Discrimination and Worker Evaluation*

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Abstract

African-Americans face shorter employment durations than apparently similar whites. We hypothesize that employers discriminate in either acquiring or acting on ability-relevant information. We construct a model with a binary information generating process, “monitoring”, at the disposal of firms. Monitoring black but not white workers is self-sustaining: new black hires are more likely to have been screened by a previous employer than white workers and therefore firms find it optimal to discriminate in monitoring. The model’s additional predictions, lower lifetime incomes and longer unemployment durations for blacks, are both strongly empirically supported.

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1 Introduction

Many African-Americans believe black workers ‘don’t get second chances’\textsuperscript{1} or that they face additional scrutiny in the workplace. Similarly, black workers are admonished to be ‘twice as good’\textsuperscript{2} in order to succeed. If black workers are subject to higher standards or scrutinized more heavily, we expect this to be reflected in more separations.

Indeed, the data support the idea of shorter employment duration\textsuperscript{3} for black workers. Bowlus, Kiefer and Neumann (2001) detect and ponder the disparity in job destruction rates; Bowlus and Eckstein (2002) estimate\textsuperscript{4} that young black male high school graduates had roughly 2/3 the job spell duration of their white counterparts, despite more of their job spells ending in unemployment. Both papers assume an exogenously higher separation rate for black workers to fit their models to the data. Lang and Lehmann (2012) show that differences in unemployment duration alone are insufficient to account for the black/white unemployment gap and therefore that black workers’ employment stints are shorter. This aspect of labor discrimination has thus far eluded theoretical explication.

In this paper, our proposed explanation for differential employment durations is, in its broadest sense and consistent with the aforementioned observations, that firms discriminate in acquisition or use of productivity-relevant information. That is, firms either learn differently about black workers or, when information regarding ability is received, they condition how they act on it on workers’ race. Crucially, we establish that such discrimination can be self-perpetuating.

We develop a model in which differences in job duration arise naturally and their relation to skill is plausible. The essence of our model is that, because black workers are more closely scrutinized, a larger share of low-performance workers will separate into unemployment. As a result, since productivity is correlated across jobs, the black unemployment pool is ‘churned’ and therefore weaker than the white unemployment pool. Since workers can, at least to some extent, hide their employment histories, race serves as an indicator of expected worker productivity. This in turn makes monitoring newly hired black (but not white) workers optimal for firms. Figure 1 illustrates employment in the two labor markets.

There are multiple equilibria in our model, a property it shares with models of rational

\textsuperscript{1}This assertion can be found in a range of occupations including football coaching (Reid, 2015), music and films (The Guardian, 2014) as well as more generally (Spencer, 2014).

\textsuperscript{2}Coates, Ta-Nehisi (2012) and Mabry, Marcus (2012)

\textsuperscript{3}Throughout this paper we refer to employment duration by which we mean the length of an employment spell rather than job duration by which we mean the time a worker spends with a particular employer. Job duration depends on, among other factors, the arrival rate of outside offers. Our model abstracts from job-to-job transitions.

\textsuperscript{4}Using the NLSY data for 1985 and 1988.
stereotyping or self-confirming expectations (Coate and Loury, 1993). However, in our model discrimination is not simply a product of coordination failure; instead, history matters. A group that begins with a low level of skills for which only the bad (monitoring) equilibrium exists will remain in that equilibrium even if its skill level rises to a level consistent with the existence of both the good and bad equilibria. Even if blacks are, on average, more skilled than whites, whites can be in the good steady-state and blacks in the bad steady-state because of a history of lower access to schooling and other human capital investments. Equalizing the human capital that blacks and whites bring to the labor market may be insufficient to equalize labor market outcomes. In contrast, in self-confirming expectations models, if we could just convince blacks to invest in themselves and employers that blacks have invested, we would immediately jump to the good equilibrium.

There is an abundance of evidence that black workers face lower wages and longer unemployment duration than white workers. Moreover, these disparities are less prevalent and, perhaps, in some cases nonexistent for the most skilled workers as measured by education or performance on the Armed Forces Qualifying Test. While there are a plethora of models intended to explain wage or unemployment differentials, none addresses both and their relation to skill.\footnote{Many models (e.g. Aigner and Cain, 1977; Becker, 1971; Bjerk, 2008; Charles and Guryan, 2011; Coate and Loury, 1993; Fryer, 2007; Lang, 1986; Lang and Manove, 2011; Lundberg and Startz, 1983; Moro and Norman, 2004) assume market clearing and therefore cannot address unemployment patterns. Search models (e.g. Black, 1995; Bowlius and Eckstein, 2002; Lang and Manove, 2003; Lang, Manove and Dickens, 2005; Rosen, 1997) can explain unemployment differentials, but assume otherwise homogeneous workers and thus cannot address wage differentials at different skill levels. Peski and Szentes (2013) treat wages as exogenous. In general, discrimination models have not addressed employment duration. See the review in Lang and Lehmann (2012).} Since in our model newly hired black workers are on average less productive than white ones, firms that expect to hire blacks anticipate less profit from a vacancy and therefore offer fewer jobs. Consequently, blacks have longer unemployment durations. Also,
since they both spend more time searching for a job and are believed to be less productive on hiring, blacks earn less over their lifetimes. Additionally, the higher level of scrutiny increases the return to skill for blacks, consistent with evidence that blacks invest more in schooling compared with apparently equivalent whites.

We derive additional implications from informal extensions to the model. The higher level of scrutiny increases the return to skill for blacks, consistent with evidence that blacks invest more in schooling compared with apparently equivalent whites. In addition, if unemployment history is partially observable, black job seekers who have experienced enough turnover may be permanently relegated to low-skill, low-wage jobs. Although we do not wish to overstate the predictive power of the model, we note that until around 1940, blacks and whites had similar unemployment rates (Fairlie and Sundstrom, 1999), while blacks faced lower wages. This is consistent with a setting in which, due to low human capital investments, blacks were assumed to have low productivity at most jobs and therefore not monitored for quality. ‘Churning’ of the black labor market would not begin until human capital investments were sufficiently high.

We believe that the broad implications of our model can be derived through a variety of formalizations. The key elements common to these are:

i. that a worker’s productivity at different firms is correlated,

ii. that workers cannot or do not signal their ability and that they can, at least imperfectly, hide their employment histories,\(^6\)

iii. that firms must therefore, to some degree, statistically infer worker ability,

iv. that further information about match productivity is costly, imperfect, or both, and

v. that this information, if obtained, may affect hiring or retention, so that firm behavior affects the average unemployed worker’s ability.

The details of our formal model are driven by our desire for a theoretically rigorous model of wage-setting in a dynamic framework with asymmetric information. Firms and workers bargain over wages and use a costly monitoring technology to assess the quality of the match, which is correlated with the worker’s underlying type. We argue that separating worker-types is impossible without commitment to monitor, as the workers with the greatest incentive to be monitored for match quality while their type is unknown to the firm (those privately sure to be a good match) are also the ones for whom monitoring is most ex-post inefficient.

\(^6\)In particular, they must sometimes be able to omit or mischaracterize prior bad matches.
Therefore, use of the monitoring technology depends on the firm’s prior: if the belief that a worker is well-matched is sufficiently high or sufficiently low, it will not be worth investing resources to determine match quality. However, if the cost of determining the match quality is not too high, there will be an intermediate range at which this investment is worthwhile. Firm beliefs about black, but not white, workers fall in this region. Consequently, they are subject to heightened scrutiny and are more likely to be found to be a poor match and fired. The increased scrutiny ensures that the pool of unemployed black workers has a higher proportion of workers who have been found to be a poor match at one or more prior jobs. And therefore employers’ expectations that black workers are more likely to be poor matches is correct in equilibrium. This, nested in a search model, generates the empirical predictions discussed above.\textsuperscript{7}

This churning equilibrium is hard to escape. This is disheartening since policy succeeding at convergence of group characteristics may fail to equate labor market outcomes. Only if the skill level of blacks is raised sufficiently above that of whites (technically the proportion of good workers is sufficiently high), does the bad equilibrium cease to exist and white and black workers receive similar treatment.

2 A simple example of churning

To provide some intuition, we first consider a simple discrete-time market in which we abstract from wage bargaining and vacancy creation decisions. These will play a central role in the full model.

A unit mass of worker is born every period. Suppose new workers have a probability \( g = \frac{2}{3} \) of being type \( \alpha \) and producing \( q_\alpha = 1 \) unit per period and with the remaining probability are type \( \beta \) and produce \( q_\beta = 0 \). Each unemployed worker who has not been publicly revealed to be type \( \beta \) is matched to a firm at the beginning of the period. Wages are set to \( w = \frac{1}{3} \) exogenously,\textsuperscript{8} to be endogenized in the full model. Firms can either hire a worker indefinitely, or hire for a single period with monitoring costing \( b = \frac{4}{3} \), which reveals a \( \beta \) employee to the firm with a probability \( \frac{1}{2} \), then firing those revealed and keeping the rest indefinitely following that. Matches do not dissolve naturally and the discount factor is \( \delta = .95 \).

To show churning can persist in environments where the market learns about worker ability rather quickly, we assume that the second revelation of a \( \beta \) worker is public. Such

\textsuperscript{7}Note that our model abstracts from moral hazard and that performance is observed objectively. MacLeod (2003) develops an interesting model in which biased subjective assessments interact with moral hazard concerns.

\textsuperscript{8}This is half the expected surplus of a new worker.
a worker is not hired again, and thus exits this labor market for one with lower wages and production that is less type-sensitive\(^9\) - unlike one revealed only once, who is re-matched next period.

Consider first a market with only first-time job seekers. A firm that hires but does not monitor a worker earns

\[
g(q_{\alpha} - w)/r + (1 - g)(q_{\beta} - w)/r = \frac{80}{9} - \frac{20}{9} = \frac{20}{3}.
\]

One that does monitor to fire revealed \(\beta\)s (recall they stay for one period) earns

\[
g(q_{\alpha} - w)/r + (1 - g)\left[\frac{1}{2}(q_{\beta} - w)/r + \frac{1}{2}(q_{\beta} - w)\right] - b = \frac{80}{9} - \frac{21}{18} - \frac{4}{3} = \frac{115}{18} < \frac{20}{3}.
\]

So, a firm will prefer not to monitor newly hired workers. Consequently, no \(\beta\) workers are revealed or fired, and all unemployed workers are first-time job seekers as assumed.

Now consider a market that has been churned by the monitoring technology. In each period, half the \(\beta\) workers who got their first job the previous period are fired and return to the job-seeking pool where they join a new batch of \(\beta\) workers of size \((1 - g)\) and \(\alpha\) workers of size \(g\); thus the probability a newly hired worker is of type \(\alpha\) is \(g_{c} = g/(g + (1 - g) + .5(1 - g)) = 4/7\).

An employer in this market who does not monitor will get a payoff of

\[
g_{c}(q_{\alpha} - w)/r + (1 - g_{c})(q_{\beta} - w)/r = \frac{160}{21} - \frac{60}{21} = \frac{100}{21},
\]

whereas one who does monitor expects a payoff of

\[
g_{c}(q_{\alpha} - w)/r + (1 - g_{c})\left[\frac{1}{2}(q_{\beta} - w)/r + \frac{1}{2}(q_{\beta} - w)\right] - b = \frac{160}{21} - \frac{21}{14} - \frac{4}{3} = \frac{201}{42} > \frac{100}{21}.
\]

Thus workers in the second, or ‘churned’, market are monitored and can be fired.

This simplistic model demonstrates how two groups with the same underlying abilities can face very different treatment, and that this process can be self-enforcing. It captures churning-induced discrimination. Since only one group suffers separations, we interpret this as an employment duration differential. To address wage and unemployment duration differentials however, we will need the main model.

This simple example also helps demonstrate an important point: history matters. It is readily confirmed that the market switches from monitoring to not monitoring when the

\(^9\)In the context of our main model, we call this degenerating into ‘dead end jobs’ in the extension presented in Section 5.5.6.
proportion of $\alpha$s in the labor market surpasses $11/19$ and that firms will make a loss if this proportion is less than $29/101$. Consider a group for which historically the proportion $\alpha$ was less than $29/101$ and was therefore employed in some other type of job. Now let improvements in human capital lead new entrants in period $t$ to have a proportion $\alpha$ equal to .3; also, let this proportion grow to $2/3$ in period $t+1$ and remain at this level thereafter. The group never exits the churning equilibrium. Despite a legacy of only one generation in which the quality of the inflow favored churning, the group would remain stuck in the churning equilibrium until some time after the proportion $\alpha$ in the new generation exceeded $33/49$.

The example also shows that it is not essential that a worker’s employment history be entirely opaque. Even though workers can only hide a single dismissal, the churning mechanism operates and induces a worse steady state.

3 The Model

We now present our model. As in the model in Section 2, employers statistically infer past employment based on race and may therefore discriminate in monitoring and retention; this can in turn churn the labor market and thus self-perpetuate. The main model’s richer ontology will now enable us to address wage setting, unemployment duration and a host of other questions.

3.1 Setup

There are two worker groups, ‘blacks’ and ‘whites’. Race is observable by the worker and employers but does not have any direct impact on production.

At all times a steady flow of new workers is born into each population group.\textsuperscript{10} A proportion $g \in (0,1)$ of new workers are type $\alpha$, for whom every job is a good match.\textsuperscript{11} The rest, referred to as type $\beta$, have probability $\beta \in (0,1)$ of being a good match at any particular job. The probability of a worker being good at a job, conditional on her type, is independent across jobs. Worker type is private to the worker. Workers begin their lives unemployed. We define the average probability of being good at a particular job among new job seekers as

$$\theta_0 = g + (1 - g) \beta.$$  \hspace{1cm} (1)

\textsuperscript{10}We do not allow for death but could do so at the cost of a little added complexity.

\textsuperscript{11}Having type $\alpha$ workers perform well at every job does not appear to be essential to the argument but does appear to be essential to having comprehensible mathematics.
Employers cannot directly observe worker type or employment history, but can instead draw statistical inferences from race.

3.2 Match Quality

Production, the payment of wages and the use of the monitoring technology occur in continuous time using a common discount rate $r$.

Workers can be either well-suited to a task (a ‘good’ match), producing $q$ per unit time; or ill-suited (a ‘bad’ match), producing expected output $q - \lambda c$ per unit time. We can interpret the lower productivity of bad workers as errors or missed opportunities, each costing the firm $c$, that arrive at a constant rate $\lambda$. Under this interpretation, opportunities for error are also opportunities to learn the quality of the match as well-matched workers are observed to avoid errors.

For monitoring to ever be useful, matches revealed to be bad must separate. To this end, we make the sufficient and simple assumption that such a match is unproductive:

\[(C1)\quad q - \lambda c \leq 0.\]

It is much stronger than necessary. In general, if the worker and firm know that the match is bad, it will be efficient for the worker to experience some unemployment in order to try a new match; this is a consequence of productivity conditional on worker type being match-specific. Assumption (C1) ensures that such separation in search of a better match is efficient regardless of the expected duration of unemployment.

Neither the employer nor a type $\beta$ worker can know the match quality without monitoring. The parties can agree to a costly regime of monitoring that may produce a fully informative, bilaterally observable signal about match quality. In keeping with the opportunities-for-errors interpretation, we assume the signal arrives at a constant hazard rate $\lambda$. The monitoring technology costs $b$ per unit time, so that the expected cost of information is $\int_0^\infty be^{-\lambda t}dt = b/\lambda$ and its expected discounted cost is $\int_0^\infty (e^{-rt}b)e^{-\lambda t}dt = b/(\lambda + r)$. The principal benefit of a signal whose arrival is exponentially distributed, rather than one that arrives deterministically, is that it makes the employment survival function more realistic.

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12 At a more informal level, we believe that workers have some ability to hide their employment history and that they will not report information speaking to their own low ability. We show the model is robust to imperfect history revelation in Section 5.5.6.

13 Alternatively, we could assume that the flows are $q - d$ and $q$ with $d \equiv \lambda c$ and that $\lambda$ is the arrival rate of opportunities to measure the flows.

14 Nothing of interest is ruled out here; if known bad matches don’t end, then costly monitoring for separation is never worth paying for.
In addition, it allows for a certain stationarity in the model: so long as no signal has arrived, the underlying incentives do not change.

3.3 Job Search

When a worker is born or her match is terminated, she becomes unemployed. Unemployed workers are stochastically matched to firms, which occurs at a constant hazard $\mu$. For the moment, we treat this rate as exogenous; it will be endogenized in Section 5.4 to address unemployment duration. When a match dissolves, transfers cease and the worker becomes unemployed. A firm does not recoup a vacancy and therefore receives a payoff of 0 on termination.\footnote{This occurs naturally due to free entry when vacancy creation is endogenized; see Section 5.4.}

In the unemployed state, workers merely search for new jobs; we normalize the flow utility from this state to 0. The value from unemployment is thus simply the appropriately discounted expected utility from job-finding and is invariant to history. The discount on job-finding is $\int_0^\infty e^{-rt} \mu e^{-\mu t} dt = \mu / (\mu + r)$; the value of a new job will depend on the equilibrium. We denote the value of the job-finding state as $U_\alpha$ for type $\alpha$ workers and $U_\beta$ for type $\beta$ workers.

3.4 Bargaining

3.4.1 Informal Description

In the interest of modeling wage determination, this section ends up being more technical than may be of interest to readers who are primarily interested in discrimination. We therefore begin with a brief intuitive discussion which we hope will be sufficient to permit such readers to skip the technical discussion.

We cannot use Nash bargaining because there is no accepted model of Nash bargaining with asymmetric information. Instead, we use a bargaining model in which workers and firms make alternating offers. We assume that the parties may unilaterally reopen bargaining at any time but with a delay. Offers take the form of a wage and monitoring regime. If the regime involves no monitoring, no new information arises. If the regime involves monitoring, bad matches will separate, and those shown to be good will renegotiate so as to not continue monitoring.

A critical question is whether the bargaining can reveal workers’ private information about their type. Intuitively, firms might propose a monitoring offer that would attract one type and then a no monitoring offer that would attract the other. Alternatively, a
worker could try to signal her type by bargaining tactics, effectively engaging in ‘money burning’. The problem is that if separation occurs, $\alpha$ workers and firms should immediately renegotiate to a no-monitoring regime with a high wage reflecting the fact that the match is known to be good. But this is also the best possible outcome for a $\beta$ worker. So, knowing that renegotiation will occur immediately, $\beta$ workers will pretend to be $\alpha$s, so such type-separating solutions fail to exist. Thus separation in this setting would require commitment, even in the face of Pareto-improving alternatives.

Since they do not wish to reveal themselves, in the solution $\beta$ workers negotiate as if they were type $\alpha$. The firm therefore evaluates offers as though the worker is an average of the two types. As in the Rubinstein (1982) model, there is no utility flow while bargaining.

When there is no monitoring and the firm believes the match is good with probability $\theta$, as the bargaining delay disappears the worker receives $w/r$ and the firm receives $(q - (1 - \theta) \lambda c - w)/r$. This is split by an average (over proposers) wage of

$$0.5(q - (1 - \theta) \lambda c),$$

as in the Nash bargaining solution. Lemma 1 shows each party values a revealed good match at $0.5q/r$.

When monitoring takes place, bargaining splits the cost equally on average. In addition to that however, a new term appears reflecting that firms and workers evaluate the probability of the match being good differently. Again abstracting from bargaining delays, the wage is

$$\frac{1}{2} (q - b - \lambda c(1 - \theta)) - \frac{(1 - \theta)}{2} \lambda \frac{q}{2r}.$$

This would also obtain as the equal-weights Nash bargaining outcome of an $\alpha$ worker bargaining with a firm with belief $\theta$ that the match is good, with 0 outside options. Since all workers bargain as $\alpha$s who know the match is good but firms have belief $\theta$, workers are more impatient to get to revelation and therefore bargain as if delays were more costly. This means that monitored workers bear not only their share of the monitoring cost but an additional “Pooling Penalty.” As in Nash bargaining, the monitoring policy is efficient from the standpoint of firms and $\alpha$ workers.

In the next subsubsections we impose conditions to ensure that monitoring is indeed optimal in the churned (black) labor market only and that type separation is infeasible. We furthermore make bargaining stationarity assumptions that empower off-path renegotiation as a way to exclude equilibria supported by either unreasonable off-path beliefs or repeated-games-style ‘punishments’. We then derive both steady-state solutions of the full alternating-offers bargaining model when the time between offers is small. Readers who are less interested
in the technical details may wish to skim the material until Section 4.

3.4.2 The Formal Bargaining Model

Wage and monitoring contracts are determined by alternating-offers bargaining with a delay of $\Delta$.\(^{16}\) An offer is a pair $(w, m) \in \mathbb{R} \times \{0, 1\}$ comprised of a wage $w$ per unit time paid continuously and a policy $m$ of using or not the monitoring technology.\(^ {17}\)

When a match is first formed, a first proposer is chosen with equal probability on the firm and worker. Production, monitoring and wages cease during bargaining. We are interested in solutions when $\Delta$ is low.

Most importantly, either partner may unilaterally choose to re-open negotiations at any time by causing a single delay of length $\Delta$ during which production and wages are suspended. Once this delay expires, the party instigating renegotiation is placed in the role of proposer.\(^ {18}\) The choice to reopen negotiation is logically simultaneous at each time, and if both partners wish to reopen negotiations at the same instant they each assume the role of proposer with probability $1/2$.

Thus, there is no commitment to any agreement. This is important. If the wage is independent of worker type, it will generally fall between the wages that would be negotiated by a known $\beta$ and by a known $\alpha$. Therefore, starting from a common wage, if a worker is revealed to be an $\alpha$, she will renegotiate to raise her wage, while the firm will renegotiate a lower wage if the worker is revealed to be a $\beta$. This creates an environment hostile to separating equilibria.

With the impermanence of deals, however, we now open ourselves to repeated-games type equilibria where the acceptance of bad offers, and intransigence in insisting on them, is enforced by off-path punishment. To recover the uniqueness of Rubinstein bargaining from this, we make an assumption:

\(^{16}\)Although the standard Mortensen and Pissarides (1994) model uses Nash bagaining, it requires symmetric information and therefore is unusable in our setting. Evidence from Hall and Milgrom (2008) suggests that their own Rubinstein variant with added pecuniary costs of delay is able to produce far more realistic unemployment predictions than Nash baganing. Our model shares the feature that enables this prediction (workers’ outside option not dampening firm payoff fluctuations).

\(^{17}\)We assume that offers entail constant wages and monitoring, a limitation. Allowing time-varying wage profiles to be offered does not affect our findings but results in the loss of some elegance. We can show that our results hold for the average wage over a small interval that is nevertheless large relative to the bargaining delay but cannot rule out wages that, for example, alternate between a high and low wage with each wage maintained for a period equal to the bargaining delay. If we further assume that wages and monitoring can be contingent on the signal arriving, we require additional assumptions on the delay, $\Delta$, to preserve our results; at the cost of considerable complexity, the equilibrium derived here is essentially unique as $\Delta \downarrow 0$.

\(^{18}\)This delay on renegotiation ensures that disagreeable offers are rejected rather than accepted with the intent to renegotiate instantly.
(S0) Stationarity: Consider histories where firm beliefs put probability 1 on a certain worker type or match quality. There are no deviating offers at such histories that if not renegotiated (in the case of uncertain match quality, until revelation) improve\textsuperscript{19} the payoff of the proposer while giving the receiver more than the once-discounted expected value at their previous offer (or, if this is the first offer, the receiver’s once-discounted value of offering first).

Stationarity allows for precisely the kind of argument present in standard Rubinstein bargaining. A party who makes offers it values at \( x \) should be willing to accept offers it values at \( e^{-r\Delta}x \). This further allows us to dispense with repeated-games type inefficient behavior, such as strategies that waste most of the surplus under the threat of wasting even more of the surplus.

At this point we want to assume that the bargaining delay is not too large for the parties to renegotiate to shut off monitoring after match quality revelation.

\[(C2)\quad e^{-r\Delta} \cdot q > q - b.\]

In fact, we want to think of the bargaining delay as being vanishingly small and do our analysis in Sections 5 and 6 treating it as such.

Bearing this in mind, we additionally postulate that

\[(C3)\quad e^{-r\Delta} > \frac{\mu}{\mu + r}\]

to ensure that for a worker, rejecting an offer and making a counter offer is, in expectation, faster than separating in order to find a new match where the worker might be the first proposer (for simplicity, we formalize this as though he will be the first proposer). Counteroffering is quicker than finding a new employer to make an offer to. Again, this condition must always be satisfied for sufficiently small \( \Delta \).

### 3.4.3 Bargaining solution with symmetric information

First, let us find the subgame perfect Nash equilibrium (SPNE) solution where bargaining occurs under symmetric information. Using S0, the parties will make stationary offers and split the output according to the Rubinstein shares, \( 1/(1 + e^{-r\Delta}) \) for the first proposer and \( e^{-r\Delta}/(1 + e^{-r\Delta}) \) for the responder. These shares are delivered via a constant wage, avoiding renegotiation in the absence of information from monitoring.

\textsuperscript{19}A delay caused by rejecting an equilibrium offer or reopening negotiations is of course factored in to deciding whether a deviating proposal is payoff-improving to the proposer.
**Known Bad Match.** By (C1), the match would forever produce a negative average flow should it persist, so it instead separates.

**Known Good Match.** The total match surplus is the discounted value of producing \( q \) for all time, \( \int_0^\infty q e^{-rt} \, dt = q/r \). The first proposer therefore earns

\[
\frac{1}{1 + e^{-r\Delta}} \cdot \frac{q}{r}.
\]

We can now show that matches in which revelation of good quality occurred via a policy of monitoring will instantly renegotiate:

**Lemma 1** If monitoring reveals a match to be good, both parties request renegotiation; they each expect a payoff of \( e^{-r\Delta} q/(2r) \) upon such revelation.

**Proof.** See A.1 ■

As the bargaining under symmetric information will be efficient, we can decouple the monitoring decision from wage setting and proceed to examine the latter.

**Known \( \beta \), no monitoring.** Worker type is commonly known to be \( \beta \), and the match is of unknown quality. The average over match qualities cost of errors per unit time is \((1 - \beta)\lambda c\). Total match surplus is therefore

\[
S_{N\beta} = \frac{q - (1 - \beta)\lambda c}{r}. \tag{2}
\]

The first proposer therefore receives

\[
\frac{1}{1 + e^{-r\Delta}} \cdot \frac{q - (1 - \beta)\lambda c}{r}. \tag{3}
\]

**Known \( \beta \), monitoring.** If the match is revealed by the signal to be bad, separation occurs; the firm receives 0 and the worker \( U_\beta \). By Lemma 1, when the match is revealed to be good, each player expects a payoff of \( q e^{-r\Delta}/(2r) \). Expected discount on revelation is \( \int_0^\infty e^{-rt} \cdot \lambda e^{-\lambda t} \, dt = \lambda/(\lambda + r) \). For expected discounted pre-revelation total wages \( W \), the worker’s total expected payoff is

\[
W + \frac{\lambda}{\lambda + r} \left( \beta \frac{q e^{-r\Delta}}{2r} + (1 - \beta) U_\beta \right). \tag{4}
\]

The firm’s payoff, remembering a separation has a value of 0, is

\[
\frac{q - (1 - \beta)\lambda c - b}{\lambda + r} - W + \frac{\lambda}{r + \lambda} \left( \beta \frac{q e^{-r\Delta}}{2r} + (1 - \beta) \cdot 0 \right). \tag{5}
\]

The total surplus from the match is therefore
\[
S_{M\beta} = \frac{q - b}{\lambda + r} - \frac{(1 - \beta)\lambda c}{\lambda + r} + \frac{\lambda \beta}{\lambda + r} \frac{qe^{-r\Delta}}{r} + \frac{\lambda (1 - \beta) U_{\beta}}{\lambda + r}. \tag{6}
\]

We can solve for the instantaneous wage, which averages (over proposers) to

\[
w_{M\beta} = .5 (q - (1 - \beta) \lambda c - b) - .5 (1 - \beta) \lambda U_{\beta}. \tag{7}
\]

Continuation play from an off-path history in any of the above cases is for both players to request immediate renegotiation and propose the equilibrium shares, unless both players are receiving greater than the receiver’s share by the current offer (in which case the status quo offer continues until revelation).

3.5 Steady State

A steady state of a labor market is a mass of \(\alpha\) job seekers, a mass of \(\beta\) job seekers and a mass of monitored \(\beta\) workers along with equilibrium firm and worker strategies that make these populations constant over time. There are two kinds of stable steady states: those in which all employees are monitored until match quality is revealed, and those in which no monitoring occurs.\(^{21}\)

Consider the case where no employees are monitored: the white labor market. Matches never deteriorate and therefore the only source of job seekers is newly born workers. In this scenario, a firm just matched with an employee infers his probability of being of type \(\alpha\) is the population prevalence \(g\); the chance of a white job-seeker being good at a job to which he is matched is therefore

\[
\theta_W = \theta_0 = g + (1 - g)\beta.
\]

Now suppose that all newly hired black employees are monitored and all bad matches are terminated. Newly matched black workers will be worse than average.

**Lemma 2** The probability a newly hired black worker is in a good match is

\[
\theta_B = \frac{\beta}{\beta g + (1 - g)} < \theta_W. \tag{8}
\]

\(^{20}\)The solution has the somewhat disturbing property that the worker’s value following separation lowers the wage. The worker is impatient for the opportunity to ‘try again’ if she turns out to be bad at the job. Similarly to most alternating offers models, the outside option does not directly affect the outcome here. However, as \(\beta\) workers will not be strategically revealed, we do not observe wages with this property.

\(^{21}\)A steady state in which only some workers are monitored until revelation is not stable as it implies indifference and a mixed strategy for the firm. A perturbation in \(\theta\) will lead to either complete or no monitoring, causing movement away from the steady state.
Proof. See A.2

Therefore, although monitoring may be individually prudent for each matched pair, it creates a negative externality by feeding a stream of workers who are worse than the population average (i.e. containing more $\beta$ types) back into the job-seeker pool. Surprisingly, the steady state $\theta_B$ of this process does not depend on the rate of information $\lambda$, the worker matching rate $\mu$ or the rate at which new workers enter the market.\footnote{This is an artifact of the assumption that workers are infinitely lived.}

3.6 Solution Concept

We are interested in solutions that fulfill the following criteria in addition to S0:

S1 Steady State: The labor market is in steady state.

S2 PBE: Firm and worker strategies form a perfect Bayesian equilibrium.

S3 Stationarity/No Dictatorial Beliefs: At no history with firm beliefs $\theta_h \in (\beta, 1)$ on the match being good can a deviating offer be made that, should it stay in place until revelation and beliefs be fixed at $\theta_h$ until revelation:

a) strictly improves the payoff of a proposing firm or $\alpha$ worker, and either

b1) if the first offer in a match, improves on the lesser of the receiving firm or $\alpha$ worker’s once-discounted first-proposer payoff or equilibrium payoff at the current node, or

b2) if a subsequent offer, gives a greater payoff to a receiving firm or $\alpha$ worker than their once-discounted expected payoff at their previous offer.

Restriction S3 requires some explanation. Its primary purpose is to provide uniqueness. It allows $\alpha$ workers to make off-equilibrium offers that are beneficial to them without having to worry about the offers’ effect on beliefs. It furthermore allows firms to make offers that the best workers should accept without those workers worrying about a deleterious effect acceptance has on beliefs.

This restriction ensures that the bargaining protocol will produce real bargaining and Rubinstein-like solutions rather than dogmatic offers backed by the threat of belief change or punishment off-path. Lacking S3, low wages could be maintained by the firm believing any deviation is due to the worker being type $\beta$. By providing for deviations from such situations without belief ramifications, we eliminate these distasteful equilibria.\footnote{Without S3, workers could be forced to accept far below half the average surplus even if match quality was unobservable in principle - despite types having identical incentives.}
3.7 Parametric Assumptions

Now we impose certain restrictions on the joint values of parameters sufficient to ensure the existence of both solutions.

For an equilibrium with no monitoring to exist for white workers, we want to assume that monitoring costs are not too low. Initially, we want to abstract from bargaining frictions; in the limit as $\Delta$ disappears, the first parameter restriction can be stated as saying that monitoring costs exceed the sum of the benefits to the firm and $\alpha$ worker.

\[
\frac{b}{\lambda} > \frac{\lambda c}{2r} (1 - \theta_W) \quad \text{Wage Increase}\]
\[
\quad \quad + \frac{\lambda c}{2r} (1 - \theta_W) \left( \frac{q}{2r} - \frac{1 - \theta_W}{2r} \right) - \theta_W (1 - \theta_W) \frac{\lambda c}{2r} \quad \text{Bad Match End Benefit}\]
\[
\quad \quad \quad \text{Revelation Gain to Firm}\]
\[
\text{Mon. cost} \quad \text{Revelation Gain to } \alpha \quad \text{Wage Increase}\]
\[
\text{(9)}
\]

Restating this limiting condition to constrain $\beta$ and $g$ rather than the monitoring costs and recalling that $\lambda c - q/2$ is guaranteed to be positive due to (C1), we get

\[
\theta_W = g + (1 - g) \beta \geq 1 - \frac{r}{\lambda} \cdot \frac{b}{\lambda c - \frac{q}{2}} \Rightarrow \quad \frac{(1 - \beta)(1 - g)}{\lambda c - \frac{q}{2r}} < \frac{b}{\lambda c - \frac{q}{2r}}. \quad \text{(10)}
\]

However, as we wish to derive equilibria when bargaining delays are nonzero, things are a bit more complicated. Expression (9), when accounting for bargaining frictions, transforms into the rather less interpretable

\[
\frac{b}{\lambda} > (1 - \theta_W) \frac{\lambda c}{r} - \frac{q(2 - e^{-r\Delta}(1 + \theta_W))}{2r}. \quad \text{(C4)}
\]

Unsurprisingly, this condition is still of the form “monitoring costs must not be too low.” On the left hand side is the instantaneous cost of information, $b/\lambda$. On the right hand side, the losses to the employer that result from bad matches, which would end if monitoring is successful, minus a measure of production lost to both separations and delays due to renegotiation when match quality revelation occurs.\(^{24}\) Strictness of the inequality ensures persistence and stability. Condition (C4) guarantees that a type $\alpha$ worker (who is more eager for monitoring) does not want to propose monitoring if, having done so, the employer will not update beliefs; firm beliefs $\theta_W$ are good enough.

Our second condition, antisymmetrically to (C4), posits that “monitoring costs

\(^{24}\)As an $\alpha$ worker and firm hold different beliefs about the probability of a bad match, the loss production is estimated using their average beliefs.
must not be too high” and ensures the monitoring equilibrium also exists:

\[
\frac{b}{\lambda} < (1 - \theta_B) \frac{\lambda c}{r} - \frac{q(2 - e^{-r\Delta}(1 + \theta_B))}{2r}.
\]

This condition establishes that, in the black (‘churned’) labor market with a pooling monitoring equilibrium, it is not profitable for \( \alpha \) types to deviate to a no-monitoring offer if this would not change the employer’s belief. In other words \( \theta_B \), the belief about the average ability in the black unemployed pool, must be sufficiently low that \( \alpha \) workers prefer to be monitored. Strictness of the inequality ensures that switching to an unchurned market is not simply a matter of switching equilibria (as the non-monitoring one will not exist here).

Combining conditions (C4) and (C5), in the limit as \( \Delta \downarrow 0 \), we get

\[
(1 - \beta)(1 - g) < \frac{b}{\lambda} \frac{\lambda c}{r} - \frac{q(2 - e^{-r\Delta}(1 + \theta_B))}{2r} < (1 - \beta)(1 - g) \frac{\beta g + (1 - g)}{\beta g + (1 - g)}.
\]

That is, the ratio of the average costs of monitoring to a measure of its benefits lies strictly between the rate of bad matches in the two markets.

The two remaining conditions rule out the complexities associated with partial pooling. We assume that the efficient outcome for a match of unknown quality in which the worker is revealed to be type \( \beta \) is to monitor. This is the case even when it takes a single delay to renegotiate:

\[
(C6) \quad \max\{S_{N\beta}, 0\} < e^{-r\Delta} \cdot S_{M\beta}
\]

where \( S_{N\beta} \) is the surplus from a known \( \beta \) match without monitoring and \( S_{M\beta} \) is the surplus from a known \( \beta \) match with monitoring, both derived in Section 3.4.2. This further allows us to say that there is positive surplus from these matches; it is not detrimental to social welfare that \( \beta \) workers are employed at all. Assuming (C6) allows us to say that since \( \alpha \)s benefit more than \( \beta \)s from monitoring, if \( \beta \)s want to reveal themselves in order to be monitored, \( \alpha \)s will want to pretend to be \( \beta \)s. Second, in the absence of (C6), it is difficult to rule out mixed strategy equilibria where some \( \beta \) workers reveal themselves to avoid monitoring costs.

Finally, we require that the black equilibrium pooling wage is no lower than the wage \( \beta \) workers could get by revealing their type:

\[
(C7) \quad \frac{e^{-r\Delta}(q - b - \lambda(1 - \theta_B)) - (1 - e^{-r\Delta}\theta_B)q e^{-r\Delta}/(2r)}{(1 + e^{-r\Delta})} > w_{M\beta}.
\]

Condition (C7) provides that \( \beta \)s must prefer pooling with monitoring to revealing their
type and being monitored. As discussed, $\beta$s gain from pooling with $\alpha$s because of believed higher productivity but lose because it is costly for them to bargain as if they were $\alpha$s. This can be rewritten as a restriction on $\beta$. If $\beta$ is near 0, $\theta_\beta$ will also be near 0, and the benefits of pooling will be diminished.

Note that if (C6) and/or (C7) is violated, it will still not be an equilibrium for all $\beta$s to reveal themselves, as any worker not doing so would earn the revealed good payoff, $.5e^{-r\Delta q/r}$. Thus the benefit of these conditions is that they allow us to rule out equilibria in which some, but not all, $\beta$s reveal their type.

It is not self-evident that (C4)-(C7) can hold simultaneously. In Section 6 we provide an example with plausible parameters in which they do.

4 Solution

We present the main results of the paper: existence and essential uniqueness of equilibria in the two markets that perpetuate their associated steady states.

4.1 The Non-Monitored Market

**Proposition 1** Assuming (C1)-(C7), the white (non-churned) labor market has a solution where the monitoring technology is not used on-path. Employed workers, regardless of type, receive their Rubinstein share of the surplus. In the limit as $\Delta \downarrow 0$ their wage is

$$w_{N\theta_w} = .5[q - (1 - \theta_W)\lambda c].$$  \hspace{1cm} (12)

**Proposition 2** The above equilibrium is unique.

All proofs are in the appendix.

The main intuition here flows from (C4), the lack of commitment and S3. On the one hand, (C4) tells us that $\alpha$ workers would only really want to deviate to a monitoring offer if they could affect beliefs by doing so - beliefs are high enough that the wage is already good in equilibrium. On the other hand, if it were possible for $\alpha$ workers to reveal themselves by making a monitoring offer, then as soon as they made it, beliefs would change, and S3 would allow them to make a new, improved no-monitoring offer also preferable to $\beta$ workers.

Interestingly, since the firm cannot learn the worker’s type in this non-churned equilibrium, type has no effect on wages. The firm’s prior, $\theta_W$, is high enough that even good workers do not wish to pay to reveal match quality.
4.2 The Monitored Market

Whether or not the equilibrium involves monitoring, $\alpha$s are always willing to pay more than $\beta$s to be monitored since they know the match is good. In the monitored (black) labor market, average job-seeker quality $\theta_B$ is low and the firm’s expected error costs, $\lambda c(1 - \theta_B)$, are high. These expected costs are shared with the workers, so both firms and $\alpha$ workers wish to reveal match quality. But then $\beta$s must follow suit lest they be revealed. All workers must therefore bargain as if they were type $\alpha$ and make offers with monitoring.

**Proposition 3** Assuming (C1)-(C7), the black (churned) labor market has a monitoring employment equilibrium with a wage limiting to

$$w_{M\theta_B} = \frac{1}{2}[q - b - \lambda c(1 - \theta_B)] - \frac{1 - \theta_B}{2} \frac{q}{\lambda q r}. \quad (13)$$

**Proposition 4** This equilibrium is unique.

Note that since in order not to reveal his type, a $\beta$-worker has to bargain as type $\alpha$, he acts as though the probability of promotion is 1 even though the firm treats him as being of average type $\theta_B$. If the worker were truly a “type $\theta_B$,” with probability of matching well of $\theta_B$, known to be one and bargained as one, the Rubinstein bargaining solution would substitute the value of unemployment as a $\theta_B$ for $q/r$ in the final term. Instead, the firm here extracts additional surplus over the baseline of a “$\theta_B$” type; as $\Delta \downarrow 0$ this limits to $0.5(1 - \theta_B)\lambda(0.5q - U_{\theta_B})$, the ‘pooling penalty’.\(^{25}\) Type $\alpha$s are hurt by pooling with $\beta$s not only because of the pooling penalty but also because the firm underestimates their output by $\lambda c(1 - \theta_B)$.

As the equilibrium strategies induce full monitoring, employees who are revealed to be in bad matches separate from the firm. This sends only $\beta$ workers back into the job-seeking pool, churning the market quality to $\theta_B$.

5 Implications for Labor Markets

The previous sections establish conditions under which there are two distinct steady-states of the labor market. In this section, we compare labor market outcomes for workers in these steady states. Our comparative statics are performed in the limit as the bargaining delay goes to 0.

\(^{25}\)The Pooling Penalty is always positive as the unemployment value of the worker is never higher than the payoff to matching well.
5.1 Employment Duration

Absent monitoring, there is no new information to dissolve the match. Therefore, taken literally, the model implies no turnover in the white equilibrium. In contrast, with monitoring, some workers prove ill-suited for the job and return to the unemployment pool. We interpret this as predicting that black workers will have lower average employment duration. Recall that workers who return to the unemployment pool are all type $\beta$. Therefore, turnover is even higher than if only new entrants were monitored.

The model, again taken literally, implies that the separation hazard for blacks is

$$h_t = \frac{(1 - \beta)(1 - g) \lambda e^{-\lambda t}}{1 - (1 - \beta)(1 - g) e^{-\lambda t}}$$

which is decreasing in $t$. We expect the prediction that the exit hazard into unemployment should decline more rapidly for blacks than for whites is robust to consideration of important real world elements not addressed by the model. Unfortunately, all the estimates of this hazard by race that we have been able to find assume a constant hazard. The closest result we know of is Bowlus and Seitz (2000) who find that this hazard is much higher for young blacks than for young whites but that this difference disappears among older workers, a finding consistent with our model but that nevertheless does not directly measure the relations between hazards and seniority.

As our model abstracts from firm-to-firm hiring, we have no prediction with regards to it. Although it may seem that firms would be out to poach black workers with high seniority (that are likely to have passed monitoring), adverse selection effects (with the worst workers more willing to leave) could unravel such effects, depending on the ability of outside employers to commit. Still, our predictions are in terms of employment, not job, duration.

5.2 Persistence of Discrimination

A key result of the churning mechanism in this paper is that deleterious steady states are persistent. In this section we show just how hard it is to transition to a good steady state. We regard this as illustrating the difficulty of addressing labor market discrimination in the context of policy, particularly policy aimed at improving the skills of black workers. The existence of a range of $g$ values for which both equilibria exist allows us to talk about persistence of the deleterious equilibrium.

Heretofore we have assumed that average skill levels for the two population groups are identical. Suppose instead that skill levels are $g_B \neq g_W$ and the initial equilibrium has monitoring of blacks but not whites. Monitoring will persist as the equilibrium in the black
labor market until \( g_B \) rises above some critical level while the no monitoring equilibrium will persist in the white market provided that \( g_W \) remains above a lower critical level. In principle, we can have the black workers in the bad equilibrium and the white workers in the good equilibrium provided that \((C4)\) is satisfied and

\[
g_B \leq \frac{g_W}{\beta + (1 - \beta) g_W}. \tag{15}
\]

To set ideas, suppose that \( g_W \) and \( \beta \) both equal .5, then we could observe the black workers in the bad equilibrium if \( g_B \) is as large as \( 2/3 \). In short, not only may discriminatory markets persist when skill levels for whites and blacks are identical, but they may persist even when black skill levels are significantly higher. Policy aimed at accomplishing convergence of labor market outcomes via changes in population skill may fail to clear the hurdle of inertia.

### 5.3 Wages

Wages are lower for black workers at the point of hiring. Not only do they pay a share of the monitoring cost, they also pay what we dubbed the Pooling Penalty. In addition, each type expects lower lifetime earnings than its white counterpart. To see this, consider the following:

(i) Rearranging \((C4)\), we have that the payoff to \( \alpha s \) is higher in the unchurned market for a no-monitoring strategy:

\[
\frac{q - \lambda c(1 - \theta_W)}{2r} > \frac{q - \lambda c(1 - \theta_W) - b + \lambda(1 + \theta_W) \frac{q}{2r}}{2(r + \lambda)}. \tag{16}
\]

But as the right-hand side of that inequality is increasing in \( \theta \), we further have

\[
\frac{q - \lambda c(1 - \theta_W) - b + \lambda(1 + \theta_W) \frac{q}{2r}}{2(r + \lambda)} > \frac{q - \lambda c(1 - \theta_B) - b + \lambda(1 + \theta_B) \frac{q}{2r}}{2(r + \lambda)} \tag{17}
\]

and therefore

\[
\frac{q - \lambda c(1 - \theta_W)}{2r} > \frac{q - \lambda c(1 - \theta_B) - b + \lambda(1 + \theta_B) \frac{q}{2r}}{2(r + \lambda)}, \tag{18}
\]

which implies that white \( \alpha s \) have a higher ex-ante payoff compared to their black counterparts. As all worker payoff derives from wages, this means that lifetime wages are lower for black \( \alpha \) workers.

(ii) On the other hand, white \( \alpha \) and \( \beta \) workers expect the same lifetime wages. Since \( \beta \)
workers value monitoring strictly less than \( \alpha \) workers and black \( \alpha \) workers are worse off than white ones, black \( \beta \) workers must expect lower lifetime wages than their white counterparts.

Significantly, the model predicts that the realized strategies produce payoffs that maximize the joint surplus of \( \alpha \)s and \( \theta \)-belief firms. This implies that as a function of market \( \theta \), payoff to newly matched firms and type \( \alpha \) workers is continuous, being an upper envelope of linear functions. However, the strategy shift produces a jump discontinuity in the payoff to \( \beta \) types forced to follow suit. Figure 2 illustrates this jump.\(^{26}\)

![Figure 2: Equilibrium \( \alpha \) and \( \beta \) payoffs as a function of average hire quality \( \theta \).](<image_url>)

In a sense, because of the sharp discontinuity in the earnings of \( \beta \)s, the model predicts that the return to skill is higher for blacks than for whites, consistent with the empirical findings in Neal and Johnson (1996) and Lang and Manove (2011). We are reluctant to push this point strongly because the evidence concerns either observable ability in the form of education or potentially observable ability in the form of performance on the Armed Forces Qualifying Test. In section 5.5 we consider the case of observable investments.

The model has no role for human capital acquisition although below we briefly discuss the possibility that workers invest in human capital before they enter the labor market. Since

\(^{26}\)Figure 2, and this discussion, only concern expected lifetime payoff starting at a new job. As time goes to infinity, any single black worker will eventually be revealed good at a match and will therefore receive a better wage than white workers. We don’t dwell on this issue as it is an artefact of the irrelevance of the outside option (much lower for black workers) in our particular bargaining model and the perfectly revealing nature of the monitoring technology. An alternate ultimate fate for workers is discussed in the model variant of section 5.5.6, where unlucky black workers can get stuck in low-wage jobs.
there is no post-employment investment in the model, conditional on seniority, there is no
return to experience for either blacks or whites. Since blacks spend more time unemployed,
we might expect that, once we allow for such investments, the return to potential experience
would be higher for whites, at least conditional on seniority. On the other hand, since the
probability that blacks, but not whites, are well-matched to their jobs increases with job
duration (through the selection effect), if we do not condition on seniority, this will tend to
give blacks a higher return to potential experience.

But even this ignores the potential complementarity between match quality and human
capital investment. On the one hand, firms are less likely to invest in workers whom they
believe may be badly matched. On the other hand, they may be more (or less) likely to
invest in black workers who have been revealed to be a good match than in white workers
whose match quality is unknown and will not be revealed.

As a consequence of these considerations, we do not view the predictions regarding the
differing effects of seniority and experience on the wages of blacks and whites to be robust.
This is less disturbing than it could have been since we interpret the empirical literature on
this issue as fairly mixed.\textsuperscript{27}

5.4 Unemployment Duration

We have so far treated the workers’ matching rate, $\mu$, as exogenous. Making the standard
assumption of free entry, we now allow firms to post and maintain vacancies at a cost $k$
per unit time. When a firm creates a vacancy, it can direct its search. This can take sev-
eral forms, most notably locating production operations in an area with specific population
characteristics or advertising the vacancy in different areas and through different media. In
general, a firm can target markets indexed by $i$ where a proportion $\rho_i$ of unemployed workers
are white. The open vacancy cost $k$ is invariant to this target choice. We assume that in each
market $i$ the bargaining equilibria and population group steady states break down along the
discriminatory lines described so far.

Define $\phi$ as market tightness and let the worker job-finding rate function follow the
commonly assumed form

$$\mu(\phi) = m \phi^\gamma$$

for constants $m > 0$ and $0 < \gamma < 1$. Note that if firms expect a match to be worth $V$, the

\textsuperscript{27}Monk (2000) finds that the experience effect on wages for blacks exceeds that for whites until roughly
fifteen years of experience while the seniority effect is larger for whites through thirteen years of seniority.
Bronars and Famulari (1997) also find that the black-white wage differential tends to fall with experience.
On the other hand, D’Amico and Maxwell (1994) find that the gap between blacks and whites widens with
experience, a result that Altonji and Blank (1999) view as confirming earlier work.
free-entry level of $\phi$ in such a market sets

$$\frac{\mu(\phi)}{\phi} V - k = 0. \quad (20)$$

So

$$\phi = \left(\frac{V m}{k}\right)^{\frac{1}{1-\gamma}}. \quad (21)$$

Therefore, $\phi$ is an increasing function of $V$.

Assuming that (C6) and (C7) hold for the entire breadth of derived matching rates, we can now derive the free-entry equilibrium level of $\mu_{\rho_i}$ for each market $i$. The payoff to a firm for matching is the same as for an $\alpha$ worker, that is, when hiring from pool $i$, the firm expects a successful match to pay

$$V_i = \rho_i \frac{q - \lambda c(1 - \theta_W)}{2r} + (1 - \rho_i) \frac{q - \lambda c(1 - \theta_B) - b + \lambda (1 + \theta_B) \frac{q}{2r}}{2(r + \lambda)}. \quad (22)$$

Since the payoff to white $\alpha$ workers is higher than for blacks, the expression above is increasing in $\rho_i$. Therefore, markets with more black workers will have a lower expected payoff for a filled vacancy. Therefore, the free-entry $\phi(\rho_i)$ and $\mu(\phi(\rho_i))$ are increasing in $\rho_i$. As average unemployment duration is $\frac{1}{\mu}$, this implies that markets with higher black concentration will experience higher average unemployment duration. In the extreme case where markets are fully segregated, that is $\rho_i \in \{0, 1\}$, we can derive the ratio of the matching rates in the two markets:

$$\frac{\mu(\phi(0))}{\mu(\phi(1))} = \left(\frac{q - \lambda c(1 - \theta_B) - b + \lambda (1 + \theta_B) \frac{q}{2r}}{q - \lambda c(1 - \theta_W)} \frac{r}{r + \lambda}\right)^{\frac{1}{1-\gamma}} < 1. \quad (23)$$

5.5 Extensions

5.5.1 Eventual revelation in all matches

We have assumed unrealistically that the match quality of workers who are not monitored is never revealed. More plausibly, heightened scrutiny speeds the rate at which match quality is revealed. In a model in which workers live forever, this change considered in isolation would eliminate our result because the composition of the jobless pool is independent of the rate at which bad matches are revealed. However, if workers do not live forever, then reducing the rate at which match quality is revealed does affect the quality of the unemployment pool, and our basic results go through.
5.5.2 Skill level and discrimination

Further, we can allow for observable heterogeneity among workers. If there are groups of workers for whom \( g \) is high, only the no-monitoring equilibrium will exist for these groups, regardless of race. This is also true at very low \( g \) and very low \( \beta \) (although we have assumed away this case to simplify the proofs). The first result is consistent with similar outcomes for blacks and whites with high levels of skill as measured by education or the Armed Forces Qualifying Test (Neal and Johnson, 1996; Lang and Manove, 2011). The latter is consistent with some evidence that the bottom of the labor market is similarly bad for blacks and whites. On the other hand, Lang and Manove find that the market learns the productivity of white but not black high school dropouts. This is consistent with an equilibrium in which white dropouts are, on average, more skilled than black dropouts and therefore in which white but not black dropouts are monitored. Nevertheless, without additional, largely \textit{ad hoc} assumptions, this story cannot account for the very high unemployment rate among black dropouts.

5.5.3 Investment in unobservable skills

We have heretofore postulated that the proportion of \( \alpha \) types is exogenous. Assume instead that some fraction of workers are innately of type \( \alpha \). Others can transform themselves from \( \beta \)s into \( \alpha \)s at some cost \( \omega \) with cdf \( F(\omega) \). Provided that the fraction of natural \( \alpha \)s satisfies (C4) and (C5), both equilibria will continue to exist. However, since in the no-monitoring equilibrium \( \alpha \)s and \( \beta \)s receive the same wage, there is no incentive to invest in becoming an \( \alpha \). In contrast, in the monitoring equilibrium, lifetime earnings are strictly higher for \( \alpha \)s than for \( \beta \)s. Thus, some individuals will have an incentive to make the investment.\(^{28}\) This prediction contrasts with Coate and Loury (1993), where black workers are less willing to invest in skills.

5.5.4 Education

Suppose now that there exists a signal,\(^{29}\) which we identify with education, that \( \alpha \) workers can purchase at some personal cost \( \kappa \sim F(\kappa) \). Assume doing so assures that any employer

\(^{28}\)It might appear that the incentive to undertake such investments would unravel the monitored equilibrium. However, if this were the case, no worker would have an incentive to invest. This raises messy dynamic issues which we sidestep by assuming that the fraction of additional workers who would choose to invest is insufficient to overturn (C5).

\(^{29}\)We analyze the case of a pure signal. If education can also turn a \( \beta \) into an \( \alpha \), the analysis is a combination of the analysis in this and the prior subsection since productive investment increases the fraction of workers who are \( \alpha \) but investment that reveals workers to be \( \alpha \) reduces the fraction of unrevealed workers who are \( \alpha \).
will be immediately aware that the worker is indeed type $\alpha$. A worker of either population will then anticipate a lifetime utility of $V_{\text{Educ}}(\kappa) = \frac{\mu q}{(2r(\mu + r))} - \kappa$. In Section 5.3 we showed that unrevealed white $\alpha$ workers receive a higher lifetime payoff than their black counterparts; therefore, the incentive for the latter to invest in education is greater. As this implies that $\kappa_W \equiv \max\{\kappa : V_{\text{Educ}}(\kappa) \geq V^\alpha_W\} < \max\{\kappa : V_{\text{Educ}}(\kappa) \geq V^\alpha_B\} \equiv \kappa_B$, we must have that $F(\kappa_W) < F(\kappa_B)$ and therefore more black workers will purchase education. In particular, there exists some range of idiosyncratic costs for which black workers will purchase education but white workers will not. This is consistent with the finding in Lang and Manove (2011) that, conditional on past test scores, blacks get more education than whites do. The intuition here is simple; if a worker of high skill is treated as if she has the average hire’s skill for her group, she has a greater incentive to reveal her high skill if that average is lower.$^{30}$

Perhaps equally importantly, this extension suggests that blacks and whites with high observable skills will have similar outcomes as discussed in the previous subsubsection.

5.5.5 Imperfect monitoring

Reader may have noted that the intuitive example presented first is distinct from the main model where monitoring resolves all uncertainty about worker type. As the example demonstrates, one can write a very similar model in which $\beta$ workers always match badly but monitoring can result in false positive good matches. Given a wage-determination mechanism with outcomes similar to our bargaining protocol, much of the analysis would remain unchanged.$^{31}$ Parameters would exist that would force monitoring on blacks but not whites, the black labor market would churn, and it would produce higher unemployment duration and lower lifetime wages for blacks. In this formulation, black workers succeeding at monitoring would only be as good as whites who had never been monitored; therefore a churned market does not necessarily produce better long-run matches or higher wages for the successfully monitored.

However, this alternate model would imply that some workers are purely parasitic and cannot be matched well, but rather aspire simply to find a job where their lack of productivity is undiscovered. An equivalent of (C6) cannot hold here and as a result we cannot rule out equilibria where negotiations sometimes break down and separation occurs without

$^{30}$Strictly speaking, this creates a feedback loop from lower wages for the uneducated to a greater measure of education. The right assumptions on $F$ rule out associated complexities.

$^{31}$Unfortunately, this alternate model would add a lot of complexity and require additional assumptions for uniqueness, due to the lack of a single posterior following successful monitoring. Bargaining strategies would have a much more tangled relation to beliefs and wages, and S3 would not be an apt tool to facilitate the task.
monitoring producing information.

5.5.6 Stigma and degeneration into lower-skilled jobs

Our model unrealistically assumes that employers have no information regarding the time that workers have been in the labor market or the number of jobs they have held. If the other aspects of our model were a rough representation of reality, it is implausible that firms would not recognize that some workers were unlikely to be new entrants and therefore very likely to $\beta$ types. Suppose also that if a worker is sufficiently likely to be a $\beta$, it is not efficient to employ or monitor him. Then workers who do not find a good match sufficiently quickly will be permanently barred from the monitoring sector.

Somewhat more formally, as an extension to the model, we can relax the assumption that past history is entirely unobservable. Assume instead that each separation has a probability $\zeta$ of becoming public common knowledge. Any worker who has a revealed separation is known to be of type $\beta$ in any new match. Thus, a newly hired worker who does not have such a stigma will be of average quality $\theta'_B = \frac{[\beta + g\zeta(1 - \beta)]}{[g\beta + (1 - g) + g\zeta(1 - \beta)]}$. If we assume $\theta'_B$ satisfies (C5), churning can persist but will be primarily a phenomenon for relatively young workers.

But what will happen to workers revealed to be $\beta$s? It is straightforward to extend the model to allow for a second occupation type ($q', c'$) lacking monitoring technology$^{32}$ that is less skill intensive than the task described so far, i.e. $q > q'$ and $q - \lambda c < q' - \lambda c'$. As unrevealed $\beta$ types are strictly better off than revealed ones in a new match of the first task, there must be $q', c'$ such that the revealed $\beta$ types prefer to enter the job market for the second occupation but the unrevealed ones do not.

In this scenario, a fraction of black workers are relegated to low-wage jobs while white workers with similar skills can always get better jobs. Furthermore, since the low-wage jobs are not monitored, they are a terminal state, with no possibility of promotion or escape.

5.5.7 Changing screening and monitoring technology

Autor and Scarborough (2008) examine the effect of bringing in a new screening process. They find that the screening process raised the employment duration of both black and white workers with no noticeable effect on minority hiring. In our model, we can think of this technology as allowing the firm to screen for job match quality prior to employment. This increases the proportion of hired blacks who become permanent workers since some bad

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$^{32}$Or, more palatably, the same technology but without the incentives to use it, as in the case of a small enough $c'$.  

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matches are not hired. If the screening mechanism is good enough, the firm will choose not to monitor the black workers it hires, and all black workers will be permanent. Formally, since all white workers are permanent in the absence of the screen, the screen does not affect this proportion. Informally, if poor matches are more likely to depart even without monitoring, then there will also be positive effects on white employment duration.\textsuperscript{33} Similarly, Wozniak (2015) shows that drug testing increases black employment and reduces the wage gap; we interpret this as confirming evidence for the notion that employers are more uncertain about the quality of black workers, and therefore that black workers benefit more from early resolution of such uncertainty.\textsuperscript{34}

We note that improved technology appears to have reduced monitoring costs. This is unambiguously good for blacks who share the cost of being monitored. Unless the reduction shifts whites into the monitoring equilibrium, they are unaffected by the cost reduction. However, if firms begin monitoring, $\alpha$ workers and the firm will initially be better off. On the other hand, $\beta$ workers will generally be worse off as they will not be able to oppose the use of monitoring without revealing themselves. In a collective bargaining setting, the union might resist monitoring. The more interesting point is that since monitoring creates an externality, it is easy to develop an example in which monitoring makes both types of workers and capital worse off in the long run.\textsuperscript{35}

\subsection*{5.6 Additional Empirical Content}

This paper’s aim is to explain the employment duration differential; its chief extra predictions are longer unemployment duration and lower lifetime income for black workers. In the interest of falsifiability, we posit here additional empirical implications that we view as following from our explanatory hypothesis and are relatively model-free.

We expect that when jobs vary with respect to the cost or accuracy of monitoring technology, black workers would skew more heavily towards those that favor monitoring. If the firm’s cost of monitoring is lower, the initial match surplus with black workers is greater with benefits split between higher initial wages and lower labor costs. If the monitoring is

\textsuperscript{33}Formally, the model would have to be modified to ensure that some $\beta$ workers are never perfectly matched and/or that some $\beta$ workers are still in bad matches when they exit the labor force.

\textsuperscript{34}Wozniak (2015) is not to be interpreted as evidence that monitoring is good for black workers on the aggregate. As in the present paper, it can beneficial on an individual level; our model, however, shows it can also create a worse externality.

\textsuperscript{35}Suppose that $g_0$ is just sufficient to sustain a no-monitoring equilibrium. A small reduction in $b$ puts the labor market into a monitoring equilibrium. If there were no subsequent churning, $\alpha$ workers and firms would experience a slight gain, but the churning will wipe this out and more. Firms always make zero profit on vacancies, but if we allow for a distribution of vacancy costs, then the rents earned by firms with low costs of creating vacancies will also fall.
faster at a firm,\textsuperscript{36} $\alpha$ workers can reveal their ability and reap better wages sooner, while the firm will keep bad matches for less time. Either case produces a comparative advantage for this firm in hiring black workers. Jobs with high monitoring potential could for example be recording employee-customer interactions, as incoming call centers do.

Our explanation for lower black employment duration involves learning about match quality. In our model, the separation hazard into unemployment at time $t$ is $h_t = (1 - \theta_B)\lambda e^{-\lambda t}/(\theta_B + (1 - \theta_B)e^{-\lambda t})$ for black workers and, rather starkly, 0 for white workers. More realistically, we expect the gap in the hazard rate to be declining in seniority.

6 Example

Here we provide a simple numerical example satisfying our conditions.

Take $r = .05$, suggesting a unit of time of about a year. $\lambda = 2$ so that the average unsuccessful job lasts six months. $\beta = .2$ and $g = .95$, implying that most workers are good matches. $c = .5$ and $q = 1$ so that bad matches produce expected flow output of 0, making separation efficient regardless of unemployment duration. Finally, $b = 1$ so that $b/\lambda = 1/2$, making the expected cost of monitoring roughly equal to the value of six months of output from a well-matched worker.

It is readily verified numerically that the numerical conditions hold.

These parameters result in a $\theta_W$ of .96 and churning produces a $\theta_B$ of .833. The value of a filled vacancy in the white market is 9.6 and in the black labor market 8.9. Postulating a Cobb-Douglas matching function with elasticity $\eta = .75$, the model predicts a black-white unemployment duration ratio of 1.25. We can now compute the ratio of white to black income PDV at birth to be 1.11.

This example illustrates that a churning equilibrium is possible even if the proportion of type $\beta$ workers is quite low in the population, and can generate reasonable income and unemployment disparities while doing so; one would do well to bear in mind, however, that our model applies conditional on observables and therefore cannot be calibrated to make economy-wide predictions.

7 Conclusions

We have developed a model that explains the black-white employment duration differential, and in the process have uncovered a mechanism that both reproduces standard empirical findings and makes novel predictions.

\textsuperscript{36}while keeping type productivity constant; that is, the firm has $\lambda' > \lambda$ and $\lambda'c' = \lambda c$.
Our model in some ways resembles models of adverse selection in the labor market. Displaced workers are worse, on average, than a randomly selected worker. However, in contrast with standard adverse selection models, firms cannot distinguish between displaced workers and other unemployed workers. Therefore displaced workers depress the wage of all unemployed workers. At the same time, our approach does not generate the asymmetry of information among firms that drives adverse selection models. If the worker is known to be good at a particular job, he will not leave for another job even if he knows that, on average, he will be good at other jobs. If the worker is bad at this particular job, he separates immediately.

To keep the analysis simple, our model assumes that workers who turn out to be well-matched remain employed forever. At first blush, this suggests that it applies only to new entrant unemployment because the market will surely recognize that a fifty-year old worker is not a new entrant. We believe it is more realistic to assume that the market cannot tell whether a fifty-year old worker who was laid-off six months ago has just been unlucky and not had any matches or has had a match that turned out to be bad. The market often cannot tell how long the worker has been unemployed. Thus we think the model is more general. In addition, it provides some insight into the scarring effect of unemployment.

Unlike most, perhaps all, existing models, ours can explain a number of empirical regularities regarding discrimination simultaneously:

1. Black workers have shorter employment durations.
2. Black workers have longer unemployment durations.
3. Black workers have lower lifetime earnings.

As written, the model has infinitely lived matches and agents so there is no unemployment rate. Allowing for deaths, we would have well-defined unemployment rates and would predict the rate for black workers is higher.

More generally, we view the main message of this paper as robust to many of the modeling decisions. The key element is that blacks are subject to more scrutiny or to a higher standard than white workers. This leads to more blacks being returned to the unemployment pool, lowering the quality of that pool and, completing the equilibrium, making tougher scrutiny optimal.

Our model also strongly suggests that history matters and that equality of opportunity is not enough to eliminate racial disparities in the labor market even if this concept is used very expansively. The fact that blacks historically had low skills leads to an equilibrium in which the pool of black job seekers has lower skills than the pool of white job seekers.
even when the distribution of skills among all workers is identical for blacks and whites. While, over time, a human capital-based policy could mitigate labor market discrimination, achieving equality in human capital may be insufficient to eliminate racial disparities in the labor market.
References


A Appendix

A.1 Lemma 1: Payoff after revelation of a good match

Consider an equilibrium match that has just been revealed to be good at \( t \). For revelation to have just occurred, the currently active offer involves monitoring.

If renegotiation occurs as per case 2 in Section 3.4.2 the proposer will receive \( q/\left(r \left(1 + e^{-r \Delta}\right)\right) \). The payoff to triggering renegotiations is obtained by discounting this by \( e^{-r \Delta} \).

Assume that renegotiation never occurs in equilibrium; then the current monitoring offer persists forever, yielding a total surplus of \( (q - b) \). Assuming that neither player wants to reopen negotiations, if the current wage in place is \( w \), we must have that

\[
\min\{\frac{w}{r}, \frac{q - b - w}{r}\} \geq \frac{1}{1 + e^{-r \Delta}} \frac{q}{r}
\]

For any current wage \( w \), the greatest \( \min\{w/r, (q - b - w)/r\} \) can be is \( (q - b)/(2r) \); thus for renegotiation to never occur we require that \( (q - b)/(2r) \geq q/\left(r \left(1 + e^{-r \Delta}\right)\right) \Leftrightarrow (1 + e^{-r \Delta})b < (1 - e^{-r \Delta})q \), which is ruled out by (C2).

But as one’s opponent reopening negotiations gives the receiver’s share of the new bargain, \( e^{-r \Delta}qe^{-r \Delta}/\left(1 + e^{-r \Delta}\right) \), it becomes even harder to satisfy the requirement to not renegotiate instantly with probability 1 if one’s opponent may trigger renegotiation; therefore both instantly triggering renegotiation is the only equilibrium. As this means that each player has a probability \( 1/2 \) of being first proposer following revelation, each player at the instant of revelation has an expected payoff of

\[
\frac{1}{2} \cdot \frac{qe^{-r \Delta}}{1 + e^{-r \Delta}} + \frac{1}{2} \cdot e^{-r \Delta} \frac{qe^{-r \Delta}}{1 + e^{-r \Delta}} = \frac{qe^{-r \Delta}}{2} \cdot \square
\]

A.2 Lemma 2: Makeup of the monitored market’s job-seeking pool

Define the quantities

- \( \xi \) Flow mass of workers born per unit time
- \( A \) Mass of unemployed black type \( \alpha \) workers
- \( B \) Mass of unemployed black type \( \beta \) workers
- \( A \) Mass of currently monitored black type \( \beta \) workers
As $g$ is the fraction of new workers that is type $\alpha$ and unemployed $\alpha$ workers are becoming employed each at a Poisson rate $\mu$ and never separate, $A$ obeys

$$\frac{dA}{dt} = \xi g - \mu A$$

Similarly, a proportion $(1 - g)$ of new workers is type $\beta$ and such unemployed workers are also being hired at a Poisson rate $\mu$ each. However, as $\Lambda$ workers who are of type $\beta$ are being monitored, a flow mass $\Lambda \lambda (1 - \beta)$ of black $\beta$ workers are separating after monitoring reveals a bad match are also coming in to the black unemployed pool. Hence, $B$ obeys

$$\frac{dB}{dt} = \xi (1 - g) - \mu B + \Lambda \lambda (1 - \beta)$$

Finally, unemployed $\beta$ workers are becoming employed with monitoring at a Poisson rate $\mu$ and once they are employed they cease being monitored when match quality is revealed, which occurs at a rate $\lambda$. Thus the mass of monitored black $\beta$ workers $\Lambda$ must satisfy

$$\frac{d\Lambda}{dt} = \mu B - \Lambda \lambda$$

Steady state implies that

$$\frac{dA}{dt} = \frac{dB}{dt} = \frac{d\Lambda}{dt} = 0$$

Solving, we obtain

$$A = \frac{\xi g}{\mu}$$

$$B = \frac{\xi (1 - g)}{\mu \beta}$$

and therefore the proportion of $\alpha$ workers in the unemployed pool is

$$\frac{A}{A + B} = \frac{\frac{\xi g}{\mu} + \frac{\xi (1 - g)}{\mu \beta}}{\frac{\xi g}{\mu} + \frac{\xi (1 - g)}{\mu \beta}} = \frac{g}{g + \frac{1}{\beta} (1 - g)}.$$

Thus, a new match from the black job-seeker pool is of average quality

$$\frac{g}{g + \frac{1}{\beta} (1 - g)} \cdot 1 + \left(1 - \frac{g}{g + \frac{1}{\beta} (1 - g)}\right) \cdot \beta = \frac{\beta}{\beta g + (1 - g)} \equiv \theta_B$$

As $\beta < 1$ this is less than $\theta_W$. □
A.3 Proof of Proposition 1

The equilibrium wage proposed is

\[ w_{work}^{N\theta_W} = \frac{1}{1+e^{-r\Delta}}(q - \lambda(1 - \theta_W)c) \]

if the worker proposes first and

\[ w_{firm}^{N\theta_W} = \frac{e^{-r\Delta}}{1+e^{-r\Delta}}(q - \lambda(1 - \theta_W)c) \]

if the firm proposes first. As there will be no revelation, these shares split the expected output (using firm beliefs) equally. This equilibrium is supported by firm beliefs that are invariant to all contingencies before revelation.

At a pre-revelation history where an off-path offer is on the table, or where one is already in place, it is accepted/not renegotiated if it does not involve monitoring and the wage \( w \) satisfies

\[ w_{work}^{N\theta_W} \geq w \geq w_{firm}^{N\theta_W} \]

If this condition does not hold at the off-path history in question, then, if in negotiations, the current proposer plays the equilibrium offer; otherwise, both players’ strategy is to instantly reopen negotiations; and when they do, the equilibrium offer will be proposed.

At off-path histories where the match is revealed to be good, if the wage \( w \) satisfies

\[ \frac{1}{1+e^{-r\Delta}}q \geq w \geq \frac{e^{-r\Delta}}{1+e^{-r\Delta}}q \]

it stays in place; otherwise, play proceeds as in Lemma 1, granting an expected \( q e^{-r\Delta}/(2r) \) to each party.

As discussed in Section 3.4.2, off-path histories that led to the revelation of a bad match lead to termination of the match.

A party who deviates before revelation can at most, therefore, transition from the receiver’s share to the proposer’s share of the match surplus, as one’s opponent’s strategy will not accept worse offers. Doing so, however, occasions a single delay, which discounts the payoff from such a deviation to exactly the receiver’s payoff, which is the least the deviator could have started with. Therefore, there is no deviation that will strictly increase the agents’ payoff and the strategies described are mutual best responses.

To show that there is no S3-type deviation that proposes monitoring, it remains to show that an \( \alpha \) worker or a firm cannot propose a mutually beneficial monitoring regime.

Lemma 1 pins down continuation payoffs from being in a match revealed to be good. Thus, an \( \alpha \) worker making an offer of \( w_u \) with monitoring in place yields to this worker, in
the absence of renegotiation until match quality revelation,

\[ V_{M\alpha}(w_u) = \frac{w_u + \lambda \frac{qe-r\Delta}{2r}}{\lambda + r}. \]  

(24)

The requirement for S3 is stated in terms of beliefs remaining constant, in this case at \( \theta_W \).

Thus, an employer accepting this offer expects a payoff of

\[ F_{M\theta_W}(w_u) = \frac{q - b - (1 - \theta_W)\lambda c - w_u + \lambda \theta_W \frac{qe-r\Delta}{2r}}{\lambda + r}. \]  

(25)

Summing (24) and (25), we get

\[ \frac{q - b - (1 - \theta_W)\lambda c + \lambda(1 + \theta_W) \frac{qe-r\Delta}{2r}}{\lambda + r} \]  

(26)

For such a deviation to violate S3 necessarily (26) has to be greater than equilibrium payoffs; this can only be the case if

\[ \frac{b}{\lambda} < \frac{(1 - \theta_W)\lambda c}{r} - \frac{q(2 - e^{-r\Delta}(1 + \theta_W))}{2r} \]  

(27)

which is precluded by assumption (C4).

In the limit as \( \Delta \downarrow 0 \), the equilibrium shares of the first proposer and receiver equalize; the limiting wage is \( w_{N\theta_W} = .5q - .5(1 - \theta_W)\lambda c \).

### A.4 Proof of Proposition 2

Consider strategies for the firm and each type of worker that may in principle involve renegotiation and different monitoring for each type of worker. Call \( \tilde{m}_\alpha \) and \( \tilde{m}_\beta \) the expected discount at the point of match quality revelation for \( \alpha \) and \( \beta \) workers given these strategies and let \( \tilde{W}_\alpha \) and \( \tilde{W}_\beta \) be the total expected discounted wages before such revelation occurs.

Equilibrium requires incentive compatibility: each worker weakly prefers the strategy they actually adopt to the other type’s; hence, \( \alpha \)’s IC requires

\[ \tilde{V}_\alpha = \tilde{W}_\alpha + \tilde{m}_\alpha \left( \frac{qe-r\Delta}{2r} \right) \geq \tilde{W}_\beta + \tilde{m}_\beta \left( \frac{qe-r\Delta}{2r} \right) \]  

(28)

and the \( \beta \)’s IC imposes

\[ \tilde{V}_\beta = \tilde{W}_\beta + \tilde{m}_\beta \left( \frac{qe-r\Delta}{2r} + (1 - \beta)U_\beta \right) \geq \tilde{W}_\alpha + \tilde{m}_\alpha \left( \frac{qe-r\Delta}{2r} + (1 - \beta)U_\beta \right) \]  

(29)
Combining the two and rearranging terms

\[
(\bar{m}_\beta - \bar{m}_\alpha)(1 - \beta) \left( U_\beta - \frac{qe^{-r\Delta}}{2r} \right) \geq \left( \bar{W}_\alpha + \bar{m}_\alpha \frac{qe^{-r\Delta}}{2r} \right) - \left( \bar{W}_\beta + \bar{m}_\beta \frac{qe^{-r\Delta}}{2r} \right) \geq 0
\] (30)

Since from (C3) \( U_\beta < \frac{qe^{-r\Delta}}{2r} \), we have that \( \bar{m}_\alpha \geq \bar{m}_\beta \).

The firm’s payoff in any such equilibrium given \((\bar{W}_\alpha, \bar{m}_\alpha, \bar{W}_\beta, \bar{m}_\beta)\) is 37

\[
\tilde{F} = g \left( \frac{q}{r}(1 - \bar{m}_\alpha) - \bar{W}_\alpha + \bar{m}_\alpha \frac{qe^{-r\Delta}}{2r} - \bar{m}_\alpha \frac{b}{\lambda} \right) + (1 - g) \left( \frac{q - (1 - \beta)\lambda c}{r} \left( 1 - \bar{m}_\beta \right) - \bar{W}_\beta + \bar{m}_\beta \beta \frac{qe^{-r\Delta}}{2r} - \bar{m}_\beta \frac{b}{\lambda} \right)
\] (31)

S3 requires that deviating first offers cannot be made that improve the payoff of an \( \alpha \) worker and the firm. As the wage can transfer surplus freely, this implies that the sum of candidate equilibrium payoffs are weakly greater than the sum of those feasible by a no-monitoring offer38:

\[
\tilde{V}_\alpha + \tilde{F} \geq \frac{q - (1 - \theta_W)\lambda c}{r}
\] (32)

Expanding,

\[
\tilde{V}_\alpha + \tilde{F} = (1 - g)(\bar{W}_\alpha - \bar{W}_\beta) + \bar{m}_\alpha ((1 + g)q e^{-r\Delta} - \frac{gb}{\lambda} - \frac{qg}{r}) + \bar{m}_\beta (1 - g) \left( - \frac{q - (1 - \beta)\lambda c}{r} + \beta \frac{qe^{-r\Delta}}{2r} - \frac{b}{\lambda} \right)
\] (33)

Retrieving

\[
\bar{W}_\alpha - \bar{W}_\beta \leq (\bar{m}_\beta - \bar{m}_\alpha) \left( \beta \frac{qe^{-r\Delta}}{2r} + (1 - \beta)U_\beta \right)
\]

by rearranging (29), we substitute it into (33) we arrive at an expression weakly greater than

37Here it is relevant to point out that if the expected discount on revelation is \( m \), then the expected cost of monitoring until revelation is \( mb/\lambda \). This is for the following reason: to say the expected discount on revelation is \( m \) is to say that for probability \( M(t) \) of monitoring at time \( t \) conditional on no revelation by \( t \),

\[
\int_{0}^{\infty} e^{-rt}e^{-\int_{0}^{t} \lambda M(t')dt'} \lambda M(t)dt = m;
\]

but the amount spent on monitoring at each time is the probability no revelation occurs until that time multiplied by the probability monitoring occurs at that time, the discount and the flow cost of monitoring; hence the total cost is \( \int_{0}^{\infty} e^{-rt}e^{-\int_{0}^{t} \lambda M(t')dt'} bM(t)dt = mb/\lambda \).

38Notice that while the candidate equilibrium is allowed to generate value from screening worker types by strategies and therefore apply monitoring more efficiently, the deviations S3 checks against are not. That it turns out such deviations are enough to destroy all equilibria but one is a product of the \( \beta \) workers’ incentives to not reveal themselves.
the LHS of (32) where the coefficient of $\tilde{m}_\beta$ is

$$(1 - g) \frac{1}{r} \left[ \lambda c (1 - \beta) - q (1 - e^{-r\Delta} \beta) + r (1 - \beta) U_{\beta} - \frac{rb}{\lambda} \right]$$

which we know from (C6) is positive. Therefore, up to the constraint imposed by (28) we have an upper bound of the LHS of (32) increasing in $\tilde{m}_\beta$. So if (32) holds for some $(\tilde{m}_\alpha, \tilde{m}_\beta)$, it must hold for $(\tilde{m}_\alpha, \tilde{m}_\beta)$. Making this substitution, we arrive at

$$\frac{q - (1 - \theta W) \lambda c}{r} (1 - \tilde{m}_\alpha) + \tilde{m}_\alpha \frac{(1 + \theta W) q e^{-r\Delta}}{2r} - \frac{b}{\lambda} \geq \frac{q - (1 - \theta W) \lambda c}{r}$$

which due to (C4) can only occur if $\tilde{m}_\alpha = 0$.

Therefore, regardless of the first proposer, all equilibria in the white labor market lack monitoring for both types of workers. We can further exclude equilibria with delay, as an S3-type deviation giving the receiver his equilibrium utility and the proposer taking the excess would be payoff-increasing in those cases.

Finally, by S3, no deviation by a first receiver that gives the first proposer his payoff when proposing, discounted, is gainful. Therefore, the first proposer’s share cannot be greater than $\frac{1}{1 + e^{-r\Delta}}$. Similarly, the initial proposer $i$ cannot be getting $x < \frac{1}{1 + e^{-r\Delta}}$, lest $j$ have a deviating offer in his own role as first proposer giving $i$ his discounted value, $e^{-r\Delta} x$ and $j$ a share of $1 - xe^{-r\Delta} > \frac{1}{1 + e^{-r\Delta}}$.

Thus, all equilibria of the white labor market reach immediate agreement with a no-monitoring offer; the wage splits the surplus along the Rubinstein shares and therefore the equilibrium of Proposition 1 is essentially (up to off-path behavior and beliefs) unique.

A.5 Proof of Proposition 3

The initial equilibrium wage proposed is

$$w_{\text{work}}^{M_{\theta B}} = \frac{[q - b - \lambda (1 - \theta_B) - (e^{-r\Delta} - \theta_B) q e^{-r\Delta} / (2r)]}{1 + e^{-r\Delta}}$$

if the worker proposes first and

$$w_{\text{firm}}^{M_{\theta B}} = \frac{[e^{-r\Delta} (q - b - \lambda (1 - \theta_B)) - (1 - e^{-r\Delta} \theta_B) q e^{-r\Delta} / (2r)]}{1 + e^{-r\Delta}}$$

if the firm proposes first. Monitoring is in use until revelation, and no renegotiation takes place until then.

If the worker rejects a firm monitoring with a wage in $[w_{\text{firm}}^{M_{\theta B}}, w_{\text{work}}^{M_{\theta B}}]$, opens renegotiation when a monitoring regime with a wage in that interval is in place, or makes or accepts a
non-monitoring offer before revelation, the firm immediately believes the worker to be type $\beta$. This change is irreversible. Otherwise, the firm has beliefs constant at $\theta_B$.

When the firm starts believing the worker to be type $\beta$, both parties immediately renegotiate to the equilibrium in Section 3.4.2.4 with monitoring and a lower wage.

As long as beliefs are $\theta_B$, agents in the role of proposer offer their $w^i_{M\theta_B}$. Monitoring offers with wages in $[w^f_{M\theta_B}, w^{work}_{M\theta_B}]$ wages are accepted by either party without renegotiation until revelation. Other offers are rejected or renegotiated, and the next offer is the proposer’s $w^i_{M\theta_B}$.

If revelation occurs, bad matches separate; good matches renegotiate as per Lemma 1.

Clearly, workers don’t want to deviate to propose in $[w^f_{M\theta_B}, w^{work}_{M\theta_B}]$ as they will lead to acceptance but a lower payoff; also, they don’t propose wages outside $[w^f_{M\theta_B}, w^{work}_{M\theta_B}]$ as the firm will reject and propose $w^f_{M\theta_B}$ in addition to suffering the delay. Thus, always proposing $w^{work}_{M\theta_B}$ is optimal. Workers won’t reject offers in $[w^f_{M\theta_B}, w^{work}_{M\theta_B}]$ or accept or propose non-monitoring offers as they don’t want to be treated as $\beta$s as per (C6) and (C7).

Firms know that by the workers’ strategy, the highest offer they can get accepted is $w^f_{M\theta_B}$ and that higher ones, or ones below $w^{work}_{M\theta_B}$, will be rejected and that the worker will counter-offer $w^{work}_{M\theta_B}$, in addition to the firm suffering a delay. Firm offers in $(w^f_{M\theta_B}, w^{work}_{M\theta_B})$ will be accepted but yield a lower payoff than $w^f_{M\theta_B}$; thus always proposing $w^f_{M\theta_B}$ is optimal for the firm. Given this, the firm accepts offers in $[w^f_{M\theta_B}, w^{work}_{M\theta_B}]$; but will reject higher ones because it can do better as proposer, and lower ones because it knows renegotiation will be imminent once they are in place. If production is occurring, the firm can gain by renegotiating if either (a) the worker will instantly renegotiate, and the firm’s first offer here is a lower wage than the worker’s (so as above, if $w < w^f_{M\theta_B}$), or if the firm’s payoff from making its offer, $w^f_{M\theta_B}$, with delay, is preferable to the current payoff; but that is precisely when the current wage $w > w^f_{M\theta_B}$.

That there is no S3-type deviation that proposes no monitoring follows from (C4).

In the limit as $\Delta \downarrow 0$, the equilibrium shares of the first proposer and receiver equalize; the limiting wage is $w_{M\theta_B} = \frac{1}{2} \left[q - b - \lambda c(1 - \theta_B)\right] - \frac{(1-\theta_B)}{2r}\lambda \frac{q}{2r}$.

A.6 Proof of Proposition 4

The proof here proceeds in the same fashion as that in A.4. Instead of comparing to a no-monitoring deviating offer we compare to a monitoring offer; therefore instead of 35 we have
\[
\frac{q - (1 - \theta_B) \lambda c}{r} (1 - \tilde{m}) + \tilde{m} \frac{(1 + \theta_B) q e^{-r \Delta}}{2r} - \frac{b}{\lambda} \geq \frac{q - (1 - \theta_B) \lambda c - b + \frac{(1 + \theta_B) q e^{-r \Delta}}{2r}}{\lambda + r} \tag{36}
\]

which due to (C5) can only be true if \( \tilde{m} \geq \frac{\lambda}{\lambda + r} \); but as \( \tilde{m} \) is an expected discount of a variable that at most arrives as a Poisson with rate \( \lambda \), this constitutes an upper bound to \( \tilde{m} \) and corresponds to full monitoring and no delay.

Therefore, only fully monitoring equilibria exist in the black labor market. Within such candidate equilibria, S3 would allow for deviation from any initial offer not corresponding to that in Proposition 3, therefore that equilibrium is unique.