Carbon Trading, Carbon Taxes and Social Discounting

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Abstract

This paper studies the optimal taxation of carbon emissions in a dynastic economy. When the welfare function places direct Pareto weights on unborn generations, the social discount rate is lower than the discount rate of the current generation. I show that this welfare criterion has important consequences for the structure of the optimal regulatory system. In particular, I show that: (*i*) the optimal carbon tax does not in general equal the social cost of carbon; (*ii*) a subsidy on oil reserves is sometimes optimal; and (*iii*) carbon trading programs should limit the award of carbon offset allowances

Keywords: Climate; Discount rates; Intergenerational Equity; Optimal Taxation; Carbon Trading.

JEL: E6, H21, H23, Q58.

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1 Introduction

The mean lifetime of anthropogenic carbon dioxide is approximately 300 years (Archer (2005)). The problem of climate change is, therefore, intergenerational in nature. This paper studies optimal carbon policy in an economy that captures both the externality and the intergenerational aspect of climate change. I consider a welfare function that places direct Pareto weights on unborn generations, as opposed to future generations receiving weight only through the altruism of the current generation. This specification delivers a social discount rate that is lower than the discount rate of private individuals. The literature has argued for and against using a low discount rate in climate-economic models. While ethical considerations underly the argument for using a low discount rate, unrealistic savings and an optimal carbon tax that is too high are the main arguments against it. This paper shows that a low discount rate has further implications for the structure of an optimal regulatory system. In particular, I show that standard policies to control carbon emissions, such as taxes or caps on *net* carbon emissions, are insufficient to achieve the social optimum. Non-standard carbon policies must be designed. These non-standard policies require mitigation technologies to be subsidized and carbon emissions to be taxed, but each at different rates: in general, the optimal subsidy for removing a ton of carbon from the atmosphere will not equal the optimal tax on creating a ton of carbon. Moreover, if carbon taxes and subsidies are set as equal, then it is optimal to also introduce a subsidy on the oil stock in situ. I also show that an optimal cap and trade scheme must include a cap on carbon-offset allowances.

I develop my argument in a model economy with altruistic generations and a climate externality. In the model, the extraction of an exhaustible resource ("oil") generates a climate externality ("carbon in the atmosphere") that can be offset with an available technology ("sequestration"). Each generation decides how much oil to consume and how much of its carbon emissions to sequester. Private individuals do not take into account the effect that both activities have on the aggregate carbon stock. Therefore, the equilibrium in the market economy is not optimal, and policy intervention is necessary.

To address intergenerational issues, I set up an economy composed of an infinite sequence of generations. Each generation consists of a continuum of altruistically linked individuals who live for only one period. I consider a welfare function that places direct weights on current as well as future unborn generations. By varying these weights, I can recover any point in the Pareto frontier between the present and future generations, and study how the optimal carbon policies are affected by alternative welfare criteria. One particular criterion is when only the current generation receives weight in the social welfare function. Future generations are still valued through the altruism of their ancestors. In this case, the social discount rate coincides with the private discount rate, and the planning problem corresponds to that of a representative infinitely lived individual. In this particular case, carbon taxes and sequestration subsidies are equal to the value of the externality, which I refer to as the "social cost of carbon". Alternatively, in a cap and trade economy, it is optimal for the government to set a cap on the net carbon emissions.

However, the social optimum associated with valuing future generations only through the altruism of the current generation is just one among many other efficient allocations. These alternative efficient allocations are each associated with positive weights on future generations in the welfare function, and each corresponds to a point on the Pareto frontier. Moreover, these welfare criteria imply a social discount rate that, at any point in time, is lower than the discount rate of the individuals in the society. The main contribution of this paper is to show that these efficient allocations cannot be implemented using standard carbon policies. First, the social cost of carbon in the planning problem is no longer a sufficient indicator of the optimal climate policies. And second, it is not enough to control the net carbon emissions. A social planner who cares about future generations wants to treat carbon emissions differently from emissions offsets.

The first main result of the paper is that, when social and private discount rates differ, an optimal tax scheme consists of a carbon tax on emissions and a subsidy on sequestration, but these two are not equal. Thus, creating a ton of carbon is no longer equivalent to removing a ton of carbon, even though both have the same effect on the aggregate externality. I provide an analytical derivation of the carbon tax formula for this case and show that the social and the private discount rates appear as an extra term in the tax rate. Importantly, the carbon tax is not equal to the social cost of carbon. This result is important because it shows that a different carbon tax *formula* (not just a higher *value* for the carbon tax) is optimal when climate-economic models use a low social discount rate. Moreover, it is not enough to solve for the path of carbon emissions in a social planner's problem and associate taxes with the social cost of carbon within that problem. When social and private discount rates differ, the design of optimal policies requires special attention.

It is important to notice that a social discount rate that is lower than the private one will have broader implications for the structure of the optimal tax system, not only for carbon policies. Specifically, it will call for a subsidization of all forms of capital accumulation in the economy. In this sense, an insight drawn from this paper is that the reserves of oil in situ are a form of capital accumulation for society and, therefore, should be treated differently from sequestration and other mitigation policies. It is a well-known principle in public finance that there are many ways to decentralize an optimal allocation. Therefore, I also show that an alternative optimal carbon policy is to set "standard" carbon emissions taxes and carbon sequestration subsidies (both equal to the social cost of carbon) and couple them with a subsidy on the reserves of oil in situ. That is, governments should pay firms for keeping fossil fuels underground. I theoretically characterize the optimal subsidy rate on oil reserves for this case and show that it is a function of the social and the private discount rates as well.

The literature that proposes a supply-side approach to environmental policies has already discussed the idea of paying firms for extracting less. Sinn (2008) considers subsidizing the oil stock in situ as an alternative to introducing a decreasing carbon tax rate. More recently, Harstad (2012) shows that it is optimal for governments to buy fossil-fuel deposits to prevent non-participants in a global climate treaty from burning the oil. This paper contributes to this literature by providing an alternative theoretical justification for this type of supply-side carbon policy. In this paper, a subsidy on the oil stock in situ is a tool to pursue climate equity.

The second main result of the paper is that an optimal cap and trade program must include a cap on carbon-offset allowances. If the government sets the net emissions caps equal to the carbon emitted in the optimal allocation, the economy will exhibit both too much depletion of fossil fuels and too much sequestration. By introducing a cap on carbon offsets that firms can use, the optimal cap and trade scheme embeds a mechanism to lock more oil under the earth's crust. It is interesting to see that this policy prescription resembles some of the features of actual policies. In particular, the European Union Emissions Trading Scheme (EU ETS), the California's greenhouse gas (GHG) cap-and-trade program and the Regional Greenhouse Gas Initiative (RGGI) set limits to the use of compliance carbon credits. Although the difficulty of quantifying and verifying the reductions in emissions coming from these projects is often the justification for these limits, this paper provides a new rationale for why these caps on carbon offsets are optimal.

The basic intuition behind the results of this paper is the simple rule in public finance that optimal policies require the number of instruments to equal the number of policy targets. Standard carbon policies provide only one instrument. If the carbon policies are meant to deal not only with the environmental externality itself but also with intergenerational equity, an additional lever is missing. Finally, I extend the basic model to include capital, an alternative sequestration technology and heterogeneous consumers, and I show that the main results and intuition carry over to these richer environments.

The paper is related to the vast literature on discounting and climate change, especially Stern (2008), Nordhaus (2007) and Weitzman (2007) among others. This paper makes two contributions to this literature. First, the paper shows that Pareto weights on future generations' welfare map onto low social discounting. This contribution is important because rationalizing a low social discount rate through the choice of Pareto weights is a well-suited approach with clear-cut policy implications. This approach to social discounting is borrowed from Bernheim (1989), Phelan (2006) and Farhi and Werning (2007), although they work in a different economic environment. Second, it shows that low social discounting maps onto optimal carbon policies. This paper is closely related to von Below (2012), who studies the implications of social discounting on the design of optimal carbon taxes. Like von Below (2012), I also find that the social and private discount rates appear as an extra term in the optimal carbon tax formula. However, this paper differs in that I also derive the policy implications of social discounting on alternative forms of climate policies such as carbon sequestration and alternative policy instruments such as oil subsidies and cap and trade schemes. Also, von Below (2012) assumes that the social discount factor is different from the private one but does not provide a rationale as I do in this paper.

The paper is related to the literature that studies environmental policies in overlapping generations models, including Bovenberg and Heijdra (1998), Hoel et al. (2015), Karp and Rezai (2014), von Below et al. (2014) and Williams III et al. (2014). This literature focuses on policies that implement a Pareto improvement from the business-as-usual scenario. These Pareto improving allocations are a restricted portion of the Pareto frontier. In contrast, this paper characterizes carbon policies as a function of the Pareto weights in the social welfare function. By varying the Pareto weights, these policies can implement any point along the Pareto frontier. This paper is also related to the literature on the optimal taxation of fossil fuels with a climate externality, including Acemoglu et al. (2012), Barrage (2014), Golosov et al. (2014), Schmitt (2014) and van der Ploeg and Withagen (2014). However, this paper differs from those papers because they do not formally study the interaction between social discounting and the optimal design of carbon policies. This paper also differs from Barrage (2014) and Schmitt (2014) in that I assume that the government has access to lump sum taxes.

This paper is related to Moreaux and Withagen (2015) regarding the interaction between an exhaustible resource and the optimal abatement policy. It differs from that paper in that they do not consider social discounting and therefore they find that the optimal carbon tax and the optimal abatement price should be the same. This paper relates to Belfiori (2015), Karp (2016), Gerlagh and Liski (2015) and Iverson (2012) in that all of them focus on the interaction between social discounting and the design of optimal carbon policies. Like these papers, I also find that, with social discounting, the optimal carbon tax is not equal to the social cost of carbon. However, this paper differs from those in several aspects. First, this paper restricts attention to geometric discounting, and it does not contemplate inconsistency and commitment problems. Secondly, this paper provides a theoretical framework that links the social discount rate with the Pareto weights on future generations' welfare. The paper is more closely related to Belfiori (2015) in this respect. Finally, this paper derives the implications of social discounting for a more general class of carbon policies, including not only the optimal carbon tax but also the subsidies to oil reserves in situ, carbon sequestration subsidies and optimal carbon trading schemes.

The remainder of the paper is organized as follows. Section 2 sets up the basic model. Section 3 solves the social planning problem. Section 4 proposes a market economy with carbon taxes. Section 5 and 6 characterize optimal carbon policies and contain the main results of the paper. Section 7 discusses an extension of the benchmark model with household's heterogeneity. Section 8 provides some conclusions from the analysis. Finally, the Appendix presents some extensions and all the mathematical proofs.

2 The Basic Model

Consider the following economy. Time is discrete and infinite, $t \in \{0, ..., \infty\}$. Each period, the economy is populated by a unit mass continuum of identical individuals, who live for one period and constitute generation t. There is a single consumption good k_t ("oil"), which is exhaustible. The economy starts with an initial stock of oil equal to k_0. Resource feasibility requires

$$c_t + k_{t+1} = k_t \tag{1}$$

for every period t, where $c_t \geq 0$ represents oil consumption.

Extraction of oil increases the amount of carbon in the atmosphere, S_t, and carbon decreases when individuals exert an effort level z_t in emissions offsets ("sequestration"). The stock of carbon in the atmosphere evolves according to

$$S_{t+1} = (1-\gamma)\,S_t + k_t - k_{t+1} - z_t \tag{2}$$

where $\gamma \in [0,1)$ is the rate of natural reabsorption of carbon and S_0 is given. Carbon in the atmosphere generates a negative externality in the economy, which is modeled as a disutility cost. In particular, an individual's utility in period t is given by $U(c_t, z_t, S_{t+1}) = u(c_t) - v(z_t) - x(S_{t+1})$. The function u is increasing, concave and twice differentiable with $\lim_{c\to 0} u'(c) = \infty$. The disutility cost functions v and x are increasing, convex and twice differentiable with $\lim_{z\to 0} v'(z) = 0$ and $\lim_{S\to 0} x'(S) = 0$. This specification of the economy, where households derive a disutility cost from sequestration, is equivalent to an alternative environment where sequestration is produced using labor. That is, the economy with a disutility cost $v(z)$ is isomorphic to an economy where sequestration is produced using the technology $z = f(An)$ where A is some productivity level and households derive disutility $v(n)$ from working. Thus, the carbon sequestration technology in this paper is consistent with previous literature, including the carbon sequestration cost function in Moreaux and Withagen (2015) and the green energy technology in Golosov et al. (2014). The main underlying assumption is that there are no scarcity effects in the carbon sequestration sector.

Over time, individuals care about their utility and that of their children. Thus, the utility of an individual born in period t is given by

$$W_t = U(c_t, z_t, S_t) + \beta W_{t+1} \tag{3}$$

where $\beta \in (0,1)$ is the altruistic weight over the child's utility. This demographic specification is consistent with one in which households consist of a single infinitely lived individual who cares about the value

$$\sum_{t=0}^{\infty} \beta^t U(c_t, z_t, S_{t+1}) \tag{4}$$

Most of the earlier literature that studies optimal environmental policy within an overlapping generations model assumes that individuals are not altruistic, see for example Bovenberg and Heijdra (1998), Hoel et al. (2015), Karp and Rezai (2014), von Below et al. (2014) and Williams III et al. (2014). One of the main lessons drawn from this related literature is that it is optimal to transfer resources from future generations to the current generation in order to compensate the current generation for the cost of pursuing climate policies. In contrast, this paper considers altruistic generations: parents care about their child and this introduces a bequest motive. Moreover, because each generation cares about the next one, altruism nests in a way that the current generation indirectly cares about the welfare of all future generations according to (4). This implies that the benefits derived from climate policies enter into the current generation's welfare even if they accrue in the distant future and compensatory transfers to the current generation are not optimal.

Social Welfare. When a representative infinitely lived individual inhabits the economy, it is natural to consider a social welfare function that coincides with (4). However, when the economy is composed of an infinite sequence of altruistically linked generations, a more general social welfare function involves weighting the utility of each generation separately. Following Farhi and Werning (2007), I consider a utilitarian criterion that weighs current as well as future unborn generations according to the following function

$$\sum_{s=0}^{\infty} \alpha_s \left[\sum_{t=s}^{\infty} \beta^{t-s} U(c_t, z_t, S_{t+1}) \right] \tag{5}$$

where $\{\alpha_s\}_{s=0}^{\infty}$ is an arbitrary weighting scheme across generations. We can further simplify the welfare function to obtain

$$\sum_{t=0}^{\infty} \rho_t U(c_t, z_t, S_{t+1}) \tag{6}$$

where $\rho_t \equiv \sum_{\tau=0}^{t} \alpha_\tau \beta^{t-\tau}$ represents the social discount function. Note that when future generations enter into the calculation of social welfare, even though each individual discounts the future by β, society as a whole does it at a different rate, given by

$$\frac{\rho_{t+1}}{\rho_t} = \beta + \frac{\alpha_{t+1}}{\rho_t} \geq \beta \tag{7}$$

Eq. (7) provides a one-to-one mapping from the Pareto weights and the private discount factor to the social discount factor. Societies, and planners, may differ on their most preferred choice of Pareto weights. However, for every choice of weights there is an associated social discount rate given by Eq. (7). It follows that, if the weighting scheme is such that all generations receive strictly positive weight, the social discount factor is higher than the private one. The planner necessarily cares more about future generations because he cares about them directly by assigning a Pareto weight α to their welfare, and also indirectly, through the altruism of their ancestors.

For the rest of the paper, I will restrict attention to welfare weights that satisfy Assumption 1. This assumption ensures that the social discount function takes the standard geometric form.

Assumption 1 (AI) *The welfare weights* $\{\alpha_t\}_{t=0}^{\infty}$ *in the social welfare function 5 satisfy one of the following conditions:*

(i) $\alpha_0 = 1$ *and* $\alpha_{t+1} = 0\ \forall t$

(ii) $\alpha_0 = \frac{1}{\hat{\beta}-\beta}$ *and* $\alpha_t = \hat{\beta}^t\ \ \forall t \geq 1$*and for some constant* $\hat{\beta} > \beta$

Condition (i) describes a social welfare criterion that places direct weight on the current generation while future unborn generations are only valued indirectly through the altruism of the current one. This is a special case in which the social and private discount rates coincide. Condition (ii) restricts Pareto weights on unborn generations to be geometric. This ensures that the social discount function is geometric too, as shown in Bernheim (1989). That is, at any point in time, the planner discounts the future at a constant rate $\hat{\beta} > \beta$.

Condition (ii) rules out hyperbolic social discounting and, more generally, any form of time-varying social discounting. A time varying social discount rate introduces further complications to the policy design problem because it becomes dynamically inconsistent. The carbon policies that arise from the solution to a Markov perfect equilibrium are not optimal but rather "constrained optimal". This means that they are the best policies that society can implement given the constraint that society cannot commit to future climate policies. Belfiori (2015), Gerlagh and Liski (2015) and Iverson (2012) study such a problem and find that standard Pigouvian taxes are not sufficient to implement the Markov perfect path of carbon emissions.

The next section characterizes the socially optimal allocation as the solution to a planning problem.

3 Social Planning Problem

The *socially optimal allocation* is the path of consumption, fossil fuel, sequestration and carbon level, $\{c_t^*, k_t^*, z_t^*, S_t^*\}_{t=0}^{\infty}$, that maximizes the social welfare function (6) subject to the carbon cycle (2), the resource constraint (1) and the initial conditions $\{k_0, S_0\}$.

It is useful to define q_t^* as the social (shadow) cost of carbon, which corresponds to the Lagrange multiplier on the carbon cycle constraint (2) . The social cost of carbon is at the center of all economic models of climate change, so it is important to understand the basics behind this concept. The social cost of carbon is derived from solving recursively the first order condition with respect to the carbon stock and, in this economy, it is equal to

$$q_t^* = \sum_{j=1}^{\infty} [\hat{\beta}\,(1-\gamma)]^{j-1} x'(S_{t+j}^*) \tag{8}$$

Essentially, it measures the marginal damage of increasing carbon emissions by an extra unit. The formula highlights the dynamic structure of the externality: carbon emissions are cumulative and add to a stock that only depreciates at a low rate. Moreover, it also highlights the reason why discounting is a controversial issue in the economics of climate change. The discount factor directly affects the social value of the externality and, hence, the desirability of any policy aimed at controlling it. Because it will be useful to derive the results, I will denote μ_t^* the social cost of carbon expressed in units of the consumption good. That is

$$\mu_t^* \equiv \frac{q_t^*}{u'(c_t^*)} \tag{9}$$

The remaining optimality conditions are standard. The marginal cost of removing carbon emissions through sequestration must be equalized to the social cost of carbon - the social value of carbon sequestration is the sum of the accumulated benefits of having one fewer unit of carbon in the atmosphere forever after. Moreover, the rate of oil depletion is driven by a version of the Hotelling rule reflecting the exhaustible nature of fossil fuels, Hotelling (1931). In this economy, optimality requires that the benefits from extracting must be net from the associated climate damages. Specifically, the following Euler equation holds

$$\hat{\beta}[u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*) \tag{10}$$

The social cost of carbon (9), the optimality condition for carbon sequestration and the Euler equation (13), together with the feasibility constraint (1) and the carbon cycle (2), fully characterize the socially optimal allocation.

The two key decisions society faces are how quickly oil reserves are used up and how much effort each generation should devote to mitigating the associated externality. Because individuals are too small to be able to affect the aggregate level of carbon, they will typically get both decisions wrong: they will consume too much and they will fail to make the optimal level of effort toward sequestration. A role for policy intervention thus arises.

4 Market Economy with Carbon Taxes

We say that a social planner's allocation is implementable if we can find policies and equilibrium prices such that the allocation and prices are a competitive equilibrium given these policies. There are typically two instruments to control carbon emissions: carbon taxes and a cap and trade system. The first is a price-based policy and the second is a quantity control. From a theoretical point of view, both instruments are equivalent in the sense that they are equally capable of implementing the desired allocation. Motivated by this well-known result, this paper studies the implications of social discounting for the optimal design of each of these alternative carbon policies. The paper proceeds as follows. This section proposes an economy with carbon taxes and characterizes a competitive equilibrium. The next section uses this decentralized environment to derive the results. Later, Section 6 proposes an economy with a cap and trade scheme and characterizes the optimal policy in that case.

Consider the following market economy. A continuum of firms of mass one operate a linear technology $f(y_t) = y_t$ to produce the consumption good. Firms own the economy's stock of fossil fuel and use it as an input for production. Extraction of fossil fuels creates carbon emissions that can be offset if firms buy sequestration services from households at a market price w_t. Firms face two forms of taxation: a carbon tax, τ_t^k, on emissions and a tax credit, τ_t^z, on sequestration. Hence, per-period profits of the firm are given by

$$\pi_t = (k_t - k_{t+1}) - w_t z_t^d - \tau_t^k (k_t - k_{t+1}) + \tau_t^z z_t^d$$

Taxes are defined in units of the consumption good. The firms must choose a sequence $\left\{k_t, z_t^d\right\}_{t=0}^{\infty}$ to maximize discounted profits given by

$$\Pi_0(k_0) = \sum_{t=0}^{\infty} (\prod_{s=0}^{t} \frac{1}{R_s}) \pi_t \tag{11}$$

with the initial stock of fossil fuel k_0 given. All profits are rebated to households as dividends.

Households derive utility from consumption of the single good in the economy and disutility from providing sequestration services z_t. As already mentioned, this specification of the economy with a disutility cost from sequestration is isomorphic to an alternative environment where sequestration is produced using labor.

Households save in a risk free asset, $b_{t+1} \geqslant 0$ which bears a gross one-period rate of return R_t. The asset can be thought of as bequests from individuals in generation t to those in the next generation. Households consume, provide sequestration services and save subject to the following set of budget constraints for $t = 0, 1, 2, ...$

$$c_t + b_{t+1} = w_t z_t + R_t b_t + T_t + \pi_t \tag{12}$$

where T_t represents a lump sum rebate from the government, π_t are the profits received from the firm and initially $b_0 = 0$. The problem of the households is to choose a sequence $\{c_t, z_t, b_t\}_{t=0}^{\infty}$ to maximize (4) subject to (15), taking prices and taxes as given.

A government collects carbon taxes and pays subsidies. Any surplus (or deficit) is rebated in a lump-sum transfer to households. The sequence of the government's budget constraints for $t = 0, 1, ...$ is given by

$$\tau_t^k (k_t - k_{t+1}) - \tau_t^z z_t^d = T_t \tag{13}$$

Finally, market clearing for every period t requires that

$$c_t = k_t - k_{t+1} \tag{14}$$

$$z_t^d = z_t \tag{15}$$

$$b_t = 0 \tag{16}$$

A *competitive equilibrium with taxes* $\left\{\tau_t^k, \tau_t^z, T_t\right\}_{t=0}^{\infty}$ is a sequence of prices $\{w_t, R_t\}_{t=0}^{\infty}$ and allocations $\left\{c_t, z_t, z_t^d, k_t, b_t\right\}_{t=0}^{\infty}$ such that: (i) given taxes and prices, the allocation solves the consumer's problem, maximizing (4) subject to (15), and the firm's problem, maximizing (14); (ii) given the allocation, transfers are such that the government budget constraint (16) is satisfied; and (iii) prices clear the markets.

Next, I characterize a competitive equilibrium with taxes. Combining the first order condition with respect to consumption for two subsequent periods delivers the following Euler equation for this economy

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \tau_t^k}{1 - \tau_{t+1}^k} \tag{17}$$

The Euler equation determines the rate of depletion of the oil stock. Note that in an economy without carbon taxes, consumers would be willing to equalize the marginal utility of consumption in present value terms. That is, the marginal utility of consumption would grow at the discount rate. This reflects the exhaustible nature of the resource and it is a version of the Hotelling rule from the perspective of the consumer. In addition, the optimality condition for carbon sequestration states that consumers will decide on consumption and sequestration in order to equalize marginal utilities to the ratio of prices, which is equal to the carbon sequestration subsidy.

The optimality condition for carbon sequestration and the Euler equation (22), together with the condition for prices in the firm's problem, the budget constraint of the government

(16) and the market clearing conditions fully characterize a competitive equilibrium with taxes.

5 Optimal Policy

This section presents the main results of the paper. Characterizing optimal policies consists of finding a set of instruments that makes the socially optimal allocation arise as the equilibrium outcome of a decentralized market. In this model, optimal policies operate through two channels: the rate of depletion of fossil fuels (which increases carbon) and the amount of sequestration (which reduces it). The laissez-faire (or business-as-usual) economy is a simple cake-eating problem where there is no sequestration, and the only relevant decision is how fast society depletes oil over time. It is easy to see that the laissez-faire rate of oil depletion is not optimal. In particular, individuals do not take into account the environmental damages that oil extraction generates. Besides, if the social welfare function gives direct weight to future generations, the utility gain from saving for the future weights β on the current generation's welfare, but it is worth $\hat{\beta}$ from a social point of view. Therefore, there is a role for policy intervention. The policies proposed in this section share the same basic intuition: optimal economic policy requires that the number of instruments equals the number of policy targets (Tinbergen (1952)). If regulating carbon emissions is not only a global externality problem but also a matter of intergenerational equity, then two policy goals are combined into one, and the typical trade-offs between efficiency and equity arise. Moreover, carbon policies designed in the usual way provide only one lever, which proves to be insufficient to achieve these two policy targets. The policies that I propose in this section provide the missing extra lever.

The following proposition presents one of the main results of this paper. It says that when social and private discount rates differ, an optimal tax scheme requires subsidizing the mitigation technology and taxing carbon emissions, but each at different rates: creating a ton of carbon is no longer equivalent to removing a ton of carbon from the atmosphere. Moreover, the optimal tax on emissions is not equal to the value of the externality that the emissions generate. The proof is relegated to the Appendix.

Proposition 1 ***(Optimal Taxes)*** *Suppose that the social welfare weights are given by AI.ii so that $\beta < \hat{\beta}$. If $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ solves the social planning problem, then it solves the* *competitive equilibrium with taxes* $\{\tau_t^k, \tau_t^z, T_t\}_{t=0}^{\infty}$ *defined as follows*

$$\tau_t^k = 1 + \left(\frac{\hat{\beta}}{\beta}\right)^t \mu_t^* - \left(\frac{\hat{\beta}}{\beta}\right)^t \tag{18}$$

$$\tau_t^z = \mu_t^* \tag{19}$$

for every period t *and all proceeds from taxation rebated/financed lump-sum through* T_t.

The optimal carbon emissions tax is not equal to the social cost of carbon. In particular, not only does the social cost of carbon appear as an essential piece of the carbon tax formula but the social and private discount rates do as well. These two components are a reflection of the two policy goals. A carbon tax that is equal to the social cost of carbon corrects for the climate externality but does not achieve the socially optimal degree of intergenerational equity. On the other hand, the optimal carbon sequestration subsidy is equal to the social cost of carbon[1]. The fundamental difference between the carbon emissions tax and the carbon sequestration subsidy is that they hold asymmetric power as a redistributive tool. In particular, carbon sequestration affects future generation's welfare only through its effect on the carbon level. Importantly, the social cost of carbon correctly reflects the cost that a high carbon level imposes on future generations. On the contrary, more carbon emissions imply more atmospheric carbon in the future but also fewer oil reserves. The social cost of carbon does not reflect the cost of leaving too little oil for generations to come, and this is picked up by the optimal carbon tax formula. The important theoretical point that the proposition makes is that a high social discount factor leads to a different carbon tax formula, not just a high tax.

Proposition 1 encompasses the special case of equal social and private discount rates in which the social planning problem is equivalent to that of a representative infinitely-lived individual. This is the social planning problem solved in many positive studies, such as that of Nordhaus and Boyer (2003). In this special case, it is easy to see that the optimal carbon emissions tax and carbon sequestration subsidy are both equal to the social cost of carbon, as standard Pigouvian taxation principles would prescribe. For their emissions, current generations pay an amount equal to the discounted value of marginal costs imposed on future generations. In the same way, they are compensated for removing carbon with an

[1]This result does not depend on sequestration being a disutility cost. I show in the appendix that the results go through in an economy where sequestration uses real resources with a fraction of output being devoted to sequestration, as modeled for example in Nordhaus and Boyer (2003).

amount equal to the discounted value of the benefits created for future generations. Two features are important to emphasize regarding this particular case of same social and private discount rates. First, it is often sufficient to control the *net* emissions of carbon. That is, a carbon emissions tax on the net carbon emissions (emissions net of sequestration) is enough to implement the optimal allocation. Second, the optimal carbon emissions tax is equal to the social cost of carbon in the social planner's problem, which allows policymakers to characterize the optimal taxes without having to solve for the competitive equilibrium.

None of these two features goes through when social and private discount rates differ. As I discussed before, the optimal tax scheme requires subsidizing sequestration and taxing emissions at different rates. Therefore, it is not enough to tax the net carbon emissions. More importantly, the social cost of carbon is no longer a sufficient indicator of the carbon price. For this reason, it is not enough to solve for the optimal path of carbon emissions that arises in a social planner's problem and argue that the carbon tax in the market economy equals the social cost of carbon in the planner's problem. When social and private discount rates differ, the design of optimal carbon policies requires special attention. To highlight the importance of this point, I state it as follows: suppose that we agree that the optimal level of carbon is the one proposed by Stern (2008) and that we introduce standard carbon taxes to achieve it. That is, carbon taxes equal to the social cost of carbon estimated in Stern's paper. The proposition says that the level of carbon will remain suboptimally high despite the policy intervention. Implementing a carbon path such as the one proposed by Stern (2008) requires the design of non-standard climate policies such as the ones this paper suggests.

There is a second subtle but important point that emerges from the proposition. The optimal policy distinguishes between the private and the social discount factors and depends on both. Therefore, introducing a high social discount factor in climate-economic models is not a relabeling of the social planning problem. It is fundamentally a different problem from the one with same social and private discount factors and, for this reason, it requires different policies. To be able to characterize them adequately, the policymaker must specify the Pareto weights in the social welfare function. These Pareto weights map into a social discount rate and, in turn, into the optimal carbon policies.

A particular feature of the optimal carbon emissions tax characterized in Proposition 1 is that it may potentially become a subsidy. It is useful to discuss some of the properties of the optimal allocation to understand this result. First, given the initial oil stock and a high enough initial carbon stock, the optimal path of carbon is decreasing over time and approaches zero as time goes to infinity[2]. By assumption, climate damages are proportional to the carbon stock and, therefore, the social cost of carbon is also decreasing over time. Consider first the climate externality in isolation (i.e. assume there is no difference in social and private discount rates). The carbon tax, in this case, is equal to the social cost of carbon. Therefore, the carbon tax is decreasing and converges to zero as the oil reserves are exhausted, and carbon either depreciates or is sequestered. Consider next the difference in social and private discount rates in isolation (i.e. assume there is no externality). In this case, a carbon emission tax is still required to achieve the socially optimal degree of intergenerational equity. It is easy to show that a decreasing tax is needed to provide incentives for oil companies to delay extraction over time[3]. Overall, it follows that the optimal carbon tax is decreasing because it reflects a decreasing social cost of carbon and, also, because it reflects the need to delay extraction further to pursue the optimal degree of intergenerational equity. Consequently, the optimal carbon tax reaches zero before the social cost of carbon does, and it potentially becomes a decreasing subsidy.

While uneasy to entertain the idea of a carbon tax possibly turning into a carbon subsidy, it is important to see that a declining subsidy still provides the right incentives to delay extraction. What matters is that the after-subsidy return from extraction is relatively higher in the future. Nevertheless, it is a standard principle in public finance that there are many ways to decentralize an optimal allocation and this is, of course, right in this model as well. In the light of the previous discussion, some readers may find Corollary 1 to be a more natural decentralization of the optimal allocation. This alternative decentralization involves the combination of standard carbon taxes and subsidies with a subsidy on fossil fuel reserves.

Corollary 1 ***(A Subsidy on Oil Reserves)*** *The optimal allocation can also be decentralized with a carbon tax, a sequestration subsidy and a subsidy τ_t^s on oil reserves defined by*

$$\tau_t^k = \tau_t^z = \mu_t^* \tag{20}$$

$$\tau_t^s = (\frac{\hat{\beta}}{\beta} - 1)(1 - \mu_t^*) \tag{21}$$

[2]If the initial carbon stock is low, it can be optimal to consume oil and pollute initially, letting the carbon stock increase for some time. Given the natural rate of carbon reabsorption and the exhaustibility nature of the oil reserves, the carbon stock eventually decreases and vanishes over time.

[3]See Sinclair (1992) and Ulph and Ulph (1994), for early contributions.

for every period t and all proceeds from taxation rebated/financed lump-sum through T_t.

This result states that if the tax on emissions is a standard carbon tax (that is, a tax equal to the social cost of carbon), then governments should pay oil companies to keep oil underground. In this case, the firm's per-period profits are given by

$$\pi_t = (k_t - k_{t+1}) - w_t z_t^d - \tau_t^k (k_t - k_{t+1}) + \tau_t^z z_t^d + \tau_t^s k_t$$

where the last term corresponds to the subsidy on the oil stock. The subsidy τ_t^s is composed of two terms, which is a reflection of the two policy goals: the first bracket in the formula reflects the difference in discounting and the second bracket indicates the value of the externality. The result resembles that in Farhi and Werning (2010), although they study a different problem. They find that, when future generations are directly valued in social welfare, it is optimal to subsidize bequests. In this model, the oil stock is a source of intergenerational transfers in addition to bequests. In this sense, the subsidy on the oil in situ plays the same role as the one on bequests in Farhi and Werning (2010). In fact, a decentralization with estate taxes is also possible in this model. This alternative decentralization requires combining negative estate taxes with standard carbon emission taxes and sequestration subsidies. This result suggests that estate taxation can potentially play a role in the portfolio of optimal climate policies.

Paying for the oil stock in situ is a policy that the literature on supply-side environmental policies advocates for, especially Harstad (2012) and also discussed in Sinn (2008). Harstad (2012) proposes buying coal as an optimal policy when countries behave non-cooperatively. In that paper, it is optimal to preserve more fossil fuel deposits to prevent non-participants in a climate coalition of nations from burning them. The rationale is different in this paper. Here, it is optimal to preserve more fossil fuel deposits because it serves as a tool to pursue intergenerational climate equity. It is interesting to see that social discounting provides a new theoretical justification for policies that the literature has already proposed in on different grounds.

It is important to notice that a social discount rate that is lower than the private one has broader implications for the structure of the optimal tax system, not only for carbon policies. Specifically, it calls for a subsidization of all forms of capital accumulation in the economy. For example, von Below (2012) and Barrage (2016) find that capital income subsidies are optimal when the government discounts the future less than households. In Appendix A, I show that this result goes through in an extended version of the benchmark model that includes a production function that uses capital and labor. In a similar way, if not oil and capital but also possibly gas and coal are used in production, the optimal carbon policy is likely to include a subsidy for the stocks of coal and gas as well. In this respect, an interesting insight of this paper is that the reserves of oil in situ are a form of capital accumulation for the society. For this reason, the reserves of oil in situ should be treated differently from sequestration and other mitigation policies.

6 Optimal Cap and Trade Scheme

A social discount rate that is lower than the private one has implications also for the design of an optimal carbon trading scheme. In particular, consider the following alternative market economy. The demographics, preferences and technologies are identical to the economy with carbon taxes. Only the policy instruments differ. Instead of carbon taxes, the government introduces a cap and trade scheme that sets a cap on *net* emissions of carbon $\{\theta_t\}_{t=0}^{\infty}$ together with a cap on sequestration $\{\phi_t\}_{t=0}^{\infty}$. The intuition for why two caps are needed, as opposed to only one cap on the net emissions of carbon, will become clear when I discuss the optimal carbon trading scheme. The government endows households with both carbon permits and sequestration licenses. Sequestration licenses are authorizations to sell sequestration services that firms can count as offsets of their emissions. If a firm wants to use sequestration as a reduction of its emissions, then it must buy sequestration from the authorized households. Households and firms are then allowed to trade carbon permits and sequestration rights at market prices $\hat{\tau}_t$ and $\hat{\tau}_t^z$, respectively.

To produce one unit of output, a firm must purchase one carbon permit. Alternatively, the firm can buy sequestration from authorized households and offset the emission. The possibility of offsetting emissions is available as long as sequestration licenses have not been exhausted. Hence, firms face the following constraints for every period t

$$\theta_t^d \geqslant k_t - k_{t+1} - z_t^d \tag{22}$$

$$\phi_t^d \geqslant z_t^d \tag{23}$$

where θ_t^d and ϕ_t^d correspond to the firm's demand for carbon permits and sequestration rights, respectively. Per-period profits of the firm are given by

$$\pi_t = (k_t - k_{t+1}) - w_t z_t^d - \hat{\tau}_t \theta_t^d - \hat{\tau}_t^z \phi_t^d \tag{24}$$

The firm must choose a sequence $\left\{k_t, z_t^d, \theta_t^d, \phi_t^d\right\}_{t=0}^{\infty}$ to maximize discounted profits subject to the constraints (27) and (28). All profits are rebated to households as dividends.

Households provide sequestration services at a competitive price w_t. Carbon permits and licenses cannot be stored. Households consume, provide sequestration services and save subject to the following set of budget constraints for $t = 0, 1, 2, ...$

$$c_t + b_{t+1} = w_t z_t + R_t b_t + \hat{\tau}_t \theta_t + \hat{\tau}_t^z \phi_t + \pi_t \tag{25}$$

where prices and taxes are defined in units of the consumption good and π_t are the profits received from the firm. The problem of the households is to choose a sequence $\{c_t, z_t, b_t\}_{t=0}^{\infty}$ to maximize (4) subject to (30).

Finally, market clearing for this economy requires that the following conditions must be satisfied for every period t

$$c_t = k_t - k_{t+1} \tag{26}$$

$$b_t = 0 \tag{27}$$

$$z_t = z_t^d \tag{28}$$

$$\theta_t = \theta_t^d \tag{29}$$

$$\phi_t = \phi_t^d \tag{30}$$

The last two equations correspond to the markets for carbon emission and sequestration permits.

A *competitive equilibrium with a cap and trade scheme* $\{\theta_t, \phi_t\}_{t=0}^{\infty}$ is a sequence of prices $\{w_t, R_t, \hat{\tau}_t, \hat{\tau}_t^z\}_{t=0}^{\infty}$ and allocations $\left\{c_t, z_t, k_t, b_t, z_t^d, \theta_t^d, \phi_t^d\right\}_{t=0}^{\infty}$ such that: (i) given prices and the caps, the allocation solves the consumer's problem, maximizing (4) subject to (30) and the firm's problem, maximizing (29) subject to (27)-(28); (ii) given the allocation, prices clear the markets.

I omit the characterization of the competitive equilibrium with a cap and trade scheme because it is easy to see that the equilibrium conditions are the same as in the tax economy except that permit prices $\hat{\tau}_t$ and $\hat{\tau}_t^z$ appear where taxes were before.

The following proposition characterizes the main features of an optimal cap and trade scheme. The main result is that, when the social discount rate is lower than the private one, it is optimal to set a cap on the carbon sequestration credits that can be used to meet the compliance obligations within the carbon trading scheme.

Proposition 2 ***(Optimal Cap and Trade)*** *Suppose that the social welfare weights are given by AI.ii so that $\beta < \hat{\beta}$. If $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ solves the social planning problem, then it solves the competitive equilibrium with a cap and trade system defined by*

$$\theta_t = k_t^* - k_{t+1}^* - z_t^*$$

$$\phi_t = z_t^*$$

for every period t.

The proposition says that when future generations are directly valued in social welfare an optimal cap and trade scheme must include a cap on carbon offset allowances. The intuition for this result is that, absent a cap on sequestration, firms burn oil too fast compared to the social optimum. Furthermore, because this implies that emissions are too high, firms will rely on sequestration to meet their cap on net emissions. By setting a cap on the amount of carbon offsets that firms can use, the government is indirectly setting a cap on fossil fuel extraction. Therefore, the optimal cap and trade scheme includes a mechanism to lock more oil under the crust of the earth and, in this way, shares some of the same spirit of the supply-side carbon policies discussed before. The cap on sequestration is the tool that is required to induce firms to deliver not only the optimal level of carbon to the next generation but also the optimal stock of fossil fuels.

It is interesting to compare this policy prescription to some of the carbon policies that are currently in place. Setting a cap on sequestration is a policy that, on the surface, seems a little perplexing. Removing carbon from the atmosphere is a social good and it seems counterintuitive that an optimal policy would set a cap on this good. However, this policy prescription resembles some of the features of actual policies. In particular, the European Union Emissions Trading Scheme (EU ETS) allows firms the use of compliance carbon credits up to a limit, which varies across member countries. California's greenhouse gas (GHG) cap-and-trade program allows the use of offset credits to meet up to 8 percent of firms' triennial compliance obligations. The Regional Greenhouse Gas Initiative (RGGI) lets regulated firms use offsets to meet up to 3.3 percent of their compliance obligations. In all cases, carbon offsets must be authorized and must meet regulatory criteria. Therefore, another way to interpret the results of this paper is that concerns about intergenerational equity provide a different rationale for why a cap on carbon offset projects is optimal.

7 Heterogeneous Households

This section extends the model to include heterogeneity in consumer's preferences, in the ownership of the exhaustible resource and the ownership of the sequestration technology. A natural conjecture is that with heterogeneous consumers, carbon policies can be used to redistribute resources further not only across generations but also across different types of consumers within a given generation. I study this conjecture in an economy with two types of individuals, indexed $i = 1, 2$. Individuals of type 1 own the fossil fuel stock and individuals of type 2 own the sequestration technology. Furthermore, individuals of type i care about their utility and that of their children according to

$$W_{it} = U^i(c_{it}, z_{it}, S_t) + \beta^i W_{it+1} \tag{31}$$

where c_{it} and z_{it} denote oil consumption and carbon sequestration of a consumer of type i. Notice that this preference specification allows for heterogeneity in altruism and the climate damages borne by each type of consumer. In a dynastic interpretation of the economy, these preferences are consistent with a dynasty who, over time, cares about the overall stream of consumption, carbon sequestration and the carbon level according to

$$\sum_{t=0}^{\infty} \beta^{it} u^i(c_{it}, z_{it}, S_{t+1}) \tag{32}$$

for $i = 1, 2$.

Social Optimum. Consider a utilitarian criterion that weighs current as well as future unborn generations according to the following function

$$\sum_{i=1}^{2} \omega_i \sum_{s=0}^{\infty} \alpha_{is} \left[\sum_{t=s}^{\infty} \beta^{i(t-s)} U^i(c_{it}, z_{it}, S_{t+1}) \right] \tag{33}$$

where ω_i represents the "intra-generational" weight that the social planner assigns to individuals of type i's welfare, and $\{\alpha_{is}\}_{s=0}^{\infty}$ represent the "inter-generational" weights that the social planner assigns to generations of type i's welfare. The welfare weights $\omega_i \in [0, 1]$ satisfy that $\omega_1 + \omega_2 = 1$. The intergenerational weights satisfy the following assumption:

Assumption 2 (A2) *The welfare weights $\{\alpha_t^i\}_{t=0,i=1,2}^{\infty}$ in the social welfare function (38) satisfy one of the following conditions:*

(i) $\alpha_0^i = 1$ *and* $\alpha_{t+1}^i = 0 \; \forall t, \forall i$

(ii) $\alpha_0^i = \frac{1}{\hat{\beta}^i - \beta^i}$ *and* $\alpha_t^i = \hat{\beta}^{it}$ $\forall t > 0$, *for some constants* $\hat{\beta}^1 > \beta^1$ *and* $\hat{\beta}^2 > \beta^2$

As in the benchmark model, the welfare weights on future generations are restricted to be either zero (in which case the social and the private discount rate is the same) or decrease geometrically over time (in which case the social discount rate is constant and social preferences are time-consistent).

The utility function is given by $U^i(c_{it}, z_{it}, S_{t+1}) = u^i(c_{it}) - v^i(z_{it}) - x^i(S_{t+1})$ for every type i with the same properties described in the benchmark model for u, v and x. We can further simplify the welfare function to obtain

$$\sum_{i=1}^{2} \sum_{t=0}^{\infty} \rho_t^i U^i(c_{it}, z_{it}, S_{t+1}) \tag{34}$$

where $\rho_t^i \equiv \omega_i \sum_{\tau=0}^{t} \alpha_{i\tau} \beta^{i(t-\tau)}$ represents the social discount function. Notice that the social discount factor differs from the private discount factor due to both the intragenerational and intergenerational Pareto weights. Notice also that, within consumer's types, the difference between the social and the private discount rate is only due to the intergenerational Pareto weights. Furthermore, weighing future generations with geometric Pareto weights, as the ones defined in Assumption 2, gives the following social welfare function[4]

$$\sum_{i=1}^{2} \sum_{t=0}^{\infty} \hat{\beta}^{it} U^i(c_{it}, z_{it}, S_{t+1}) \tag{35}$$

Therefore, social welfare is the weighted sum of consumer's utilities across generations and consumer's types.

The social planning problem is to choose the path of consumption, fossil fuel, sequestration and carbon level $\{c_t^{i*}, k_t^{i*}, z_t^{i*}, S_t^*\}_{t=0, i=\{1,2\}}^{\infty}$ that maximizes the social welfare function (40) subject to the carbon cycle

$$S_{t+1} = (1-\gamma)S_t + \sum_i (k_{it} - k_{it+1} - z_{it}) \tag{36}$$

the resource constraint

$$\sum_i c_{it} = \sum_i (k_{it} - k_{it+1}) \tag{37}$$

and the initial conditions $\{k_0^i, S_0\}_{i=1,2}$. The first-order conditions for the social planning problem imply that the marginal utility of consumption is equalized across consumer's types

[4]This social welfare function derives from performing intergenerational discount factor calculations similar to the ones in Section 2 and Farhi and Werning (2007).

and that the marginal cost of carbon sequestration is equal to the social cost of carbon. The social cost of carbon is calculated in the usual way, and it is given by

$$\mu_t^* = \frac{\sum_i \sum_{j=0}^{\infty} \omega_i \hat{\beta}^{i(t+j)} (1-\gamma)^j x^{i\prime}(S_{t+j})}{\lambda_t} \tag{38}$$

in every period t, where λ_t is the social value of oil. Thus, μ_t^* is the social cost of carbon in oil units. Additionally, the following intertemporal Euler equation drives the pace of oil depletion

$$\frac{\hat{\beta}^i u_c^{i\prime}(c_{it+1})}{u_c^{i\prime}(c_{it})} = \frac{1-\mu_t^*}{1-\mu_{t+1}^*} \tag{39}$$

for $i = 1, 2$ and for every period t.

Optimal Carbon Taxes. Consider a decentralized environment where consumers of type 1 own the oil stock and consumers of type 2 own the carbon sequestration technology. Consumers save in a risk-free asset b_t which bears a gross one-period rate of return R_t. The asset is a bequest from individuals in generation t to those in the next generation. There is a government that sets a carbon tax on oil extraction and a carbon sequestration subsidy and rebates the tax proceedings in a lump sum tax/subsidy to consumers. For convenience, I assume that the government taxes oil consumption instead of oil extraction and distributes the tax revenue equally among consumers. Oil owners choose the path of oil extraction, bequest and consumption subject to the following budget constraint

$$(1+\tau_t^c)c_{1t} + b_{1t+1} \leq k_t - k_{t+1} + R_t b_{1t} + T_{1t} \tag{40}$$

in every t. Thus, the problem of oil owners consumers is to maximize (37) for $i = 1$ subject to (45). Additionally, type 2 consumers choose consumption and carbon sequestration in order to maximize (37) for $i = 2$ subject to the following budget constraint

$$(1+\tau_t^c)c_{2t} + b_{2t+1} \leq \tau_t^z z_{2t} + R_t b_{2t} + T_{2t} \tag{41}$$

in every t. As in the benchmark model, oil is the only consumption good, and it is the numeraire. In a broader sense, oil consumption can be thought of as energy consumption. Type 2 consumers must buy energy from oil owners. Notice that both consumers face the same carbon tax. The next corollary shows that a version of Proposition 1 holds in this environment, although with one qualification. As in the benchmark model, the optimal carbon tax does not equal the social cost of carbon while the carbon sequestration subsidy does. The optimal carbon tax follows a more general formula with both the social cost of carbon and the private and social discount factors playing a role in it. The qualification is that the result does not hold for any choice of geometric Pareto weights. Specifically, Corollary 2 adds a restriction on type 2 consumer's welfare weights that can enter into the social welfare calculation. This restriction guarantees that all consumers face the same carbon emissions tax and carbon sequestration subsidy. In particular, notice that the private discount factor, and the utility function, can potentially be different across consumer's types. In this case, a common carbon tax is optimal only if the planner adjusts the intergenerational Pareto weights to equalize the marginal utility of consumption across types[5]. Corollary 2 formalizes this result. The proof is in the mathematical appendix.

Corollary 2 *Suppose that the social welfare weights satisfy A2.ii and* $\alpha_{2t} = \alpha_{1t}\frac{\beta^2}{\beta^1}$ *for every* t*. The optimal allocation* $\{c_t^{i*}, z_t^{i*}, k_t^{i*}, S_t^*\}_{t=0,i=1,2}^{\infty}$ *can be decentralized in a competitive market with carbon taxes defined as follows*

$$1 + \tau_t^c = \left(\frac{\beta^1}{\hat{\beta}^1}\right)^t \frac{1}{1-\mu_t^*} \tag{42}$$

$$\tau_t^z = \mu_t^*(1+\tau_t^c) \tag{43}$$

for every period t *and all proceeds from taxation equally rebated lump-sum to consumers.*

The carbon tax formula in the equation (64) is the same as the one in (23) in Proposition 1. To see that, notice that oil consumption taxes τ^c and oil extraction taxes τ^k can be expressed as a function of each other: $\tau_t^k = \frac{\tau_t^c}{1+\tau_t^c}$. Without loss of generality, the oil owner's discount factor appears in the carbon tax formula. Alternatively, the non-oil owner's discount factor can enter into the formula, adjusting the weights on type 1 consumer's welfare accordingly[6]. What matters is that the marginal utility of oil consumption is the same across consumers.

The restriction on Pareto weights that qualifies the results can be understood building on the central intuition of the paper regarding the number of policy targets and instruments. In particular, there is potentially a third policy goal when consumers are heterogeneous. As in the benchmark model, the optimal carbon policies are designed to achieve efficiency and intergenerational equity. Likewise, when consumers are heterogeneous, a third policy goal is to achieve intra-temporal equity between consumer's types. With three policy goals, two

[5]If the tax code allows carbon tax rates to differ across consumers types, the extra condition on the Pareto weights is not necessary. I study this particular case in the online appendix.

[6]If the private discount factor is the same across consumer types, then the intergenerational Pareto weights are the same too. Notice that because α_{1t} satisfy AI.ii, the welfare weights α_{2t} are also geometric.

instruments (a carbon tax and a carbon sequestration subsidy) are in general not enough. In particular, two instruments can implement just one point on the Pareto frontier, which is the one that corresponds to Pareto weights that satisfy Assumption 2. For a more general specification of weights, another lever is required[7]. This new lever can be, for example, a carbon tax that is different across consumers so that oil companies and non-oil owners bear a different carbon tax. I study this implementation in the online appendix. I also show that a decentralization with a subsidy on oil reserves in situ is optimal in the economy with heterogeneous consumers as well.

Overall, this section shows that the main results of the paper go through to an economy with heterogeneous consumers: when carbon policies are designed to achieve efficiency and some form of equity (either inter-generational or intra-generational equity or both), the carbon tax is in general not equal to the social cost of carbon. The optimal carbon tax formula includes aspects of the distributional goals it seeks to achieve. For this reason, policymakers need to pay special attention to the design of optimal carbon policies.

8 Concluding Comments

The choice of the social discount rate in climate-economic models has sparked a heated debate. Stern (2008) argues for a low discount rate based on a desire for intergenerational equity. Nordhaus and Boyer (2003), Weitzman (2007), and many others criticize this point of view. They argue that the discount rate should be consistent with market returns. This paper shows that a social welfare function that assigns Pareto weights to the welfare of future generations maps onto low social discounting. Thus, the discussion is better framed around the question of what the appropriate Pareto weights are rather than what the proper social discount rate is. In that sense, there is no right or wrong answer. Further, the paper shows that social discounting maps onto optimal carbon policies. Carbon policies designed in the standard way (i.e., carbon taxes equal to the social cost of carbon) do not implement the optimal path of carbon when the social and private discount rates are different. I theoretically characterize the optimal carbon policies in this case. A common feature of these policies is that they treat direct abatement (i.e. a reduction in carbon emissions) differently from other alternative

[7]Similarly, in the benchmark model, standard carbon taxes can implement just one point on the Pareto frontier, which is the one that corresponds to assigning no direct weight to future generation's welfare. For a more general specification of weights, another lever is required.

abatement measures such as carbon sequestration. The reason is that a reduction in carbon emissions and an increase in carbon sequestration affect future generations differently. In particular, carbon sequestration affects future generation's welfare only through its effect on the carbon level. On the contrary, more carbon emissions imply more atmospheric carbon but also fewer oil reserves, both of which have a negative impact on future generation's well-being. Therefore, a reduction in carbon emissions is more effective to pursue an optimal degree of climate equity. Some recent papers, see for example Allen (2016) and Moreaux and Withagen (2015) among others, argue that carbon sequestration is likely to be a central instrument of climate policy. The results in this paper suggest that the role of carbon sequestration is more limited when climate policies are meant to attain intergenerational equity, besides dealing with the climate externality itself.

The main results of the paper are an application of Tinbergen (1952) important principle of equal policy targets and tools. Although policymakers usually consider carbon emissions to be an externality problem, questions of equity across generations, and potentially across different population groups, can make climate change a broader problem that will require a more comprehensive set of instruments to tackle it adequately.

A Model Assumptions and Extensions

I have derived the results of the paper in a stylized model economy that has just enough ingredients to make a theoretical point. This section presents a general specification of the model economy that includes: (i) a production function that uses capital and labor; (ii) a climate externality that takes the form of an output loss; and (iii) a sequestration technology that uses final output. This model economy shares some of the main features of the DICE model in Nordhaus and Boyer (2003). I show that the results go through in the more general model. In particular, I show that when social and private discount rates differ, the optimal carbon tax is not equal to the social cost of carbon and it is not equal to the subsidy on sequestration.

Model. Consider the following economy. There is a single consumption good that can be thought of as "energy". Energy is produced out of capital and labor. Energy production increases carbon in the atmosphere, S_t. Carbon generates a climate externality that takes the form of an output loss and it is captured by the damage function $x(S)$. The function x is increasing, convex, twice differentiable and $x(0) = 1$. Thus, the aggregate output function is given by

$$Y_t = x(S_{t+1})F(k_t, n_t) \tag{44}$$

Carbon in the atmosphere decreases with sequestration. When a fraction $v(z_t)$ of output is used in sequestration, carbon in the atmosphere decreases by z_t units. Therefore, the feasibility constraint in this economy is given by

$$c_t + k_{t+1} = [1 - v(z_t)]x(S_{t+1})F(k_t, 1) \tag{45}$$

where I have assumed full depreciation of capital and I have normalized the labor supply to unity. Moreover, carbon in the atmosphere follows the following process

$$S_{t+1} = (1-\gamma)S_t + F(k_t, 1) - z_t \tag{46}$$

where $\gamma \in [0, 1)$ is the rate of natural reabsorption of carbon and S_0 is given. Finally, an individual's utility in period t is given by $u(c)$. The function u is increasing, concave and twice differentiable with $\lim_{c\to 0} u'(c) = \infty$. The demographics and the social welfare function correspond to the ones described in section 2.

The *socially optimal allocation* is the path of consumption, sequestration, capital and carbon stock, $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, that maximizes the social welfare function (6) subject to the carbon cycle (51), the resource constraint (50) and the initial conditions $\{k_0, S_0\}$.

I omit the characterization of the optimal allocation here. However, I describe below the expression for the social cost of carbon that comes out from this setting because it will be important for the design of the optimal taxes. The social cost of carbon is given by the Lagrange multiplier on the carbon cycle constraint and it equals

$$\Lambda_t^* = -\frac{\sum_{j=0}^{\infty}[\beta(1-\gamma)]^j u'(c_{t+j})[1 - v(z_{t+j})]x'(S_{t+1+j})F(k_{t+j}, 1)}{u'(c_t)} \tag{47}$$

The social cost of carbon measures the marginal damage of increasing carbon emissions by an extra unit. Because carbon only depreciates slowly, this marginal damage equals the discounted sum of all future output losses. Because it will be useful to derive the results, I denote Λ_t^* the social cost of carbon expressed in units of the consumption good.

Market Economy. In a decentralized environment, consumers choose how much energy to consume and how much to invest in capital subject to the following sequence of budget constraints for every period t

$$c_t + k_{t+1}^s = r_t k_t^s + w_t n_t^s + \pi_t + T_t \tag{48}$$

where T_t is a lump sum tax/rebate received from the government and π_t are dividends.

Firms face two forms of taxation: a carbon tax, τ_t^k, on emissions and a tax credit, τ_t^z, on carbon sequestration. Firms choose a sequence $\left\{n_t^d, k_t^d, z_t\right\}_{t=0}^{\infty}$ in order to maximize profits given by

$$\pi_t = [1 - v(z_t)]x(S_{t+1})F(k_t^d, n_t^d) - w_t n_t^d - r_t k_t^d - \tau_t^k F(k_t^d, n_t^d) + \tau_t^z z_t \tag{49}$$

The last two terms in the profit function represent the tax bill. All profits are rebated to households as dividends.

A government collects carbon taxes and pays subsidies. Any surplus (or deficit) is rebated in a lump-sum transfer to households. The sequence of the government's budget constraints for $t = 0, 1, ...$ is given by

$$\tau_t^k F(k_t^d, n_t^d) - \tau_t^z z_t = T_t \tag{50}$$

Finally, market clearing for every period t requires that the equation (50) together with the market clearing for the capital stock

$$k_t^d = k_t^s \tag{51}$$

and the labor market are satisfied.

A *competitive equilibrium with taxes* $\{\tau_t, \tau_t^z, T_t\}_{t=0}^{\infty}$ is a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$ and allocations $\left\{c_t, z_t, k_t^s, k_t^d, n_t^d, n_t^s\right\}_{t=0}^{\infty}$ such that: (*i*) given taxes and prices, the allocation solves the consumer's problem, maximizing (4) subject to (53), and the firm's problem, maximizing (54); (*ii*) given the allocation, transfers are such that the government budget constraint (55) is satisfied; and (*iii*) prices clear the markets.

Optimal Policy. The next proposition shows that the main result of the paper goes through in this more general setting. In particular, I show that, when the social and private discount rates differ, the optimal tax policy requires subsidizing sequestration and taxing carbon emissions, but each at different rates: the optimal subsidy for removing a ton of carbon from the atmosphere will in general not equal the optimal tax for creating a ton of carbon. Moreover, the optimal carbon tax does not equal to the social cost of carbon. The proof is included in the Appendix B.

Proposition 3 *Suppose that the social welfare weights are given by AI.ii so that $\beta < \hat{\beta}$. If $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$ solves the social planning problem, then it solves the competitive equilibrium* *with taxes* $\{\tau_t^k, \tau_t^z, T_t\}_{t=0}^{\infty}$ *defined as follows*

$$\tau_{t+1}^k = \frac{\hat{\beta}}{\beta}\Lambda_{t+1}^* - (\frac{\hat{\beta}}{\beta} - 1)[1 - v(z_{t+1}^*)]x(S_{t+2}^*) \tag{52}$$

$$\tau_t^z = \Lambda_t^* \tag{53}$$

for every period t*, with* $\tau_0^k = 0$ *and all proceeds from taxation rebated/financed lump-sum through* T_t*.*

The optimal tax on emissions is not equal to the value of the externality that the emissions generate. While the optimal subsidy for sequestration equals the social cost of carbon, the optimal carbon tax is composed of two terms. The first term in the carbon tax formula (57) captures the carbon externality and the second term captures the effect of social discounting. As it was the case for the stylized economy presented in section 2, there are two potentially opposing effects driving the optimal carbon tax. On the one hand, energy production must be taxed to control the climate externality. On the other hand, the capital stock should be subsidized to take into account the different discount rates. That is, it is socially optimal to save more capital for future generations when the social discount rate is lower. Absent a direct subsidy for the capital stock, the carbon tax picks up this effect too. Depending on how large the climate externality is, the overall optimal carbon tax could potentially become a subsidy.

It is easy to show that an alternative tax policy would be to introduce a time-invariant subsidy on capital income equal to

$$\tau^s = [\frac{\hat{\beta}}{\beta} - 1]$$

and a carbon tax and subsidy for sequestration, both equal to the social cost of carbon. It is also easy to see that when the social and private discount rates are the same, the optimal carbon tax and the optimal carbon sequestration subsidy are both equal to the social cost of carbon (the Pigouvian rate).

I limited this appendix to the study of the optimal taxation policies and I have not discussed the optimal cap and trade program that corresponds to this more general economic environment. However, it is easy to see that the mechanism that drives the results is still present in this more general setting and an optimal cap on carbon offset allowances is likely to be optimal here too.

B Mathematical Appendix

Proof of Proposition 1. The proof consists of showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, when taxes and subsidies are set optimally. It is useful to rewrite taxes in terms of allocations (instead of shadow prices) in the following way

$$1 - \tau_t^k = \left(\frac{\hat{\beta}}{\beta}\right)^t \left[1 - \frac{v'(z_t^*)}{u'(c_t^*)}\right]$$

The intertemporal condition (22), evaluated at the star allocation, is

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1 - \tau_t^k}{1 - \tau_{t+1}^k}$$

Plug the optimal tax rate to obtain

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{\left(\frac{\hat{\beta}}{\beta}\right)^t \left[1 - \frac{v'(z_t^*)}{u'(c_t^*)}\right]}{\left(\frac{\hat{\beta}}{\beta}\right)^{t+1} \left[1 - \frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)}\right]}$$

Rearranging terms we obtain

$$\hat{\beta}[u'(c_{t+1}^*) - v'(z_{t+1}^*)] = u'(c_t^*) - v'(z_t^*)$$

which is satisfied by the efficient allocation because it coincides with (13). Consider now the equilibrium condition for carbon sequestration evaluated at the star allocation

$$\frac{u'\left(c_t^*\right)}{v'\left(z_t^*\right)} = \frac{1}{\tau_t^z}$$

This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel (19) is already satisfied by the efficient allocation. Given the sequence of taxes $\left\{\tau_t^k, \tau_t^z\right\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government (16) is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers $b_{t+1} = R_t b_t$ for all t, which, together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial $b_0 = 0$. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm. This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied.

Proof of Corollary 1. The proof consists of showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, when taxes and subsidies are set optimally. The intertemporal condition for this tax economy, evaluated at the star allocation, is

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)}=\frac{1-\tau_t^k}{1-\tau_{t+1}^k+\tau_{t+1}^s}$$

Plug the optimal tax rate to obtain

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)}=\frac{1-\frac{v'(z_t^*)}{u'(c_t^*)}}{1-\frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)}+\left(\frac{\hat{\beta}}{\beta}-1\right)\left[1-\frac{v'(z_{t+1}^*)}{u'(c_{t+1}^*)}\right]}$$

Rearranging terms we obtain

$$\hat{\beta}[u'(c_{t+1}^*)-v'(z_{t+1}^*)]=u'(c_t^*)-v'(z_t^*)$$

which is satisfied by the efficient allocation because it coincides with (13). Consider now the equilibrium condition for carbon sequestration evaluated at the star allocation

$$\frac{u'\left(c_t^*\right)}{v'\left(z_t^*\right)}=\frac{1}{\tau_t^z}$$

This condition holds by definition of the optimal taxes. The market clearing condition for fossil fuel (19) is already satisfied by the efficient allocation. Given the sequence of taxes $\left\{\tau_t^k,\tau_t^z,\tau_t^s\right\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government $p_t\tau_t^k(k_t-k_{t+1})-p_t\tau_t^z z_t^d-p_t\tau_t^s k_t=T_t$ is satisfied. Plugging transfers and profits into the budget constraint of the consumer delivers $b_{t+1}=R_t b_t$ for all t, which, together with the market clearing condition for bonds, delivers the equilibrium sequence for the bonds and the interest rate, given initial $b_0=0$. Finally, the sequence of prices that supports this equilibrium is obtained from the optimizing behavior of the firm. This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied.

Proof of Proposition 2. The proof is by construction. The constraints (27) and (28), together with the market clearing condition for the caps, eq. (34) and (35), imply that the competitive allocation coincides with the optimal. Equilibrium permit prices are then recovered from the following equilibrium conditions

$$\frac{v'\left(z_t\right)}{u'\left(c_t\right)}=w_t \tag{54}$$

and

$$\frac{\beta u'\left(c_{t+1}\right)}{u'\left(c_t\right)}=\frac{1-\hat{\tau}_t}{1-\hat{\tau}_{t+1}} \tag{55}$$

Proof of Corollary 2. The proof consists of showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^{i*}, k_t^{i*}, z_t^{i*}, S_t^*\}_{t=0,i=\{1,2\}}^{\infty}$ when taxes and subsidies are optimal. Because only type 2 consumers own the sequestration technology, the intratemporal equilibrium condition for carbon sequestration is given by

$$\frac{U_z^2(c_{2t}, z_{2t}, S_{t+1})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{\tau_t^z}{1+\tau_t^c}$$

Plug the optimal tax rates to obtain

$$\frac{U_z^2(c_{2t}, z_{2t}, S_{t+1})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \mu_t^*$$

which is satisfied by the optimal allocation. The intertemporal equilibrium condition for type 2 consumers is

$$\frac{\beta^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{1+\tau_{t+1}^c}{1+\tau_t^c}$$

Plug the optimal carbon tax rate to obtain

$$\frac{\beta^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{\beta^1}{\hat{\beta}^1}\frac{1-\mu_t^*}{1-\mu_{t+1}^*}$$

which equals

$$\frac{\hat{\beta}^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{1-\mu_t^*}{1-\mu_{t+1}^*}$$

by assumption 2. Furthermore, this intertemporal optimality condition is satisfied by the optimal allocation. It remains to show that the equilibrium conditions for type 1 consumers are also satisfied by the optimal allocation. The intertemporal equilibrium condition for type 1 consumers is

$$\frac{\beta^1 U_c^1(c_{1t+1}, z_{1t+1}, S_{t+2})}{U_c^1(c_{1t}, z_{1t}, S_{t+1})} = \frac{1+\tau_{t+1}^c}{1+\tau_t^c}$$

Plug the optimal carbon tax rate to obtain

$$\frac{\beta^1 U_c^1(c_{1t+1}, z_{1t+1}, S_{t+2})}{U_c^1(c_{1t}, z_{1t}, S_{t+1})} = \frac{\beta^1}{\hat{\beta}^1}\frac{1-\mu_t^*}{1-\mu_{t+1}^*}$$

The private discount rate cancels out from both side of the equality. Therefore, this optimality condition is satisfied by the optimal allocation. The market clearing condition for oil satisfies (42). Given the sequence of taxes $\{\tau_t^c, \tau_t^z\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government is satisfied. By assumption $T_{it} = T_t/2$ for all i. Because the oil stock and bequests are alternative savings assets for type 1 consumers, an arbitrage condition must hold. It follows that $R_{t+1} = 1$. This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied.

Proof of Proposition 3 in Appendix A. The proof consists of showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^*, z_t^*, k_t^*, S_t^*\}_{t=0}^{\infty}$, when taxes and subsidies are set optimally. The intertemporal equilibrium condition evaluated at the star allocation is given by

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1}{\left[[1-v(z_{t+1}^*)]x(S_{t+1}^*) - \tau_{t+1}^k\right] F_k'(k_{t+1}^*, 1)}$$

Plug the optimal tax rate characterized in equation 57 to obtain

$$\frac{\beta u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1}{\left[[1-v(z_{t+1}^*)]x(S_{t+1}^*) - \frac{\hat{\beta}}{\beta}\Lambda_{t+1}^* + (\frac{\hat{\beta}}{\beta} - 1)[1-v(z_{t+1}^*)]x(S_{t+2}^*)\right] F_k'(k_{t+1}^*, 1)}$$

After some algebra manipulation, the intertemporal equilibrium condition becomes

$$\frac{\hat{\beta} u'\left(c_{t+1}^*\right)}{u'\left(c_t^*\right)} = \frac{1}{\left[[1-v(z_{t+1}^*)]x(S_{t+1}^*) - \Lambda_{t+1}^*\right] F_k'(k_{t+1}^*, 1)}$$

which is also satisfied by the optimal allocation.

Consider now the intratemporal equilibrium condition that characterizes the optimal decision for carbon sequestration. The intratemporal equilibrium condition is given by

$$v'(z_t)x(S_{t+1})F(k_t) = \tau_t^z$$

while the intratemporal optimality condition in the planner's problem is given by

$$v'(z_t^*)x(S_{t+1}^*)F(k_t^*) = \Lambda_t^*$$

Both conditions coincide when the subsidy for sequestration is set equal to equation 58.

By definition, the market clearing condition is satisfied by the optimal allocation. Given the sequence of taxes $\left\{\tau_t^k, \tau_t^z\right\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government (55) is satisfied. Finally, the budget constraint of the consumer is satisfied by Walras law. This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied by the optimal allocation.

Table 1: **Calibration.**

Parameter	Value	Target
α_t	0.9990^t	Consistent with social discount rate in Stern (2007)
β	0.9852	Nordhaus (2007)
ψ_s	2.4e-6	Welfare loss associated with a 1.74% output loss
ψ_z	0.3	Initial per unit carbon tax equal to \$100 per ton of carbon
γ	0.0023	300 years average lifetime of atmospheric carbon

C Online Appendix

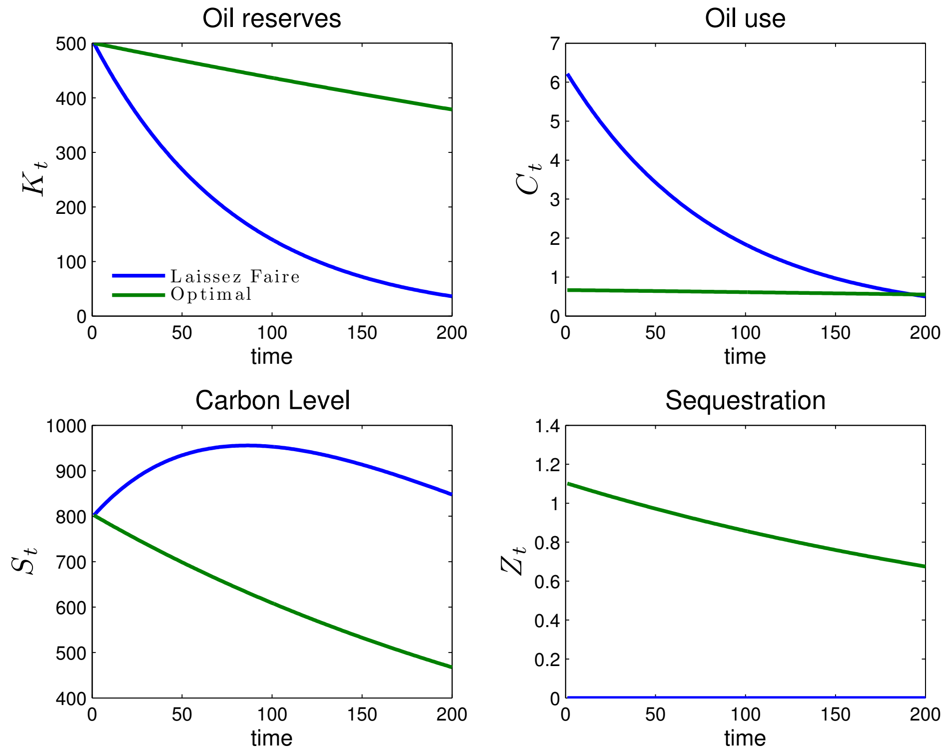
C.1 A Numerical Example

This section presents a numerical example. The goal is not to provide a comprehensive quantitative evaluation but to highlight the effects of different social and private discount rates on the optimal climate policies. I take a period in the model to be one year. I consider an annual private discount rate equal to 1.5% and Pareto weights that decrease geometrically over time. The choice of Pareto weights and the private discount rate imply an annual social discount rate equal to 0.1%, which is consistent with the social discount rate in Stern (2007). I assume a per-period utility function given by $u(c, z, S) = log(c) - \psi_z \frac{z^2}{2} - \psi_s \frac{S^2}{2}$. The parameter ψ_s summarizes the cost of climate change in the model, and it is calibrated so that the welfare loss that arises from doubling the atmospheric carbon from pre-industrial levels is equivalent to the welfare loss obtained from a 1.74% annual decrease in consumption. This calibration roughly matches the output loss associated with a $2.5C^{\circ}$ increase in temperature estimated by Nordhaus (2008). Normalizing the price of fossil fuel to \$461 per ton of carbon, the sequestration cost parameter ψ_z is calibrated to match an initial per unit carbon tax equal to \$100 per ton of carbon. According to Archer (2005), the carbon that does not stay in the atmosphere forever has a mean lifetime of about 300 years. Thus, the natural rate of carbon reabsorption γ is set to match 300 years of average lifetime of atmospheric carbon. Table 1 summarizes the calibration.

Figure 1 depicts oil consumption and oil reserves in situ in the optimal allocation and laissez-faire. As expected, the optimal policy leads to a much smaller use of oil. Moreover, the optimal allocation displays a roughly constant consumption profile while oil depletion is much faster in laissez faire. That is to say that the optimal policies lock more oil under the earth's crust and for a longer time frame. The graph also plots the path for atmospheric carbon and carbon sequestration. Under the benchmark calibration, the carbon level is monotonically decreasing, implying that atmospheric carbon has already reached the optimal peak. Using Nordhaus's mapping from the stock of atmospheric carbon to the temperature, the increase in the mean temperature associated with the optimal carbon level is less than $1.5°C$. On the contrary, the laissez faire economy continues on an increasing carbon path for about a century, and the temperature increase is above $2°C$. Carbon eventually decreases due to its natural rate of reabsorption. With optimal policies, the carbon level also decreases due to carbon sequestration. Under the benchmark calibration, carbon sequestration starts off high and decreases over time. In an optimal cap and trade scheme, optimal net emissions are negative, and the cap on carbon sequestration starts at approximately 1.6% of allowances and decreases over time.

In Figure 2, I plot the per-unit optimal carbon tax and sequestration subsidy under different assumptions about the private discount rate. The figure also shows the ad-valorem optimal subsidy on oil reserves. The left panel depicts the optimal carbon tax and sequestration subsidy when the private and social discount rates are the same. Both tax rates are equal to the social cost of carbon. The social cost of carbon is decreasing because the optimal path of atmospheric carbon is falling over time. For this reason, the carbon tax and carbon subsidy are both decreasing. The panel in the middle depicts the optimal carbon tax for different assumptions on the private discount rates. The picture shows that social discounting introduces a wedge between the social cost of carbon (blue line) and the optimal carbon tax. On one extreme, I plot the case when social and private discount rates are the same which corresponds to the same tax rate as in the left panel, but on a different scale. On the other extreme, I consider a private discount rate consistent with that in Nordhaus (2007) and a social discount rate equal to 0.1%. Between these two extreme cases, I also consider some intermediate values of the private discount rate. When the social and private discount rates take values that roughly represent Stern's and Nordhaus's point of view (0.1% and 2.5%, respectively), the optimal carbon tax decreases quickly over time and turns into a carbon subsidy in about 50 years. For intermediate values of the private discount rate, the carbon

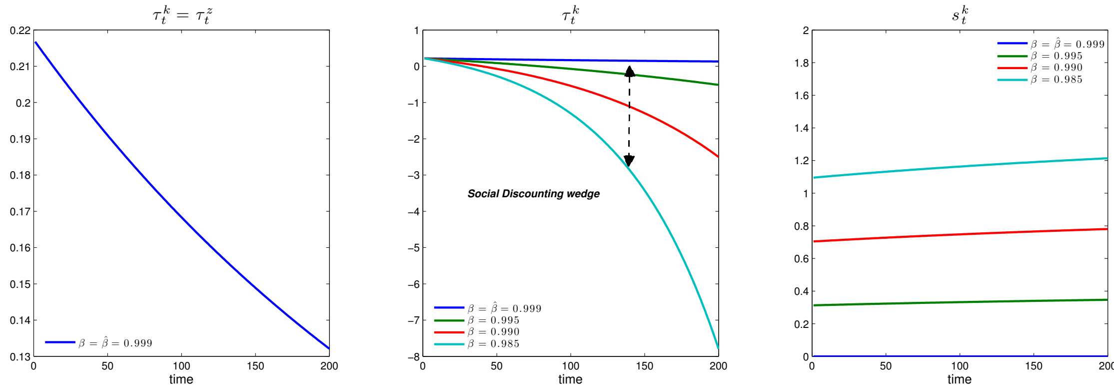
Figure 1: **Optimal allocation vs Laissez Faire**



tax decreases at a lower rate and does not become negative for at least a century. Notice that it is the relative tax rate between two consecutive periods what matters and even a negative tax generates incentives to delay extraction as long as it decreases over time. Nevertheless, a negative carbon tax is a policy that seems, at the very least, difficult to implement, and this numerical example does illustrate some of the shortcomings of using social discounting in climate-economic models.

Finally, the panel on the right shows the optimal subsidy on oil reserves. The figure shows that the optimal subsidy is about 1.1% when the private discount rate is lowest, and it slightly increases over time. The optimal subsidy rate is rather small when compared to the optimal capital income subsidies found in Barrage (2016), which start at 30% and increases to 65% by the end of the century.

Figure 2: **Social Discounting and the Optimal Carbon Policies**



C.2 Heterogeneous Households

This section shows that an implementation of the optimal allocation using a subsidy on the stock of oil in situ and standard carbon taxes and sequestration subsidies is optimal when consumers are heterogeneous. The section also studies the optimal carbon policy when the tax system allows the carbon emissions tax to differ across consumers types.

Corollary 3 characterizes the optimal policy with a subsidy on oil reserves.

Corollary 3 *Suppose that the social welfare weights satisfy A2.ii and* $\alpha_{2t} = \alpha_{1t}\frac{\beta^2}{\beta^1}$ *for every t. The optimal allocation* $\{c_t^{i*}, z_t^{i*}, k_t^{i*}, S_t^*\}_{t=0,i=1,2}^{\infty}$ *can be decentralized in a competitive market with carbon taxes defined as follows*

$$1 + \tau_t^c = \frac{1}{1 - \mu_t^*} \tag{56}$$

$$\tau_t^z = \mu_t^*(1 + \tau_t^c) \tag{57}$$

$$s_t^k = \frac{\hat{\beta}^1}{\beta^1} - 1 \tag{58}$$

for every period t and all proceeds from taxation equally rebated lump-sum to consumers.

Although type 2 consumers do not receive the subsidy directly (because they do not own oil), the subsidy translates into a higher return on bequests by an arbitrage condition that must hold in equilibrium. To see this, notice that oil owners see bequests and the oil stock as two alternative saving assets. For both assets to be held in equilibrium, the return on the assets must be equalized. Notice also that the qualification on the Pareto weights still holds in this alternative implementation. Therefore, although the carbon policies in Corollary 3 achieve some degree of intergenerational equity across types, the policies cannot pursue any arbitrary intergenerational equity goal. Alternative equity goals are possible, but an additional instrument is required. Corollary 4 studies this case. Notice that this corollary only requires that the intergenerational Pareto weights decrease geometrically over time[8].

Corollary 4 *Suppose that the social welfare weights satisfy A2.ii for every t. The optimal allocation* $\{c_t^{i*}, z_t^{i*}, k_t^{i*}, S_t^*\}_{t=0,i=1,2}^{\infty}$ *can be decentralized in a competitive market with carbon taxes defined as follows*

$$1 + \tau_t^i = \left(\frac{\beta^i}{\hat{\beta}^i}\right)^t \frac{1}{1 - \mu_t^*} \tag{59}$$

$$\tau_t^z = \mu_t^*(1 + \tau_t^i) \tag{60}$$

for every period t *and all proceeds from taxation equally rebated lump-sum to consumers.*

This result shows that the carbon tax borne by oil owners should be different from the carbon tax paid by non-oil owners whenever the policies pursue redistributive goals. In particular, suppose for simplicity that all consumers discount the future at the same rate, the corollary implies that the more society cares about non-oil owner's welfare, the higher a tax oil owners should pay.

C.2.1 Proofs

Proof of Corollary 3. The proof consists of showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^{i*}, k_t^{i*}, z_t^{i*}, S_t^*\}_{t=0,i=\{1,2\}}^{\infty}$ when taxes and subsidies are optimal. The intertemporal equilibrium condition for type 1 consumers who face a carbon tax and receive a oil stock subsidy is

$$\frac{\beta^1 U_c^1(c_{1t+1}, z_{1t+1}, S_{t+2})}{U_c^1(c_{1t}, z_{1t}, S_{t+1})} = \frac{1}{1 + s_{t+1}} \frac{1 + \tau_{t+1}^c}{1 + \tau_t^c}$$

Plug the optimal carbon tax rate to obtain

$$\frac{\beta^1 U_c^1(c_{1t+1}, z_{1t+1}, S_{t+2})}{U_c^1(c_{1t}, z_{1t}, S_{t+1})} = \frac{\beta^1}{\hat{\beta}^1} \frac{1 - \mu_t^*}{1 - \mu_{t+1}^*}$$

The private discount rate cancels out from both side of the equality. Therefore, this equilibrium condition coincides with the optimal Euler equation for type 1 consumers. The intertemporal equilibrium condition for type 2 consumers is

$$\frac{\beta^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{1}{R_{t+1}} \frac{1 + \tau_{t+1}^c}{1 + \tau_t^c}$$

[8] For ease of notation, I drop the superscript "c" from the carbon tax rate.

By arbitrage, $R_t = s_t$ for all t. Therefore, after plugging the optimal carbon tax rate, we obtain

$$\frac{\beta^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{\beta^1}{\hat{\beta}^1} \frac{1-\mu_t^*}{1-\mu_{t+1}^*}$$

which equals

$$\frac{\hat{\beta}^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{1-\mu_t^*}{1-\mu_{t+1}^*}$$

given the assumption on Pareto weights. Furthermore, this equilibrium condition coincides with the optimal Euler equation for type 2 consumers. The intratemporal equilibrium condition for carbon sequestration is given by

$$\frac{U_z^2(c_{2t}, z_{2t}, S_{t+1})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{\tau_t^z}{1+\tau_t^c}$$

Plug the optimal tax rates to obtain

$$\frac{U_z^2(c_{2t}, z_{2t}, S_{t+1})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \mu_t^*$$

which is satisfied by the optimal allocation. Given the sequence of taxes $\{\tau_t^c, \tau_t^z\}_{t=0}^{\infty}$, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government is satisfied. By assumption $T_{it} = T_t/2$ for all i. This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied.

Proof of Corollary 4. The proof consists of showing that all conditions for an equilibrium are satisfied by the optimal allocation $\{c_t^{i*}, k_t^{i*}, z_t^{i*}, S_t^*\}_{t=0,i=\{1,2\}}^{\infty}$ when taxes and subsidies are optimal. Because only type 2 consumers own the sequestration technology, the intratemporal equilibrium condition for carbon sequestration is given by

$$\frac{U_z^2(c_{2t}, z_{2t}, S_{t+1})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{\tau_t^z}{1+\tau_t^2}$$

Plug the optimal tax rates to obtain

$$\frac{U_z^2(c_{2t}, z_{2t}, S_{t+1})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \mu_t^*$$

which is satisfied by the optimal allocation. The intertemporal equilibrium condition for type 2 consumers is

$$\frac{\beta^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{1}{R_{t+1}} \frac{1+\tau_{t+1}^2}{1+\tau_t^2}$$

By arbitrage, $R_{t+1} = 1$. Plug the optimal carbon tax rate to obtain

$$\frac{\beta^2 U_c^2(c_{2t+1}, z_{2t+1}, S_{t+2})}{U_c^2(c_{2t}, z_{2t}, S_{t+1})} = \frac{\beta^2}{\hat{\beta}^2} \frac{1-\mu_t^*}{1-\mu_{t+1}^*}$$

The private discount rate cancels out from both side of the equality. Therefore, this optimality condition is satisfied by the optimal allocation. The intertemporal equilibrium condition for type 1 consumers is

$$\frac{\beta^1 U_c^1(c_{1t+1}, z_{1t+1}, S_{t+2})}{U_c^1(c_{1t}, z_{1t}, S_{t+1})} = \frac{1+\tau_{t+1}^1}{1+\tau_t^1}$$

Plug the optimal carbon tax rate to obtain

$$\frac{\beta^1 U_c^1(c_{1t+1}, z_{1t+1}, S_{t+2})}{U_c^1(c_{1t}, z_{1t}, S_{t+1})} = \frac{\beta^1}{\hat{\beta}^1} \frac{1-\mu_t^*}{1-\mu_{t+1}^*}$$

The private discount rate cancels out from both side of the equality. Therefore, this optimality condition is satisfied by the optimal allocation. Given the sequence of taxes and subsidies, transfers $\{T_t\}_{t=0}^{\infty}$ are defined so that the budget constraint of the government is satisfied. By assumption $T_{it} = T_t/2$ for all i. This completes the proof that all conditions for a competitive equilibrium with taxes are satisfied.

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