Scooping Up Own Debt On the Cheap: The Effect Of Corporate Bonds Buyback on Firm's Credit Condition

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Abstract

The paper constructs a structural model to study the effect of corporate bonds buyback on the firm's credit conditions. The model implies that the firm strategically choose how much debt to buy back and the buyback reduces the firm's probability of default. In contrast to commonly perceived deleverage channel, the model highlights a novel channel that buying back bonds on the cheap transfers value from bondholders to equity holders and incentivizes the equity holders to choose a much lower assets value to declare default. The lowered default boundary furthermore reduces debt overhang and increases return to equity. The virtuous cycle does not stop until the marginal benefit of bonds buyback equals its marginal cost. The model also implies that when bonds market liquidity dries up, the firm should buy back more bonds, as the shortage of liquidity is independent of the firm's fundamental but depresses the market price of bonds. The paper also provides empirical evidences for the implications.

*Department of Economics, University of Illinois. E-mail: huixu2@illinois.edu. An earlier version of this paper was titled "Bonds Buyback, Credit Risk and Liquidity Risk". I am grateful to my committee members: George Pennacchi (Chair), Timothy Johnson, Heitor Almeida, Dan Bernhardt and Alexei Tchistyi for their help and guidance. I also thank Neil Pearson, Joshua Pollet and Yufeng Wu and Finance seminar participants at University of Illinois for their for valuable comments. All errors remain my own.

It is well known that firms have trouble issuing new bonds to raise funding during economic recessions . However, an intriguing fact is that many firms are engaged in bonds buyback at the same time. As noted by *the Wall Street Journal* during the burst of dot-com bubble (Newswires (2000)):

" It's not just investors who are bargain-hunting amid the beaten-down sectors of the corporate-bond market. Companies themselves are beginning to buy back their own debt at discount prices"

...

"Stater Brothers Holdings Inc., a Southern California supermarket chain with strong single-B ratings, saw its bonds fall 20 points to about 80 cents on the dollar after reporting a net loss of \$9.1 million in May. Confident that it would be able to engineer a turnaround, it retired about \$11 million of its debt, realizing an extraordinary gain of \$1.1 million."

The same also has occurred during the Great Recession. An article in *the Wall Street Journal* (Ng (2009)) has noted that

"A number of corporations are quietly buying back bonds on the cheap in the open market as the financial system works its way out of crisis mode. They are taking advantage of depressed prices to save millions of dollars in interest and debt-repayment costs."

Not only bonds buyback is an important corporate finance strategy at micro level, it is also an economically important macroeconomic factor. Using debt repurchase data cover the 1996-2011 period, Julio (2013) documented that total debt repurchase activity has been increasing over time, from \$11.7 billion in 1996 to \$65.3 billion in 2011. In addition, it also shows cyclical pattern at an aggregate level. Begenau and Salomao (2014), Covas and Haan (2011) and Jermann and Quadrini (2012) found that debt repurchase is countercyclical. The countercyclicality of debt repurchase might also contribute to the procyclicality of debt maturity. Chen, Xu, and Yang (2012) found that the average debt maturity is longer in economic expansions than in recessions. This is not surprising given the fact that firms tend to buy more long-term bonds than short-term ones when buying back bonds: Julio (2013) showed that the average maturity of debt drops from 10.84 years to 6.90 years after repurchase.

Despite the significant role played by debt buyback in the financial markets and in the economy, few academic literature has studied it. In this paper, I provide a dynamic structural model for corporate debt buyback. The paper focuses on two questions. The first question is how debt buyback affects on firm's default decision and credit rating. The second question is how liquidity, which is usually scarce during recession, affects firm's buyback decision. Indeed, the two questions are related. As previous literature including He and Xiong (2012), Ericsson and Renault (2006) and Chen, Lesmond, and Wei (2007) pointed out, credit risk and liquidity risk are intricately interconnected. However, to better understand their connection qualitatively and quantitatively, one also needs to consider firm's strategies thoroughly such as rollover and debt buyback when they face those risks.

To study these questions, I employ and augment Leland and Toft (1996) model and provide a much more general framework accounting for *both* debt rollover and repurchase. As in Leland's model, the firm's assets are exogenous and follow a geometric Brownian motion. However, the firm commits itself to a stationary debt maturity structure not only by issuing new debt but also buying back outstanding debt. Depending on the firm's current assets, issuing new debt, paying off maturing debt and buying back outstanding debt can result in capital gain or loss which equity holders have to assume. Any gain would be paid out to equity holders right away and any loss would be paid off by a new contribution from equity holders. The equity holders decide to default when assets drop to an endogenous threshold chosen by the equity holders; i.e. when the equity value reaches zero and the firm stops servicing the debt.

As for the first question, I find that the firm strategically chooses how much debt to buy back and the buyback program reduces the firm's probability of default and consequently improves its credit ratings, relative to the case where the firm does not buy back any outstanding bonds. Moreover, the effect of corporate debt buyback on the firm's credit risk changes with leverage. The model shows that higher leveraged firms are more actively engaged in bonds buyback and the firm's credit condition improves more as a result of the buyback program. The model also shows that debt buyback strategy allows firm to employ more debt and the optimal leverage ratio is higher than what early models predicted.

With regard to the second question, I discuss how market liquidity condition affects equity holder's optimal buyback strategy and, as a feedback, how the strategy dampens the adverse effect of liquidity drought. As market liquidity is well known to be pro-cyclical, the connection between market liquidity and buyback also shed light on the countercyclicality of debt buyback. The model shows that as market liquidity dries up, the firm tends to buy back more bonds from secondary market. As a consequence, the firm opportunistically exploiting the market liquidity condition reduces the unfavorable impact of liquidity deterioration on the firm's credit condition. Following He and Xiong (2012), we quantify the effect of bonds buyback on the firm's credit risk. Depending on the size of liquidity shock, the buyback can reduce the credit spread in an amount of around 10 to 15 basis points for Investment-Grade A firm; and reduce the credit spread in an amount of around 20 to 60 basis points for Speculative Grade BB firms. This also echoes the empirical discovery in Julio (2013) that firm with lower credit ratings are more likely to repurchase debt from secondary market.

I discuss the underlying mechanism and economic intuition behind the results. In the model, equity holders would stop servicing the debt and declare bankruptcy when the assets value is too low. Buying back outstanding bonds when their market price is low can transfer value from bondholders to equity holders. The increased equity value therefore incentivizes equity holders willing to bail out the firm until a much lower assets value and reduces the credit risk of the firm overall. We characterize the value transfer to equity holders and consistent with the results aforementioned, as liquidity cost rises, equity holders tend to buy back more bonds and total value transfer is more substantial. In addition to the value transfer, reduced debt overhang acts as an amplification mechanism. We shows that reduced overhang improves return of equity when assets value becomes higher in the future and thus amplifies the initial value transfer effect. However, as the firm buys back more outstanding bonds, the premium for investors to sell or tender their bonds also goes up. The benefit and cost of bonds buyback eventually determines the optimal buyback strategy.

Our model assumes that the equity holders make the decision on how much debt to buy back. Like most decision variables in corporate finance e.g. investment and leverage, the amount (or proportion) of debt to repurchase also subjects to agency cost.An interesting yet understudied question is to gauge the agency cost on debt buyback. To do so, we compare the equity maximizing debt buyback with the firm-value maximizing debt buyback for firms of different leverages. The model suggests that equity holders tend to under-buy-back the debt for low leverage firms while over-buy-back the debt if the leverage is high. Equity holders choose how much debt to buy back and endogenous default threshold jointly to maximize the equity value. When leverage is low, market value of the debt is less discounted relative to the principal and value transfer is limited, and as a result equity holders would like to choose a smaller proportion of the debt to repurchase and higher default boundary. Although a slight more repurchase of debt increases overall debt value, it will hurt equity holders. When the leverage is high, debt value is greatly discounted and value transfer is considerable. Equity holders thus have incentive to buy back much of the outstanding debt from the secondary market. However, a significant amount of the value becomes deadweight loss during the buyback transaction and it is not efficient tot the firm overall.

Given the importance of debt financing in US financial market and economy relative to equity as well as the huge literature on share repurchase, the early research on debt buyback is really scarce. Kruse, Nohel, and Todd (2014) analyzes the impact of debt tender offers on stock market. They found that debt tender offers increase return of equity in general. Mao and Tserlukevich (2014) builds up a static model and found that debt is cheaper when the outstanding bouds are held by many dispersed creditors. They also argued that debt repurchase increase firm value ex-ante as it makes capital structure more flexible. Another strand of literature focused on debt restructuring when the firm is financial distress. Gertner and Scharfstein (1991) analyzes the condition under which debt-for-equity is exchange is profitable. Cornett and Travlos (1989) analyzes wealth transfer between different security holders during debt restructuring. The present paper is the first dynamic model to study debt repurchase and quantify the effect of debt repurchase on the firm's credit risk. The model also identifies the lower default boundary chosen by equity holders as a new mechanism for firm value to increase ex-ante.

Our model also contributes to a large literature on corporate debt maturity structure, rollover risk and liquidity risk. Among early studies, Almeida, Campello, Laranjeira, and Weisbenner (2009) shows firms with large amount of bonds that matured during the 2008 crisis reduced more investments than the others. Harford, Klasa, and Maxwell (2014) analyzes that firm can use cash holdings to mitigate rollover risk when the debt has a short maturity. Acharya, Gale, and Yorulmazer (2011) explains frequent rollover is one important factor that leads to a sudden freeze in the availability of short-term, secured borrowing. These papers mostly focus on the unfavorable impact of liquidity risk on the firm via debt rollover. Yet, as mentioned before, in reality the firm use sophisticate corporate finance strategies, such as issuing bonds of different maturities(Choi, Hackbarth, and Zechner (2014),Norden, Roosenboom, and Wang (2016)) and buying back cheap bonds, to lessen the adverse effect or even take advantage of the depressed market situation. Accordingly, a more precise assessment of the effect of liquidity risk as well as firm's debt maturity structure calls for a model featuring the strategies.

The remainder of the article proceeds as follows. In section I, I introduce several stylized facts about debt buyback to motivate the model; Section II provides a general framework to study bonds rollover and buyback; Section III parameterizes the general framework and studies the effect of debt buyback on the firm's credit risk and liquidity risk; Section IV discusses other important variables firms have to take into consideration when repurchasing bonds from market and concludes.

I. Stylized Facts

Our model aims at replicating several stylized facts on debt buyback. To motivate our model, we draw existing empirical literature on macroeconomics, finance, accounting and law, and summarize several stylized facts on debt buyback in this section.

1. **Debt buyback is countercyclical**

   Literature on macroeconomics and business cycles have widely documented that debt buyback is countercyclical. Using Flow of Funds Accounts of the Federal Reserve Board, Jermann and Quadrini (2012) shows that debt repurchase is strongly countercyclical. In Figure 1 , we plot both debt repurchase as well as credit spread for BofA Merrill Lynch US corporate AA and B firms. Credit spread is also known for moving counter-cyclically over business cycle and it is a critical variable representing cost of debt finance that firms have to take into account. It clearly shows that strong countercyclicality of debt repurchase activity by firms. Begenau and Salomao (2014) uses CRSP/Compustat Merged Fundamentals Quarterly reached the similar conclusion for both small and large firms.

The macroeconomic literature usually focus on firm's trade-off between equity and debt over business cycle and emphasize the effect of borrowing constraint on the trade-off. Moreover, they also usually define debt repurchase as reduction in outstanding debt. A reduction in outstanding debt does not necessarily mean that firms are "repurchasing" debt. It could simply means that firm temporarily suspend issuing new debt after outstanding debt matures. To examine the actual debt repurchase activity in details, Figure 2 plots the yearly data of open market repurchase and tender offers firm engage in bonds market from Julio (2013), along with credit spread of BofA Merrill Lynch AA and B firms. Table V also calculate the correlation coefficients between open market repurchase, tender offers and credit spread. The correlation coefficients show that the debt buyback activity is still countercyclical.

2. **Debt buyback can be achieved in various ways**

When firms try to buy back debt from secondary market, there are numerous ways to do so. The typical methods include open market repurchase, tender offer, debt-for-equity exchange.

Open market repurchase means the issuer firm directly buys back bonds from secondary market. The firm can remain anonymous and take advantage of the distressed debt pricing. However, open market repurchase can only buy back a small proportion of outstanding bonds in a short time and subject to a serious of legal restrictions. For the firm to buy back a larger proportion of bonds in a short time, tender offer is often employed. Moreover, if ever there are covenants restricting debt repurchase, a consent solicitation approved by certain number of bonds holders can slack the covenant. To tender a large amount of bonds, the firm has to offer a compelling premium to the bondholders and this makes tender offer very costly.

Debt-for-equity exchange (or debt-to-equity swap) is often used by firms in financial distress or short of cash holdings (Butler (2010)). Debt-for-equity exchange is similar to tender offer in many aspects. However, by exchanging debt for equity, the firm avoids using cash. All these methods have their own advantages and disadvantages and are substitutes for each other. For example, Table V shows the correlation between open market repurchase and tender offers are negatively correlated, implying the firm substitutes one with the other during the sample period.

The debt buyback approach in our model can be interpreted as either open market repurchase or tender offer or debt-for-equity swap[1]. As in most Leland-type models, our model does not feature cash holdings. We discuss the role of cash hoardings in debt buyback in Section IV.

3. **Debt buyback is correlated with firm and debt characteristics**

Julio (2013) found that debt buyback improves the firm's investment distortions. Firms with higher leverage are more likely to repurchase debt, as the improvement is more salient for higher leverage firms. He also discovered that average credit ratings for repurchased bonds prior to the repurchase are declining and while the credit ratings stabilize and even increases following the buyback. Xu (2014) found similar pattern, although she mainly focused on callable bonds.

Debt characteristics also affect firm's buyback activities. Julio (2013) found that firms bought back more long-term bonds than short term bonds, with average maturity being 10.84 years for repurchased debt prior to repurchase whilst the average maturity shortens to 6.9 years after repurchase. Moreover, firms are more likely to buy back convertible bonds through open market repurchase instead of tender offer. Open market repurchase consists of 40% convertible bonds while tender offer is only composed of 6.7%.

[1]Another important way to buy back bonds is to write the call provision in the initial contract at date-0, i.e. issuing callable bonds. This feature makes callable bonds different from other buyback methods we mentioned here. For details of pricing callable bonds and their effect on the firm's default risk, see Acharya and Carpenter (2002), Jarrow, Li, Liu, and Wu (2010) and Leland (1998)

II. The Model

In this section, I provide a general framework to study bonds rollover and buyback based on Leland and Toft (1996).

A. *Firm and Assets*

Unlevered value of firm's assets V follows a geometric Brownian motion given by

$$\frac{dV}{V} = (r - \delta)dt + \sigma dz \tag{1}$$

where $\{z_t\}$ is a standard Brownian motion. r is the risk-free interest rate; δ is the payout rate; σ is the volatility of asset value; r, σ, δ are assumed to be constants.

B. *Debt Structure*

Suppose firm has 1 unit of outstanding debt in total and time-to-maturities, s, are distributed on a finite interval $[0, T]$. To isolate the effect of maturity structure, all of the outstanding bonds are assumed to be of equal seniority. 1 unit of outstanding debt allows us to use a probability density function $\kappa_t(s)$ to denote the amount (or, fraction) of debt maturing in s periods from date t. Specifically, there are bonds in the amount of $\kappa_t(s)\, ds$ with time-to-maturity s at date t, and $\kappa_t(s)$ satisfies

$$\int_0^T \kappa_t(s) ds = 1 \tag{2}$$

Also let $\mathscr{K}_t(s)$ denote the corresponding cumulative distribution function. We make the following assumptions.

ASSUMPTION 1: *Equity holders control the firm and commit to a stationary structure through continuously repurchasing debt as well as rolling over maturing debt. See Figure 3.*

ASSUMPTION 2: *The debt structure $\kappa_t(s)$ takes U-shape. Formally, $\kappa_t(s) \leq 0\,, \forall s \in [0, T^*]$ and $\kappa_t(s) \geq 0\,, \forall s \in [T^*, T]$.*

Consistent with empirical evidences, a U-shape $\kappa_t(s)$ implies that the firm issues more short-term debt and buys back more long-term debt. The second assumption is not critical, as the model can be modified easily to accommodate *any* debt structure $\kappa_t(s)$.

[Place Figure 3 about here]

A stationary debt structure means that

$$\kappa_t(s) = \kappa_{t+\triangle t}(s), \forall \triangle t \tag{3}$$

During $[t, t+\triangle t]$, debt repurchasing and issuing does not change the total debt outstanding. Hence for the debt maturity structure considered here $\kappa_t(s)$, we have

$$\underbrace{\int_0^{\triangle t} \kappa_t(s)ds}_{1} + \underbrace{\left(\int_{T^*}^{T} \kappa_t(s)ds - \int_{T^*-\triangle t}^{T-\triangle t} \kappa_{t+\triangle t}(s)ds\right)}_{2} = \underbrace{\left(\int_0^{T^*-\triangle t} \kappa_{t+\triangle t}(s)ds - \int_{\triangle t}^{T^*} \kappa_t(s)ds\right)}_{3} + \underbrace{\int_{T-\triangle t}^{T} \kappa_{t+\triangle t}(s)ds}_{4} \tag{4}$$

At date t, the debt structure is given by $\kappa_t(s)$. The first term accounts for the debt that matures during $[t, t+\triangle t]$; The second term accounts for debt buyback from the secondary market. Note the debt with time-to-maturities ranging over $[T^*, T]$ at date t will have time-to-maturities ranging over $[T^*-\triangle t, T-\triangle t]$ at date $t+\triangle t$. This explains the shift of lower and upper bounds of integrals in term (2); The third term accounts for the debt rollover: debt that just matured recently is refinanced by issuing new debt with time-to-maturities ranging over $[\triangle t, T^*]$ at date t; The fourth term accounts for newly issued debt with maturity T. The rollover and buyback also shows in Figure 3.

By changing the bounds of integrals, (4) is equivalent to

$$\int_0^{\triangle t} \kappa_t(s)ds + \int_{T^*}^{T} \left(\kappa_t(s) - \kappa_{t+\triangle t}(s-\triangle t)\right) ds = \int_{\triangle t}^{T^*} \left(\kappa_{t+\triangle t}(s-\triangle t) - \kappa_t(s)\right) ds + \int_{T-\triangle t}^{T} \kappa_{t+\triangle t}(s)ds \tag{5}$$

Substitute (5) with (3), it yields:

$$\int_0^{\triangle t} \kappa_t(s)ds + \int_{T^*}^{T} \Big(\kappa_t(s) - \kappa_t(s-\triangle t)\Big) ds = \int_{\triangle t}^{T^*} \left(\kappa_t(s-\triangle t) - \kappa_t(s)\right) ds + \int_{T-\triangle t}^{T} \kappa_t(s)ds \tag{6}$$

Differentiate with respect to $\triangle t$ on both sides of (6) and let $\triangle t \to 0$, we have

$$\kappa_t(0) + \int_{T^*}^{T} \kappa_t'(s)ds = \int_0^{T^*} (-\kappa_t'(s))ds + \kappa_t(T) \tag{7}$$

The left side of Equation (7) is the total reduced bonds, including bonds that just matured and bought back. The right side of Equation (7) is the total bonds newly issued.

Equation (7) implies a particular way the firm manages the debt maturity structure by debt buyback and rollover. At each instant dt, $\kappa(0)\,dt$ amount of debt matures; for the debt with time-to-maturities $s \in [T^*, T]$, the firm buys an amount of $\kappa'(s)\,ds\,dt$ back from the open market; The firm also issues new bonds with time-to-maturities $s \in (0, T^*]$ in an amount of $-\kappa_t'(s)\,ds\,dt$, and new bonds with maturity T in an amount of $\kappa(T)\,dt$. Eventually, the firm manages to maintain a stationary debt structure represented by $\kappa_t(s)$. Since the debt maturity structure $\kappa_t(s)$ is time-homogeneous and does not depend on t, we will drop subscript t and use $\kappa(s)$ to denote it below.

C. Secondary Market

We follow Amihud and Mendelson (1986) and He and Xiong (2012), assuming an illiquid secondary bond market. Each bond investor subjects to an idiosyncratic Poisson liquidity shock with intensity λ. Upon the arrival of the liquidity shock, the bond investor has to sell his bond holdings at a fractional cost of k, and exit the market. The presence of liquidity shock, on one hand, causes higher discount and reduces the market value of new bonds; On the other hand, from the perspective of the firm, it is an great opportunity to buy back the bonds on fire sale. Intuitively, buying back bonds on cheap has many benefits, such as decreasing the leverage ratio, reducing the repayment burden in the future and alleviating debt overhang effect against new investment. Henceforth, we assume that the firm can buy back a proportion of the bonds sold by bond investors who got struck by liquidity shocks.

We implicitly assume that bond investors do not care to whom they sell the bonds, upon the arrival of the liquidity shock. This is consistent with the real bonds market. The secondary bond market is highly illiquid and fragmented than the stock market(Acharya, Amihud, and Bharath (2013), Bao, Pan, and Wang (2011), Bushman, Le, and Vasvari (2010)). As documented by Levy and Shalev (2013), the transaction of corporate bonds in the secondary market usually takes place between two deals over the phone. The dealer who sells the bonds is not aware of who the end counterparty to the transaction is, whether the other dealer is buying corporate bonds on behalf of himself or as an agent for a different party.

Although firms can buy back bonds quietly by open market repurchase, it is subject to negotiation and the amount of bonds is limited. By paying a premium, firms can buy back a larger amount of bonds in a shorter time via tender offer. In addition, the information about the stealthy repurchase is usually disclosed in the following statements and sophisticate bondholders will take the information into consideration in the future. Either way, firms are likely to pay a fractional cost ϕ, which is higher than $1-k$ received by bond investors, for each share of bonds bought back.

Therefore, as can be seen below, given the firm's default boundary, the debt buyback in the secondary market does not change the way investors value the bonds. However, the debt buyback has an significant effect on the firm's endogenous default boundary and as a result, affects the market value of the bonds through bondholder's rational expectation.

D. *Debt Valuation*

In this subsection, we characterize the value of bonds. Let V_B be the assets value when equity holders choose to default. Taking V_B as given, the current market value of one unit of debt, $d(V,s;V_B)$, with a time-to-maturity of s, coupon payment of c and a principal value of p when current assets is V satisfies the following partial differential equation (P.D.E):

$$r\cdot d(V,s;V_B) = c-\lambda\cdot k\cdot d(V,s;V_B)-\frac{\partial d(V,s;V_B)}{\partial s}+\frac{\partial d(V,s;V_B)}{\partial V}(r-\delta)V+\frac{1}{2}\frac{\partial^2 d(V,s;V_B)}{\partial V^2}\sigma^2V^2 \quad (8)$$

To pin down the bond price, two boundary conditions are needed. When time-to-maturity $s=0$, the bond investors can claim the principal value p if the assets value V is greater than the default threshold V_B, i.e.

$$d(V,0;V_B) = p, \forall V \ \geq V_B \quad (9)$$

The other boundary condition describes the payoff to bondholders when equity holders choose to default. Since all bonds are of equal seniority, the assets value that goes to bonds with time-to-maturity s upon default is $\kappa(s)\, V_B$. Noting the total amount of bonds with time-to-maturity s is $\kappa(s)$, each unit of bonds will receive V_B as a consequence, i.e.

$$d(V_B, s; V_B) = V_B, \forall s \in [0, T] \tag{10}$$

The solution to (8) with (9) and (10) is given by

$$d(V, s; V_B) = \frac{c}{r + \lambda k} + e^{-(r+\lambda k)s} \left(p - \frac{c}{r + \lambda k}\right) (1 - F(s)) + \left(\alpha V_B - \frac{c}{r + \lambda k}\right) G(s) \tag{11}$$

where

$$F(s) = N(h_1(s)) + (\frac{V}{V_B})^{-2a} N(h_2(s)) \tag{12}$$

$$G(s) = (\frac{V}{V_B})^{-a+z} N(q_1(s)) + (\frac{V}{V_B})^{-a-z} N(q_2(s)) \tag{13}$$

$$q_1(s) = \frac{(-b - z\sigma^2 s)}{\sigma\sqrt{s}}; q_2(s) = \frac{(-b + z\sigma^2 s)}{\sigma\sqrt{s}} \tag{14}$$

$$h_1(s) = \frac{(-b - a\sigma^2 s)}{\sigma\sqrt{s}}; h_2(s) = \frac{(-b + a\sigma^2 s)}{\sigma\sqrt{s}} \tag{15}$$

$$a = \frac{r - \delta - \frac{\sigma^2}{2}}{\sigma^2}; b = \ln(\frac{V}{V_B}); z = \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{\frac{1}{2}}}{\sigma^2} \tag{16}$$

The result is similar to the bond price derived in Leland and Toft (1996) and He and Xiong (2012). Yet, the total market value of outstanding debt, $D(V, s; V_B)$ depends on the debt maturity structure $\kappa(s)$ the firm chooses to maintain.

$$D(V; V_B) = \int_0^T \kappa(s) d(V, s; V_B)\, ds \tag{17}$$

E. *Equity Valuation*

In this subsection, we will derive the equity value E and endogenous default threshold V_B. As there are transaction costs in trading bonds, part of the firm value accrues to neither bondholders nor equity holders. To derive equity value E, note that E satisfies the following differential equation

$$\begin{aligned} rE = & \underbrace{(r-\delta)VE_V + \frac{\sigma^2}{2}V^2E_{VV}}_{1} + \underbrace{\delta V - (1-\pi)c}_{2} + \\ & \underbrace{\kappa(T)d(V,T;V_B) + \int_0^{T^*}(-\kappa'(s))d(V,s;V_B)ds}_{3} \underbrace{-\kappa(0)p}_{4} - \underbrace{\phi\int_{T^*}^{T}\kappa'(s)d(V,s:V_B)ds}_{5} \end{aligned} \tag{18}$$

The left hand side of (18) is the required return of holding equity; Term (1) on right hand side of (18) is equity change caused by underlying assets fluctuation. Term (2) is the payout plus tax benefits of debt minus coupon payment; Term (3) is the market value of newly issued bonds [2]; Term (4) is equity holders' payment on principal due; Term (5) is equity holder's expense on bonds buyback ($\phi > 0$). Note that (18) reduces to equation (11) in He and Xiong (2012) by letting $\kappa = \frac{1}{T}$.

E.1. A measure of maturity risk

Before solving (18) and deriving default boundary V_B, we first examine the terms (3),(4) and (5), as they are the terms of rollover and buyback and affect equity holders' decision of default. Assuming firms pay competitive price when buying back the bonds, i.e. $\phi = 1$, the second line of equation (18) becomes

$$\kappa(T)d(V,T;V_B) - \kappa(0)p - \int_0^T \kappa'(s)d(V,s;V_B)ds \tag{19}$$

Note that $d(V,0;V_B) = p$. Integrating by parts, (19) can be rewritten as

$$\int_0^T \kappa(s)\frac{\partial d(V,s;V_B)}{\partial s}\,ds \tag{20}$$

Note (20) does not rely on Assumption (2) and can be interpreted as the maturity risk of debt equity holders face. It is the weighted average of sensitivity of debt market value with respect to time-to-maturity. However, as we assume that firms pay a premium when purchasing bonds in the secondary market, $\phi > 1-k$ in (18). Therefore, term (3),(4) and (5) in (18) do not necessarily have a simple form like (20) any more.

[2]The firm might also incur cost when issuing bonds, such as underwriter compensation. I leave out the cost of issuing bonds as it is not the focus of the current paper.

E.2. Equity Valuation

We can solve the equity value, E, in the closed form by guess and verify. The expression of E is provided in the appendix. The endogenous default boundary V_B satisfies smooth-pasting condition

$$E_V|_{V=V_B} = 0 \tag{21}$$

and is given in Theorem II.1.

Theorem II.1.

$$V_B = \frac{\frac{(1-\pi)c+\kappa(0)p-\kappa(T)Q_1(T)+\int_0^{T^*}\kappa'(s)Q_1(s)ds+\phi\int_{T^*}^{T}\kappa'(s)Q_1(s)ds}{\eta} + \left(\kappa(T)Q_2(T) - \int_0^{T^*}\kappa'(s)Q_2(s)\,ds - \phi\int_{T^*}^{T}\kappa'(s)Q_2(s)\,ds\right)}{\frac{\delta}{\eta-1} + \alpha\left(\kappa(T)(B(-u,T)+B(u,T)) - \int_0^{T^*}\kappa'(s)(B(-u,s)+B(u,s))ds - \phi\int_{T^*}^{T}\kappa'(s)(B(-u,s)+B(u,s))ds\right)} \tag{22}$$

where

$$Q_1(T) = \left(\frac{c}{r+\lambda k} + e^{-(r+\lambda k)T}(p - \frac{c}{r+\lambda k})\right) \tag{23}$$

$$Q_2(T) = \left(p - \frac{c}{r+\lambda k}\right)(b(-a,T) + b(a,T)) + \frac{c}{r+\lambda k}(B(-u,T) + B(u,T))) \tag{24}$$

$$b(u,s) = \frac{e^{-(r+\lambda k)s}}{z+u}\left(N\left(u\sigma\sqrt{m}\right) - e^{rs}N\left(-z\sigma\sqrt{m}\right)\right) \tag{25}$$

$$B(u,s) = \frac{1}{z+u}\left(N\left(u\sigma\sqrt{s}\right) - e^{\frac{1}{2}(z^2-u^2)\sigma^2 s}N\left(-z\sigma\sqrt{s}\right)\right) \tag{26}$$

$$\eta = z - a, a = \frac{r - \delta - \frac{\sigma^2}{2}}{\sigma^2}, z = \frac{\sqrt{a^2\sigma^4 + 2r\sigma^2}}{\sigma^2}, u = \frac{\sqrt{a^2\sigma^4 + 2(r+\lambda\,k)\sigma^2}}{\sigma^2} \tag{27}$$

III. Debt Buyback

We are interested in the effect of debt buyback on the default decision by equity holders and how the effect changes with market liquidity risk. To focus on the questions and put the model into work, we make two additional assumptions. The goal of the assumptions is to facilitate calibration exercise while keep intuitive interpretation.

ASSUMPTION 3:

$$\kappa(s) = \frac{\beta e^{\beta s}}{e^{\beta T} - 1} \tag{28}$$

See Figure 4. The choice of debt maturity structure echoes Poisson random maturity model of Leland (1994a) and Leland (1998). Also, it is consistent with the empirical evidence that firms buy back more long-term bonds. More importantly, the probability density function considered above has the following property

$$\frac{\kappa'(s)}{\kappa(s)} = \beta \tag{29}$$

It comes with a simple interpretation that the firm early finances and buys back bonds in the proportion of β, and rolls over maturing bonds by issuing new bonds with maturity T. Julio (2013) documented that β is between 5% to 10%. Later we will see that the calibrated model predicts that the β chosen by equity holders lie within this range.

ASSUMPTION 4: $\phi(\beta) = (1-k)e^{\psi\beta}$, *where* $\psi > 0$

$\phi(\beta)$ also has an intuitive interpretation. Note that

$$\frac{\partial\beta}{\frac{\partial\phi}{\phi}} = \frac{1}{\psi} \tag{30}$$

Hence, given the interpretation of β in (29), $\frac{1}{\psi}$ measures the price elasticity of debt buyback. The base $1-k$, which is received by bondholders when forced to sell, is the lowest price the firm can get. The parameter ψ and k will be calibrated later.

A. Model Calibration

To compare and clearly see the effect of debt buyback, I adopt most of parameters from He and Xiong (2012). I set $T = 6$, meaning the firm issues bonds with time-to-maturities spanning from 0 to 6 years. We also set the principal $P = 61.68$. Coupon c is determined in a way such that the new debt is issued at par under the condition that the firm is not engaged in debt buyback, i.e. $d(V, T; VB)|_{\beta=0} = p$. Powers and Mann (2005) found that bondholders respond to higher tender premiums by tendering a greater percentage of their bonds and a 1% increase in tender premium increases the tendering rate by approximately 9%. Thus, we set $\psi = 0.11$.

The calibrated parameters are listed in Table I.

Baseline Parameters	
Debt Tax Benefit Rate	$\pi = 0.27$
Assets Volatility	$\sigma = 0.23$
Interest Rate	$r = 8.0$ %
Payout Rate	$\delta = 2.0$ %
Bankruptcy Recovery Rate	$\alpha = 0.6$
Liquidity Cost	$k = 0.01$
Liquidity Shock Intensity	$\lambda = 1.0$
Current (date-0) Assets Value	$V = 100.0$
Maturity	$T = 6.0$
Coupon	$c = 6.12$
Debt Principal	$p = 61.68$
Price elasticity	$\psi = 0.11$

Table I

B. Default Boundary and Credit risk

In this subsection, I examine how debt buyback affects firm's decision on default and credit risk.

Figure 5 plot the endogenous default boundary V_B and equity value E for different β, given other parameters listed in Table I. 5(a) shows that there is an optimal β^* that maximizes equity value *ceteris paribus*. Specifically, the equity value first increases with β until $\beta^* = 6.85\%$ and then starts to decrease. The default boundary, which shows in 5(b), follows an inverse pattern: it first decreases with β and then starts to increase and has a minimum around $\beta = 20\%$. We will talk about these two extrema in subsection III.E, as the difference between them clearly shows agency cost.

Figure 6 examines the relationship between leverage, equity maximizing β^* and default boundary V_B. I vary debt principal p and search for β^* that maximizes the equity value. Coupon c is determined such that new debt is issued at par given p and β. Consistent with empirical evidence, the model predicts that equity holders are more actively engaged in debt buyback as leverage increases. Figure 6 shows that the proportion of debt repurchased steadily increases from 4% to 8% as leverage rises. Figure 7 compares the default boundary V_B when equity holders choose to buy back debt in the proportion of β^* to the one when they do not buy back at all. The conclusion is that strategic buyback always lowers the default boundary and the effect is more salient as leverage increases. The relationship has important implication on the firm's optimal leverage. Early studies on firm's optimal leverage mostly focus on firm's debt rollover without considering that the firm can also repurchase debt from secondary market. The flexible debt buyback strategies imply that the firm can probably employ more debt than what early models predicted.

C. Liquidity and Debt Buyback

One feature of debt buyback is its countercyclicality: firms tend to buy back more debt during recession. On the other hand, market liquidity is pro-cyclical(Eisfeldt (2004),Brunnermeier and Pedersen (2009),Næs, Skjeltorp, and Ødegaard (2011)). This implies that market liquidity might impact equity holder's choice on debt buyback. I formally explore the relationship in this subsection.

Figure 8(b) shows how equity maximizing β^* changes with market liquidity. 8(a) plots β^* with respect to different liquidity shock intensity λ; 8(b) plots β^* with respect to liquidity cost k. They show that β^* increases with both λ and k yet the rates are different: β^* increases much faster with k. Although higher λ and k both lower the market price of bonds, higher k also lowers $1-k$, the base of buyback price the firm has to pay and thus triggers the firm to buy back more debt from secondary market.

As mentioned before, bonds buyback strategy can potentially increase the optimal leverage of the firm. To see this, I compute the optimal leverage following Leland and Toft (1996). I look for p^* that maximizes equity value plus aggregate debt value, given the coupon such that the new debt is issued at par, i.e.

$$\begin{aligned} &\max_p E(p;V,V_B)+D(p;V,V_B) \\ &\qquad s.t. \quad d(V,T;V_B)=p \end{aligned} \tag{31}$$

The market leverage is then defined as

$$\frac{D(p^*;V,V_B)}{E(p^*;V,V_B)+D(p^*;V,V_B)} \tag{32}$$

Figure 9 plots the optimal leverage with respect to liquidity cost k for $\beta = 0$ and $\beta = 4\%$. In either case, the optimal leverage decreases with liquidity cost k. However, as expected, the debt buyback strategy allows the firm to issue more debt and increases optimal leverage as a result.

Not only market liquidity affects the equity holder's choice on debt buyback, debt buyback also alters the effect of liquidity risk on the firm. Liquidity risk, interacting with default risk, determines the credit spread of a firm together with default risk. To better illustrate the impact of debt buyback, I follow He and Xiong (2012) and compare responses of firms with investment-grade A and speculative-grade BB and different buyback strategies to liquidity shock represented by an increase in k. Specifically, A-rated firms have $\sigma = .21$ and $k = 0.5\%$; BB-rated firms have $\sigma = .23$ and $\kappa = 1\%$. Other parameters are adopted from Table I. . For each type credit rating of firms, I consider two maturities: $T = 6$ and $T = 10$. Principal p and coupon c are determined such that new bonds are issued at par with a credit spread of 100 bps for A-rated firms and with a credit spread of 330 bps for BB-rated firms, given that the firm is not engaged in debt buyback for each maturity T, i.e. p and c are the solutions to

$$\begin{cases} d(V, T; V_B)|_{\beta=0} = p \\ \frac{c}{y}(1 - e^{-yT}) + pe^{-yT} = p \end{cases}$$

where y is the bonds yield. I then let equity holders choose β^* and see how credit spread changes with β^*.

Table II shows the result. Debt buyback reduces the adverse effect of liquidity cost increase on firm's credit spread, compared to the case where firm does not buy back any debt at all. The effect is much stronger for speculative-grade BB bonds than investment-grade A bonds. However, the effect across different maturities (T=6 vs. T=10) is mixed and does not show a clear pattern.

D. Mechanism

So far, we have seen that the results of the calibrated model are in line with the empirical evidences. One might wonder the channel through which debt buyback strategy affects the firm's credit risk and values. In this subsection, we focus on two mechanisms: value transfer from debt holders to equity holders and reduced debt overhang.

Panel A: Investment-Grade A

	k=0.5%				k=1%				k=2%			
	$\beta=0$	$\beta=\beta^*$			$\beta=0$	$\beta=\beta^*$			$\beta=0$	$\beta=\beta^*$		
Maturity (Years)	Spread (bps)	Spread (bps)	△ Spread (bps)	△ Spread (fraction)	Spread (bps)	Spread (bps)	△ Spread (bps)	△ Spread (fraction)	Spread (bps)	Spread (bps)	△ Spread (bps)	△ Spread (fraction)
T=6	100	93.06	-6.94	-6.94%	155.35	146.17	-9.19	-5.91%	265.77	251.54	-14.23	-5.35%
T=10	100	93.38	-6.62	-6.62%	153.47	144.09	-9.38	-6.11%	259.49	244.96	-14.53	-5.60%

Panel B: Speculative-Grade BB

	k=1%				k=2%				k=4%			
	$\beta=0$	$\beta=\beta^*$			$\beta=0$	$\beta=\beta^*$			$\beta=0$	$\beta=\beta^*$		
Maturity (Years)	Spread (bps)	Spread (bps)	△ Spread (bps)	△ Spread (fraction)	Spread (bps)	Spread (bps)	△ Spread (bps)	△ Spread (fraction)	Spread (bps)	Spread (bps)	△ Spread (bps)	△ Spread (fraction)
T=6	330	305.13	-24.87	-7.54%	458.94	422.47	-36.47	-7.95%	700.58	640.46	-60.12	-8.58%
T=10	330	311.82	-18.18	-5.51%	449.61	413.61	-36.00	-8.01%	670.58	609.19	-61.39	-9.15%

Table II

D.1. Value Transfer

In the model, equity holders decide to stop servicing the debt and liquidate the firm when the assets value hits a boundary V_B and the equity value becomes zero. However, when liquidity cost is high, the market value of the debt is also very low. Buying back outstanding bonds on the cheap therefore can transfer value from bondholders to equity holders and increase equity value, compared to the case where the equity holders do not buy back any bonds at all. The transferred value from debt holders thus incentivize the equity holders to bail out the firm to a lower assets value. To see this, the value transferred from debt holders to equity holders at date 0 is

$$\beta\, p - \phi(\beta) \int_0^T \beta\, \kappa(s)\, d(V, s; V_B)\, ds \tag{33}$$

(33) uses the fact that $\kappa'(s) = \beta\kappa(s)$. By buying back outstanding debt in the proportion of β, equity holders have to pay $\phi(\beta)\int_0^T \beta\, \kappa(s)\, d(V, s; V_B)\, ds$ but avoid the principal payment in the amount of $\beta\, p$. Thus (33) represents the value transferred from debt holders to equity holders.

Figure 10 shows the value transfer with respect to different buyback strategies β for $k = 0.01$ and $k = 0.012$. It confirms the idea that buying back debt on the cheap transfers value from debt holders to equity holders. Also, as liquidity cost k gets higher, an increase in β will transfer more value in favor of equity holders, resulting in lower default boundary V_B. However, the buyback strategy and more value transfer does not overturn the adverse effect of higher liquidity cost k on the firm's default boundary and credit risk. Figure 11 plots default boundary V_B with respect to liquidity cost k, with equity holders choosing β^*. It shows that V_B still increases with liquidity cost k. Together, it explains the pattern we have seen in Table II: credit spread decreases when equity holders choose $\beta = \beta^*$ from $\beta = 0$, given k; but increases with k.

D.2. Amplification: Reduced Overhang

Figure 5(a) shows the β^* the equity holders would choose when $k = 0.01$, which is much greater than the β maximizing value transferred from debt holders to equity holders. This implies that there must be other amplification mechanism of the initial value transfer effect. I will argue that the mechanism is reduced debt overhang effect.

Debt-overhang, stated formally in Myers (1977), refers to the fact that part of earnings generated by potential new projects is appropriated by existing debt holders and reduces equity holders' incentive to invest on the projects. The effect is more salient when the firm is under financial distress. Diamond and He (2014) also showed that a higher default threshold is another form of debt overhang in the model with endogenous default boundary. Formal modeling debt overhang requires to specify the firm's production technology. Here I follow Diamond and He (2014) and use the sensitivity of market value of the new debt with respect to current assets value to measure debt-overhang effect, i.e.

$$\frac{\partial d(V, T; V_B)}{\partial V} \tag{34}$$

it measures how much assets value change accrues to debt holders.

Figure 12 plots debt overhang effect $\frac{\partial d(V,T;V_B)}{\partial V}$ as well as endogenous default boundary V_B with respect to debt buyback proportion β. Both variables synchronize to decrease at first and then increase with β. The synchronization reflects the fact that β only affects $d(V, T; V_B)$ via V_B. From (11):

$$\frac{\partial \frac{\partial d(V,T;V_B)}{\partial V}}{\partial \beta} = \frac{\partial \frac{\partial d(V,T;V_B)}{\partial V}}{\partial V_B} \frac{\partial V_B}{\partial \beta} \tag{35}$$

Proposition III.1. *If* $p > \frac{c}{r+\lambda k} > \left(1 + \frac{1}{2a}\right) \alpha V_B$ *and* $a = \frac{r-\delta-\frac{\sigma^2}{2}}{\sigma^2} > 0$, $\frac{\partial \frac{\partial d(V,T;V_B)}{\partial V}}{\partial V_B} > 0$

Proposition III.1 indicates that given debt principal and bankruptcy cost are sufficiently high, a lower default boundary reduces debt overhang. Suppose equity holders start from $\beta = 0$, an slight increase in β transfers value from debt holders to equity holders and lowers V_B ($\frac{\partial V_B}{\partial \beta} < 0$). As a consequence, the lowered V_B reduces the debt overhang effect ($\frac{\partial \frac{\partial d(V,T;V_B)}{\partial V}}{\partial V_B} > 0$). The reduced overhang improves return of equity when assets value becomes high in the future, which incentives equity holders to incur more cost and buy back more debt. Reduced debt overhang amplifies the initial effect of value transfer of debt buyback. In the end, β^* is the optimal point where marginal cost of debt buyback equals its marginal benefit from the perspective of equity holders.

E. *Agency Cost on Debt Buyback*

Hitherto, we have retained the assumption 1 that equity holders choose the debt buyback strategy β. When equity holders make decisions, they do not take into account the externalities of their decisions on debt holders , resulting conflict of interest between equity and debt holders and agency cost. The two famous and well-studied problems on conflict of interest are excessive risk taking (Jensen and Meckling (1976)) and debt overhang (Myers (1977)). In this subsection, we show that agency cost also reflects on the deb buyback strategy and equity maximizing β^* deviates from what is optimal for the entire firm, i.e. equity value plus aggregate debt value.

To gauge the agency cost, I consider equity maximizing β^* and firm value maximizing β^{**} for different principal p outstanding.

$$\begin{aligned} \beta^* &= \arg\max\ E(\beta; p, V) \\ \beta^{**} &= \arg\max\ \{E(\beta; p, V) + D(\beta; p, V)\} \end{aligned} \tag{36}$$

Figure 13 plots β^* and β^{**} as a function p. Interestingly, although equity holders maximizing β^* deviates from β^{**}, the sign of the deviation depends on the leverage: when the leverage is low, $\beta^{**} > \beta^*$, meaning equity holders tend to under-buy-back the bonds compared to what is optimal to the firm; when the leverage is high, $\beta^{**} < \beta^*$ and they tend to over-buy-back the bonds.

To further understand the economic reasons, I consider two specific cases where $p = 80$ (high leverage) and $p = 45$ (low leverage). Table III lists the buyback proportion β, endogenous default boundary V_B, equity value E, debt value D when equity holders choose β^* or firm chooses β^{**} for each p, respectively. If there were no transaction cost, the firm value would have equaled the asset value plus the value of tax benefits minus the value of bankruptcy costs (Leland (1994b)). Or in other words,

$$DWL = V + \frac{\tau c}{r}\left[1 - \left(\frac{V}{V_B}\right)^{-(a+z)}\right] - \alpha V_B \left(\frac{V}{V_B}\right)^{-(a+z)} - (E + D) \tag{37}$$

represents the deadweight loss that occurs during buyback and sales of bonds. This reflects in the last row of Table III.

When maximizing their security value, equity holders decide the endogenous default boundary V_B based on smooth-pasting condition (21) given β. Then equity holders choose the pair $(\beta, V_B(\beta))$ that yield the highest equity value. When leverage is low, the market value of the debt and bonds buyback cost is high relative to the principal outstanding , and thus the value transferred is limited. Therefore, equity holders only would like to buy back a smaller proportion of bonds and choose a higher default boundary, compared to what is optimal to the entire firm. Optimal firm buyback strategy β^{**} requires equity holders to buy more, as it reduces V_B and increases debt value and the increased value exceeds the buyback cost.

When leverage is high, the market value of the debt is low *ceteris paribus* and value transferred to equity holders from buyback is high. Under such circumstance, equity holders choose a higher β and lower default boundary V_B, compared to what is optimal to the firm. Nevertheless, most of the value eventually does not go to equity holders but is lost in the transaction. A decrease in β therefore cuts the transaction cost and increase the firm value overall.

F. Empirical Evidence

One implication of the model is that the firm should buy back more bonds when bonds market liquidity dries up. In this section, we provide some empirical evidences. The evidences serve to peep "the tip of the iceberg" and are no way in place of a rigorous empirical study.

Agency Cost

		V_B	Equity	Debt	DWL
p=80	$\beta^* = 0.118$	60.241	29.269	71.846	3.368
	$\beta^{**} = 0.111$	60.371	29.264	71.858	3.307
	$\triangle(\beta^* \to \beta^{**})$	0.13	-0.005	0.012	-0.061
p=45	$\beta^* = 0.022$	37.807	61.809	50.087	4.937
	$\beta^{**} = 0.055$	37.491	61.734	50.241	4.913
	$\triangle(\beta^* \to \beta^{**})$	-0.316	-0.075	0.154	-0.024

Table III

Figure 14 plots the debt repurchase data from Jermann and Quadrini (2012) and liquidity measure from Corwin and Schultz (2012) spanning from 2004 to 2010. Corwin and Schultz (2012) developed a bid-ask spread estimator from bonds daily high and low prices and is one of the best performed bonds market liquidity proxies along with Roll (1984) and Hasbrouck (2009) [3]. It clearly shows that debt repurchase positively co-moves with illiquidity in the bonds market, with correlation being 0.55. Debt repurchases reaches the peak around 2008-2009 when the corporate bonds are highly discounted.

Nevertheless, in Jermann and Quadrini (2012) debt repurchase is defined as "the reduction in outstanding debt (or increase if negative)" and measured by the negative of "net increase in credit markets instruments of nonfinancial business" in the Flow of Funds accounts of the Federal Reserve Board. This also includes the instances that firms halt new bonds issuance when existing bonds mature. To better match the liquidity environment described in the model to the reality, we focus on bonds tender offer within a window period from 2004 to 2005. The choice is based on two considerations. First, by considering bonds tender offer, we focus on "clean" bonds buyback and rule out the cases where firms suspend issuing new bonds when existing bonds mature. Second, In 2005 May 5th, bonds issued by GM and Ford was downgraded by junk status by S&P. While the downgrade was expected by investors , the timing still came as a shock to the bonds market

[3] The three measures are highly correlated (Schestag, Schuster, and Uhrig-Homburg (2016)), so it is not critical which measure to use.

(Acharya, Almeida, Ippolito, and Perez (2014)). As a result, many insurance companies, pension funds etc. were forced to liquidate the bonds holdings issued by GM and Ford as regulations prevent them from holding junk-rated securities (Acharya, Schaefer, and Zhang (2015)). This bonds market liquidity shock exactly captures what is described in the model. And moreover, it occurs solely within the bonds market and rules out confounding factors in other large scale economic or financial crisis (e.g. the Great Recession) that could also possibly cause debt buyback.

We employ data of total tender amount and the number of bond issues tendered from Mergent Fixed Income Securities Database (FISD). Figure 15(a) shows that the total amount of bonds tender offers strongly and positively co-moves with illiquidity measures, both peaking around May, 2005 when the bonds market liquidity shock occurs. The correlation is 0.07. Figure 15(b) shows the number of bonds tender offers in 2004-2005, with the most tender offers occurring in June, 2005.

A deeper analysis on bonds market liquidity, firm's tender offer decision and tender offer premium calls for data on market price of bonds. However, the data on market price of bonds were sparse back in 2005. The National Association of Securities Dealers (NASD) did not report and publicize information on market transaction of bonds until July 1, 2002, and at the beginning the reporting merely covered investment-grade bonds with initial issuance size greater than 1 billion. The project started to cover 99% of the public transactions beginning from Feburary 7, 2005. (Bao et al. (2011)). Furthermore, linking to Compustat for firm's characteristics results in even less observations. The data deficiency makes it challenging to analyze the causal relationship between bonds market liquidity and firm's tender offer decision. We call for attention of future empirical studies for a more rigorous analysis.

IV. Conclusion

In this paper, we present a model to study bonds buyback, an important yet somehow overlooked corporate finance strategy. Moreover, debt buyback is also a major macroeconomic variable at an aggregate level. We focus on its link to the firm's default risk and market liquidity. Firms strategically choose how much debt to buy back and the decision increases with market liquidity cost. The model shows that bonds buyback can help to reduce the firm's default risk and lessen the adverse effect of liquidity risk on the firm. The reason lies in the fact that debt buyback transfers value from debt holders to equity holders and incentivize equity holders to bail out the firm to a much lower assets level. The higher liquidity cost is, the more the market price of debt is discounted and therefore more value transferred to equity holders. In addition, the lower default boundary also reduces the debt overhang effect and increase the return of equity.

There are two issues our model does not cover. First, the model does not leave room for cash. When the firm buys back debt from secondary market, it is more likely that the firm will use cash hoard. Imperfect capital market makes it costly to issue more equity. Especially, as noted in Myers and Majluf (1984), if firm is short of cash and has to issue more equity to finance, the firm will pass profitable opportunity with asymmetric information. Cash hoard lessens firm's reliance on equity issuance to raise capital. Second, as an assumption to derive closed-form endogenous default boundary V_B, the firm issues new debt and buys back old debt such that the total outstanding principal remains the same. However, the firm usually buys back debt as a way to deleverage. The deleverage has two countervailing effects. On one hand, it reduces total debt outstanding, mitigates debt overhang to a much larger extent, resulting a higher return of equity and lower default boundary; On the other hand, the value transfer from bond holders to equity holders also decrease with leverage and it makes equity holders to buy back less bonds. To analyze the roles of cash and leverage, a more delicate and comprehensive model is needed. We leave these questions to future research.

Appendix

A. *Proof of Theorem II.1*

I take a guess-verify approach to solve the equity value E. Note that in He and Xiong (2012), equity value E satisfies

$$rE = (r-\delta)VE_V + \frac{\sigma^2}{2}V^2E_{VV} + \delta V - (1-\pi)c + \underbrace{\kappa(T)d(V,T;V_B) - \kappa(0)p}_{*} \tag{38}$$

In (18), the underlined part is

$$\kappa(T)d(V,T;V_B) + \int_0^{T^*}(-\kappa'(s))d(V,s;V_B)ds \; - \kappa(0)p - \phi\int_{T^*}^{T}\kappa'(s)d(V,s:V_B)ds$$

Every part including the integral is a linear operator of $d(V,\cdot;V_B)$. Therefore, we conjecture that equity value E satisfying (18) is given by

$$\begin{aligned}
E = {} & V - \frac{\delta V_B}{z\sigma^2}\left(\frac{V}{V_B}\right)^{-\gamma}\frac{1}{\gamma+1} - \frac{1}{z\sigma^2}\left(\frac{1}{\eta} + \frac{1-\left(\frac{V}{V_B}\right)^{-\gamma}}{\gamma}\right)\left((1-\pi)c + \kappa(0)p - \kappa(T)\left(\frac{c}{r+\lambda k} + e^{-(r+\lambda k)T}(p - \frac{c}{r+\lambda k})\right) + \right.\\
& \left.\int_0^{T^*}\left(\frac{c}{r+\lambda k} + e^{-(r+\lambda k)s}(p-\frac{c}{r+\lambda k})\right)\kappa'(s)ds + \phi\int_{T^*}^{T}\left(\frac{c}{r+\lambda k} + e^{-(r+\lambda k)s}(p - \frac{c}{r+\lambda k})\right)\kappa'(s)ds\right) + \\
& \frac{1}{z\sigma^2}\left[\kappa(T)\left(e^{-(r+\lambda k)T}(p-\frac{c}{r+\lambda k})A(T) - (\alpha V_B - \frac{c}{r+\lambda k})A(T)\right) - \int_0^{T^*}\kappa'(s)\left(e^{-(r+\lambda k)s}(p-\frac{c}{r+\lambda k})A(s) - (\alpha V_B - \frac{c}{r+\lambda k})A(s)\right)ds\right.\\
& \left. -\phi\int_{T^*}^{T}\kappa'(s)\left(e^{-(r+\lambda k)s}(p-\frac{c}{r+\lambda k})A(s) - (\alpha V_B - \frac{c}{r+\lambda k})A(s)\right)ds\right]
\end{aligned} \tag{39}$$

It is easy to verify the conjecture by plugging it into (18).

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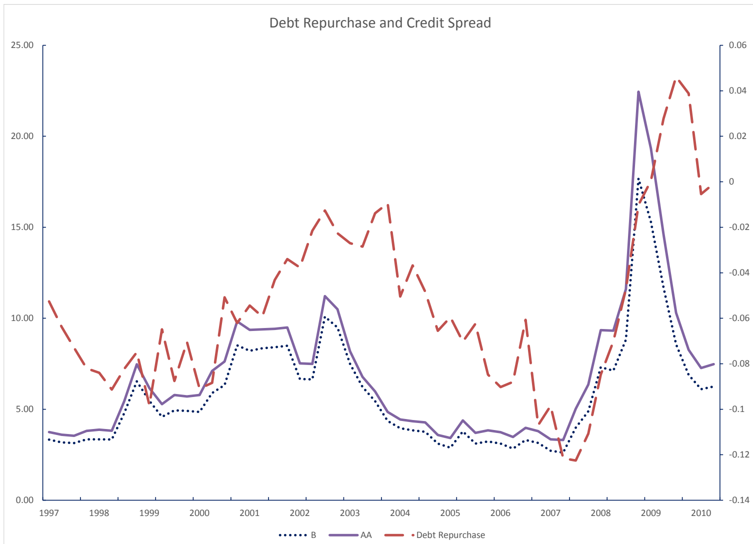


Figure 1. Time series of debt repurchase and credit spread of BofA Merrill Lynch US corporate AA and B firms. The debt repurchase data is from Jermann and Quadrini (2012) and credit spread data are from Federal Reserve Bank of St. Louis

Correlation Coefficients

	Debt Repurchase	B	AA
Debt Repurchase	1		
B	0.5870	1	
AA	0.3882	0.8637	1

Table IV

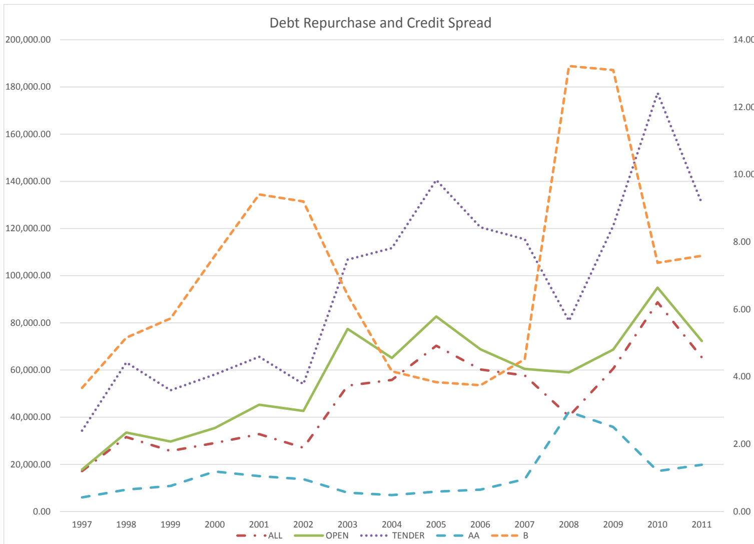


Figure 2. Time series of open market repurchase, tender offers and credit spread of BofA Merrill Lynch US corporate AA and B firms. Yearly data of open market repurchase and tender offers are from Julio (2013). Credit spread data are from Federal Reserve Bank of St. Louis.

Correlation Coefficients

	OPEN	*TENDER*	*AA*	*B*
OPEN	1.0000			
TENDER	-0.2057	1.0000		
AA	0.2419	0.0405	1.0000	
B	0.4453	-0.2263	0.8365	1.0000

Table V

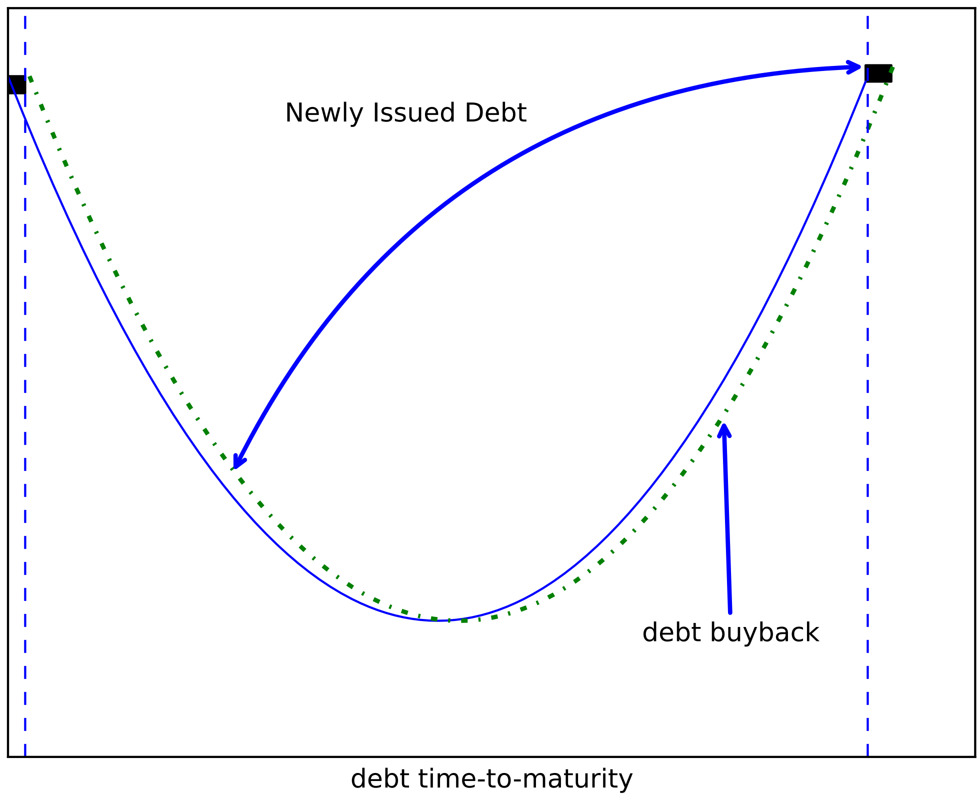


Figure 3. Debt Rollover and Buyback

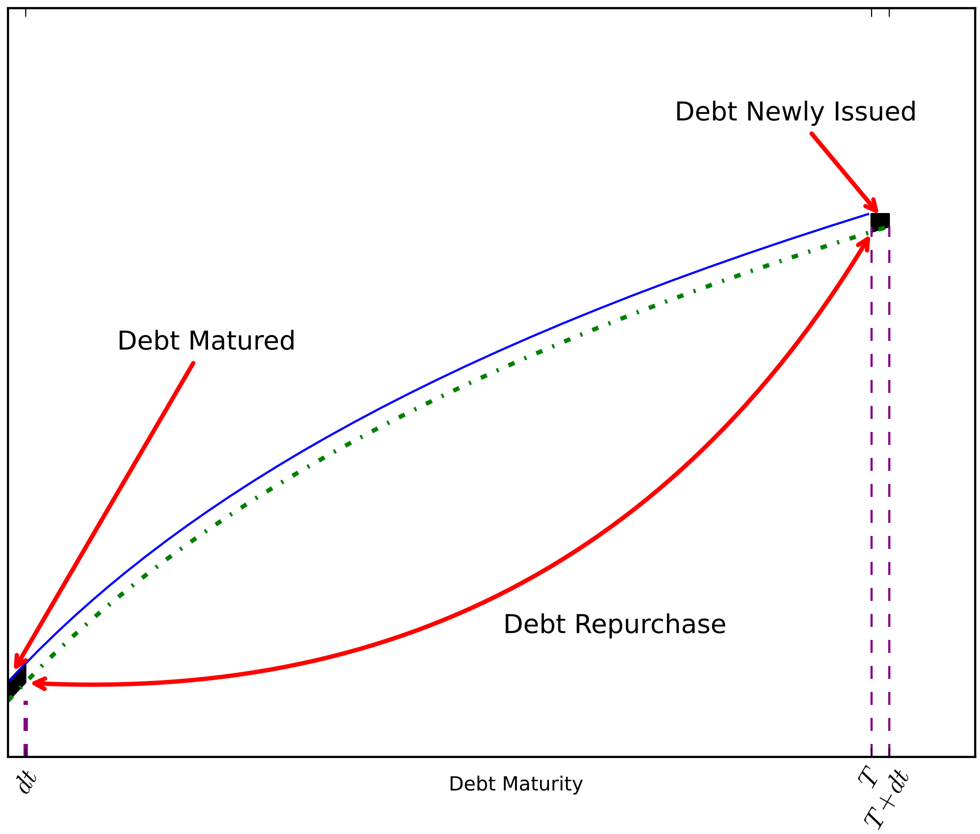
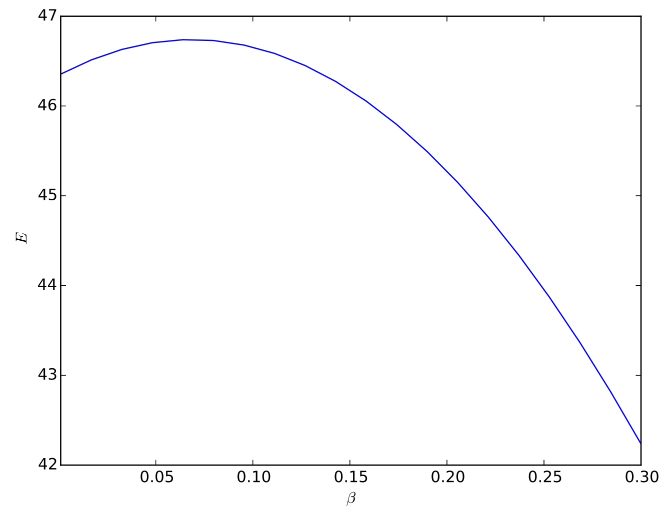
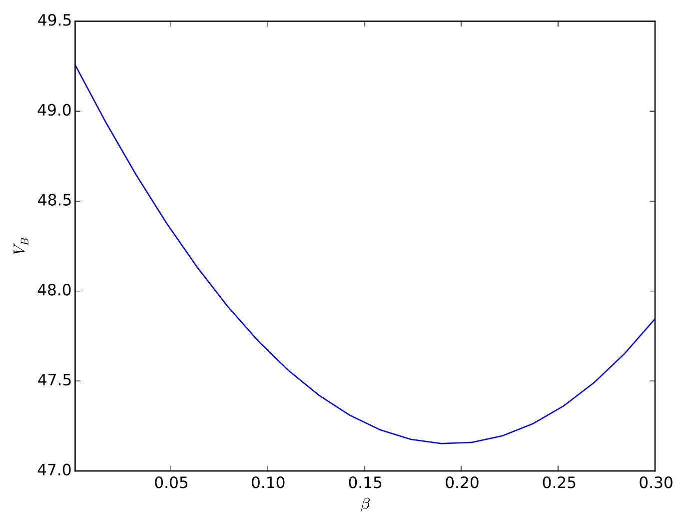


Figure 4. Debt Buyback



(a)



(b)

Figure 5. Equity Value E and Endogenous Default boundary V_B when the firm buys back different proportion (β) of outstanding bonds. The parameters are listed in I.

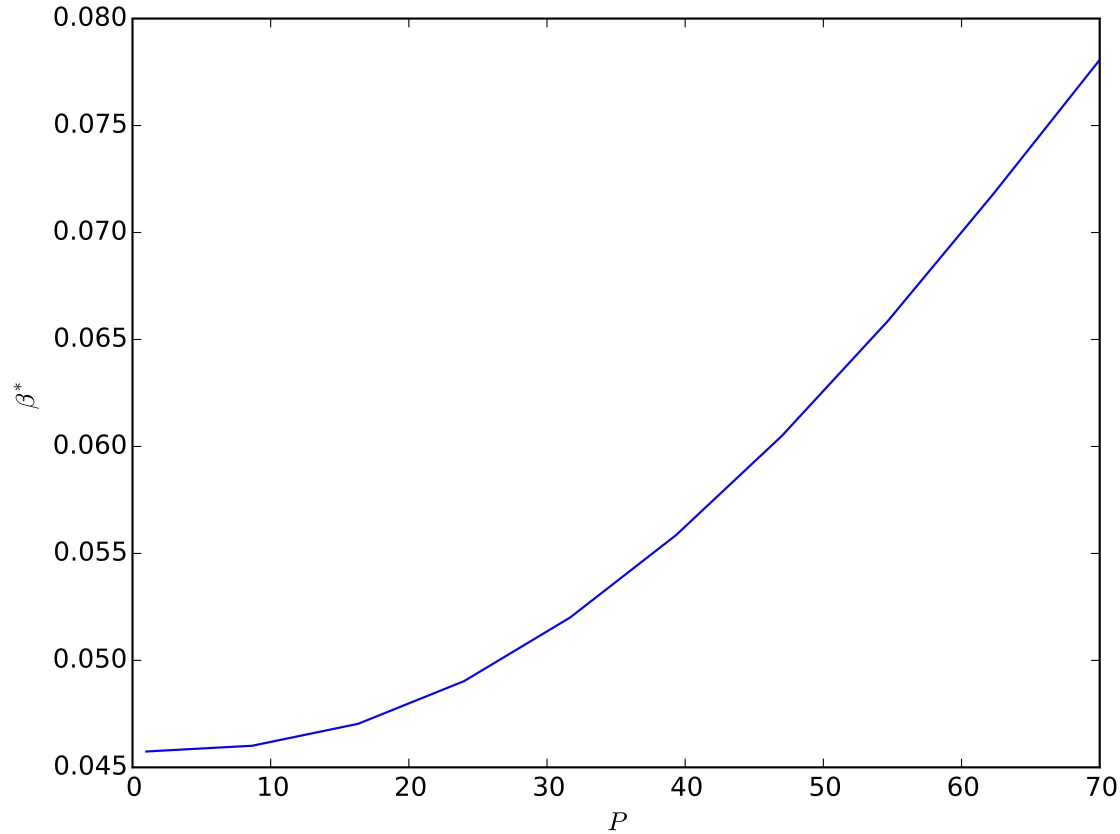


Figure 6. Equity maximizing β^* for different principal outstanding p. The coupon c is determined such that new debt is issued at par given p and β. The rest of the parameters are listed in I

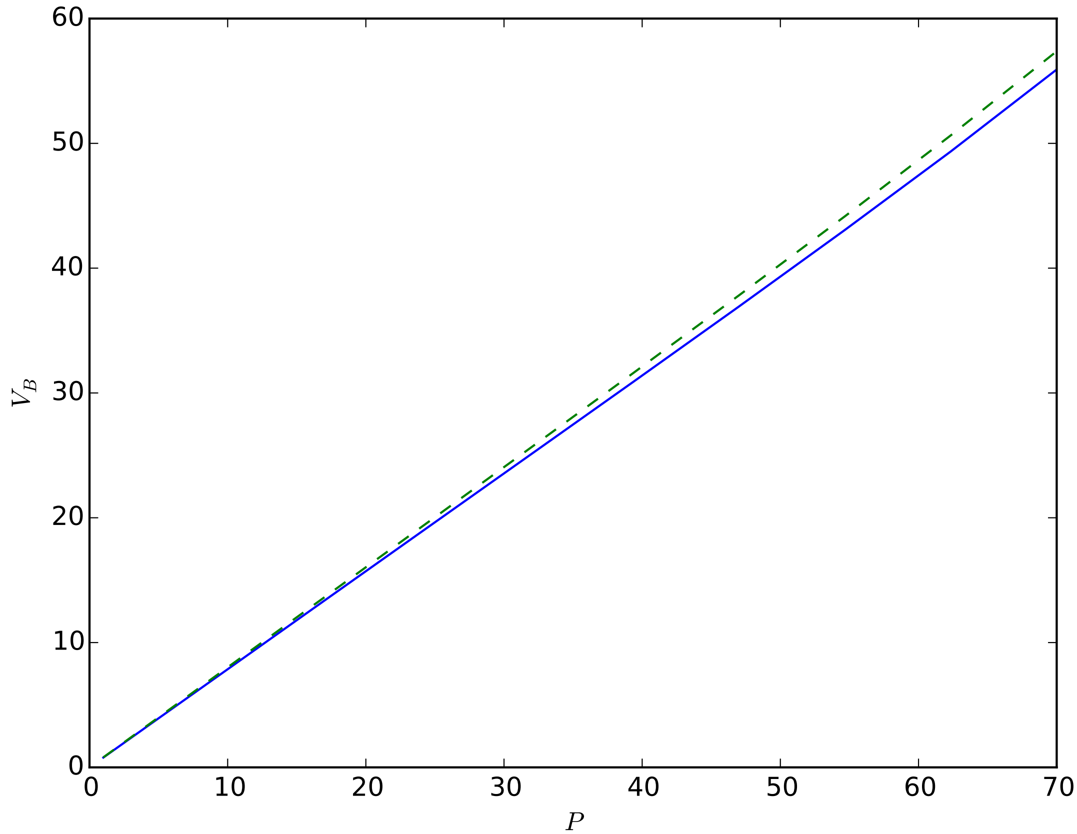
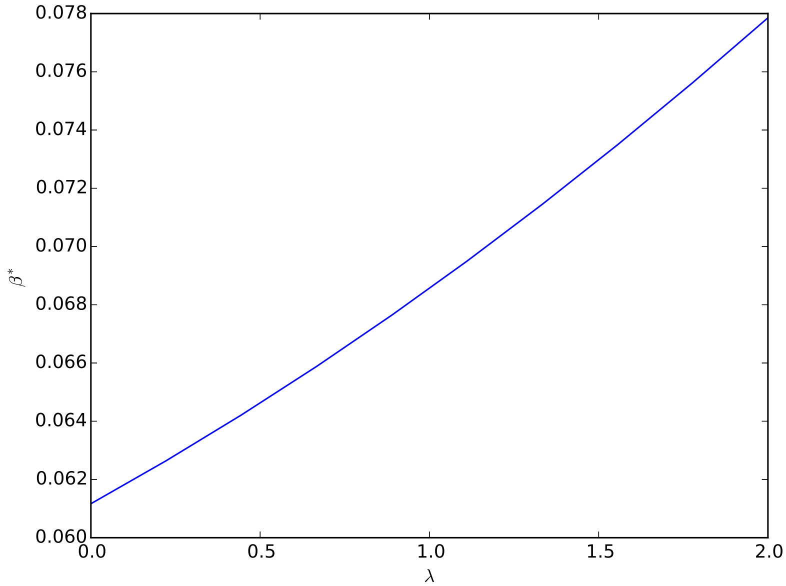
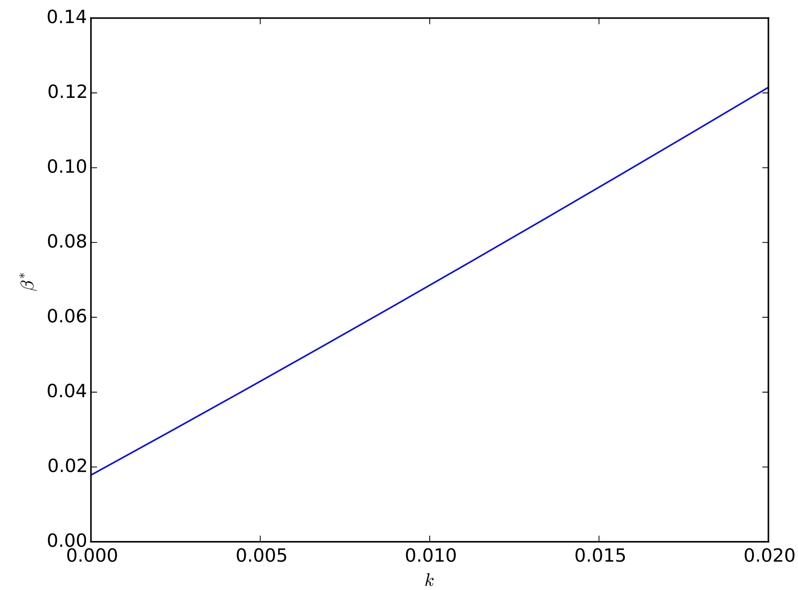


Figure 7. Default Boundary V_B when equity holders optimally buy back debt (solid line) compared to the one when equity holders do not buy back at all (dashed line).



(a)



(b)

Figure 8. Equity maximizing β^* for different liquidity shock frequency λ and bonds transaction cost k. The rest of the parameters are listed in I.

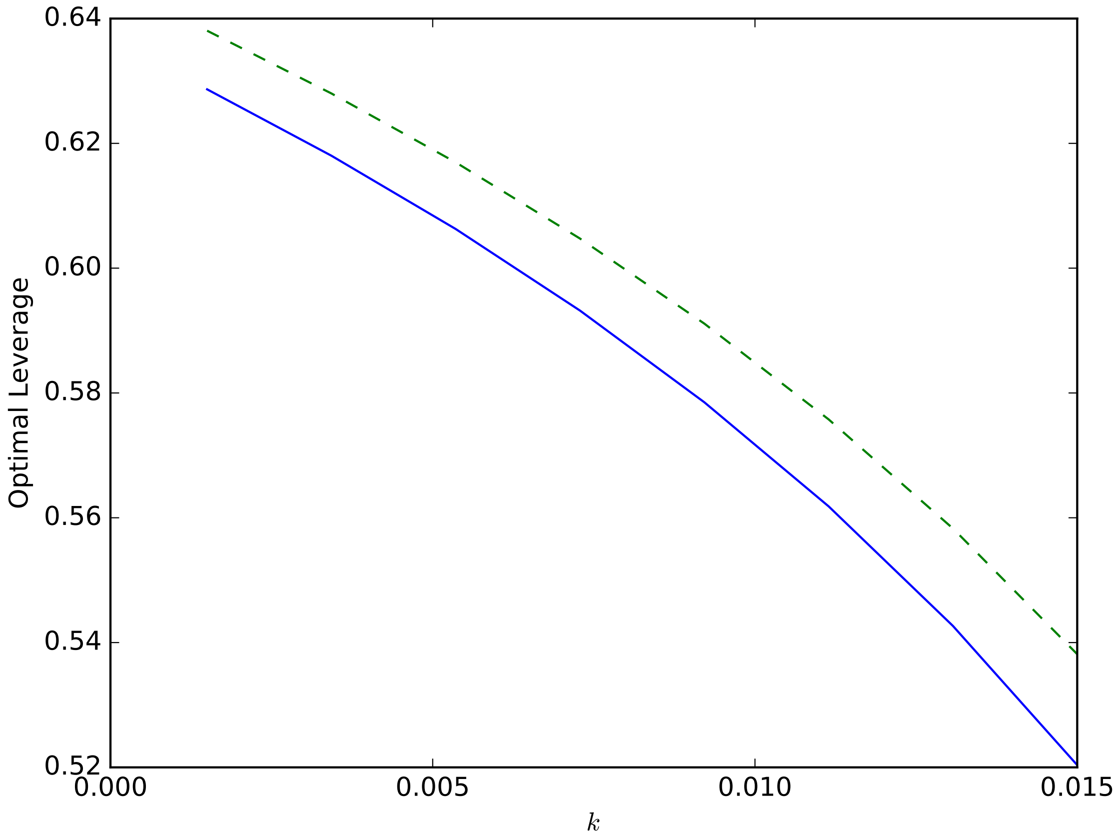


Figure 9. Optimal Leverage Ratio for different liquidity cost k when firms do not buy back bonds at all (solid line) or buy back 4% annually (dashed line).

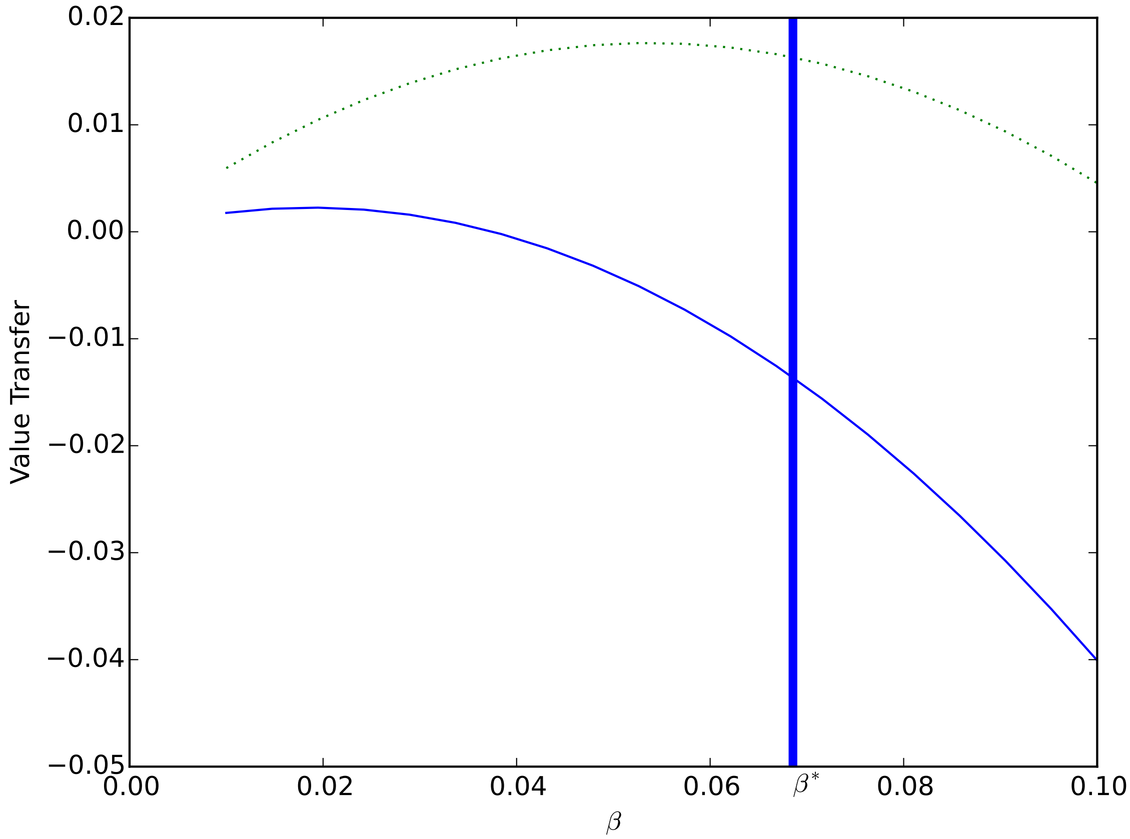


Figure 10. Value transfer from debt holders to equity holders for different buyback strategy β when bonds transaction cost $k = 0.01$ (solid line) and $k = 0.012$ (dashed line). The value transfer is defined as $\beta\, p - \phi(\beta) \int_0^T \beta \kappa(s) d(V, s; V_B)$.

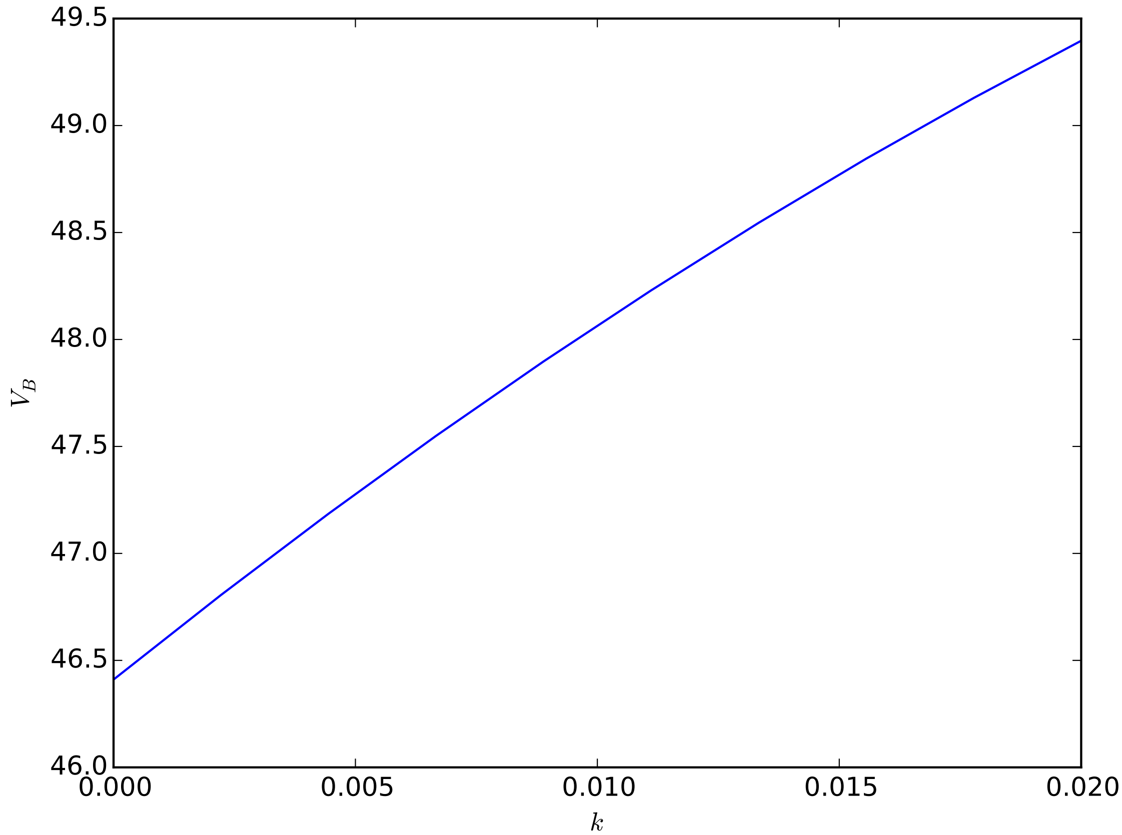


Figure 11. Endogenous Default boundary V_B as a function of liquidity shock cost k when equity holders optimally buy back outstanding bonds.

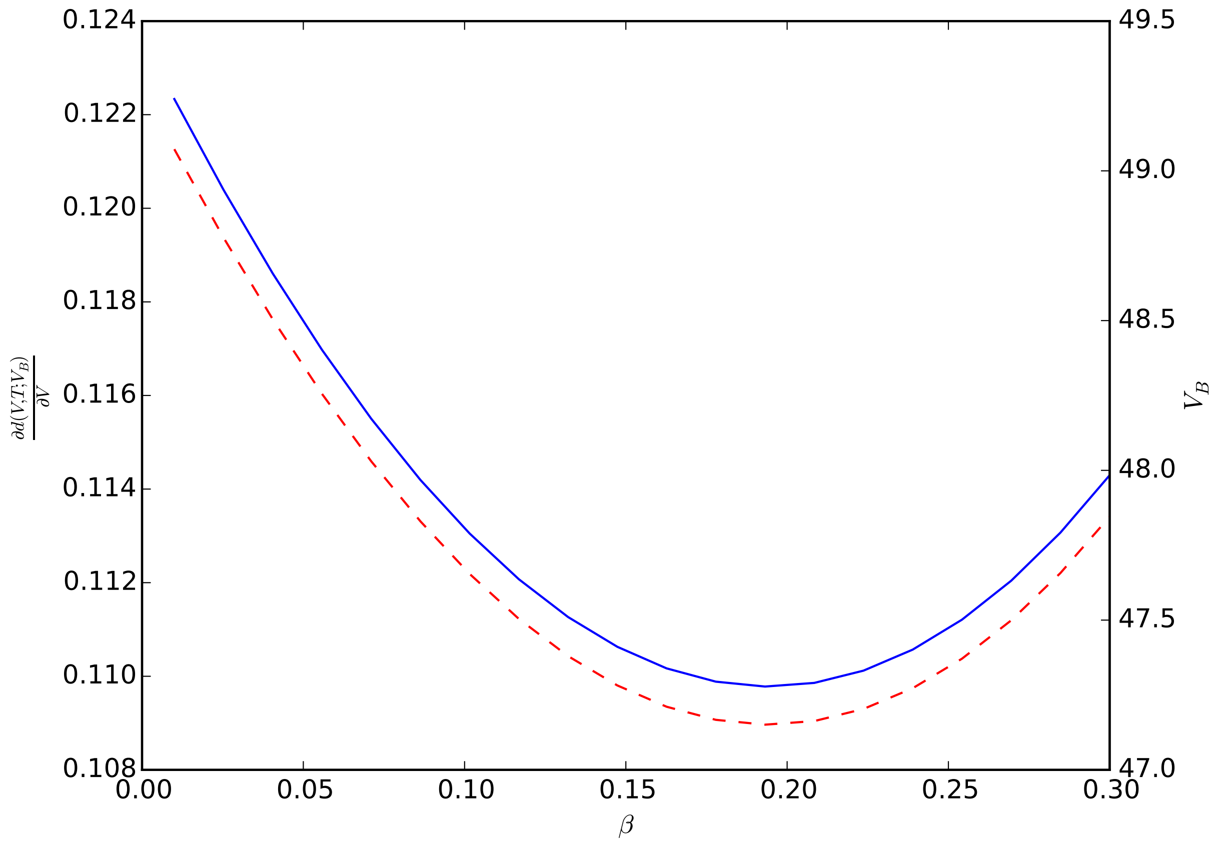


Figure 12. Endogenous Default boundary V_B (dashed line, right axis)and debt overhang $\frac{\partial d(V,T;V_B)}{\partial V}$ (solid line, left axis) for different buyback strategy β.

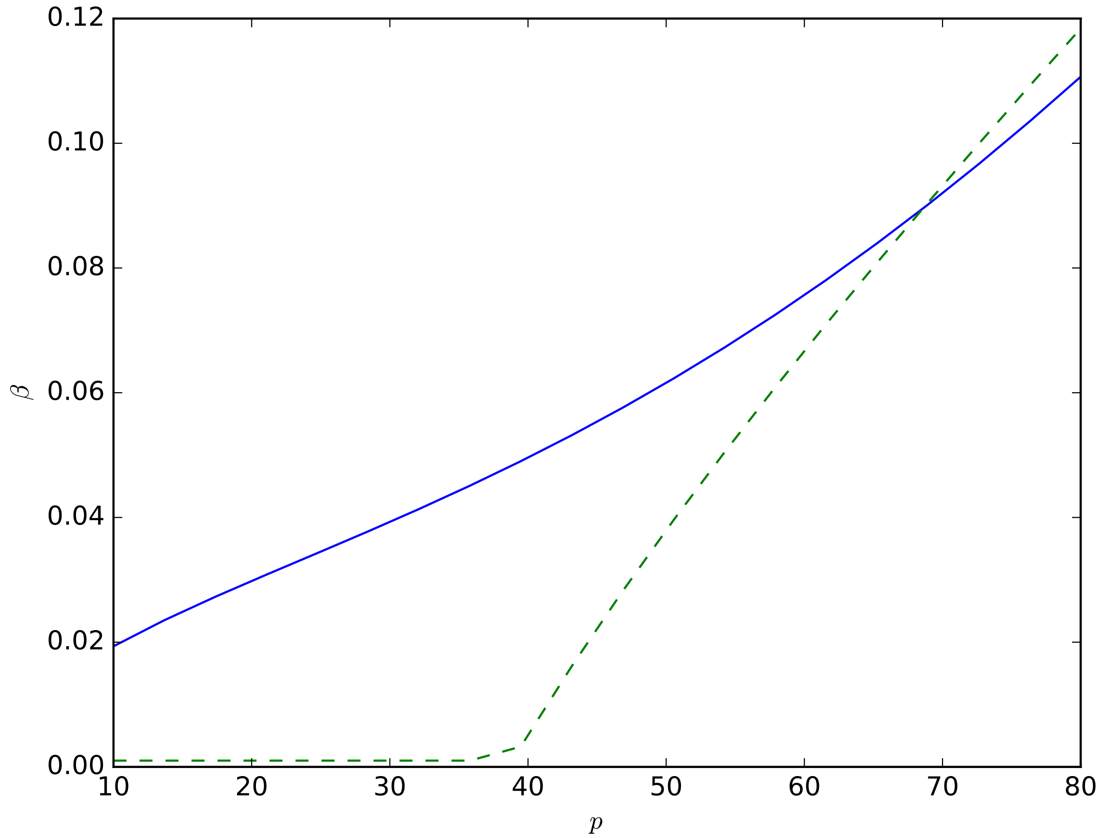


Figure 13. Equity maximizing β^* (dashed line) and firm value maximzing β^{**} (solid line) as a function of debt principal outstanding p.

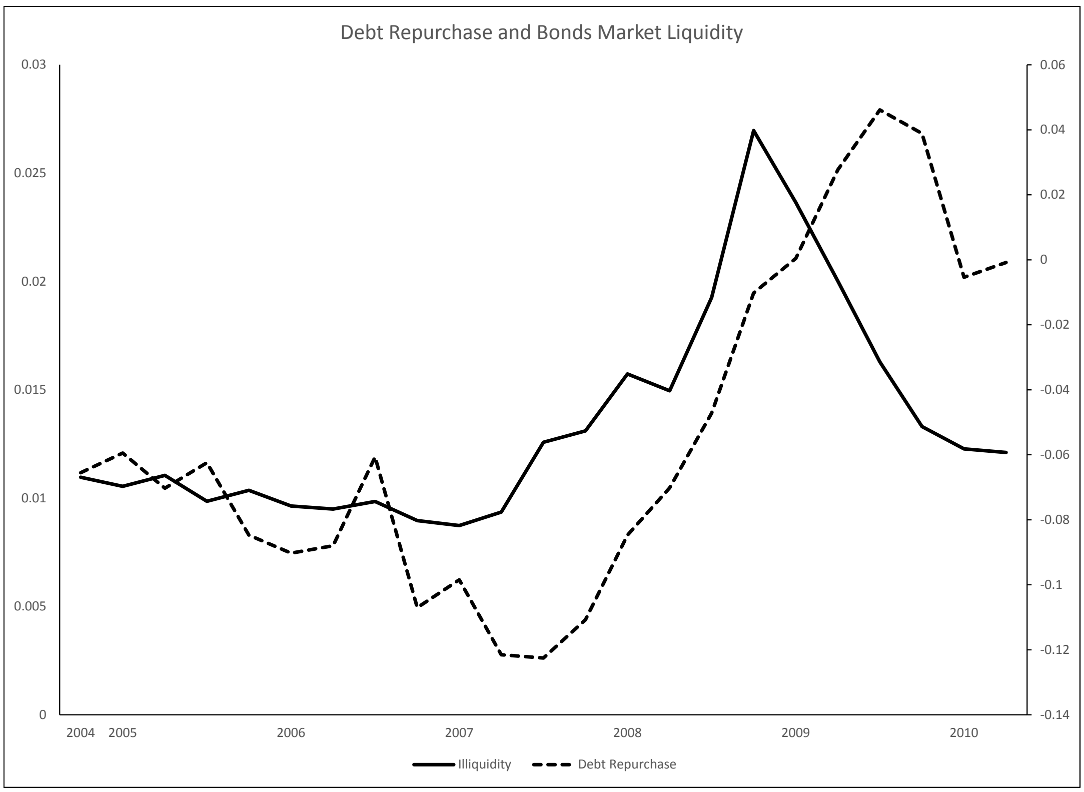
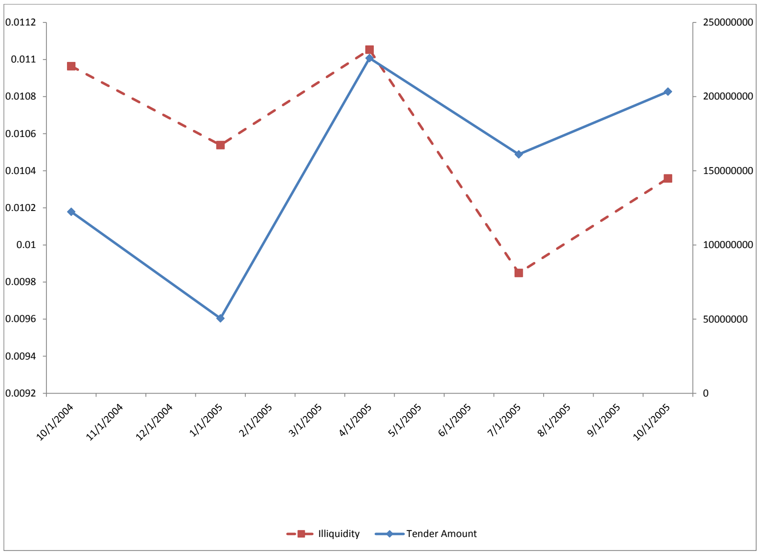
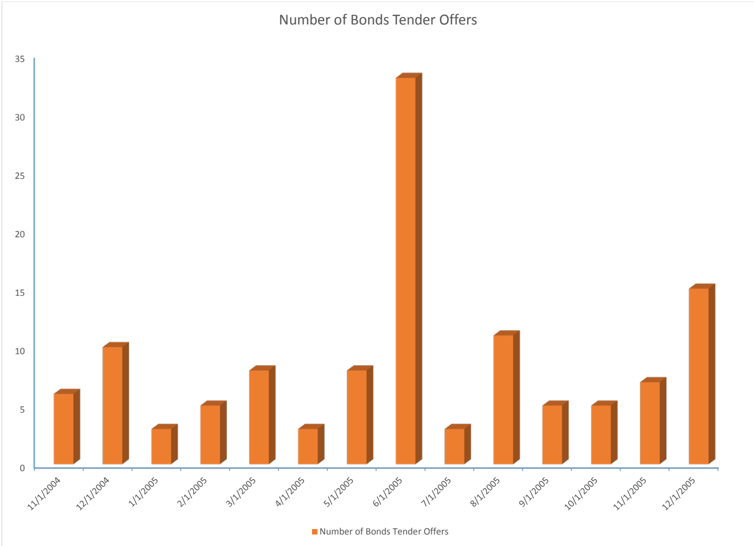


Figure 14. Debt Repurchase from Jermann and Quadrini (2012) (dashed line, left axis) and bonds market liquidity measure from Corwin and Schultz (2012) (solid line, right axis)



(a)



(b)

Figure 15. Figure 15(a) shows liquidity measure from Corwin and Schultz (2012) (dashed line, left axis) and the total amount of bonds tendered in 2004-2005 (solid line, right axis). Figure 15(b) shows the number of bonds tender offers in 2004-2005.