Will yield factors tell more? a generalized affine HJM model with unspanned stochastic volatility

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Motivation and Features
This paper investigates the relationship between yields and volatility factors. It is motivated by the literature that incorporates features of curve-fitting models, like Diebold and Li (2006) and that study the unspanned stochastic volatility, like Collins-Duffie and Goldstein (2003). Stochastic volatility factors in this paper indirectly affect yields through yield factors.

The model has several distinctive features.
- The yield factors exhibit level, slope, and curvature shapes.
- The volatility factors are partially spanned by yields.
- The volatility factors are the kernels of the shadow means of yield factors.

The Model
The model follows Heath et al. (1992) forward rate-of-framework by specifying a general form of forward rate volatility.

Model Specification
The forward rate volatility is defined as:

\[ \sigma(t, s) = \sigma_0 + \sigma_1 (t - s)^{1/2} + \sigma_2 (t - s)^{2/3} \]

and the market price of risk as a simplified extended affine form,

\[ \theta(t) = \theta_0 + \theta_1 (t - s)^{1/2} + \theta_2 (t - s)^{2/3} \]

Empirical Analysis
The model is tested with weekly data of USD LIBOR/swap rates from Jan. 2002 to Nov. 2011. Extended Kalman filter is implemented in the quasi-maximum likelihood estimation.

Yield Factor and Shadow Mean
The yield and volatility factors are extracted with extended kalman filter. The commonest structure of the yield factors and their shadow means (equation 8) is changed during and after the Great Recession in 2007-2008 (Figure 1, 3). The fitted and one-period ahead forecast errors are highly comparable to other studies. Yield and volatility factors are extracted with the extended kalman filter.

Unspannedness
On average, two-thirds of the stochastic volatility can not be spanned with yields and volatility factors. Therefore, two-thirds of the stochastic volatility can not be spanned with yields and volatility factors. The model provides a convenient way to diagnose the shadow means of yield factors.

Conclusions
The model provides a convenient way to diagnose the dynamics of yield and volatility factors. It shows that two-thirds of stochastic volatility can not be spanned with yields. Yield factors and their shadow means changed concoment patterns during and after the Great Recession. It also reveals market overreactions during the recession time.