Robust Consumption and Energy Decisions

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Abstract

We study a simple model of economic growth where society's preferences are a function of consumption per capita and climate quality; and the specification of the climate dynamics is inspired by recent work in climate science. The model is estimated to establish a reference model and we develop a new method that determines the reasonable size of a set of surrounding models which are difficult to distinguish from the reference model. We show that robust agents who deny the effects of climate change on the economy, behave more like agents who believe climate changes are real. This happens because robust non-believers design policies that hedge against their worst case model which does include an anthropogenic effect of their emissions on climate and these changes in climate have negative effects on preferences and productivity.

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This article conducts an empirically disciplined robustness analysis in the context of a simple Ramsey, Cass, Koopmans model of economic growth where society's preferences at any date are a Constant Elasticity of Substitution (CES) function of consumption per capita; and climate quality and the specification of the climate dynamics is inspired by recent work in climate science by Matthews et al. (2009) and Matthews, Solomon, and Pierrehumbert (2012). The model is estimated to establish a reference model which is surrounded by a set of alternative models which are difficult to distinguish from the reference model. In the reference model, climate changes affect both productivity and preferences, and we demonstrate that the two effects have different implications for optimal choices.

We develop a new method to calibrate robustness parameters in the spirit of computing detection probabilities but much less computationally demanding. This computation empirically disciplines the robustness analysis via a parameter that determines the size of the set of surrounding models. We show that robust agents who deny the effects of climate change on the economy, behave more like agents who believe climate changes are real.

Overview

We begin by providing an historical context, informal motivation, and an overview of our main results.

Climate Change and Uncertainty

Uncertainty is the hallmark of global climate change and the analysis of policies to address it. While the basic physical principles governing the response of the planetary atmosphere to increasing concentrations of greenhouse gases (GHGs) have been known since the nineteenth century, the detailed workings of the climate system and how it will be affected by increasing GHGs produced by human society remain imperfectly understood (APS 2013). Moreover, the capacity of numerical general circulation (climate) models to accurately predict the future course of the global climate system over multiple decades or longer is very limited, and subject to significant intra- and inter-model uncertainty.

Analogously, in the economics of climate change there has been considerable debate regarding appropriate principles for analyzing the costs and benefits of GHG abatement, particularly regarding the problem of discounting over the very long run. Even conditional on discounting and other assumptions, however, economic and policy analysis of abatement strategies continues to be subject to extreme uncertainty.

This state-of-affairs and the reasons behind it are receiving heightened attention from both scientists and economists, focusing implicitly or explicitly on "integrated assessment (IA)" models, which represent both earth system and economic dynamics, and their interactions, in reduced forms, and which are the predominant analytical tools for policy analysis of climate change on a global scale. Although uncertainty has been addressed in some of the IA literature, since their initial development more than two decades ago integrated assessment models have been and continue to be primarily deterministic. Pindyck (2013) argues that IA models "are of little or no value for evaluating alternative climate change policies," essentially because they fail to acknowledge and address fundamental uncertainties in both the workings of the climate system, and the future economic damages that may result from climate change. In a broad-ranging critique of IA modeling, Stern (2013) calls for a "new generation" of such models that would, among other improvements, be developed explicitly within a risk-management framework rather than on the deterministic foundations of the current generation.

Similarly, Roe (2013) suggests that in view of the persistent scientific and economic uncertainty pertaining to global climate change, particularly including key IA modeling assumptions, technical and quantitative analysis to develop policies may have "...reached the point of diminishing returns." He argues, moreover, that the appropriate path forward for deliberating on and developing policies to address climate change is therefore to give significantly greater weight to moral and ethical considerations.

Roe's argument is especially important in the case of poorer parts of the world where food insecurity due to the damaging impact of climate change on agriculture looms large.

Burke and Emerick (2016) and Deschenes and Greenstone (2012) show that climate change impacts on agriculture are substantial. Diffenbaugh et al. (2012) and Schlenker and Roberts (2009) show that variance of damages to agriculture, especially yields on major crops, is likely to increase as global warming proceeds. This is so primarily because plants have an ideal temperature during their growing season and global warming is likely to produce longer clustered periods of exceedances of ideal growing temperature during the critical growing season for food crops.

Since agriculture is a larger share of output in poorer countries the increasing trend of damages and increasing variance of damages to agriculture raises ethical issues of inequality, food insecurity, and much more since the wealthier developed nations can be accused by the poorer developing world of having caused most of the climate change problem in the first place and food security is critical to the welfare of peoples everywhere. While our model does not explicitly include increased weights in the welfare measure on poorer countries, because it is aggregated to the world level, our robustness analysis and estimation could be extended to a multi-regional multisector model.

It is important to emphasize that, notwithstanding their critical perspectives on IA modeling, Pindyck and Stern hold the mainstream economic opinion that large-scale GHG abatement starting in the present has already been demonstrated to be fully justified on cost-benefit grounds. Nevertheless, their observations and Roe's observations highlight the importance of uncertainty analysis in climate economics and modeling as well as the treatment of moral and ethical issues.

Building on previous research in integrated assessment and several other areas of economics as well as in climate science, this article addresses both of these topics. To analyze uncertainty, we apply a methodology developed by macroeconomists based on the concept of "robustness to model uncertainty" (Hansen and Sargent 2008). In this approach, decision-makers employ mathematical models of systems such as economies but acknowledge the possibility that their chosen model may not in fact be an accurate representation of the given system. That is, they confront fundamental model uncertainty, and seek to make decisions that are robust to this type of uncertainty – i.e., that will yield acceptable outcomes even in the case of an incorrect model. To incorporate the moral and ethical dimensions of climate change, we explicitly include the state of the climate in decision-making regarding the abatement of GHG emissions – that is, in addition to strictly economic considerations.

There have been several previous papers adapting robustness analysis of this type to the economics of climate change. There has also been previous work in environmental economics on how "environmental quality" affects decision-making. However, the work we describe in this article is the first to combine these two approaches. Moreover, our particular treatment of robustness analysis is new to the integrated assessment literature. In addition, we adopt a reduced-form representation of the climate based on the work of Matthews et al. (2009) and Matthews, Solomon, and Pierrehumbert (2012), and this feature is also new.

Existing Deterministic and Stochastic Models

High-dimensional economic-climate models, which link partial or general equilibrium models of the world economy with intermediate complexity climate models and other parts of the carbon cycle as well as ecosystem models, are primarily deterministic. The economic components of these models are based on a calibration philosophy that does not in most instances include statistical procedures for parameterization and associated uncertainty quantification (Dawkins, Srinivason, and Whalley 2001). Moreover, the size of these large models generally precludes the use of stochastic optimization methods – a consequence of the "curse of dimensionality."

In parallel to the development of these large IA models, a substantial body of work has been conducted using lower-dimensional IA models following the Ramsey-Cass-Koopmans (RCK) optimal control framework, in which a perfectly foresighted representative decision-maker chooses dynamic paths of consumption and investment – in the context of a representative production function which yields output given capital and labor inputs – to maximize discounted intertemporal utility. This research has to a very large extent been based directly or indirectly on the DICE (Dynamic Integrated Climate Economy) model of Nordhaus (2008), which has come to play a paradigmatic role in this field. In brief, the DICE template augments the basic RCK – optimal growth model with a reduced-form specification of the climate; economic activity produces GHG emissions as a by-product, thereby increasing global temperature, which in turn acts to reduce output. Abating these emissions – trading off with consumption and investment – is then an additional decision dimension. In the DICE paradigm also, most analysis has been deterministic. The tractability of the RCK approach, however, has facilitated various forms of stochastic analysis by a number of researchers. Following are a number of key examples.

Nordhaus and Popp (1997) developed the "PRICE" (PRobabilistic Integrated model of Climate and Economy) variation of DICE and used it to compare five methods of estimating the value of information regarding eight uncertain parameters, analyzed singly and jointly. Kolstad (1996) created and solved a stochastic version of DICE to analyze the influence optimal policy of learning about damages caused by climate change. Extending this work, Kelly and Kolstad (1999) implemented a stochastic variant of DICE, solved by dynamic programming, to conduct a Bayesian analysis of learning about the relationship between GHG levels and global mean temperature changes, in the presence of a stochastic shock to temperature. Keller, Bolker, and Bradford (2004) adapted earlier versions of DICE (Nordhaus 2008, and references) to include a climate-related environmental threshold – the collapse of the Atlantic thermohaline circulation due to temperature increase - learning, and uncertainty in the climate sensitivity, and solved this model using a global optimization method. Crost and Traeger (2011) developed a version of DICE in a recursive dynamic programming framework with uncertainty in damages and Epstein-Zin utility to study the different effects of risk, risk aversion, and aversion to intertemporal substitution. Jensen and Traeger (2013) use the stochastic DICE framework to study how uncertainty in long-run economic growth affects optimal climate policy.

The most ambitious extension of a DICE type framework to the stochastic case is the work of Cai, Judd, and Lontzek (2012a), hereafter, "CJL". Their reduced-form climate has three layer carbon cycle dynamics and a two layer atmosphere and ocean temperature dynamics. When these state variables are added to the state variables from the economic dynamics, there are a total of 8 state variables. The Cai, Judd, and Lontzek (2012b) paper extends their work to include abrupt changes in climate dynamics, e.g. tipping points and the impact this possibility has upon the solution of the model. Tipping points can be viewed as a form of "catastrophic" climate change and are, indeed, catastrophic, if they are large enough (Cai, Judd, and Lontzek 2013a,b, Lenton and Ciscar 2013). The CJL model is solved by a sophisticated (and quick) optimization algorithm that they have developed which is quite readily adaptable to other dynamic models.

Robust Models

Broadly speaking, the work we have sketched above is in the domain of "parametric uncertainty analysis." That is, within a given model structure, key inputs or parameters – such as those describing the dynamics of the climate system or the economy – are assumed to be stochastic and to have associated probability distributions. The decision-making agent(s) represented in the model then act according to, for example, expected utility maximization or some other stochastic optimization procedure. This type of approach reflects long-standing analytical frameworks in economics including dynamic stochastic general equilibrium modeling.

By contrast, as noted previously the robustness analysis methodology developed by Hansen and Sargent posits that the underlying structure of the model itself is uncertain – a state-of-affairs that well characterizes both climate economics and climate science. One way of framing this assumption is to suppose that there is a set of "candidate" probability distributions representing model characteristics - described as a situation of "ambiguity" – and that the decision-maker is averse to this ambiguity and acts accordingly. Building on technical tools from fields including, risk-sensitive optimal control, Hansen and Sargent have created a theory to analyze this category of problem and, in particular, to identity robust decision rules given model uncertainty or ambiguity.

Several researchers have introduced robustness and ambiguity aversion into climate economics and integrated assessment modeling. Following Hansen and Sargent (2001), Hennlock (2008, 2009) and Sterner and Hennlock (2011) incorporate robustness with respect to uncertainty regarding the product of climate sensitivity and equilibrium radiative forcing, in a model with both "clean" and "dirty" energy sectors, both of which have a form of endogenous technical change. Lemoine and Traeger (2011) adapt DICE to include an uncertain tipping point and learning about the threshold that triggers it, and aversion to ambiguity regarding the threshold's distribution. Li, Narajabad, and Temzelides (2014) adapt the model of Golosov et al. (2014), assuming that climate change directly damages – i.e., reduces – the capital stock, with include model uncertainty embodied in a stochastic parameter governing the magnitude of this effect, and analyze robustness with respect to this uncertainty in a dynamic two-person zero-sum game, pitting the social planner against a malevolent agent (who controls the capital stock damage).

Traeger's "GAUVAL" model (Traeger 2015) uses risk-sensitive control to model robustness but he, like all of the above treatments, does not estimate his climate economics model on a data set like we do here. Hence, Traeger (2015), like the work reviewed above, does not empirically discipline his robustness analysis like we do here.

In our article, the basic economic dynamics are specified as a conventional aggregative growth model with capital accumulation, here representing the global economy, and capital, labor, and fossil fuel inputs into production. Basic climate dynamics are specified as a trend in the temperature anomaly driven by cumulative fossil fuel emissions which is a specification inspired by Matthews et al. (2009) and Matthews, Solomon, and Pierrehumbert (2012).

There are many other complementary approaches for robustness, such as Bayesian approaches, which are described in an appendix.

Cumulative Climate Response

Using numerical simulations performed by a set of general circulation (climate) models, Matthews et al. (2009) and Matthews, Solomon, and Pierrehumbert (2012) have shown within a large range of cumulative emissions the increase in global average yearly temperature caused by increasing GHG emissions is approximately linearly related to cumulative emissions – they designate the slope parameter in this relationship the "Cumulative Climate Response (CCR)." The estimated value of this quantity varies across the set of climate models they simulated, and this variation, documented by Matthews et al., can be used as a measure of fundamental uncertainties in climate models. Given this wide variation in CCR's we can infer that climate scientists may not agree on what baseline model to estimate. While it is beyond the scope of this article to fully explore this question, potential lack of agreement on a baseline model prompts us to discuss other methods of dealing with robustness analysis.

Uncertainty is incorporated into the model by adding stochastic shocks to both the economic (production) and the climate – temperature – dynamics. These shocks represent the decision-makers doubts about the underlying models' specification of economic production, technology change, climate change, and the economic costs of climate change. These doubts are addressed by incorporating robustness into the decision rule used to solve the model.

Robustness and Consensus Policies

We consider several different reference models. In order to make key points quickly we shall initially assume that the known reserve of fossil fuels is infinite and that extraction costs of fossil fuels is zero. One of the reference models, call it the "believer" model, uses optimal parameter estimates which imply that anthropogenic climate change has a large effect on productivity and preferences. Hence, in the "believer" model, optimal policy is to tightly constrain fossil fuel emissions even though extraction costs are zero. Another, polar opposite, reference model, call it the ‘non-believer” model, assumes there is no anthropogenic climate change effect on production and climate quality. Hence, in the non-believer model the optimal policy is to use an infinite amount of fossil fuels. Thus we see that in the absence of robustness, optimal policy under these two reference models is drastically different. Now suppose that policy makers have small doubts that the reference model is correct. As compared to the non-robust case, policy makers who use the “believer” reference model with optimal estimates will only slightly change their behavior, whereas policy makers who use the “non-believer” model, i.e. who deny the effects of climate change on production and air quality will make huge changes in their behavior because, under robustness, they optimize against a “worst case” possibility that anthropogenic climate change is real and might have a large negative effect on production and preferences if an infinite amount of emissions occur. Figure 4 shows that robustly optimal energy use is strongly restrained in order to keep total emissions low enough to hedge against small doubts about the specification of its baseline model. We also show that the resulting optimal decisions of the two robust agents are somewhat similar which suggests that if policy makers could agree to adopt robust decision making procedures then there would be much less disagreement about optimal policies. Of course the case of infinite known reserves and zero extraction costs is an extreme case, the point remains that since both believers and non-believers optimize against a worst case model where anthropogenic climate change has negative impacts on preferences and productivity, their policy actions are more similar under robustness than under non-robustness.

Empirically Disciplined Robustness

While the qualitative direction of the results we present are intuitive we believe that what is important is our illustration of how to develop an empirically disciplined quantitative approach to robustness. While we realize that our contribution is very limited we believe that more extensive and complete quantitative approaches to empirically disciplined robustness analysis in more realistic climate economics models will end up following a template much like ours.

Since an empirically disciplined robustness analysis may not be familiar to many readers we lay out the explicit steps.

Step (1): Formulate a completely specified climate-economic model with explicitly parameterized climate and economic dynamics that include the impacts of climate on preferences and productivity. We call our completely specified model the reference model and it is presented in climate-economic model section when the parameter θ (discussed below), is zero.

Step (2): Express observable implications of the model involving unknown parameters and estimate the parameters. We estimate the model using generalized method of moments in the estimating the climate-economic model section. Alternative estimation methods, such as maximum likelihood, also could be used.

Step (3): Choose parameter values based on the estimates computed in Step (2). We choose parameter values in the parameter selection section and in most cases set the parameters equal to their estimated values.

Step (4): Specify perturbations of the economic climate dynamics that are parameterized by the scalar, θ. The magnitude of the perturbations are found by solving a minimization problem for a given θ. When $\theta = 0$ there are no perturbations and as θ rises the perturbations increase. In order to save space and avoid presenting similar things twice, we discuss the perturbations in the climate-economic model section at the same time that we discuss the reference model.

Step (5): For many different values of θ compute the perturbations and find the maximum value of θ such that the perturbations can not be rejected from the data. A robust agent worries that the perturbations generated by the maximum value of θ are possible. We approximate the maximum value of θ in the appropriate levels of robustness subsection of the parameter selection section.

Step (6): For the maximum value of θ, compute optimal policy by solving a robust control problem. We present several examples of optimal policy in the simulation section.

A Climate-Economic Model

This section develops our reference model and introduces our robust perturbations.

Preferences are time additively separable where period preferences are a power function of a CES function of consumption per capita and climate quality. Using a power function of a CES function allows us to discuss the impact of different values of these two key parameters of preferences on the robustly optimal path of economic development and change in the temperature anomaly of the Intertemporal Elasticity of Substitution (IES) (related to the parameter in the power function) and the Elasticity of Substitution (related to the parameter in the CES function) between consumption per capita and our measure of climate quality. Our measure of climate quality declines as the temperature anomaly rises.

Since it was beyond the scope of this article to gather the data necessary to produce a serious index of climate quality, we simply assume that "climate quality" is proportional to the inverse of the productivity damage measure used in this article. Given this rather artificial measure of "climate quality" we regard the current article as a preliminary attempt to focus attention on the potential importance of the elasticity of substitution between consumption per capita and some measure of climate quality in preferences, as well as the usual focus of climate change on productivity. Indeed it can be shown that if the elasticity of substitution between consumption per capita and climate quality is less than one, then no matter how much consumption per capita grows over time, the maximal utility is bounded above because climate quality itself is bounded above. This latter statement is true for any measure of climate quality.

More formally, let c denote consumption per capita and Q a measure of climate quality.

The period utility of a representative individual is

$$U(c,Q) = \frac{u(c,Q)^{1-\gamma} - 1}{1-\gamma} \tag{1}$$

where

$$u(c,Q) = \left[\varphi c^{\tau} + (1-\varphi)Q^{\tau}\right]^{\frac{1}{\tau}}. \tag{2}$$

Hence the Intertemporal Elasticity of Substitution (IES) between utilities between periods is IES=$1/\gamma$, and the elasticity of substitution between consumption per capita and climate quality is $1/(1-\tau)$. We introduce a parameter, θ, for the robustness analysis, where the "size" of the set of departures from the baseline model increases as θ increases. E.g. $\theta = 0$, indicates that we have no doubts at all about the baseline model and our doubts about our specification increase as θ increases. When $\gamma = 1$ we interpret preferences are logarithmic. An appendix presents one possible argument for the inclusion of climate quality in preferences based on ethical considerations.

In our dynamic finite horizon model preferences are

$$\sum_{t=t_0}^{t_0+J-1} \beta^t \left[U\left(\frac{C_t}{L_t}, Q_t\right) + \frac{\beta}{2\theta} \sum_{i=m,a,d} G_{i,t}^2 \right] + \beta^J W\left(K_{t_0+J}, R_{t_0+J}, M_{t_0+J}, S_{t_0+J}, A_{t_0+J}, G_{d,t_0+J-1}, L_{t_0+J}\right) \tag{3a}$$

and the constraints are for $t = t_0, t_0 + 1, \dots t_0 + J - 1$

$$\log K_{t+1} = \log \bar{K}_{t+1} + \epsilon\, \sigma_k e_{k,t+1} \tag{3b}$$

$$R_{t+1} = R_t - F_t + \mu_r \tag{3c}$$

$$M_{t+1} = (1 - \kappa_m) M_t + \lambda F_t - \sigma_m\, G_{m,t} \tag{3d}$$

$$S_{t+1} = (1 - \kappa_s) S_t + \epsilon\, \sigma_s e_{s,t+1} \tag{3e}$$

$$\log A_{t+1} = \log A_t + \mu_a - \sigma_a G_{a,t} + \epsilon\, \sigma_a e_{a,t+1} \tag{3f}$$

$$\log D_t = (\omega_d - \sigma_d\, G_{d,t-1})\, |T_t - \underline{T}|^p \tag{3g}$$

$$\log L_{t+1} = \log(1 + n) + \log L_t + \epsilon\, \sigma_l e_{l,t+1} \tag{3h}$$

where

$$Y_t = \frac{A_t}{D_t} K_t^{\alpha} F_t^{\nu} L_t^{1-\alpha-\nu} \tag{3i}$$

$$\bar{K}_{t+1} = \phi_t\, [Y_t + (1 - \delta) K_t] \tag{3j}$$

$$C_t = (1 - \phi_t)\, [Y_t + (1 - \delta) K_t] \tag{3k}$$

$$T_t = \underline{T} + M_t + S_t \qquad\qquad Q_t = \frac{1}{D_t} \tag{3l}$$

$$1 \geq \phi_t \geq 0 \qquad\qquad F_t, R_{t+1} \geq 0 \tag{3m}$$

At time t, we interpret C_t as consumption, L_t as population (which we assume is equal to the labor force), K_t as capital, Y_t as output, T_t as temperature, $Y_t + (1-\delta)K_t$ as resources, $(1 - \phi_t)$ as the fraction of resources consumed, $\log \bar{K}_{t+1}$ as the mean of next period's log capital ($\log K_{t+1}$), F_t as fossil fuel usage, R_t as the stock of remaining available fossil fuels, M_t as man made climate changes, S_t as short run shocks to temperature, A_t as productivity, and D_t, as damages to productivity.

We let $e_{k,t+1}$, $e_{s,t+1}$, $e_{a,t+1}$, and $e_{l,t+1}$ be i.i.d. standard normal random variables. The parameter ϵ multiplies the shocks and facilities a small-noise expansion described in later sections. We let β be a discount factor which includes terms related to population growth.[1]

We assume $p \geq 0$, $0 < \beta < 1$, $\tau \leq 1$, $\gamma > 0$, $\alpha > 0$, $\nu > 0$, and $(\alpha + \nu) < 1$. We let $\underline{T}$ be the temperature level at a pre-industrial date far in the past.

We let J be the social planner's horizon and we let the terminal value function be

$$W(K, R, M, S, A, G_{d,-1}, L) = U\left(\frac{C}{L}, Q\right) \tag{4}$$

where

$$T = \underline{T} + M + S, \qquad D = \exp\left[\left(\omega_d - \sigma_d G_{d,-1}\right)|T - \underline{T}|^p\right], \qquad Q = \frac{1}{D}, \tag{5}$$

$$Y = \frac{A}{D} K^{\alpha} F^{\nu} L^{1-\alpha-\nu}, \qquad C = Y + (1-\delta)K, \qquad F = R. \tag{6}$$

The terminal value function assumes all remaining energy is immediately used in production and all remaining capital is immediately consumed.

The social planner wants to maximize the expected value of 3a by choice of adaptive process for ϕ_t and F_t; and minimize it by choice of adaptive process for $G_{m,t}$, $G_{a,t}$ and $G_{d,t}$ subject to the constraints 3b through 3m. Recall that robustness analysis uses the device of the minimizing agent, solely as a mechanism to construct a policy that works well for all departures from the estimated baseline model that lie in a constraint set whose size is determined by the estimated uncertainty in the estimated parameters of the baseline model (Hansen and Sargent 2008, espcially Chapter 9).

Because realistic values of κ_m are known to be near zero by climate scientists, our specification of man-made damages (M_t) approximately captures the CCR model of Matthews et al. (2009) and Matthews, Solomon, and Pierrehumbert (2012).

Our reference model is general enough to include the case where the production function is for agriculture where damages from climate change can be particularly large because most of agriculture is conducted outside and is exposed to the weather in contrast to a lot of industrial production where adaptations such as air conditioning can shield workers from extreme heat which would otherwise hurt productivity. In the case of agriculture Diffenbaugh et al. (2012, page 514) say, for the example of corn, "The climate change impact is driven primarily by intensification of severe hot conditions in the primary corn growing region of the US, which causes US corn price volatility to increase sharply in response to global warming projected over the next three decades."

There are both conditional mean and conditional variance effects at work here but Diffenbaugh et al. (2012) suggest that the volatility rise may contribute the most to damage effects as global warming proceeds. Although corn is only one crop, other agricultural crops are impacted by excessive clusters of hot days with temperatures beyond the ideal growing range in a similar manner (Diffenbaugh et al. 2012, Burke and Emerick 2016).

Indeed Burke and Emerick (2016) argue that even the industrial sector can be negatively impacted by warming. Of course air conditioning and other forms of adaptation can mitigate damaging effects of warming on industrial labor. While some adaptation in agriculture obviously occurs, e.g. by changing the mix of crop varieties grown as climate change proceeds, it may be more difficult to adapt in areas that are already warm because we have little experience with agriculture if those areas become even warmer.

Estimating the Climate-Economic Model

This section estimates the parameters in our climate-economic model using Hansen's (1982) Generalized Method of Moments (GMM). Tables 1-5 display GMM estimates of parameters the determine world population growth, capital accumulation, temperature change, output, and preferences. All of our estimates assume the reference model is correct. In the reference model, $\epsilon = 1$ and there is no robustness ($\theta = 0$), so that all of the G's are zero. An appendix describes the data.

Estimating Population Growth

We estimate n and σ_l using GMM from the moment conditions

$$E\left[\log L_{t+1} - \log L_t - \log(1+n)\right] = 0, \tag{7}$$

$$E\left[(\log L_{t+1} - \log L_t - \log(1+n))^2 - \sigma_l^2\right] = 0 \tag{8}$$

which follow from Equation 3h. Since there are two moments and two parameters, the parameters are exactly identified and the GMM test of overidentifying restrictions is not available. The parameter estimates are presented in Table 1 and are

$$n = \exp\left[\bar{E}\left(\log L_{t+1} - \log L_t\right)\right] - 1, \tag{9}$$

$$\sigma_l = \sqrt{\bar{V}\left(\log L_{t+1} - \log L_t - \log(1+n)\right)} \tag{10}$$

where $\bar{E}$ and $\bar{V}$ denote the sample mean and sample variance.[2]

Estimating the Capital Evolution Process

We estimate δ and σ_k from the moment conditions:

$$E\left[\log K_{t+1} - \log \bar{K}_{t+1}\right] = 0, \tag{11}$$

$$E\left[\left(\log K_{t+1} - \log \bar{K}_{t+1}\right)^2 - \sigma_k^2\right] = 0, \tag{12}$$

using GMM where

$$\bar{K}_{t+1} = Y_t + (1-\delta)K_t - C_t. \tag{13}$$

Although there does not exist a simple closed from expression for the estimates, the parameters are exactly identified and estimates are presented in Table 2.

Estimating the Temperature Equations

An appendix shows that

$$\mathcal{T}_t = (\kappa_s - \kappa_m)\lambda \sum_{j=h}^{t-1} (1-\kappa_m)^{t-j-1} F_j + \lambda F_t + (1-\kappa_s) T_t + \kappa_s \underline{T} \tag{14}$$

is the conditional mean of time $t+1$ temperature, using information at time t, where we set $h = 1751$. We estimate $\lambda, \kappa_s, \underline{T}$, and σ_s with GMM using the moments

$$E \begin{bmatrix} (T_{t+1} - \mathcal{T}_t) \otimes z_{1,t} \\ (T_{t+1} - \mathcal{T}_t)^2 - \sigma_s^2 \end{bmatrix} = 0 \tag{15}$$

with the instruments

$$z_{1,t} = \begin{bmatrix} 1 & T_t & F_t & F_{t-1} \end{bmatrix}, \tag{16}$$

and a fixed diagonal weighting matrix, W, where for $i = 1, 2 \ldots 4$, the (i,i) element of W is equal to the inverse of the sample mean of $z_{1,it}^2$. Here $z_{1,it}$ is the value of the ith instrument at time t. When $\lambda = 0$, the value of κ_m does not matter. If $\kappa_s = 0$, then $\underline{T}$ is not identified.

Estimates are presented in Table 3. Although the model performs poorly on the GMM test of overidentifying restrictions, there is some evidence that the model has weak explanatory power for data since 1952. Our estimates of λ are typically around 0.0028 and consistent with previous studies. For example, Matthews et al. (2009) report values of about 0.0017 based on numerical climate model simulations. Also, Leduc, Matthews, and de Elía (2016) find values up to 0.0030 for higher latitude regions.[3] However, our estimates should be viewed with some caution since our instruments T_t and F_t may not be stationary.

Estimating the Output Equations

An appendix shows

$$\log A_{t+1} - \log A_t = \mathcal{M}_{t+1} + \mathcal{E}_{t+1} = \mu_a + \sigma_a e_{a,t+1} \tag{17}$$

where $\mathcal{E}_{t+1}$ captures the change in log productivity due to temperature changes and $\mathcal{M}_{t+1}$ represents other changes:

$$\mathcal{M}_{t+1} = \log \frac{Y_{t+1}}{Y_t} - \alpha \log \frac{K_{t+1}}{K_t} - \nu \log \frac{F_{t+1}}{F_t} - (1 - \alpha - \nu) \log \frac{L_{t+1}}{L_t}, \tag{18}$$

$$\mathcal{E}_{t+1} = \omega_d \left| T_{t+1} - \underline{T} \right|^p - \omega_d \left| T_t - \underline{T} \right|^p. \tag{19}$$

For several different values of p and ν, we estimate ω_d, μ_a, and σ_a using the moments

$$E \begin{bmatrix} (\mathcal{M}_{t+1} + \mathcal{E}_{t+1} - \mu_a) \otimes z_{2,t} \\ (\mathcal{M}_{t+1} + \mathcal{E}_{t+1} - \mu_a)^2 - \sigma_a^2 \end{bmatrix} = 0, \tag{20}$$

with the instruments

$$z_{2,t} = \begin{bmatrix} 1 & \log \frac{Y_t}{Y_{t-1}} & \log \frac{K_t}{K_{t-1}} & \log \frac{F_t}{F_{t-1}} & \log \frac{L_t}{L_{t-1}} & \log \frac{K_t}{Y_t} \end{bmatrix}', \tag{21}$$

and a fixed diagonal weighting matrix, W, where for $i = 1, 2 \ldots 6$, the (i, i) element of W is equal to the inverse of the sample mean of $z_{2,it}^2$. Here $z_{2,it}$ is the value of the ith instrument at time t. The $(7, 7)$ element of weighting matrix W is one. Although one could argue that

$$(T_t - \underline{T})^p - (T_{t-1} - \underline{T})^p \tag{22}$$

would be a good candidate for an instrument, we choose not to use it because it varies with values of p and complicates comparison of the performance of different values of p. We fix the value of $\underline{T} = 13.74$ using the value of one of its estimates from the temperature equations. We also fix α at 0.4.

Table 4 presents estimates for $p = 2$ and $p = 4$. For comparison purposes we also present results when $p = 1$. In this case, we interpret $\mathcal{E}_{t+1}$ as

$$\omega_d \left(T_{t+1} - T_t\right) \tag{23}$$

to avoid possible discontinuities in derivatives.[4]

There is some evidence that the model with $p = 2$ and $\nu = 0.25$ provides a reasonable representation of past data. However, we find conflicting evidence on the value of ω_d. Although its estimates are generally not significantly different from zero, the p-values of models tell us that the model with $p = 2$ and $\omega_d = 0.2997$ is difficult to reject whereas the model with $\omega_d = 0$ is easily rejected. On the basis of these results, depending upon which test is used, one could say there is some evidence that ω_d is likely to be between approximately -0.20 and 0.70.

It is important to realize that there are many possible ways our model of economic climate change could be misspecified. For example, it is possible that increases in temperature adversely affect growth rates through channels other than D_t (Moyer et al. 2014). To some extent, our agents are robust to growth rate effects through the minimizing choice of $G_{a,t}$.

Calibrating Preference Parameters

In this section we estimate τ and β for several different fixed values of φ. We begin by showing that out model implies that there are two rates of return that satisfy the usual Euler equations that all available rates of return satisfy. We then estimate parameters using GMM, following Hansen and Singleton (1982) and many subsequent authors.

We make two assumptions for the analysis in this section:

Assumption 1. *The parameter p is a positive even integer.*

Assumption 2. *The derivative of the agent's value function with respect to reserves is zero, at all dates.*

Assumption 1 guarantees that $|T_t - \underline{T}|^p$ is differentiable with respect to T_t. Assumption 2 guarantees that the economy is not resource constrained. Given the parameter values we use in most of our examples in later sections, Assumption 2 is an implication of the model, but it does not necessarily hold for all parameter values and all initial conditions. Assumption 2 is useful in this section to simplify the calibration but we do NOT impose this assumption in other sections of this article.

We write the utility function at time t as

$$U_t \equiv U\left(\frac{C_t}{L_t}, Q_t\right) = \frac{\left(\varphi\left[\frac{C_t}{L_t}\right]^\tau + (1-\varphi)Q_t^\tau\right)^{\frac{1-\gamma}{\tau}}}{1-\gamma} \tag{24}$$

and use the following notation for derivatives:

$$U_{xt} = \frac{\partial U\left(\frac{C_t}{L_t}, Q_t\right)}{\partial X_t} \qquad \text{where } X_t = C_t \text{ or } Q_t. \tag{25}$$

Let

$$\mathcal{S}_t = \beta \frac{U_{ct}}{U_{ct-1}} \tag{26}$$

be a stochastic discount factor (Hansen and Renault 2010). In an appendix, we derive the moment conditions

$$E_{t-1}\mathcal{S}_t R_{kt} = 1, \qquad\qquad E_{t-1}\mathcal{S}_t R_{dt} = 1 \tag{27}$$

where

$$R_{kt} = \alpha \frac{Y_t}{\bar{K}_t} + (1-\delta)\left(\frac{K_t}{\bar{K}_t}\right) \tag{28}$$

is the gross return on capital and

$$R_{dt} = \frac{Y_t F_{t-1}}{Y_{t-1} F_t} \left[1 - \kappa_m + \left(\frac{\omega_d \, p \, \lambda}{\nu} \right) (T_t - \underline{T})^{p-1} \left(\frac{Q_t U_{qt}}{Y_t U_{ct}} + 1 \right) F_t \right] \tag{29}$$

is a fictitious return related to optimal energy usage.[5] These moments conditions are for a non-robust version of the model in which $G_{m,t}, G_{a,t}$, and $G_{d,t}$, are zero. The moment conditions are plausibly stationary versions of the usual consumption Euler equation and a corresponding equation for optimal energy usage.

For various values of φ, Table 5 estimates τ and β using the moments

$$E \begin{bmatrix} (\mathcal{S}_t R_{kt} - 1) \otimes z_{3,t-1} \\ (\mathcal{S}_t R_{dt} - 1) \otimes z_{3,t-1} \end{bmatrix} = 0 \tag{30}$$

with the instruments,

$$z_{3,t-1} = \begin{bmatrix} 1 & \frac{Y_{t-1}}{K_{t-1}} & \frac{Y_{t-1}}{Y_{t-2}} & \frac{F_{t-1}}{F_{t-2}} & \frac{C_{t-1} L_{t-2}}{C_{t-2} L_{t-1}} \end{bmatrix}' \tag{31}$$

and a fixed diagonal weighting matrix, W, where for $i = 1, 2 \ldots 5$, the (i, i) and $(5+i, 5+i)$ elements of W are equal to the inverse of the sample mean of $z^2_{3,it-1}$. Here $z_{3,it-1}$ is the value of the ith instrument at time t. We fix $p = 2$, $\gamma = 1$, $\omega_d = 0.2997$, $\nu = 0.25$, $\lambda = 0.0028$, $\kappa_m = 0.001$, $\underline{T} = 13.74$, $\alpha = 0.4$, and $\delta = 0.0573$ in all specifications.

We find some evidence for values of φ around 0.8 and values of τ around -1.3. However, as is typical of many dynamic economic models, there is strong evidence to suggest that the model is misspecified as the GMM test of overidentifying restrictions test fails for all specifications. The estimated parameter values should be viewed as calibrated, or as rough approximations, and not statistically justified estimates.

Although we only estimate preference parameters for a non-robust representative agent, since optimal decisions do not change very much as a reasonable amount of robustness is introduced into the economy, when $\gamma = 1$ and $\omega = 0.2997$, the presented estimated values are also reasonable approximations for the preference parameters of a robust representative agent. As we describe in later sections robust decision rules can be very different for other parameter values (such as when $\omega_d = 0$), so its not always the case that a non-robust model can be used to calibrate a robust model.[6]

For interpretation, we can decompose R_{dt} into three components. The first component includes terms that represent the effect of temperature change on productivity:

$$R_{at} = \frac{Y_t F_{t-1}}{Y_{t-1}} \left(\frac{\omega_d \, p \, \lambda}{\nu} \right) (T_t - \underline{T})^{p-1} \,. \tag{32}$$

This term would be zero if ω_d was always zero or if D_t did not affect output. The second component represents the *direct* effects of temperature change on preferences:

$$R_{qt} = \frac{Y_t F_{t-1}}{Y_{t-1}} \left(\frac{\omega_d \, p \, \lambda}{\nu} \right) (T_t - \underline{T})^{p-1} \frac{Q_t U_{qt}}{Y_t U_{ct}} \tag{33}$$

$$= \frac{F_{t-1}}{Y_{t-1}} \left(\frac{\omega_d \, p \, \lambda}{\nu} \right) (T_t - \underline{T})^{p-1} \left(\frac{1-\varphi}{\varphi} \right) C_t \left(\frac{Q_t L_t}{C_t} \right)^{\tau} . \tag{34}$$

This term would be zero if $\omega_d = 0$ or if climate quality did not directly affect preferences. The third component represents the contribution of future damages (beyond time $t+1$):

$$R_{ht} = \frac{Y_t F_{t-1}}{Y_{t-1} F_t} (1 - \kappa_m) \,. \tag{35}$$

This term would be zero if man made temperature increases only lasted one period, which happens when $\kappa_m = 1$. However, κ_m is generally thought to be near zero by climate scientists, so this term is likely to be large.

Figure 1 graphs the three components of R_{dt} using our estimates and actual data. We see that until about 1970 the R_{at} and R_{qt} components are almost identical. Starting in the late 1970s, R_{qt} grows at a fast rate and starts to dominate R_{at}. R_{at} is roughly constant between 1952 and 2000; and starts to fall in the 21st century.

Parameter Selection

In this section, we describe the parameter values and initial conditions used in simulations discussed in subsequent sections.

Climate and Preference Parameters

We combine our estimates for 1952-2011 with standard calibrations to set parameter values that we believe are useful for future policy evaluation.

For the parameter values directly related to temperature we set

$$\lambda = 0.0028, \qquad \kappa_s = 0.77, \qquad \kappa_m = 0.001, \qquad \underline{T} = 13.74, \qquad \sigma_s = 0.0943 \tag{36}$$

from the estimates for 1952-2011 of the temperature equation when κ_m is fixed at 0.001. Although the estimates when κ_m is fixed at zero have a slightly higher p-value, there is scientific evidence to suggest that man made damages should depreciate at a small rate over time. In addition, our estimates for the time period 1882-2011 suggest that κ_m may be much higher than zero.

For the parameter values directly related to output and damages, we set

$$p = 2, \quad \omega_d = 0.2997, \quad \mu_a = 0.0103, \quad \alpha = 0.4, \quad \nu = 0.25, \quad \sigma_a = 0.0397 \tag{37}$$

from Table 4, Panel C, the third row. Since we can not reject the hypothesis that the parameter ω_d is zero, in some examples set $\omega_d = 0$. A social planner who denies that temperature changes effect the economy would set $\omega_d = 0$.

We set the population growth parameters and capital evolution parameters as

$$n = 0.0172, \qquad \sigma_l = 0.002, \qquad \delta = 0.0573, \qquad \sigma_k = 0.0217 \tag{38}$$

from their estimates. We set

$$\beta = 0.969 \qquad \varphi = 0.8 \tag{39}$$

from the estimate of preference parameters with the highest p-value (See the 8th row of Table 5). In most of our examples we set $\tau = -1.3$ (which is τ's estimated value in the same estimation) though we do consider other values of τ. We also usually set $\gamma = 1$ but briefly consider other values.

We have very little information about the other parameters and we set them as:

$$\sigma_m = 0.0001, \qquad \sigma_d = 0.2350, \qquad \mu_r = 0. \tag{40}$$

We select the values of σ_m and σ_d, to be similar to the standard errors of estimates of λ and ω_d. This is a reasonable setting if the parametric models for temperature and damages are correct and we are mainly worried about parameter uncertainty. Though, these values perhaps underestimate the values of σ_m and σ_d if we are worried that the parametric specification is wrong.

In all of our examples, $\epsilon = 1$ and we consider several different values for θ.

Appropriate Levels of Robustness

How robust should the representative agent be? In this section, we present a procedure for determining reasonable levels of robustness, using past data. A reasonable amount of robustness generates perturbations of the reference model that can not be rejected from past data whereas an unreasonably large amount of robustness generates perturbations that can be rejected.

We let

$$\mathcal{G}^{\theta}_{1,t+1} = \sigma_m \, G_{m,t} \tag{41}$$

$$\mathcal{G}^{\theta}_{2,t+1} = \sigma_a G^{\theta}_{a,t} + \sigma_d \, G^{\theta}_{d,t} \, |T_t - \underline{T}|^p - \sigma_d \, G^{\theta}_{d,t-1} \, |T_{t+1} - \underline{T}|^p \tag{42}$$

where the superscript θ indicates that the value of θ is fixed at a particular value. Here $\mathcal{G}^{\theta}_{1,t+1}$ are the robust perturbations to man-made damages generated by θ and $\mathcal{G}^{\theta}_{2,t+1}$ are the robust perturbations to the change in log productivity generated by θ.

We introduce a parameter ϱ and estimate ϱ with GMM using the stacked moment conditions from the temperature and output equations:

$$m^{\theta}_{t+1} = \begin{bmatrix} \left(T_{t+1} + \varrho \, \mathcal{G}^{\theta}_{1,t+1} - \mathcal{T}_t\right) \otimes z^{\theta}_{1,t} \\ \left(T_{t+1} + \varrho \, \mathcal{G}^{\theta}_{1,t+1} - \mathcal{T}_t\right)^2 - \sigma_s^2 \\ \left(\mathcal{M}_{t+1} + \mathcal{E}_{t+1} + \varrho \, \mathcal{G}^{\theta}_{2,t+1} - \mu_a\right) \otimes z^{\theta}_{2,t} \\ \left(\mathcal{M}_{t+1} + \mathcal{E}_{t+1} + \varrho \, \mathcal{G}^{\theta}_{2,t+1} - \mu_a\right)^2 - \sigma_a^2 \end{bmatrix} \tag{43}$$

for a given value of θ, where the other parameters are held fixed at the values described in our parameter selection section, unless otherwise stated.[7] The instruments, $z^{\theta}_{1,t}$ and $z^{\theta}_{2,t}$, may depend on the fixed value of θ.[8] If the robust perturbations generated by θ were the best description of the world then estimates of ϱ would be near one. However, since the reference model is a reasonably good description of the world we expect estimates of ϱ to be near zero.

Although a robust agent expects estimates of ϱ to be near zero, he wants to know if past data says that ϱ could be as large as one. We assume the robust agent uses standard hypothesis tests based on estimates of ϱ and its standard error to evaluate the possibility that ϱ is one. If the standard error of an estimate of ϱ is large enough so that we can not reject its value being one, we say that the robust perturbations generated by θ are reasonable. If we can reject ϱ being one then the robust perturbations are too large and the value of θ should be reduced. For a given θ, if we can not reject values of φ much larger than one then (although the robust perturbations are reasonable for this θ), the value of θ should be increased because larger robust perturbations also will be reasonable.

This approach to determining appropriate values of the parameters shares many features of the detection probability approach advocated by Hansen and Sargent (2008, Chapter 9). Our approach is computationally simpler because, for a given θ, it only requires solving the model numerically J times, where J is the horizon.[9] One drawback of our approach is that we rely on asymptotically justified standard errors and do not fully take into the limited amount of data available.

The reasonableness of perturbations in part depends on agents preferences. Some robust agents may be worried about extreme perturbations which are likely to occur with 10% probability, based on estimates from previous data. Others may view 10% percent as too extreme and only worry about perturbations that could occur with 30% probability. We adopt a middle ground and assume agents should worry about perturbations that can occur with about 20% probability.

When the estimate of ϱ is very near zero,[10] if the standard error of ϱ is 0.780 then we expect perturbations as large as those generated by θ to occur with probability 10%. If the standard error of ϱ is 1.188 then we expect perturbations as large as those generated by θ to occur with probability 20%. If the standard error of ϱ is 1.907 then we expect perturbations as large as those generated by θ to occur with probability 30%. If the estimate of ϱ is not near zero then these critical values need to be adjusted.

The results in Table 6 show that estimates of ϱ are very near zero, when the the damages parameter ω_d is set at its estimated value 0.2997. When τ is between -1.3 and -1.0 reasonable values of θ are between 0.1 and 0.2. We can reject the perturbations generated by $\theta \geq 2$ as being unreasonable, whereas the perturbations generated by $\theta \leq 1$, although reasonable are not large enough, for a social planner who worries about perturbations that can occur with about 20% probability.

Table 6 also shows that when the social planner uses a reference model different from the estimated model, then larger values of ϱ are possible. For example, when $\omega_d = 0$ and

$\tau = -1.3$, values of ϱ are much bigger than zero. In this case, when $\theta = 0.1$ or 0.2 estimates of ϱ are even greater than one and thus regardless of the value of ϱ's standard error, the social planner should be worried about the perturbations generated by 0.1 and 0.2, since his estimated ϱ says that even larger distortions are the most likely outcome. A social planner willing to worry about perturbations that can occur with 20% probability will set θ to be between 0.3 and 0.4 when $\omega_d = 0.0$.[11]

Initial Conditions

For our simulations in subsequent sections, we let time begin in the year 2011 and set the initial conditions accordingly. We set the values of capital and reserves to be their actual values in 2011 (using the measurements described in our data appendix):

(44) $$K_{2011} = 158.72, \qquad L_{2011} = 3.67, \qquad R_{2011} = 1635.72.$$

The initial value of S is chosen so that temperature in the model directly matches actual temperature in 2011:

(45) $$S_{2011} = T_{2011} - M_{2011} - \underline{T} = -0.126$$

where

(46) $$M_{2011} = \lambda \sum_{j=1751}^{2010} (1 - \kappa_m)^{t-j-1} F_j = 0.9860$$

and where $T_{2011} = 14.6$ is temperature in 2011. The initial value of A is chosen so that output matches actual output in 2011:

(47)

$$\log A_{2011} = \log Y_{2011} + \log D_{2011} - \alpha \log K_{2011} - \nu \log F_{2011} - (1-\alpha-\nu) \log L_{2011} = 1.0357$$

where

$$\log D_{2011} = \omega_d \left| T_{2011} - \underline{T} \right|^p = 0.2217 \tag{48}$$

and where $Y_{2011} = 47.35$ and $F_{2011} = 9.45$ are the actual values of output and carbon usage in 2011.

Simulations

This section conducts a multitude of simulations for various values of the robustness parameter ranging from near zero robustness (where the analyst is almost certain she has the "right" specification, i.e. the reference specification is correct) to a sizable amount of robustness where doubts are much larger but within the range of empirically disciplined plausible doubts.

Figures 2 thru 6 plot energy usage, consumption per-worker, temperature, output and capital in the reference model, starting in 2011. The social planner uses robust decision rules each period, although the reference model is correct and the minimizing distortions do non effect future state variables. The simulations are designed to roughly match the mean dynamics of the system. Every period the social planner uses the optimal decision rules for the stochastic problem, but the shocks ($e_{k,t+1}$, $e_{s,t+1}$, $e_{a,t+1}$, and $e_{l,t+1}$) end up always being zero. Thus, we simulate the system:

$$K_{t+1} = \phi_t \left[Y_t + (1-\delta) K_t \right], \qquad R_{t+1} = R_t - F_t + \mu_r, \tag{49}$$

$$M_{t+1} = (1-\kappa_m) M_t + \lambda F_t, \qquad S_{t+1} = (1-\kappa_s) S_t \tag{50}$$

$$A_{t+1} = A_t \exp\left(\mu_a\right), \qquad L_{t+1} = (1+n) L_t, \tag{51}$$

where

$$(52) \quad T_t = \underline{T} + M_t + S_t, \qquad D_t = \exp\left(\omega_d \left|T_t - \underline{T}\right|^p\right), \qquad Q_t = \frac{1}{D_t},$$

$$(53) \quad Y_t = \frac{A_t}{D_t} K_t^{\alpha} F_t^{\nu} L_t^{1-\alpha-\nu}, \qquad C_t = (1-\phi_t)\left[Y_t + (1-\delta)K_t\right],$$

and where ϕ_t and F_t are the optimal decision rules in the robust stochastic economy.[12] The initial date $t_0 = 2011$ and the horizon $J = 160$. Although the social planner, imagines the world as ending in 2171 we only report the values of the state for the first 100 years, in long-horizon graphs, and the first 50 years, in short-horizon graphs.

We solve the robust model using a version of the small noise algorithm presented in Anderson, Hansen, and Sargent (2012). In the algorithm, we compute a Taylor Series approximation for the optimal decision rules around a deterministic dynamic game in which $\epsilon = 0$. By expanding around a deterministic dynamic game rather than a deterministic optimal control problem, we generally achieve more accurate solutions. See Section 9.1 of Anderson, Hansen, and Sargent (2012) for a detailed description of the algorithm.

Simulations Using Optimal Estimates

Figure 2 presents long-horizon simulations, for several values of the robustness parameter θ, for our leading choice of parameter values listed in our parameter selection section with $\gamma = 1$, $\tau = -1.3$, and $\omega_d = 0.2997$. We see that the evolution of energy usage, consumption per-person, temperature, and output do not very much as θ increases. Figure 3 presents short horizon simulations under alternative parameter values. A higher and positive value of τ leads to higher fuel usage and higher consumption per-worker. When $\gamma = 0.5$, the results are almost identical to the log case. When $\gamma = 5$, fuel usage only slightly changes but consumption per-worker is noticeably smaller.

Simulations Assuming Temperature Changes Have No Economic Impact

Figure 4 presents simulations for a robust social planner who believes that temperature changes have no effects on productivity or climate quality ($\omega_d = 0$). Although this social planner denies the effects of temperature change, he wants to be robust to the possibility that he is wrong and temperature changes do affect productivity and air quality. The time-paths of optimal energy usage, consumption per-worker, temperature and output vary significantly as θ increases. For very small θ, energy usage can be constrained by the stock of reserves but for $\theta \geq 0.1$ energy usage is mitigated mainly by the fear that the reference model is wrong.

We see that robust agents who deny temperature effects on the economy choose similar (but not identical) policies to robust (or non-robust) agents who believe temperature changes affect productivity and preferences. For example, when $\theta = 0.4$, initial choices of energy usage and consumption per-worker in 2011 are in the same ballpark.[13]

Separating the Productivity and Preference Effects

In this section we discuss the different implications of the productivity and preference effects. The first row of Figure 5 uses our optimal parameter estimates and corresponds to the plots in Figure 2. The second row removes the productivity effect so that

$$Y_t = A_t K_t^{\alpha} F_t^{\nu} L_t^{1-\alpha-\nu} \tag{54}$$

every period. The third row removes air quality from preferences every period from preferences so that:

$$u\left(c, Q\right) = c. \tag{55}$$

In the second row, we see that the preference effect on its own leads to a gloomy outcome. The social planner knows that utility will eventually be limited by air quality and optimally decides on a large value of consumption per-worker now, which drastically decreases over time. The social planner chooses to not accumulate much capital and capital falls thru time. In later years (not plotted) consumption per-worker continues to fall at a fast rate after 2060. In the third row we see that the productivity effect on its own, leads to a much more gradual decrease in consumption per-worker and a large increase in capital. The increase in capital partially offsets the decrease in productivity caused by temperature change. Interestingly in the first row, where both effects are present, consumption per-worker falls only slightly in the initial periods before eventually rising after about year 2033. The first row shows that the combination of the productivity and preference effects are much different than a simple linear combination of the separate effects. For example, consumption per-worker, after an initial fall, returns to small but persistent growth in about 2033; whereas with only the preference effect, or only the productivity effect, consumption per-worker continues to fall over time.

Figure 6 considers the case of a planner who denies temperature changes affect productivity and air quality; and sets $\omega_d = 0$. The planner, however, does wish to be robust to the possibility that he is wrong and there are economic damages from climate change. The first row of Figure 6 corresponds to the plots in Figure 4. The second row has only the preference effect. Like the second row in Figure 5 consumption per-worker and capital fall rapidly. The third row has only the productivity effect. We see that for values of $\theta \geq 0.01$ consumption is rising over time. For $\theta = -0.00001$, consumption initially rises even faster and then starts to fall. Capital rises at a fast rate.

Conclusions

This article has developed the first, to our knowledge, example of an empirically disciplined robustness analysis in climate economics. It is also the first model in climate economics to use a specification of climate dynamics built on foundations laid by recent work on the Cumulative Climate Response, CCR (Matthews et al. 2009, Matthews, Solomon, and Pierrehumbert 2012) which shows that the increase in the temperature anomaly to approximately linearly proportional to cumulated emissions. We estimate a baseline model of economic growth dynamics and climate dynamics and calibrate robustness parameters to empirically discipline the size of the set of perturbations from the baseline model.

The data on the economic impacts of climate change is sparse and subject to disagreement and many interpretations. We suggest that although reasonable economists can doubt that temperature changes affect productivity and preferences, robust economists will roughly agree on optimal strategies. We show, in a polar example of infinite known fossil fuel reserves and zero extraction costs, that although non-robust climate believers and climate deniers choose drastically different policies, robust climate believers and climate deniers choose somewhat similar policies. Our results suggest that if a consensus to use robust policies emerges, then there may be much less disagreement between climate believers and deniers about policy.

Our results suggest that including both preference and productivity effects of climate change is important and our simulations suggest that preferences for air-quality have a different impact on optimal decisions than productivity damages. Our empirical calibration of preferences shows that the preference affects are becoming more important over time.

While our model is very stylized and very simple it is rich enough to expose the economic importance of changes in the IES, ES, output elasticity w.r.t. capital, output elasticity w.r.t. to labor and energy, as well as the empirically disciplined size of the perturbation set around the estimated baseline version of our model. Even though our model is very simple it required development of computational methods that yield workably useful results using only laptop computers. Better computational results will need more computer power.

Future research is needed to introduce recursive preferences, e.g., as in Hansen and Sargent (2008, Chapter 14) and spatial transport of heat across space, e.g. as in Brock et al. (2013). Extension of our work to the case of recursive preferences is important because this allows effects of changing IES to be separated from effects of changing risk aversion. More research is also needed to produce better measures of climate quality than we used here. It would also be valuable to introduce endogenous technical change, adaptation to climate change as well as mitigation, better representation of climate dynamics, and backstop technologies. However, our model, as is, was rich enough to expose the importance of the economic forces inherent in the robust formulation of economic-climate models.

Appendix

All the appendices are designed to be available online only.

Other Approaches To Robustness

We note that complementary techniques exist for dealing with issues closely related to the type of robustness considered here, e.g. using Bayesian methods, for addressing model uncertainty have been proposed in a series of papers also in the context of macroeconomics and growth. Brock and Durlauf (2001) discussed Bayesian Model Uncertainty, Leamer's extreme bounds analysis, and versions of ambiguity aversion that modify Bayesian Model Averaging. They also conduct an illustrative application to the impact of ethnolinguistic heterogeneity to African economic growth in comparison to other countries around the world. Brock, Durlauf, and West (2003, 2007) discussed application of closely related approaches to economic growth policy and especially for macroeconomic policy, e.g. the setting of "Taylor" type rules for monetary policy.

Brock, Durlauf, and West (2007) argue that in some cases where the scientific team does not wish to take a stand on the preferences of the policy maker, it should simply prepare a graphical summary, called an "action dispersion, welfare dispersion plot," that illustrates, for each model in the model uncertainty set, the optimal action chosen, the optimal welfare produced by that optimal action, and an empirically disciplined credibility number (e.g. a relative likelihood computed from data). In this way the policy maker can see how optimal actions, optimal welfares, and credibility numbers are dispersed in the model uncertainty set. In this way the policy maker's attention is drawn towards the cluster of models that have the most credibility given the data and is not unduly distracted by models that have little support in the data. This kind of plot can illustrate quickly the type of uncertainty management problem the policy maker faces for the case of one parameter rules in monetary policy; two and three parameter rules have also been used. Another example of the Bayesian Model Uncertainty approach was an application by the Bank of England; Cogley et al. (2011), for the setting of Central Bank policy. Cogley and Sargent (2005) do an interesting Bayesian Model Uncertainty study where the posterior probabilities over three rival models having some a priori credibility in economic science are updated over time by a policymaker in an optimal learning framework. Brock and Durlauf (2015) compare and contrast these various approaches to dealing with "sturdy" policy choices that perform well over a range of uncertainties, e.g. model uncertainties, that policy makers must face, as well as discuss critiques of received approaches to this basic problem in policy analysis.

Preferences and Climate Ethics

IA models based on the RCK-DICE framework have analyzed optimal dynamic GHG abatement as a problem of balancing economic consumption and well-being between present and future generations, taking account of the costs both of climate change damages and of abatement policies. In particular, the standard model specification omits the possibility that human society may explicitly value climate or environmental quality as distinct from economic consumption. This has led Roe and others to point out that in fact, future generations may not regard a high level of consumption as adequate "compensation" for a degraded climate. In turn, this possibility has been one argument used to support approaches such as a "precautionary" framing of abatement policy that would dictate present-day efforts to reduce GHG emissions substantially more aggressive that those justified on cost-benefit grounds in many IA analyses. This is one example of the view that shortcomings of economic methodology justify a turn to instead relying upon ethical and moral criteria to formulate climate policy that would avoid an unacceptably high probability of catastrophic climate change. Many thoughtful commentators suggest that it is simply wrong for the State to take a life and, likewise, it is simply wrong for today's generations to bequeath a planet with a degraded climate to future generations.

If, on the basis of such concerns society were to take a "moral imperative" position on climate change as suggested by Roe (2013), what imperative should be used? How would this kind of approach actually be implemented in policy? In the IA modeling context, the most common approach to this question has been to lower the pure rate of time preference in models based on RCK-DICE, thereby giving greater weight to future economic outcomes, including damages from climate change, and therefore justifying more stringent GHG emissions abatement. However, as has been pointed out by Nordhaus and Dasgupta, without other changes to the assumptions of such models, this can result in internal inconsistencies that yield model outputs that actually weaken the case for more aggressive climate policy, are contrary to empirical evidence, or both.

However, such ethical concerns are indeed within the purview of economic analysis, and, correctly applied, economic methods can yield valuable insights about them and show in a clear and rigorous way they might inform policy.

For example, Hoel and Sterner (2007), Sterner and Persson (2008) study the "environmental quality" problem, analyzing the impacts of relative prices between consumption goods and environmental goods for the discounting process. A potentially attractive criterion is sustainability in genuine wealth across generations as argued by Arrow et al. (2012). They create a measurement of well-being which includes health capital, human capital, consumption of material goods per capita, natural capital, environmental quality, etc., which can also include components of climate quality. They argue that proper policy with respect to future generations requires that their measure of "comprehensive wealth" rises over future generations.

The analysis presented in this article addresses some of the ethical concerns about climate change by taking into account climate quality in the utility function of the representative decision-maker in our model, where the elasticity of substitution between climate quality and consumption per capita can be less than one. In this case the utility is eventually bounded above independently of how high consumption per capita rises due to economic growth. This captures the moral intuition that there are fundamental limits to the substitutability of economic consumption for climate quality.

Data

The data used in the estimations is yearly and is measured as follows:

1. Temperature: We measure temperature in degrees Celsius using data on "Combined Land-Surface Air and Sea-Surface Water Temperature Anomalies" downloaded from `http://data.giss.nasa.gov/gistemp/` on January 8, 2016. We use the average temperature over the calendar year. A direct link to the data is here `http://data.giss.nasa.gov/gistemp/tabledata_v3/GLB.Ts+dSST.txt`.

2. Energy: We measure energy in billions of metric tons of carbon, using data on total world carbon usage was downloaded from `http://cdiac.ornl.gov/trends/emis/meth_reg.html` on January 8, 2016. A direct link to the data is here `http://cdiac.ornl.gov/ftp/ndp030/global.1751_2011.ems`.

3. Reserves: We set $\underline{R}$ to 2000 billions of metric tons of carbon and interpret this as a measure of the stock of reserves at the beginning of 1751. Then using data on carbon usage we deduct the cumulative carbon usage to determine reserves. Reserves in year t are

$$R_t = \underline{R} + \mu_r(t - 1751) - \sum_{i=1751}^{t-1} F_i \tag{56}$$

when $t \geq 1751$.

4. Output, capital, consumption, and population: The data used on output, capital, consumption, and population is measured with estimates from version 8.1 of the Penn World Table (Feenstra, Inklaar, and Timmer 2015), downloaded on January 8, 2016. We measure output using data on the real side of output in trillions of 2005 US

dollars (series “RGDPO”, rescaled). We measure capital using data on the capital stock in trillions of 2005 US dollars (series “CK”, converted to constant PPP and rescaled)[14]. We measure (total) consumption with the sum of private consumption (series “CSH_C” times ”RGDPO”, rescaled) and 70% of government consumption (0.7 times series “CSH_G” times “RGDPO”, rescaled). Since government consumption partially includes government investment, we only include 70% of government consumption in total consumption. We measure population in billions (series “POP”, rescaled). We sum up output, capital, consumption, and population for each country for which data in all years (1950 to 2011) is available. If output, capital, consumption, or population data is missing in one or more years for a country, then that country is excluded from the data set for all years.

Of the 167 countries included in the Penn World table, the following 54 countries have the necessary data: Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Canada, Colombia, Costa Rica, Cyprus, Democratic Republic of the Congo, Denmark, Ecuador, Egypt, El Salvador, Ethiopia, Finland, France, Germany, Guatemala, Honduras, Iceland, India, Ireland, Israel, Italy, Japan, Kenya, Luxembourg, Mauritius, Mexico, Morocco, Netherlands, New Zealand, Nigeria, Norway, Pakistan, Panama, Peru, Philippines, Portugal, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Thailand, Trinidad and Tobago, Turkey, Uganda, United Kingdom, United States, Uruguay, and Venezuela.

The Conditional Mean of Temperature

Substituting $S_t = T_t - M_t - \underline{T}$, $S_{t+1} = T_{t+1} - M_{t+1} - \underline{T}$, and Equation 3d into Equation 3e and rearranging yields

$$T_{t+1} = \mathcal{T}_t + \sigma_s e_{s,t+1} \tag{57}$$

where we define:

(58a) $$\mathcal{T}_t = M_{t+1} + (1-\kappa_s)(T_t - M_t) + \kappa_s \underline{T}$$

(58b) $$= (\kappa_s - \kappa_m)M_t + \lambda F_t + (1-\kappa_s)T_t + \kappa_s \underline{T}.$$

We assume $M_h = 0$ for some date h far in the past, and solve Equation 3d backward. In this article, we take $h = 1751$. For $t > h$ the solution is

(59) $$M_t = \lambda \sum_{j=h}^{t-1} (1-\kappa_m)^{t-j-1} F_j.$$

We substitute the result into Equation 58b to yield the conditional mean of time $t+1$ temperature:

(60) $$\mathcal{T}_t = (\kappa_s - \kappa_m)\lambda \sum_{j=h}^{t-1} (1-\kappa_m)^{t-j-1} F_j + \lambda F_t + (1-\kappa_s)T_t + \kappa_s \underline{T}.$$

The Change in Log Productivity

We begin by solving Equation 3i for A_t and replacing D_t with the expression in Equation 3g:

(61) $$A_t = \frac{Y_t \exp\left(\omega_d \left|T_t - \underline{T}\right|^p\right)}{K_t^\alpha F_t^\nu L_t^{1-\alpha-\nu}}$$

when $G_{d,t-1} = 0$ and $\epsilon = 0$. Using an analogous expression for A_{t+1} we write

(62) $$\log A_{t+1} - \log A_t = \mathcal{M}_{t+1} + \mathcal{E}_{t+1}$$

where we define

$$(63)\qquad \mathcal{M}_{t+1} = \log \frac{Y_{t+1}}{Y_t} - \alpha \log \frac{K_{t+1}}{K_t} - \nu \log \frac{F_{t+1}}{F_t} - (1-\alpha-\nu) \log \frac{L_{t+1}}{L_t},$$

$$(64)\qquad \mathcal{E}_{t+1} = \omega_d \left|T_{t+1} - \underline{T}\right|^p - \omega_d \left|T_t - \underline{T}\right|^p.$$

$\mathcal{E}_{t+1}$ captures the change in log productivity due to temperature changes and $\mathcal{M}_{t+1}$ represents other changes. From Equation 3f, we know that

$$(65)\qquad \mathcal{M}_{t+1} + \mathcal{E}_{t+1} = \mu_a + \sigma_a e_{a,t+1}.$$

Moment Conditions From Optimization

In this appendix, we write the value function at time t as

$$(66)\qquad V_t \equiv \begin{cases} V\left(K_t, R_t, M_t, S_t, A_t, G_{d,t-1}, L_t, t\right) & t < t_0 + J - 1 \\ W\left(K_t, R_t, M_t, S_t, A_t, G_{d,t-1}, L_t\right) & t = t_0 + J \end{cases}$$

and use the following notation for derivatives:

(67)

$$V_{xt} = \frac{\partial V\left(K_t, R_t, M_t, S_t, A_t, G_{d,t-1}, L_t, t\right)}{\partial X_t} \qquad \text{where } X_t = K_t, R_t, M_t \text{ or } A_t \text{ when } t < t_0 + J - 1.$$

We derive moment conditions for a non-robust version of the model in which the objective at time t can be written as:

$$(68)\qquad V_t = \max_{\phi_t, F_t} \left[U_t + \beta E_t V_{t+1}\right]$$

where E_t denotes expectations with respect to time t information. For $t = t_0, t_0+1, \ldots t_0+$

$J-1$, the time $t+1$ values of the state are:

$$K_{t+1} = \phi_t \left[Y_t + (1-\delta) K_t \right] \exp\left(\sigma_k e_{k,t+1} \right), \tag{69}$$

$$R_{t+1} = R_t - F_t + \mu_r, \tag{70}$$

$$M_{t+1} = (1-\kappa_m) M_t + \lambda F_t, \tag{71}$$

$$S_{t+1} = (1-\kappa_s) S_t + \sigma_s e_{s,t+1}, \tag{72}$$

$$A_{t+1} = A_t \exp\left(\mu_a + \sigma_a e_{a,t+1} \right), \tag{73}$$

$$L_{t+1} = (1+n) L_t \exp\left(\sigma_l e_{l,t+1} \right), \tag{74}$$

where

$$D_t = \exp\left(\omega_d \left| T_t - \underline{T} \right|^p \right), \qquad Q_t = \frac{1}{D_t}, \tag{75}$$

$$C_t = (1-\phi_t) \left[Y_t + (1-\delta) K_t \right], \tag{76}$$

$$Y_t = \frac{A_t}{D_t} K_t^{\alpha} F_t^{\nu} L_t^{1-\alpha-\nu}, \tag{77}$$

$$T_t = \underline{T} + M_t + S_t, \tag{78}$$

$$C_t, F_t, R_{t+1}, K_{t+1} \geq 0, \qquad 1 \geq \phi_t \geq 0. \tag{79}$$

In this version of the problem we have assumed $\epsilon = 1$; and $G_{m,t}, G_{a,t}$, and $G_{d,t}$ are zero.

Euler equation

The first order condition for ϕ_{t-1} and the envelope condition for k_t can be written as:

$$U_{ct-1} = \beta E_{t-1} \zeta_t V_{kt} \tag{80}$$

$$V_{kt} = \left[\alpha \frac{Y_t}{K_t} + (1-\delta) \right] U_{ct} \tag{81}$$

where

$$\zeta_t = \frac{K_t}{\bar{K}_t} = \exp\left(\sigma_k e_{kt}\right). \tag{82}$$

Combining Equations 80 and 81 yields a version of the usual consumption Euler equation in a production economy:

$$U_{ct-1} = \beta E_{t-1}\left(\zeta_t\left[\alpha\frac{Y_t}{K_t} + (1-\delta)\right]U_{ct}\right) \tag{83}$$

$$= \beta E_{t-1}\left[\alpha\frac{Y_t}{\bar{K}_t} + (1-\delta)\left(\frac{K_t}{\bar{K}_t}\right)\right]U_{ct}. \tag{84}$$

By defining a stochastic discount factor

$$\mathcal{S}_t = \beta\frac{U_{ct}}{U_{ct-1}} \tag{85}$$

and the gross return on capital

$$R_{kt} = \alpha\frac{Y_t}{\bar{K}_t} + (1-\delta)\left(\frac{K_t}{\bar{K}_t}\right) \tag{86}$$

we can write the Euler equation as:

$$E_{t-1}\mathcal{S}_t R_{kt} = 1. \tag{87}$$

The first order condition for energy

The first order condition for energy, F_t, is:

$$\frac{\nu Y_t U_{ct}}{F_t} = \beta E_t\left[V_{rt+1} - \lambda V_{mt+1}\right]. \tag{88}$$

Since assumption 2 guarantees that $V_{rt+1} = 0$, we write the first order condition as

$$z_t = -\beta E_t V_{mt+1} \tag{89}$$

where we define

$$z_t = \frac{\nu Y_t U_{ct}}{\lambda F_t}. \tag{90}$$

Below we will also use a lagged version of Equation 89 which say that $z_{t-1} = -\beta E_{t-1} V_{mt}$.

The envelope conditions for M_t is

$$V_{mt} = U_{qt} \frac{\partial Q_t}{\partial T_t} - \frac{Y_t U_{ct}}{D_t} \frac{\partial D_t}{\partial T_t} + (1 - \kappa_m) \beta E_t V_{mt+1} \tag{91}$$

where

$$\frac{\partial D_t}{\partial T_t} = \omega_d p \left(T_t - \underline{T}\right)^{p-1} D_t \tag{92}$$

$$\frac{\partial Q_t}{\partial T_t} = -\frac{1}{D_t^2} \frac{\partial D_t}{\partial T_t} \tag{93}$$

$$= \omega_q p \left(T_t - \underline{T}\right)^{p-1} Q_t \tag{94}$$

with

$$\omega_q = -\omega_d. \tag{95}$$

Using Equation 89 and the derivatives above, we rewrite the envelope condition in Equation 91 as

$$V_{mt} = -(1 - \kappa_m) z_t + U_{mt} \tag{96}$$

where we define

$$U_{mt} = p\left(T_t - \underline{T}\right)^{p-1}\left(\omega_q Q_t U_{qt} - \omega_d Y_t U_{ct}\right) \tag{97}$$

$$= -\omega_d p\left(T_t - \underline{T}\right)^{p-1}\left(Q_t U_{qt} + Y_t U_{ct}\right). \tag{98}$$

Using the definition of z_{t-1}, we write Equation 96 as:

$$-z_{t-1} = -(1-\kappa_m)\beta E_{t-1} z_t + \beta E_{t-1} U_{mt} \tag{99}$$

where we have taken expected values at time $t-1$ and multiplied all terms by β. Dividing both sides by z_{t-1} gives us

$$(1-\kappa_m)\beta E_{t-1}\left(\frac{z_t}{z_{t-1}}\right) - \beta E_{t-1}\left(\frac{U_{mt}}{z_{t-1}}\right) = 1 \tag{100}$$

Since

$$\frac{z_t}{z_{t-1}} = \frac{Y_t U_{ct} F_{t-1}}{Y_{t-1} U_{ct-1} F_t} \tag{101}$$

and

$$\frac{U_{mt}}{z_{t-1}} = -\left[\frac{\omega_d\, p\, \lambda\left(T_t - \underline{T}\right)^{p-1}\left(Q_t U_{qt} + Y_t U_{ct}\right) F_{t-1}}{\nu Y_{t-1} U_{ct-1}}\right] \tag{102}$$

we can write this moment as:

$$\beta E_{t-1}\left[\frac{(1-\kappa_m) Y_t U_{ct} F_{t-1}}{Y_{t-1} U_{ct-1} F_t} + \frac{\omega_d\, p\, \lambda\left(T_t - \underline{T}\right)^{p-1}\left(Q_t U_{qt} + Y_t U_{ct}\right) F_{t-1}}{\nu Y_{t-1} U_{ct-1}}\right] = 1 \tag{103}$$

By defining a fictitious gross return:

$$R_{dt} = \frac{(1-\kappa_m)Y_t F_{t-1}}{Y_{t-1}F_t} + \frac{\omega_d\, p\, \lambda\, (T_t - \underline{T})^{p-1} \left(Q_t \frac{U_{qt}}{U_{ct}} + Y_t\right) F_{t-1}}{\nu Y_{t-1}} \tag{104}$$

$$= \frac{Y_t F_{t-1}}{Y_{t-1}F_t}\left[1 - \kappa_m + \frac{\omega_d\, p\, \lambda\, (T_t - \underline{T})^{p-1} \left(\Gamma_t + 1\right) F_t}{\nu}\right] \tag{105}$$

where

$$\Gamma_t = \frac{Q_t U_{qt}}{Y_t U_{ct}} \tag{106}$$

$$= \frac{Q_t (1-\varphi) Q_t^{\tau-1}}{Y_t \varphi \frac{C_t^{\tau-1}}{L_t^{\tau}}} \tag{107}$$

$$= \left(\frac{1-\varphi}{\varphi}\right)\left(\frac{C_t}{Y_t}\right)\left(\frac{Q_t L_t}{C_t}\right)^{\tau}, \tag{108}$$

we can write the Euler equation for energy as:

$$E_{t-1}\mathcal{S}_t R_{dt} = 1 \tag{109}$$

where we have used the stochastic discount factor stated in Equation 85.

For interpretation we note that we can write:

$$U_{ct} = \varphi\, u_t^{1-\gamma-\tau} \frac{C_t^{\tau-1}}{L_t^{\tau}} \tag{110}$$

$$U_{qt} = (1-\varphi) u_t^{1-\gamma-\tau} Q_t^{\tau-1} \tag{111}$$

$$U_{mt} = p\, (T_t - \underline{T})^{p-1}\, u_t^{1-\gamma-\tau} \left[(1-\varphi)\omega_q Q_t^{\tau} - \varphi\, \omega_d Y_t \frac{C_t^{\tau-1}}{L_t^{\tau}}\right] \tag{112}$$

where

$$u_t = \left[\varphi \left(\frac{C_t}{L_t}\right)^{\tau} + (1-\varphi) Q_t^{\tau}\right]^{\frac{1}{\tau}}. \tag{113}$$

Table 1: **Estimates of Population Growth**

Parameter Estimates	
n	σ_l
0.0172	0.002
(0.0008)	(0.0004)

Note: This table uses GMM to estimate the mean, n, and standard deviation, σ_l, of annual world population growth rates from 1952 to 2011. Asymptotically valid standard errors are listed in parentheses below estimates and are computed using the method of Newey and West (1987) with 10 lags.

Table 2: **Estimates of the Capital Evolution Process**

Parameter Estimates	
δ	σ_k
0.0573	0.0217
(0.0037)	(0.0037)

Note: This table uses GMM to estimate the yearly depreciation rate, δ, and the standard deviation of expected next year's world capital, σ_k, from 1952-2011. Asymptotically valid standard errors are listed in parentheses below estimates and are computed using the method of Newey and West (1987) with 10 lags.

Table 3: **Estimates of the Temperature Equation**

Time period	Parameter Estimates					Model Test	
(for $t+1$)	λ	κ_s	κ_m	$\underline{T}$	σ_s	J-stat	P-value
1882-2011	0	-0.0157	—	13.5304	0.1226	10.9767	0.0041
		(0.0258)		(1.0984)	(0.0079)		
	0.0023	0.0568	0	13.7941	0.1183	1.3165	0.2512
	(0.0014)	(0.1911)		(0.3094)	(0.0065)		
	0.0024	0.0560	0.001	13.7927	0.1184	1.2234	0.2687
	(0.0014)	(0.1894)		(0.312)	(0.0064)		
	0.0032	0.0619	0.01	13.7733	0.1180	0.6929	0.4052
	(0.0015)	(0.1734)		(0.2479)	(0.0056)		
1952-2011	0	-0.0229	—	13.7683	0.1114	45.7019	0
		(0.0181)		(0.2934)	(0.006)		
	0.0027	0.7640	0	13.7415	0.0943	0.0711	0.7898
	(0.0001)	(0.1215)		(0.0318)	(0.005)		
	0.0028	0.7716	0.001	13.7407	0.0943	0.0839	0.7721
	(0.0001)	(0.1202)		(0.0318)	(0.005)		
	0.0036	0.7489	0.01	13.7353	0.0944	0.1610	0.6882
	(0.0002)	(0.1123)		(0.0332)	(0.0051)		

Note: This table present first stage GMM estimates of the temperature equation with annual data. Asymptotically valid GMM standard errors are listed in parentheses below estimates and are computed using the method of Newey and West (1987) with 10 lags. Parameters without standard errors are fixed. The J-stats measure moment condition errors and the corresponding p-values indicate the likelihood of observing errors at least this large.

Table 4: **Estimates of the Output Equation**

	Parameter Estimates			Model Test	
p	ω_d	μ_a	σ_a	J-stat	P-value
Panel A: $\nu = 0.10$					
—	0	0.0081	0.0139	13.9688	0.0158
		(0.0025)	(0.0014)		
1	0.1704	0.0105	0.0252	2.7531	0.6000
	(0.1527)	(0.0026)	(0.0153)		
2	0.2508	0.0114	0.0343	1.5638	0.8153
	(0.2031)	(0.0035)	(0.0235)		
4	0.1351	0.0092	0.0233	4.8623	0.3017
	(0.1886)	(0.0038)	(0.0183)		
Panel B: $\nu = 0.20$					
—	0	0.0069	0.0132	18.0523	0.0029
		(0.0024)	(0.0013)		
1	0.2063	0.0098	0.0286	1.7000	0.7907
	(0.1701)	(0.0026)	(0.0180)		
2	0.2787	0.0106	0.0373	0.7346	0.9470
	(0.2205)	(0.0038)	(0.0261)		
4	0.1027	0.0078	0.0195	5.6841	0.2240
	(0.1430)	(0.0034)	(0.0127)		
Panel C: $\nu = 0.25$					
—	0	0.0063	0.0130	19.8381	0.0013
		(0.0024)	(0.0013)		
1	0.2237	0.0094	0.0304	1.5253	0.8222
	(0.1794)	(0.0027)	(0.0193)		
2	0.2997	0.0103	0.0397	0.6847	0.9532
	(0.2350)	(0.0040)	(0.0281)		
4	0.0824	0.0070	0.0174	6.7851	0.1477
	(0.1166)	(0.0031)	(0.0095)		

Note: This table present first stage GMM estimates of the world output equation with annual data from 1952-2011. Asymptotically valid GMM standard errors are listed in parentheses below estimates and are computed using the method of Newey and West (1987) with 10 lags. Parameters without standard errors are fixed. The J-stats measure moment condition errors and the corresponding p-values indicate the likelihood of observing errors at least this large. A dash indicates that the value of p does not matter since $\omega_d = 0$. Standard errors are not adjusted for the pre-estimation of some of the variables.

Table 5: **Calibration of Preference Parameters**

Parameter Estimates			Model Test	
φ	τ	β	J-stat	P-value
0.1	0.3332	0.9434	67.4499	0
	(0.2010)	(0.0030)		
0.2	-0.0463	0.9503	45.5878	0
	(0.2133)	(0.0035)		
0.3	-0.2947	0.9546	32.2615	0.0001
	(0.2208)	(0.0040)		
0.4	-0.4965	0.9579	24.0948	0.0022
	(0.2265)	(0.0043)		
0.5	-0.6803	0.9607	19.1975	0.0138
	(0.2315)	(0.0047)		
0.6	-0.8628	0.9634	16.5023	0.0357
	(0.2363)	(0.0050)		
0.7	-1.0605	0.9661	15.1745	0.0558
	(0.2413)	(0.0054)		
0.8	-1.3001	0.9690	14.6749	0.0658
	(0.2471)	(0.0059)		
0.9	-1.6577	0.9727	14.8111	0.0629
	(0.2552)	(0.0065)		

Note: This table provides estimates of the (non-) robust first order conditions for optimization, with world annual data from 1952-2011. Asymptotically valid GMM standard errors are listed in parentheses below estimates and are computed using the method of Newey and West (1987) with 10 lags. Parameters without standard errors are fixed. The J-stats measure moment condition errors and the corresponding p-values indicate the likelihood of observing errors at least this large, if the model is correct. Standard errors are not adjusted for the pre-estimation of some of the variables. The value of γ is fixed at one in all rows.

Table 6: **Robustness Calibration**

θ / τ	$\gamma = 1$ and $\omega_d = 0.2997$					$\gamma = 1$ and $\omega_d = 0$				
	-1.3	-1.0	-0.1	0.1	0.5	-1.3	-1.0	-0.1	0.1	0.5
0.100	0.02	0.01	0.01	0.01	0.01	3.71	2.05	0.71	0.65	0.59
	(1.71)	(1.22)	(0.60)	(0.56)	(0.51)	(2.19)	(1.28)	(0.42)	(0.39)	(0.35)
0.200	0.01	0.01	0.00	0.00	0.00	1.51	0.90	0.36	0.33	0.30
	(0.78)	(0.56)	(0.30)	(0.28)	(0.26)	(0.95)	(0.56)	(0.21)	(0.20)	(018)
0.300	0.00	0.00	0.00	0.00	0.00	0.85	0.53	0.24	0.22	0.20
	(0.44)	(0.34)	(0.20)	(0.19)	(0.18)	(0.54)	(0.33)	(0.14)	(0.13)	(0.12)
0.400	0.00	0.00	0.00	0.00	0.00	0.50	0.34	0.18	0.17	0.16
	(0.29)	(0.23)	(0.15)	(0.15)	(0.14)	(0.32)	(0.21)	(0.11)	(0.10)	(0.09)
0.500	0.00	0.00	0.00	0.00	0.00	0.33	0.24	0.14	0.14	0.13
	(0.19)	(0.16)	(0.12)	(0.12)	(0.11)	(0.21)	(0.15)	(0.09)	(0.08)	(0.08)

Note: This table computes estimates of ϱ and its standard error for various values of θ, τ, and ω_d, using world annual data from 1952 to 2011. The other parameter values are set at the values listed in the parameter selection section. We use a fixed weighting matrix which is a combination of the fixed weighting matrices used in Tables 3 and 4. Asymptotically valid GMM standard errors are listed in parentheses below estimates and are computed using the method of Newey and West (1987) with 10 lags. The standard errors are not adjusted for the pre-estimation of the other parameters.

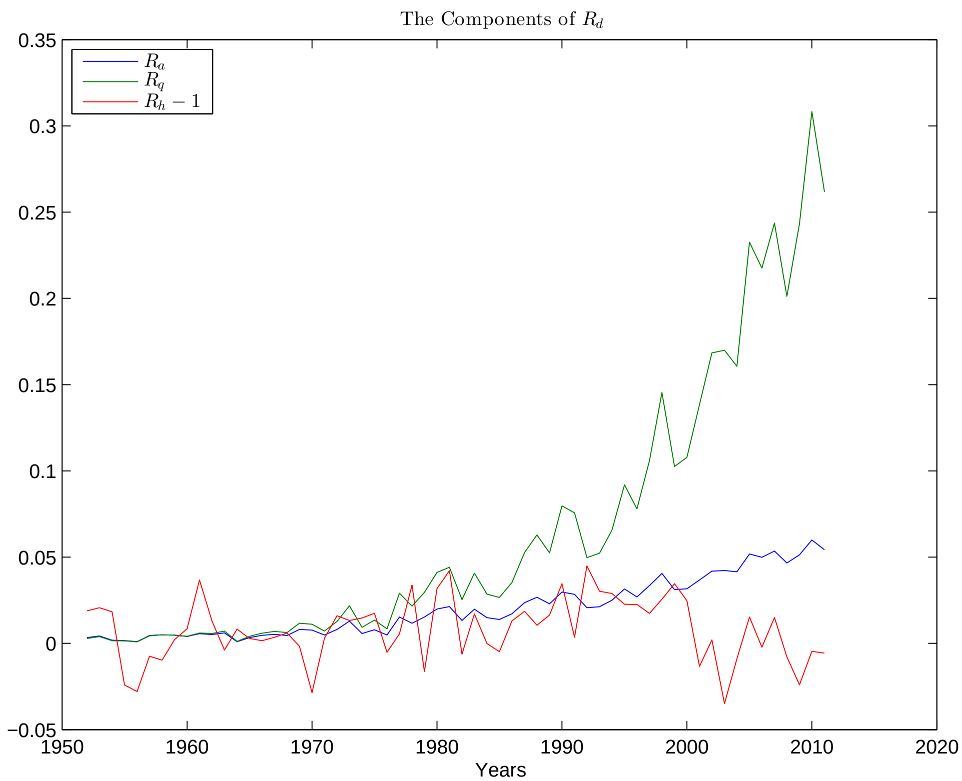

Figure 1: **The components of** R_{dt}

Note: This figure plots the three components of R_{dt} using our optimal parameter estimates and actual data when $\gamma = 1$, $\tau = -1.3$, $\omega_d = 0.2997$.

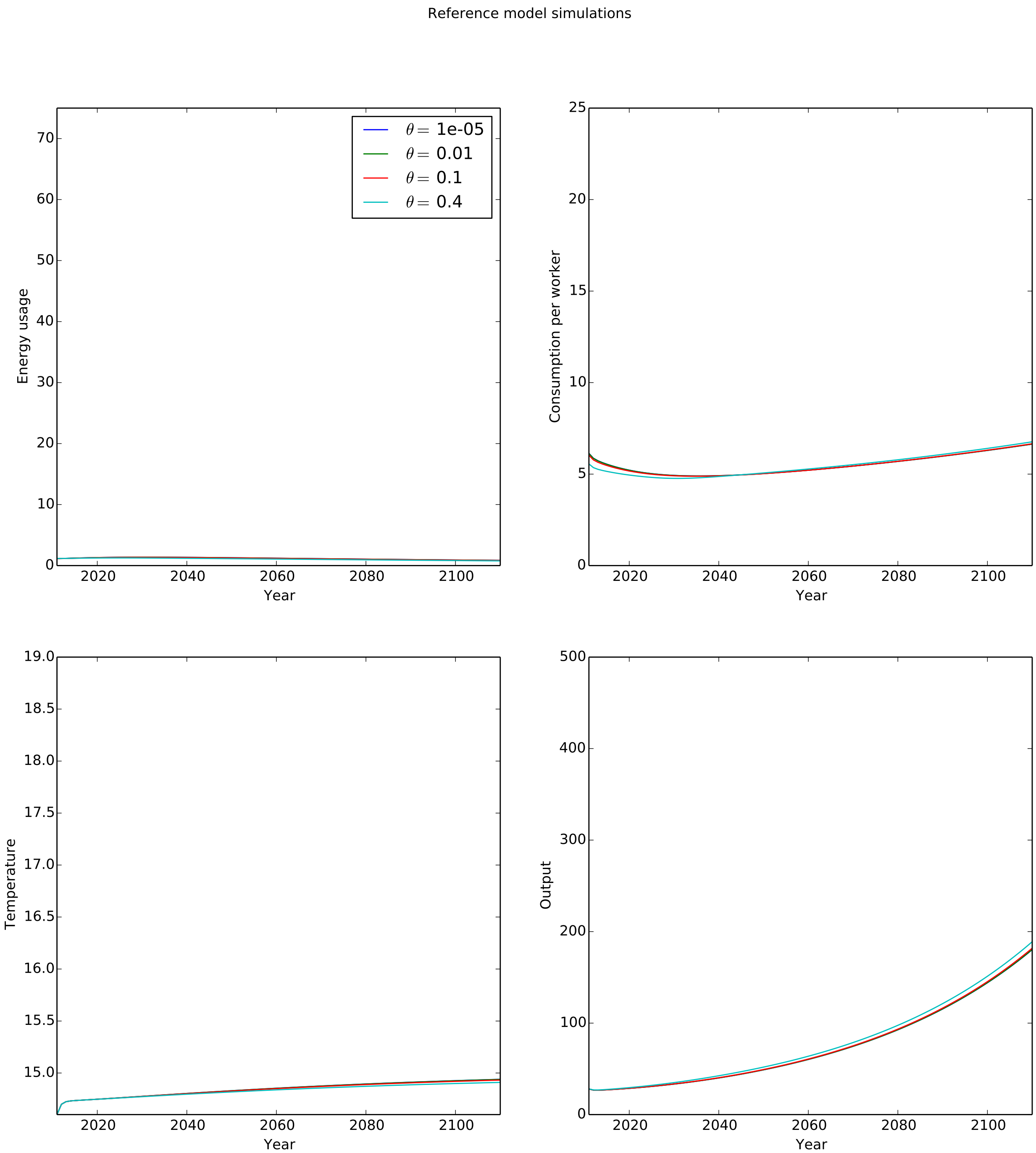

Figure 2: **Long horizon simulations using the optimal estimates**

Note: This figure simulates optimal energy usage (F_t), consumption per-worker (C_t/L_t), temperature (T_t), and output (Y_t) for the parameter values described in the parameter selection section when $\gamma = 1$, $\tau = -1.3$, $\omega_d = 0.2997$ for four different values of θ. The simulations are almost identical for the four values of θ.

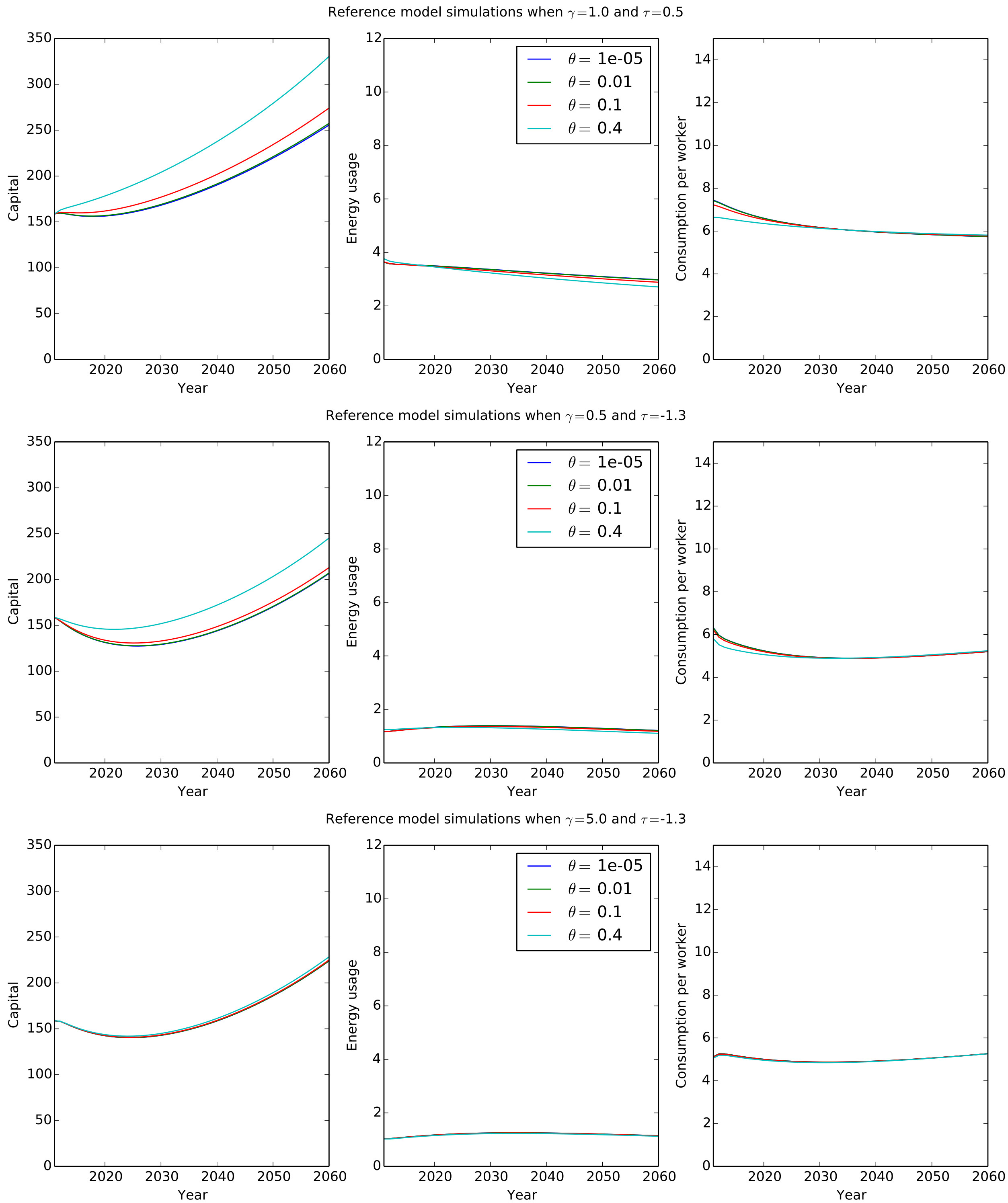

Figure 3: **Short horizon simulations using alternative parameter values**

Note: Each row simulates capital (K_t), energy usage (F_t), and consumption per-worker (C_t/L_t) using different parameters. Row 1 lets $\tau = 0.5$, row 2 lets $\gamma = 0.5$, and row three lets $\gamma = 5.0$. The other parameters are the same as in Figure 2 and the first row of Figure 5.

Reference model simulations

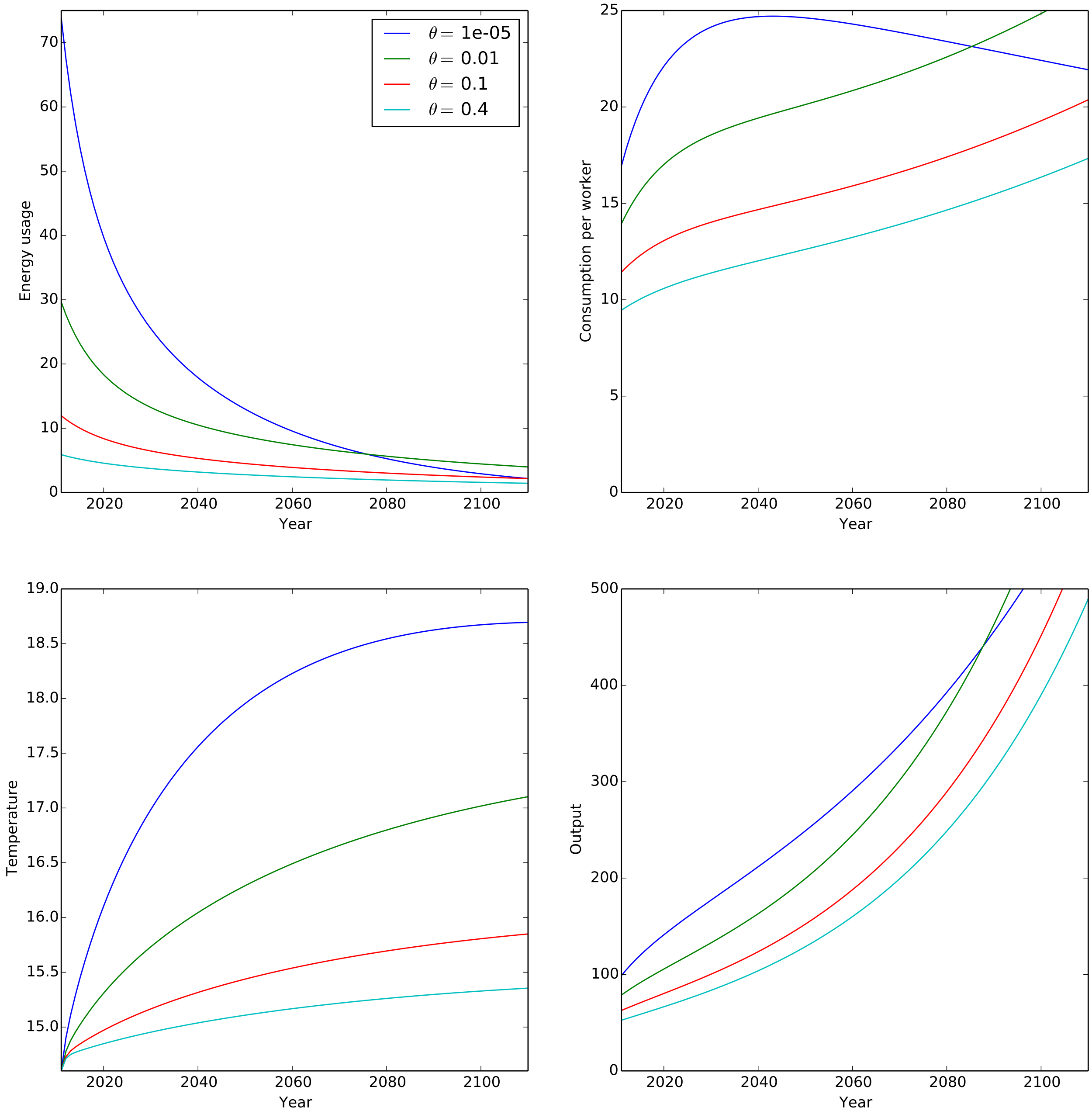

Figure 4: **Long horizon simulations with** $\omega_d = 0$

Note: This figure simulates optimal energy usage (F_t), consumption per-worker (C_t/L_t), temperature (T_t), and output (Y_t) $\gamma = 1$, $\tau = -1.3$, $\omega_d = 0$.

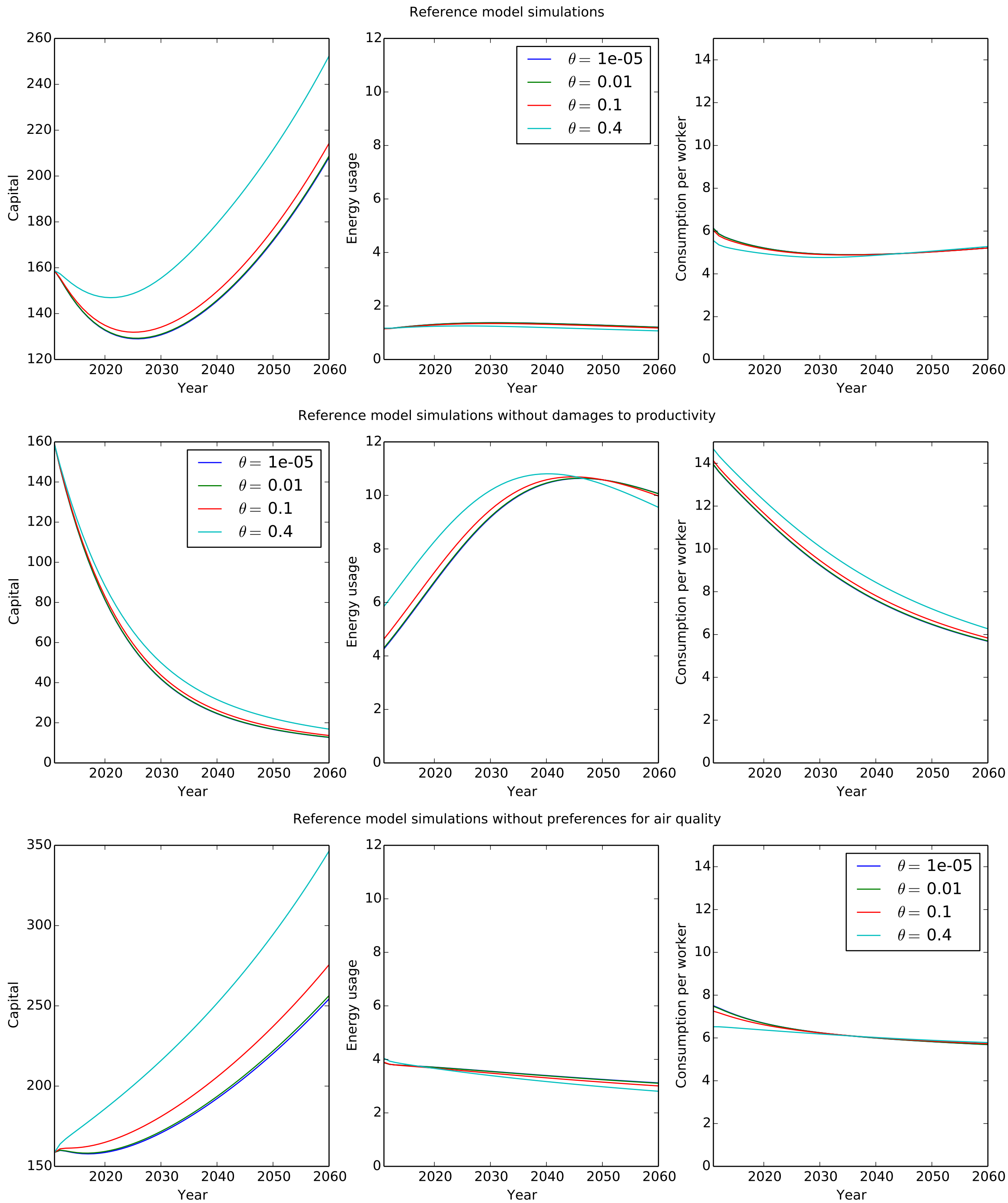

Figure 5: **Short horizon simulations using the optimal estimates**

Note: Each row simulates capital (K_t), energy usage (F_t), and consumption per-worker (C_t/L_t) under different assumptions using the same parameters values as Figure 2. Row 1 lets environmental damages affect productivity and preferences. Row 2 lets environmental damages only affect preferences. Row 3 lets environmental damages only affect productivity.

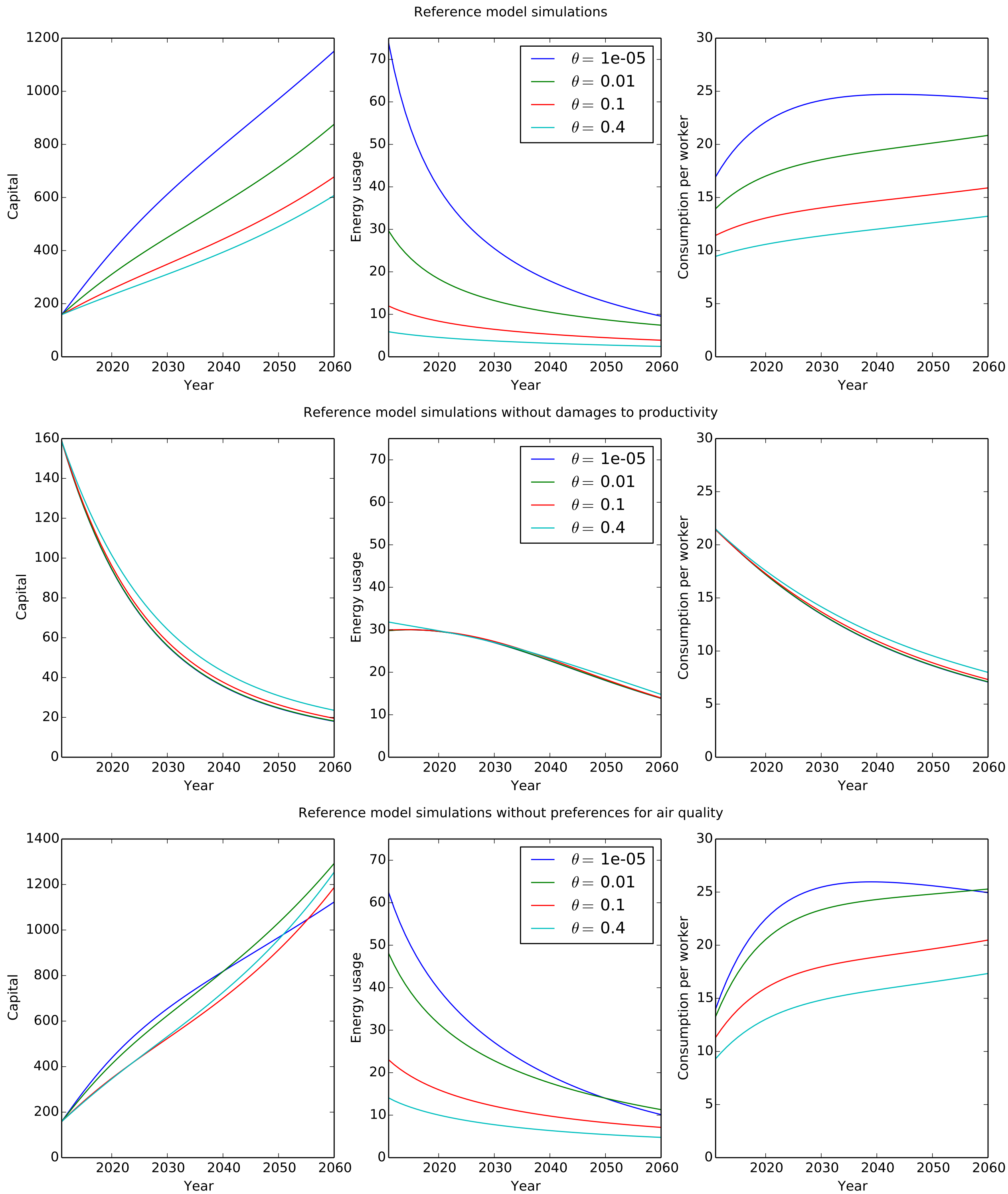

Figure 6: **Short horizon simulations when** $\omega_d = 0$

Note: Each row simulates capital (K_t), energy usage (F_t), and consumption per-worker (C_t/L_t) under different assumptions using the same parameters values as Figure 4. Row 1 lets environmental damages affect productivity and preferences. Row 2 lets environmental damages only affect preferences. Row 3 lets environmental damages only affect productivity.

Notes

[1]We interpret β as the subjective discount factor times $(1+n)$.

[2]To compute the sample variance, we divide by the sample size.

[3]As described in our data appendix the units of λ are Celsius per billion metric tons of carbon. Many authors use different units such as Celsius per trillion metric tons of carbon. A λ of 0.0028 corresponds to 2.8 Celsius per trillion metric tons of carbon.

[4]In our data sample T_t is always greater than $\underline{T}$ so that $\mathcal{E}_{t+1} = \omega_d (T_{t+1} - T_t)$. However, our model predicts that its possible that $T_{s+1} < \underline{T}$ and $T_s \neq T_{s+1}$ for some s, in which case $\mathcal{E}_{s+1} \neq \omega_d (T_{s+1} - T_s)$.

[5]By fictitious return, we mean that this is not necessarily a return on asset that agents can invest in usual financial markets. However, the return satisfies the same equation that investable assets satisfy, and our model is consistent with there either being, or not being, an investable asset with this return.

[6]See Figures 2 and 4; and the discussion in our simulation section.

[7]The moment conditions when $\varrho = 1$ and $\epsilon = 1$ can be derived in a similar way to the moment conditions in earlier sections.

[8]For example, $G^\theta_{a,t}, G^\theta_{b,t-1}$, and $G^\theta_{d,t}$ are good candidates to supplement the instruments, $z_{2,t}$. In our results, we set $z^\theta_{1,t} = z_{1,t}$ and $z^\theta_{2,t} = z_{2,t}$; and do not use additional instruments.

[9]We solve the model numerically using the method described in our simulation section. Its not computationally feasible for us to reliably compute detection probabilities using ordinary workstations with a limited number of processors.

[10]By very near zero, we mean between -0.0001 and 0.0001.

[11]The probability that ϱ is greater than equal to one when $\theta = 0.3$ is about 39%. The probability that ϱ is greater than equal to one when $\theta = 0.4$ is about 6%.

[12]In the simulation, $\bar{K}_t$ equals K_t for all t.

[13]Note that Figures 2 and 4 plot many quantities though time, assuming that the different reference models in each case are correct. This makes it difficult to compare future choices. However, in unreported results the decision rules, as a function of the current state, are in the same ballpark for climate believers and robust climate deniers, when $\theta = 0.4$.

[14]Series “CK” is converted to constant purchasing power parity (PPP) by multiplying by series “RGDPO” and dividing by series “CGDPO.”

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