WINNING BY DEFAULT: WHY IS THERE SO LITTLE COMPETITION IN GOVERNMENT PROCUREMENT?

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Abstract. In government procurement, contracts generally have a small number of participating bidders, and it is not uncommon that only one bidder is allowed to bid. To understand the determinants of competition in government procurement, we develop, identify, and estimate a principal-agent model in which the procurer chooses whether to solicit bids and how much effort to exert to attract more bids, and then she negotiates with bidders to choose a winner and reach an agreement on a contract. We find that the cost of soliciting, identifying, and processing an additional bid is relatively large compared to the cost savings from competition. By strategically setting the contract terms, the procurer can extract informational rent from bidders, as an alternative to attracting more bidders.

1. Introduction

In recent ten years, the market for the United States federal government procurement is worth over $460 billion annually, which constitutes about 18% of the federal government spending. Despite its vast size, the extent of competition for a procurement contract is not very intense. Contracts generally have a small number of participating bidders, and it is not uncommon that only one bidder is allowed to bid. In this paper, we develop, identify, and estimate a procurement model to better understand the extent of competition observed in the data.

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There are three important institutional features of federal government procurement that have received relatively little attention from the literature. First, for each procurement contract, the extent and method by which the contract will be competed is chosen by contracting officers who are hired by the government. The regulations allow them to eliminate certain bidders from consideration, although full and open competition is encouraged.

Second, a sealed-bid auction is not always a dominant procedure to choose a contractor, depending on the nature of the products or services to be procured. An alternative solicitation procedure is by *negotiation*, through which the proposals submitted by contractors are evaluated, negotiated, and selected. After the request for proposals is posted, the qualified contractors can submit their proposals, which will be reviewed in detail to determine which proposals are within a competitive range. Discussions may then be carried out with the contractors within the competitive range, and the contractor whose proposal is found to be most advantageous to the procuring agency will be selected. During the discussions, the contract terms and prices are considered together.

Third, contract modifications frequently occur after a contract initiates. Some modifications reflect bilateral agreements, often resulting from change of orders, while other modifications do not require such agreements and are determined unilaterally by the procurer. The existing literature has focused on the former type of contract modifications, and the latter has received little attention.

In this paper, we construct a principal-agent framework that incorporates these features. The procurer chooses the extent of competition, i.e., the eligibility conditions and the expected number of bidders, and negotiates with the bidders on the contract terms, which determine the circumstances under which each of the two types of contract modifications may occur. We model the negotiations as the procurer's offering a menu of contracts to the bidders with a hidden type (cost). When the number of bidders is very small, which is often the case with the government procurement, we show that this procedure is more profitable to the procurer than a standard auction.

When the procurer chooses whether to solicit bids, she faces the trade-off between cost savings from more competition in the open, formal solicitation procedure and potential benefits from bypassing such procedure. These potential benefits include (unverifiable) quality of work and less administrative costs, as well as rents from corruption. A similar trade-off exists when choosing the level of efforts to attract more bids.

Given the observables in the data, the model is nonparametrically identified. However, the number of observations in our dataset is not large enough to nonparametrically estimate the model. Instead, we estimate the model parameters using a simulated GMM estimator. We focus on the definitive, negotiated contracts with a large size, from $0.3 to $5 million for customized or commercially unavailable IT/Telecommunications services, which were awarded during FY 2004-2012.

We conduct counter-factual analyses of the estimated model to determine the factors that affect the extent of competition, in particular, (i) cost savings from more competition, (ii) per-bidder bid solicitation and processing costs, and (iii) the cost of implementing a formal solicitation process. Note that we cannot separately distinguish the benefits from quality of work and the rents from corruption, but we can identify and estimate the sum of various benefits of not using the solicitation procedure, net of administrative costs due to the requirement for justification and risk of being caught for corruption. Our counterfactual scenarios include first-price sealed bid auctions as opposed to negotiations, mandatory solicitation, minimum number of bids, and exogenous changes to per-bidder bid costs or the formal solicitation process costs.

We find that the cost of soliciting, identifying, and processing an additional bid is relatively large compared to the cost savings from competition. By strategically setting the contract terms, the procurer can extract informational rent from bidders, as an alternative to attracting more bidders.

Our paper is related to the large literature on procurement and auctions. One strand of the literature explains why less competition does not necessarily lower the payoff of the auctioneer in independent private value auctions. Li and Zheng (2009) show that when the number of bidders is endogenously determined, the equilibrium bidding behavior can become less aggressive as the number of potential bidders increases. Krasnokutskaya and Seim (2011) study a bid preference program, and Athey, Coey and Levin (2013) compare a set-asides program and the bid subsidy program. Both papers show the importance of allowing endogenous entry when assessing restrictive competition policies.

An important contribution of our paper is that we build and estimate a model where the procurer is assumed to optimally choose the extent of competition. In this regard, Bandiera, Prat and Valletti (2009) and Coviello, Guglielmo and Spagnolo (2014) are closely related to our paper. Bandiera, Prat and Valletti (2009) develop a formal framework for distinguishing active waste and passive waste in the total government cost of procurement, and separately estimate them exploiting a policy experiment in Italy's public procurement system. Active waste entails utility for the public decision makers, part of which is related to favoritism in our paper, while passive waste does not, such as bid processing and solicitation costs. Coviello, Guglielmo and Spagnolo (2014) study government discretion on public goods provision in terms of whether or not to impose entry restrictions, and document the casual effect of increasing such discretion on procurement outcomes using a database for public procurement in Italy.

Another strand of the literature studies nonstandard contractor selection procedures, such as scoring auctions (Asker and Cantillon (2010)), multi-attribute auctions (Krasnokutskaya, Song and Tang (2013)), or negotiations (Bajari, McMillan and Tadelis (2008)), where the price is not the only factor in selecting a contractor. We consider an optimal direct revelation mechanism in a competitive environment, studied by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). We extend their models by allowing the procurer to choose the optimal extent of competition. In that sense, our paper also belongs to the literature on the identification of principal-agent models, for example, Perrigne and Vuong (2011).

Lewis and Bajari (2014) and Bajari, Houghton and Tadelis (2014) are related to our paper in that they study the contract modifications or price adjustments after the winning contractor is chosen and the project initiates. We distinguish the contract modifications into two categories depending on whether the modification requires a bilateral agreement or not. The former type of contract modifications may reflect incomplete contract designs, as studied by Bajari, Houghton and Tadelis (2014). The latter type of modifications often include administrative changes, related to the contingency plans agreed in the original contract. We interpret that these modifications occur due to the cost changes related to the unknown type of the contractors. Bajari, Houghton and Tadelis (2014), on the other hand, study these modifications in a moral hazard framework. One key difference between our paper and other existing papers on contract modifications is that we endogenize the terms of contract modifications.

Lastly, our paper is related to the political economy literature on how the federal government funds are allocated to the state or local governments or the private entities. Knight (2005) shows that members in the transportation committee secure higher project spending than do members from other districts. De Figueiredo and Silverman (2006) find that universities represented by a House or Senate Appropriations Committee member receive benefits regarding earmarks.

The rest of the paper is organized as follows. In Section 2, we present the key data features of the US procurement regarding the extent of competition, the solicitation procedure, and ex-post price adjustments. We then discuss how these features are incorporated in the theoretical model described in Section 3. The identification of the model follows in the next section, and the estimation is discussed in Section 5. In Section 6, we discuss the sample selection for the empirical analyses and provide summary statistics of the data. The estimates and the counterfactual results are shown in the same section, and then Section 7 concludes.

2. The US Federal Procurement

In this section, we describe the institutional backgrounds that motivate our modeling of the US federal procurement. We focus on how the extent of competition, solicitation procedure, and ex-post contract modifications are determined in practice.

In providing descriptive statistics regarding these institutional features, we analyze the data on federal government contracts and their modifications from the Federal Procurement Data System - Next Generation. We focus on *definitive* contracts that were initiated during the period of FY 2004–2012. Definitive contracts have specified terms and conditions, as opposed to indefinite delivery, indefinite quantity contracts. The former contracts tend to be much bigger in terms of payment size than the latter. For example, in FY 2010, \$507 billion (94%) of the total amount of money that the government was obliged to pay, \$540 billion, is for definitive contracts.[1] We further restrict our attention to the contracts with the actual obliged payment of \$300,000 or more. This size threshold is chosen because the contracts of an anticipated size less than $\$300,000$ are generally expected to be reserved for small business concerns.[2] Note that we use the actual payment, not the expected payment, for the threshold–this is because the anticipated payment amount does not appear in the data.

2.1. **Restriction of Competition.**

The full and open competition without exclusion is default in the acquisition process. However, the federal regulations specify the circumstances under which contracting officers are allowed to provide for full and open competition after excluding one ore more sources or even to choose a contractor without competition.[3] When they impose such entry restrictions, they are required to provide a

[1]In the FY 2010 contract data, there are about 2.8 million unique contracts. About 60% of them are definitive contracts (over 1.7 million contracts). The average obligated amount of money for a definitive contract during the one-year period is \$296,000, while that for an indefinite delivery, indefinite quantity contracts is \$31,000.

[2]FAR 13.003(b)(1) states that acquisitions of supplies or services that have an anticipated dollar value exceeding \$3,000 but not exceeding \$150,000 are reserved exclusively for small business concerns and shall be set aside. For certain supplies or services, the upper limit can be \$300,000, according to FAR 2.101.

[3]See FAR 6.202–8 for the list of circumstances under which full and open competition after excluding one or more sources is allowed. See FAR 6.302 for the list of seven different circumstances under which no competition is allowed, and FAR 6.303-4 describes the procedures for written justifications and approvals.

Table 1. The Extent of Competition

Extent Competed	Num.	Total Size ($ Billion)	Median Size ($K)	Ratio of One Bid
Full & open competition	83,372 (40%)	671 (43%)	802.4	30.6%
Restricted by regulation	41,929 (20%)	121 (8%)	793.1	98.8%
Restricted to small business	41,266 (20%)	101 (7%)	745.0	20.7%
Restricted by discretion	43,008 (21%)	654 (42%)	66.5	89.9%
Total	209,575 (100%)	1,546 (100%)	756.7	54.5%

Note: This table is based on 209,575 definitive contracts that satisfy the following two criteria: (i) initiated during FY 2004–2012 and (ii) the actual size is greater than or equal to $300,000 at the nominal value. The size of the contracts is the CPI-adjusted total amount of money that the government is obligated to pay to the contractors, where CPI of December 2010 is 100.

documentation signed by the head of the agency that describes the estimated reduction in overall costs and how the estimate was derived.

As can be seen in Table 1, less than half of the contracts in the data were fully competed. As a result, out of $1.5 trillion spent on large contracts that initiated during the period of study, about 57% was spent on contracts with entry restrictions. We categorize the entry restrictions into three cases: (i) regulatory, (ii) small business concerns, and (iii) discretionary restrictions. The first case is related to international agreements or statutes, which authorize or require that acquisition be made from a specified source or through another agency. The entry restrictions related to small businesses could be under contracting officers' discretion to a certain extent, but there are statutory requirements for small business concerns.[4] The remaining reasons for entry restrictions could be considered as "discretionary". They include (i) the source has submitted an unsolicited research proposal, (ii) a follow-on contract for the continued development or production of a major system or highly specialized equipment, (iii) the existence of limited rights in data, patent rights, copyrights, or secret processes, or (iv) an acquisition that uses a brand-name description or other purchase description to specify a particular brand-name, product, or feature of a product, peculiar to one manufacturer, to name a few. Also, national security, unusual and compelling urgency, or public interest can also be cited as a reason for entry restrictions.[5] Table 2 show the various reasons why there was discretionary entry restrictions.

[4]The statutes or the programs are section 8(a) of the Small Business Act, the Small Business Innovation Research Program, the Historically Underutilized Business Zones Act of 1997, the Veterans Benefits Act of 2003, (Economically Disadvantaged) Women-owned Small Business Program, and the Disaster Relief Act Amendments of 1974.

[5]For example, FAR 6.302-7 states that full and open competition need not be provided for when the disclosure of the agency's needs may compromise the national security unless the agency is permitted to limit the number of sources from which it solicits bids or proposals.

Table 2. Reasons for Discretionary Entry Restrictions

Reasons	Num.	Total Size ($ B)	Median Size ($K)
Only one available source	24,750 (58%)	488 (75%)	656.3
Follow-on contract	2,512 (6%)	74 (11%)	783.1
Urgency	4,799 (11%)	29 (4%)	816.8
Other	3,614 (8%)	47 (7%)	639.0
Unspecified	7,333 (17%)	16 (2%)	618.8
Total	43,008 (100%)	654 (100%)	666.5

Note: This table is based on 43,008 contracts that (i) initiated during FY 2004–2012, (ii) the actual size is greater than or equal to $300,000 at the nominal value, and (iii) were either not competed or competed after exclusion of sources. "Only one available source" include patent or data rights, unique source, unsolicited research proposal, brand, and other justifiable reasons. "Other" reasons include national security, public interest, authorized resale, standardization, maintaining alternative sources, and simplified acquisition. For contracts that were competed after exclusion of sources, the reasons for such exclusion are often omitted in the data, and these are categorized as "Unspecified" in the table. The size of the contracts is the CPI-adjusted total amount of money that the government is obligated to pay to the contractors, where CPI of December 2010 is 100.

2.2. **Solicitation Procedure and the Number of Bids.** When full and open competition with or without exclusion of sources is employed, sealed bidding and negotiation are both acceptable procedures to solicit bids. Sealed bids are used if (i) time permits the solicitation, submission, and evaluation of sealed bids, (ii) the award will be made on the basis of price and other price-related factors, (iii) it is not necessary to conduct discussions with the responding contractors about their bids, and (iv) there is a reasonable expectation of receiving more than one sealed bid. When these conditions are not met, negotiated acquisitions can be used instead.[6]

As can be seen Table 3, the most prevalent solicitation procedure is negotiation. Negotiated acquisition was used for about 59% of the definitive contracts with a size greater than or equal to $300,000 that were competed for award during the period of study, accounting for $560 billion of government spending. Sealed bidding, on the other hand, was used much less frequently, accounting for $48 billion. In this paper, we study negotiated acquisitions.

In a negotiated acquisition, a contracting agency issues a request for proposal (RFP), upon which interested contractors submit their proposals. A typical RFP describes (i) the requirement, (ii) the anticipated terms and conditions that will apply to the contract, (iii) the information required to be in the bidder's proposal, and (iv) the proposal evaluation criteria. RFPs can be posted at the federal business opportunities website, faxed, mailed, or presented orally. After receipt of proposals, award can be

[6]See FAR 6.4 for the conditions under which either of the two procedures is chosen. Although there are other procedures, such as two step, architect-engineer, and basic research, amongst others, sealed bidding and negotiation are the major solicitation procedures. See Table 3.

Table 3. Solicitation Procedure and the Number of Bids

Solicitation Procedure	Num.	Total Size ($ Billion)	Num. of Bids Average	Ratio of One Bid
Negotiation	51,355 (59%)	560 (83%)	6.0	36%
Sealed-bid	13,783 (16%)	48 (7%)	5.3	6%
Simplified acquisition	9,864 (11%)	15 (2%)	4.4	32%
Other procedures	7,830 (9%)	29 (4%)	14.5	28%
Unspecified	4,342 (5%)	19 (3%)	5.6	12%
Total	87,174 (100%)	942 (100.0%)	6.4	29%

Note: This table is based on 87,174 contracts that (i) initiated during FY 2004–2012, (ii) the actual size is greater than or equal to $300,000 at the nominal value, and (iii) were competed with or without exclusion of sources. "Other solicitation procedures" include two step, architect-engineer, and basic research, amongst others. The size of the contracts is the CPI-adjusted total amount of money that the government is obligated to pay to the contractors, where CPI of December 2010 is 100.

made with or without discussions. If discussions are to be conducted, the agency first determines *competitive range* and then negotiate with the bidders within the range. The discussions may apply to price, schedule, technical requirements, type of contract, or other terms of a proposed contract.[7]

In evaluating the proposals, the agency's objective is to select the proposal that represents the *best value*. Therefore, the relative importance of price in choosing the winner may vary. There are two selection processes: *tradeoff* and *lowest price technically acceptable* selection processes. A *tradeoff* process allows the government to accept other than the lowest priced proposal. On the other hand, the proposal with the lowest price is chosen as long as the proposal meets or exceeds the acceptability standards for non-cost factors in a *lowest price technically acceptable* selection process. The factors that may be considered other than price include past performance, compliance with solicitation requirements, technical excellence, management capability, personnel qualifications, and prior experience. The less definitive the requirement, the more development work required, or the greater the risk of unsuccessful contract performance, the more technical or past performance considerations may play a dominant role in selecting the winner, as opposed to price.

As can be seen in Table 3, the average number of bids is over 6, but 29% of the competed contracts were awarded to a single bidder. Including non-competed contracts, only one contractor was considered for 54% of the contracts in the data. Putting it differently, over one trillion dollars was obligated to pay contractors that won a contract by default during the period of the study.

[7]See FAR 15 for details of the negotiated acquisition process.

The number of bids can be affected by the efforts of the contracting agency. Prior to issuing a RFP, the agency may exchange information with industry prior to receipt of proposals. The early exchanges of information can take the form of industry conferences, public hearings, market research, one-on-one meetings with potential contractors, draft request for proposals (RFP), or request for information (RFI). Furthermore, the agency may publish a pre-solicitation notice that provides a general description of the scope or purpose of the acquisition and invites potential contractors to submit information. The notice contains information to permit a potential contractor to make an informed decision about whether to participate in the acquisition. The agency evaluates all responses in accordance with the criteria stated in the notice, and advises each respondent in writing either that it will be invited to participate in the resultant acquisition or, based on the information submitted, that it is unlikely to be a viable competitor.

These activities before issuing a RFP could help decrease the burden of contractors to search for a suitable contract to apply for and to prepare for their proposal. Note that these can be costly to the contracting agency. Furthermore, an additional proposal incurs an additional administrative cost of evaluating the proposal and a larger risk of receiving a bid protest. In FY 2012, the Government Accountability Office received 2,475 bid protest cases. Although the office upholds only a small number of bid protests, they may still have a big impact.[8]

2.3. **Ex-post Price Adjustment.** The agreed-upon price at the time of award, *base price*, can be different from the actual price at the end of the contract, *final price*. To construct the base and the final prices from the administrative data, we focus on the amount of money that the government is obligated to pay: the base price is the total amount of money that is obligated to the government on the beginning date of the contract; and the final price is the sum of all obligated amount of money regarding that contract.[9] The final prices are often larger than than the base prices. As can be

[8]Federal Times reported in July 2013 on how bid protests are slowing down procurements. The article quoted Mary Davie, assistant commissioner of the Office of Integrated Technology Services at the General Services Administration: "We build time in our procurement now for protests. We know we are going to get protested."

[9]In the data, there are three variables on the price of each contract action: (i) base and all options value, which is defined to be the mutually agreed upon total contract value including all options; (ii) base and exercised options value, which is defined to be the contract value for the base contract and any options that have been exercised; and (iii) action obligation, which is the amount that is obligated or de-obligated by the transaction. For our analysis, the first two variables are more suitable than the obligated value. However, either of the base and all or exercised options value variables is reported in the base contract record for only 39% of the contracts in the dataset.

TABLE 4. Ex-post Price Adjustments

Price Adjustment	Num.	Total Size ($Billion)	Adjustment/ Total	Ratio of Firm-fixed
No price adjustments	65,102 (37%)	80 (6%)	0%	92%
Bilateral only	37,654 (31%)	270 (19%)	21%	89%
Unilateral only	40,458 (14%)	113 (8%)	54%	68%
Both types of adjustments	32,066 (19%)	941 (67%)	68%	69%
Total	175,280 (100 %)	1,404 (100%)	30%	82%

Note: This table is based on 175,280 contracts that (i) initiated during FY 2004–2012, (ii) the actual size is greater than or equal to $300,000 at the nominal value, and (iii) either competed and then negotiated or not competed. The fourth columns shows the average ratio of the size of adjustment to the total size of a contract. The last column shows the ratio of contracts that are under the "firm-fixed" pricing.

seen in Table 4, the difference between the base and the final prices, which we call *ex-post price adjustment*, is nonzero for about 63% of the large-sized definitive contracts in the data that were either competed and then negotiated or not competed.

In the table, we divide the contracts in the data into four categories by whether or not *bilateral* or *unilateral* price adjustments occurred. A bilateral adjustment is signed by both the contractor and the contracting officer. They are used to make negotiated equitable adjustments resulting from the issuance of a change order, to definitize letter contracts, and to reflect other agreements of the parties modifying the terms of contracts. A unilateral adjustment is, on the other hand, is signed only by the contracting officer. This does not require an additional agreement because it is due to the predetermined terms of a contract. The ex-post price adjustment records in the data fall into one of the following categories: (i) additional work, (ii) supplemental agreement for work within scope, (iii) exercise an option, (iv) change order, (v) definitized letter contracts, all five of which are considered as *bilateral* in the table, and the remaining *unilateral* categories, such as an administrative action, an exercise of an option, and close-out.[10]

The size of price adjustments are significant relative to the total size of a contract. As can be seen in Table 4, for contracts with both types of price adjustments, the average ratio of the amount of total price adjustment to the total final price is 72%. Looking at the contracts with either type of the modifications only, the average ratio is 35% and 47%, respectively.

[10]The administrative actions include funding, representation, transfer, and novation actions. Some administrative actions could be to simply fix a mistaken record. For that reason, we consider that the first two administrative actions that are followed by a bilateral modification without any price adjustments are related to that bilateral modification.

TABLE 5. Non-repeat vs. Repeat Contractors

Contractors by Num. of Contracts	Num. of Contractors	Num. of Contracts	Median Size ($k)	Std. Dev. Size ($m)
Non-repeat contractors	1,848 (63%)	1,905 (26%)	810.8	9.1
Repeat contractors (≤ 5)	865 (30%)	2,366 (33%)	1,058.4	31.3
Repeat contractors (> 5)	212 (7%)	2,971 (41%)	986.8	44.8
Total	2,925 (100%)	7,242 (100%)	961.0	34.2

Note: This table is based on 7,242 contracts that (i) initiated during FY 2004–2012, (ii) the actual size is greater than or equal to $300,000 at the nominal value, and (iii) the product/service code starts with D3, services related to IT and telecommunications. The size of the contracts is the CPI-adjusted total amount of money that the government is obligated to pay to the contractors, where CPI of December 2010 is 100.

The ex-post price adjustment reflects the terms of the contract that allow the final price to vary with the observed outcomes of the project. Such contract terms could depend on contract types, ranging from *firm fixed price*, in which the contractor has full responsibility for the performance costs and resulting profit or loss, to *cost plus fixed fee*, in which the contractor has minimal responsibility for the performance costs and the negotiated fee is fixed. In between are the various incentive contracts. However, as can be seen in Table 4, even for firm fixed price contracts, the ex-post price adjustments are not uncommon. Because ex-post price adjustments, even the unilateral ones, are frequent in firm-fixed price contracts, we focus on the price adjustments, rather than the stated contract type.

2.4. **Collusion and Dynamics.** When it comes to the lack of observed competition, one of the most likely causes is collusion among contractors. We claim that collusion is less likely in the large-sized definitive contracts for the US federal government. It is mainly because of the large heterogeneity of contracts that appear infrequently. As discussed in Porter and Zona (1993), this makes it difficult for contractors to maintain a collusive relationship.

For example, among the 2,925 contractors that have won at least one definitive IT and telecommunications contract of size greater than or equal to $300,000, 63% of them won only one contract during the period of study, as can be seen in Table 5. It is true that the 7% of the contractors, who won more than 5 contracts during the 9-year period, won a large share of the contracts, 41%. However, the contracts they won are not necessarily much larger than those won by less frequently winning contractors, and there is a large heterogeneity, as measured by the standard deviation of the contract sizes. Indeed, the top contractors in the IT and telecommunications industry have distinct areas of advantages. The top six contractors in terms of the

Table 6. Contract Performance and Reputation

Ratio of Contracts with	Among Contracts Performed by		Mean Difference
	Non-repeat or Repeat (≤ 5)	Repeat (> 5)	
Bilateral price adjustments	42.8% (0.7%)	33.9% (0.8%)	8.9%*** (1.2%)
Unilateral price adjustments	65.0% (0.7%)	56.0% (0.9%)	8.9%** (1.2%)
Delays	63.4% (0.7%)	51.5% (0.9%)	11.9%*** (1.2%)

Note: The standard errors are in parentheses, and asterisk marks represent the statistical significance level of 1% (***) and 5% (**). A contract is considered to be "delayed" if the actual duration is 10% or more longer than the initially-agreed duration. This table is based on 7,242 contracts that (i) initiated during FY 2004–2012, (ii) the actual size is greater than or equal to $300,000 at the nominal value, and (iii) the product/service code starts with D3, services related to IT and telecommunications.

number of contracts are AT&T, Verizon, Computer Science Corp., Lockheed Martin, and Wolverine Services. Therefore, although collusion cannot be ruled out completely, these facts suggest that the first-order issues regarding the observed extent of competition are related to the government actions and the availability of reliable, competent contractors of specific expertise, rather than collusion.

As can be seen in Table 5, about 7% of contractors that won more than 5 large IT and telecommunications contracts during the 9 years collectively won 41% of contracts. These contractors seem to perform better than non-repeat ones, if we measure performance by the probability that ex-post price adjustments and delays occur, as documented in Table 6. The contracts performed by these repeat contractors are less likely to experience price adjustments, both bilateral and unilateral, and delays than the rest.[11]

However, these repeat contractors won 55% of large IT and telecommunications contracts with full and open competition while won 40% of contracts with limited or no competition due to the contracting officers' discretion, as documented in Table 7. This implies that discretionary restrictions in competition are not necessarily associated

[11]The delays are defined to be the difference between the base and the final durations. The former is determined by the difference of the expected completion date and the starting date as in the base contract record. In the data, there are three variables on the dates of each contract action: (i) effective date, which is the date that the parties agree will be the starting date for the contract requirements; (ii) current completion date, which is the scheduled completion date for the base contract and any options exercised at time of award; and (iii) ultimate completion date, which is the estimated or scheduled completion date including the base contract and all options. For the "expected completion date" in our analysis, we use the current completion date variable, and for the "starting date", we use the effective date of the base contract. The final duration is the difference of the expected completion date of the last contract modification record and the starting date of the base contract record.

Table 7. The Extent of Competition and Reputation

Extent Competed	Won by			Total
	Non-repeat	Repeat (≤ 5)	Repeat (> 5)	
Full & open competition	519 (24%)	449 (21%)	1,206 (55%)	2,174 (100%)
Restricted by regulation	500 (21%)	997 (43%)	814 (35%)	2,311 (100%)
Restricted to small business	314 (40%)	305 (39%)	167 (22%)	786 (100%)
Restricted by discretion	572 (29%)	615 (31%)	784 (40%)	1,971 (100%)
Total	1,905 (26%)	2,366 (33%)	2,971 (41%)	7,242 (100%)

Note: This table is based on 7,242 contracts that (i) initiated during FY 2004–2012, (ii) the actual size is greater than or equal to $300,000 at the nominal value, and (iii) the product/service code starts with D3, services related to IT and telecommunications.

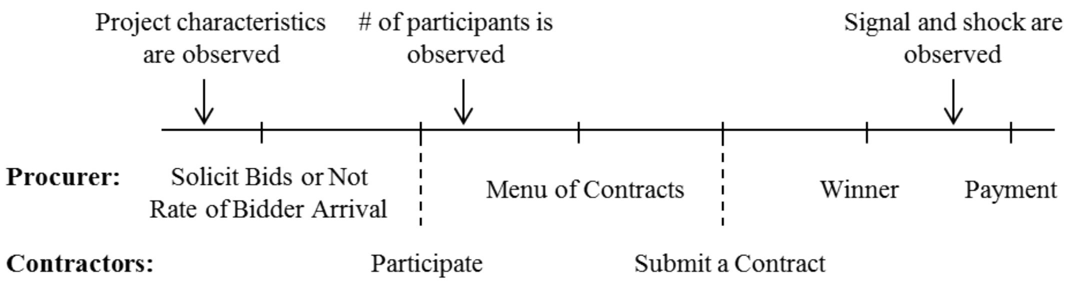
with dynamic incentive schemes. This motivates our static model described in the next section.

3. Model

This section lays out our model and shows how its components relate to the institutional features described in the previous section describing the data. The model comprises: a timeline for the procurement process; payoffs to the procurer and the contractors; a description of how information disseminates throughout the process; factors that determine whether the project will be competed; costs for soliciting bids in competitive projects; constraints the procurer must respect given the objectives and private information of contractors; a menu of contracts the procurer offers contractors to select from; and finally the priorities the procurer uses to select the winning contractor from competing bids. After outlining the timeline, information and payoffs to the contracting parties, we use backwards induction to explain the rest of the model and solve its equilibrium.

3.1. **Timeline.** Figure 1 lays out the timeline in the model. When a project is realized, its characteristics and a default contractor are observed. The procurer decides whether to hold a competitive solicitation process that may attract multiple bidders, or award the contract to the default contractor without the solicitation process. When the number of bidders is known, the procurer issues a menu of contracts and a preference ordering over the menu items. Contractors simultaneously select a contract, and the procurer chooses a winner by following the preference ordering. Upon completion of the project, both parties observe the project outcomes, which affect the final payment to the contractor. The project outcomes that are unanticipated at the time of signing the initial contract may ensue contract modifications that require a bilateral agreement.

Figure 1. Timeline of the Procurement Process in the Model



On the other hand, the initial contract specifies the contingency payment schedule for anticipated project outcomes, thus these outcomes do not require further negotiation.

This timeline of the model represents the observed institutional features in the data. First, a large proportion of contracts did not have full and open competition due to discretion of the contracting officers, as documented in Table 1. We consider the contracts which attracted only one bid, with the extent of competition being neither full nor open for reasons unrelated to regulations or small business concerns to have been "awarded to a default contractor" and the rest to have been "competed" in our model.

Second, the negotiation procedure is modeled as a principal-agent problem where the principal, or the procurer, offers a menu of contracts to agents. In negotiated procurements, selection of the contract terms can be a matter for negotiation, as described in the Federal Acquisition Regulation 16.1. The regulations state that "negotiating contract type and negotiating prices are closely related and should be considered together."

In our analysis, we divide the contracts into two broad contract categories, fixed or variable, based on whether or not ex-post price adjustments are allowed. Both types of contracts may have bilateral price adjustments. The difference comes from that unilateral price-adjustments are allowed for variable contracts only. They specify the contingencies under which unilateral adjustments may occur. This categorization of contracts do not coincide with the nomenclature of fixed vs. cost plus contracts, as can be seen in the last column of Table 4.

Lastly, the contingencies under which a unilateral price adjustment may occur and that are observed by both the procurer and the contractor do not necessarily include the cost of a project to the contractor, although these contingencies can be correlated with the project cost. This is because the contracting agency does not always observe the cost of completing a project, and therefore, the final payment is often not based on the actual, realized cost. The government may require the contractors to disclose in writing their cost accounting practices and to comply with the Cost Accounting Standards. However, only 10% of the contracts in our data have such requirements in place.

3.2. **Contractor Types and Information.** The total cost of completing a given procurement project is the sum of the expected cost, $\alpha \in \mathcal{R}^+$ for the low-cost contractors and $\alpha + \beta > \alpha$ for the hight-cost ones, and ex-post cost changes due to stochastic realizations of demand and supply shocks, denoted by $\epsilon \in \mathcal{R}$. A project specific parameter $\pi \in [0, 1]$ denotes the proportion of the low-cost contractors in the population.

The realization of (α, β, π) is observed by the procurer at the beginning of the process, and by the contractors before they bid. After the project is initiated, both the procurement officer and the winning contractor observe ϵ. However, the expected cost to the contractors is hidden information; only the contractor knows whether she is low-cost or not. We also assume that ϵ is distributed independently of the expected cost.

When the project is completed, a signal, denoted by s, is revealed to both the procurer and the winning contractor. Let $\underline{F}(s)$ denote the cumulative distribution function for s conditional on the winning contractor being low-cost; the corresponding function for a high-cost winning contractor is $\overline{F}(s)$. We assume both distribution functions are differentiable with densities $\underline{f}(s)$ and $\overline{f}(s)$ respectively. The signal is informative but imperfect: in particular $\underline{F}(s)$ and $\overline{F}(s)$ are defined on common support denoted by S, but $\underline{F}(s) \neq \overline{F}(s)$ for some $s \in S$.

In our empirical analysis, π is treated as a project specific unobserved continuous variable: after conditioning on the number of bidders, the contract type, and other observed characteristics, we allow the winning bid to vary continuously across observations with π. Since only the winning bid is observed and both types of awards vary continuously with an unobserved variable specific to each observation, the data on awards is saturated in a statistical sense. Intuitively, we lack another source of variation to distinguish a third type of contractor from the other two. For this reason we restrict the number of contractor types to two, whose costs are permitted to vary with π. However, to simplify the exposition, and without loss of generality to the model, we suppress the dependence of costs on π until the section on identification, and postpone our analysis of how equilibrium contracts vary with π until then.

3.3. **Payoffs.** Payment to a winning contractor has two parts: a base price p, and a variable component Δ. Liquidity concerns, or the cost of working capital, lead the winning contractor to discount the variable part of the payoff, and enlarge unanticipated cost adjustments. We denote the value to a low-cost contractor of completing a project with payment schedule (p, Δ) by:

$$p + \psi(\Delta - \epsilon) - \alpha,$$

where $\psi(\cdot)$ is a continuous real-valued function defined on $\mathcal{R}$, with $\psi(0) = 0$, $\psi'(0) = 1$, $\psi' > 0$, and $\psi'' < 0$. The value of the project to a high-cost contractor is defined by subtracting β from the expression above.

The procurer, on the other hand, does not have liquidity concerns. It is straightforward to show that since ϵ is independent of the contractor's expected cost, it is optimal to fully insure contractors against ϵ on a cost-plus basis. For this reason we define $q \equiv \Delta - \epsilon$ and solve the optimal contract menu in terms of (p, q), recognizing that the variable component of every contract is simply $\Delta = q + \epsilon$.

The objective of the procurer is to minimize the total expected cost of the project, which is the sum of the expected payment to a contractor and the cost of soliciting bids. The latter cost derives from multiple sources: reduced quality by allowing potentially low-quality contractors to participate, administrative burden related to formal solicitation procedure, and opportunity costs related to corruption.

Finally, we assume that, leaving aside payments by contractors that arise from projects that are less costly than originally expected (which occur when $\epsilon < 0$), there exists a maximal penalty the procurer can impose on contractors, denoted by $M \in \mathcal{R}^-$, such that $q \geq M$.

3.4. **Contract Menu and Selection Mechanism.** Let $n \in \{1, 2, \ldots\}$ denote the number of contractors who bid. Given the project specifications $\left\{\alpha, \beta, \pi, \underline{F}(s), \overline{F}(s)\right\}$, we show that it is optimal to offer a menu of two contracts: a preferred fixed contract in which the price only depends on n; and a variable contract, in which the base price is a constant, and the variable component we called q only depends on s. Therefore we can express the optimal menu as a triplet $\{\underline{p}_n; \overline{p}, q(s)\}$. Presented with an optimally designed contract menu, bidders truthfully reveal their cost type through their contract selection, low-cost bidders choosing $\underline{p}_n$, high-cost bidders choosing $(\overline{p}, q(s))$. The procurer only selects a bidder choosing $(\overline{p}, q(s))$ if no bidder chooses $\underline{p}_n$. In the case of a tie, the procurer randomly selects a winner. We now provide some intuition for the formulas determining $\{\underline{p}_n; \overline{p}, q(s)\}$. An appendix contains the proof of the main theorem that fully characterizes the optimal contract.

Since bidders reveal their type through their choice of a contract in equilibrium, and the probability that a low-cost contractor bids is $1-(1-\pi)^n$, the expected transfer to a winning contractor is:

$$[1-(1-\pi)^n]\,\underline{p}_n+(1-\pi)^n\left[\overline{p}+\int q(s)\overline{f}(s)ds\right]. \tag{1}$$

Appealing to the revelation principle the procurer is limited to choosing $\underline{p}_n$ and $(\overline{p},q(s))$ subject to three constraints: that both contractor types are willing to bid if presented with the opportunity, called individual rationality, and that the contract menu induces the low-cost contractor to reveal his true type, called incentive compatibility.[12]

Individual rationality and incentive compatibility. The individual rationality constraints for the two types are:

$$\underline{p}_n\geq\alpha, \tag{2}$$

$$\overline{p}+\int\psi[q(s)]\overline{f}(s)ds\geq\alpha+\beta. \tag{3}$$

To derive the incentive compatibility constraint for the low-cost type, we first compute the probability of a bidder winning if he chooses the fixed contract when the $n-1$ bidders follow their equilibrium strategy, which is:

$$\underline{\phi}_n\equiv\sum_{k=0}^{n-1}\binom{n-1}{k}\frac{\pi^k(1-\pi)^{n-1-k}}{k+1}=\frac{1}{n\pi}\sum_{j=1}^{n}\binom{n}{j}\pi^j(1-\pi)^{n-j}=\frac{1-(1-\pi)^n}{n\pi}. \tag{4}$$

If the bidder chooses the variable contract instead, the probability of winning is:

$$\overline{\phi}_n\equiv\frac{(1-\pi)^{n-1}}{n}. \tag{5}$$

Thus a low-cost contractor prefers $\underline{p}_n$ to $(\overline{p},q(s))$ if and only if:

$$\underline{\phi}_n\{\underline{p}_n-\alpha\}\geq\overline{\phi}_n\{\overline{p}+\int\psi[q(s)]\underline{f}(s)ds-\alpha\}. \tag{6}$$

Interior solution. We show below that given $\left(\alpha,\pi,\underline{F}(s),\overline{F}(s)\right)$ the low-cost contractor makes strictly positive rents from the optimal menu if the cost differential is high enough; at the optimum $\underline{p}_n>\alpha$ for big β. Moreover $q(s)>M$ for all $s\in S$ providing $|M|$ is sufficiently large. Because much of the intuition for the solution to the optimal menu comes from the interior solution obtained from a simpler related problem, that is ignoring the constraints that $\underline{p}_n\geq\alpha$ and $q\geq M$ altogether, we now temporarily choose

[12]We show that the menu of contracts that satisfy the three conditions, IR's for both types and IC for the low-cost type, automatically satisfies the incentive compatibility condition for high-cost contractors.

$\underline{p}_n$ and $(\overline{p}, q(s))$ to minimize (1) subject to (3) and (6) alone. To further elaborate, define $l(s) \equiv \underline{f}(s)/\overline{f}(s)$ and $h(\psi'(q)) \equiv q$. In words $l(s)$ is the likelihood ratio, and $h : \mathcal{R}^+ \to \mathcal{R}$ is the inverse of the first derivative of $\psi(q)$. If an interior solution to this simpler minimization problem exists, the remaining two constraints, (3) and (6) are met with equality. Given $q(s)$, the two unknowns $\overline{p}$ and $\underline{p}_n$ in the constraints can then be solved directly, yielding:

$$\underline{p}_n = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta + \int \psi[q(s)]\,[l(s)-1]\,\overline{f}(s)ds\right) \equiv \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}(\beta-\gamma), \tag{7}$$

$$\overline{p} = \alpha + \beta - \int \psi[q(s)]\overline{f}(s)ds. \tag{8}$$

Substituting the solutions for $\overline{p}$ and $\underline{p}_n$ into (1) concentrates the procurer's objective function to:

$$\alpha + (1-\pi)^{n-1} \int \{\beta + [\pi l(s) - 1]\,\psi[q(s)] + (1-\pi)q(s)\}\,\overline{f}(s)ds.$$

Rearranging the first order condition for $q(s)$ yields:

$$q(s) = h\left[\frac{1-\pi}{1-\pi l(s)}\right]. \tag{9}$$

In equilibrium the value of fixed contracts decline with the number of bids, but regardless of the number of bids, the expected utility of high-cost contractors is their reservation value of losing the procurement auction. Indeed the variable contract does not depend on the number of bidders. As can be seen in (8), $\overline{p}$ does not depend on n. Since n does not differentially affect the expressions containing $q(s)$ inside the integral, the optimal choice of the variable component does not depend on n either, as can be seen in (9).

Since $h(1) = 0$ and its derivative is negative, it now follows from (9) that $q(s) \gtreqless 0$ as $l(s) \lesseqgtr 1$ with $q(s) = 0$ if and only if $l(s) = 1$; that is if s is more likely to be generated by a high-cost contractor than a low-cost one, then the variable component for s is positive, and vice-versa. Comparing the expected transfers for the two types:

$$\underline{p}_n < \alpha + \beta - \gamma < \alpha + \beta < \alpha + \beta + \int \{q(s) - \psi[q(s)]\}\,\overline{f}(s)ds \equiv \alpha + \beta + r, \tag{10}$$

proving that procuring a low-cost contractor is cheaper than a high-cost one. Note that the first inequality results from two observations: $\pi(1-\pi)^{n-1}/\{1-(1-\pi)^n\} < 1$ for $n > 1$ and the IR condition for the low-cost contractors. These two observations guarantee that $\beta - \gamma > 0$. The second inequality results from the IC condition for the low-cost contractors when $n = 1$, and the last inequality from $q > \psi(q)$ for all $q \neq 0$.

We interpret r as the risk premium paid to a high-cost contractor to take a risky contract that deters a low-cost contractor, and γ as the amount extracted from a low-cost contractor when there is only one bidder. These inequalities demonstrate that when solving the optimal menu problem the procurement officer balances the gains of extracting rent from the low-cost contractor with the losses of the risk premium she pays to a high-cost contractor. Substituting (7), (8) and the definition of r in (10) into (1) yields:

$$\alpha + (1-\pi)^{n-1}\left[\beta - \pi\gamma + (1-\pi)r\right] \equiv \alpha + (1-\pi)^{n-1}\left[\beta + \Gamma\right]. \tag{11}$$

Thus Γ is the expected net benefit from using the signal when there is only one bidder. If the signal was useless, meaning $\underline{F}(s) = \overline{F}(s)$ and $l(s) = 1$ for all $s \in S$, then $\Gamma = 0$ and the optimal menu reduces to a menu of two fixed contracts. This menu consists of one preferred contract, $\alpha + \pi(1-\pi)^{n-1}\beta/[1-(1-\pi)^n]$, which is (7) with $r = 0$, and a default contract of $\alpha + \beta$, which is selected only if no bidder chooses the preferred lower price contract. Thus Γ is bounded above by zero. We show in the appendix that Γ is strictly negative whenever $\underline{F}(s)$ and $\overline{F}(s)$ are not identical. As n increases the value of using a signal falls to zero, and competition between bidders and the menu converges to the two fixed contracts. In this way the simplified problem captures a basic intuition permeating through our analysis: in her quest to extract rent from low-cost bidders when faced with the constraint of having to accept a high-cost bidder as a last resort, the procurement officer uses signals in contract menu design to discriminate between different types of bidders, potentially as a partial substitute to increasing the number of bidders.

The optimal menu. To complete the optimal menu design we now impose the additional constraints, that $\underline{p}_n \geq \alpha$ and $q \geq M$, and formally state the theorem. The motivation for a maximal penalty arises to finesse situations where it might be optimal to impose a steep penalty on a high-cost winner in the event of a very unlikely outcome when $l(s)$ is very high in order to achieve an outcome very close to first best; in practice this framework would be hard to distinguish from a full information procurement auction in which low-cost contractors are awarded α and high-cost contractors $\alpha + \beta$. Given some finite upper bound on the penalty M, we define a threshold likelihood ratio as:

$$\tilde{l}(\pi, M) \equiv \frac{1}{\pi} - \frac{1-\pi}{\pi\psi'(M)}.$$

We prove in the appendix that the $q(s)$ defined by the first order condition (9) is optimal if and only if $l(s) \leq \tilde{l}(\pi, M)$, while M is optimal for $l(s) > \tilde{l}(\pi, M)$.

The participation constraint for the low-cost contractors, (2), may or may not be binding at the optimum. To preserve (6), the procurer faces a trade off between surrendering more surplus to low-cost bidders, $\underline{p}_n - \alpha$, versus paying higher risk premia to high-cost winners, r. As π increases, the probability of paying a risk premium declines and the probability of surrendering surplus increases. This prompts the procurer to offer menus with less attractive fixed contracts by reducing $\underline{p}_n$, and more volatile variable contracts and simultaneously increasing r. Indeed for sufficiently high values of π, it may be optimal to reduce $\underline{p}_n$ to α, and eliminate all surplus to the low-cost contractors by loading the entire loss associated with incentive compatibility on to r, which is then paid out infrequently to high-cost winners. In this case (2) is met with equality. We show that there exists a threshold of π, denoted by $\tilde{\pi}$, at or above which the participation constraint for the low-cost contractors binds, defined as the unique root to:

$$\beta - \int_{l(s)<\tilde{l}(\pi,M)} \psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)[\overline{f}(s)-\underline{f}(s)]ds - \psi(M)\int_{l(s)\geq\tilde{l}(\pi,M)}[\overline{f}(s)-\underline{f}(s)]ds. \tag{12}$$

Theorem 3.1. *The optimal menu of contracts consists of two contracts, a fixed contract* $\underline{p}_n$ *defined in* (7)*, and a variable contract denoted by* $(\overline{p},\widetilde{q}(s))$*, where* $\overline{p}$ *is defined in* (8) *and:*

$$\widetilde{q}(s) = \begin{cases} h\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-\min\{\pi,\tilde{\pi}\}l(s)}\right) & \text{if } l(s) \leq \tilde{l}(\min\{\pi,\tilde{\pi}\},M), \\ M & \text{if } l(s) > \tilde{l}(\min\{\pi,\tilde{\pi}\},M). \end{cases}$$

3.5. **Type and Extent of Competition.** To implement the project, at least one contractor must bid. This is our point of departure. The procurer decides whether or not to solicit extra bids. If she chooses to hold a competitive solicitation procedure (as opposed to awarding a contract to a default contractor), she also chooses the level of solicitation, denoted by $\lambda \in \mathcal{R}^+$, which is the arrival rate of a Poisson probability distribution for the extra number of bids. We now discuss how these two choices affect the type and extent of competition.

Competitive solicitation. When procurement is competed, more than one bidder may participate. To solicit and process each bid is costly. Let $\kappa > 0$ denote such cost per bid. To solicit one additional bid, the procurer may conduct market research and post an early advertisement or a pre-solicitation notice before posting a formal solicitation. Furthermore, the cost of delaying the project start date to solicit more bids is not negligible. This point is related to the fact that "urgency" is a frequently cited reason for no competition, as can be seen in Table 2. The cost of processing an additional bid includes the cost of reading a proposal, clarifying ambiguous language in the proposal about undertaking the project, and assessing the attributes of the bidding contractor. In addition to these various administrative costs of bid solicitation and processing, broadening the pool of potential bidders can be costly because this may necessitate a large compromise in quality. This may explain why "only one available source' or "follow-on contract" are valid reasons for no competition.

The expected total cost of competed procurement, denoted by $U(\lambda)$, is the sum of transfers and extra bid processing costs, integrated over the number of extra bidders:

$$U(\lambda) \equiv \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} \left\{ \kappa n + \left[1 - (1-\pi)^{n+1}\right] \underline{p}_{n+1} + (1-\pi)^{n+1} \left[\overline{p} + \int q(s) \overline{f}(s) ds \right] \right\}.$$

Substituting the expressions for $\underline{p}_{n+1}$ and $\overline{p}$ into $U(\lambda)$, and integrating over n yields an expression for $U(\lambda)$, which is maximized over λ to obtain the optimal solicitation rate, which we denote by λ_c. If a sufficiently high proportion of contractors are low-cost, or conversely high-cost, the procurer does not solicit extra bids; at $\pi = 0$ and at $\pi = 1$ they are all the same. Within the midrange of π, she may optimally solicit extra bids providing processing costs are sufficiently low. Two countervailing forces affect how λ shifts in response to increments in π: on the one hand, the marginal value of an extra bidder increases because there is a higher likelihood of attracting a low-cost contractor; on the other hand, fewer resources on soliciting extra bids are required to attract the same expected number of low-cost contractors.

Lemma 3.1.

$$U(\lambda) = \alpha + \kappa\lambda + e^{-\lambda\pi}(\beta + \Gamma). \tag{13}$$

Define real valued function λ^* *on* $[0, 1]$ *as:*

$$\lambda^* = \frac{1}{\pi} \left[\ln(\pi) + \ln(\beta + \Gamma) - \ln(\kappa) \right] \tag{14}$$

Then $\lambda_c = \lambda^*$ *if* $\lambda^* > 0$ *and* $\lambda_c = 0$ *if* $\lambda^* \leq 0$.

Choosing between competed and non-competed contract menus. Let η denote the cost of awarding a contract to a default contractor without competition after setting aside differences in transfers and bid solicitation and processing costs. The sign of η can be negative or positive, depending on the strength of administrative and political considerations, including for example (the negative of) bribery, protests from excluded bidders, reputation side effects (positive from corruption, or negative from selecting favorites on the basis of efficiency or quality). The expected total cost of non-competed procurement, denoted by U_0, is the sum of direct costs from not holding a competitive solicitation and the expected amount of transfer. Appealing to (13):

$$U_0 \equiv U(0) + \eta = \alpha + \beta + \Gamma + \eta.$$

The procurer chooses to hold a competitive solicitation if and only if the expected total procurement cost is lower with the formal solicitation procedure than without it, that is if and only if $U_0 \geq U(\lambda_c)$. If $\lambda^* \leq 0$, then the choice reduces to the sign of η. Alternatively if $\lambda^* > 0$, then appealing (14), a competitive solicitation is chosen if and only if:

$$\eta \geq \frac{\kappa}{\pi}\left[1 + \ln(\pi) + \ln(\beta + \Gamma) - \ln(\kappa)\right] - \beta - \Gamma. \tag{15}$$

4. Identification

Since our empirical work models costs (α, β, κ) to depend on π, we now express them as $\alpha(\pi)$, $\beta(\pi)$ and $\kappa(\pi)$. Similarly we make explicit the dependence of $\underline{p}_n$, $\overline{p}$, $q(s)$ and γ by writing $\underline{p}_n(\pi)$, $\overline{p}(\pi)$, $q(s, \pi)$ and $\gamma(\pi)$ respectively. Thus the primitives of the econometric structure comprise the distribution of the proportion of the low-cost contractors, $F_\pi(\pi)$, expected contractor costs, $\alpha(\pi)$ and $\beta(\pi)$, the cumulative distribution function of factors affecting the procurer's decision about whether to allow competition, denoted by $F_\eta(\eta)$, the costs of solicitation $\kappa(\pi)$, and finally the distribution function of signals from both types of contractor, $\underline{F}(s)$ and $\overline{F}(s)$.

We seek to identify the primitives of the model from observations on: whether the contract was competed, which we denote by setting $c = 1$, or not (setting $c = 0$); how many bids were tendered, n; whether the winning bid is a variable contract, denoted by setting $v = 1$, or not (setting $v = 0$); the signal s, observed regardless contract type; the fixed price $\underline{p}_n$ if the winning contract is a fixed contract; and the base price $\overline{p}$ and the ex-post unilateral price adjustment q if the winning contract is variable. Note that we separately observe price changes arising from bilateral contract modifications, ϵ and q.

After conditioning on observed project characteristics, such as industry and project characteristics, the assumption that π is a constant is strongly rejected: it is identified off the proportion of variable contracts $(1 - \pi)^n$, and thus over-identified from variation in n. For this reason we treat π is an unobserved continuous variable filtering through the equilibrium analysis and complicating identification.

To preserve tractability, our empirical analysis makes the following two assumptions about the unobserved variables:

A1: $s \perp (\pi, \eta)$ and $\eta \perp \pi$.

A2: $F_\pi(\pi)$ is strictly increasing for all $\pi \in \Pi$.

We also simplify the analysis by restricting the parameter space so that an interior solution invariably attains, meaning neither the individual rationality constraint for the low-cost contractor nor the maximal penalty constraint bind. In addition we assume that as the proportion of the low-cost contractors increases, the expected cost of the project to either type declines.

A3: $\Pi \subset (0, \min\{\tilde{\pi}, 1\})$, and $l\,(s) \leq \widetilde{l}(\pi, M)$ for all $(s, \pi) \in S \times \Pi$.
A4: $\alpha(\pi)$ and $\beta(\pi)$ are nonincreasing in π.
A5: $\alpha(\pi)$ and $\beta(\pi)$ satisfy the following inequality for all π:

$$\frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \frac{h'\,[\psi'(M)]\,[\psi'(M)]^2[1 - \psi'(M)]}{\pi(1 - \pi)} < 0.$$

In identification we exploit three monotonicity properties exhibited by the optimal menu. Lemma 4.1 shows both the absolute value of the variable component and the rent extracted from the low cost contractor, is increasing in the proportion of the low-cost contractors. Intuitively, the expected cost of paying a higher risk premium on high-cost contractors declines with the probability of not finding a low-cost contractor, leading to a cheaper fixed contract on the optimal contract menu.

Lemma 4.1. *If **A3** holds then* $\partial\,|q(s;\pi)|\,/\partial\pi > 0$ *and* $\partial\gamma(\pi)\,/\partial\pi > 0$.

Lemma 4.2 shows the fixed contract declines in π. Holding signaling distributions $\underline{F}(s)$ and $\overline{F}\,(s)$ fixed, the procurer unambiguously ranks projects by π. Intuitively, the cost of paying a higher risk premium to high-cost contractors declines with the probability of not finding a low-cost contractor, and this saving is enhanced when **A4** holds:

Lemma 4.2. *If **A3** and **A4** hold then* $\partial \underline{p}_n(\pi)\,/\partial\pi < 0$ *for all* $n \in \{1, 2, \ldots\}$.

For identification purposes, we require $\overline{p}$ to be monotone in π. On the one hand $\overline{p}$ might increase as $|q|$ increases with π for given s (in order to reduce $\underline{p}_n$ without violating the incentive compatibility constraint of the low cost contractor), so that the individual participation constraint of the high cost contractor is maintained. On the other hand, **A5** suffices to prove $\overline{p}$ is monotone decreasing in π. In this case as π increases, a larger portion of the payment is shifted to the variable component and away from the base component.

Lemma 4.3. *If **A3** and **A5** hold then* $\partial\overline{p}\,(\pi)\,/\partial\pi < 0$.

4.1. **Liquidity Cost Function.** Identification of $\psi(q)$ is based on the rate at which q decreases as $l(s)$ increases for any given π, an equation derived from totally differentiating the first order condition with respect to q and l. Since π is unobserved, this equation is redefined in terms of $\overline{p}$, which we assume is monotone in π; we write $\pi = \pi(\overline{p})$ and rearrange the first order condition to define:

$$l^*(q,\overline{p}) \equiv \frac{1}{\pi(\overline{p})} - \frac{1-\pi(\overline{p})}{\pi(\overline{p})\psi'(q)} \text{ with } \frac{\partial l^*(q,\overline{p})}{\partial q} = \left[\frac{1-\pi(\overline{p})}{\pi(\overline{p})}\right]\frac{\psi''(q)}{[\psi'(q)]^2} < 0. \tag{16}$$

Since s only enters the optimal contract only through the likelihood ratio we can summarize outcomes of variable contracts in terms of $(\overline{p},q,l)$ rather than $(\overline{p},q,s)$, where $l = l(s)$. Formally:

Lemma 4.4. *If **A5** holds, then $(\overline{p},q,l) = (\overline{p},q,l^*(q,\overline{p}))$ for all variable contract outcomes $(\overline{p},q,s)$ and for all $l^*(q,\overline{p}) \neq 1$:*

$$\psi''(q) = \left[\frac{1-\psi'(q)}{1-l^*(q,\overline{p})}\right]\psi'(q)\frac{\partial l^*(q,\overline{p})}{\partial q} \tag{17}$$

The first order ordinary differential equation (17) can be uniquely solved in $\psi'(q)$ with normalizing constant $\psi'(0) = 1$. It now follows that $\psi(\cdot)$ is solved from the other normalizing constant for the liquidity cost function, that $\psi(0) = 0$. Since $l^*(q,\overline{p})$ is identified off variable cost contract outcomes $(\overline{p},q,l)$, so is $\psi(q)$.

4.2. **Project-related Costs for Contractors.** We identify project contracting costs, $\alpha(\pi)$ and $\beta(\pi)$, by exploiting $\overline{p}^*(q,s)$, as well as an identified mapping for $\underline{p}_n$, denoted by $\underline{p}_n^*(\pi,c)$. The derivation of $\underline{p}_n^*(\pi,c)$ is based on Lemma 4.2, that $\underline{p}_n$ its monotone (declining) in π for each $c \in \{0,1\}$. Let $f_{\pi|c,n,v}(\pi\,|c,n,v)$ denote the probability density function of π conditional on (c,n,v), and denote by $G_{\underline{p}_n|c}(\cdot\,|c)$ the cumulative distribution function for $\underline{p}_n$ conditional on whether or not the contract is competed, c. Since $\underline{p}_n$ is strictly declining in π, we may define $\underline{p}_n^*(\pi,c)$ as:

$$\underline{p}_n^*(\pi,c) \equiv G^{-1}_{\underline{p}_n|c}\left(\int_{\pi}^{\pi_{\max}} f_{\pi|v,n,c}(x\,|0,n,c)\,dx\middle|c\right). \tag{18}$$

Since $\psi'(\cdot)$ is identified, so is π corresponding to each variable contract $(\overline{p},q,s)$ defined through the first order condition (9) by:

$$\pi_{q,s} \equiv \frac{1-\psi'[q(s)]}{1-l(s)\psi'[q(s)]}.$$

Conditioning on c and n, we interpret $\pi_{q,s}$ as a random draw from the $f_{\pi|c,n,v}(\pi\,|c,n,1)$ probability density. The integrand in (18) is linked to $f_{\pi|c,n,v}(\pi\,|c,n,1)$ by the conditional probability of a high-cost contractor winning given π and n and the odds ratio related to contract types conditional on the mode of competition and the number of bids, defined as:

$$\varphi_{c,n} \equiv \Pr(v=1|c,n)\,/\Pr(v=0|c,n)\ .$$

That linkage yields a representation for $\underline{p}_n^*(\pi,u)$ in terms of $\pi_{q,s}$ draws, $G_{\underline{p}_n|c}(\cdot\,|c)$ and $\varphi_{u,n}$, all identified objects, thus ensuring $\underline{p}_n^*(\pi,c)$ is too.

Lemma 4.5. *If **A1** through **A4** hold, then* $\underline{p}_n = \underline{p}_n^*(\pi,c)$ *for all outcomes of a fixed contract* $\left(n,\underline{p}_n\right)$*, and*

$$\underline{p}_n^*(\pi,c) \equiv G^{-1}_{\underline{p}_n|c}\left(\varphi_{c,n}E_{\pi_{q,s}|v,n,c}\left[1\left\{\pi_{q,s}\geq\pi\right\}\frac{1-\left(1-\pi_{q,s}\right)^n}{\left(1-\pi_{q,s}\right)^n}\middle|1,n,c\right]\middle|c\right).$$

To identify $\alpha(\pi)$ and $\beta(\pi)$ we substitute $\underline{p}_n^*(\pi,c)$ for $\underline{p}_n$ and $\overline{p}^*(q,s)$ for $\overline{p}$ in (7) and (8) for any s and any $n\in\{2,3,\ldots\}$, rearrange the resulting expressions and substitute out q using (9) to obtain:

$$\alpha(\pi) = \frac{1-(1-\pi)^n}{1-(1-\pi)^{n-1}}\underline{p}_n^*(\pi,c) - \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^{n-1}}\underline{p}_1^*(\pi,c), \tag{19}$$

$$\beta(\pi) = \overline{p}^*\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right],s\right)+\int\psi\left(h\left[\frac{1-\pi}{1-\pi l(t)}\right]\right)\overline{f}(t)\,dt-\alpha(\pi). \tag{20}$$

Thus $\alpha(\pi)$ and $\beta(\pi)$ are both identified from (19) and (20).

In fact, these derivations establish some over-identifying restrictions. First, the optimal setting of $\underline{p}_1$ does not depend on whether competition is restricted or not, $\underline{p}_1^*(\pi,0)=\underline{p}_1^*(\pi,1)$. Note that this equality has empirical content as $G_{\underline{p}_n|c}(\cdot\,|0)\neq G_{\underline{p}_n|c}(\cdot\,|1)$ due to selection issues. Furthermore varying $n\in\{2,3,\ldots\}$ in (19) and (20) yields further testable restrictions. Finally setting $n=1$ in (7) and (8), the binding constraints used to derive $\overline{p}$ and $\underline{p}_n$, substituting out the parameters α and β , which enter only as the sum $\alpha+\beta$ in both expressions when $n=1$, and then substituting $\overline{p}^*(q,s)$ for $\overline{p}$ and $\underline{p}_1^*(\pi,c)$ for $\underline{p}_1$ yields:

$$\int\psi\left(h\left[\frac{1-\pi}{1-\pi l(t)}\right]\right)\underline{f}(t)\,dt=\underline{p}_1^*(\pi,c)-\overline{p}^*\left[\frac{1-\pi}{1-\pi l(s)},s\right]. \tag{21}$$

Varying π in (21) provides further over-identifying information for $\psi(q)$.

4.3. **Costs and Benefits Borne by Procurer.** Variation in the number of bids partially identifies the solicitation cost function $\kappa(\pi)$ in competitive procurement auctions. Whether or not the procurer holds a competitive auction partially identifies the cumulative distribution function for the net cost of awarding a contract to a default contractor without competition, $F_\eta(\eta)$. To emphasize its dependence on π, let $\lambda(\pi)$

denote the optimal solicitation rate in competed contracts. It is the expected number of bids generated by a Poisson process, solved in terms of the underlying densities $f_{\pi|c}\left(\pi\,|1\right)$ and $f_{\pi,n|c}(\pi,n+1\,|1)$.

$$\lambda(\pi)=\sum_{n=0}^{\infty}\frac{nf_{\pi,n|c}(\pi,n+1\,|1)}{f_{\pi|c}\left(\pi\,|1\right)}, \tag{22}$$

Note that $f_{\pi|c}\left(\pi\,|1\right)$ and $f_{\pi,n|c}(\pi,n{+}1\,|1)$ are derived from $\Pr\left[v\,|n,c\right]$ and $f_{\pi|v,n,c}\left(\pi\,|v,n,c\right)$, whose identification has been established in the proof of Lemma 4.5.

Write $\Gamma(\pi)$ to express the dependence of Γ on π, defined in (11). If $\lambda(\pi)=0$, then a lower bound from $\kappa(\pi)\geq\pi\Gamma(\pi)$ is obtained. Otherwise $\lambda(\pi)>0$ and we rearrange (14) to obtain:

$$\kappa\left(\pi\right)=\pi\Gamma(\pi)\exp\left[-\pi\lambda(\pi)\right]. \tag{23}$$

Finally $F_{\eta}\left(\cdot\right)$, the distribution for the unobserved variable affecting contract type, is partially identified from procurer choices, through variation in π transmitted through the identified costs to both parties. The procurer chooses not to hold a competitive solicitation if the expected total cost of doing so is less than or equal to the alternative. Let:

$$\Omega(\pi)\equiv\frac{\kappa(\pi)}{\pi}\left\{1+\ln\left(\pi\right)+\ln\left[\Gamma(\pi)\right]-\ln\left[\kappa(\pi)\right]\right\}-\Gamma(\pi).$$

The optimal choice rule is that a contract is not competed if and only if $\eta\leq\Omega(\pi)$. Note that from $f_{\pi|c,n,v}\left(\pi\,|c,n,v\right)$, we identify $\Pr(c=0|\pi)$ for $\pi\in\Pi$. Therefore, by exploiting the variation in π, we identify $F_{\eta}(\eta)$ for $\eta\in\Omega^{*}\equiv\left\{\Omega\left(\pi\right):\kappa(\pi)<\pi\Gamma(\pi)\right\}$ and $\eta=0$. Specifically, if $\kappa(\pi)<\pi\Gamma(\pi)$, then $\Pr(c=0|\pi)=F_{\eta}[\Omega(\pi)]$; otherwise, $Pr(c=0|\pi)=F_{\eta}(0)$.

4.4. **Summary Statement on Identification.** Taken together, the six lemmas in this section provide the critical arguments for establishing the model is identified. Summarizing, denote the set of models under consideration by:

$$\Theta\equiv\left\{F_{\pi}\left(\pi\right),F_{\eta}\left(\eta\right),\underline{F}_{s}(s),\overline{F}_{s}(s),\psi\left(q\right),\alpha\left(\pi\right),\beta\left(\pi\right),\kappa\left(\pi\right)\right\}.$$

Suppose the data set, denoted by I, comprises observations:

$$i\equiv\{c,n,v,s,v\overline{p},vq,(1-v)\,\underline{p}_{n}\}.$$

Assume each $i\in I$ is generated by an independent draw of $(\pi,\eta,s,\tau,\varepsilon)$ from a particular element in the set $\theta\in\Theta$. The following theorem encapsulates the results of this section by completing the remaining arguments used to establish identification:

Theorem 4.1. *If **A1** through **A5** hold, then $\theta \in \Theta$ is identified by the process generating observations $i \in I$.*

5. Parameterization of the Model and Estimation

We assume that the distribution of π is $Beta(\alpha_\pi, \beta_\pi)$ on $(\pi_{\min}, \pi_{\max})$. The extra net cost of bypassing the formal solicitation procedure, η, is assumed to follow $N(\mu_\eta, \sigma_\eta)$ with $\sigma_\eta > 0$. The distribution of the signal for the low-cost contractors is:

$$\underline{F}_s(s) = \begin{cases} \underline{\rho} & \text{if } s = 0, \\ (1-\underline{\rho})\underline{G}(s) & \text{if } s > 0, \end{cases}$$

where $\underline{G}(s)$ is the CDF of a Gamma distribution with shape parameter $\underline{\alpha}_s > 0$ and scale parameter $\underline{\beta}_s > 0$. The counterpart for the high-cost contractors is similarly assumed:

$$\overline{F}_s(s) = \begin{cases} \overline{\rho} & \text{if } s = 0, \\ (1-\overline{\rho})\overline{G}(s) & \text{if } s > 0, \end{cases}$$

where $\overline{G}(s)$ is the CDF of a Gamma distribution with shape parameter $\overline{\alpha}_s > 0$ and scale parameter $\overline{\beta}_s > 0$.

We assume that the contractor costs, α for the low-cost contractors and $\alpha + \beta$ for the high-cost contractors, are linear in π.

$$\begin{aligned} \alpha(\pi) &= \alpha_0 + \alpha_1 \pi, \\ \beta(\pi) &= \beta_0 + \beta_1 \pi, \end{aligned}$$

where the parameters guarantee that the costs are always positive regardless of the value of π. Furthermore, we assume that $\alpha_1 \leq 0$, and $\beta_1 \leq 0$ in order to use the monotonicity result of Lemmas 4.2 and 4.3.

We also assume that the per-bid solicitation and processing cost κ is quadratic in π.

$$\kappa(\pi) = \kappa_0 + \kappa_1(\pi + \kappa_2)^2.$$

We restrict our attention to nonnegative bid solicitation and processing costs for all π.

Lastly, we assume that the maximal penalty is a constant, denoted by γ, and we assume $\psi(\cdot)$ takes the parametric form:

$$\psi(q) = -\psi_0 \exp(-q/\psi_0) + \psi_0,$$

where $\psi_0 > 0$.

We estimate the parameters of the model by an efficient simulated GMM estimator. The moment conditions are motivated by the identification argument: the joint probabilities regarding entry restrictions, number of bids, and contract type; some moments of the joint distribution of signals and contract type; and the quantiles of contract prices conditional on contract type and number of bids. Under certain regularity conditions, our estimator is asymptotically normally distributed, and the standard errors are based on the asymptotic variance. In Appendix, we define the estimator and provide more details on the estimation procedure.

6. Results

6.1. **Scope of the Analysis.** For our analysis, we focus on definitive contracts that initiated during FY2004–2012 and resulted in the government obligation of $300,000 or more as of the end of FY2014.[13] Our sample selection rules are (i) the procurer either solicited bids for full and open competition or restricted entry for discretionary reasons, other than regulatory or small business concerns, (ii) the contract terms were negotiated and the goods/services for the contract are commercially unavailable, (iii) the project was completed before FY2014 and performed in the U.S. continent, (iv) the expected duration of the contract is at least 30 days, and (vi) without any inconsistent records on the contract. There are in total 37,186 such contracts in the data, costing the government $331 million in total.

We further restrict our attention to contracts of certain *sub-sectors*. We define a *sub-sector* by the first two digit of the product/service code and whether or not the contracting agency is related to defense.[14] We narrow down to the industries with at least 200 fixed and variable contracts respectively in the data. There are 14 *sub-sectors* that satisfies this criterion, including 14,844 individual contracts with $153.3 billion in 2010 dollars in total.

6.2. **Definition of the Variables.** Given the observed variables in the data, we define the variables of the model as follows. We define contract i is restricted ($c_i = 0$) if there was no or limited solicitation procedure and only one bidder was considered. The definition of the contract type, fixed ($v_i = 0$) or variable ($v_i = 1$), relies on whether or not there was a unilateral modification. For fixed contracts or those with bilateral

[13]There are 209,575 such contracts in the data, in total $1.5 trillion in 2010 dollars.

[14]These product/service codes are used to record the products and services being purchased by the federal government. They are periodically updated, and the manual for the codes can be found online. We consider the following agencies are military-related: Department of State, Department of Homeland Security, National Aeronautics and Space Administration (NASA), Defense Nuclear Facilities Safety Board, and Department of Defense.

Table 8. Summary Statistics: IT and Telecommunications Service

	Civil			Military		
	Mean	SD	Median	Mean	SD	Median
Prices ($K, 2010)						
Base price	744.56	796.86	472.49	921.26	980.68	497.32
Final price	1,387.09	2,247.00	778.53	2,390.71	4,377.99	1,179.12
Unilateral price change	393.00	670.35	0.00	463.80	775.74	110.92
Bilateral price change	249.53	1,830.96	0.00	1,005.64	4,029.80	0.00
Duration (Days)						
Base duration	420.72	382.73	364	406.15	373.61	364
Final Duration	746.07	709.21	379	756.56	609.75	551
Unilateral duration change	263.94	481.69	0.00	215.47	397.38	0.00
Bilateral duration change	70.98	253.26	0.00	142.69	398.58	0.00
Contract type						
Fixed-price	0.45	0.49	0	0.37	0.48	0
Competition						
Entry restrictions	0.63	0.48	1	0.72	0.44	1
Number of bids	2.40	6.25	1	1.55	1.82	1

Note: These statistics are based on 718 IT and telecommunications service contracts and 325 military ones, all of which satisfy the sample selection rule described in the text. In this table, we present the summary statistics for 563 civil and 237 military IT and telecommunications service contracts whose total payment excluding the bilateral price change ranges between 0.3 and 5 million 2010 dollars.

modifications only, the difference in the final and the base prices is denoted by ϵ_i. For variable contracts of those with unilateral modifications, the base price is $\overline{p}_i$, the sum of all changes in the price related to unilateral modifications is q_i, and the difference between the final price and $\overline{p}_i + q_i$ is ϵ_i. We consider the length of delay related to unilateral modifications, divided by the base duration of the contract, as an observed signal, s_i. If the final duration is shorter than the expected duration, $s_i = 0$.

Our analyses can be used for each of the 14 *sub-sectors*, but we perform our analyses using the IT and telecommunications service contracts. We present the summary statistics of the variables for such contracts whose total transfer excluding bilateral price changes being in the range of $0.3 and $5 million in 2000 dollars in our sample in Table 8.

6.3. **Parameter Estimates.** The parameter estimates for the civl IT and telecommunications service contracts are presented in Table 9. The average proportion of low-cost contractors is estimated to be 0.32, and here we provide the estimation results evaluated at the average value of the proportion of low-cost contractors. The results indicate that absent cost shocks, it takes $0.8 million for efficient contractors to complete a procurement project, while it takes $2.4 million for inefficient contractors.

TABLE 9. Parameter Estimates

Description	Parameter	Estimate	Parameter	Estimate
Low project cost (in $M)	α_0	2.2281	α_1	-3.6546
		(0.0962)		(0.5071)
Extra project cost (in $M)	β_0	0.7804	β_1	-1.3895
		(0.0789)		(0.3699)
Bid cost (in $M)	κ_0	-0.0576	κ_1	0.7500
		(0.0246)		(0.1944)
	κ_2	0.1000		
		(0.0255)		
Maximum penalty (in $M)	γ_0	-0.4506		
		(194.81)		
Liquidity cost (in $M)	ψ_0	7.1882		
		(9.2736)		
Distribution of π	α_π	5.5110	β_π	2.5814
		(0.9882)		(0.7684)
	$\pi_{\max}$	0.4940		
		(0.0765)		
Distribution of η	μ_η	-0.0170	σ_η	0.0142
		(0.0082)		(0.0025)
Distribution of signal	$\underline{\rho}$	0.9131	$\overline{\rho}$	0.3757
		(0.1618)		(0.1160)
	$\underline{\alpha}_s$	1.3070	$\underline{\beta}_s$	0.4482
		(3.9394)		(0.8172)
	$\overline{\alpha}_s$	0.8857	$\overline{\beta}_s$	4.4350
		(0.6301)		(2.8071)

Note: The estimates are based on the civil IT and telecommunications service contracts in our sample. Numbers in parentheses are asymptotic standard errors.

The per-bidder bid solicitation and processing cost is $51,080, which is 3.1% of the cost differential between the low-cost and the high-cost contractors. This cost may include administrative cost as well as the opportunity cost of having to hire a potentially low-quality contractor by broadening the pool of qualified contractors.

Our estimates indicate that the average *private* monetary net benefit of bypassing the formal solicitation procedure is $22,200. The benefits may result from saving administrative costs, selecting favorites on the basis of efficiency or quality, and receiving bribery, while the costs may include the expected cost of protests from excluded bidders. We consider this value as an upper bound of corruption assuming the *private* cost of bypassing the formal solicitation procedure is negligible.

Using the estimated parameters, we simulate the data and calculate some key moments displayed in Table 10. The overall fit of the simulated data to the actual data is good in both the level and the trend. The table shows the actual and predicted

Table 10. Model Fit

	Observed	Predicted
Probability of		
Entry restriction	0.6306	0.6450
One bid conditioning on competition	0.2356	0.2834
Up to two bids conditioning on competition	0.3510	0.4558
Up to five bids conditioning on competition	0.8317	0.8208
Fixed contracts conditioning on entry restriction	0.4366	0.3628
Fixed contracts conditioning on one bid	0.4490	0.4135
Fixed contracts conditioning on up to two bids	0.4110	0.4438
Fixed contracts conditioning on up to five bids	0.4971	0.5408
Average transfer ($M) of fixed contracts		
Conditioning on entry restriction	0.8712	0.8438
Conditioning on competition	1.1613	1.1437
Average transfer ($M) of variable contracts		
Conditioning on entry restriction	1.2573	1.2462
Conditioning on competition	1.3636	1.4845

moments regarding the extent of competition, the contract types, and the contract prices.

6.4. **Counterfactual Analyses.** Using the estimated model, we conduct various counterfactual analyses. We first decompose the effects of three potential reasons why the observed extent of competition is relatively small: (i) cost savings from more competition, (ii) per-bidder bid solicitation and processing costs, and (iii) the net direct benefit of bypassing the solicitation procedure. In particular, we consider three counterfactual scenarios. Under Scenario 1, there exists a 10% decrease in the bid solicitation and processing costs. Under Scenario 2, the average benefit from bypassing solicitation is decreased by 10%. Under the current regulations, the contracting officers are allowed to restrict entry, choose the level of effort to attract bids, and to negotiate with the contractors. An alternative to such delegation is to impose a one-size-fits-all rule, such as mandatory solicitation (Scenario 3), a required minimum number of bids (Scenario 4), and no negotiation on contract terms where unilateral modifications are not allowed and a first-price sealed bid auction is used instead (Scenario 5). The results are presented in Table 11.

7. Conclusion

In this paper, we study the determinants of entry requirements and the number of participating bidders in government procurement auctions. To understand the determinants of competition in government procurement, we develop, identify, and estimate

Table 11. Counterfactual Analyses 1: Sources of Limited Competition

	Data	Model	Change in Each Scenario				
			(1)	(2)	(3)	(4)	(5)
Prob. of restriction	0.63	0.64	-0.05	-0.02	-0.64	-0.64	-0.28
Average number of bids	2.40	1.87	+0.21	+0.01	+0.18	+0.69	+0.83
Prob. of fixed contracts	0.45	0.44	+0.03	+0.001	+0.03	+0.17	+0.56
Average costs ($K)							
Transfer	1,137	1,152	-9.95	-0.74	-12.06	-43.87	+21.92
Bid costs	-	32.29	+5.33	+0.58	+10.13	+48.26	+43.72
Entry restriction costs	-	-13.80	+1.00	+1.25	+13.80	+13.80	+5.32
Efficiency loss costs	-	2.90	-0.24	-0.01	-0.22	-0.63	-2.90
Average *total* costs ($K)							
If restriction cost is private	-	1,185	-4.57	-0.16	-1.93	+4.39	+65.64
If restriction cost is public	-	1,171	-3.57	1.09	+11.87	+18.19	+70.95

Note: Under Scenario 1, there exists a 10% decrease in the bid solicitation and processing costs. Under Scenario 2, the average benefit from bypassing solicitation is decreased by 10%. Under Scenario 3, using the formal solicitation procedure is mandatory. Under Scenario 4, the required minimum number of bids is 2. Under the last scenario, the contract negotiation is not allowed so that a first-price sealed-bid auction is used. See Appendix for more detailed description of the simulation of each scenario.

a principal-agent model in which the procurer chooses whether to solicit bids and how much effort to exert to attract more bids, and then she negotiates with bidders to choose a winner and reach an agreement on a contract. We find that the cost of soliciting, identifying, and processing an additional bid is relatively large compared to the cost savings from competition. By strategically setting the contract terms, the procurer can extract informational rent from bidders, as an alternative to attracting more bidders.

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Appendix

A. Proofs.

A.1. Proof of Theorem 3.1. The following five lemmas collectively prove Theorem 3.1. The first lemma shows that variable contracts are only offered in conjunction with fixed contracts, not by themselves.

Lemma 7.1. *The equilibrium contract menu includes a fixed contract.*

Proof. The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is a variable contract. Denote by $\{p, q(s)\}$ one of the contracts on the menu. There are three cases to consider.

First, suppose $\overline{\mathbb{E}}\{\psi[q(s)]\} \equiv \int \psi[q(s)]\overline{f}(s)ds > \underline{\mathbb{E}}\{\psi[q(s)]\} \equiv \int \psi[q(s)]\underline{f}(s)ds$. Then, the procurer can offer an additional, fixed contract of $p' = p + \overline{\mathbb{E}}\{\psi[q(s)]\}$. The high-cost contractor would accept the contract, but the low-cost contractor will not. By strict concavity of $\psi(\cdot)$, we have $\overline{\mathbb{E}}\{\psi[q(s)]\} < \overline{\mathbb{E}}\{q(s)\}$. Therefore, the expected payoff of the procurer increases when the high-cost contractor accept the fixed contract with any positive probability.

Second, suppose $\overline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{\psi[q(s)]\}$. The procurer can offer an additional, fixed contract of $p' = p + \underline{\mathbb{E}}\{\psi[q(s)]\}$. The low-cost contractor would accept the contract, but the high-cost contractor will not. Since $\underline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{q(s)\}$, the expected payoff of the procurer increases when the low-cost contractor to accept the new contract with any positive probability.

Lastly, suppose $\overline{\mathbb{E}}\{\psi[q(s)]\} = \underline{\mathbb{E}}\{\psi[q(s)]\}$. The procurer can offer instead an fixed contract of $p' = p + \underline{\mathbb{E}}\{\psi[q(s)]\}$. Both types of contractor would accept the contract. Since $\underline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{q(s)\}$ the expected payoff of the procurer increases when either or both contractor types to accept the new contract with any positive probability. □

Given Lemma 7.1, an optimal menu of contracts includes at least one fixed contract. We show that low-cost contractors never selects a variable contract.

Lemma 7.2. *It is optimal for the procurer to offer a menu of contracts that induces the low-cost contractor to select a fixed contract with probability one.*

Proof. Suppose not; i.e., the low-cost contractors select a variable contract with positive probability. Then by Lemma 7.1, the menu must include a fixed contract that is selected by the high-cost contractors. In that case, the fixed-price must be $\alpha + \beta$ so that the individual rationality constraint for the high-cost contractor is satisfied. Notice that the individual rational constraint for the low-cost contractors is satisfied with strict inequality; otherwise, the low-cost contractor will select the fixed contract instead. Given this, the procurer's problem boils down to choosing the terms of the variable contract, p and $q(\cdot)$ to minimize expected total transfer:

$$\phi_n \{p + \underline{\mathbb{E}}[q(s)]\} + (1 - \phi_n)(\alpha + \beta), \tag{24}$$

where ϕ_n is the probability that a contractor that chooses a variable contract becomes a winner when n bidders participate, subject to the incentive compatibility constraint for the low-cost contractor, which is:

$$\underline{\phi}_n (p - \alpha + \underline{\mathbb{E}}\{\psi[q(s)]\}) \geq \overline{\phi}_n \beta, \tag{25}$$

where $\underline{\phi}_n$ and $\overline{\phi}_n$ denote the subjective probability that a contractor that chooses the variable contract (or the fixed contract) wins when n bidders participate. Since the individual rationality constraint is satisfied with strict inequality, the incentive compatibility constraint must bind. Solving for p when (25) holds with equality,

$$p = \frac{\overline{\phi}_n}{\underline{\phi}_n}\beta + \alpha - \underline{\mathbb{E}}\{\psi[q(s)]\}$$

Substituting for p in (24) and simplifying we obtain:

$$\alpha + \phi_n \underline{\mathbb{E}}\{q(s) - \psi[q(s)]\} + \beta\left(1 - \phi_n + \phi_n \frac{\overline{\phi}_n}{\underline{\phi}_n}\right),$$

which is minimized with respect to $q(s)$ for each $s \in S$. Since $q(s) \gtrless \psi[q(s)]$ when $q(s) \lessgtr 0$ and $q(0) = \psi[q(0)]$, $q(s) = 0$ for all $s \in S$. This leads to a contradiction. □

Lemma 7.3. *If two fixed contracts are offered, then it is optimal to offer* $\alpha + \beta$ *and* $\alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$, *where the first priority going to contractors submitting the latter and the second priority to those who submit the former.*

Proof. To ensure the project is undertaken, the procurer must meet the individual rationality (henceforth IR) constraint of the high-cost contractor, and the cheapest fixed price contract meeting this constraint is $\alpha+\beta$. To meet the incentive compatibility

(IC) constraint of a low-cost contractor, the procurer must offer terms that are at least as profitable as $\overline{\phi}(n)\beta$, which are the expected profits to an efficient contractor from selecting $\alpha+\beta$. Letting p denote any price that solves the IC constraint:

$$\underline{\phi}_n(p-\alpha) \geq \overline{\phi}_n\beta$$

Appealing to (4) and (5), this inequality can be expressed as:

$$p-\alpha \geq \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$$

which is minimized by setting $p=\alpha+\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$. □

This leaves us two generic possibilities on the optimal menu of contracts. Either two fixed contracts comprise the optimal menu, or it consists of a fixed contract designed for the low-cost contractors and one or more variable contracts designed for the high-cost contractors. If the signal was of very high quality and most of the contractors were low-cost, we might expect the procurer to extract all the rent from the low-cost contractors, and limit his losses to the risk premium paid to the high-cost contractors. As proved in Theorem 3.1, this is indeed the case.

In preparation for that theorem we now define the expression:

$$H(\pi) \equiv \int_{l(s)<\tilde{l}(\pi,M)} \psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)[\overline{f}(s)-\underline{f}(s)]ds+\psi(M)\int_{l(s)\geq\tilde{l}(\pi,M)}[\overline{f}(s)-\underline{f}(s)]ds, \tag{26}$$

where the cutoff $\tilde{l}(\pi,M)$ is defined in the text. Lemma 7.4 shows that if signals are informative, then the expression $\beta-H(\pi)$ has a unique root, denoted by $\tilde{\pi}$.

Lemma 7.4. *A unique probability denoted by $\tilde{\pi}>0$ solves $\beta=H(\pi)$.*

Proof. Note from equation (26) that $H(0)=0$. We show that $H(\cdot)$ is strictly increasing in π. To see this, we rewrite $H(\pi)$ by

$$H(\pi)=\int\tilde{H}(\pi,s)\overline{f}(s)ds,$$

where $\tilde{H}(\pi,s)$ is defined by

$$\tilde{H}(\pi,s)=\begin{cases}\psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)\{1-l(s)\} & \text{if } l(s)<\tilde{l}(\pi,M),\\ \psi(M)\{1-l(s)\} & \text{otherwise.}\end{cases}$$

When $l(s)\geq\tilde{l}(\pi,M)$, $\partial\tilde{H}(\pi,s)/\partial\pi=0$. Otherwise, we can see that

$$\frac{\partial}{\partial\pi}\tilde{H}(\pi,s)=-\psi'\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)h'\left(\frac{1-\pi}{1-\pi l(s)}\right)\frac{[l(s)-1]^2}{[1-\pi l(s)]^2}>0.$$

Therefore, $H(\pi)$ is strictly increasing in π. □

Now we characterize the optimal menu of contracts when it consists of one fixed contract and one variable contract.

Lemma 7.5. *Suppose the optimal menu of contracts consists of one fixed contract, denoted by* $\underline{p}_n$*, and one variable contract, denoted by* $\{\overline{p}, q(\cdot)\}$*. The ex-post price adjustment schedule,* $q(\cdot)$*, is:*

$$q(s) = \begin{cases} h\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-\min\{\pi,\tilde{\pi}\}l(s)}\right) & \text{if } l(s) \le \tilde{l}(\min\{\pi,\tilde{\pi}\}, M), \\ M & \text{if } l(s) > \tilde{l}(\min\{\pi,\tilde{\pi}\}, M). \end{cases} \tag{27}$$

The base price of the variable contract is:

$$\overline{p} = \alpha + \beta - \int \psi[q(s)]\overline{f}(s)ds. \tag{28}$$

The price of the fixed contract is:

$$\underline{p}_n = \alpha + \frac{\pi\left(1-\pi\right)^{n-1}}{1-\left(1-\pi\right)^n}\left[\beta - \int \psi[q(s)]\left\{\overline{f}(\mathbf{s}) - \underline{f}(s)\right\}ds\right]. \tag{29}$$

Proof. The principal designs a menu of two contracts that minimizes the expected transfer:

$$\left[1-(1-\pi)^n\right]\underline{p}_n + (1-\pi)^n\left[\overline{p} + \overline{\mathbb{E}}(q(s))\right]. \tag{30}$$

subject to the constraints that the low-cost contractors select the fixed contract, the high-cost contractors select the variable contract, and the winning contractor never declares bankruptcy. A necessary condition of the optimal menu is that the IR constraint the high-cost contractors holds with equality (otherwise the base price $\overline{p}$ could be further reduced, reducing the price and strengthening the IC constraint for the low-cost contractors). Solving for $\overline{p}$ yields (28). The IC constraint for the low-cost contractors is:

$$\underline{\phi}_n(\underline{p} - \alpha) \ge \overline{\phi}_n\{\overline{p} + \underline{\mathbb{E}}[\psi(q)] - \alpha\},$$

Substituting for $\overline{p}$ using equation (28), $\overline{\phi}_n$ using (5), and $\underline{\phi}_n$ using (4), the IC inequality simplifies to:

$$\underline{p}_n \ge \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta - \int \psi(q(s))[1-l(s)]\overline{f}(s)ds\right). \tag{31}$$

Note that the IC for the high-cost contractors will be satisfied with strict inequality at the optimum by Lemma 7.2. Therefore, at least one of the two remaining constraints, IR and IC for the low-cost contractors, must bind. Otherwise, the price of the fixed contract could be reduced, earning the procurer higher revenue. This leads us to consider the following three cases separately.

Case 1: IC binds but IR does not Solving for $\underline{p}_n$ from the IC constraint, and substituting the resulting expressions for $\underline{p}_n$ and $\overline{p}$, obtained from equations and (28) and (29), into the expected total cost for the procurer, we obtain:

$$\begin{aligned} & [1-(1-\pi)^n]\left\{\alpha+\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta-\int\psi(\overline{q}(s))[1-l(s)]\overline{f}(s)ds\right)\right\} \\ & +(1-\pi)^n\left\{\alpha+\beta+\int[q(s)-\psi(q(s))]\overline{f}(s)ds\right\} \\ = \;& \alpha+(1-\pi)^{n-1}\left\{\beta-\pi\int\psi(\overline{q}(s))[1-l(s)]\overline{f}(s)ds+(1-\pi)\int[q(s)-\psi(q(s))]\overline{f}(s)ds\right\}. \end{aligned}$$

The (scaled) Lagrangian for the cost minimization problem can now be expressed as:

$$L=-\pi\int\psi(q(s))[1-l(s)]\overline{f}(s)ds+(1-\pi)\int[q(s)-\psi(q(s))]\overline{f}(s)ds-\int\varkappa_1(s)\left[q(s)-M\right]\overline{f}(s)ds,$$

where $\varkappa_1(s)\geq 0$ denotes the Kuhn Tucker multiplier for the linear constraint $q(s)\geq M$. The first order condition for $q(s)$ is:

$$-\pi\psi'(q(s))[1-l(s)]+(1-\pi)\left[1-\psi'(q(s))\right]-\varkappa_1(s)=0.$$

Rearranging terms we obtain:

$$\psi'\left[q(s)\right]=\frac{1-\pi-\varkappa_1(s)}{1-\pi l(s)}. \tag{32}$$

If $l(s)<\tilde{l}(\pi,M)$, then $q(s)=h\left[\frac{1-\pi}{1-\pi l(s)}\right]>M$ and $\varkappa_1(s)=0$ solve equation (32). If $l(s)\geq\tilde{l}(\pi,M)$, then $\varkappa_1(s)>0$ and $q(s)=M$ solve the equation.

Case 2: IR binds but IC does not When IR binds, $\underline{p}_n=\alpha$. Substituting for $\underline{p}_n$ and $\overline{p}$, using equation (28), the expected total transfer (30) simplifies to:

$$\alpha+(1-\pi)^n\left\{\beta+\int\left\{q(s)-\psi[q(s)]\right\}\overline{f}(s)ds\right\}. \tag{33}$$

Substituting for $\underline{p}_n$ in inequality (31) yields:

$$\beta\leq\int\psi(q(s))[1-l(s)]\overline{f}(s)ds. \tag{34}$$

Notice the solution to this problem depends on neither π nor n. If IR binds but IC does not, then the first order condition for the Kuhn Tucker formulation is:

$$1-\psi'(q(s))=\varkappa_1(s).$$

If $q(s)>M$, then the complementary slackness condition requires $\varkappa_1(s)=0$, and hence $1=\psi'(q(s))$ or $q(s)=0$. Therefore, either $q(s)=M$, and the marginal benefit of imposing a harsher signal would exceed its cost were it not for the bankruptcy constraint, or $q(s)=0$. Let us define S_M as the set of signals such that $q(s)=M$ and let μ denote $\Pr(s\in S_M)$. Note that for any $\mu\in[0,1]$, both IR constraints and the IC constraint for the inefficient are satisfied. The total expected transfer can now be written as

$$\alpha+(1-\pi)^n\left\{\beta+[M-\psi(M)]\mu\right\}.$$

Notice that the above transfer is increasing in μ, while $\mu=0$ does not satisfy the IC condition for the efficient, or inequality (34). This implies that when both IR constraints bind, the IC for the efficient must bind.

Case 3: Both IR and IC bind If (34) holds with equality, the (scaled) Lagrangian for the minimization problem can be written as:

$$\begin{aligned} L &= \int \left(q(s)-\psi[q(s)]\right)\overline{f}(s)ds-\int \varkappa_1(s)\left[q(s)-M\right]\overline{f}(s)ds \\ &\quad +\varkappa_2\left\{\beta-\int \psi[q(s)][1-l(s)]\overline{f}(s)ds\right\}. \end{aligned}$$

The first order condition with respect to $q(s)$ is:

$$1-\psi'[q(s)]-\varkappa_1(s)-\varkappa_2\psi'[q(s)][1-l(s)]=0.$$

This can be written as:

$$\psi'[q(s)]=\frac{1-\varkappa_1(s)}{1+\varkappa_2[1-l(s)]}. \tag{35}$$

Substituting for $\varkappa_2=\tilde{\pi}/(1-\tilde{\pi})$ in equation (35) follows that the solution for $q(s)$ in this case can be obtained as in (27).

We have ruled out the second case, implying that the IC for the low-cost contractors always binds at the optimum. The IR constraint for the low-cost contractors does not always bind, i.e.

$$\beta-\int \psi[q(s)][1-l(s)]\overline{f}(s)ds=\beta-H(\pi)\le 0,$$

where $H(\pi)$ is defined in equation (26). As shown in Lemma 7.4, $\beta - H(0) = \beta > 0$, $H(\cdot)$ is increasing in π, and there always exists a unique root of $\beta - H(\pi)$, $\tilde{\pi}$. Therefore, if $\pi < \tilde{\pi}$, then the IR does not bind; otherwise, it binds. This completes the proof. □

We now show that the menu of contracts characterized in Lemma 7.3 is always dominated by that of Lemma 7.5 if the signals are informative. In other words, the procurer is better off exploiting the signals.

Lemma 7.6. *Suppose $\overline{F}(s) \neq \underline{F}(s)$ for some signal s in the support. Then the menu of contracts characterized in Lemma 7.5 minimizes the total expected transfer.*

Proof. Given that there are two types, the optimal menu includes two contracts. By Lemma 7.1, we have shown that at least one of them must be a fixed contract, and it is optimal to induce the low-cost contractors to choose a fixed contract in the menu, as shown in Lemma 7.2. There are two possibilities: one is to offer two fixed contracts, as characterized in Lemma 7.3, and the other is to offer one fixed contract for the low-cost contractors and one variable contract for the high-cost contractors, as characterized in Lemma 7.5. We show that the latter is cheaper than the former.

The expected total cost of offering the two fixed contracts to n bidders, as characterized in Lemma 7.3, denoted by T_n^F, is:

$$T_n^F = (1-\pi)^n(\alpha+\beta) + [1-(1-\pi)^n]\,\alpha + \pi(1-\pi)^{n-1}\beta = \alpha + (1-\pi)^{n-1}\beta.$$

Denoting by T_n^V, the total cost of offering the menu of contracts of Lemma 7.5 is:

$$\begin{aligned} T_n^V &= [1-(1-\pi)^n]\left[\alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\{\beta-\gamma\}\right] + (1-\pi)^n[\alpha+\beta+r] \\ &= \alpha + (1-\pi)^{n-1}\{\beta - \pi\gamma + (1-\pi)r\}. \end{aligned}$$

Thus $T_n^V < T_n^F$ if and only if:

$$\Gamma \equiv -\pi\gamma + (1-\pi)r < 0.$$

This condition is satisfied if and only if $T_1^V < T_1^F = \alpha + \beta$.

We complete the proof by showing that this inequality holds. The proof is done by construction that it is less profitable to offer one fixed contract than a menu of two contracts.

For some $\epsilon > 0$, we define $S \equiv \{s : \overline{f}(s) - \underline{f}(s) > \epsilon\}$. Let the probability that a signal is in S conditional on that the contractor is low-cost as γ_1 and that conditional on that the contractor is high-cost as γ_2. If $\overline{F}(s) \neq \underline{F}(s)$ for some signal s in the support, there exists $\epsilon > 0$ such that $\gamma_1 \neq 0$ and $\gamma_2 \neq 0$. Note that $\gamma_2 > \gamma_1$. For any

$\delta > 0$ choose $\mu(\delta)$ for a two-part variable contract in which $\overline{p} = c + \beta$ and:

$$q(s) = \begin{cases} \delta \text{ if } s \in S, \\ \mu(\delta) \text{ if } s \notin S, \end{cases}$$

where

$$\gamma_2 \psi(\delta) + (1 - \gamma_2) \psi(\mu(\delta)) = 0.$$

Note that the above equation implies that $\mu(\delta) < 0$. Because $\psi(\cdot)$ is strictly increasing, $\mu(\delta)$ is uniquely defined by the equation:

$$\mu(\delta) = \psi^{-1} \left[\frac{-\gamma_2}{1 - \gamma_2} \psi(\delta) \right],$$

and is twice differentiable with:

$$\mu'(\delta) = \frac{-\gamma_2}{1 - \gamma_2} \frac{\psi'(\delta)}{\psi'(\mu(\delta))},$$

where $\mu(0) = 0$. The fixed contract takes the form:

$$\begin{aligned} \underline{p} &= \alpha + \beta + \gamma_1 \psi(\delta) + (1 - \gamma_1) \psi(\mu(\delta)), \\ &= \alpha + \beta + \gamma_1 \psi(\delta) - (1 - \gamma_1) \left(\frac{\gamma_2}{1 - \gamma_2} \right) \psi(\delta). \end{aligned}$$

Note that the incentive compatibility constraint is satisfied with equality by the low-cost contractor and strict inequality by the high-cost contractor because $\gamma_1 < \gamma_2$. Similarly, the participation constraint is satisfied with equality by the high-cost contractors and strict inequality by the low-cost contractors as long as $\delta > 0$ is small enough. The expected price to the procurer is:

$$\begin{aligned} \mathbb{E}(T|\delta) &= \alpha + \beta + \pi \left[\gamma_1 \psi(\delta) + (1 - \gamma_1) \psi(\mu(\delta)) \right] + (1 - \pi) \left[\gamma_2 \delta + (1 - \gamma_2) \mu(\delta) \right], \\ &= \alpha + \beta + \pi \left[\gamma_1 \psi(\delta) - \frac{(1 - \gamma_1) \gamma_2}{1 - \gamma_2} \psi(\delta) \right] + (1 - \pi) \left[\gamma_2 \delta + (1 - \gamma_2) \mu(\delta) \right]. \end{aligned}$$

We now show this expression is decreasing in the neighborhood of $\delta = 0$. Differentiating with respect to δ yields:

$$\frac{\partial \mathbb{E}(T|\delta)}{\partial \delta} = \pi \left[\gamma_1 \psi'(\delta) - \frac{(1 - \gamma_1) \gamma_2}{1 - \gamma_2} \psi'(\delta) \right] + (1 - \pi) \left[\gamma_2 - \gamma_2 \frac{\psi'(\delta)}{\psi'(\mu(\delta))} \right].$$

Evaluating $\frac{\partial \mathbb{E}(T|\delta)}{\partial \delta}$ at $\delta = 0$ gives us:

$$\frac{\partial \mathbb{E}(T|\delta = 0)}{\partial \delta} = \pi \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} < 0,$$

which shows that a fixed contract fails to meet a first order necessary condition. □

A.2. Proof of Lemma 3.1. Let T_n denote the transfer to the winning contractor when n bidders participate:

$$T_n \equiv [1-(1-\pi)^n]\,\underline{p}_n + (1-\pi)^n \left[\overline{p} + \int q(s)\overline{f}(s)ds\right].$$

Substituting for $\underline{p}_n$ and $\overline{p}$ in T_n gives

$$\begin{aligned} T_n &= \alpha + \pi\,(1-\pi)^{n-1}\,(\beta-\gamma) + (1-\pi)^n\,(\beta+r)\,, \\ &= \alpha + (1-\pi)^{n-1}\,(\beta+\Gamma). \end{aligned}$$

Integrating over the number of solicited bidders (one less the total) yields:

$$\begin{aligned} U(\lambda) &\equiv \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!}\,[\kappa n + T_{n+1}] = \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!}\,[\kappa n + \alpha + (1-\pi)^n\,(\beta+\Gamma)] \\ &= \alpha + \kappa\lambda + e^{-\lambda\pi}(\beta+\Gamma). \end{aligned}$$

The second derivative of $U(\lambda)$ is $\pi^2 e^{-\lambda\pi}(\beta+\Gamma) > 0$, proving $U(\lambda)$ is convex in $\lambda \in \mathcal{R}$. Also $U(\lambda) \to \infty$ as $\lambda \to \infty$ and as $\lambda \to -\infty$. Therefore $U(\lambda)$ attains a global minimum at its unique stationary point, found by solving the first order condition $\kappa = \pi e^{-\lambda\pi}(\beta+\Gamma)$, which we denote by:

$$\lambda^* = \frac{1}{\pi}\left\{\ln(\pi) + \ln[\beta+\Gamma] - \ln(\kappa)\right\}.$$

The constraints and interpretation of the model restrict λ to be nonnegative but λ^* might be positive or negative. If $\lambda^* > 0$ then λ^* solves the constrained problem of minimizing $U(\lambda)$ with respect to positive values of λ. If $\lambda^* < 0$, then $U(0) < U(\lambda')$ for all $\lambda' > 0$.

A.3. Proof of Lemma 4.1. If **A3** holds, then $q(s)$ satisfies the first order condition, (9). Rewriting (9) while replacing $q(s)$ by $q(s,\pi)$ to emphasize the dependence of q on π,

$$\psi'[q(s,\pi)]\,[1-\pi l(s)] = 1-\pi.$$

Note that $q(s,\pi) = 0$ if $l(s) = 1$ and $q(s,\pi) > 0$ if $l(s) < 1$. Similarly $q(s,\pi) < 0$ if $l(s) > 1$. Totally differentiating the first order condition with respect to π yields:

$$\psi''[q(s,\pi)]\,\frac{\partial q(s,\pi)}{\partial \pi}\,[1-\pi l(s)] - \psi'[q(s,\pi)]\,l(s) = -1.$$

Rearranging to make $\partial q(s,\pi)/\partial\pi$ the subject of the equation gives:

$$\frac{\partial q(s,\pi)}{\partial \pi} = \frac{l(s)-1}{\psi''[q(s,\pi)]\,[1-\pi l(s)]^2}.$$

Noting $\psi''(\cdot) < 0$ it follows that $\partial q(s,\pi)/\partial\pi > 0$ when $l(s) < 1$ and $\partial q(s,\pi)/\partial\pi < 0$ when $l(s) > 1$. Therefore,

$$\frac{\partial q(s,\pi)}{\partial \pi} \begin{array}{l} > 0 \text{ if } q(s,\pi) > 0, \\ = 0 \text{ if } q(s,\pi) = 0, \\ < 0 \text{ if } q(s,\pi) < 0. \end{array}$$

as was to be proved.

A.4. Proof of Lemma 4.2. Rewriting (7) while making the dependence of $\underline{p}_n$, $q(s)$, α, and β on π:

$$\underline{p}_n(\pi) = \alpha(\pi) + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left[\beta(\pi) - \int \psi(q(s,\pi))[\overline{f}(s) - \underline{f}(s)]ds\right].$$

To show that $\underline{p}_n'(\pi) < 0$ we consider the two expressions involving π separately. First:

$$\frac{\partial}{\partial\pi}\ln\left[\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\right] = \frac{1-n\pi-(1-\pi)^n}{\pi(1-\pi)\left[1-(1-\pi)^n\right]}$$

Note that the derivative is zero at $n=1$ and that at $n=2$ is $-\pi^2$, which is negative. Now suppose it is negative for all $n \in \{2,\ldots,n_0\}$. Then for n_0+1 the denominator is clearly positive and the numerator is:

$$1-(n_0+1)\pi-(1-\pi)(1-\pi)^{n_0} < \pi(1-\pi)^{n_0} - \pi < 0.$$

The first inequality follows from an induction hypothesis, and the second one from the inequalities $0 < \pi < 1$. Therefore $\pi(1-\pi)^{n-1}/\pi(1-\pi)^{n-1}$ is decreasing in π for all $n > 1$.

Second, we note that:

$$\frac{\partial}{\partial\pi}\int \psi\left[q(s,\pi)\right]\left[1-l(s)\right]\overline{f}(s)\,ds = \int \psi'\left[q(s,\pi)\right]\frac{\partial q(s,\pi)}{\partial\pi}\left[1-l(s)\right]\overline{f}(s)\,ds$$

$$= \int (1-\pi)\frac{\partial q(s,\pi)}{\partial\pi}\left[\frac{1-l(s)}{1-\pi l(s)}\right]\overline{f}(s)\,ds = \int \frac{(\pi-1)\left[1-l(s)\right]^2}{\psi''\left[q(s,\pi)\right]\left[1-\pi l(s)\right]^3}\overline{f}(s)\,ds > 0.$$

The second equality follows from using the first order condition to substitute out $\psi'[q(s,\pi)]$. Note that we can use the first order condition because **A3** holds. The third equality results from the expression we derived for $\partial q(s,\pi)/\partial\pi$. The last inequality appeals to the concavity of $\psi(\cdot)$. Finally note that since the participation constraint is satisfied with an inequality for the low-cost contractor under **A3**.

$$\beta(\pi) - \int \psi\left[q(s,\pi)\right]\left[1-l(s)\right]\overline{f}(s)\,ds > 0$$

for all $\pi \in \Pi$. Hence, if $\alpha(\pi)$ and $\beta(\pi)$ are nonincreasing in π, as assumed in **A4**, the following inequality holds as claimed.

$$\begin{aligned}\frac{\partial}{\partial\pi}\underline{p}_n(\pi) &= \alpha'(\pi) + \frac{\partial}{\partial\pi}\left[\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\right]\left\{\beta(\pi) - \int \psi\left[q(s,\pi)\right]\left[1-l(s)\right]\overline{f}(s)\,ds\right\} \\ &+ \frac{\pi(1-\pi)^n}{1-(1-\pi)^n}\left\{\beta'(\pi) - \int \frac{\left[1-l(s)\right]^2}{\psi''\left[q(s,\pi)\right]\left[1-\pi l(s)\right]^3}\overline{f}(s)\,ds\right\} < 0.\end{aligned}$$

A.5. Proof of Lemma 4.3. Recall that

$$\overline{p} = \alpha + \beta - \int \psi\left[q(s)\right]\overline{f}(s)\,ds.$$

Differentiating with respect to π yields:

$$\begin{aligned}\frac{\partial\overline{p}}{\partial\pi} &= \frac{\partial\alpha}{\partial\pi} + \frac{\partial\beta}{\partial\pi} - \int \psi'\left[q(s)\right]\frac{\partial q(s;\pi)}{\partial\pi}\overline{f}(s)\,ds \\ &= \frac{\partial\alpha}{\partial\pi} + \frac{\partial\beta}{\partial\pi} - \int h'\left[\frac{1-\pi}{1-\pi l(s)}\right]\frac{1-\pi}{1-\pi l(s)}\frac{(1-\pi)\,l(s) - \left[1-\pi l(s)\right]}{\left[1-\pi l(s)\right]^2}\overline{f}(s)\,ds \\ &= \frac{\partial\alpha}{\partial\pi} + \frac{\partial\beta}{\partial\pi} + \int h'\left[\frac{1-\pi}{1-\pi l(s)}\right]\frac{1}{\left[1-\pi l(s)\right]^2}\frac{(1-\pi)}{1-\pi l(s)}\left[1-l(s)\right]\overline{f}(s)\,ds\end{aligned}$$

First note that the integral is negative for $l(s) < 1$ and positive for $l(s) > 1$ because $h'(\cdot) < 0$ and $\pi l(s) < 1$ by **A3**. Therefore,

$$\frac{\partial\overline{p}}{\partial\pi} < \frac{\partial\alpha}{\partial\pi} + \frac{\partial\beta}{\partial\pi} + \int_{l(s)>1} h'\left[\frac{1-\pi}{1-\pi l(s)}\right]\frac{(1-\pi)\left[1-l(s)\right]}{\left[1-\pi l(s)\right]^3}\overline{f}(s)\,ds.$$

We define $m(\pi, l)$ by

$$m(\pi, l) \equiv h'\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)\left[1-l\right]}{\left[1-\pi l\right]^3}.$$

It can be seen that the derivative of $m(\pi, l)$ with respect to the second argument is positive if $h'' > 0$ and $l > 1$.

$$\begin{aligned}&\frac{\partial m(\pi, l)}{\partial l} \\ &= h''\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)^2\left[l-1\right]\pi}{\left[1-\pi l\right]^5} - h'\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)}{\left[1-\pi l\right]^3} + 3h'\left[\frac{1-\pi}{1-\pi l}\right]\frac{\pi(1-\pi)\left[1-l\right]}{\left[1-\pi l\right]^4} \\ &= h''\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)^2\left[l-1\right]\pi}{\left[1-\pi l\right]^5} + h'\left[\frac{1-\pi}{1-\pi l}\right](1-\pi)[2\pi(1-l)+\pi-1].\end{aligned}$$

Therefore,

$$\frac{\partial \overline{p}}{\partial \pi} < \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + h' \left[\frac{1-\pi}{1-\pi \tilde{l}\left(\pi, M\right)} \right] \frac{\left(1-\pi\right)\left[1-\tilde{l}\left(\pi, M\right)\right]}{\left[1-\pi \tilde{l}\left(\pi, M\right)\right]^3}. \tag{36}$$

Using the definition of $\tilde{l}\left(\pi, M\right) \equiv \frac{1}{\pi} - \frac{1-\pi}{\pi \psi'(M)}$, we have

$$\begin{aligned} 1-\widetilde{l}\left(\pi, M\right) &= 1-\frac{1}{\pi}+\frac{1-\pi}{\pi \psi'\left(M\right)} = \frac{\pi \psi'\left(M\right)-\psi'\left(M\right)+\left(1-\pi\right)}{\pi \psi'\left(M\right)} \\ &= \frac{\left(1-\pi\right)\left(1-\psi'\left(M\right)\right)}{\pi \psi'\left(M\right)}, \\ 1-\pi \widetilde{l}\left(\pi, M\right) &= 1-1+\frac{1-\pi}{\psi'\left(M\right)} = \frac{1-\pi}{\psi'\left(M\right)}. \end{aligned}$$

Using these, we simplify the RHS of (36) as:

$$\frac{\partial \overline{p}}{\partial \pi} < \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \frac{h'\left[\psi'(M)\right]\left[\psi'(M)\right]^2\left[1-\psi'(M)\right]}{\pi(1-\pi)}.$$

Therefore, if **A5** holds, $\frac{\partial \overline{p}}{\partial \pi} < 0$.

A.6. Proof of Lemma 4.4. Appealing to **A5**, $\overline{p}$ is monotone decreasing in π, implying the existence of a mapping $\pi\left(\overline{p}\right)$ such that $l^*\left(q, \overline{p}\right)$ defined in (16) satisfies:

$$\psi'\left(q\right) = \frac{1-\pi}{1-\pi l^*\left(q, \overline{p}\right)}$$

Making π the subject we obtain:

$$\pi = \frac{1-\psi'\left(q\right)}{1-l^*\left(q, \overline{p}\right)\psi'\left(q\right)}$$

Differentiating with respect to q holding π and $\overline{p}$ constant yields:

$$\psi''\left(q\right) = \frac{\pi}{1-\pi l^*\left(q, \overline{p}\right)} \psi'\left(q\right) \frac{\partial l^*\left(q, \overline{p}\right)}{\partial q}$$

Using these two equations we substitute π out to obtain (17).

A.7. Proof of Lemma 4.5. The joint probability that the contract type is fixed and $\pi \leq \pi^*$ can be expressed as:

$$\begin{aligned} \Pr\left\{\pi \leq \pi^*, v=0 \,|c, n\right\} &= F_{\pi|c,n,v}\left(\pi^* \,|c, n, 0\right) \Pr\left(v=0 \,|c, n\right) \\ &= \int_{\pi=\underline{\pi}}^{\pi^*} f_{\pi|c,n}\left(\pi \,|c, n\right)\left[1-\left(1-\pi\right)^n\right] d\pi. \end{aligned} \tag{37}$$

Taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}\left(\pi^* \left| c,n,0\right.\right)\Pr\left(v=0\left|c,n\right.\right)=f_{\pi|c,n}\left(\pi^*\left|c,n\right.\right)\left[1-\left(1-\pi^*\right)^n\right]. \tag{38}$$

Similarly:

$$\begin{aligned}\Pr\left\{\pi\leq\pi^*,v=1\left|c,n\right.\right\} &= F_{\pi|c,n,v}\left(\pi^*\left|n,v=1\right.\right)\Pr\left(v=1\left|c,n\right.\right)\\ &= \int_{\pi=\underline{\pi}}^{\pi^*} f_{\pi|c,n}\left(\pi\left|c,n\right.\right)\left(1-\pi\right)^n d\pi,\end{aligned}$$

and taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}\left(\pi^*\left|c,n,1\right.\right)\Pr\left(v=1\left|c,n\right.\right)=f_{\pi|c,n}\left(\pi^*\left|c,n\right.\right)\left(1-\pi^*\right)^n. \tag{39}$$

Rearranging the quotient of (7) and (39) to make $f_{\pi|u,n,d}\left(\pi^*\left|c,n,v=0\right.\right)$ the subject of the resulting equation, and relabeling π^* as x, we obtain:

$$f_{\pi|c,n,v}\left(x\left|c,n,0\right.\right)=\varphi_{c,n}\frac{1-\left(1-x\right)^n}{\left(1-x\right)^n}f_{\pi|c,n,v}\left(x\left|c,n,1\right.\right), \tag{40}$$

where $\varphi_{c,n}\equiv\Pr\left\{v=1\left|c,n\right.\right\}/\Pr\left\{v=0\left|c,n\right.\right\}$, as defined in the text. Therefore:

$$\begin{aligned}\int_{\pi}^{\overline{\pi}} f_{\pi|c,n,v}\left(x\left|c,n,0\right.\right)dx &= \int_{\underline{\pi}}^{\overline{\pi}} 1\left\{\pi\leq x\right\}f_{\pi|c,n,v}\left(x\left|c,n,0\right.\right)dx\\ &= \varphi_{c,n}\mathbb{E}_{x|c,n,v=1}\left[1\left\{\pi\leq x\right\}\frac{1-\left(1-x\right)^n}{\left(1-x\right)^n}\right],\end{aligned} \tag{41}$$

where x is a random variable drawn from $f_{\pi|c,n,v}\left(\cdot\left|c,n,1\right.\right)$ and $\mathbb{E}_{x|c,n,v=1}\left[\cdot\right]$ is its associated expectations operator. Substituting (41) into the definition of $\underline{p}_n^*\left(\pi,u\right)$ given in (18) we obtain:

$$\underline{p}_n^*\left(\pi,c\right)=G^{-1}_{\underline{p}_n|c}\left(\varphi_{c,n}E_{x|c,n,v=1}\left[1\left\{\pi\leq x\right\}\frac{1-\left(1-x\right)^n}{\left(1-x\right)^n}\right]\middle| c\right)$$

Noting that by definition $\pi_{q,s}$ is drawn from $f_{\pi|c,n,v}\left(\cdot\left|c,n,1\right.\right)$, the lemma is proved by setting $x=\pi_{q,s}$ in the equation above.

A.8. Proof of Theorem 4.1. We prove identification for the framework augmented by (τ,ε) by following the sequence of lemmas in the text. (i) From the text, $((1-d)\widehat{\overline{p}}$, $(1-d)\,q$, and $(1-d)\,s$ are observed. From the first part of **A5** ϕ is identified. Hence from the second part of **A5** the signaling densities are identified. (ii) We assume conditions on $\psi(q)$ that guarantee $l^*\left(q,\overline{p}\right)$ defined in (16) is uniformly Lipschitz continuous in q. Then the Picard–Lindelöf theorem applies thus proving the differential equation (17) has a unique solution given the normalizing constant $\psi'\left(0\right)=1$. Hence $\psi\left(\cdot\right)$ is identified from the other normalizing constant $\psi(0)=0$. (iii) Noting that $\pi_{q,s}$, defined by (need to number) is identified from $((1-d)\,q,(1-d)\,s)$ is identified, and can be interpreted as a random draw from the $f_{\pi|c,n,v}\left(\pi\,|c,n,1\right)$ it now follows that $f_{\pi|c,n,v}\left(\pi\,|c,n,1\right)$ is identified. From (7):

$$f_{\pi|c,n,d}\left(\pi\,|c,n,1\right)=\varphi_{c,n}\frac{\left[1-(1-\pi)^{n}\right]}{(1-\pi)^{n}}f_{\pi|c,n,d}\left(\pi\,|c,n,0\right)$$

Since $\varphi_{c,n}\equiv\Pr(v=1|c,n)\,/\Pr(v=0|c,n)$ is identified so is $f_{\pi|u,n,d}\left(v\,|u,n,1\right)$. (iv) Since $\underline{p}_{n}^{*}\left(\pi,u\right)$ can be expressed in terms of $\pi_{q,s}$ draws, $G_{\underline{p}_{n}|c}\left(\cdot\,|c\right)$ and $\varphi_{c,n}$, which are all identified objects, so is $\underline{p}_{n}^{*}\left(\pi,c\right)$. (v) The identification of the mappings $\alpha\left(\pi\right)$ and $\beta\left(\pi\right)$ follow from the analysis given in the text surrounding (19) and (20). (vi) To identify $\lambda(\pi)$, we show the expressions in the numerator $f_{\pi,n|c}\left(\pi,n+1\,|1\right)$ and denominator $f_{\pi|c}\left(\pi\,|1\right)$ of (22) are identified. Note first that the identification of $f_{\pi|c,n,d}\left(\pi\,|c,n,d\right)$ is established above in the proof of Lemma 4.5, while $\Pr\left\{d\,|c,n\right\}$is directly identified. But:

$$f_{\pi,n|c}\left(\pi,n\,|c\right)=\sum\nolimits_{\tau=0}^{1}f_{\pi|c,n}\left(\pi\,|c,n\right)f_{n|c,\tau}\left(n\,|c\right)$$

and:

$$f_{\pi|c}\left(\pi\,|1\right)=\sum\nolimits_{n=0}^{\infty}\Pr\left\{n\,|1\right\}f_{\pi,n|c}\left(\pi,n+1\,|1\right)$$

where:

$$f_{\pi|c,n}\left(\pi\,|c,n\right)=\Pr\left\{d=0\,|c,n\right\}f_{\pi|c,n,d}\left(\pi\,|c,n,0\right)+\Pr\left\{d=1\,|c,n\right\}f_{\pi|c,n,d}\left(\pi\,|c,n,1\right)$$

(vii) Finally the text explains the partial identification of $\kappa(\pi)$ and how $F_{\eta}\left(\eta\right)$ is identified for $\eta=0$ and $\eta\in\Omega^{*}$.

B. Estimation. Let us denote the vector of the parameters of the model by θ. Our estimator minimizes a weighted sum of squared distances:

$$g_n(\theta)'Wg_n(\theta),\ \text{with}\ g_n(\theta)=\frac{1}{n}\sum_{t=1}^{n}g(w_t;\theta),$$

where W is a symmetric positive-definite weighting matrix. The $g(w_t;\theta)$ vector is related to the moment conditions that are motivated by the identification argument. Specifically, one sub-vector of the $g(w_t;\theta)$ vector is related to the joint probabilities regarding entry restrictions (c), number of bids (n), and contract type (v):

$$\begin{bmatrix} (1-c_t)-\mathbb{E}(1-c|\theta) \\ c_t 1\{n_t=1\}-\Pr(c=1,n=1|\theta) \\ c_t 1\{n_t\le 2\}-\Pr(c=1,n\le 2|\theta) \\ c_t 1\{n_t\le 5\}-\Pr(c=1,n\le 5|\theta) \\ (1-c_t)(1-v_t)-\Pr(c=0,v=0|\theta) \\ c_t(1-v_t)-\Pr(c=1,v=0|\theta) \\ c_t(1-v_t)\{n_t=1\}-\Pr(c=1,v=0,n=1|\theta) \\ c_t(1-v_t)\{n_t\le 2\}-\Pr(c=1,v=0,n\le 2|\theta) \\ c_t(1-v_t)\{n_t\le 5\}-\Pr(c=1,v=0,n\le 5|\theta) \end{bmatrix}.$$

Another sub-vector is related to some moments of the joint distribution of signals, contract type, and the sign of the ex-post price adjustments:

$$\begin{bmatrix} (1-v_t)1\{s_t=0\}-\Pr(v=0,s=0|\theta) \\ v_t 1\{s_t=0\}-\Pr(v=1,s=0|\theta) \\ v_t 1\{s_t=0,q_t\ge 0\}-\Pr(v=1,s=0,q\ge 0|\theta) \\ v_t 1\{s_t>0,q_t\ge 0\}-\Pr(v=1,s>0,q\ge 0|\theta) \\ (1-v_t)s_t-\mathbb{E}((1-v)s|\theta) \\ (1-v_t)s_t^2-\mathbb{E}((1-v)s^2|\theta) \\ v_t s_t-\mathbb{E}(vs|\theta) \\ v_t s_t^2-\mathbb{E}(vs^2|\theta) \end{bmatrix}.$$

Lastly, the $g(w_t;\theta)$ vector includes the quantiles of the contract prices conditional on (c,n). For example, we consider the following vector regarding the price of the fixed contracts for the quantile values of $a=\{0.25,0.5,0.75\}$:

$$\begin{bmatrix} (1-v_t)(1-c_t)1\{\underline{p}_t\le Q_a(\underline{p}|c=0;\theta)\}-a\Pr(c=0,v=0|\theta) \\ (1-v_t)c_t 1\{\underline{p}_t\le Q_a(\underline{p}|c=1;\theta)\}-a\Pr(c=1,v=0|\theta) \\ (1-v_t)c_t 1\{n_t=1\}1\{\underline{p}_t\le Q_a(\underline{p}|c=1,n=1;\theta)\}-a\Pr(c=1,n=1,v=0|\theta) \\ (1-v_t)c_t 1\{n_t\le 2\}1\{\underline{p}_t\le Q_a(\underline{p}|c=1,n\le 2;\theta)\}-a\Pr(c=1,n\le 2,v=0|\theta) \\ (1-v_t)c_t 1\{n_t\le 5\}1\{\underline{p}_t\le Q_a(\underline{p}|c=1,n\le 5;\theta)\}-a\Pr(c=1,n\le 5,v=0|\theta) \end{bmatrix},$$

where $Q_a(\underline{p}|c,n;\theta)$ denote the a^{th} quantile value of $\underline{p}$ conditioning on (c,n) for a given value of θ. Similarly, we consider the vectors regarding the sum of the base price and the price adjustment of the variable contracts. Note that the moments as a function of θ are calculated using simulation. In our estimation, the simulation size is 5,000.

Let us denote the estimated parameters based on a weighting matrix by $\tilde{\theta}_n$. The asymptotic variance of $\sqrt{n}g_n(\theta_0)$, S, can be estimated by:

$$\hat{S} \equiv \frac{1}{n}\sum_t g(w_t,,\tilde{\theta}_n)g(w_t;\tilde{\theta}_n)'.$$

Then we re-estimate the parameters using the optimal weighting matrix $\hat{S}^{-1}$. Let us denote this efficient simulated GMM estimator by $\hat{\theta}_n$.

Under certain regularity conditions, the efficient simulated GMM estimator is asymptotically normally distributed. A consistent estimator of the asymptotic variance of $\sqrt{n}(\hat{\theta}_n - \theta_0)$ is:

$$\left(\frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'}' \hat{S}^{-1} \frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'}\right)^{-1}.$$

Since the moments are calculated by simulation, we use a numerical derivative of $g_n(\cdot)$.

C. Counterfactual Analyses. For the first three scenarios, we simulate our model with different parameters. For Scenario 1, we decrease κ_0 and κ_1 by 10 percent; and for Scenario 2, we increase the mean of η distribution, μ_η,by 10 percent of its absolute value.

For the next three scenarios, we use the estimated parameters but consider a different set of assumptions. In Scenario 3 where the formal solicitation is mandatory, the contractor officer must solicit bids and choose the optimal rate of bid arrival conditioning on solicitation, regardless of the realized value of η. In Scenario 4 where the required minimum number of bids is 2, the contractor officer is provided with two random bidders for each procurement project in the beginning. If she chooses the rate of additional bid arrival to be $\tilde{\lambda}$, the total cost of the procurement is

$$U(\tilde{\lambda}) \equiv \sum_{n=0}^{\infty} \frac{\tilde{\lambda}^n e^{-\tilde{\lambda}}}{n!}\left\{\kappa(n+1) + \left[1-(1-\pi)^{n+2}\right]\underline{p}_{n+2} + (1-\pi)^{n+2}\left[\overline{p} + \int q(s)\overline{f}(s)ds\right]\right\}.$$

Substituting for $\underline{p}_{n+2}$ and $\overline{p}$ gives:

$$\begin{aligned} U(\tilde{\lambda}) &= \sum_{n=0}^{\infty} \frac{\tilde{\lambda}^n e^{-\tilde{\lambda}}}{n!}\left\{\kappa(n+1) + \alpha + (1-\pi)^{n+1}(\beta+\Gamma)\right\} \\ &= \alpha + \kappa(\tilde{\lambda}+1) + e^{-\tilde{\lambda}\pi}(1-\pi)(\beta+\Gamma). \end{aligned}$$

By taking FOC, we derive the optimal $\tilde{\lambda}$, $\tilde{\lambda}^*$ as:

$$\tilde{\lambda}^* = \max\left\{0, \frac{\ln(\pi) + \ln(1-\pi) + \ln[\beta+\Gamma] - \ln\kappa}{\pi}\right\}.$$

In Scenario 5, we consider a first-price sealed bid auction where the bidders observe the total number of participating bidders. If only one bidder participates, his optimal bid is $\alpha + \beta$. Following Lemma 7.3, when there are more than one bidder, a low-cost bidder will bid at $\alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$ and a high-cost bidder will bid at $\alpha + \beta$. The cost of procurement when restricting entry is:

$$\alpha + \beta + \eta,$$

while the counterpart when soliciting bids at the rate of λ is:

$$\begin{aligned} U(\lambda) &\equiv \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} \left\{ \kappa n + \left[1 - (1-\pi)^{n+1}\right] \left(\alpha + \frac{\pi(1-\pi)^n}{1-(1-\pi)^{n+1}} \beta \right) + (1-\pi)^{n+1} (\alpha + \beta) \right\} \\ &= \alpha + \kappa\lambda + e^{-\lambda\pi}\beta. \end{aligned}$$

Given this, the optimal additional bid arrival rate, λ^*, is the maximum of 0 and $(\ln(\pi\beta) - \ln\kappa)/\pi$, and accordingly, imposing entry restrictions is optimal if $\eta \leq \kappa\lambda^* + (e^{-\lambda^*\pi} - 1)\beta$.