advantage is particularly important for previous owners, suggesting that underlying switching costs are more important for older borrowers with longer prior experience. In comparison, the effect of income on the loyalty premium is positive, but much smaller (less than \$0.5/month).

Search cost distribution Table 2 reports the parameters of the average search-costs. Recall that we use an exponential distribution, and model the mean as a log-linear function of income and prior-ownership status. We find that search costs are increasing in income and ownership

Table 4: Search and interest costs for searchers and non-searchers

VARIABLES		Mean	SD	P25	P50	P75
Non-searchers	Total search cost	2.3	1.3	1.47	2	2.82
	Interest cost	44.2	18.3	30.1	40.9	55.3
	Loan size	130	55.1	87.1	120	165
Searchers	Total search cost	.549	.443	.203	.461	.809
	Interest cost	45.6	19	30.6	42.6	57.6
	Loan size	141	58.6	94.2	132	180
Total	Total search cost	1.15	1.19	.323	.784	1.58
	Interest cost	45.1	18.8	30.4	42	56.9
	Loan size	138	57.7	91.4	128	175

Units: $\$/1,000$. The search and interest costs correspond to the total over the term of the mortgage contract (60 months).

experience. New home-buyers are estimated to have lower search costs on average (8.6%), and a 1% increase in income leads to 0.4% increase in the average search cost of consumers. This is consistent with an interpretation of search costs as being proportional to the time cost of collecting multiple quotes.

Since search costs are not paid on a monthly basis, Table 4 summarizes the simulated distribution of search costs expressed over the 5-year term of the mortgage contract.[22] The bottom panel reports the unconditional distribution, and the top two panels illustrate the selection effect of consumers' search decisions. On average, we estimate that the cost of searching for multiple offers is equal to $1,150 (with a median of $784). The difference between searchers and non-searchers is substantial. We estimate that the search cost of "searchers" is $549, while "non-searchers" decided to accept the initial offer in order to avoid paying on average $2,300 in search costs.

To put these numbers in perspective, we also report in Table 4 the total interest cost over 5 years, as well as the total loan size. While the search cost estimates are nominally very important, they represent on average only 2.5% of the overall cost of the contracts (i.e. $2.5\% = 1.15/45.1$).

An important feature of the model, is that consumers financing larger loans are more likely to search. This is because the gains from search are increasing in loan size, while the search cost is fixed. As a results, in Table 4 we find that that searchers finance loans that are on average $11,000 larger than non-searchers, and incur 3% larger total interest costs. This is *despite* paying on average 20 basis-points lower rates.

Are these number realistic? Hall and Woodward (2012) calculate that a U.S. home buyer could save an average of $983 on origination fees by requesting quotes from two brokers rather than one. Our estimate of the search cost distribution is consistent with this measure. Our results are also

[22] Most of mortgage contracts in Canada involve substantial financial penalties for borrowers who decide to pre-pay their mortgage before the end of the 5-year term period. Borrowers are free to switch lenders after this period. It is therefore reasonable to use the term period length as the planning horizon.

Table 5: Summary statistics for simulated and observed data

		Spread (bps)	Discounts (bps)	1(Discount=0)	Payment ($/Month)	1(Loyal)	Network size (relative)
Observed	Mean	119.5	95.3	0.127	924.6	0.651	1.599
	SD	59.3	45.4	0.333	385.0	0.477	1.015
	P25	81.0	70.0		619.3		0.989
	P50	115.0	95.0		857.9		1.370
	P75	161.0	125.0		1169.0		1.931
Simulated	Mean	119.4	92.2	0.092	962.8	0.670	1.678
	SD	62.0	53.4	0.289	397.3	0.470	1.136
	P25	78.1	51.0		647.4		0.969
	P50	123.0	86.7		896.2		1.400
	P75	165.1	126.7		1218.6		2.087

The simulated sample is obtained by simulating 300,000 contracts from the baseline model, and dropping consumers who fail to qualify for a loan (5.5%).

comparable to those in Allen et al. (2014a), where, using a simpler complete-information analogue to the bargaining model employed here, results suggest that for the Canadian mortgage market search costs represent about 4% of the overall cost.

How do our results compare to existing estimates of search costs in the literature? Perhaps the closest point of comparison comes from Honka's (2014) analysis of the insurance market. She estimates the cost of searching for policies to be $35 per online search and a little over $100 per offline search. These numbers represent roughly 6% and 20% of annual insurance premia respectively, and are therefore somewhat larger than the 2.5% reported above.

We can also compare our findings to those of Saiz (2015), Hortaçsu and Syverson (2004) and Hong and Shum (2006). Saiz (2015) studies the New York City trade-waste market in which businesses contract with waste cartels for waste disposal and finds that search costs represent between 30% and 50% of total expenses. Hortaçsu and Syverson (2004) estimate a median search cost of 5 bps, yielding a ratio of 8%. The average search cost across the four books considered by Hong and Shum (2006) is $1.58 (for non sequential search), yielding a ratio of 33%.

Although somewhat lower, our estimates of the cost of search are comparable with those found in the literature. This is despite the fact that, because of the negotiation process, it is more complicated to obtain information about mortgage prices than about most products studied up until now.

6.2 Goodness of fit

We next provide a number of tests for the goodness of fit of the baseline model. To do so, we simulate 100,000 contracts from the model. We follow these steps:

1. Sample individual shocks from the estimated distributions: $(c_i, \omega_{i1}, \ldots, \omega_{in}, \kappa_i)$,

2. Sample borrower characteristics from the empirical distribution: $\big(L_i, \bar{p}_{t(i)}, x_i, h(i)\big)$,

3. Solve the model and compute the endogenous outcomes: $\big(p_i^0, p_i^*, 1(\kappa_i < \bar{\kappa}_i(p^0)), b_i\big)$,

4. Drop consumers who failed to qualify for a loan at any bank—about 5.5% of consumers.

Table 5 presents summary statistics for the key endogenous outcomes of the model. The top panel summarizes the observed sample, and the bottom panel summarizes the simulated data set. Overall, the baseline model is able to match very well the unconditional distribution of interest-rate spread (i.e. transaction rate minus bond-rate) and monthly payments. The predicted and observed discount distributions are also very similar, but the model tends to under-predict the median discount (86.7 versus 95 bps), as well as the fraction of borrowers paying the posted rate (i.e. 9.2% vs 12.7%).

Figure 1 shows that these shortcomings can largely be explained by the fact that the predicted distribution of discounts is smoother than the empirical distribution. For instance, when we group discounts into 25 bps bins, the model accurately predicts the fraction of consumers receiving zero or very small discounts, suggesting that few consumers in the observed sample receive discounts between 0 and 12 bps. Similarly, the empirical distribution of discounts exhibits a large mass around 100 bps, and as a result the density is sharply decreasing between 0 and 50. This is consistent with some lenders using 100 bps as a focal point discount. The model does not have any such prediction. Instead, the model predicts a smoother decrease in the density between 0 and 50, and a less pronounced peak at 100 pbs.

The last two columns of Table 5 highlight how well the model matches aggregate lender choice decisions. The model slightly over-predicts the fraction of loyal consumers (i.e. 67% instead of 65%), as well as the fact that borrowers tend to choose lenders with larger than average branch network sizes (i.e. 1.678 instead of 1.599). In addition, the model reproduces very well the lenders' aggregate market shares (available upon request).

In Table 6, we contrast the predicted search probabilities from the model, with the average frequencies reported in the national survey of new home buyers. We report the probabilities by income, city-size, and region. The last two columns correspond to auxiliary moments in the likelihood function.

The first column reports the predictions from the baseline specification. On average, the model predicts that 65.7% of consumers reject the initial offer and search, compared to 62.5% in the survey. This difference is significantly different from zero at a 10% significance level.

Figure 1: Predicted and observed distribution of negotiated discounts

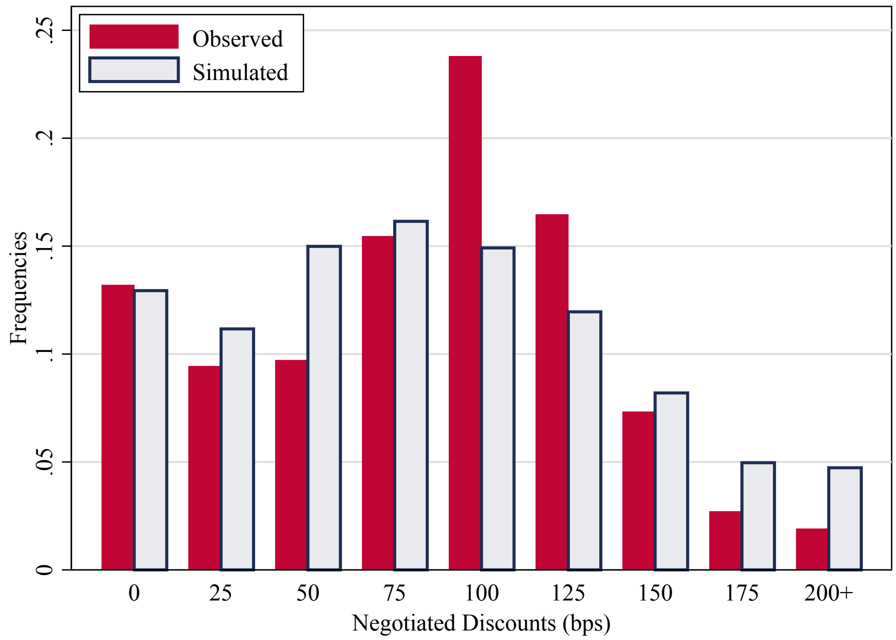


Table 6: Observed and predicted search probability by demographic groups

	Model predictions			Survey data	
	Baseline	Zero moment weight	Large moment weight ($100 \times n$)	Freq.	n
City size					
Pop. > 1M	0.673	0.717^b	0.661	0.660	338
1M > Pop. > 100K	0.657	0.695	0.639	0.654	268
Pop. ≤ 100K	0.628^b	0.655^a	0.584	0.560	275
Regions					
East	0.626^a	0.656^a	0.582	0.557	289
West	0.651	0.688^c	0.628	0.643	327
Ontario	0.673	0.713	0.659	0.668	265
Income					
> \$60K	0.639^a	0.670^a	0.586	0.579	400
≤ \$60K	0.669	0.712^b	0.670	0.666	441
Total	0.657^c	0.694^a	0.635	0.625	841

The simulated sample is obtained by simulating 100,000 contracts from the model, and dropping consumers who fail to qualify for a loan (5.5%). Source: FIRM survey by Ipso Reid. Null hypothesis: Survey average = Model average. Significance levels: a = 1%, b = 5%, c = 10%. P-values are calculated using the asymptotic standard-errors of the survey.

The model reproduces the general patterns of the survey across different regions and city sizes, but tends to under-estimate the amount of heterogeneity across demographics groups. For instance, the survey suggests that there is a 10 percentage point difference in the search probabilities for small and large cities, while the model implies that the difference is 5%. Similarly, both the model and the survey predict that high income borrowers search more. The baseline specification predicts that low income borrowers search with probability 63.9% compared to 59.6% in the survey. Most of the differences between the model-predicted probabilities and the survey results are not statistically significant.

In columns (2) and (3) we report the predicted search probabilities from the two alternative specifications in Table 11, which vary the weight placed on the search moments. As discussed above, when the search moments are not used in the estimation (middle column), the model tends to predict a larger search probability (69.4), but reproduces the same qualitative patterns from the survey. The differences are now statistically significant from zero in 6 out of 10 cases. In contrast, by assigning larger weights to the search moments (third column), the model is able to reproduce almost perfectly the survey predictions.

Finally, in Table 7 we evaluate the ability of the model to reproduce the reduced-form relationships observed in the data between rates, loyalty, and transaction characteristics. Regressions in columns (1)-(3) are estimated using the observed sample, and regressions in columns (4)-(6) are estimated using the simulated sample from the baseline specification. In columns (1) and (4), we regress discounts on characteristics, using the sample of consumers paying *less* than the posted-rate. In the remaining columns we estimate linear probability models describing the probability of paying the posted-rate ((2) and (5)), and the probability of remaining loyal to the home-bank ((3)and (6)).

In general, the model does a good job at predicting the relationship between discounts and financial attributes. The R^2s from the different specifications are nearly identical, suggesting that the model predicts more or less the same magnitude of residual rate dispersion observed in the data. The regression coefficients for loan-to-value and FICO scores are also similar in the simulated and observed samples, and the model captures well the non-linear relationship between income/loan size, and discounts (see marginal effects at the bottom). Similarly, the effect of competition on the probability of obtaining a discount and the magnitude of discounts has the same sign and similar magnitudes in both samples.

The fit of the model is not as good when it comes to the rate difference between loyal and switching consumers. In the data we estimate that loyal consumers obtain on average 9.1 bps lower discounts than do switching consumers, and are 2.3% more likely to pay the posted rate. In contrast, the model predicts that loyal consumers have 16 bps lower discounts, and a 5.9% greater probability of paying the posted-rate. This is because the model is restrictive in terms of the timing of moves, such that "switching" consumers must have rejected an initial offer and must pay

Table 7: Reduced-form discount and loyalty rate regressions

	Observed Sample			Simulated Sample		
	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Discount	1(Disc.=0)	Loyal	Discount	1(Disc.=0)	Loyal
Loyal dummy	-0.091^{a}	0.023^{a}		-0.16^{a}	0.059^{a}	
	(0.0075)	(0.0053)		(0.0022)	(0.0011)	
Total loan (X 100K)	0.010	0.0083	0.011	0.0060	-0.0015	-0.016^{a}
	(0.018)	(0.013)	(0.017)	(0.0055)	(0.0034)	(0.0047)
Annual income (X 100K)	0.20^{a}	-0.13^{a}	-0.0033	0.23^{a}	-0.098^{a}	0.018^{b}
	(0.034)	(0.025)	(0.030)	(0.010)	(0.0068)	(0.0086)
Loan/Income	0.100^{a}	-0.081^{a}	-0.050^{a}	0.11^{a}	-0.060^{a}	-0.015^{a}
	(0.014)	(0.0096)	(0.013)	(0.0042)	(0.0026)	(0.0036)
Previous home-owner	-0.022^{a}	0.0036	0.11^{a}	-0.017^{a}	0.013^{a}	0.13^{a}
	(0.0080)	(0.0057)	(0.0072)	(0.0024)	(0.0014)	(0.0020)
FICO (mid-point)	0.61^{a}	-0.31^{a}	0.27^{a}	0.73^{a}	-0.23^{a}	-0.0056
	(0.045)	(0.034)	(0.045)	(0.014)	(0.0085)	(0.012)
Loan to Value Ratio	-0.63^{a}	0.37^{a}	-0.14	-0.77^{a}	0.19^{a}	-0.030
	(0.11)	(0.073)	(0.10)	(0.034)	(0.017)	(0.029)
$LTV = 0.95$	-0.023^{b}	0.013^{c}	-0.021^{b}	-0.034^{a}	0.012^{a}	0.0033
	(0.0098)	(0.0072)	(0.0094)	(0.0030)	(0.0017)	(0.0026)
Posted-rate spread	0.30^{a}	-0.13^{a}	-0.039^{c}	0.70^{a}	-0.20^{a}	-0.0023
	(0.023)	(0.015)	(0.021)	(0.0069)	(0.0041)	(0.0060)
Bond rate	0.27^{a}	-0.13^{a}	-0.039^{c}	0.48^{a}	-0.14^{a}	0.0076
	(0.023)	(0.015)	(0.021)	(0.0070)	(0.0039)	(0.0061)
Relative network size	-0.0094^{b}	0.019^{a}	0.026^{a}	0.0055^{a}	0.00064	0.048^{a}
	(0.0045)	(0.0036)	(0.0045)	(0.0013)	(0.00077)	(0.0014)
Nb. Lenders (log)	0.085^{a}	-0.070^{a}	-0.076^{a}	0.059^{a}	-0.040^{a}	-0.17^{a}
	(0.021)	(0.016)	(0.019)	(0.0062)	(0.0045)	(0.0052)
Observations	17,531	20,619	20,619	244,212	269,303	283,476
R-squared	0.132	0.079	0.095	0.139	0.065	0.065
Marginal effect: income	-0.16	0.17	0.18	-0.18	0.12	0.071
Marginal effect: loan	-0.34	0.18	-0.19	-0.43	0.054	-0.065

The simulated sample is obtained by simulating 300,000 contracts from the model, and dropping consumers who fail to qualify for a loan (5.5%). Sample selection: All specifications exclude mortgages originated from lender(s) with missing loyalty variable, and specifications (1) and (4) exclude transactions with zero discounts. Dependent variables: Discount = Posted rate - negotiated negotiated rate (if $\bar{r} > r_i$), 1(Disc.=0) = Indicator variable equal to 1 Discount> 0, Loyal = indicator variable equal to 1 if lender is h. Control variables: Income, loan size, loan/income, 5-year bond-rate, region fixed-effects, year/quarter fixed-effects. Robust standard errors in parentheses. Significance levels: ^{a} = 1%, ^{b} = 5%, ^{c} = 10%.

a competitive price. In practice, the timing of moves probably differs across consumers, in ways that we cannot measure.

This type of measurement error likely explains why the model does a relatively poor job of matching the reduced-form loyalty probability regression. Since search cost and loyalty premium depend on the ownership-status of borrowers, the model is able to reproduce very well the fact that previous owners are over 10% more likely to remain loyal to their home banks. The model is also able to match the sign of the relationships between loyalty and key attributes of the transaction: consumers are more likely to switch in more competitive markets, more likely to remain loyal to a large network institution, more likely to switch when financing a larger loan, and more likely to remain loyal when of high income. However, the magnitudes of these marginal effects are not always accurate.

7 Search frictions and market power

In this section, we use the model to quantify the effect of search frictions and market power on consumer surplus and firms' profits. In the model, market power and search frictions are tightly linked, since lenders are able to use the initial quote to screen high search-cost consumers. We start by quantifying the welfare impact of search frictions by computing the equilibrium allocation of contracts absent search costs. We then quantify the importance of market power in the industry, by focusing on the incumbency advantage.

7.1 Quantifying the effect of search frictions on welfare

The presence of search costs lowers the welfare of consumers for three distinct reasons. First, it imposes a direct burden on consumers searching for multiple quotes. Second, it can prevent non-searching consumers from matching with the most efficient lender in their choice set, thereby creating a misallocation of buyers and sellers. Lastly, it opens the door to price discrimination, by allowing the initial lender to make relatively high offers to consumers with poor outside options and/or high expected search costs. These factors can be identified by decomposing the change in consumer surplus caused by the presence of search frictions:

$$\begin{aligned}
\Delta \text{CS}_i &= \underbrace{\bar{v} - p_i - 1\left(\kappa_i < \bar{\kappa}(p_i^0)\right)\kappa_i}_{\text{CS}_i} - \underbrace{(\bar{v} - \tilde{p}_i)}_{\widetilde{\text{CS}}_i} \\
&= [\tilde{c}_{i,b} - c_{i,b}] - (m_i - \tilde{m}_i) - 1\left(\kappa_i < \bar{\kappa}(p_i^0)\right)\kappa_i \\
&= \Delta V_i - \Delta m_i - 1\left(\kappa_i < \bar{\kappa}(p_i^0)\right)\kappa_i, \qquad (25)
\end{aligned}$$

where the $\sim$ superscript indicates the equilibrium outcomes without search cost, $\bar{v}$ is the WTP for mortgages (policy invariant), $V_{ij} = \bar{v} - c_{ij}$ is the transaction surplus (excluding the search cost),

Table 8: Decomposing the effect of search frictions on welfare

		Consumer surplus change: Zero search-cost				Change	Change: CS
		Misc. ($/M.)	Disc. ($/M.)	Search ($/M.)	Total ($/M.)	Interest Cost ($)	Market-Power ($/M.)
		(1)	(2)	(3)	(4)	(5)	(6)
Zero changes	%	0.83	0.68	0.32	0.02	0.68	0.00
Distribution: Non-zero changes							
	Mean	-16.32	11.37	9.49	-13.03	1569	-15.12
	P10	-33.44	-7.59	1.46	-26.55	344	-34.24
	P50	-11.70	12.43	7.86	-10.56	1591	-9.89
	P90	-2.19	28.48	19.01	-1.68	2697	-1.53
Cumulative	$	-2.73	3.64	6.42	-12.80	503	-15.12
	%	0.21	0.28	0.50	1.00		

Each entry corresponds to an average over 300,000 simulated contracts. Statistics in lines 2-5 are calculated using the samples of consumers facing non-zero changes. Cumulative changes are the sum of all changes divided by the total number of qualifying consumers. The welfare decomposition in columns (1)-(3) corresponds to: $\Delta \text{CS}_i = \Delta V_i - \Delta m_i - \Delta \kappa_i S_i = \text{Misallocation}_i - \text{Discrimination}_i - \text{Search cost}_i$. The last row reports the contribution of each component, in percentage of the cumulative change. Column (5) summarizes the effect of search frictions on the total interest payment over 5 years: Total interest cost ($\kappa_i > 0$) - Total interest cost ($\kappa_i = 0$). The last column reports the *further* reduction in consumer surplus arising from the presence of market power in the second-stage of the game: $CS(\kappa_i = 0, m_i > 0) - CS(\kappa_i = 0, m_i = 0)$.

$m_{ij} = p_i - c_{ij}$ is the profit margin, and $1\left(\kappa_i < \bar{\kappa}(p_i^0)\right)$ is an indicator variable equal to one if the consumer rejects the initial offer. As before, we assume that the WTP for mortgages is large enough that the same group of consumers would enter the housing market with or without search frictions.

We label the three components *misallocation*, *discrimination*, and *search cost*, respectively. The sum of the first and third components measures the change in total welfare caused by search frictions. The discrimination component is related to the surplus split between firms and consumers.

We simulate the counter-factual experiments as before. The only difference between the baseline and the *zero search-cost* environments is that, absent search frictions, consumers do not obtain an initial quote. As a result, the posted rate becomes the reservation price in the competition stage.

Table 8 presents the main simulation results. Columns 1 through 3 show the change in the misallocation, discrimination and search cost components respectively, while column 4 presents the total change in consumer surplus. To illustrate the heterogeneity across consumers, the first line reports the fraction of simulated consumers experiencing zero changes, and the next four describe the conditional distribution of non-zero changes. To calculate the cumulative changes, we average the changes across all qualifying consumers. The percentage shares of each component are expressed relative to the cumulative changes.

We estimate that the cumulative reduction in consumer surplus associated with search frictions is equal to $12.80 per month, or 2% of the total interest cost of mortgages in our data-set. The largest component (50%) is attributed to the sunk cost of searching, followed by the increase in margins associated with price discrimination (28%), and the misallocation (21%). Over 98% of consumers are affected. The sum of the misallocation and discrimination components corresponds to the effect of search frictions on monthly payments alone: \$6.37/month on average per borrower. This leads to an increase in interest payments of \$503 over 5 years (column (5)), or \$1,569 for consumers who are directly impacted by the price change.

The sum of the misallocation and search-cost components corresponds to the total welfare cost of search frictions (i.e. \$9.15/month per borrower). For these two components, the fraction of zero changes measures the percentage of buyers and sellers that are matched efficiently and the fraction of non-searchers in the presence of search frictions, respectively. Search frictions cause 17% of transactions to be misallocated, despite the fact that more than 32% of consumers are not searching. Note that the difference between these two fractions would be close to zero if the loyalty premium were null. Since banks' fixed-effects are not highly dispersed, this difference results mostly from the fact that consumers visit the highest expected surplus seller first, which reduces the fraction of inefficient matches.

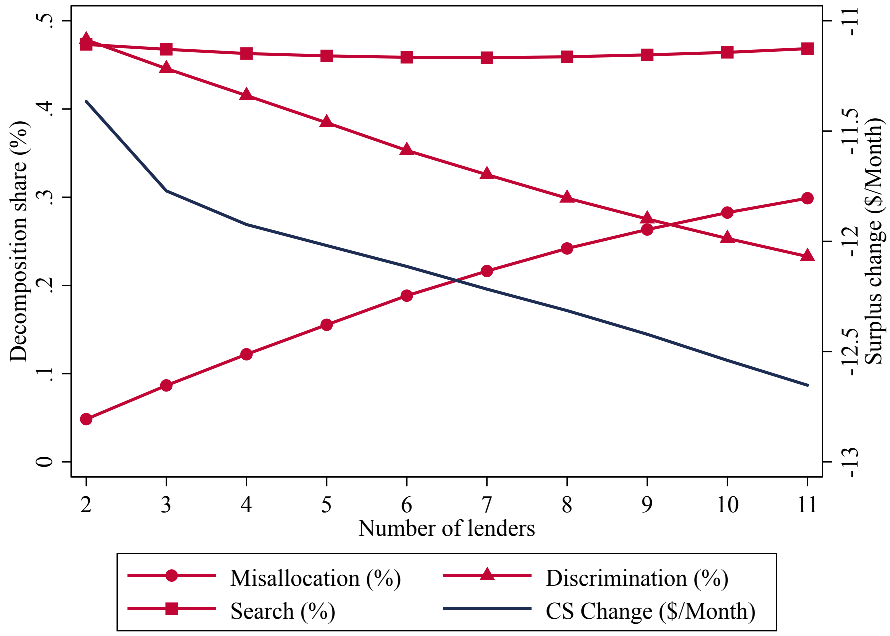
Focusing directly on the change in profit margins, Column (2) shows that the relatively small contribution of the price discrimination component is explained by the fact that some consumers pay *higher* markups in the frictionless market. The median change in profit margins is equal to \$12.43 per month; significantly more than the median increase in search costs (i.e. 12.43 vs 7.86). However, the 10th percentile consumer benefits from a \$7.59 reduction in profit margins, which brings the cumulative effect down to \$3.64.

To understand this heterogeneity, recall that the initial quote is used both as a screening tool, and as a price ceiling in the competition stage. The home bank is in a monopoly position in the first stage, and can set individual prices based on consumers' expected outside options. This is analogous to first-degree price discrimination, and strictly increases the expected profit of the home bank. This adverse effect is weighed against the fact that the initial offer can be recalled, and therefore protects consumers against excessive market power in the competition stage. In the zero search-cost environment, the price ceiling is on average higher (i.e. it is the posted-rate), which explains why some consumers experience an increase in profit margins after eliminating search frictions.

To put these numbers in perspective, column (6) summarizes the distribution of consumer-surplus changes arising from eliminating market power entirely, relative to the zero-cost environment. That is, we calculate the difference in consumer surplus between the zero search-cost environment, and an environment with no search frictions **and** zero profits margins. This is equivalent to shifting the bargaining power entirely to consumers in the competition stage, and therefore maximizing the surplus of consumers.

Relative to the baseline environment, eliminating market power **and** search frictions would increase consumer surplus by \$27.92/month on average (i.e. 12.80+15.12). Therefore, eliminating

Figure 2: The effect of competition on the welfare cost of search frictions



Each entry corresponds to an average over 100,000 simulated contracts. The welfare decomposition corresponds to: $\Delta \text{CS}_i = \Delta V_i - \Delta m_i - \Delta \kappa_i S_i = \text{Misallocation}_i - \text{Discrimination}_i - \text{Search cost}_i$. The decomposition is repeated for each alternative market-structure (i.e. $n = 1, \ldots, 11$).

search frictions would allow consumers to reach 46% of their maximum surplus.

What is the effect of competition on the welfare cost of search frictions? The answer to this question is ambiguous, because competition limits the ability of firms to price discriminate, while increasing the gains from search. Furthermore, the link between the probability of searching and market structure is ambiguous.

To shed light on this issue, we repeat the same welfare-cost decomposition for counter-factual market structures, which vary the number of rival lenders from $n = 1, \ldots, 11$. To isolate the effect of competition, we "homogenize" banks by setting the bank fixed-effects to zero, and setting the branch-network size to the average for all banks. In order to eliminate any composition effect, we restrict the simulated sample to consumers who qualify for a loan in every experiment.

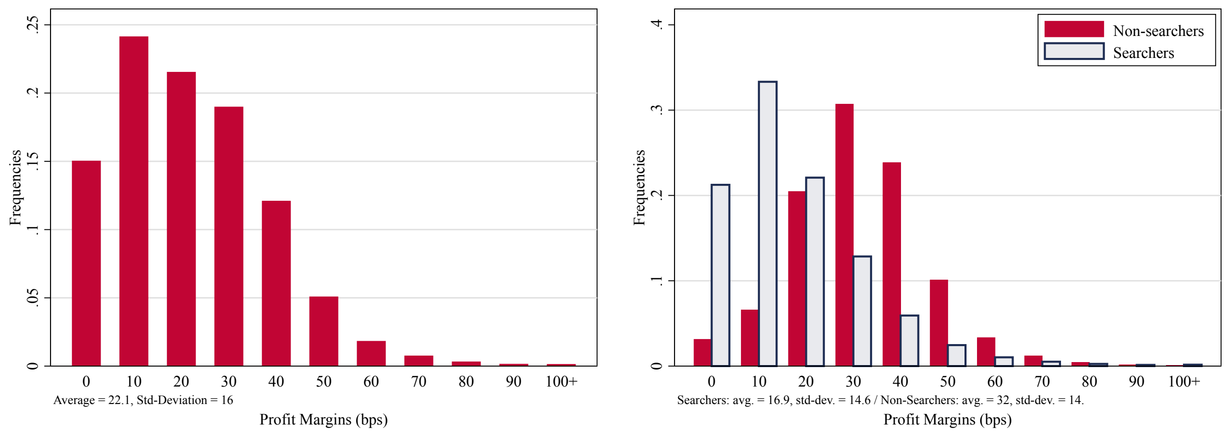
Figure 2 plots the cumulative change in consumer surplus (solid line) against the number of lenders, as well as the percentage share of each component (i.e. misallocation, discrimination and search-cost). We find that increasing the rival number of lenders from 1 to 11 **increases** the welfare cost of search frictions by about 11% (i.e. from -11.37 to -12.66). Therefore competition tends to modestly amplify the importance of search frictions.

This relatively small adverse effect of competition masks two opposite forces. Increasing the number of rival competitors from 1 to 11 decreases the price discrimination component by more than

Figure 3: Distribution of profit margins

(a) Full sample

(b) Searchers and non-searchers



50% (from 48% to 23%), but also increases by a factor of six the contribution of the misallocation component (from 5% to 30%). The increase in misallocation comes from the fact that the gains from search are rapidly increasing in the number of options, while the search probability is only weakly increasing in the size of the choice set. We find that 60% of consumers search in duopoly markets, compared to 66% in markets with 11 lenders. As a results the contribution of the search-cost component to the cumulative consumer surplus change remains almost constant at about 47%.

These results can be compared to those of Gavazza (2016), who performs a similar decomposition of the effect of search frictions on welfare in decentralized asset markets. Using data from the business aircraft market, he finds that, relative to his estimated model, when search costs are set to zero welfare falls slightly (by roughly $1 million per quarter). This small decrease is the result of a reduction in direct search costs (of about $6 million), a reduction of misallocation ($3 million) and an offsetting increase in dealer costs ($11 million).

7.2 Quantifying the importance of market power

Overall, we find that the market is competitive. Figure 3a plots the distribution profit margins. The average profit margins is 22.1 bps, which corresponds to a Lerner index of 3.2%. This is consistent with our earlier findings that mortgage contracts are fairly homogenous across lenders, and search-costs represent a small share of consumers' overall mortgage spending. It is also fairly consistent with the findings in Allen et al. (2014a), which suggest margins of around 35 bps before the merger and 40 bps afterwards.

This implies that a large fraction of the observed spread between negotiated rates and the 5-year bond-rate corresponds to transaction costs. In particular, we estimate that each contract costs roughly 100 bps to originate, beyond the financing cost, which is proxied by the bond rate. This cost stems from a variety of sources: the compensation of loan officers (bonuses and commissions), the premium associated with pre-payment risks, transaction costs associated with the securitization of contracts, as well as upstream profit margins from financing.

The distribution of profit margins is also very dispersed. The coefficient of dispersion of profit margins is equal to 72%, and the range exceeds 100 bps. Figure 3b shows that part of this dispersion is caused by heterogeneous search efforts. On average, firms charge a markup that is 90% larger on consumers who are not searching (i.e. 32.1/16.9). The margin distribution for searchers also exhibits an important mass between 0 and 20 bps, and the median margin among searchers is only 13 bps (compared to 32 bps in the non-searcher sample).

The dispersion in profit margins also reflects the fact that market-power arises from a variety of sources: (i) price discrimination, (ii) loyalty premium, (iii) observed cost differences (i.e. bank fixed effects and network size), and (iv) idiosyncratic cost differences (i.e. ω_{ij}).

The last two components ensure positive profits margins in the competition stage. On average, the difference between the lowest and second-lowest cost among rival lenders is equal to \$15.70/month. This is the profit margin that would be realized if the home bank were not present and there were no posted-rate (i.e. ceiling), and therefore can be thought of as an upper bound on the market power of rival banks. In practice, rival lenders earn slightly less: banks' average profits from switching consumers are \$14.99/month (or 17.1 bps), compared to \$20.22/month for loyal consumers (or 24.6 bps).

The profit gain from loyalty corresponds to an *incumbency advantage:* Banks with a large consumer base have more market power because of a first-mover advantage and loyalty premium (or differentiation). We find that the loyalty premium is substantial: the average home-bank cost advantage is 33% larger than the standard-deviation of idiosyncratic cost differences. As a result the home bank is able to retain, on average, 51% of searching consumers. The first-mover advantage arises because the home bank is in a quasi-monopoly position in the first-stage of the game, and can price discriminate between consumers based on heterogeneity in their expected reservation prices. The ability to make the first quote allows the home bank to charge a higher markup **and** retain a larger fraction of consumers who, absent search costs, would choose another lender.

To measure the source and magnitude of the incumbency advantage, we use the simulated model to evaluate the correlation between the size of a lender's consumer base and its profitability. In the model, the consumer base of a given bank is defined as the share of consumers with whom it has an existing day-to-day banking relationship, and this base determines the fraction of consumers in a given market who start their search with the bank (i.e. ψ_{ij}). Recall that this matching probability is defined at the level of a neighborhood, income group (low, medium and high), and year. We use this definition to construct markets that each have a homogenous consumer base distribution, and we construct measures of profits and concentration at this level of aggregation. Doing so yields $8,075$ unique markets.

To construct a measure of consumer base that is comparable across markets, we compute, for each market i, the ratio of the matching probability of lender j over the average matching probability among banks in the market:

$$\text{Matching probability ratio} = \frac{\psi_{ij}}{\bar{\psi}_i}.$$

Table 9a summarizes the distribution of contracts and profits across different types of lenders. The table ranks banks from the smallest consumer base (i.e. between 0 and 25% of the average size in the same market), to the largest (i.e. between 4 and 7 times the average size). As we saw earlier, most consumers choose a mortgage lender with a large branch presence. This is reflected in the distribution of contracts shown in column (1): 46% of contracts are issued by banks with a consumer base between 1 and 2 times larger than the average bank in their market.

Columns (2) and (3) report the weighted average share of profits and contracts generated by each bank type. To get this number, for each market, we calculate the average share of profits and contracts generated by lenders with consumer bases belonging to one of the 6 categories. We then aggregate these shares across markets, using the total number of contracts originated in each market as weight.

If there were no relationship between banks' consumer bases and mortgages, contracts and profits would be uniformly distributed across categories (i.e. would be about 11% on average). The resulting distributions are significantly more concentrated. Banks in the top category (4 to 7) earn, on average, 62% of the profits generated in their respective markets, compared to only 2%, on average, for the smallest banks. Note that the average profit share increases very quickly with the size of the consumer base.

In addition, the distribution of profits is more concentrated than the distribution of contracts. On average the top lenders originate 54% of contracts, but earn 62% of the profits. This pattern reflects the fact that banks with a large consumer base charge, on average, higher markups. Column (5) shows that the average profit margin for banks in the top category is equal to 30.7 bps, compared to only 16.6 bps for banks in the bottom category. This discrepancy is largely explained by the difference in markups between searchers and non-searchers. Banks in the smallest consumer-base category earn on average 90% of their profits from consumers reaching the second stage of the game, compared to 40% for banks in the largest category. This confirms the importance of the first-mover advantage as a source of market power for large consumer base lenders.

Identifying the relative importance of the first mover advantage and differentiation is not an easy task however, since the two interact to generate a correlation between profitability and size of consumer base. For instance, the profit gain from being able to make the first offer depends on the amount of differentiation, since lower-cost banks have more leverage in the initial negotiation stage. Similarly, the presence of a cost advantage reduces the incentive for consumers to search, and increases the fraction of profits generated from price discrimination.

Table 9: Incumbent advantage and market power

(a) Distribution of bank profitability and consumer base in the baseline environment

Consumer base	Matching probability ratio	Sample frequency (1)	Within market shares: Profits (2)	Within market shares: Contracts (3)	Second stage profits (%) (4)	Margins (bps) (5)
Small	0 to 1/4	0.05	0.02	0.02	0.90	16.6
	1/4 to 1/2	0.04	0.04	0.05	0.62	19.1
	1/2 to 1	0.17	0.07	0.08	0.51	18.9
	1 to 2	0.46	0.16	0.16	0.50	20.4
	2 to 4	0.25	0.34	0.30	0.51	24.0
Large	4 to 7	0.04	0.62	0.54	0.40	30.7

(b) Distribution of bank profitability in the baseline and counterfactual environments

Statistics	Variables	Baseline	CF-1 $\Delta_i = 0$	CF-2 $\psi_i = 1/N$	CF-3 $\psi_i = 1/N$ & $\Delta_i = 0$
Ratio: Large base/Small base					
	Margins (bps)	1.851	1.369	1.485	1.145
	Profit shares	35.717	11.652	17.159	6.699
	Contract shares	24.582	9.769	13.204	6.243
Full sample averages					
	Search probability	0.656	0.774	0.838	0.822
	2^{nd} stage profits (%)	0.531	0.727	0.809	0.784
	Margins (bps)	22.07	18.56	21.34	18.60
	Match prob. ratio	1.709	1.546	1.605	1.439

(c) Decomposition of the incumbency advantage

Large/Small Ratio	Incumbency adv. Base − CF-3	Loyalty premium CF-2− CF-3	Price discrimination CF-1− CF-3	Interaction
Margins	0.707	0.340	0.224	0.142
		(0.48)	(0.32)	(0.2)
Profit share	29.018	10.460	4.954	13.605
		(0.36)	(0.17)	(0.47)
Contract share	18.339	6.961	3.526	7.852
		(0.38)	(0.19)	(0.43)

Each entry in Table 9a is the weighted average outcome of lenders belonging to each category (rows). The weights are proportional to the number of contracts originated in each market (i.e. neighborhood/year/income). Variable definitions: Matching probability ratio = Consumer base of bank j / Average consumer base; Sample frequency = Market share of lenders in each category; Second-stage profit (%) = Average share of profits originating from the searching consumers; Within market share = Average share of profits or contracts generated by lenders in each category; Margins = $r_i - c_i$ in percentage basis points; Ratio: Large base/Small base = Ratio of the average outcomes of lenders in the large group, over those in the small group. Counter-factual environments: (1) Zero home-bank cost advantage, (2) Uniform matching probability, (3) combination of (1) and (2).

To measure each of the components that generate the incumbency advantage, we simulate a series of counter-factual experiments aimed at varying the first-mover advantage and the differentiation component independently. In particular, to eliminate the differentiation component, CF-1 simulates a model in which the cost-advantage of the home bank is set to zero, which is analogous to separating the provision of mortgages from other banking services. We eliminate the ability of firms to screen high search-cost consumers by imposing a uniform matching probability and breaking the link between the ability to make the first offer and the size of the consumer base (CF-2). Finally, CF-3 combines the previous two environments by assuming a uniform matching probability and zero loyalty premium.[23]

Table 9b displays the results from this exercise. The top panel summarizes the amount of concentration in the industry, as well as the dispersion in profit margins between large and small banks. The bottom panel describes some of the key equilibrium outcomes in the baseline and counter-factual environments.

The ratio of the profit margin of large and small banks is a measure of the incumbency advantage: how much more market power do banks with large consumer bases have relative to banks with small consumer bases. In the baseline environment, we estimate that large banks' profit margins are 85.1% larger. Eliminating the first-mover and the loyalty premium shrinks the margin difference to 14.5% (CF-3), and so this is a measure of the market power of large banks that stem solely from brand and branch network differences.[24] The difference, or $0.707 = 1.851 - 1.145$, is explained by the incumbency advantage.

The first column of Table 9c summarizes the incumbency advantage in terms of profit margins, profit shares, and market shares (or contract). Columns (2) to (4) use the uniform matching probability (CF-2) and the zero loyal premium (CF-1) counter-factual environments to decompose the incumbency advantage into three terms:

$$\underbrace{\text{Incumbency advantange}}_{0.707} = \underbrace{\text{Loyalty premium}}_{0.34} + \underbrace{\text{Price discrimination}}_{0.22} + \underbrace{\text{Interaction}}_{0.14}.$$

Therefore, relative to CF-3, almost 50% of the market-power of large banks is caused by the home-bank cost advantage, just over 30% by the first-mover advantage, and the remaining 20% is explained by the interaction of both elements.

The interaction term originates from the joint equilibrium effect of differentiation and the order of moves on the search probability. As the middle panel indicates, the combined effect of the home-bank cost and first-mover advantage is to lower the search probability from 82.2% to 65.6%, which increases the profit margin ratio by an extra 14 percentage points through a change in the composition of loyal borrowers. Independently, the two elements have little or no effect on the search probability relative to the CF-3 environment.

[23] An alternative approach for eliminating the first-mover advantage is to set consumer search costs to zero. We chose instead to modify the order of moves by setting $\phi_{ij} = 1/N$, since doing so does not fundamentally change the degree of competition in the market. The zero search-cost counterfactual yields very similar conclusions. Results are available from the authors upon request.

[24] All ratios would be equal to one if the difference between lenders were caused only by idiosyncratic cost differences.

The concentration of profits and contracts is similarly impacted. Eliminating both the loyalty premium and the first-mover advantage substantially reduces the concentration of profits: large banks' share of profits is 35.72 times larger than that of small banks in the baseline, compared to only 6.70 times in CF-3. As with margins, the loyalty premium alone explains a bigger share of the drop in concentration (36%) than the first-mover advantage (17%). However, unlike with margins, a larger portion of the profit share ratio is explained by the interaction of differentiation and discrimination: 47% of the profit share difference between large and small banks is explained by the interaction term. This is because the increase in the search probability from letting the most efficient lender make the first offer has a very large effect on banks' retention probability, and therefore on their overall profitability.

8 Conclusion

The paper makes three main contributions. The first is to provide an empirical framework for studying markets in which prices are negotiated. The second is to show that search frictions are important and generate significant welfare losses for consumers that can be decomposed into misallocation, price discrimination, and direct search cost components. We also show that the welfare loss is mitigated by switching costs (loyalty premium) and posted prices, but amplified by competition. Finally, the paper also demonstrates the importance of having a large consumer base for market power, and decomposes the effect into a first-mover advantage and brand loyalty. We find that brand loyalty is the most important source of market power, but that search frictions play an important role through the first-mover advantage.

Although the overall fit of our model is good, the goodness of fit analysis highlights several caveats. First, reduced-form estimates using the data show that loyal consumers pay around 8 bps more, while the model predicts more than 35 bps. This difference is directly related to our modeling assumptions: the timing and order of search are the same for all consumers, and all consumers have a single home bank. These are simplifying assumptions that closely link search and switching in the model.

Similarly, the model tends to over-estimate the impact of competition on rates. This likely reflects the fact that that market structure is assumed to be independent of consumers' unobserved attributes, up to regional fixed-effects. If this is not the case, our estimates of firms' cost differences, which determine markup levels, would suffer from a attenuation bias, and therefore our results would correspond to a lower bound on the size of profit margins in this market.

A related interpretation of the small reduced-form effect of competition on rates and discounts, is that consumers face heterogenous consideration sets, conditional on being located in the same postal-code area. This would create measurement error in the choice-set of consumers. Because lenders are ex-ante heterogeneous, it is computationally prohibitive to incorporate this type of measurement error in the model. Moreover, we do not have access to data on the set, or identity of lenders considered by borrowers. For recent work along these lines, see Honka et al. (2016).

Finally, in order to keep the model tractable, we decided to focus only on branch-level transactions, and ignore contracts that are negotiated through brokers. Brokers, act as intermediaries and can potentially lower the search cost of individuals by searching over a larger set of lenders. Since brokers are used by approximately 25% of borrowers it would be important to understand better the role they play in this environment. In an ongoing project we are working on modeling the behavior of these intermediaries.

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A Data description

Our data-set consists of a 10% random sample of insured contracts from CMHC. It covers the period from 1992 to 2004. We restrict our analysis to the 1999-2002 period for two reasons. First, between 1992 and 1999, the market transited from one with a larger fraction of posted-price transactions and loans originated by trust companies, to a decentralized market dominated by large multi-product lenders. Our model is a better description of the latter period. Second, between November 2002 and September 2003, TD-Canada Trust experimented with a new pricing scheme based on a "no-haggle" principle. Understanding the consequences of this experiment is beyond the scope of this paper, and would violate our confidentiality agreement.

We also have access to data from Genworth Financial, but do not use these contracts in the paper, since we are missing some key information for these contracts. We obtained the full set of contracts originated by the 12 largest lenders and further sampled from these contracts to match Genworth's annual market share.

Both insurers use the same guidelines for insuring mortgages. First, borrowers with less than 25% equity must purchase insurance.[25] Second, borrowers with monthly gross debt service (GDS) payments that are more than 32% of gross income or a total debt service (TDS) ratio of more than 40% will almost certainly be rejected. Crucial to the guidelines is that the TDS and GDS calculations are based on the posted rate and not the discounted price. Otherwise, given that mortgages are insured, lenders might provide larger discounts to borrowers above a TDS of 40 in order to lower their TDS below the cut-off. The mortgage insurers charge the lenders an insurance premium, ranging from 1.75 to 3.75% of the value of the loan – lenders pass this premium onto borrowers. Insurance qualifications (and premiums) are common across lenders and based on the posted rate. Borrowers qualifying at one bank, therefore, know that they can qualify at other institutions, given that the lender is protected in case of default.

[25]This is, in fact, not a guideline, but a legal requirement for regulated lenders. After our sample period, the requirement was adjusted and today borrowers with less than 20% equity must purchase insurance.

Table 10: Definition of Household / Mortgage Characteristics

Name	Description
FI	Type of lender
Source	Identifies how lender generated the loan (branch, online, broker, etc)
Income	Total amount of the borrower(s) salary, wages, and income from other sources
TDS	Total debt service ratio
GDS	Gross debt service
Duration	Length of the relationship between the borrower and FI
R-status	Borrowers residential status upon insurance application
FSA	Forward sortation area of the mortgaged property
Market value	Selling price or estimated market price if refinancing
Applicant type	Quartile of the borrowers risk of default
Dwelling type	10 options that define the physical structure
Close	Closing date of purchase or date of refinance
Loan amount	Dollar amount of the loan excluding the loan insurance premium
Premium	Loan insurance premium
Purpose	Purpose of the loan (purchase, port, refinance, etc.)
LTV	Loan amount divided by lending value
Price	Interest rate of the mortgage
Term	Represents the term over which the interest rate applies to the loan
Amortization	Represents the period the loan will be paid off
Interest type	Fixed or adjustable rate
$CREDIT$	Summarized application credit score (minimum borrower credit score).

Some variables were only included by one of the mortgage insurers.

B Model predictions

We rewrite equation 4 as:

$$p^* = \begin{cases} p^0 & \text{If } \omega_{(1)} > p^0 - c - \lambda \\ c + \lambda + \omega_{(1)} & \text{If } -\tilde{\Delta} < \omega_{(1)} < p^0 - c - \lambda \\ c - \tilde{\Delta} & \omega_{(2)} > -\tilde{\Delta} > \omega_{(1)} \\ c + \omega_{(2)} & \text{If } \omega_{(2)} < -\tilde{\Delta}, \end{cases}$$

and describe the model by the following functions:

- The expected second stage price:

$$\begin{aligned} E(p^*|p^0, s) &= p^0(1 - G_{(1)}(p^0 - c - \lambda)) + \int_{-\tilde{\Delta}}^{p^0-c-\lambda} (c + \lambda + \omega_{(1)}) dG_{(1)} \\ &\quad + (c - \tilde{\Delta}) \left[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta}) \right] + \int_{-\infty}^{-\tilde{\Delta}} (c + \omega_{(2)}) dG_{(2)} \end{aligned}$$

- The expected second stage profit of the home bank:

$$E(\pi_h^*|p^0, s) = (p^0 - c + \Delta)(1 - G_{(1)}(p^0 - \lambda - c)) + \int_{-\tilde{\Delta}}^{p^0-c-\lambda} (\omega_{(1)} + \tilde{\Delta}) dG_{(1)}$$

- The search-cost threshold is:

$$\bar{\kappa}(p^0, s) = p^0 - E(p^*|p^0, s) - \lambda G_{(1)}(-\tilde{\Delta})$$

- The first-order condition (FOC):

$$\begin{aligned} f(p^0, s) &= 1 - H(\bar{\kappa}(p^0, s)) - (p^0 - c + \Delta) H'(\bar{\kappa}(p^0, s)) \frac{\partial \bar{\kappa}(p^0, s)}{\partial p^0} \\ &\quad + H'(\bar{\kappa}(p^0, s)) \frac{\partial \bar{\kappa}(p^0, s)}{\partial p^0} E(\pi_h^*|p^0, s) + H(\bar{\kappa}(p^0, s)) \frac{\partial E(\pi_h^*|p^0, s)}{\partial p^0} = 0 \end{aligned}$$

B.1 Comparative statics

Using these functions, we can derive the following comparative statics. The derivative of $E(p^*|p^0, s)$ wrt p^0 is given by:

$$\begin{aligned} \frac{\partial E(p^*|p^0, s)}{\partial p^0} &= 1 - G_{(1)}(p^0 - c - \lambda) - p^0 g_{(1)}(p^0 - c - \lambda) + g_{(1)}(p^0 - c - \tilde{\Delta})[c + \lambda + p^0 - c - \lambda] \\ &= 1 - G_{(1)}(p^0 - c - \lambda) \end{aligned}$$

The derivative of $E(p^*|p^0, s)$ wrt c is given by:

$$\begin{aligned}
\frac{\partial E(p^*|p^0,s)}{\partial c} &= p^0 g_{(1)}(p^0-c-\lambda) - (c+\lambda+p^0-c-\lambda)g_{(1)}(p^0-c-\lambda) + \left[G_{(1)}(p^0-c-\lambda) - G_{(1)}(-\tilde{\Delta})\right] \\
&\quad + \left[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\right] + G_{(2)}(-\tilde{\Delta}) \\
&= G_{(1)}(p^0-c-\lambda)
\end{aligned}$$

The derivative of $E(p^*|p^0, s)$ wrt λ is given by:

$$\begin{aligned}
\frac{\partial E(p^*|p^0,s)}{\partial \lambda} &= p^0 g_{(1)}(p^0-c-\lambda) - (c+\lambda+p^0-c-\lambda)g_{(1)}(p^0-c-\lambda) \\
&\quad (c+\lambda-\tilde{\Delta})g_{(1)}(-\tilde{\Delta}) + \left[G_{(1)}(p^0-c-\lambda) - G_{(1)}(-\tilde{\Delta})\right] \\
&\quad - \left[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\right] - (c-\tilde{\Delta})\left[g_{(1)}(-\tilde{\Delta}) - g_{(2)}(-\tilde{\Delta})\right] - (c-\tilde{\Delta})g_{(2)}(-\tilde{\Delta}) \\
&= \lambda g_{(1)}(-\tilde{\Delta}) + \left[G_{(1)}(p^0-c-\lambda) - G_{(1)}(-\tilde{\Delta})\right] - \left[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\right]
\end{aligned}$$

The derivative of $E(p^*|p^0, s)$ wrt Δ is given by:

$$\begin{aligned}
\frac{\partial E(p^*|p^0,s)}{\partial \Delta} &= (c+\lambda-\tilde{\Delta})g_{(1)}(-\tilde{\Delta}) - \left[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\right] \\
&\quad -(c-\tilde{\Delta})\left[g_{(1)}(-\tilde{\Delta}) - g_{(2)}(-\tilde{\Delta})\right] - (c-\tilde{\Delta})g_{(2)}(-\tilde{\Delta}) \\
&= \lambda g_{(1)}(-\tilde{\Delta}) - \left[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\right]
\end{aligned}$$

The difference is given by:

$$\frac{\partial E(p^*|p^0,s)}{\partial \lambda} - \frac{\partial E(p^*|p^0,s)}{\partial \Delta} = G_{(1)}(p^0-c-\lambda) - G_{(1)}(-\tilde{\Delta})$$

The derivative of the expected profits wrt to p^0 is:

$$\begin{aligned}\frac{\partial E(\pi^*|p^0,s)}{\partial p^0} &= 1-G_{(1)}(p^0-\lambda-c)-(p^0-c+\Delta)g_{(1)}(p^0-c-\lambda)+(p^0-c-\lambda+\Delta+\lambda)g_{(1)}(p^0-c-\lambda)\\ &= 1-G_{(1)}(p^0-\lambda-c)\end{aligned}$$

The derivative of the expected profits wrt to c is:

$$\begin{aligned}\frac{\partial E(\pi^*|p^0,s)}{\partial c} &= -(1-G_{(1)}(p^0-\lambda-c))+(p^0-c+\Delta)g_{(1)}(p^0-c-\lambda)-(p^0-c-\lambda+\Delta+\lambda)g_{(1)}(p^0-c-\lambda)\\ &= G_{(1)}(p^0-\lambda-c)-1\end{aligned}$$

The derivative of the expected profits wrt to λ is:

$$\begin{aligned}\frac{\partial E(\pi^*|p^0,s)}{\partial \lambda} &= (p^0-c+\Delta)g_{(1)}(p^0-\lambda-c)-(p^0-c-\lambda+\Delta+\lambda)g_{(1)}(p^0-c-\lambda)\\ &\quad +(-\tilde{\Delta}+\tilde{\Delta})g_{(1)}(-\tilde{\Delta})+\left[G_{(1)}(p^0-c-\lambda)-G_{(1)}(-\tilde{\Delta})\right]\\ &= \left[G_{(1)}(p^0-c-\lambda)-G_{(1)}(-\tilde{\Delta})\right]\end{aligned}$$

The derivative of the expected profits wrt to Δ is:

$$\begin{aligned}\frac{\partial E(\pi^*|p^0,s)}{\partial \Delta} &= 1-G_{(1)}(p^0-\lambda-c)+(-\tilde{\Delta}+\tilde{\Delta})g_{(1)}(-\tilde{\Delta})+\left[G_{(1)}(p^0-c-\lambda)-G_{(1)}(-\tilde{\Delta})\right]\\ &= 1-G_{(1)}(-\tilde{\Delta})\end{aligned}$$

The difference is negative:

$$\frac{\partial E(\pi^*|p^0,s)}{\partial \lambda}-\frac{\partial E(\pi^*|p^0,s)}{\partial \Delta} = G_{(1)}(p^0-c-\lambda)-1<0$$

and,

$$\begin{aligned}\frac{\partial E(\pi^*|p^0,s)}{\partial p^0} &= \frac{\partial E(\pi^*|p^0,s)}{\partial \Delta}-\frac{\partial E(\pi^*|p^0,s)}{\partial \lambda}\\ &= -\frac{\partial E(\pi^*|p^0,s)}{\partial \lambda}+1-G_{(1)}(-\tilde{\Delta})\end{aligned}$$

The derivative of $\bar{\kappa}(p^0, s)$ wrt p^0 is:

$$\frac{\partial \bar{\kappa}(p^0, s)}{\partial p^0} = 1 - \frac{\partial E(p^*|p^0, s)}{\partial p^0} = G_{(1)}(p^0 - c - \lambda) > 0$$

The derivative of $\bar{\kappa}(p^0, s)$ wrt c is:

$$\frac{\partial \bar{\kappa}(p^0, s)}{\partial c} = -\frac{\partial E(p^*|p^0, s)}{\partial c} = -G_{(1)}(p^0 - c - \lambda) < 0$$

The derivative of $\bar{\kappa}(p^0, s)$ wrt λ is:

$$\begin{aligned}
\frac{\partial \bar{\kappa}(p^0, s)}{\partial \lambda} &= -\frac{\partial E(p^*|p^0, s)}{\partial \lambda} - G_{(1)}(-\tilde{\Delta}) + \lambda g_{(1)}(-\tilde{\Delta}) \\
&= -\lambda g_{(1)}(-\tilde{\Delta}) - \Big[G_{(1)}(p^0 - c - \lambda) - G_{(1)}(-\tilde{\Delta})\Big] + \Big[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\Big] \\
&\quad -G_{(1)}(-\tilde{\Delta}) + \lambda g_{(1)}(-\tilde{\Delta}) \\
&= -\Big[G_{(1)}(p^0 - c - \lambda) - G_{(1)}(-\tilde{\Delta})\Big] - G_{(2)}(-\tilde{\Delta}) < 0
\end{aligned}$$

The derivative of $\bar{\kappa}(p^0, s)$ wrt Δ is:

$$\begin{aligned}
\frac{\partial \bar{\kappa}(p^0, s)}{\partial \Delta} &= -\frac{\partial E(p^*|p^0, s)}{\partial \Delta} + \lambda g_{(1)}(-\tilde{\Delta}) \\
&= -\lambda g_{(1)}(-\tilde{\Delta}) + \Big[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\Big] + \lambda g_{(1)}(-\tilde{\Delta}) \\
&= \Big[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\Big]
\end{aligned}$$

The difference is:

$$\begin{aligned}
\frac{\partial \bar{\kappa}(p^0, s)}{\partial \lambda} - \frac{\partial \bar{\kappa}(p^0, s)}{\partial \Delta} &= -\Big[G_{(1)}(p^0 - c - \lambda) - G_{(1)}(-\tilde{\Delta})\Big] - G_{(2)}(-\tilde{\Delta}) - \Big[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\Big] \\
&= -G_{(1)}(p^0 - c - \lambda)
\end{aligned}$$

B.2 Proofs of Proposition 1 and Corollary 1

Using these results, we can prove Proposition 1 and Corollary 1. In order to do so we need to derive expressions for the second-order condition (SOC) and the partials of the FOC wrt c, λ and Δ.

The FOC can be re-arranged as follows:

$$\begin{aligned}
f(p^0, s) &= 1 - H(\bar{\kappa}(p^0, s)\left[1 - \frac{\partial E(\pi_h^*|p^0, s)}{\partial p^0}\right] - H'(\bar{\kappa}(p^0, s))\frac{\partial \bar{\kappa}(p^0, s)}{\partial p^0}\left[\pi^0 - E(\pi_h^*|p^0, s)\right] \\
&= 1 - H(\bar{\kappa}(p^0, s))G_{(1)}(p^0 - c - \lambda) - H'(\bar{\kappa}(p^0, s))G_{(1)}(p^0 - c - \lambda)\left[\pi^0 - E(\pi_h^*|p^0, s)\right] \\
&= 1 - G_{(1)}(p^0 - c - \lambda)\left(H(\bar{\kappa}) + H'(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right]\right)
\end{aligned}$$

where $\bar{\kappa} \equiv \bar{\kappa}(p^0, s)$, $\pi^0 \equiv p^0 - c + \Delta$, and $E(\pi_h^*) \equiv E(\pi_h^* | p^0, s)$.

The SOC is given by:

$$\begin{aligned}
f_{p^0}(p^0, s) &= -g_{(1)}(p^0 - c - \lambda)\left(H(\bar{\kappa}) + H'(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right]\right) \\
&\quad -G_{(1)}(p^0 - c - \lambda)\left(H'(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial p^0} + H''(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial p^0}\left[\pi^0 - E(\pi_h^*)\right] + H'(\bar{\kappa})\left[1 - \frac{\partial E(\pi_h^*)}{\partial p^0}\right]\right) \\
&= -g_{(1)}(p^0 - c - \lambda)\left(H(\bar{\kappa}) + H'(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right]\right) - G_{(1)}(p^0 - c - \lambda)^2 \left(\begin{array}{c} 2H'(\bar{\kappa}) \\ +H''(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right] \end{array}\right)
\end{aligned}$$

The derivative of the FOC wrt c is:

$$\begin{aligned}
f_c(p^0, s) &= g_{(1)}(p^0 - c - \lambda)\left(H(\bar{\kappa}) + H'(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right]\right) \\
&\quad -G_{(1)}(p^0 - c - \lambda)\left(H'(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial c} + H''(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial c}\left[\pi^0 - E(\pi_h^*)\right] + H'(\bar{\kappa})\left[-1 - \frac{\partial E(\pi_h^*)}{\partial c}\right]\right) \\
&= g_{(1)}(p^0 - c - \lambda)\left(H(\bar{\kappa}) + H'(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right]\right) + G_{(1)}(p^0 - c - \lambda)^2 \left(\begin{array}{c} 2H'(\bar{\kappa}) \\ +H''(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right] \end{array}\right)
\end{aligned}$$

The derivative of the FOC wrt λ is:

$$\begin{aligned}
f_\lambda(p^0, s) &= g_{(1)}(p^0 - c - \lambda)\left(H(\bar{\kappa}) + H'(\bar{\kappa})\left[\pi^0 - E(\pi_h^*)\right]\right) \\
&\quad -G_{(1)}(p^0 - c - \lambda)\left(H'(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial \lambda} + H''(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial \lambda}\left[\pi^0 - E(\pi_h^*)\right] + H'(\bar{\kappa})\left[-\frac{\partial E(\pi^*)}{\partial \lambda}\right]\right)
\end{aligned}$$

The derivative of the FOC wrt Δ is:

$$f_\Delta(p^0, s) = -G_{(1)}(p^0 - c - \lambda)\left(H'(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial \Delta} + H''(\bar{\kappa})\frac{\partial \bar{\kappa}}{\partial \Delta}\left[\pi^0 - E(\pi_h^*)\right] + H'(\bar{\kappa})\left[1 - \frac{\partial E(\pi^*)}{\partial \Delta}\right]\right)$$

B.2.1 Proof of Proposition 1

We first prove Proposition 1.

Proof of Proposition 1. From above, we have that

$$f_c(p^0, s) \quad = \quad -f_{p^0}(p^0)$$

Therefore,

$$\frac{dp^0}{dc} = -\frac{f_c(p^0)}{f_{p^0}(p^0)} = 1$$

This proves that the initial quote is additive in c as claimed. □

B.2.2 Proof of Proposition Corollary 1

Next, we prove Corollary 1(i)

Proof of Corollary 1(i). The proof consists of three steps:

Step 1: If p^0 is additive in c, then, from equation (26), since p^0 and the value of the home bank and the second highest utility lender are additive in c, we have that $E(p^*|\bar{p}, s)$ is also additive in c.

Step 2: Since only the difference between p^0 and $E(p^*|\bar{p}, s)$ matters for determining the threshold of consumers, the search probability H is independent of c. Specifically, if p^0 is in the interior, then the search threshold is implicitly defined by equation 6 and depends only on n: $\bar{\kappa}(n)$. If, on the other hand, $p^0 = \bar{p}$, then $\bar{\kappa}(p^0, c, n) = E(p^*|\bar{p}, c, n) - \bar{p}$.

This completes the proof of Corollary 1(i). □

Proof of Corollary 1(ii). To prove Corollary 1(ii) we need to show that the marginal effect of λ and Δ on $\bar{\kappa}(p^0, s)$ are equal in equilibrium. The total derivatives are given by:

$$\begin{aligned}
\frac{d\bar{\kappa}(p^0, s)}{d\lambda} &= \frac{\partial\bar{\kappa}(p^0, s)}{\partial\lambda} + \frac{\partial\bar{\kappa}(p^0, s)}{\partial p^0}\frac{dp^0}{d\lambda} \\
&= -\left[G_{(1)}(p^0 - c - \lambda) - G_{(1)}(-\tilde{\Delta})\right] - G_{(2)}(-\tilde{\Delta}) + \frac{dp0}{d\lambda}\left(1 - \frac{\partial E(p^*|p^0, s)}{\partial p^0}\right) \\
\frac{d\bar{\kappa}(p^0, s)}{d\Delta} &= \frac{\partial\bar{\kappa}(p^0, s)}{\partial\Delta} + \frac{\partial\bar{\kappa}(p^0, s)}{\partial p^0}\frac{dp^0}{d\Delta} \\
&= \left[G_{(1)}(-\tilde{\Delta}) - G_{(2)}(-\tilde{\Delta})\right] + \frac{dp0}{d\Delta}\left(1 - \frac{\partial E(p^*|p^0, s)}{\partial p^0}\right)
\end{aligned}$$

The difference between the two is:

$$\begin{aligned}
\frac{d\bar{\kappa}(p^0, s)}{d\lambda} - \frac{d\bar{\kappa}(p^0, s)}{d\Delta} &= -G_{(1)}(p^0 - c - \lambda) + \left(1 - \frac{\partial E(p^*|p^0, s)}{\partial p^0}\right)\left[\frac{dp0}{d\lambda} - \frac{dp0}{d\Delta}\right] \\
&= -G_{(1)}(p^0 - c - \lambda) + G_{(1)}(p^0 - c - \lambda)\left[\frac{f_\Delta(p^0, s) - f_\lambda(p^0, s)}{f_{p^0}(p^0)}\right]
\end{aligned}$$

Therefore the proof of Corollary 3.2 requires that:

$$\frac{f_\Delta(p^0,s) - f_\lambda(p^0,s)}{f_{p^0}(p^0)} = 1$$

The difference between $\frac{f_\Delta(p^0,s)}{f_{p^0}(p^0)}$ and $\frac{f_\lambda(p^0,s)}{f_{p^0}(p^0)}$ is:

$$\begin{aligned} f_\Delta(p^0,s) - f_\lambda(p^0,s) &= -g_{(1)}(p^0-c-\lambda)\left(H(\bar\kappa) + H'(\bar\kappa)\left[\pi^0 - E(\pi_h^*)\right]\right) \\ &\quad -G_{(1)}(p^0-c-\lambda)\begin{pmatrix} H'(\bar\kappa)\left[\frac{\partial\bar\kappa}{\partial\Delta} - \frac{\partial\bar\kappa}{\partial\lambda}\right] + H''(\bar\kappa)\left[\frac{\partial\bar\kappa}{\partial\Delta} - \frac{\partial\bar\kappa}{\partial\lambda}\right]\left[\pi^0 - E(\pi_h^*)\right] \\ +H'(\bar\kappa)\left[1 - \frac{\partial E(\pi^*)}{\partial\Delta} + \frac{\partial E(\pi^*)}{\partial\lambda}\right] \end{pmatrix} \\ &= -g_{(1)}(p^0-c-\lambda)\left(H(\bar\kappa) + H'(\bar\kappa)\left[\pi^0 - E(\pi_h^*)\right]\right) \\ &\quad -G_{(1)}(p^0-c-\lambda)^2\left(2H'(\bar\kappa) + H''(\bar\kappa)\left[\pi^0 - E(\pi_h^*)\right]\right) \\ &= f_{p^0}(p^0,s) \end{aligned}$$

Therefore,

$$\frac{f_\Delta(p^0,s) - f_\lambda(p^0,s)}{f_{p^0}(p^0,s)} = 1.$$

□

Proof of Corollary 1(iii). To prove that the distribution of prices for switchers is a function only of the sum of the two loyalty terms (i.e. $\tilde\Delta$), we consider the expected second stage price paid by switchers conditional on c.

$$E(p_S^*|p^0,s,S) = \frac{1}{G_{(1)}(-\tilde\Delta)}\left[(c-\tilde\Delta)\left[G_{(1)}(-\tilde\Delta) - G_{(2)}(-\tilde\Delta)\right] + \int_{-\infty}^{-\tilde\Delta}(c+\omega_{(2)})dG_{(2)}\right]$$

Since λ and Δ do not enter this equation separately, the distribution of prices for switchers is a function only of their sum $\tilde\Delta$. □

Proof of Corollary 1(iv). The average transaction price paid by loyal consumers is given by:

$$\begin{aligned} E(p|p^0,s,L) &= E(p^0(1-H) + HE(p^*|p^0,s,L)) \\ &= E(p^0 - H(p^0 - E(p^*|p^0,s,L)) \end{aligned}$$

We want to show that $\frac{dE(p|p^0,s,L)}{d\lambda} \neq \frac{dE(p|p^0,s,L)}{d\Delta}$.

$$\begin{aligned} \frac{dE(p|p^0,s,L)}{d\lambda} - \frac{dE(p|p^0,s,L)}{d\Delta} &= E\left[\frac{dp^0}{d\lambda} - \frac{dp^0}{d\Delta} - H\left[\frac{dp^0}{d\lambda} - \frac{dp^0}{d\Delta} - \left(\frac{dE(p_L^*|p^0,s,L)}{d\lambda} - \frac{dE(p_L^*|p^0,s,L)}{d\Delta}\right)\right]\right] \\ &= E(1 - H(1 - E(p^*|p^0,s,L)) \end{aligned}$$

Therefore, to show that the average transaction price paid by loyal consumers is affected asymmetrically we need to compare $\frac{dE(p_L^*|p^0,s,L)}{d\lambda}$ and $\frac{dE(p_L^*|p^0,s,L)}{d\Delta}$, where $E(p_L^*|p^0,s,L)$ is the expected second stage price for loyals and is given by:

$$E(p_L^*|p^0, s, L) = \frac{1}{1 - G_{(1)}(-\tilde{\Delta})}\left[p^0(1 - G_{(1)}(p^0 - c - \lambda)) + \int_{-\tilde{\Delta}}^{p^0 - c - \lambda}(c + \lambda + \omega_{(1)})dG_{(1)}\right].$$

We have that

$$\frac{dE(p_L^*|p^0, s)}{d\lambda} = \frac{\partial E(p_L^*|p^0, s, L)}{\partial \lambda} + \frac{\partial E(p_L^*|p^0, s, L)}{\partial p^0}\frac{dp^0}{d\lambda},$$

$$\frac{dE(p_L^*|p^0, s)}{d\Delta} = \frac{\partial E(p_L^*|p^0, s, L)}{\partial \Delta} + \frac{\partial E(p_L^*|p^0, s, L)}{\partial p^0}\frac{dp^0}{d\Delta}.$$

The derivative of $E(p_L^*|p^0, s, L)$ wrt λ is given by:

$$\begin{aligned}\frac{\partial E(p_L^*|p^0, s, L)}{\partial \lambda} &= \frac{-g_{(1)}(-\tilde{\Delta})}{[1 - G_{(1)}(-\tilde{\Delta})]^2}\left[p^0(1 - G_{(1)}(p^0 - c - \lambda)) + \int_{-\tilde{\Delta}}^{p^0 - c - \lambda}(c + \lambda + \omega_{(1)})dG_{(1)}\right] \\ &+ \frac{1}{1 - G_{(1)}(-\tilde{\Delta})}\left[(c + \lambda - \tilde{\Delta})g_{(1)}(-\tilde{\Delta}) + \left[G_{(1)}(p^0 - c - \lambda) - G_{(1)}(-\tilde{\Delta})\right]\right]\end{aligned}$$

The derivative of $E(p_L^*|p^0, s, L)$ wrt Δ is given by:

$$\begin{aligned}\frac{\partial E(p_L^*|p^0, s, L)}{\partial \Delta} &= \frac{-g_{(1)}(-\tilde{\Delta})}{[1 - G_{(1)}(-\tilde{\Delta})]^2}\left[p^0(1 - G_{(1)}(p^0 - c - \lambda)) + \int_{-\tilde{\Delta}}^{p^0 - c - \lambda}(c + \lambda + \omega_{(1)})dG_{(1)}\right] \\ &+ \frac{1}{1 - G_{(1)}(-\tilde{\Delta})}\left[(c + \lambda - \tilde{\Delta})g_{(1)}(-\tilde{\Delta})\right]\end{aligned}$$

The difference is given by:

$$\frac{\partial E(p_L^*|p^0, s, L)}{\partial \lambda} - \frac{\partial E(p_L^*|p^0, s, L)}{\partial \Delta} = \frac{1}{1 - G_{(1)}(-\tilde{\Delta})}\left[G_{(1)}(p^0 - c - \lambda) - G_{(1)}(-\tilde{\Delta})\right]$$

We also have that $\frac{\partial E(p_L^*|p^0,s,L)}{\partial p^0} = \frac{1}{1-G_{(1)}(-\tilde{\Delta})}\left[1 - G_{(1)}(p^0 - c - \lambda)\right]$, such that

$$\frac{\partial E(p_L^*|p^0, s, L)}{\partial p^0}\frac{dp^0}{d\lambda} - \frac{\partial E(p_L^*|p^0, s, L)}{\partial p^0}\frac{dp^0}{d\Delta} = \frac{1}{1 - G_{(1)}(-\tilde{\Delta})}\left[1 - G_{(1)}(p^0 - c - \lambda)\right],$$

since $\frac{dp^0}{d\lambda} - \frac{dp^0}{d\Delta} = 1$.

Combining these differences we have that

$$\begin{aligned}
\frac{dE(p_L^*|p^0,s,L)}{d\lambda} - \frac{dE(p_L^*|p^0,s,L)}{d\Delta} &= \frac{1}{1-G_{(1)}(-\tilde{\Delta})}\left[G_{(1)}(p^0-c-\lambda) - G_{(1)}(-\tilde{\Delta})\right] \\
&\quad + \frac{1}{1-G_{(1)}(-\tilde{\Delta})}\left[1-G_{(1)}(p^0-c-\lambda)\right] \\
&= \frac{1-G_{(1)}(-\tilde{\Delta})}{1-G_{(1)}(-\tilde{\Delta})} = 1
\end{aligned}$$

Therefore

$$\frac{dE(p|p^0,s,L)}{d\lambda} - \frac{dE(p|p^0,s,L)}{d\Delta} = 1$$

□

C Identification in English Auctions with Additively Separable Unobserved Heterogeneity

We follow Quint (2015) to show that a model with additively-separable unobserved heterogeneity is identified using only minimal variation in the number of bidders. Our case differs from Quint in that the setting is one of procurement, and also that one of the firms (the home bank) has a known additive component and the sample we observe is for cases where this firm *does not* win the auction. As a result, we do not observe the entire distribution of transaction prices.

C.1 Model

As in Quint (2015) we assume symmetric independent private costs with additively separable unobserved heterogeneity. We have a single home bank and $N-1$ other bidders in the auction. Bidder j's private cost is:

$$c_j = \begin{cases} c - \tilde{\Delta} & \text{If } i \text{ is the home bank} \\ c + \omega_j & \text{Otherwise,} \end{cases} \tag{26}$$

where c is a common term (observed by all bidders, but not the econometrician) and $\{\omega_j\}$ is $i.i.d$ and independent of c. $\tilde{\Delta}$ is known by all.

In following Quint, for simplicity, in the proof, we assume that c and $\{\omega_j\}$ are both discrete-valued. We assume that c is bounded below and above by $\underline{c}$ and $\bar{c}$. We further suppose that variation in N is exogenous, and that there is not a binding reserve price. Finally, we assume that ties in the auction are broken in favor of the home bank.

The winning bid in each auction is equal to the second-lowest cost. We denote by T the transaction price of the auction.

Theorem: If N varies exogenously and takes at least two values, then observations of T and N identify c and a truncated distribution of ω_j up to $\bar{c} - \tilde{\Delta}$.

C.2 Proof

Let $Pr(T = \cdot|N, \text{switch})$ denote the distribution of transaction prices given that N bidders participated and the home bank did not win the auction (ie. that the consumer switched banks). Let $c_k = Pr(c = k)$ and $w_k = Pr(\omega = k)$. Also, define $w_{<l} = Pr(\omega_j < l) = \sum_{l'<l} w_{l'}$ Let $\omega^{(1)}$ and $\omega^{(2)}$ denote the first and second lowest of the ω observed in an auction and $Pr(\omega^{(2)} = \cdot|N, \text{switch})$ its distribution conditional on N bidders and the home bank not winning. Then we have:

Lemma: Fix m and m', with $m' > m \geq 2$. For any $k \geq 0$, the parameters $c_{\bar{c}-k}$ and $w_{<-\tilde{\Delta}-k}$, can be recovered from the moments $Pr(T = \bar{c} - \tilde{\Delta} - k|m, \text{switch})$, $Pr(T = \bar{c} - \tilde{\Delta} - k|m', \text{switch})$, and the parameters $\{c_{\bar{c}-l}, w_{-\Delta-l}\}_{l<k}$.

C.2.1 Step 1: Recover $c_{\bar{c}}$ and $\omega_{<-\tilde{\Delta}}$ from the data

$c_{\bar{c}}$ and $w_{<-\tilde{\Delta}}$ are pinned down by $Pr(T = \bar{c} - \tilde{\Delta}|m, \text{switch})$, $Pr(T = \bar{c} - \tilde{\Delta}|m', \text{switch})$. Since c is bounded above by $\bar{c}$,

$$\begin{aligned}
Pr(T = \bar{c} - \tilde{\Delta}|N, \text{switch}) &= Pr(c = \bar{c})Pr(\omega^{(2)} \geq -\tilde{\Delta}| - \tilde{\Delta} > \omega^{(1)}, N) \\
&= Pr(c = \bar{c})\frac{Pr(\omega^{(2)} \geq -\tilde{\Delta} > \omega^{(1)}|N)}{Pr(-\tilde{\Delta} > \omega^{(1)}|N)} \\
&= c_{\bar{c}}\frac{Pr(\omega^{(2)} \geq -\tilde{\Delta} > \omega^{(1)}|N)}{Pr(-\tilde{\Delta} > \omega^{(1)}|N)} \\
&= c_{\bar{c}}\frac{(N-1)w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{N-2}}{1-(1-w_{<-\tilde{\Delta}})^{N-1}}
\end{aligned}$$

The denominator is explained by the fact that the first order statistic for the ω will be less than $-\tilde{\Delta}$ as long as it is not the case that all non-home bank lenders have draws greater than or equal to $-\tilde{\Delta}$. The numerator is the probability that just one non-home bank lender is below and all other non-home bank lenders have draws equal to or greater than $-\tilde{\Delta}$. Then the home bank loses and the first order statistic is below $-\tilde{\Delta}$ and the second order statistic is equal to or above.

Since both $Pr(T = \bar{c} - \tilde{\Delta}|m, \text{switch})$ and $Pr(T = \bar{c} - \tilde{\Delta}|m', \text{switch})$ are observed, we can calculate:

$$\begin{aligned}
\frac{Pr(T = \bar{c} - \tilde{\Delta}|m', \text{switch})}{Pr(T = \bar{c} - \tilde{\Delta}|m, \text{switch})} &= \frac{c_{\bar{c}}\frac{Pr(\omega^{(2)} \geq -\tilde{\Delta} > \omega^{(1)}|m')}{Pr(-\tilde{\Delta} > \omega^{(1)}|m')}}{c_{\bar{c}}\frac{Pr(\omega^{(2)} \geq -\tilde{\Delta} > \omega^{(1)}|m)}{Pr(-\tilde{\Delta} > \omega^{(1)}|m)}} \\
&= \frac{c_{\bar{c}}\frac{(m'-1)w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{m'-2}}{1-(1-w_{<-\tilde{\Delta}})^{m'-1}}}{c_{\bar{c}}\frac{(m-1)w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{m-2}}{1-(1-w_{<-\tilde{\Delta}})^{m-1}}}
\end{aligned} \tag{27}$$

Once $w_{<-\tilde{\Delta}}$ has been recovered, we can use $Pr(T = \bar{c} - \tilde{\Delta}|N) = c_{\bar{c}}\frac{(N-1)w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{N-2}}{1-(1-w_{<-\tilde{\Delta}})^{N-1}}$ to solve for $c_{\bar{c}}$.

We define a function $\gamma : [0, 1] \rightarrow \Re^+$ and need to show that it is invertible such that $w_{<-\tilde{\Delta}}$ can be recovered.

$$\begin{aligned}
\gamma(x) &= \frac{\frac{(m'-1)x(1-x)^{m'-2}}{1-(1-x)^{m'-1}}}{\frac{(m-1)x(1-x)^{m-2}}{1-(1-x)^{m-1}}} \\
&= \frac{m'-1}{m-1}\frac{(1-x)^{m'-m} - (1-x)^{m'-1}}{1-(1-x)^{m'-1}}
\end{aligned} \tag{28}$$

We want to show that this is strictly monotone in x for $m' > m$ for all x between 0 and 1. Differentiating we get,

$$\gamma'(x) = \frac{m'-1}{m-1}\frac{\left(-(m'-1)(1-x)^{m'-2} + (m'-m)(1-x)^{m'-m-1}\right)\left((1-x)^{m'-1} - 1\right) + \left((1-x)^{m'-1} - (1-x)^{m'-m}\right)(m'-1)(1-x)^{m'-2}}{\left((1-x)^{m'-1}-1\right)^2} \tag{29}$$

This is negative for x between 0 and 1. The proof is in the subappendix.

C.2.2 Step 2: Prove the case of $\bar{c} - \tilde{\Delta} - k, k > 0$

$$\begin{aligned} Pr(T = \bar{c} - \tilde{\Delta} - k|N, \text{switch}) &= Pr(c = \bar{c} - k)Pr(\omega^{(2)} \geq -\tilde{\Delta}|N, \text{switch}) \\ &+ \sum_{l=1}^{k} Pr(c = \bar{c} - (k-l))Pr(\omega^{(2)} = -\tilde{\Delta} - l|N, \text{switch}) \end{aligned} \tag{30}$$

We already know $Pr(\omega^{(2)} \geq -\tilde{\Delta}|N, \text{switch})$. We just need to find:

$$\begin{aligned} Pr(\omega^{(2)} &= -\tilde{\Delta} - l|N, \text{switch}) \\ &= \Big((1 - w_{<-\tilde{\Delta}-l})^{N-1} - (1 - w_{<-\tilde{\Delta}-l+1})^{N-1} - (N-1)(w_{<-\tilde{\Delta}-l+1} - w_{<-\tilde{\Delta}-l})(1 - w_{<-\tilde{\Delta}-l+1})^{N-2} \\ &+ (N-1)((w_{<-\tilde{\Delta}-l})((1 - w_{<-\tilde{\Delta}-l})^{N-2} - (1 - w_{<-\tilde{\Delta}-l+1})^{N-2})\Big)/(1 - (1 - w_{<-\tilde{\Delta}})^{N-1}) \end{aligned}$$

The first line in the numerator is the probability that the lowest and second-lowest of the ω_i are both equal to $-\tilde{\Delta} - l$ and the second part is the probability that exactly one ω_i is below $-\tilde{\Delta} - l$ and at least one of the remaining ω_i is exactly $-\tilde{\Delta} - l$.

We rewrite equation 30 as follows:

$$\begin{aligned} &Pr(T = \bar{c} - \tilde{\Delta} - k|N, \text{switch}) - \sum_{l=1}^{k-1} c_{\bar{c}-(k-l)} Pr(\omega^{(2)} = -\tilde{\Delta} - l|N, \text{switch}) \\ &= c_{\bar{c}-k} Pr(\omega^{(2)} \geq -\tilde{\Delta}|N, \text{switch}) + c_{\bar{c}} Pr(\omega^{(2)} = -\tilde{\Delta} - k|N, \text{switch}) \end{aligned}$$

$$\frac{Pr(T = \bar{c} - \tilde{\Delta} - k|m, \text{switch}) - \sum_{l=1}^{k-1} c_{\bar{c}-(k-l)} Pr(\omega^{(2)} = -\tilde{\Delta} - l|m, \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m, \text{switch})} = c_{\bar{c}-k} + c_{\bar{c}} \frac{Pr(\omega^{(2)} = -\tilde{\Delta} - k|m, \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m, \text{switch})}$$

$$\frac{Pr(T = \bar{c} - \tilde{\Delta} - k|m', \text{switch}) - \sum_{l=1}^{k-1} c_{\bar{c}-(k-l)} Pr(\omega^{(2)} = -\tilde{\Delta} - l|m', \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m', \text{switch})} = c_{\bar{c}-k} + c_{\bar{c}} \frac{Pr(\omega^{(2)} = -\tilde{\Delta} - k|m', \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m', \text{switch})}$$

Taking the difference yields:

$$\begin{aligned} &\frac{1}{c_{\bar{c}}}\Big[\frac{Pr(T = \bar{c} - \tilde{\Delta} - k|m', \text{switch}) - \sum_{l=1}^{k-1} c_{\bar{c}-(k-l)} Pr(\omega^{(2)} = -\tilde{\Delta} - l|m', \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m', \text{switch})} \\ &- \frac{Pr(T = \bar{c} - \tilde{\Delta} - k|m, \text{switch}) - \sum_{l=1}^{k-1} c_{\bar{c}-(k-l)} Pr(\omega^{(2)} = -\tilde{\Delta} - l|m, \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m, \text{switch})}\Big] \\ &= \frac{Pr(\omega^{(2)} = -\tilde{\Delta} - k|m', \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m', \text{switch})} - \frac{Pr(\omega^{(2)} = -\tilde{\Delta} - k|m, \text{switch})}{Pr(\omega^{(2)} \geq -\tilde{\Delta}|m, \text{switch})} \end{aligned}$$

We define a function $\gamma : [0, 1] \rightarrow \Re^{+}$ and need to show that it is invertible such that $w_{<-\tilde{\Delta}-k}$ can be recovered.

$$\begin{aligned} \gamma(x) &= \frac{(1-x)^{m'-1} - (1 - w_{<-\tilde{\Delta}-k+1})^{m'-1} - (m'-1)(w_{<-\tilde{\Delta}-k+1} - x)(1 - w_{<-\tilde{\Delta}-k+1})^{m'-2} + (m'-1)((x)((1-x)^{m'-2} - (1 - w_{<-\tilde{\Delta}-k+1})^{m'-2})}{(m'-1)w_{<-\tilde{\Delta}}(1 - w_{<-\tilde{\Delta}})^{m'-2}} \\ &- \frac{(1-x)^{m-1} - (1 - w_{<-\tilde{\Delta}-k+1})^{m-1} - (m-1)(w_{<-\tilde{\Delta}-k+1} - x)(1 - w_{<-\tilde{\Delta}-k+1})^{m-2} + (m-1)((x)((1-x)^{m-2} - (1 - w_{<-\tilde{\Delta}-k+1})^{m-2})}{(m-1)w_{<-\tilde{\Delta}}(1 - w_{<-\tilde{\Delta}})^{m-2}} \end{aligned}$$

and

$$\begin{aligned}
\gamma'(x) &= \frac{-(m'-2)(m'-1)x(1-x)^{m'-3}}{(m'-1)w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{m'-2}} - \frac{-(m-2)(m-1)x(1-x)^{m-3}}{(m-1)w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{m-2}} \\
&= \frac{-(m'-2)x(1-x)^{m'-3}}{w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{m'-2}} - \frac{-(m-2)x(1-x)^{m-3}}{w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})^{m-2}} \\
&= \frac{-(m'-2)x(\frac{1-x}{1-w_{<-\tilde{\Delta}}})^{m'-3}}{w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})} - \frac{-(m-2)x(\frac{1-x}{1-w_{<-\tilde{\Delta}}})^{m-3}}{w_{<-\tilde{\Delta}}(1-w_{<-\tilde{\Delta}})}
\end{aligned}$$

We have that $w_{<-\tilde{\Delta}} > x = w_{<-\tilde{\Delta}-k}$ and so $(\frac{1-x}{1-w_{<-\tilde{\Delta}}})^{N-3} > 1$ such that $(\frac{1-x}{1-w_{<-\tilde{\Delta}}})^{m'-3} > (\frac{1-x}{1-w_{<-\tilde{\Delta}}})^{m-3}$. Therefore, the numerator of the first term is bigger than the numerator of the second. Since the first term is negative, the derivative is overall negative.

C.3 Subappendix

We want to show that the following expression is monotone on $[0,1]$:

$$\gamma(x) = \frac{\frac{(m'-1)x(1-x)^{m'-2}}{1-(1-x)^{m'-1}}}{\frac{(m-1)x(1-x)^{m-2}}{1-(1-x)^{m-1}}}$$

Using $1-x=y$, rewrite this as:

$$\gamma(x) = \frac{m'-1}{m-1}\frac{y^{m'-1}-y^{m'-m}}{y^{m'-1}-1}$$

Differentiating wrt y we get

$$\gamma'(x) = \frac{m'-1}{m-1}\frac{\left((m'-1)\,y^{m'-2}-(m'-m)\,y^{m'-m-1}\right)\left(y^{m'-1}-1\right)-\left(y^{m'-1}-y^{m'-m}\right)(m'-1)\,y^{m'-2}}{\left(y^{m'-1}-1\right)^2}$$

The denominator is positive and so we are only interested in the sign of the numerator, which we can simplify as:

$$\begin{aligned}
&\left(\left(m'-1\right)y^{m'-2}-\left(m'-m\right)y^{m'-m-1}\right)\left(y^{m'-1}-1\right)-\left(y^{m'-1}-y^{m'-m}\right)\left(m'-1\right)y^{m'-2} \\
= \quad & -y^{m'-m-2}\left(m'y^m-my^{m'}-m'y+my+y^{m'}-y^m\right)
\end{aligned}$$

And so the sign depends on the term in brackets, which we label $G(y)$

$$G(y) = m'y^m-my^{m'}-m'y+my+y^{m'}-y^m = \left(m'-1\right)y^m-(m-1)\,y^{m'}-\left(m'-m\right)y$$

We need to show that this is $G(y)<0$ for $0<y<1$. We will show that $G\left(y\right)$ can be written as:

$$-\left(m-1\right)y\left(y-1\right)^{2}\sum_{i=0}^{m'-3}k_{i}y^{i}$$

with $k_i > 0$ for all i. This proves that $G\left(y\right) < 0$ for all $0 < y < 1$.

We have

$$\left(m-1\right)\left(2y^{2}-y-y^{3}\right)\sum_{i=0}^{m'-3}k_{i}y^{i}=\left(m'-1\right)y^{m}-\left(m-1\right)y^{m'}-\left(m'-m\right)y$$

For $y^{m'}$, let's match the coefficients: we need that $-\left(m-1\right)k_{m'-3}=-\left(m-1\right)$ or $k_{m'-3}=1$

For $y^{m'-1}$, we need that $2\left(m-1\right)k_{m'-3}-\left(m-1\right)k_{m'-4}=0$,or $k_{m'-4}=2$.

For $y^{m'-2}$, we need that $-\left(m-1\right)k_{m'-3}-\left(m-1\right)k_{m'-5}+2\left(m-1\right)k_{m'-4}=0$, or $k_{m'-5}=3$

So for $m'-r > m$, we find $k_{m'-r}=r-2>0$.

Next, we have that for y^{m}, $\left(m-1\right)\left(-k_{m-3}-k_{m-1}+2k_{m-2}\right)=m'-1$.

Then, for $m'-r>m$, we have $k_{m'-r}=r-2$, so $k_{m-1}=m'-m-1$ and $k_{m-2}=m'-m$.

Thus $\left(m-1\right)k_{m-3} = -\left(m'-1\right)-\left(m-1\right)\left(m'-m-1\right)+2\left(m-1\right)\left(m'-m\right)$, and we get $k_{m-3}=\frac{m-2}{m-1}\left(m'-m\right)>0$.

For y, $-\left(m-1\right)yk_{0}=-\left(m'-m\right)y$ and $k_{0}=\left(m'-m\right)/\left(m-1\right)>0$.

For y^{2}, $2\left(m-1\right)k_{0}-\left(m-1\right)k_{1}=0$, $k_{1}=2k_{0}>0$.

For y^{3} $2\left(m-1\right)k_{1}-\left(m-1\right)k_{2}-\left(m-1\right)k_{0}=0$, $k_{2}=3k_{0}>0$.

In general we have that $k_{r}=(r+1)k_{0}>0$ for $r<m-3$.

D Full derivation of the likelihood function

The model is described by the following random variables:

- Lending cost: $c_{ij} = c_i + \omega_{ij}$.
- Home bank cost differential: $\Pr(\omega_{ih} < z) = G_h(z|\sigma_\omega)$.
- Home bank quality: λ.
- Rival banks cost differential: $\Pr(\omega_{ij} < \omega) = G_j(\omega|\sigma_\omega)$.
- Common cost component: $\Pr(c_i < c|x_i) = F(c|x_i, \beta, \sigma_\epsilon)$.
- Search cost: $\Pr(\kappa_i < \kappa|z_i^1) = H(\kappa|z_i^1)$.
- Set of rival banks: $n(i)$.
- Posted rate at negotiated date: $\bar{p}_{t(i)}$.

To simplify the notation, we abstract completely from consumer i's information. This gives us three distributions: $G_j(\cdot)$, $F(\cdot)$ and $H(\cdot)$. Also, we use the subscript b to index the chosen lender, and $-b$ to index the most efficient lender other than b in the choice-set of consumers. Moreover, since Δ is realized and common knowledge at the time of negotiation, we will treat it as a constant.

Let p^0 denote the realized initial quote. If the home bank qualifies for a loan (i.e. $c - \Delta < \bar{p}$), the auction outcome is:

$$p^* = \begin{cases} p^0 & \text{If } \omega_{(1)} > p^0 - c - \lambda, \\ c + \omega_{(1)} + \lambda & \text{If } -\Delta - \lambda < \omega_{(1)} < p^0 - c - \lambda, \\ c + \min\{\omega_{(2)}, -\Delta - \lambda\} & \text{If } -\Delta - \lambda > \omega_{(1)}. \end{cases}$$

If the home bank fails to qualify ($c - \Delta > \bar{p}$), the reserve price is $p^0 = \bar{p}$ and the auction outcome is:

$$p^* = \begin{cases} \bar{p} & \text{If } \omega_{(2)} > p - c > \omega_{(1)}, \\ c + \omega_{(2)} & \text{If } \omega_{(2)} < \bar{p} - c. \end{cases}$$

The initial quote is the following step function:

$$p^0 = \begin{cases} +\infty & \text{If } c > \bar{p} + \Delta, \\ \bar{p} & \text{If } \bar{p} > c > \bar{p} - \mu_i, \\ c + \mu_i & \text{If } c < \bar{p} - \mu_i. \end{cases}$$

where $\mu_i \equiv \mu(n, \Delta, \lambda)$ is the initial quote markup (interior solution). Consumers decide to search based on a cut-off rule: $\kappa_i < \bar{\kappa}(s)$ where $s = (c, \bar{p}, \lambda, \Delta, n)$. To highlight the dependence of the search threshold with c and simplify the notation, we use $H(\bar{\kappa}(s)) \equiv H(\bar{\kappa}(c))$ to denote equilibrium search probability. The other state variables are implicit in the consumer index i.

To construct the likelihood we consider three separate cases: (i) Switching consumers, (ii) Loyal consumers going to the auction, and (iii) Loyal consumers accepting the initial quote. Within each of these cases, it is useful to consider separately consumers paying the posted rate and consumers obtaining a discount.

Case 1: Switching consumers

The population of switchers receiving a discount and dealing with lender b is composed of searchers who rejected p^0 (or didn't receive an initial quote) and whose home bank was not competitive at the auction stage. Recall that the price that switchers pay is given by:

$$p_i = \begin{cases} c - \Delta - \lambda & \text{If } \omega_{i,b} < -\Delta - \lambda < \omega_{i,-b} \text{ and } c < \bar{p} + \Delta \\ c + \omega_{i,-b} & \text{Else.} \end{cases}$$

The joint probability of observing an unconstrained price lower than p from a switching consumer is given by:

$$\begin{aligned}
&\Pr(P_i < p, B_i = b, B_i \neq h) \\
&= \int_{-\infty}^{\bar{p}+\Delta} \int_{\omega_{i,-b}} \int_{\omega_b} 1(c + \min\{-\Delta - \lambda, \omega_{i,-b}\} < p) 1(\omega_{i,b} < \min\{-\Delta - \lambda, \omega_{i,-b}\}) H(\bar{\kappa}(c)) dG_{-b} dG_b dF \\
&\quad + \int_{\bar{p}+\Delta}^{\infty} \int_{\omega_{i,-b}} \int_{\omega_b} 1(c + \omega_{i,-b} < p) 1(\omega_{i,b} < \omega_{i,-b}) dG_{-b} dG_b dF \\
&= \int_{-\infty}^{\bar{p}+\Delta} \int_{-\Delta-\lambda}^{+\infty} 1(c < p + \Delta + \lambda) G_b(-\Delta - \lambda) H(\bar{\kappa}(c)) dG_{-b} dF \\
&\quad + \int_{-\infty}^{\bar{p}+\Delta} \int_{-\infty}^{-\Delta-\lambda} 1(\omega_{i,-b} < p - c) G_b(\omega_{i,-b}) H(\bar{\kappa}(c)) dG_{-b} dF + \int_{\bar{p}+\Delta}^{\infty} \int_{-\infty}^{p-c} G_b(\omega_{i,-b}) dG_{-b} dF \\
&= \int_{-\infty}^{\min\{p+\Delta+\lambda, \bar{p}+\Delta\}} (1 - G_{-b}(-\Delta - \lambda)) G_b(\omega_h - \lambda) H(\bar{\kappa}(c)) dF \\
&\quad + \int_{-\infty}^{\bar{p}+\Delta} \int_{-\infty}^{\min\{-\Delta-\lambda, p-c\}} G_b(\omega_{i,-b}) H(\bar{\kappa}(c)) dG_{-b} dF + \int_{\bar{p}+\Delta}^{\infty} \int_{-\infty}^{p-c} G_b(\omega_{i,-b}) dG_{-b} dF
\end{aligned}$$

The derivative of the previous probability with respect to p yields the likelihood contribution of unconstrained switching consumers:

$$\begin{aligned}
l_i(p,b) &= 1(\bar{p} > p + \lambda)(1 - G_{-b}(-\Delta - \lambda)) G_b(-\Delta - \lambda) H(\bar{\kappa}(p + \Delta + \lambda)) f(p + \Delta + \lambda) \\
&\quad + \int_{-\infty}^{\bar{p}+\Delta} 1(p - c < -\Delta - \lambda) g_{-b}(p - c) G_b(p - c) H(\bar{\kappa}(c)) dF + \int_{\bar{p}+\Delta}^{\infty} g_{-b}(p - c) G_b(p - c) dF \\
&= 1(\bar{p} > p + \lambda) \left[\begin{array}{l} (1 - G_{-b}(-\Delta - \lambda)) G_b(-\Delta - \lambda) H(\bar{\kappa}(p + \Delta + \lambda)) f(p + \Delta + \lambda) \\ + \int_{p+\Delta+\lambda}^{\bar{p}+\Delta} g_{-b}(p - c) G_b(p - c) H(\bar{\kappa}(c)) dF \end{array} \right] \\
&\qquad + \int_{\bar{p}+\Delta}^{\infty} g_{-b}(p - c) G_b(p - c) dF \\
&= 1(\bar{p} > p + \lambda)(1 - G_{-b}(-\Delta - \lambda)) G_b(-\Delta - \lambda) H(\bar{\kappa}(p + \Delta + \lambda)) f(p + \Delta + \lambda) \\
&\quad + \int_{p+\Delta+\lambda}^{\infty} g_{-b}(p - c) G_b(p - c) H(\bar{\kappa}(c)) dF
\end{aligned}$$

The likelihood is more straightforward for switching consumers paying the posted rate. This event occurs only if: (i) the home bank fails to quality, (ii) the lowest cost lender is the only qualifying firm. The probability of observing this event is the likelihood contribution of constrained switching consumers:

$$
\begin{aligned}
l_i(p=\bar{p},b) &= \int_{c>\bar{p}+\Delta}\int_{\omega_{i,-b}}\int_{\omega_{i,b}} 1(\omega_{i,-b}>\bar{p}-c)1(\omega_{i,b}<\bar{p}-c)dG_b dG_{-b}dF \\
&= \int_{\bar{p}+\Delta}^{\infty} G_b(\bar{p}-c)(1-G_{-b}(\bar{p}-c))dF \qquad (31)
\end{aligned}
$$

Case 2: Loyal consumers going to the auction

We consider first the case of loyal consumers going to the auction and receiving a discount. The population of consumers receiving a discount and staying with their home bank despite rejecting the initial offer is composed of consumers: (i) who qualify at their home bank, (ii) for whom the home bank cost advantage is large enough, and (iii) who chose to search. Two events can lead to this case:

$$
p_i = \begin{cases} c+\mu & \text{If } c+\mu<\bar{p} \text{ and } \omega_{(1)}>\mu-\lambda, \\ c+\omega_{(1)}+\lambda & \text{If } c<\bar{p}+\Delta \text{ and } -\Delta-\lambda<\omega_{(1)}<p^0(c)-c-\lambda, \end{cases}
$$

where $p^0(c)=\min\{\bar{p},c+\mu(c,\Delta,\lambda)\}$ is the initial quote function.

The joint probability of observing a price lower than p from a loyal consumer is given by:

$$
\begin{aligned}
\Pr(P_i<p,B_i=h) &= \int_{-\infty}^{\bar{p}-\mu}\int_{\mu-\lambda}^{\infty} 1(c<p-\mu)H(\bar{\kappa}(c))dG_{(1)}dF \\
&\quad + \int_{-\infty}^{\bar{p}+\Delta}\int_{-\Delta-\lambda}^{p^0(c)-c-\lambda} 1(\omega_1<p-c-\lambda)H(\bar{\kappa}(c))dG_{(1)}dF \\
&= \int_{-\infty}^{p-\mu}[1-G_{(1)}(\mu-\lambda)]H(\bar{\kappa}(c))dF \\
&\quad + \int_{-\infty}^{\bar{p}+\Delta}\left[G_{(1)}(\min\{p-c-\lambda,p^0(c)-c-\lambda\})-G_{(1)}(-\Delta-\lambda)\right]H(\bar{\kappa}(c))dF \\
&= \int_{-\infty}^{p-\mu}[1-G_{(1)}(\mu-\lambda)]H(\bar{\kappa}(c))dF \\
&\quad + \int_{\bar{p}-\mu}^{\bar{p}+\Delta} 1(c<p+\Delta)\left[G_{(1)}(p-c-\lambda)-G_{(1)}(-\Delta-\lambda)\right]H(\bar{\kappa}(c))dF \quad \text{[Note: } p^0=\bar{p}] \\
&\quad + \int_{-\infty}^{\bar{p}-\mu}\left[G_{(1)}(\min\{p-c-\lambda,\mu-\lambda\})-G_{(1)}(-\Delta-\lambda)\right]H(\bar{\kappa}(c))dF \quad \text{[Note: } p^0<\bar{p}]
\end{aligned}
$$

The derivative of this probability with respect to p corresponds to the likelihood of unconstrained loyal consumers at the auction:

$$\begin{aligned}
l_i(p,b=h) &= (1-G_{(1)}(\mu-\lambda))H(\bar{\kappa}(p-\mu))f(p-\mu) \\
&\quad +1(p+\Delta>\bar{p}-\mu)\int_{\bar{p}-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) \\
&\quad +\int_{-\infty}^{\bar{p}-\mu} 1(p-c-\lambda<\mu-\lambda)1(p-c-\lambda>-\Delta-\lambda)g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) \\
&= (1-G_{(1)}(\mu-\lambda))H(\bar{\kappa}(p-\mu))f(p-\mu) \\
&\quad +1(p+\Delta>\bar{p}-\mu)\int_{\bar{p}-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) \\
&\quad +\int_{p-\mu}^{\min\{\bar{p}-\mu,p+\Delta\}} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) \\
&= \begin{cases} (1-G_{(1)}(\mu-\lambda))H(\bar{\kappa}(p-\mu))f(p-\mu) & \\ \quad +\int_{\bar{p}-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) & \text{If } p+\Delta>\bar{p}-\mu \\ \quad +\int_{p-\mu}^{\bar{p}-\mu} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) & \\ & \\ (1-G_{(1)}(\mu-\lambda))H(\bar{\kappa}(p-\mu))f(p-\mu) & \text{If } p+\Delta<\bar{p}-\mu \\ \quad +0+\int_{p-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) & \end{cases} \\
&= (1-G_{(1)}(\mu-\lambda))H(\bar{\kappa}(p-\mu))f(p-\mu)+\int_{p-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c)
\end{aligned}$$

The likelihood contribution for constrained consumers is given by the probability of observing a loyal consumer going to the auction and not receiving a discount:

$$l_i(p=\bar{p},b=h)=\int_{\bar{p}-\mu}^{\bar{p}+\Delta}(1-G_{(1)}(\bar{p}-c-\lambda))H(\bar{\kappa}(c))dF(c). \tag{32}$$

Case 3: Loyal consumers accepting the initial quote

In this case, the transaction price is equal to $p^0(c)=\min\{\bar{p},c+\mu(c,\Delta,\lambda)\}$. The likelihood contribution is therefore given by a truncated distribution:

$$l_i(p,b=h)=\begin{cases}\int_{\bar{p}-\mu}^{\bar{p}+\Delta}\left[1-H(\bar{\kappa}(c))\right]dF(c) & \text{If } p=\bar{p} \\ f(p-\mu)\left[1-H(\bar{\kappa}(p-\mu))\right] & \text{If } p<\bar{p}.\end{cases}$$

Combined likelihood function

Conditional on h, the likelihood contribution function is given by:

$$l_i(p,b;h) = \begin{cases} 1(\bar{p} > p+\lambda) \begin{bmatrix} (1-G_{-b}(-\Delta-\lambda))G_b(-\Delta-\lambda) \\ \times H(\bar{\kappa}(p+\Delta+\lambda))f(p+\Delta+\lambda) \\ + \int_{p+\Delta+\lambda}^{\bar{p}+\Delta} G_b(p-c)H(\bar{\kappa}(c))g_{-b}(p-c)dF \end{bmatrix} & \text{If } b \neq h \text{ and } p < \bar{p}, \\ \quad + \int_{\bar{p}+\Delta}^{+\infty} G_b(p-c)g_{-b}(p-c)dF & \\ & \\ \int_{\bar{p}+\Delta}^{\infty} G_b(\bar{p}-c)(1-G_{-b}(\bar{p}-c))dF & \text{If } b \neq h \text{ and } p = \bar{p}, \\ & \\ (1-G_{(1)}(\mu-\lambda))H(\bar{\kappa}(p-\mu))f(p-\Delta-\mu) & \\ \quad + \int_{p-\mu}^{p+\Delta} g_{(1)}(p-c-\lambda)H(\bar{\kappa}(c))dF(c) & \text{If } b = h \text{ and } p < \bar{p} \\ \quad + f(p-\mu)\left[1-H(\bar{\kappa}(p-\mu))\right] & \\ & \\ \int_{\bar{p}-\mu}^{\bar{p}+\Delta} \left[1-H(\bar{\kappa}(c))\right] dF(c) & \\ \quad + \int_{\bar{p}-\mu}^{\bar{p}+\Delta} (1-G_{(1)}(\bar{p}-c-\lambda))H(\bar{\kappa}(c))dF(c) & \text{If } b = h \text{ and } p = \bar{p}. \end{cases}$$

E Robustness

Table 11: Maximum likelihood estimation results with or without extra weight on search moments

	Specification 4		Specification 5		Specification 6		Specification 7	
	est	se	est	se	est	se	est	se
Heterogeneity								
Common shock (σ_c)	0.356	(0.003)	0.357	(0.003)	0.365	(0.004)	0.365	0.004
Idiosyncratic shock (σ_ω)	0.120	(0.003)	0.112	(0.003)	0.108	(0.002)	0.111	0.002
Avg. search cost (log)								
α_0	-1.644	(0.042)	-1.766	(0.045)	-1.711	(0.023)	-1.766	0.045
α_{inc}	0.399	(0.068)	0.325	(0.066)	-0.363	(0.024)	0.325	0.066
α_{owner}	0.004	(0.08)	0.103	(0.082)	0.319	(0.038)	0.103	0.082
Home-bank WTP								
λ_0			0.011	(0.008)			-0.051	0.008
λ_{owner}			-0.019	(0.008)			-0.032	0.008
λ_{inc}			0.023	(0.011)			0.074	0.01
Home-bank cost-adv.								
Δ_0	0.176	(0.009)	0.154	(0.01)	0.18	(0.006)	0.23	0.01
Δ_{owner}	0.078	(0.005)	0.089	(0.009)	0.066	(0.004)	0.097	0.009
Δ_{inc}	0.019	(0.007)	-0.001	(0.011)	0.008	(0.006)	-0.061	0.011
Cost function								
Intercept	5.548	(0.253)	5.532	(0.228)	5.414	(0.223)	5.455	0.243
Bond rate	0.308	(0.02)	0.308	(0.025)	0.291	(0.028)	0.29	0.027
Range posted-rate	-0.144	(0.018)	-0.145	(0.017)	-0.153	(0.018)	-0.153	0.018
Total loan	-0.200	(0.127)	-0.201	(0.073)	-0.276	(0.064)	-0.27	0.078
Income	-0.207	(0.026)	-0.218	(0.027)	-0.199	(0.027)	-0.261	0.028
Loan/Income	-0.102	(0.01)	-0.103	(0.01)	-0.114	(0.011)	-0.118	0.011
Previous owner	0.055	(0.008)	0.059	(0.009)	0.043	(0.007)	0.065	0.008
House price	0.202	(0.116)	0.202	(0.066)	0.279	(0.057)	0.281	0.07
FICO	-0.652	(0.037)	-0.656	(0.038)	-0.68	(0.039)	-0.681	0.04
LTV	1.062	(0.32)	1.063	(0.156)	1.303	(0.141)	1.314	0.172
$1(LTV = 95\%)$	0.029	(0.01)	0.029	(0.008)	0.03	(0.008)	0.029	0.008
Rel. network size	-0.046	(0.002)	-0.043	(0.002)	-0.04	(0.002)	-0.041	0.002
Range of Bank FE	[-0.103 , 0.070]		[-0.096 , 0.066]		[-0.084 , 0.070]		[-0.086 , 0.072]	
Quarter-year FE	Y		Y		Y		Y	
Region FE	Y		Y		Y		Y	
Sample size	26,218		26,218		26,218		26,218	
LLF/N	-2.046		-2.045		-1.989		-1.988	
Search moments weight	0		0		100		100	
Likelihood ratio test ($\chi^2(3)$)	43.226				55.890			

Units: \$/100 per month. Average search cost function: $\log \alpha(z_i^1) = \alpha_0 + \alpha_{inc} \log(\text{Income}_i) + \alpha_{owner}\text{EHB}_i$. Home-bank willingness-to-pay: $\lambda(z_i^2) = L_i \times (\lambda_0 + \lambda_{inc}\text{Income}_i + \lambda_{owner}\text{Previous owner}_i)$. Home-bank cost advantage: $\Delta(z_i^2) = L_i \times (\Delta_0 + \Delta_{inc}\text{Income}_i + \Delta_{owner}\text{Previous owner}_i)$. Cost function: $c_{ij} = L_i \times (c_i + \omega_{ij})$, where $c_i \sim N(x_i\beta, \sigma_c^2)$ and $\omega_{ij} \sim \text{T1EV}\left(\bar{\xi}_j + \xi_{\text{branch}}\text{Rel. network size}_{ij} - \gamma\sigma_\omega, \sigma_\omega\right)$. The likelihood-ratio test reported in the last row test Model 1 and 2 against Model 3 (alternative hypothesis). The 1% significance level critical value is 16.266. Specification 2 is our **baseline** model.