

Nonlinear Relationship between Oil Price and Exchange Rate by FDA and Copula

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Abstract

This article investigates the relationship between daily crude oil prices and exchange rates. Functional data analysis is used to show the clustering pattern of exchange rates and oil prices over the time period through high dimensional visualizations. We select exchange rates for important currencies related to crude oil prices by using the objective Bayesian variable selection method. The selected sample data exhibits non-normal distribution with fat tails and skewness. Under the nonnormality of the return series, we use copula functions that do not require to assume the bivariate normality to consider marginal distribution. In particular, our study applies the popular and powerful statistical methods such as Gaussian copula partial correlations and Gaussian copula marginal regression. We find evidence of significant dependence for all considered pairs, except for the Mexican peso-Brent. Our empirical results also show that the rise in the WTI oil price returns is associated with a depreciation of the US dollar.

JEL Classification: C32; F31.

Keywords: Nonlinear Relationship; Functional Data Analysis, Bayesian Variable Selection, Copula.

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1 Introduction

Crude oil is a unique commodity in that it plays an important role in the economy. Specifically, the oil prices are associated with production costs to oil importing countries. Oil exporting countries are also affected by their crude oil revenues. Therefore, any shock to global oil markets affects the world economy. However, the oil price shock is transmitted to the real economy through countries' exchange rates. Given this situation, policy makers and investors consider not only oil price fluctuations, but also exchange rates movements. In this article, we analyze how the oil and currency markets are linked by using functional data analysis (FDA) and copula methods.

FDA has recently begun to receive attention in the literature, particularly in the financial market analysis. The basic idea behind FDA is to create functional data from discrete observations. Then, a multivariate analysis technique is used to extract information from the multilevel functional data. [Kneip and Utikal \(2001\)](#) examine density families using functional principal component analysis (FPCA), which is one of the FDAs. [Tsay \(2016\)](#) notes that FPCA can be used to analyze big data with dependency. FPCA is the most popular technique for the statistical analysis of financial data due to its ability to capture the directions of variation and reduce dimensions of data. To our knowledge, our study is the first use of FDA to analyze nonlinear dependencies between oil prices and exchange rates. Besides the discussion in [Ramsay and Silverman \(2005\)](#), other relevant research in FPCA includes [Silverman \(1996\)](#), and [Reiss and Ogden \(2007\)](#).

Correlation coefficient under the normal distribution assumption plays a critical role in

achieving an optimal investment portfolio in financial theory. However, when a joint distribution is not normally distributed, a linear correlation coefficient is no longer an appropriate measure to describe the relationship. As we will discuss later, our return series exhibits non-normal distribution with fat tails and skewness. Indeed, there is growing concern that the joint distribution of many financial series may not be elliptical. We also observe the obvious clusters of our sample return series in high dimensional visualizations obtained by using FPCA. Given such empirical evidence, this article investigates the relationship between crude oil prices and exchange rates by using Gaussian copula partial correlations and Gaussian copula marginal regression (GCMR). The reason is that there are well-known advantages of the copula methods: first, copulas can be a non-linear measure of dependence while correlation describes only a linear dependence, second, copulas can also show rich patterns of tail dependence.

Recently, [Wu, Chung, and Chang \(2012\)](#) have studied the tail dependence structures of crude oil and the U.S. dollar exchange rate by using copula-based GARCH models, which outperform the dynamic conditional correlation multivariate GARCH models. Their empirical study shows that there is no apparent dependence structure between the two return series. [Akram \(2004\)](#) provides empirical evidence that there is a non-linear negative relationship between oil prices and the Norwegian exchange rate. The relationship varies based on the level and trend of oil prices. [Chen and Chen \(2007\)](#) present that there is a cointegrating relationship between real oil prices and real exchange rates. They also show that real oil prices have a forecasting power of real exchange rates.

The bulk of the empirical research also studies the relationship between oil price changes and interest rates. For instance, [Cognigni and Manera \(2008\)](#) and [Arora and Tanner \(2013\)](#) find that two return series' movements are correlated, and the direction of their movements are affected by economic factors. In recent work, [Kim \(2016\)](#) uses the univariate asymmetric threshold GARCH and copula models to investigate the relationship among three return series such as Brent oil prices, U.S. 10-year treasury constant maturity rates and exchange rates of the ten currencies. In particular, the Gaussian copula partial correlation and semi parametric Bayesian Gaussian copula estimation are used to examine the conditional relationships among the three log returns.

Some of the previous empirical literature on the comovements between oil prices and dollar exchange rates provides evidence of a positive link. By contrast, some studies show that the rise in oil prices is associated with a dollar depreciation (see [Aloui *et al.* 2013](#) for a summary of the key findings of previous studies). The conflicting findings may be explained by the fact that the value of the US dollar changes significantly against the currencies of net oil exporting or importing countries as oil prices fluctuate. [Lizardo and Mollick \(2010\)](#) find that an increase in real oil prices cause the currencies of oil exporting countries to appreciate and the currencies of oil importing countries to depreciate against the U.S. dollar. In this study, we use exchange rates against the euro, whose countries are neither net exporters nor significant importers of oil. Further, we consider a number of currencies rather than a dollar specifically in order to better identify the interdependence between two markets. Our study mainly finds the relationship between oil-exchange rate pairs. Indeed, our results obtained

from advanced statistical models support the body of previous findings that crude oil price increases are associated with depreciations of major currencies.

The layout of the paper is as follows. Section 2 introduces the econometric methodologies, such as FDA and copula methods, in particular, Gaussian copula partial correlation and Gaussian copula marginal regression. In Section 3, we begin with FPCA to visualize clustering the return series. Then, we analyze nonlinear dependence structures between oil and exchange rate markets by exploring various models. Concluding remarks are presented in Section 4.

2 Econometric Methodology

2.1 Functional Data Analysis

Among FDA techniques, FPCA overcomes the curse of dimensionality, and it also provides a much more informative way of examining the sample covariance structure than PCA proposed by [Person \(1901\)](#). Furthermore, it is an effective statistical method for explaining the variance of components because of the use of non-linear eigenfunctions. Because traditional PCA only shows the clustering pattern of the whole data at a certain time, FPCA is the more suitable method to know the clustering pattern of the time-course data over the given time period.

Let $y_i(t_j)$ be the random variable that represents the observed returns of financial assets i for period t_j , which can be stated as $y_i(t_j) = x_i(t_j) + e_i(t_j)$, with $x_i(t_j)$ denoting their underlying smooth functions and $e_i(t_j)$ indicating the unobserved error components. The

functional form of $x_i(t)$ is given by the sum of the weighted basis functions, $\phi_k(t)$, across the set of times T .

$$x_i(t) = \sum_{k=1}^K c_{ik} \phi_k(t), \quad (1)$$

where K is a number of basis functions. To obtain a smooth function which fits well into the observed return series, $y_i(t_j)$, we consider the following smoothing criterion:

$$SSE(\mathbf{y}|\mathbf{c}) = \sum_{i=1}^n \sum_{j=1}^T \left[y_i(t_j) - \sum_{k=1}^K c_{ik} \phi_k(t_j) \right]^2 = (\mathbf{y} - \mathbf{\Phi}\mathbf{c})'(\mathbf{y} - \mathbf{\Phi}\mathbf{c}), \quad (2)$$

where $\mathbf{\Phi}$ is a $K \times T$ matrix, with $\Phi_{kj} = \phi_k(t_j)$.

2.2 Copula Methods

A copula is a function C whose domain is the entire unit square with the following properties:

- $C(u, 0) = C(0, v) = 0$ for all $u, v \in [0, 1]$
- $C(u, 1) = C(1, u) = u$ for all $u, v \in [0, 1]$
- $C(u_1, v_1) - C(u_1, v_2) - C(u_2, v_1) + C(u_2, v_2) \geq 0$ for all $u_1, u_2, v_1, v_2 \in [0, 1]$, whenever $u_1 \geq u_2, v_1 \geq v_2$.

[Sklar \(1973\)](#) shows that any bivariate distribution function, say F_{XY} , can be represented as a function of its marginals, F_X and F_Y , by using a two-dimensional copula $C(\cdot, \cdot)$, i.e.,

$$F_{XY}(x, y) = C(F_X(x), F_Y(y)). \quad (3)$$

If F_X and F_Y are continuous, then C is unique. $C(u, v)$ is ordinarily invariant. That is, if $\psi(x)$ and $\phi(y)$ are strictly increasing functions, the copula of $(\psi(x), \phi(y))$ is also that of (X, Y) . Hence, if each marginal of $F_{XY}(x, y)$ is continuous, then by choice of $\psi = F_X(x)$ and $\phi(x) = F_Y(y)$, every copula is a distribution function whose marginals are uniform on the interval $[0, 1]$. As such, it represents the dependence mechanism between two variables by eliminating the influence of the marginals and hence of any monotone transformation on the marginals.

Let X, Y , be random variables with continuous distribution functions F_X, F_Y and copula C . Then the Pearson's correlation coefficient r and the Spearman's ρ are given, respectively, by

$$r(X, Y) = \frac{1}{D(X)D(Y)} \int_0^1 \int_0^1 [C(u, v) - uv] dF^{-1}_X(u) dF^{-1}_Y(v)$$

$$\rho_C = 12 \int_0^1 \int_0^1 [C(u, v) - uv] dudv$$

where D stands for the standard deviation ([Schweizer and Wolff 1981](#); [Whitt 1976](#)).

One of the simple ways to measure the relationship between two random variables is the Pearson partial correlation approach. However, the statistic will not be obtained when the first or second moment of the random variables does not exist. [Joe \(2006\)](#) shows that the correlation matrix can be parameterized based on the partial correlations and that the partial correlations remains unconstrained on $[0, 1]$, implying that the likelihood is easily obtained. [Kim et al. \(2011\)](#) propose the Gaussian copula partial correlation based on vine copula in order to analyze histone gene data. Regular vine distribution is flexible for multivariate

dependence models. In addition, [Smith and Vahey \(2016\)](#) present that the Gaussian copula is efficient when it is represented as a vine copula. [Bedford and Cooke \(2002\)](#) show that every unique correlation matrix is determined by assigning values from $(-1, 1)$ to the partial correlations corresponding to edges on the vine. Therefore, independent correlation matrices are obtained by using a partial correlation regular vine. A detailed discussion of the Gaussian copula partial correlation approach can be found in [Kim *et al.* \(2011\)](#).

Another methodology employed in this study to measure the relationship is the Gaussian copula regression method, where dependence is expressed in the correlation matrix of a multivariate Gaussian distribution ([Song 2000; Masarotto and Varin 2012](#)). Let $F(\cdot|x_i)$ be a marginal cumulative distribution, which depends on a vector of covariates x_i . If we consider a set of n dependent variables Y_i , then the joint cumulative distribution function is in the Gaussian copula regression given by

$$\Pr(Y_1 \leq y_1, \dots, Y_n \leq y_n) = \Phi_n(\epsilon_1, \dots, \epsilon_n; \mathbf{P}),$$

where $\epsilon_i = \Phi^{-1}\{F(y_i|x_i)\}$. $\Phi(\cdot)$ and $\Phi_n(\cdot; \mathbf{P})$ indicate the univariate and multivariate standard normal cumulative distribution functions, respectively. \mathbf{P} denotes the correlation matrix of the Gaussian copula. [Masarotto and Varin \(2012\)](#) suggest an equivalent formulation of the Gaussian copula model that links each variable Y_i to a vector of covariates x_i as follows.

$$Y_i = h(\mathbf{x}_i, \epsilon_i),$$

where ϵ_i denotes a stochastic error. In particular, the Gaussian copula regression model assumes that $h(x_i, \epsilon_i) = F^{-1}\{\Phi(\epsilon_i)|x_i\}$ and ϵ has a multivariate standard normal distribution with correlation matrix \mathbf{P} . The advantages of using GCMR are to preserve the marginal univariate distributions for each data and to have multivariate normal errors for the joint distribution.

3 Empirical Results

3.1 Data and Descriptive Statistics

Our dataset contains daily crude oil prices such as Brent and WTI and foreign exchange rates for the most traded currencies against the euro.¹ In particular, we select the forty most actively traded currencies listed in the footnote from January 3, 2013 to October 6, 2014 in order to perform FPCA.² An increase in the nominal exchange rates considered reflects a depreciation of those currencies to the euro. Let S_t be an observed daily foreign exchange rate process in discrete time, $t = 1, 2, \dots, n$ and $r_t = \log\left(\frac{S_t}{S_{t-1}}\right)$ is the rates of return of the exchange rates at time t . We also compute the returns on crude-oil price indices with the same in manner.

As with the preliminary data analysis, we employ FPCA to determine factors (i.e. principal components) explaining the total variation of the return series. The key con-

¹The dataset is obtained from the currency database retrieval system provided by Professor Werner Antweiler's website at UBC's Sauder School of Business, (<http://fx.sauder.ubc.ca/data.html>).

²Argentina, Australia, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech, Denmark, Hungary, Iceland, India, Indonesia, Israel, Japan, Kuwait, Malaysia, Mexico, New Zealand, Norway, Peru, Philippines, Poland, Republic of Korea, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Taiwan, Thailand, Turkey, the United Kingdom, the United States, and Vietnam

cept in FPCA is that we decompose density variations into a set of orthogonal principal component functions that maximize the variance along each component. To do this, we estimate return density functions by using a nonparametric method, and then we extract common structures from the estimated functions.³ Given the functional form in equation (1), a Fourier basis is used to represent smooth functions as a basis function in this study due to its flexibility and computational advantages. In particular, we have $K = 5$ and $T = 433$ in equation (2). Therefore, the Fourier series of functional forms are $\phi_1(t) = 1, \phi_2(t) = \sin(\omega t), \phi_3(t) = \cos(\omega t), \phi_4(t) = \sin(2\omega t),$ and $\phi_5(t) = \cos(2\omega t),$ where the parameter $\omega = \frac{2\pi}{T}$. For further details, see [Ramsay and Silverman \(2005\)](#).

We estimate the vector of coefficients \mathbf{c} in equation (2) by minimizing the smoothing criterion. In particular, we utilize the generalized cross-validation measure GCV developed by [Craven and Wahba \(1979\)](#):

$$GCV(\lambda) = \left(\frac{n}{n - df(\lambda)} \right) \left(\frac{SSE}{n - df(\lambda)} \right), \quad (4)$$

where $df(\lambda)$ is a measure of the effective degrees of freedom of the fit defined by smoothing parameter λ , and the best value for λ is the one that minimizes the criterion. We obtain the smoothing parameter of $\lambda = 10^{22.9}$ by using our sample data. Given the estimates $\hat{\mathbf{c}}$, we are able to obtain the smoothed return series $\hat{\mathbf{y}} = \mathbf{\Phi}\hat{\mathbf{c}}$ in equation (2).

³[Kneip and Utikal \(2001\)](#) also discusses, in detail, the validity of such function decomposition.

Table 1: FPCA component variance proportions

	1 st component	2 nd component	3 rd component
Variance Proportion (VP)	59.47 %	28.85 %	11.67 %
Cumulative VP	59.47 %	88.32 %	100.00 %

After having $\widehat{y}_i(t)$, the next step is to seek a set of orthogonal functions, $\psi_j(t)$ such that:

$$\langle \psi_j(t), \psi_k(t) \rangle = \int \psi_j(t)\psi_k(t)dt = 0, \text{ for all } j \neq k, \text{ and}$$

$$\|\psi_j(t)\|^2 = \langle \psi_j(t), \psi_k(t) \rangle = 1 \text{ for all } j.$$

For example, $\psi_1(t)$ can be achieved by maximizing the following objective function:

$$\sum_i (\langle \widehat{y}_i(t), \psi_1(t) \rangle)^2 = \sum_i \left(\int \widehat{y}_i(t)\psi_1(t)dt \right)^2,$$

subject to the constraint $\|\psi_1(t)\|^2 = 1$. Note that the function $\psi_1(t)$ is the first principal component. In this study, there is no variation left in the return series after $\psi_3(t)$.

Table 1 shows the proportion of total variation in individual exchange and oil price returns explained by the components. Each principal component reveals its percentage contribution to the whole density variation. The rankings are based on each factor's corresponding contributions to explain the density variation. The first dominant, principal component alone accounts for the 59.47%, the second principal component explains the 28.85%, and the third principal component accounts for the 11.67% of the whole variance proportion of FPCA. Note that the first three principal components account for the 100% of the whole variability.

Through visualizations, we illustrate the relationship between the most traded country currencies and two main crude oil prices: Brent and WTI. In particular, by using the first

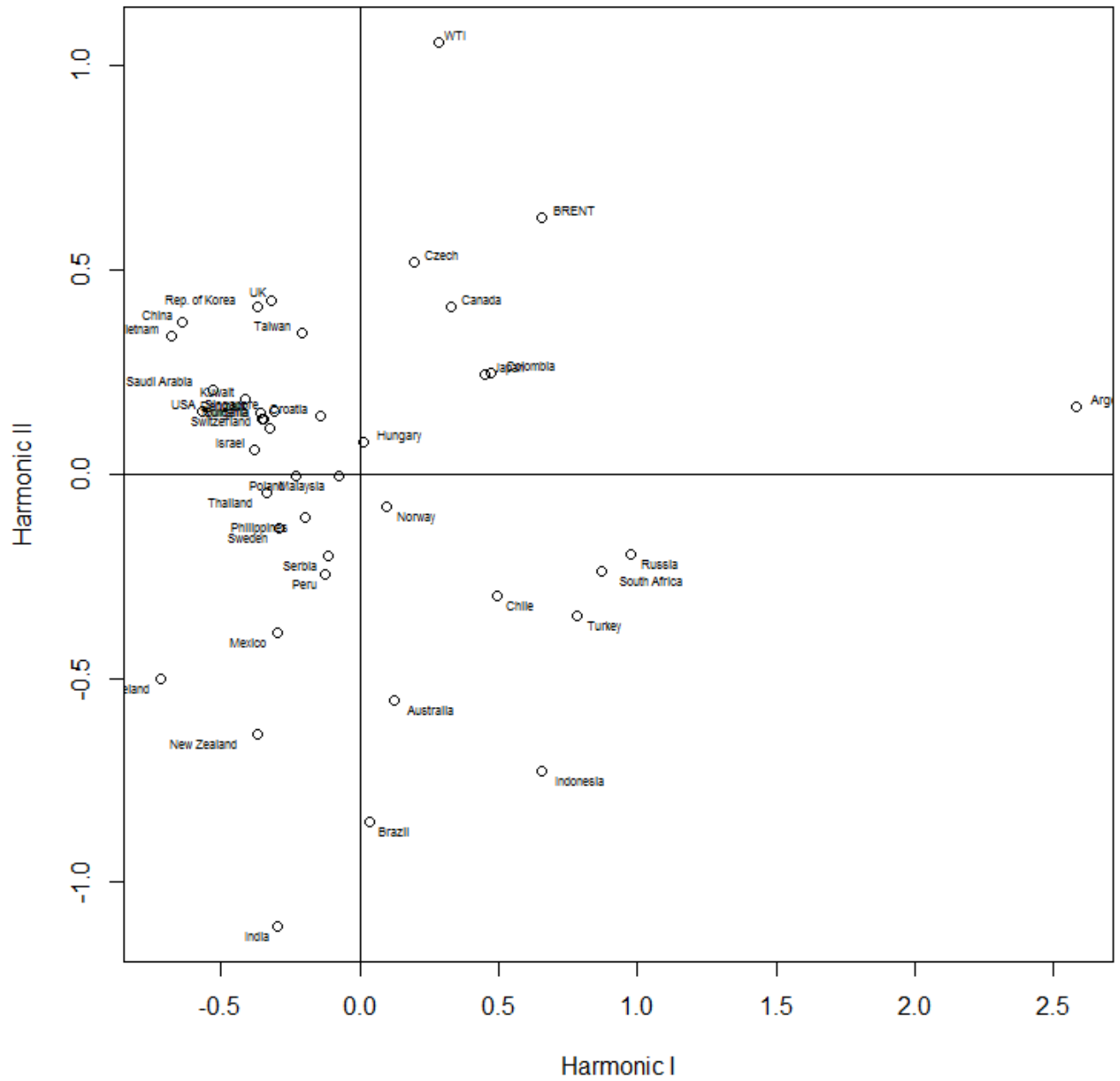


Figure 1: 2D FPCA plot of the 40 most traded country currencies with Brent and WTI

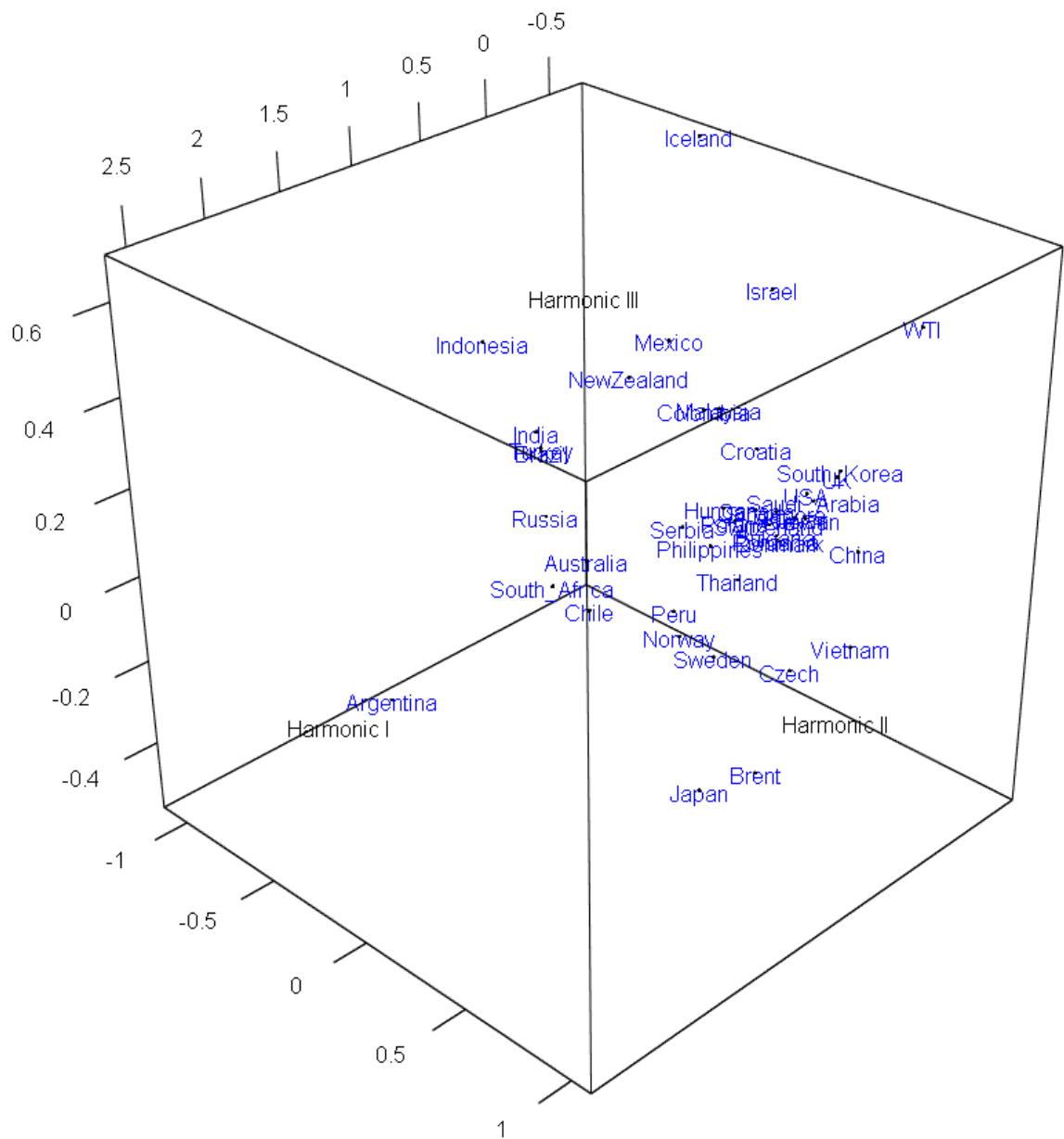


Figure 2: 3D FPCA plot of the 40 most traded country currencies with Brent and WTI

two main important components, we show that Brent and WTI are located in the same plane in Figure 1. The two-dimensional FPCA plot captures the limited view of the clusters among international currency exchange rates and oil price returns. For the detailed visualization of the relationship of the all return series, we provide a three-dimensional FPCA plot with the first three main components in Figure 2. This figure shows obvious clusters of the exchange rate returns of the U.S. and Saudi Arabia coupling with the crude oil prices, implying that those return series move together over our sample period. The observations presented in the two figures motivate our research to investigate whether there is a more prominent relationship between oil and foreign exchange markets.

For the detailed study of the relationship between each crude oil price and exchange rates, we choose the most influential variables related to the returns of Brent oil prices. We use the Bayesian variable selection method of the objective Bayesian model proposed by [Bayarri et al. \(2012\)](#). Our empirical analysis restricts our attention to exchange rates of twenty-four countries, which are considered major oil producing and consuming countries.⁴ We run a Gibbs sampling scheme to determine the optimal model on the data set by using the R package *BayesVarSel*. See [Kim \(2016\)](#) for more details about this method. The *GibbsBus* command in the R package *BayesVarSel* provides approximate posterior distribution. We set the possible prior distribution for regression parameters within each model as “Constant” and set possible prior distribution over the model space as “*gZellner*.” The number of iterations is 20,000 times after the 5,000 number of iterations at the beginning of the Markov Chain

⁴The twenty-four countries are as follows: Australia, Brazil, Canada, China, Denmark, India, Indonesia, Israel, Japan, Kuwait, Malaysia, Mexico, New Zealand, Norway, Republic of Korea, Russia, Saudi Arabia, Singapore, Sweden, Switzerland, Taiwan, Turkey, the U.K., and the U.S. Indeed, the selected countries are considered either net oil-importing or net oil-exporting countries.

Table 2: Bayesian variable selection: Brent

	Incl.prob.	Highest.Posterior.Prob.	Median.Posterior.Prob.
Intercept	1.000	*	*
Australia	0.049		
Brazil	0.169		
Canada	0.121		
China	0.129		
Denmark	0.095		
India	0.049		
Indonesia	0.047		
Israel	0.127		
Japan	0.054		
Kuwait	0.283		
Malaysia	0.057		
Mexico	0.900	*	*
New Zealand	0.049		
Norway	0.051		
Russia	0.490	*	
Saudi Arabia	0.508	*	*
Singapore	0.070		
South Korea	0.067		
Sweden	0.053		
Switzerland	0.072		
Taiwan	0.133		
Turkey	0.918	*	*
UK	0.082		
USA	0.221		

Monte Carlo (MCMC) that are dropped. With 25 covariates including the intercept, there are 16,777,216 possible competing models.

The obtained results, reported in Table 2, show the inclusion probability of all covariates, highest posterior probability, and median probability in the Bayesian model selection. The findings clearly support that the exchange rate returns of Mexico, Russia, Saudi Arabia, and Turkey currencies are critical factors in determining the returns of Brent oil prices. Table 3 displays summary statistics and distributional characteristics for the daily returns over the sample period. All return series are close to the zero mean, while their standard deviations are much greater than their means in absolute value, indicating that the mean values are not

Table 3: Descriptive statistics of log-return series

	Mean	Median	Minimum	Maximum	St.D	Skewness	Kurtosis
Brent	0.040	0.034	-3.827	3.953	1.079	0.111	3.687
Mexico	0.001	-0.005	-1.838	2.654	0.588	0.264	4.265
Russia	0.053	0.034	-1.538	1.861	0.572	0.070	3.451
Saudi Arabia	-0.010	-0.029	-1.595	1.487	0.466	-0.001	4.002
Turkey	0.046	0.024	-1.917	4.100	0.643	0.690	6.673

significantly different from zero. The table also shows that returns are skewed and exhibit excess kurtosis. This finding indicates that returns follow a fatter-tailed density function.⁵ In particular, the probability of extremely positive realizations for the returns is higher than that of a normal distribution, except for the exchange rate of Saudi Arabia.

In a similar manner, the empirical analysis is redone to obtain the estimates by using the WTI returns. Tables [A.1-A.2](#) in Appendix report the most influential currencies on WTI prices, such as the Brazil real and the US dollar and their summary statistics. The skewness statistic of WTI is positive, thereby indicating that the oil price returns are skewed to the right relative to the normal distribution. With respect to the excess kurtosis statistics, the values of both return series are significantly positive, thereby implying that the distribution of returns have heavier tails than the normal distribution.

3.2 Dependence with Copula

As described in the previous subsection, our sample dataset exhibits non-normal dependent observations. As such, our study also further investigates the nonlinear dependence between crude oil prices and exchange rates by a copula approach, which does not require indepen-

⁵If returns are normally distributed, then the kurtosis should be 3.

Table 4: Gaussian copula partial correlation coefficients: Brent

	(Brent, Mexico)	(Brent, Russia)	(Brent, Saudi Arabia)	(Brent, Turkey)
Mexico		0.174	0.245	-0.055
Russia	0.194		0.277	0.028
Saudi Arabia	0.137	0.187		0.053
Turkey	0.258	0.246	0.298	
(Mexico, Russia)			0.241	-0.081
(Mexico, Saudi Arabia)		0.139		-0.061
(Mexico, Turkey)		0.190	0.242	
(Russia, Saudi Arabia)	0.096			-0.041
(Russia, Turkey)	0.223		0.275	
(Saudi Arabia, Turkey)	0.161	0.191		
(Mexico, Russia, Saudi Arabia)				-0.084
(Mexico, Russia, Turkey)			0.236	
(Mexico, Saudi Arabia, Turkey)		0.158		
(Russia, Saudi Arabia, Turkey)	0.139			

dently and identically normally distributed observations. We look at the partial correlation coefficients by using the Gaussian copula partial correlation method (Kim *et al.* 2011), and we look at the posterior correlation coefficients by using the semi-parametric Bayesian Gaussian copula method (Hoff, 2007).⁶ Finally, GCMR is used to describe the oil-exchange rate dependence over our sample period. Estimation and inference for parameters of the Gaussian copula are available via an MCMC algorithm based on Gibbs sampling.

First, we adopt the Gaussian copula partial correlation method proposed by Kim *et al.* (2011) in order to characterize a dependence between crude oil prices and exchange rate returns.⁷ Table 4 summarizes the results of the Gaussian copula partial correlation coefficients. For the most considered pairs conditional on particular currencies, the statistics

⁶Hoff (2007) provides the semi-parametric inference for copula models via a type of rank likelihood function for the associated parameters. The semi-parametric inference is based on a generalization of marginal likelihood, called an extended rank likelihood, which does not depend on the univariate marginal distributions of the data.

⁷Note that we consider only the return pairs based on possible combinations between the Brent oil prices and the selected currencies by the Bayesian variable selection method.

Table 5: Posterior quantiles of correlation coefficients: Brent

	2.5% quantile	50% quantile	97.5% quantile
(Mexico, Brent)	0.19	0.27	0.36
(Russia, Brent)	0.14	0.25	0.33
(Saudi Arabia, Brent)	0.24	0.32	0.40
(Turkey, Brent)	0.02	0.10	0.19

indicate a positive relationship, except for the (Brent, Turkey) pairs. There is a negative association between the Brent oil prices and the Turkish lira conditional on other foreign exchanges. This is highly significant for the Brent and Saudi Arabia given other currencies. As the value of Saudi Arabia currency drops due to the lower oil prices, some investors could expect the world’s largest oil exporter to eventually reduce its oil production. Therefore, more businesses that need to buy significant quantities of crude oil in the future can take up a position in the Brent crude oil future markets to insure against the possibility that future crude oil price movements will make a transaction unprofitable. As such, these markets play a role influencing current oil prices.

Second, Table 5 shows the posterior quantiles of correlation coefficients of Brent and a currency by using the semi-parametric Bayesian Gaussian copula method (Hoff, 2007). This method allows us to take into account the nonlinear dependence and non-normalities when obtaining the copula correlation estimates. They consistently display a positive association of the return series across quantiles. Comparing the findings of Table 4, the copula correlations can be reminiscent of the dependence between Brent oil prices and exchange rates even though these estimations are primitives.

Third, we consider the regression analysis for a more complete discussion of the degree of interdependence among the return series. We use GCMR analysis to describe the depen-

Table 6: ARMA(p,q) models for return series

	df	AIC	BIC	Log Likelihood
ARMA(0,0)	6	1,221.833	1,246.257	-604.917
ARMA(1,0)	7	1,221.580	1,250.075	-603.790
ARMA(0,1)	7	1,221.593	1,250.089	-603.797
ARMA(1,1)	8	1,223.579	1,256.145	-603.789

dence between oil prices and exchange rate returns. [Song \(2000\)](#) and [Masarotto and Varin \(2012\)](#) provide a flexible general framework for modeling any response data type, which is particularly suitable for our data analysis. We use the *gcmr* R package, and inference is performed through a likelihood approach. Note that computation of the exact likelihood is possible only for continuous responses, otherwise the likelihood function is approximated by an importance sampling procedure proposed by [Masarotto and Varin \(2012\)](#). Before the GCMR analysis is conducted to measure the interdependence among the selected variables by the Bayesian variable selection method, we compare four different linear models with the error correlation structure ARMA(p, q), where $p = 0, 1$ and $q = 0, 1$. The obtained results, reported in [Table 6](#), show that the ARMA(1, 0) model is appropriate for the return series based on the usual criteria such as Akaike information criterion (AIC) and the Bayesian information criterion (BIC).⁸ Given the availability of the estimates of the ARMA(1, 0) model, we turn to estimate the GCMR, whose error term u_t follows an AR(1) process.

$$\text{Brent}_t = \beta_0 + \beta_1 \times \text{Mexico}_t + \beta_2 \times \text{Russia}_t + \beta_3 \times \text{Saudi Arabia}_t + \beta_3 \times \text{Turkey}_t + u_t,$$

where $u_t = \phi u_{t-1} + \omega_t$, and $\omega_t \sim iid N(0, \sigma_\omega^2)$.

[Table 7](#) summarizes our estimates reflecting dependence parameters of the return series

⁸Note that for all the estimates in the linear model with error correlation structure ARMA(1, 0) are statistically significant at the 5% significance level. The results are available upon request.

Table 7: Estimates of the GCMR with ARMA(1, 0) error structure: Brent

	Estimate	S.E.	t-value	p-value (two-sided)
(Intercept)	0.048	0.051	0.949	0.343
Mexico	0.406	0.114	3.551	0.000
Russia	0.270	0.102	2.654	0.008
Saudi Arabia	0.617	0.114	5.391	0.000
Turkey	-0.368	0.097	-3.786	0.000
AR(1) (ϕ_1)	0.072	0.048	1.501	0.134

between oil prices and exchange rates. For all the return series, the coefficients are statistically significant. We see that the estimated coefficients are positively significant for all considered pairs, except for the Brent-Turkish lira. Notice that the returns of exchange rates of Saudi Arabia, which is a leading oil exporter, have significant impact on the returns of Brent oil prices. The Brent index actually serves two thirds of the world’s internationally traded crude oil supplies. The depreciation of the Saudi riyal against the euro is a key factor in driving up the Brent oil price during the sample period.

We turn to estimate parameters β by using WTI crude oil prices.⁹ We continue to focus on the currencies selected by the Bayesian variable selection procedures. In particular, we estimate the GCMR with ARMA(1, 0) error correlation structure and the results are reported in Table 8.

$$\text{WTI}_t = \beta_0 + \beta_1 \times \text{Brazil}_t + \beta_2 \times \text{USA}_t + u_t,$$

where $u_t = \phi u_{t-1} + \omega_t$, and $\omega_t \sim iid N(0, \sigma_\omega^2)$.

From the findings in the table, we see that the returns of the exchange rates of the US

⁹Tables A.3-A.4 in the Appendix present the Gaussian copula partial correlation coefficients and the posterior quantiles of the correlation coefficients for WTI.

Table 8: Estimates of the GCMR with ARMA(1, 0) error structure: WTI

	Estimate	S.E.	t-value	p-value (two-sided)
(Intercept)	-0.006	0.056	-0.098	0.922
Brazil	0.306	0.080	3.844	0.000
USA	0.612	0.141	4.331	0.000
AR(1) (ϕ_1)	-0.004	0.048	-0.085	0.932

dollar and the Brazil real are positive and statistically significant at 1% significance level. It turns out that an increase in the price of WTI is associated with a depreciation of the US dollar against the euro. The negative relationship between the price of oil and the value of the dollar can be explained by the historical event in the US during our sample period. The Federal Reserve expanded the US money supply by purchasing US government bonds, known as quantitative easing from 2011 to 2013, and announced that it would continue to pursue the policy at least through 2014. The monetary policy would fuel inflation, and more investors feared the risk of high inflation. Therefore, commodities including oil or gold are preferred as means of a hedge against inflation risk. Also investors with appreciated currency, the euro, are likely to find oil relatively less expensive and their demand of oil commodity will increase, which in turn leads to the rise in the price of oil.

4 Conclusion

This article has investigated the relationship between oil prices and exchange rates. We have first shown that crude oil prices are related with exchange rates of the actively traded currencies by using 2-D and 3-D visualizations. The figures have shown the obvious clustering

pattern of our daily return series. By considering the stylized feature of our data, we have selected the most influential currencies related to Brent and WTI oil prices, separately, by using the Bayesian variable selection technique.

We have employed a copular approach to further investigate the nonlinear dependence between oil returns and exchange rate returns. Specifically, we have obtained the partial correlation coefficients by using the Gaussian copula partial correlation method and the posterior correlation coefficients by using the semi-parametric Bayesian Gaussian copula method. They have consistently presented a positive relationship of the return series across quantiles.

Finally, our study has provided strong evidence of significant dependence for all considered pairs by using GCMR analysis. Given the conflicting findings in previous studies, our empirical evidence has strongly supported that increases in the WTI oil prices are associated with a depreciation of the dollar. In addition, it has turned out that a significant depreciation of the considered currencies against the euro leads to higher crude oil prices, except for the Mexican peso-Brent pair during our sample period.

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A Appendix

Table A.1: Bayesian variable selection: WTI

	Incl.prob.	Highest.Posterior.Prob.	Median.Posterior.Prob.
Intercept	1.000	*	*
Australia	0.154		
Brazil	0.807	*	*
Canada	0.147		
China	0.369		
Denmark	0.049		
India	0.047		
Indonesia	0.048		
Israel	0.048		
Japan	0.053		
Kuwait	0.116		
Malaysia	0.049		
Mexico	0.257		
New Zealand	0.058		
Norway	0.074		
Russia	0.089		
Sadui Arabia	0.350		
Singapore	0.056		
South Korea	0.050		
Sweden	0.049		
Switzerland	0.136		
Taiwan	0.139		
Turkey	0.163		
UK	0.088		
USA	0.533	*	*

Table A.2: Descriptive statistics of log-return series

	Mean	Median	Minimum	Maximum	St.D	Skewness	Kurtosis
WTI	-0.004	0.054	-4.172	6.072	1.244	0.282	4.624
Brazil	0.027	0.034	-3.129	2.741	0.759	-0.262	4.394
USA	-0.011	0.008	-1.604	1.545	0.429	-0.172	4.537

Table A.3: Gaussian copula partial correlation coefficients: WTI

	(WTI, Brazil)	(WTI, USA)
Brazil		0.181
USA	0.172	

Table A.4: Posterior quantiles of correlation coefficients: WTI

	2.5% quantile	50% quantile	97.5% quantile
(Brazil, WTI)	0.18	0.25	0.34
(USA, WTI)	0.16	0.27	0.34