Extrapolative Expectations and the Second-Hand Market for Ships

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Abstract. This article investigates the joint behaviour of vessel prices, net earnings, and second-hand activity in the dry bulk shipping industry. We develop and estimate empirically a behavioural asset pricing model with microeconomic foundations that can account for some distinct characteristics of the market. Namely, among other features, our partial equilibrium model reproduces the actual volatility of vessel prices, the average trading activity in the market, and the positive correlation between net earnings and second-hand transactions. In order to explain the formation of vessel prices, we depart from the rational expectations benchmark of the model, incorporating extrapolative expectations on the part of investors. In contrast to the majority of financial markets’ behavioural models, however, in our environment agents extrapolate fundamentals, not past returns. This form of extrapolation is consistent with the nature of the industry. Accordingly, we introduce two types of investors who hold heterogeneous beliefs about the cash flow process. Formal estimation of the model indicates that a heterogeneous beliefs environment, where both agent types extrapolate fundamentals, while simultaneously under(over)estimate their competitors’ future demand responses, can explain the positive relation between net earnings, prices and second-hand vessel transactions. To the best of our knowledge, the second-hand market for vessels had never been examined from the perspective of a structural, behavioural economic model in the shipping literature before.

Keywords: Asset Pricing, Vessel Valuation, Biased Beliefs, Cash Flow Extrapolation, Heterogeneous-agent, Trading Activity

I. Introduction

As it is well-established in the equity markets literature, the majority of rational expectations models fail to explain numerous empirical regulations related to asset prices. Among others, a prominent example is the “excess volatility puzzle” (Leroy and Porter, 1981), according to which actual asset prices exhibit significantly higher volatility compared to the rational model-implied ones. Additional stylised facts are the positive correlation between trading volume and asset prices (Barberis et al, 2015b) and the strong positive relation between the aggregate dividend yield and future returns in the post-WWII U.S. equity markets (Campbell and Shiller, 1988a; Fama and French, 1988b; Cochrane, 2011). For the purpose of explaining these findings, researchers in the last decade have developed heterogeneous beliefs economic models that incorporate behavioural biases, mainly termed as heuristics (Barberis et al, 2015a).
Heterogeneous beliefs models, however, have not been widely applied for the modelling of other asset classes, and in particular vessels. Shipping is a very important sector of the world economy, since 90% of the world trade is transported by sea and shipping is justifiably considered as a leading indicator of world economic activity (Killian, 2009). The fact that vessels are assets with finite lives whose value depreciates over time provides different challenges in the econometric modelling of the market, compared to the case of an infinitely lived financial asset. Hence, from both a theoretical and practical perspective, it is important to understand the pricing and trading dynamics of this asset class.

In this article, we develop a heterogeneous beliefs model that can explain numerous empirical findings related to the sale and purchase market for second-hand vessels. The empirical estimation focuses on the dry bulk segment of the shipping industry, since it constitutes by far the largest sector in terms of both cargo carrying capacity and quantity transported (Alizadeh and Nomikos, 2010). Furthermore, investigating the dry bulk shipping market, as opposed to the tanker and container ones, provides us with the opportunity to employ a significantly larger dataset. In the context of this article, we concentrate on the Handysize sector, however, our model's predictions have been tested and accordingly, can be extended to the entire dry bulk shipping industry.

The proposed partial equilibrium framework explains the observed price behaviour of second-hand vessels and in particular their “excess volatility”. Most importantly, our model provides a plausible economic interpretation for specific features corresponding to the trading activity of vessels. We reproduce and justify the stylised fact that trading activity is positively related to both market conditions and absolute changes in net earnings between two consecutive periods. In our sample, the two correlation coefficients are equal to 0.53 and 0.65, respectively. In other words, investors appear to trade more aggressively during prosperous market conditions, but also when net earnings have significantly changed compared to the previous period. Interestingly, formal estimation of the model shows that the positive correlation between net earnings and trading activity is accompanied by low average volume of transactions in the market. In addition, therefore, our model implicitly captures the fact that second-hand markets are rather illiquid; during the period 1995-2014, the average annual trading activity was roughly 5.8% of the corresponding fleet size. Finally, the proposed framework accounts also for the empirical findings that high net earnings-price ratios negatively forecast future net earnings growth, and that the bulk of the ratio’s volatility is attributed to expected cash flow variation, not time-varying expected returns (Nomikos and Moutzouris, 2015).

Our discrete time environment consists of two agent types, “conservatives” and “extrapolators”, the relative population fractions of which remain constant over time (Barberis et al, 2015a). In the model, annual shipping net earnings are the sole state variable, observed at each period by the entire investor population. Accordingly, similar to Barberis et al (2005b), when valuing the asset at each
period, agents maximise recursively a constant absolute risk aversion (CARA) utility function, which is defined over next period’s wealth. In addition, for the purpose of being consistent with the nature of the industry, both agents face short-sale constraints. While both types value vessels based on the evolution of fundamentals, they are characterised by bounded rationality which stems from two facts.

First, market participants form extrapolative expectations regarding the cash flow process; the conservative at a lesser degree compared to the extrapolator. From a psychological perspective, the extrapolation of fundamentals can be the result of several heuristic-driven biases. The most frequently incorporated bias is the one known as “representativeness heuristic”, according to which, individuals believe that small samples are representative of the entire population (Tversky and Kahneman, 1974). Similarly, investors may suffer from the “availability heuristic” which causes subjects to overweigh readily available information (Shefrin, 2000). Alternatively, market agents may fall into the “this time is different belief” (Reinhart and Rogoff, 2009). In our model, there is no need to specify the exact form of psychological bias, since the presence of either of these heuristics will lead shipping agents to extrapolate current market conditions (Fuster et al, 2011).

Second, each agent’s investment strategy is independent of the other’s. In particular, both agents assume that, in all future periods, the other type will maintain his per-capita fraction of the risky asset supply (Barberis et al, 2015b). From a psychological point of view, this misbelief can be driven by a bias known as “competition neglect” (Camerer and Lovallo, 1999, Kahneman, 2011). Following Glaeser (2013) and Greenwood and Hanson (2015), this bias may be the result of bounded rationality which leads agents to form forecasts about competitors’ reactions incorporating a simplified economic framework instead of a more elaborate model of the market. In our case, each agent under(over)estimates the future demand responses of the other type.

Finally, at each period, the equilibrium price of the vessel is defined through a market clearing condition. Having expressed the equilibrium vessel price and second-hand activity as functions of the state variable, we can then estimate the parameters of interest that allow us to capture the previously analysed stylised facts. In particular, we are interested in the population fraction and the perceived net earnings persistence corresponding to each agent type.

A first-order effect of the proposed framework is that in the presence of extrapolative expectations in the market, vessel prices become more sensitive to the prevailing cash flow. As a result, the extrapolative model-generated price deviates from the asset’s fundamental value whenever the corresponding cash flow variable deviates from its steady state. This fact implies an immediate over- or under-valuation of the vessel, which in turn generates excess price volatility. In line with the literature, we define as fundamental asset value the one generated by the rational benchmark of the model, or equivalently, the price of the asset in a counterfactual economy where all agents form
rational expectations about the net earning process. However, one should not confuse the notion of over-or under-valuation in our context with the use of the term in the empirical asset pricing literature (Cochrane, 2011). In the latter, the term overvaluation means that the asset’s price is high compared to the corresponding cash flow variable. Interestingly enough, in shipping, when the asset is overvalued compared to its fundamental price, it is in general undervalued compared to the corresponding cash flow variable (Papapostolou et al, 2014, and Nomikos and Moutzouris, 2015). This stylised fact is in line with our model’s predictions.

While a homogeneous-agent setting with extrapolative expectations could capture the observed price behaviour, it would not be sufficient to justify the second-hand market transactions. Hence, trading activity in our framework is the consequence of heterogeneous valuations of the asset by market participants. While there can be alternative explanations for trading activity (e.g. limits to arbitrage, portfolio diversification policies, other information frictions) or “excess price volatility”, the motivation provided has the advantage of simultaneously explaining in a sufficient manner numerous empirical regularities. Furthermore, the economic interpretation of the model and the respective results are plausible and in line with the nature of the shipping industry. To the best of our knowledge, this is the first time in the shipping literature that a structural model incorporates the coexistence of heterogeneous beliefs agents for the purpose of explaining the joint behaviour of observed vessel prices and second-hand vessel transactions.

Regarding the existing shipping literature, Beenstock (1985) and Beenstock and Vergottis (1989) construct and estimate a rational expectations general equilibrium model relating vessel prices to cash flows, new building and scrapping activity, and demand for shipping services. The homogeneous-agent setting, however, does not allow for the explanation of the second-hand market activity. Furthermore, as the authors argue, their model is a simplified version of reality since it does not capture the possibility of extrapolative expectations on the part of investors. Greenwood and Hanson (2015) incorporate this behavioural bias by developing an elaborate microeconomic model that reproduces several features of the industry. In their homogeneous-agent framework, agents extrapolate current demand conditions while simultaneously neglect their competitors’ supply responses. The combined effect of those behavioural biases is overinvestment and overvaluation of the asset during prosperous market conditions and vice versa. The behavioural mechanism proposed here is similar to that of Greenwood and Hanson (2015) but also allows for the degree of extrapolation to depend on the agent type. Consequently, the heterogeneous nature of our model allow us to simultaneously capture the observed volatility of prices and the relation between net earnings and second-hand activity in the market.
Kalouptsidi (2014) examines the implications of demand uncertainty and construction lags on ship prices, new building and scrapping activity, and market participants’ surplus. Using a rational expectations model, she shows that the required time to build and the level and volatility of vessel prices are positively related. Similar to the previous articles, however, the assumption of a unique agent type excludes the modelling of the sale and purchase market. Finally, while Nomikos and Moutzouris (2015) address the question regarding the volatility of valuation ratios as a whole, they do not examine either the underlying mechanism behind the volatility of asset prices per se or the relation between prices, earnings and second-hand activity.

Our paper looks at the main features of heterogeneous-agent models but also introduces important modifications which are required in order to capture stylized features of the shipping markets. In particular, recent articles, mainly in equity but also in commodity markets (Ellen and Zwinkels, 2010), have attempted to explain empirical asset pricing findings using heterogeneous beliefs models in which a fraction of the population forms biased expectations about future returns. Barberis et al (2015a) develop an extrapolative capital asset pricing model (X-CAPM) that explains the volatility of the aggregate stock market. Furthermore, Barberis et al (2015b) incorporate a heterogeneous-belief extrapolative model of returns in order to analyse the formation of asset bubbles in equity markets. Some key features of their environment and model’s solution are very closely related to the one presented in this article. However, in contrast to Barberis et al, in our model there is cash flow and not return extrapolation, the corresponding asset is affected by economic depreciation due to wear and tear, and most importantly, all agents are extrapolators, however, at a different degree. The latter feature allows us to capture also severe undervaluation phenomena and moreover to explain the positive correlation between market conditions and trading activity. As a by-product, our model also reproduces the relatively low liquidity of the shipping markets.

The remainder of this article is organised as follows. Section II introduces the environment of our economy and the solution of the theoretical model. Section III presents the dataset employed along with the empirical estimation of the model and a formal analysis of the produced results. In addition, it provides an economic interpretation of the results. Section IV examines several alternative hypotheses regarding the investor population composition. Section V concludes.

II. Environment and Model Solution

Consider a discrete-time environment where the passage of time is denoted by $t$. The economy consists of two asset classes: the first one is risk-free while the second one is risky. The risk-free asset can be thought of as an infinitely lived financial instrument in perfectly elastic supply, earning an exogenously determined constant rate of return equal to $R_f$. The risky asset class consists of otherwise
identical vessels which are further categorised according to their age. Importantly, all age classes have fixed per capita supply over time, equal to \(Q\). In what follows, we restrict our attention to the modelling of the market for 5-year old vessels. As we illustrate in the following, in order to analyse the trading activity corresponding to this age class, we need to examine also the demand and price mechanism related to the 6-year old vessel. Notice that exactly the same principles apply for the valuation of the other age classes. Following market practice, we assume that a newly-built vessel has an average economy life of 25 years. At the end of this period, the asset is scrapped; hence, it exits from the economy. Accordingly, setting the time-step of the model, \(\Delta t\), equal to one year, implies that a 5-year old asset has \(T = 20\) periods of remaining economic activity. We focus on 5-year old second-hand vessels, instead of new buildings as one of the main objectives of our framework is the modelling of trading activity in the sale-and-purchase market for ships.

An inherent characteristic of the shipping industry is that forthcoming period’s net earnings are \(\mathcal{F}_t\)-measurable (Nomikos and Moutzouris, 2015). Assuming no default on the part of the charterer, the ship owner at time \(t\) knows precisely his net earnings for the period \(t \rightarrow t + 1\), defined as \(\Pi_t\). In equity markets terminology the asset is said to be trading “cum dividend”. Therefore, the owner of the 5-year old vessel at time \(t\) is entitled to an exogenously determined stream of annual net earnings, \(\{\Pi_n\}_{t}^{t+T}\). In the context of our theoretical model, net earnings are the sole state variable, the evolution of which is assumed to be following a mean-reverting process

\[
\Pi_{t+1} = (1 - \rho_0)\bar{\Pi} + \rho_0\Pi_t + \epsilon_{t+1},
\]

where \(\bar{\Pi}\) is the long-term mean, \(\rho_0 \in [0,1]\), and \(\epsilon_{t+1} \sim N(0, \sigma^2)\), \(i.i.d.\) over time. Notably, though, in contrast to \(\bar{\Pi}\), parameters \(\rho_0\) and \(\sigma^2\) are not public information.

The economy consists of two investor types, \(i\): “conservatives” and “extrapolators”, denoted by \(c\) and \(e\), respectively. We normalize the investor population related to each asset age-class to a unit measure, and we further assume that the fractions of conservatives, \(\mu^c\), and extrapolators, \(\mu^e\), are fixed both across all age classes and through each specific asset’s life. In what follows, we set \(\mu^c = \mu\); hence, \(\mu^e = 1 - \mu\). The assumption that each unit measure of investors is related to one and only

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1 In reality, the agreed time-charter rates are usually received every 15 days (sometimes also in advance). As a result, the probability of default on the part of the charterer is reduced. Moreover, an extensive network of competitive brokers and the fact that ship owners normally lease their vessels to solvent charterers assure transparency and low probability of default. In contrast, a time-charter lease with a less creditworthy charterer will incur higher rates in order to compensate the owner for the higher probability of default on the part of the charterer. Finally, additional contractual agreements included in the charter party, ensure that the owner will receive the full time-charter rate agreed.

2 For illustrational purposes, from the point of view of agent \(i\), the other agent is denoted by \(-i\).

3 This assumption is in line with both the characteristics of the industry, and the existing literature (Kalouptsidi, 2014), where ship owning companies are assumed to be operating on a “one firm-one ship” basis (Stopford, 2009).
asset age-class is equivalent to assuming that each unit mass of investors has a finite life equal to the corresponding life of the vessel. We impose this assumption in order not to overcomplicate mathematically the model; in particular, it permits us to treat each age-class in isolation. Accordingly, we can derive analytical theoretical predictions and closed-form results for many quantities of interest.

The difference between the two agent types lies in the alternative ways in which they form expectations about future cash flows. Specifically, compared to extrapolators, conservatives’ perception is closer (in principle, it might be even identical) to equation 1. Therefore, since conservatives might also form extrapolative expectations, the terms “conservative” and “extrapolator” are used in a comparative manner. From a psychological perspective, as analysed in the Introduction, both types suffer from a heuristic-driven bias, which in turn leads to the extrapolation of current cash flows. In order to capture mathematically this behavioural bias, we assume that in agent \( i \)’s mind, net earnings related to the valuation of the 5-year old vessel evolve according to

\[
\Pi_{t+1} = (1 - \rho_i)\bar{\Pi} + \rho_i \Pi_{t} + \varepsilon_{t+1}^i,
\]

in which \( 0 \leq \rho_c < \rho_e < 1 \), and \( \varepsilon_{t+1}^i \sim N\left(0, \vartheta_{5}^\varepsilon \sigma_{\varepsilon}^2\right) \), \( i.i.d. \) over time, where \( 0 < \vartheta_{5}^e < \vartheta_{5}^c \). The strictly positive parameter \( \vartheta_{5}^c \) adjusts the (true) variance of the cash flow shock according to agent’s \( i \) perspective, while the subscript denotes the current age-class of the vessel being valued.

The conservative agent parameters, \( \mu, \rho_c, \) and \( \vartheta_{5}^c \) characterise completely the information structure of our model. When \( \mu = 1, \rho_c = \rho_0, \) and \( \vartheta_{5}^c = 1 \), all agents have perfect information about the economy. We define this case as the benchmark “rational” economy of our model, and we term this agent type as fundamentalist, \( f \); hence, \( \rho_f = \rho_0 \) and \( \vartheta_{5}^f = 1 \). When \( \mu = 1, \rho_c \neq \rho_0, \) and \( \vartheta_{5}^c \neq 1 \) or \( \mu = 0 \), all agents have imperfect information about the economy. However, in all cases above, there is no information asymmetry among agents; hence, there is no trading activity in the market. Finally, when \( \mu \in (0,1) \), that is, when heterogeneous investors coexist in the market, information is both imperfect and asymmetric (Wang, 1993). As a result, trading activity is generated in the economy.

The timeline of the model is as follows. At each point \( t \), \( \Pi_t \) is realised and observed by all market participants. Furthermore, the 25-year old age class is scrapped and replaced by newly built vessels (i.e. the 0-year old age class). Accordingly, both agent types determine their time \( t \) demands for each.

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4 Essentially, we have a model of overlapping generations; however, there is no interaction among different generations.
age class asset with the aim of maximizing a constant absolute risk-aversion (CARA) utility function, defined over next period’s wealth. For the 5-year old vessel, this corresponds to

$$\max_{N^i_{5,t}} E^i_t \left[ -e^{-\alpha t w^i_{t+1}} \right],$$

(3)

where $\alpha$ and $N^i_{5,t}$ are investor $i$’s coefficient of absolute risk-aversion and time $t$ per-capita demand for the 5-year old vessel, respectively. Agent $i$’s next period’s wealth, $w^i_{t+1}$, is given by

$$w^i_{t+1} = (w^i_t - N^i_{5,t} P^i_{5,t}) (1 + R^i_t) + N^i_{6,t} (\Pi^i_t + P^i_{6,t+1}),$$

(4)

in which, $P^i_{5,t}$ and $P^i_{6,t+1}$ are the prices of the 5- and 6-year old vessel at $t$ and $t + 1$, correspondingly.\(^5\)

In what follows, we normalise the rate of return of the risk-free asset to zero (Wang, 1993, and Barberis et al, 2015b).\(^6\) Therefore, investor $i$’s objective becomes

$$\max_{N^i_{5,t}} E^i_t \left[ -e^{-\alpha t w^i_{t+1} + N^i_{5,t} (\Pi^i_t + P^i_{6,t+1} - P^i_{5,t})}} \right].$$

(5)

Accordingly, the time $t$ cum dividend price of the 5-year old vessel is endogenously determined, through the market clearing condition

$$\mu N^i_{5,t} + (1 - \mu) N^i_{6,t} = Q,$$

(6)

where $Q$ is the fixed per capita supply of the risky asset. The assumption of fixed per capital supply over the vessel’s life can be justified by the fact that we are interested in the modelling of a real asset with economic depreciation. Therefore, the supply of the age-specific asset cannot increase over time. Furthermore, since scrapping very rarely occurs before the 20th year of a vessel’s life,\(^7\) we assume that the supply of the risky asset cannot be reduced either. The immediate effect of this assumption is that our model does not endogenise the scrapping decision.

Finally, trading activity corresponding to time $t$ takes place in the market. In shipping, this activity refers to the sale and purchase market for second-hand vessels. Notice that, since this is a discrete time model, we impose the assumption that trading occurs instantaneously at each point $t$. In analogy

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\(^5\) In principle, at each $t$, each agent could invest a fraction of his wealth in every age-class of the risky asset. However, in order not to overcomplicate the analysis and obtain closed-form solutions for the demand functions, we assume that, at each $t$, a new unit mass of investors solely interested in 5-year old vessels enters in the industry. Accordingly, at $t + 1$ that investor population will be solely interested in the 6-year old asset class, while a new unit mass related to the 5-year old class will enter in the market (Figure 1).

\(^6\) It is straightforward to incorporate the risk-free parameter in the analysis, and moreover, to adjust for a time-varying risk-free return. We expect, however, the results to be qualitatively the same.

\(^7\) In practice, the supply of the fleet may be reduced due to accidents and losses as well as conversions of vessels to other uses; these in general constitute an insignificant proportion of the fleet and thus are not considered here.
to heterogeneous beliefs models for equity markets (Barberis et al., 2015b), trading activity is estimated through

\[ V_{t-1 \rightarrow t} \equiv V_t = \mu^i |N^i_{6,t} - N^i_{5,t-1}|, \tag{7} \]

where \( N^i_{6,t} \) is agent \( i \)'s time \( t \) per-capita demand for the 6-year old vessel. Intuitively, we define as trading activity the agent-specific change in demand for the risky asset between points \( t - 1 \) and \( t \), multiplied by the respective population fraction. Since supply is fixed, the quantity sold by one agent is equal to the quantity acquired by the other. Notice that in the equity markets literature, where researchers are interested in the behaviour of the same, infinitely lived intangible asset over time, there is no need for the age subscript in (7). Since, however, vessels are real assets with limited economic lives, their values are substantially affected by economic depreciation. In particular, at each point in time, a 6-year old vessel is less valuable than an identical 5-year one. Therefore, we need to estimate the demand functions for both the 5 and 6-year old vessels, at each time \( t \). Figure 1 summarises the timeline of the model.

**Figure 1: Timeline of the Model.**

\[ \text{\不足 \( \Pi_{t-1} \) is realised.} \]

- 5-year population determines \( N^i_{5,t-1} \) and \( P_{5,t-1} \). At \( t - 2 \), this group had determined \( N^i_{5,t-2} \) and \( P_{5,t-2} \).
- 6-year population determines \( N^i_{6,t-1} \) and \( P_{6,t-1} \). At \( t - 2 \), this group had determined \( N^i_{6,t-2} \) and \( P_{6,t-2} \).

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\[ \text{Trading activity for the 6-year old vessel:} \]

\[ V_{t-1} = \mu^i |N^i_{6,t-1} - N^i_{5,t-2}|. \]

\[ \text{Trading activity for the 5-year old vessel:} \]

\[ V_t = \mu^i |N^i_{5,t} - N^i_{5,t-1}|. \]

Appendix A shows that the time \( t \) per-capita demand of agent \( i \) for the 5-year old vessel is

\[ N^i_{5,t} = \frac{1 - \rho^2}{1 - \rho^2} (\Pi_t - \Pi) + 21 \Pi - X^i_{5} \sigma^2 Q - P_{5,t} \]

where
\[
\begin{align*}
Y^i_S &= \left( \frac{1 - \rho_i^{20}}{1 - \rho_i} \right)^2 \alpha^i \vartheta^i \\
X^i_S &= \left[ \frac{20}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{20})(1 + 2 \rho_i - \rho_i^{20})}{(1 + \rho_i)(1 - \rho_i)^3} \right] \alpha^i \vartheta^i \\
N^i_{S,t} &= \max \left\{ \frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21 \bar{\Pi} - X^i_S \sigma^2 \xi Q - P_{S,t} \right\}, \quad (8b)
\end{align*}
\]

Furthermore, in order to be consistent with the nature of the industry, we impose short-sale constraints for each investor type. Hence, following Barberis \textit{et al} (2015b), equation \(8a\) becomes

\[
N^i_{S,t} = \max \left\{ \frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21 \bar{\Pi} - X^i_S \sigma^2 \xi Q - P_{S,t} \right\}, \quad (9)
\]

Equation \(9\), along with the market clearing condition \(6\), determine the equilibrium 5-year old vessel price at each \(t\).

Note that in order to derive the agent-specific demand functions, we have assumed that apart from the extrapolation of fundamentals, both types of agent suffer from an additional form of bounded rationality. Namely, agent \(i\), instead of taking into account the strategy of agent \(-i\), that is, trying to forecast the evolution of \(-i\)’s demand, makes the simplifying assumption that in all future periods \(-i\) will just hold his per-capita fraction of the risky asset supply constant at \(\mu^{-i} Q\). (Barberis \textit{et al}, 2015b). Equivalently, \(i\) assumes that \(-i\)’s future demand is independent of the corresponding future net earnings variable. Therefore, we can argue that each agent \(i\)’s optimisation problem is not a function of agent \(-i\)’s strategy. From a psychological perspective, this misbelief can be driven by a bias known as “competition neglect” (Camerer and Lovallo, 1999, Kahneman, 2011). Following Glaeser (2013) and Greenwood and Hanson (2015), this bias may be the result of bounded rationality in which agents form forecasts about competitors’ reactions by incorporating a simplified economic framework instead of a more elaborate dynamic supply and demand model of the market. In our case, each agent under(over)estimates the future demand responses of the other type. The assumed form of competition neglect implies that each agent type expects the price of the risky asset to revert to its fair value (fair according to their beliefs) within one period. As a result, agent \(i\) trades more aggressively against any mispricing compared to the case they were explicitly forecasting agent’s \(-i\) future demand responses.

From an economic perspective, the quantity \(\frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21 \bar{\Pi} - X^i_S \sigma^2 \xi Q\) in the numerator of \((8a)\) is the expected income for investor \(i\) from holding the vessel for 1 period; namely, from the 5\(^{th}\) to the 6\(^{th}\) year of its economic life. Hence, the numerator corresponds to the expected one-period net income for investor \(i\). Accordingly, the numerator is scaled by the product \(Y^i_S \sigma^2 \xi\), which consists of
investor’s risk aversion and the perceived aggregate effect of the conditional one-period variances of the cash flow shock (i.e. the risk agent \(i\) is bearing for the 1-year investment according to his perception).

Since extrapolators have “more incorrect” beliefs about the net earnings process compared to conservatives, one would expect that their expectations about future returns will be more inaccurate compared to the ones of conservatives. While this belief is in general valid, the assumption of competition neglect complicates further the issue. As we illustrate in Section III, the discrepancy between agent-specific returns expectations and realised returns depends on both the agent’s beliefs about the net earnings process and the relative population fractions. Namely, the fact that each agent neglects the strategy of the other implies that conservatives do not explicitly attempt to exploit the “more incorrect” beliefs of extrapolators. In contrast, as mentioned above, conservatives’ valuation and in turn their investment strategy are based on the misbelief that the price of the vessel will revert to its fair value (fair according to their beliefs) within one period. If this were not the case, then indeed, irrespective of the relative population fractions, conservatives would always have significantly more precise expectations than extrapolators.

Section III illustrates that for a model parameterisation that reproduces sufficiently well the empirical results, conservatives always form substantially more accurate forecasts of future returns compared to extrapolators. As a result, the investment strategy of the former is noticeably less risky than the latter’s, as measured by the one period change in wealth. Consequently, it might be the case that in the long horizon, extrapolators’ wealth has become severely reduced compared to that of conservatives, and in turn, they are not able to support an investment in the risky asset.

Notice, however, that due to the exponential utility assumption, the demand function is independent of the respective wealth level. This property of the exponential utility function allows us to abstract from the “survival on prices” effect (Barberis et al, 2015a). Hence, it permits us to focus solely on the pricing and trading implications of the heterogeneous-agent economy. Nevertheless, from an economic perspective, even if extrapolators are not able to invest due to limited wealth, it is not unrealistic to assume that they will be immediately replaced by a new fraction of extrapolator investors with exactly the same characteristics (Barberis et al, 2015a). In shipping, this cohort could correspond to diversified investors with substantial cash availability (e.g. private equity firms), but little or no prior experience of the industry.

Equations 6 and 8a suggest that if the market consisted of only one agent type, the equilibrium price of the 5-year old vessel would be
\begin{equation}
\hat{p}_{5,t}^i = \frac{1-\rho_i^{21}}{1-\rho_i} (\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - \left[X_5^i + Y_5^i\right]\sigma_\varepsilon^2 Q. \tag{10}
\end{equation}

Incorporating the unconditional volatility operator on both sides of equation 10, we obtain

\begin{equation}
\sigma(\hat{p}_{5,t}^i) = \frac{1-\rho_i^{21}}{1-\rho_i} \sigma(\Pi_t). \tag{11}
\end{equation}

We observe that in this simplified case, vessel price volatility depends on the volatility of the net earnings variable and on the value of \( \rho_i \). Since \( \frac{1-\rho_i^{21}}{1-\rho_i} \) is a strictly increasing function of \( \rho_i \) in the interval \([\rho_0, 1)\), the higher the perceived persistence of net earnings, that is, the higher the degree of extrapolation, the higher the volatility of generated prices. Furthermore, as we discuss in the empirical estimation of the model, the unconditional mean of the net earnings variable is set equal to its long-term mean; that is, \( \mathbb{E}[\Pi_t] = \bar{\Pi} \). Hence, taking unconditional expectations on both sides of equation 10 yields

\begin{equation}
\mathbb{E}[\hat{p}_{5,t}^i] = 21\bar{\Pi} - \left[X_5^i + Y_5^i\right]\sigma_\varepsilon^2 Q. \tag{12}
\end{equation}

In what follows, it will be useful from both a theoretical and empirical perspective to examine the benchmark rational economy of our model. Recall that in this scenario, denoted by \( f \), the market consists solely of agents who know precisely the actual stochastic process that governs the evolution of net earnings. Accordingly, the equilibrium price and the unconditional volatility, and mean of 5-year old vessel prices are obtained from equations 10, 11, and 12, respectively, for \( i = f \). Finally, as equations 8a and 8b indicate, fundamentalists’ perception of the risk they are bearing is given by the product \( \left(\frac{1-\rho_0^{20\sigma}}{1-\rho_0}\right)\sigma_\varepsilon^2 \). In this benchmark case, this perception is correct. In the presence of extrapolators, though, it is just an approximation since future asset prices will also depend on extrapolators’ future demand responses and not just on the riskiness of cash flows.

In a similar manner, if the market consists solely of one extrapolator type, denoted by \( e \); that is, if all market participants form expectations about net earnings based on (2), the equilibrium price and the unconditional volatility, and mean of vessel prices are obtained from (10), (11), and (12), respectively, after substituting \( e \) for \( i \). The most interesting scenario, however, is the one where heterogeneous-belief agents coexist in the market. In the context of our model, second-hand activity is the result of heterogeneous estimation of the asset’s worth. Hence, while a homogeneous extrapolative environment can account for the actual volatility of vessel prices, it cannot explain the
observed second-hand transactions. As mentioned above, we define the two agent types as conservatives, \( c \), and extrapolators, \( e \). Notice that a special case of the heterogeneous environment is when conservatives form expectations based on equation 1, that is, when they are fundamentalists.

**Proposition: Equilibrium price for 5-year old vessels.** In the environment presented above, a market-clearing price for the 5-year old vessel, \( P_{5,t}^{c+e} \), always exists. The equilibrium price of the vessel depends on the prevailing market conditions. We denote the net earnings thresholds at which extrapolators and conservatives related to the 5-year old vessel class exit the market by \( \Pi_5^e \) and \( \Pi_5^c \), respectively.

First, when

\[
\Pi_5^c = \bar{\Pi} + \frac{(X_5^e - X_5^c - Y_5^c)\sigma^2 Q}{1 - \rho_e^{21} - \frac{1 - \rho_e^{21}}{1 - \rho_c}} < \Pi_t < \frac{(X_5^e - X_5^c + Y_5^e - Y_5^c)\sigma^2 Q}{1 - \rho_e^{21} - \frac{1 - \rho_e^{21}}{1 - \rho_c}} = \Pi_5^e,
\]

both agents are present in the market, and the market clearing price, denoted by \( P_{5,t}^{c+e} \), is equal to

\[
P_{5,t}^{c+e} = 21\bar{\Pi} + \frac{\mu Y_5^e}{\mu Y_5^e + (1 - \mu)Y_5^c} \left[ \frac{1 - \rho_e^{21}}{1 - \rho_c} \right] (\Pi_t - \bar{\Pi}) - \frac{\mu Y_5^e X_5^c + (1 - \mu)Y_5^e X_5^e}{\mu Y_5^e + (1 - \mu)Y_5^c} \sigma^2 Q.
\]

Second, in the case where

\[
\Pi_t \leq \bar{\Pi} + \frac{(X_5^e - X_5^c - Y_5^c)\sigma^2 Q}{1 - \rho_e^{21} - \frac{1 - \rho_e^{21}}{1 - \rho_c}} = \Pi_5^e,
\]

extrapolators exit the market, and the clearing price, \( P_{5,t}^{c} \), is given by

\[
P_{5,t}^{c} = 21\bar{\Pi} + \frac{1 - \rho_c^{21}}{1 - \rho_c} (\Pi_t - \bar{\Pi}) - \frac{X_5^c + \frac{Y_5^c}{\mu}}{\sigma^2 Q}.
\]

Third, in the scenario where

---

8 We exclude limits to arbitrage, individual liquidity concerns (Kalouptsidi, 2014), and investors’ diversification policies from being potential causes of second-hand transactions.

9 This Proposition is similar to Proposition 1 in Barberis et al (2015b).
\[
\Pi^e_5 = \overline{\Pi} + \frac{(X^e_5 - X^c_5 + Y^e_5 \sigma^2 \epsilon)Q}{1 - \rho_{21}^{e} - \frac{1 - \rho_{c}^{21}}{1 - \rho_{e}^{21}}} \leq \Pi_t, \quad (15a)
\]

conservatives exit the market, and the equilibrium price, \(P^*_x^e\), is given by

\[
P^*_x^e = 21\overline{\Pi} + \frac{1 - \rho_{m}^{21}}{1 - \rho_{m}^{21}} (\Pi_t - \overline{\Pi}) - \left[ \frac{X^e_5 + Y^e_5}{1 - \mu} \right] \sigma^2 \epsilon Q. \quad (15b)
\]

The intuition behind the equilibrium prices described through equations 13b, 14b and 15b is the following one. As the first term of each equation indicates, the price of the vessel heavily depends on the long-term mean of the cash flow variable, multiplied by the total number of payments to be received until the end of the asset’s economic life. The second term corresponds to the effect of the product of the perceived persistence of the net earnings variable times its current deviation from the long-term mean. Essentially, this term is responsible for the main bulk of over(under)valuation in the price of the risky asset. Evidently, as we move from equation 14b to 13b to 15b, the degree of the perceived aggregate extrapolation increases, which in turn implies an increase in the coefficient of the deviation term. Furthermore, in analogy to the standard present value formula with constant required returns (Shiller, 1981), the last term of the equilibrium price equations corresponds to the aggregate discounting (i.e. the perceived market risk in equilibrium), by which future cash flows are reduced in order for investors to be compensated for the risk they bear (Wang, 1993).

In a similar manner, we derive the demand functions for 6-year old vessels. In addition, we assume that agents become more “suspicious” or equivalently, more risk averse, as the specific asset’s age grows. This “suspicion” stems from the fact that they realise that the evolution of net earnings does not evolve precisely in the way they expected in the previous period. As a result, agents indirectly respond by increasing the perceived risk associated with this particular investment. In order not to overcomplicate things, we model the update in agents’ beliefs in a simple and straightforward manner. Namely, we assume that agent \(i\), at time \(t\), increases the value of the perceived cash flow shock variance corresponding to the valuation of the 6-year old vessel, \(\sigma^2 \epsilon^2\), compared to the one incorporated for the valuation of the 5-year old asset, at \(t - 1\). Hence, for a given \(t\), investors related to different vessel-age classes have different beliefs about the variance of the error term. Of course, in the special case where conservatives are fundamentalists, the agent knows the precise stochastic process; hence, no variance update occurs between periods \(t - 1\) and \(t\). Alternatively, we could have assumed that agent \(i\) becomes more risk averse, which would imply an increase of the CARA from period \(t\) to \(t + 1\). Both methods, however, yield exactly the same results. Finally, notice that this
assumption is not important either for the theoretical or the empirical predictions of our model. We impose it, however, in order for the steady state equilibrium of our economy to be well-defined from a mathematical perspective. Even if we do not impose this assumption, the steady state equilibrium restrictions will hold approximately and our results will be essentially the same.

Therefore, according to agent $i$, net earnings related to the valuation of the 6-year old vessel evolve through

$$
\Pi_{t+1} = (1 - \rho_i)\bar{\Pi} + \rho_i \Pi_t + \varepsilon_{t+1}^i,
$$

in which $\rho_0 \leq \rho_c < \rho_e < 1$, and $\varepsilon_{t+1}^i \sim N(0, \vartheta_6^i \sigma^2_e)$, i.i.d. over time, where $0 < \vartheta_5^e < \vartheta_6^e$. Despite their increased “suspicion”, however, agents remain irrational, since they still do not form unbiased forecasts of either the cash flow process or their competitors’ demand responses. Finally, following precisely the same procedure as for the 5-year old asset, agent $i$’s time $t$ demand for the 6-year old vessel given by

$$
N^i_{6,t} = \max\left\{\frac{1 - \rho_i^{20}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 20\bar{\Pi} - X^i_6 \sigma^2_e Q - P^t_6, 0\right\},
$$

where

$$
\begin{align*}
X^i_6 &= \left[\frac{19}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{19})(1 + 2\rho_i - \rho_i^{19})}{(1 + \rho_i)(1 - \rho_i)^3}\right],
Y^i_6 &= \left(\frac{1 - \rho_i^{19}}{1 - \rho_i}\right)^2 \alpha^i \vartheta_6^i.
\end{align*}
$$

Since agents adjust upwardly the perceived riskiness of the investment, $\vartheta_6^i$ is higher compared to $\vartheta_5^i$. For our parameter values, this in turn implies that $Y^i_6 \sigma^2_e > Y^i_5 \sigma^2_e$. Therefore, as mentioned above, the expected one-period net income related to the 6-year old investment is scaled by a higher quantity, compared to the respective 5-year old one.

**Corollary 1:** Market clearing price for 6-year old vessels. Extending the arguments illustrated in the Proposition, it is straightforward to show that also in the case of the 6-year old vessel, a market-clearing price, $P^*_6$, always exists. First, in the case where both agents are present in the market, that is, when

$$
\Pi^e_6 = \bar{\Pi} + \frac{(X^e_6 - X^c_6 - \frac{Y^e_6}{\mu^e}) \sigma^2_e Q}{1 - \rho_e^{20}} < \Pi_t < \bar{\Pi} + \frac{(X^e_6 - X^c_6 + \frac{Y^e_6}{1 - \mu^e}) \sigma^2_e Q}{1 - \rho_c^{20}} = \Pi^c_6,
$$

**(17a)**
the price is given by
\begin{align*}
P^{e,c+e}_{6,t} &= 20\bar{\Pi} + \frac{\mu Y^e_6 Y^c_6 e + (1 - \mu)Y^c_6 Y^e_6 e}{\mu Y^e_6 + (1 - \mu)Y^c_6} \sigma^2 Q. \tag{17b}
\end{align*}

Second, when only conservatives hold the vessel, that is, when
\begin{align*}
\Pi_t &\leq \bar{\Pi} + \frac{(X^e_6 - X^c_6 - Y^e_6 c)}{1 - \rho^2 e - 1 - \rho^2 c} = \Pi^e_6, \tag{18a}
\end{align*}
the price is given by
\begin{align*}
P^{e,c}_{6,t} &= 20\bar{\Pi} + \frac{1 - \rho^2 e}{1 - \rho c} (\Pi_t - \bar{\Pi}) - \left[ X^e_6 + \frac{Y^e_6 c}{\mu} \right] \sigma^2 Q. \tag{18b}
\end{align*}

Third, in the scenario where only extrapolators hold the risky asset; namely, when
\begin{align*}
\Pi^e_6 &= \bar{\Pi} + \frac{(X^e_6 - X^c_6 + Y^e_6 c)}{1 - \rho^2 e - 1 - \rho^2 c} \leq \Pi_t, \tag{19a}
\end{align*}
the price equals
\begin{align*}
P^{e,c}_{6,t} &= 20\bar{\Pi} + \frac{1 - \rho^2 e}{1 - \rho c} (\Pi_t - \bar{\Pi}) - \left[ X^e_6 + \frac{Y^e_6 c}{1 - \mu} \right] \sigma^2 Q. \tag{19b}
\end{align*}

\begin{align*}
\end{align*}

Notice that, as mentioned above, due to the exponential utility assumption, the demand functions and the equilibrium asset prices, for each age-class, are unaffected by the relative levels of wealth.

**Corollary 2: Steady state equilibrium.** We define the “steady state” of our economy as the one in which the net earnings variable is equal to its long-term mean, \( \bar{\Pi} \). As equation 1 indicates, the economy reaches this state after a sequence of zero cash flow shocks. In the steady state the price of the risky asset, for each age-class, is equal to its respective fundamental value. Furthermore, under a
specific condition analysed below, both fundamental agents and extrapolators are present in the market, and each type holds the risky asset in analogy to his fraction of the total population. Accordingly, the “steady state” equilibrium price of the 5-year old vessel, $P_5^*$, is given by

$$P_5^* = 21\bar{\Pi} - [X_5^i + Y_5^i]\sigma_x^2 Q,$$  \hspace{1cm} (20a)

under the restriction

$$X_5^c + Y_5^c = X_5^e + Y_5^e = X_5^f + Y_5^f.$$  \hspace{1cm} (20b)

In a similar manner, the “steady state” equilibrium price of the 6-year old vessel is

$$P_6^* = 20\bar{\Pi} - [X_6^i + Y_6^i]\sigma_x^2 Q,$$  \hspace{1cm} (21a)

under the restriction

$$X_6^c + Y_6^c = X_6^e + Y_6^e = X_6^f + Y_6^f.$$  \hspace{1cm} (21b)

Therefore, if in two consecutives dates the net earnings variable is equal to its long-term mean, the change in the price of the asset is given by

$$P_6^* - P_5^* = -\bar{\Pi} - [X_6^i + Y_6^i - (X_5^i + Y_5^i)]\sigma_x^2 Q.$$  \hspace{1cm} (22)

Notice that the right hand side of (22) is negative and corresponds to the one-year economic depreciation in the value of the vessel. Finally, in this scenario, there is no activity in the second-hand market, since the change in share demand of each agent is equal to zero.

The equilibrium conditions above imply that our model parameters are nested. This interrelationship can be illustrated in a simple manner through the following system of equations

$$\alpha^i = \frac{21\bar{\Pi} - P_5^*}{\frac{20 + (1 - \rho_i^{20})^2}{(1 - \rho_i)^2} - \frac{(1 - \rho_i^{20})(1 + 2\rho_i - \rho_i^{20})}{(1 + \rho_i)(1 - \rho_i)^3}}\frac{\rho_i}{\sigma_x^2 Q},$$  \hspace{1cm} (23a)

and

\textsuperscript{10} The consequence of the update mechanism analysed above is that restriction 21b holds with exact equality. Otherwise, the restriction holds as an approximate equality.
\[ \alpha' = \frac{20\bar{\Pi} - \bar{P}_6^*}{(1 - \rho_i)^2} \left[ \frac{19 + (1 - \rho_i^{19})^2}{(1 - \rho_i^1)^2} - \frac{(1 - \rho_i^{19})(1 + 2\rho_i - \rho_i^{19})}{(1 + \rho_i)(1 - \rho_i)^3} \right] q_i^f \sigma_z^2 Q. \]  

(23b)

The implications of this fact are analyzed in the empirical estimation of the model.

**Corollary 3: Deviation from the fundamental value.** Whenever the value of the net earnings variable deviates from its long-term mean, the model-generated price of the 5-year old vessel deviates from the fundamental value as well. In the following, we denote the degree of deviation by \( D_t \); namely, a positive (negative) value of \( D_t \) corresponds to over (under) valuation of the asset. In order to derive the equations that quantify the degree of deviation from the fundamental value, we have to distinguish between three cases.

First, in the case where both agents are present in the market, we can estimate the deviation, denoted by \( D_t^{c+e} \), by subtracting the fundamental price from the one indicated by equation 13b. Accordingly, we obtain

\[ D_t^{c+e} = \frac{(1 - \mu) \left( \frac{1 - \rho_e^{21}}{1 - \rho_e} - \frac{1 - \rho_c^{21}}{1 - \rho_c} \right) Y^c}{\mu Y^e_5 + (1 - \mu) Y^c_5} (\Pi_t - \bar{\Pi}). \]  

(24)

Since the fraction is always positive, the sign of the price deviation solely depends on the sign of the net earnings deviation. Therefore, during prosperous (deteriorating) market conditions, the risky asset is over (under) priced.

Second, when only conservatives exist in the market, the deviation, \( D_t^c \), is estimated by subtracting \( P_t^f \) from equation 14b. This yields

\[ D_t^c = \left( \frac{1 - \rho_c^{21}}{1 - \rho_c} - \frac{1 - \rho_f^{21}}{1 - \rho_f} \right) (\Pi_t - \bar{\Pi}) - \left[ X^e_5 + \frac{Y^e_5}{\mu} - X^f_5 - Y^f_5 \right] \sigma_z^2 Q, \]  

(25)

which is always negative, since \( \Pi_t \leq \Pi^e_5 < \bar{\Pi} \) and by condition 20b the second term is always negative. Therefore, when only conservative agents are present in the market, the vessel is undervalued. Notice that since extrapolators constitute a fraction of the investor population, even if conservative agents hold rational beliefs, that is \( \rho_c = \rho_f \), the vessel is still undervalued. Namely, the right hand side of 25 is always negative, as implied by the fact that \( \mu < 1 \) along with the equilibrium restriction 20b.

Third, in the scenario in which only extrapolators are present, the discrepancy, \( D_t^e \), is calculated by subtracting the fundamental price from equation 15b. This yields
The second-hand market for ships

\[ D^e_t = \left( \frac{1 - \rho e^{21}}{1 - \rho e} - \frac{1 - \rho f^{21}}{1 - \rho f} \right) (\Pi_t - \bar{\Pi}) - \frac{\mu Y^e_5}{1 - \mu} \sigma^2_e Q. \]  \tag{26}

The first term of the right hand side of 26 is always positive in this scenario, since \( \Pi_t \geq \Pi^e_5 > \bar{\Pi} \). Therefore, deviation of vessel prices is a strictly increasing function of net earnings’ discrepancy in this interval. Furthermore, while the second term is negative, it is straightforward to illustrate that in the corresponding interval (i.e. \( \Pi_t \geq \Pi^e_5 \)), its absolute value is always smaller than the first term. Therefore, \( D^e_t \) is always positive, and when market conditions significantly improve, the degree of overvaluation becomes severe.

**Corollary 4: Sensitivity of exit points to the fraction of conservatives.** As conditions 14a and 15a suggest, the agent-specific exit points differ due to the quantities \( \frac{Y^e_f}{\mu} \) and \( \frac{Y^e_e}{1 - \mu} \). The implication of this fact is that whenever \( \frac{Y^e_5}{\mu} \neq \frac{Y^e_e}{1 - \mu} \), there is no symmetry around \( \bar{\Pi} \) between the two points. As a result, the positive and negative shock cases are not mirror images of each other.

Taking the first partial derivative of the extrapolators’ 5-year exit point with respect to the fraction of conservatives yields

\[ \frac{\partial \Pi^e_5}{\partial \mu} = \frac{1}{\mu^2} \cdot \frac{Y^e_5 \sigma^2_e Q}{1 - \rho_e^{21} - \frac{1 - \rho e^{21}}{1 - \rho c^{21}}}. \]  \tag{27}

which for \( \rho_c < \rho_e \) is strictly positive. As a result, extrapolators’ exit point increases with the relative proportion of conservative investors in the market. Equivalently, the higher the fraction of conservatives, the more prone extrapolators are to exit from the market during deteriorating conditions.

Similarly, the first partial derivative of conservatives’ exit point with respect to their relative fraction is equal to

\[ \frac{\partial \Pi^e_5}{\partial \mu} = \frac{1}{(1 - \mu)^2} \cdot \frac{Y^e_5 \sigma^2_e Q}{1 - \rho_e^{21} - \frac{1 - \rho e^{21}}{1 - \rho c^{21}}}. \]  \tag{28}

which for \( \rho_c < \rho_e \) is strictly positive. Consequently, conservatives’ exit point increases with their relative proportion in the market. Hence, the higher their fraction, the less prone they are to exit the market during prosperous conditions. The same principles apply for the 6-year old vessel valuation. Hence, the asymmetry increases as \( \mu \) deviates from the midpoint 0.5.

\[ \blacksquare \]
Furthermore, as we demonstrate graphically in the next section, each agent $i$’s exit point is a strictly decreasing function of their own persistence, and a strictly increasing function of agent $-i$’s perceived persistence.

**Corollary 5: Trading volume and net earnings.** To begin with, incorporating equations 9 and 16a in (7) results in

\[ V_t = \mu^i \left| \max \left\{ \frac{1 - \rho_i^{20}}{1 - \rho_i} (\Pi_t - \overline{\Pi}) + 20\overline{\Pi} - X_i^t \sigma_e^2 Q - P_{6,t}, 0 \right\} \right| \]

Due to the short-sale constraints, the agent-specific demand functions are not strictly monotonic with respect to the net earnings variable in the entire $\Pi_t$ domain; namely, strict monotonicity disappears whenever the constraints are binding. Accordingly, in order to examine the trading activity variable, we have to distinguish between several cases.

In the first scenario, both agents are present in the market for two consecutive dates. Equivalently, conservative agents’ demands for 5 and 6-year old vessels are positive. Incorporating the equilibrium prices from (13b) and (17b) in equation 29, and applying straightforward algebra, we obtain

\[ V_t = \mu^i \left| A_6^i \Pi_t - A_5^i \Pi_{t-1} + (A_6^i - A_5^i)\overline{\Pi} \right|, \]

where

\[ A_6^i \Pi_t - A_5^i \Pi_{t-1} + (A_6^i - A_5^i)\overline{\Pi} = N_{6,t}^i - N_{5,t-1}^i, \]

is agent $i$’s change in demand for the asset between periods $t - 1$ and $t$. Te agent-specific constants are given by

\[ A_5^i \in \begin{cases} \frac{(1 - \mu) \left( \frac{1 - \rho_c^{21}}{1 - \rho_c} - \frac{1 - \rho_e^{21}}{1 - \rho_e} \right)}{[\mu Y_{5}^e + (1 - \mu)Y_{5}^c] \sigma_e^2} < 0 \end{cases}, \]

\[ A_6^i \in \begin{cases} \frac{(1 - \mu) \left( \frac{1 - \rho_c^{20}}{1 - \rho_c} - \frac{1 - \rho_e^{20}}{1 - \rho_e} \right)}{[\mu Y_{6}^e + (1 - \mu)Y_{6}^c] \sigma_e^2} < 0 \end{cases}, \]

and
\[
\begin{cases}
A_5^{e} = \frac{\mu (1 - \rho_e^{21})}{1 - \rho_e} - \frac{1 - \rho_e^{21}}{1 - \rho_e} > 0, \\
A_6^{e} = \frac{\mu (1 - \rho_e^{20})}{1 - \rho_e} - \frac{1 - \rho_e^{20}}{1 - \rho_e} > 0,
\end{cases}
\]

Since trading volume in the market is the same irrespective of the agent type, in the following we examine this variable from the conservative agent's perspective. Accordingly, equation 30d becomes

\[
V_t = \mu |A_6^{e} \Pi_t - A_5^{e} \Pi_{t-1} + (A_6^{e} - A_5^{e})\bar{\Pi}|. \quad (30e)
\]

The second scenario is when both agents are present at time \( t - 1 \), but conservatives exit at \( t \). In this case, trading activity is

\[
V_t = \mu |A_6^{e} (\Pi_{t-1} - \bar{\Pi}) + Q|. \quad (31)
\]

In the third scenario, conservatives are not present in the market at time \( t - 1 \), but both agent types are active at \( t \). Accordingly, equation 29 becomes

\[
V_t = \mu |A_6^{e} (\Pi_t - \bar{\Pi}) + Q|. \quad (32)
\]

The fourth scenario refers to the case where both agents are present in the market at time \( t - 1 \), but extrapolators exit at \( t \). In this case, trading activity is given by

\[
V_t = \mu \left|A_6^{e} (\Pi_{t-1} - \bar{\Pi}) - \frac{(1 - \mu)}{\mu} Q\right|. \quad (33)
\]

The fifth scenario is when only conservatives are present in the market at time \( t - 1 \), but both types at \( t \). Therefore, equation 29 becomes

\[
V_t = \mu \left|A_6^{e} (\Pi_t - \bar{\Pi}) - \frac{(1 - \mu)}{\mu} Q\right|. \quad (34)
\]

In the sixth (seventh) scenario, only agents of type \( i \) are present in the market at time \( t - 1 \), and only of type \(-i\) at \( t \). Namely, (39) simplifies to

\[
V_t = Q. \quad (35)
\]
Furthermore, if in two consecutive dates agents $i$ are out of the market, there is no trading activity. Finally, if $\mu^i = 0$, or equivalently, $\rho_c = \rho_e$, the market clearing condition along with equations 9 and 16a suggest that there are no second-hand transactions in the economy.

As it becomes evident from Corollary 5, short-sale constraints have a major implication for the relationship between trading activity and net earnings. In order to simplify this point, let’s define trading activity as in the equity markets literature; namely, we set $N_{i,t}^C = N_{i,t}^E$. Equivalently, we substitute $A_{5,c}^C$ for $A_{6,c}^C$ in equations 30e, 32, and 34, above. Therefore, in the absence of short-sale constraints, trading activity would always be equal to $\mu|A_{5,c}^C||\Pi_t - \Pi_{t-1}|$. As a result,

$$corr(|\Pi_t - \Pi_{t-1}|, V_t) = corr(|\Pi_t - \Pi_{t-1}|, \mu|A_{5,c}^C||\Pi_t - \Pi_{t-1}|) = 1.$$  

In other words, if there are no constraints, absolute net earnings changes are perfectly correlated with trading activity. Due to the existence of short-sale constraints, however, the two variables are significantly less correlated.

Furthermore, Corollary 4 demonstrates that both exit points increase (decrease) with the fraction of conservatives (extrapolators) and the perceived persistence on behalf of extrapolators (conservatives). Accordingly, the higher the values of the exit points, the more agents coexist during prosperous market conditions and the less they interact during a downfall. Taken together, these two observations suggest that a high value of $\mu$, along with a significant spread between $\rho_c$ and $\rho_e$ will simultaneously result in positive correlation between current net earnings and trading activity and less than perfect correlation between absolute net earnings changes and trading activity. These theoretical predictions are confirmed in the empirical estimation of the model.

III. Empirical Estimation of the Model in the Dry Bulk Shipping Industry

In this section, we discuss the dataset employed and the construction of the variables of interest. Accordingly, we evaluate empirically the theoretical predictions of our model by performing a large number of simulations. In order to provide a deeper intuition of the results, we implement impulse response and sensitivity analyses. Finally, we discuss our findings from an economic and industrial perspective.

III.A. Data on Net Earnings, Prices, and Trading Activity

The dataset employed consists of annual observations on second-hand vessel prices, 1-year time-
charter rates,\textsuperscript{11} the evolution of fleet capacity, and second-hand vessel transactions, for the Handysize dry bulk sector. Our main shipping data source is Clarksons Shipping Intelligence Network 2010. In addition, data for the U.S. Consumer Price Index (CPI) are obtained from Thomson Reuters Datastream Professional.

In line with the existing literature (Greenwood and Hanson, 2015), we impose the assumption that vessels operate in consecutive one-year time-charter contracts. In this type of arrangement, only the operating and maintenance costs are borne by the ship owner. After discussions with industry participants, we have approximated the summation of daily operating and maintenance costs for the representative 5-year old vessel (i.e. $5,500 in December 2014 values). Following Stafford et al (2002), we assume that for a given vessel, operating and maintenance costs increase with inflation. Accordingly, we define the December 2014 nominal figures as our benchmark real values. In addition, we assume that vessels spend 10 days per annum in maintenance and repairs (Stopford, 2009). During this out-of-service period, ship owners do not receive the corresponding time-charter rates, but bear the operating and maintenance expenses. Furthermore, our net earnings estimation accounts also for the commission that the brokering house receives for bringing the ship owner and the charterer into an agreement; namely, we set this fee equal to 2.5% of the daily time-charter rate.\textsuperscript{12} As it is common in the shipping literature, interest and tax expenses are ignored from the analysis. Therefore, similar to Nomikos and Moutzouris (2015), the annual net earnings variable for the period $t \rightarrow t + 1$ is estimated through

\begin{equation}
\Pi_t \equiv \Pi_{t \rightarrow t+1} = 355 \cdot 0.975 \cdot TC_{t \rightarrow t+1} - 365 \cdot OPEX_{t \rightarrow t+1},
\end{equation}

where $TC_{t \rightarrow t+1}$ and $OPEX_{t \rightarrow t+1}$ refer to the corresponding daily time-charter rates, and the summation of daily operating and maintenance costs, respectively.\textsuperscript{13}

The shipping log net earnings yield, and the 1-period log net earnings growth are given by

\begin{equation}
\pi_t - p_t = \ln(\Pi_t) - \ln(P_{5,t}) = \ln\left(\frac{\Pi_t}{P_{5,t}}\right),
\end{equation}

\begin{equation}
\pi_{t+1} - \pi_t = \ln\left(\frac{\Pi_{t+1}}{\Pi_t}\right).
\end{equation}

\textsuperscript{11} Since the 5-year old vessel prices refer to a 32,000 dead weight tonnage (dwt) carrier, while the time-charter rates to a 30,000 dwt one, following Greenwood and Hanson (2015), we multiply the initial rate series by 32/30.
\textsuperscript{12} Roughly, this fee is the sum of the brokerage and address commissions.
\textsuperscript{13} Our estimation procedure implicitly assumes that net earnings realised by a specific vessel are not a function of her age. This adjustment, however, does not have a qualitative impact on the results.
respectively. In addition, the one-year horizon log return is defined as

\[ r_{t+1} = \ln \left( \frac{\Pi_t + P_{6,t+1}}{P_{5,t}} \right). \]  

(39)

where \( P_{5,t} \) and \( P_{6,t+1} \) refer to the current and future period’s price of the 5 and 6-year old vessel, correspondingly. Since Clarksons do not provide us with the 6-year old vessel price time-series, \(^{14}\) we set \( P_{6,t} = 0.95 P_{5,t} \) in order to estimate the actual one-period returns. \(^{15}\)

Regarding the construction of the annual trading activity variable for period \( t - 1 \rightarrow t \), \( V_t \), we impose the following assumptions. First, in order to be consistent with our model’s definition of trading activity, we scale the number of total second-hand transactions taking place within the period of interest by the fleet size in the beginning of the respective period. Second, while we are interested in the trading activity related to the period between the 5th and the 6th year of the asset, Clarksons only provide us with the total number of transactions realised during the corresponding time interval. Accordingly, we assume that this scaled figure is representative for each vessel age interval. This simplifying assumption, however, is not expected to have any qualitative impact on the results.

Table 1 summarises descriptive statistics related to annual net earnings, 5-year old vessel prices, and annual trading activity, from 1989 to 2014. \(^{16}\) Panels A and B of Figure 2 illustrate the relation between trading activity and net earnings and trading activity and absolute one-year changes in net earnings, respectively.

Table 1: Descriptive statistics for vessel prices, net earnings, and trading activity.

<table>
<thead>
<tr>
<th>Variable</th>
<th>( T )</th>
<th>Mean</th>
<th>SD</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
<th>( \rho^{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pi ) ($m)</td>
<td>26</td>
<td>3.10</td>
<td>2.39</td>
<td>2.42</td>
<td>9.96</td>
<td>0.91</td>
<td>0.58</td>
</tr>
<tr>
<td>( P ) ($m)</td>
<td>26</td>
<td>22.86</td>
<td>7.65</td>
<td>22.32</td>
<td>50.23</td>
<td>13.43</td>
<td>0.50</td>
</tr>
<tr>
<td>( V )</td>
<td>20</td>
<td>0.058</td>
<td>0.020</td>
<td>0.054</td>
<td>0.099</td>
<td>0.031</td>
<td>0.11</td>
</tr>
</tbody>
</table>

\(^{14}\) In particular, our second-hand price dataset consists of observations for 5, 10, 15, and 20-year old vessels. 

\(^{15}\) Since Clarksons only provide us with 5 and 10-year old vessel prices, we estimate the average ratio of 10-year to 5-year old vessel prices; in our dataset this is approximately equal to 0.75. Accordingly, adopting a straight-line depreciation scheme implies \( P_{6,t} = 0.95 P_{5,t} \).

\(^{16}\) Trading activity is available from January 1995 and onwards.
Panel A: Relation between Trading Activity and Net Earnings.

Panel B: Relation between Trading Activity and Absolute Changes in Net Earnings.

Figure 2: Net Earnings and Trading Activity.

Panel A depicts the relation between annual trading activity and annual net earnings. Panel B depicts the relation between annual trading activity and absolute changes in annual net earnings. The sample runs from 1995 to 2014. Annual trading activity is expressed as a percentage of the fleet in the beginning of the corresponding period. Prices and net earnings are expressed in December 2014 million dollars.
III.B. Simulation Methodology and Results

In this subsection, we evaluate empirically the predictions of our model, for several combinations of the three main parameters of interest, \( \{\mu, \rho_c, \rho_e\} \), using numerical simulations. Accordingly, we compare the model-generated moments to the actual ones. In particular, using equation 1, we generate 10,000 sample paths for the economy described in Section II, where each path corresponds to 100 periods. If somewhere in a simulation, either the net earnings variable or vessel prices attain a negative value, we discard this path.\(^{17}\) Accordingly, we estimate the average of each statistic under consideration, across all valid paths (Barberis et al., 2015a).

In order to conduct the simulations, we have to calibrate two sets of model parameters. The first set contains the asset-level parameters, \( \{\bar{\Pi}, \rho_0, \sigma_*^2, Q, T\} \), and remains the same irrespective of the population composition and characteristics. We set \( \bar{\Pi} = 3.1 \); that is, the long-term mean is assigned the value of the mean of the net earnings variable (Table 1). The coefficient of persistence is approximated through the actual 1-year autocorrelation coefficient of the net earnings variable; namely, \( \rho_0 = 0.58 \) (Table 1). Furthermore, we set the standard deviation of the error term equal to 1 in order to reduce the number of discarded paths, but at the same time, ensure a sufficient degree of net earnings volatility. The value of \( \sigma_*^2 \) per se, has no direct impact on the estimation and the results remain qualitatively the same for different plausible values of \( \sigma_*^2 \).\(^{18}\) We set the fixed per capita supply, \( Q \),\(^{19}\) and the remaining economic life of the 5-year old vessel, \( T \), equal to 1 and 20, respectively. Finally, recall that we have normalised the rate of return of the risk-free asset, \( R^f \), to zero.

The second set includes the agent-specific parameters \( \mu, \rho_i, \theta_i^f, \theta_i^c \), and \( \alpha_i \) for \( i \in \{f, c, e\} \). Notice that the relative fraction of each investor type and the remaining agent-specific parameters are independent. Therefore, we can test our model’s predictions by simultaneously changing the values of \( \mu \) and \( \rho_i \). Regarding the parameter \( \mu \), we arbitrarily choose values within the interval \([0,1]\). In the limiting cases where \( \mu = 0 \) or \( \mu = 1 \) (or equivalently, \( \rho_c = \rho_e \)), the model is simplified to a homogeneous environment. While fundamentalists’ characteristics are fixed by definition, the ones related to extrapolators are recalibrated each time, depending on the scenario choice. As mentioned in Section II, due to the exponential utility assumption, the equilibrium conditions are independent of the initial wealth levels. Therefore, we do not have to assign a value to \( w_0^i \).

\(^{17}\) We impose this restriction in order to be able to perform the predictive regressions, which use log quantities as variables. Even if we do not discard these paths, the remaining results remain essentially the same, both quantitatively and qualitatively. Furthermore, notice that in the unrestricted simulation exercise, while net earnings turn negative relatively frequently, the number of negative prices is insignificant. This feature is not inconsistent with reality, since during market troughs investors can realise operating losses, however, the price of the asset cannot be negative.

\(^{18}\) In the next section, we estimate a value of \( \sigma_*^2 \), instead of assuming one.

\(^{19}\) This normalisation, does not have any quantitative or qualitative implications on the predictions of our model.
Accordingly, we begin by calibrating the “fundamental” parameters. Since fundamentalists form expectations about future net earnings based on the true stochastic process (i.e. equation 1), $\rho_f, \vartheta_f^i$, and $\vartheta_6^i$ are assigned the values of 0.58, 1, and 1, respectively. The last parameter is the value of the coefficient of absolute risk aversion, $\alpha_f$. Corollary 2 and the steady state equilibrium conditions imply that $\alpha_f$ should satisfy equation 23a, with $i = f$. Equivalently, the steady state equilibrium price of the 5-year old vessel, $P_5^*\gamma^\ast$, and $\alpha_f$ are nested; hence, we can calibrate either of the two values. In line with (12), we set the steady state equilibrium price equal to the sample arithmetic mean of the 5-year old vessel prices, $P_5^* = 22.86$ from Table 1, and accordingly, from (23a) we obtain $\alpha_f = 0.42$. At the same time, $\alpha_f$ should satisfy condition 23b. Hence, rearranging (23b), we estimate the steady state equilibrium price for the 6-year old vessel, $P_6^* = 22.14$.

The agent-specific extrapolator parameters, are estimated in a similar manner. In particular, these parameters are nested by the steady state equilibrium conditions. Therefore, for any chosen value of the key parameter of interest $\rho_i$, the values of the products $\alpha_i \vartheta_5^i$ and $\alpha_i \vartheta_6^i$ are endogenously determined through conditions 23. Accordingly, it suffices to arbitrarily fix either the parameter $\vartheta_5^i$ or $\vartheta_6^i$ or $\alpha_i$. Notably, this choice does not have any qualitative or quantitative implication on the results, since only the value of the product matters, not the values of the components per se. Accordingly, we set conservatives’ and extrapolators’ coefficients of absolute risk aversion equal to 0.35 and 0.15, respectively. Essentially, we assume that extrapolators are more risk tolerant than conservatives, an assumption consistent with their investment policies as we illustrate in subsection III.E. Finally, depending on the choice of agent $i$’s perceived persistence, $\rho_i$, the equilibrium conditions assign the corresponding values to $\vartheta_5^i$ and $\vartheta_6^i$. Equivalently, we could have set $\alpha_f = \alpha_i = 0.42$. As mentioned above, the results remain the same, since for the chosen value of $\rho_i$, $\vartheta_5^i$ and $\vartheta_6^i$ will be adjusted accordingly through the steady state equilibrium conditions. Finally, by construction, $\vartheta_6^i > \vartheta_5^i$. As analysed in section II, this feature reflects the fact that as the investor related to the specific age-class vessel gets older, he becomes more “suspicious” regarding the determination mechanism of net earnings, or equivalently more risk averse. Hence, his immediate response is to increase the perceived riskiness of the investment through the variance of the cash flow shock, albeit, at a very slow rate.

For the ease of reference, Table 2 summarises the model parameters. Notice that we list the parameters $\vartheta_5^i$ and $\vartheta_6^i$ only for the fundamentalist, since in the cases of conservatives and extrapolators, they depend solely on the choice of $\rho_i$. Having calibrated the required parameters, we can estimate the moments of interest for each scenario under consideration. In Table 3, we present our model’s predictions for several combinations of the agent-specific parameters of interest $\{\mu, \rho_c, \rho_e\}$. In addition, the right-most column illustrates the actual values of the quantities of interest.
Table 2: Parameter values.

The table summarises the assigned values regarding the long-term of the net earnings variable, $\bar{\Pi}$; the actual autocorrelation of net earnings, $\rho_0$; the variance of the net earning shock, $\sigma^2_\epsilon$; the vessel supply, $Q$; the remaining economic life of the 5-year old vessel, $T$; the risk-free rate, $R_f$; the fraction of conservatives in the investor population, $\mu$; the coefficient of absolute risk aversion of fundamentalists, $\alpha^f$; the perceived persistence of fundamentalists, $\rho^f$; the 5- and 6-year variance adjustment coefficients of fundamentalists, that is, $\vartheta^f_5$ and $\vartheta^f_6$, respectively; the coefficient of absolute risk aversion of conservatives, $\alpha^c$; the perceived persistence of conservatives, $\rho^c$; the coefficient of absolute risk aversion of extrapolators, $\alpha^e$; and the perceived persistence of extrapolators, $\rho_e$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assigned Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\Pi}$</td>
<td>3.1</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>0.58</td>
</tr>
<tr>
<td>$\sigma^2_\epsilon$</td>
<td>1</td>
</tr>
<tr>
<td>$Q$</td>
<td>1</td>
</tr>
<tr>
<td>$T$</td>
<td>20</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0</td>
</tr>
<tr>
<td>$\mu$</td>
<td>${0.1, 0.5, 0.95}$</td>
</tr>
<tr>
<td>$\alpha^f$</td>
<td>0.42</td>
</tr>
<tr>
<td>$\rho^f$</td>
<td>0.58</td>
</tr>
<tr>
<td>$\vartheta^f_5$</td>
<td>1</td>
</tr>
<tr>
<td>$\vartheta^f_6$</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha^c$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\rho^c$</td>
<td>${0.58, 0.65, 0.75}$</td>
</tr>
<tr>
<td>$\alpha^e$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>${0.9, 0.99}$</td>
</tr>
</tbody>
</table>

To begin with, an apparent feature of the simulation results is that the average price and the average earnings yield are approximately the same, irrespective of the parameterisation. By construction, since the steady state equilibrium price was set equal to the sample mean of the actual vessel prices, the average price across all simulated paths was expected to be very close to this mean. Furthermore, due to the equilibrium restrictions and in particular the system of equations 23, this statistic was expected to be approximately the same across all parametrisations. Consequently, also the average net earnings yield is roughly the same in all model-generated scenarios and the actual case. Importantly, since net earnings are the sole state variable in the model, the autocorrelations of 5-year old prices and net earnings are closely related, irrespective of the scenario. In addition, as it is well-documented in the related literature (Alizadeh and Nomikos, 2007), net earnings and vessel prices, exhibit a high degree of correlation. Taken together, these facts explain why this statistic is approximately the same across all scenarios, but also very close to the actual value.

The price volatility statistic is defined as the ratio of the standard deviation of 5-year old vessel prices in the extrapolative heterogeneous-agent economy (for the particular agent-specific
parameterisation) to the standard deviation of the fundamental value of the 5-year old asset, for a given net earnings shock sequence. Recall that the latter value corresponds to the 5-year old vessel price in a counterfactual economy where all agents form rational expectations about the net earning process. When this ratio is higher than 1, we incorporate the term “excess volatility” in the sense that the heterogeneous-agent model prices are more volatile than the corresponding ones in the benchmark rational economy (Barberis et al., 2015). Equivalently, this statistic aims to capture the stylised fact that actual vessel prices are significantly more volatile compared to the values indicated by optimally forecasted future net earnings (Greenwood and Hanson, 2015).

In order to assign a benchmark value to this statistic, we incorporate the following argument. As equation 11 suggests, in the counterfactual fully rational economy, the volatility of vessel prices should be given by

$$\sigma(p^{f}_{5,t}) = \frac{1 - \rho_f^2 \sigma(\Pi_t)}{1 - \rho_f \sigma(\Pi_t)}.$$ 

Substituting in this formula the actual volatility of net earnings from Table 1, we estimate the fundamental value for our actual data; that is, $p^{f}_{5,t} = 5.71$. However, this value is significantly lower compared to the actual volatility of vessel prices in the data, $P_{5,t} = 7.65$. Specifically, the price volatility ratio is approximately equal to 1.34. In line with Barberis et al. (2015), since in our framework the volatility of generated prices is directly related to the volatility of net earnings, it is more appropriate to examine this statistic and in turn approach the “excess volatility” phenomenon by referring to the same sequence of net earnings shocks. Accordingly, our simulation results suggest that this statistic is positively related to the perceived autocorrelation coefficient of both agents, and negatively related to the relative fraction of conservatives. Equivalently, as illustrated in section II, the higher the average degree of net earnings extrapolation in the market, the higher the volatility of vessel prices.

Interestingly, the results regarding the net earnings yields regressions confirm a well-analysed argument in the recent shipping literature. Namely, the fact that vessel prices and net earnings are highly correlated does not imply that these two variables change proportionately (Greenwood and Hanson, 2015). If this were the case, earnings yields would be constant across time; hence, earnings yields would have no predictive power. In reality, however, yields fluctuate significantly over time which in turn implies that this variable has strong predictive power. Accordingly, as Nomikos and Moutzouris (2015) demonstrate, the bulk of variation in shipping earnings yields reflects varying expected net earnings growth, not time-varying expected returns. In particular, shipping earnings
yields strongly and negatively forecast future net earnings growth, while future returns appear to be unpredictable, especially in the 1-year horizon case.

This stylised fact is the combined effect of the mean-reverting character of net earnings and the strong positive correlation between net earnings yields and net earnings. The latter observation is a consequence of the fact that the absolute growth rate of net earnings is higher than the respective absolute growth rate of prices. In essence, the average investor anticipates up to a certain degree the mean-reverting character of net earnings, and as a result, vessels are not overvalued “too much” in equilibrium. Therefore, while net earnings and prices move together over time, the rate of change of the former is substantially larger. As a result, earnings yields are high (low) during a market downturn (upturn). Accompanied by the mean reversion of net earnings, the mechanism explains the predictive power of the yield. In contrast, if the average degree of extrapolation in the market were extreme, that is, if investors valued vessels even more naively, the growth rate of prices would be higher than the one of net earnings; as a result, net earnings yields and net earnings would be negatively correlated and the yield would be strongly positively related with net earnings growth. Furthermore, due to the increased sensitivity of prices in that case, the earnings yield would also be strongly positively related with returns.

Column 2 of Table 3 illustrates the latter scenario. Namely, the presence of extreme extrapolative expectations in the market results in strong positive predictability of both net earnings growth and returns. Furthermore, in this case, returns explain a higher fraction of earnings yields variance compared to net earnings growth. These results are in sharp contrast with reality. However, as the existence of extrapolative expectations is attenuated, the model-implied predictive regressions results approach the empirical values.

Moreover, we observe that the R-squared of the net earnings growth regressions is significantly high, across all scenarios; namely, around 0.27. Since the exogenous net earnings variable is always generated through equation 1, it exhibits the same highly volatile behaviour, irrespective of the parametrisation. Hence, in all cases, a very large fraction of earnings yields’ volatility can be attributed to variation in expected net earnings growth. Equivalently, variation in current net earnings yields is always a good predictor of variation in future net earnings growth.

Accordingly, we examine our model’s predictions regarding the market trading activity. Recall that in our model, trading activity is the result of heterogeneous beliefs in the investor population. Hence,

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20 According to the Campbell-Shiller variance decomposition, regression coefficients can be interpreted as fractions of net earnings yield variation attributed to each source. Notice that the elements of this decomposition do not have to be between 0 and 100%. Accordingly, the higher the absolute value of returns coefficient compared to the growth one, the larger the fraction of earnings yield volatility it can explain (Nomikos and Moutzouris, 2015).
Table 3: Model predictions for the quantities of interest.

This table summarises the heterogeneous-agent model’s predictions for the quantities of interest presented in the left column. The right column presents the empirical values of these quantities for the period 1989-2014. Notice that trading activity data are available from 1995. Columns 2-7 report the average value of each quantity across 10,000 simulated paths, for a given parametrisation depending on the population characteristics. The basic model parameters are presented in Table 2. The first row refers to the mean of the 5-year old vessel prices; the second row refers to the 1-year autocorrelation of 5-year old vessel prices; the third row refers to the ratio of the standard deviation of 5-year old vessel prices in the extrapolative heterogeneous-agent economy to the standard deviation of the fundamental value of the 5-year old asset, under the same net earning sequence. The latter value corresponds to the 5-year old price in a counterfactual economy where all agents form rational expectations about the net earning process. Hence, this value and its standard deviation are estimated through equations 10 and 11, respectively, for $i = f$. The fourth row refers to the mean of ratio of net earnings to the 5-year old vessel price, as estimated through (37). The fifth and sixth rows report the slope coefficient and R-squared, respectively, of a regression of future one-period log net earnings growth on the log net earnings yield: $\pi_{t+1} - \pi_t = \alpha_{\Delta \pi} + \beta_{\Delta \pi} \cdot (\pi_t - p_{5,t}) + \epsilon_{\Delta \pi,t+1}$, where the log net earnings growth is estimated through (38). The seventh and eighth rows report the slope coefficient and the R-squared, respectively, of a regression of future one-period log returns on the log net earnings yield: $r_{t+1} = \alpha_{\Delta \pi} + \beta_{\Delta \pi} \cdot (\pi_t - p_{5,t}) + \epsilon_{\Delta \pi,t+1}$, where log returns are estimated through (39). Finally, rows nine, ten, and eleven present the mean of the annual trading activity, estimated through (29), the correlation between net earnings and trading activity, and the correlation between trading activity and absolute one-year changes in the net earnings variable, respectively.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$(\mu, \rho_c, \rho_e)$</th>
<th>Actual Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(0.1, 0.58, 0.9)$</td>
<td>$(0.5, 0.58, 0.9)$</td>
</tr>
<tr>
<td>Average Price</td>
<td>23.90</td>
<td>24.08</td>
</tr>
</tbody>
</table>
one should expect that the average trading activity should increase with the degree of heterogeneity, and decrease with the difference in the population fractions. Our numerical results suggest that this is precisely the case. In particular, when both types have a strong presence in the market, and a noticeable belief disagreement, trading activity is substantial (column 3). In contrast, when one investor type constitutes the vast majority of the population, trading activity is significantly reduced. Furthermore, in line with Corollaries 4 and 5, we observe that the correlation between current net earnings and trading activity increases with both the fraction of conservatives, and the degree of investors’ heterogeneity regarding the perceived persistence of the net earnings variable. As we illustrate in the sensitivity and impulse response analyses that follow, both agents’ exit points and as a result, the correlation coefficient, are extremely sensitive to parameter $\mu$. Keeping the values of $\rho_c$ and $\rho_e$ constant, we see that for $\mu$ equal to 0.1, 0.5, and 0.95, the respective correlation coefficients are negative, approximately zero, and positive (columns 2-4). Similarly, for fixed $\mu = 0.95$, columns 4-7 suggest that the correlation coefficient is positively and negatively related to $\rho_e$ and $\rho_c$, respectively. Furthermore, as expected, the correlation between absolute net earnings changes and trading activity is very high across all parameterisations, albeit, significantly less than 1. This is a combined effect of the asymmetry in investors’ exit points and the short-sale constraints.

As it becomes evident from the analysis above, in order to simultaneously approach all statistics of interest, the fraction of conservative investors in the market has to be very large. In addition, there has to be significant heterogeneity of beliefs among the two investor types. Finally, also conservatives must hold slightly extrapolative beliefs. In conclusion, the parameterisation $\{0.95, 0.65, 0.99\}$ appears to be able to capture sufficiently almost all stylised facts under consideration. Of course, if we adjust slightly this set of parameters following the principles of the analysis above, we will probably be able to approach the actual moments even better. However, the results and the realised patterns will be qualitatively the same. Unfortunately, due to the short-sale constraint, we are not able to obtain analytic closed-form expressions for the moments of interest; hence, we cannot apply a more elaborated estimation approach.

From an economic perspective, this approximate parameterisation is in accordance with the nature of the shipping industry, which consists of a large number of established private shipping companies. In some instances, ship owning families have been present in the market for more than a century (Stopford, 2009) and have strong prior experience about the supply and demand model of the shipping industry. As a result, they can perform “more rational” forecasts about future market conditions, compared to relatively new entrants in the market. The remaining fraction of the investor population can be attributed to new entrants, such as diversified investors (e.g. private equity firms) or speculators, with small or no previous experience of the market. It has been well-documented that
during prosperous periods, new entrants impressed by the high prevailing earnings and the short-term realised returns are eager to buy vessels, which accordingly, are more than keen to sell as conditions begin to deteriorate. In contrast, there are many cases where traditional investors have realised significant returns by selling vessels at the peak of the market, and buying at the trough (this strategy is known as “playing the cycles”, Stopford, 2009).

As analysed in the following subsections, our model accounts for this fact through the two exit points; namely, extrapolators (conservatives) exit during deteriorating (prosperous) conditions. Finally, while in principle, conservatives can form expectations for future net earnings based on the true process, that is, be fundamentalists, the moments of interest are better matched when they also hold extrapolative beliefs, however, at a significantly smaller degree. This feature is consistent with reality, since no matter how experienced investors are, it is highly unlikely they can forecast precisely the evolution of cash flows in such a volatile industry.

III. Sensitivity Analysis

The aim of this subsection is to provide a deeper intuition of the mechanism that creates the positive correlation between net earnings and trading activity in our model. In order to achieve that we illustrate the relation between the key model parameters, \( \{\mu, \rho_c, \rho_e\} \), and both 5-year old agents’ exit points, \( \Pi_5 \). Accordingly, we analyse how these parameters affect the trading activity variable. With these results in hand, it is easier to interpret the relation between the investor population characteristics, market conditions, and trading activity.

We begin by examining the sensitivity of the 5-year old exit points to the three key parameters. Note that exactly the same analysis holds for the other age-classes’ exit points. In each case, we allow the relevant parameter of interest to vary, while keeping the remaining two fixed. The corresponding fixed parameters are based on the parameterisation \( \{0.95, 0.65, 0.9\} \). As equations 27 and 28 indicate, both agents’ exit points are strictly increasing functions of conservatives’ persistence. Panel A of Figure 3 demonstrates this relation for \( \rho_c = 0.65, \rho_e = 0.9, \) and \( \mu \in [0.05, 0.95] \). Since the corresponding exit points in the two tails attain extreme values, we have restricted the domain of the function to be between 0.05 and 0.95 in the graph. The described pattern, however, extends to the entire interval \((0,1)\). For the ease of exposition, the black solid line represents the steady state of our model, i.e. \( \Pi_5 = \bar{\Pi} \). As it becomes evident from the graph, as \( \mu \) deviates from the midpoint 0.5, the asymmetry between the two exit points becomes substantial. Namely, when \( \mu \) attains the value of 0.95, extrapolators exit rapidly as market conditions deteriorate, while conservatives remain active in the market even for very large net earnings values.

Panel B of Figure 3 depicts the sensitivity of both agents’ exit points to the perceived persistence
Figure 3: Sensitivity of Exit Points to Parameter Values.

Panel A: Sensitivity of exit points to the fraction of conservatives. Panel A illustrates the sensitivity to the fraction of conservatives, for $\rho_c = 0.65$ and $\rho_e = 0.9$. Panel B shows the sensitivity to extrapolators’ perceived persistence, for $\mu = 0.95$ and $\rho_c = 0.65$. Panel C demonstrates the sensitivity to conservatives’ perceived persistence, $\mu = 0.95$ and $\rho_e = 0.9$. The horizontal solid black line in each panel shows the steady state value of the net earnings variable.
of extrapolators. Namely, we plot both agents’ exit points for $\mu = 0.95$, $\rho_c = 0.65$, and $\rho_e \in [0.8,1)$. As it becomes apparent from the graph, conservatives’ exit point is a strictly increasing function of $\rho_e$, while extrapolators’ exit point strictly decreases with $\rho_e$. Similar to the previous case, we have restricted the domain of the function in the graph. However, the same principles apply for the entire interval $(\rho_c, 1)$. Finally, panel C of Figure 3 shows the sensitivity of both agents’ exit points to the perceived persistence of conservatives, for $\mu = 0.95$, $\rho_e = 0.9$, and $\rho_c \in [\rho_0, 0.85]$. Evidently, conservatives’ exit point strictly decreases with $\rho_c$, while extrapolators’ exit point is a strictly increasing function of $\rho_c$. Notice that this is true for the entire interval $\rho_c \in [\rho_0, \rho_e)$.

The analysis above shows that a large fraction of conservatives, combined with a high $\rho_e$ and a low $\rho_c$ result in a 5-year old exit point for the extrapolators that is very close to the steady state equilibrium, $\bar{\Pi}$. Furthermore, while the exit point of conservatives is a decreasing function of $\rho_e$ and an increasing function of $\rho_c$, the effect of the very large value of $\mu$ dominates, that is, it yields a very high 5-year exit point for conservatives. Namely, for the choice of parameter values $\{0.95,0.65,0.9\}$, we obtain $\Pi^e_5 = 2.44$ and $\Pi^c_5 = 15.75$. In order to illustrate the degree of this asymmetry recall that in the steady state of the model $\bar{\Pi} = 3.1$. Accordingly, a slight (roughly half a standard deviation) negative shock suffices to force extrapolators to exit. In contrast, an extreme (almost 13 standard deviations) positive shock is required in order for conservatives to exit. In essence, this parameterisation implies that conservatives are always present in the market, while conservatives’ optimal investment policy is to remain inactive during market troughs.

Figures 4-6 illustrate the immediate effect of the key parameter values to market trading activity. Specifically, we examine the dependence of the (relative) magnitude of trading activity during prosperous and deteriorating market conditions on the parameters of interest. In order to capture both cases, we perturb the steady state equilibrium with a positive and a negative two standard-deviation shock. For these two given shocks we allow each time the corresponding parameter of interest to vary, while holding the remaining two parameters fixed. Figure 4 depicts the sensitivity of trading activity to the fraction of conservatives, for $\rho_c = 0.65$ and $\rho_e = 0.9$. In order to illustrate clearly the effect on the asymmetry of trading activity across market conditions, Panel B plots the ratio of trading activity after the positive shock to trading activity after the negative disturbance, as a function of each key parameter. Notice that for $\mu = 0$ and $\mu = 1$, there is no heterogeneity among agents; hence, no trading activity occurs in the market. Furthermore, for $\mu = 0.5$, trading activity is approximately the same after the two shocks. However, for values of $\mu$ higher than 0.5, this ratio becomes higher than one, and is strictly increasing with $\mu$.

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21 In Panel B of Figure 3, the function is not continuous at 0 and 1, since at these points there is no trading activity in the market; hence, the ratio is not defined. Therefore, we have plotted the function in the interval $(0,1)$. 
Figure 4 presents the relation between the fraction of conservatives and trading activity after both a positive and negative two standard-deviation shock, for $\rho_c = 0.65$ and $\rho_e = 0.9$. Panel A illustrates the dependence of the absolute trading activity to the fraction of conservatives, for both cases. Panel B shows the relation between the fraction of conservatives and the ratio of trading activity after the positive shock to trading activity after the negative shock.
It is straightforward to interpret this pattern. In line with Panel A of Figure 3, for large values of $\mu$, extrapolators exit relatively easily after a negative net earnings shock. When they exit, as is the case after this 2 standard-deviation negative shock, extrapolators’ demand and in turn their holdings become zero. In addition, as Corollary 2 suggests, in the steady state equilibrium both agents hold the risky asset in analogy to their population fraction. This implies that before the negative shock, extrapolators’ holdings were $(1 - \mu)Q$. Therefore, in line with equation (33) of Corollary 4, for large values of $\mu$, trading activity after the negative shock equals $(1 - \mu)Q$. Since $\mu$ is large, however, the resulting activity is relatively small. In contrast, after a positive shock, both agents are present in the market and trading activity is given by equations 30a and 30b, for $\Pi_{t-1} = \bar{\Pi}$ and $\Pi_t = \bar{\Pi} + 2$. The fact that $A_c^e \cong A_c^c$ implies $V_t \cong 2\mu|A_c^c|$, which for this set of parameter values is significantly higher than $(1 - \mu)Q$. It is this mechanism that creates the asymmetry between the two cases. Of course, for a shock of smaller absolute value than the one inducing the exit of extrapolators, trading activity in the positive and negative cases would be approximately the same.

Figure 5 illustrates the relation between trading activity and the perceived persistence of extrapolators, for $\mu = 0.9$ and $\rho_c = 0.65$. Recall that the cornerstone of the proposed framework is that trading occurs due to the heterogeneity of beliefs among investors. Therefore, in the limiting case where $\rho_e = \rho_c = 0.65$ agents hold homogeneous beliefs; as a result, no trading activity is realised in the market. As the perceived persistence of extrapolators deviates from $\rho_c$, the heterogeneity of beliefs and in turn trading activity in the market increases. Up to a limiting value of extrapolators’ persistence, denoted by $\rho_e^*$, trading activity in the positive and negative shock cases is approximately the same. Equivalently, as Panel B illustrates, in the interval $[\rho_c, \rho_e^*)$ the ratio is approximately equal to 1. In the interval $(\rho_e^*, 1)$, however, trading activity after the positive shock is higher compared to the one after the negative shock, and the ratio is a strictly increasing function of $\rho_e$.

In line with Panel B of Figure 3, this result is a consequence of the fact that, ceteris paribus, extrapolators’ exit point is a strictly increasing function of $\rho_e$. In particular, when $\rho_e = \rho_e^*$, extrapolators’ 6-year old exit point is equal to $\bar{\Pi} - 2 = 1.1$. Accordingly, trading activity after the negative shock is equal to $(1 - \mu)Q$, which for our parameter values is 0.05. As $\rho_e$ further increases, extrapolators’ exit point increases as well, nevertheless, trading activity after the negative shock is bounded; namely, it cannot be higher than 0.05. Therefore, as illustrated in Panel A of Figure 5, in the interval $(\rho_e^*, 1)$ trading activity corresponds to the line $V = 0.05$. In the positive shock case, however, trading activity is approximately equal to $2\mu|A_c^c|$ which is, ceteris paribus, an increasing function of $\rho_e$. Consequently, it is the continuously differentiable upward sloping curve in Panel A of Figure 5.

Finally, Figure 6 illustrates the relation between trading activity and the perceived persistence of conservatives, for $\mu = 0.95$ and $\rho_e = 0.9$. In the limiting case where $\rho_c = \rho_e = 0.9$ agents hold
Figure 5: Sensitivity of Trading Activity to Extrapolators’ Persistence.

Figure 5 presents the relation between extrapolators’ persistence and trading activity after both a positive and negative two standard-deviation shock, for $\mu = 0.95$ and $\rho_c = 0.65$. Panel A illustrates the dependence of the absolute trading activity to extrapolators’ persistence, for both cases. Panel B shows the relation between extrapolators’ persistence and the ratio of trading activity after the positive shock to trading activity after the negative shock.
homogeneous beliefs; consequently, there is no trading activity in the market. As the perceived persistence of conservatives deviates from $\rho_e$, that is, as $\rho_c$ decreases, the heterogeneity of beliefs and in turn trading activity in the market increases. Similar to the previous paragraph, there exists a limiting value of conservatives’ persistence, denoted by $\rho_c^*$, such that in the interval $[\rho_c^*, \rho_e)$ trading activity in the positive and negative shock cases is approximately the same. In the interval $[\rho_0, \rho_c^*)$, however, trading activity after the positive shock is higher compared to the one after the negative shock, and the ratio is a strictly decreasing function of $\rho_c$.

In line with Panel C of Figure 3, this is a consequence of the fact that, ceteris paribus, conservatives’ exit point is a strictly decreasing function of $\rho_e$. Specifically, when $\rho_c = \rho_c^*$, conservatives’ 6-year old exit point is equal to $\overline{\Pi} - 2 = 1.1$. Accordingly, trading activity after the negative shock is given by $(1 - \mu)Q$, which for our parameter values equals 0.05. As $\rho_c$ further decreases, conservatives’ exit point increases, however, trading activity after a negative shock is bounded; namely, it cannot be higher than 0.05. Hence, as Panel A of Figure 7 depicts, in the interval $[\rho_0, \rho_c^*)$ trading activity corresponds to the line $V = 0.05$. In the positive shock case, however, trading activity is approximately equal to $2\mu|\Lambda_c^e|$ which is, ceteris paribus, a decreasing function of $\rho_c$. Consequently, it is the continuously differentiable downward sloping curve in Panel A of Figure 6.

### III.D. Impulse Response Functions

With the sensitivity analysis results in hand, we now examine the effect of a one-time shock to the net earnings variable. Specifically, in line with the previous subsection, we present model-implied impulse response functions in the scenario where the incorporated model parameters are the ones from the simulation exercise with $(\mu, \rho_c, \rho_e) = \{0.95, 0.65, 0.9\}$. Figures 7 and 8 illustrate the behaviour of net earnings, 5-year old vessel prices, vessel demand, and trading activity after a two-standard deviation positive and negative shock, respectively. In panel B of each figure, apart from the model-generated 5-year old price (i.e. the market clearing price of the risky asset), we also present the respective agent-specific valuations. The latter are generated through equation 10 and refer to the “fair” value of the asset from each agent’s perspective. In addition, for comparative purposes, we plot the fundamental value of the asset. As Corollary 2 suggests, in the steady state equilibrium all four valuations coincide.

We begin by examining the positive shock case. At $t = 0$, the net earnings variable is equal to its long-term mean, $\overline{\Pi}$, and consequently the model is in its steady state. Accordingly, the agent-specific valuations of the vessel coincide, and as a result, all agents’ have the same per capita demand for the asset. Furthermore, assuming that also in the previous period the model was in its steady state, there is no trading activity in the market. At $t = 1$, we perturb the steady state equilibrium by generating a
Figure 6: Sensitivity of Trading Activity to Conservatives’ Persistence.

Figure 7 presents the relation between conservatives’ persistence and trading activity after both a positive and negative two standard-deviation shock, for $\mu = 0.95$ and $\rho_e = 0.9$. Panel A illustrates the dependence of the absolute trading activity to conservatives’ persistence, for both cases. Panel B shows the relation between conservatives’ persistence and the ratio of trading activity after the positive shock to trading activity after the negative shock.
positive $2 million shock (i.e. a two standard-deviation shock). The immediate first order effect of the shock is the increase of current net earnings by this amount. Due to the mean reversion of the net earnings variable, this shock is completely attenuated within roughly 10 years. However, extrapolators expect net earnings to revert to their steady state value in more than 20 periods, while conservatives in roughly 12 (Panel A of Figure 7). \(^{22}\)

As a result, the agent-specific valuations of the risky asset are higher compared to its fundamental value (Panel B of Figure 7). Hence, also the equilibrium price of the vessel deviates from its fully rational value. The degree of overvaluation is quantified through equation 24. Notice that due to the very large fraction of conservatives in the market, the equilibrium price is very close to the conservative valuation, but slightly higher, while extrapolators’ valuation is significantly higher than both. In turn, this implies that extrapolators consider the 5-year old asset to be undervalued while conservatives believe the opposite, i.e. that the asset is overvalued with respect to their subjective “fair” valuation. Essentially, agents compare their valuation of the asset to its equilibrium price\(^{23}\) and not to the fundamental price of the asset (which by not being fundamentalists, they totally ignore). As a result, extrapolators (conservatives) increase (decrease) their demand for the asset compared to the steady state of the economy (Panel C of Figure 7). Exactly the same arguments hold for the valuation of the corresponding 6-year old vessel at \(t = 1\). Therefore, also extrapolators’ (conservatives’) demand for the 6-year old vessel is increased (decreased), compared to their demand for the respective 5-year old vessel one period before.

Finally, as an immediate effect of this change in demand, there is trading activity between periods 0 and 1 in the second-hand market for vessels (Panel D of Figure 7). In particular, extrapolators increase their proportion of fleet supply, while conservatives do the opposite. At this point, recall from the sensitivity analysis that there exists a limiting value of the net earnings variable, \(\Pi^C_5\), at which conservatives related to the 5-year old vessel exit the market (\(\Pi^C_6\) for the 6-year old vessel, respectively). For the parameter values incorporated in this section, both exit points are higher than the corresponding net earnings variable at \(t = 1\); \(\Pi_{t=1} \equiv 5.1\), while \(\Pi^C_5 \equiv 15.75\) and \(\Pi^C_6 \equiv 16.10\). Therefore, both 5- and 6-year old conservatives’ demands are positive, and as a result, 5 and 6-year old vessel prices are given by equations 13b and 17b, respectively. Accordingly, trading activity is estimated through 30e.

In year 2, the net earnings variable decreases, evolving towards its long-term mean. However, it

\(^{22}\) By definition, the actual evolution of the net earnings variable coincides with the one perceived by the fundamentalist.

\(^{23}\) In order to be precise, agents compare their expected one-period income from the asset to its equilibrium price. This comparison is quantified for the 5- and 6-year old vessels through the numerator of the fraction inside the maximum function in equations 8a and 16a, respectively.
Figure 7 displays model-implied impulse response functions following one positive $2 million shock to net earnings. Panels A-D correspond to the case of heterogeneous extrapolators, related to the simulation exercise. Panel A illustrates the actual evolution of net earnings, and the evolution perceived by each extrapolator type. Panel B shows the model-generated 5-year old vessel prices, and the agent-specific valuation. Panel C demonstrates the agent-specific share demands for the 5 and 6-year old vessels. Finally, Panel D, plots the trading activity in the market. The horizontal solid black line in each panel shows the steady state value of the corresponding variable.
remains above $\bar{\Pi}$. As a result, both agents’ subjective valuations are once again higher compared to the fundamental value of the asset. Hence, in equilibrium, the vessel is still overpriced compared to its fundamental value. Furthermore, extrapolators’ valuation is substantially higher than both the equilibrium price and conservatives’ valuation. Importantly, though, due to the decrease in the net earnings variable, extrapolators now consider the vessel significantly less overvalued compared to what they did one period before. This is true for both the 5- and 6-year age classes. As a result, the demand of extrapolators for the 6-year old vessel is lower than their previous year’s demand for the corresponding 5-year old asset. For conservatives, the inverse is true.\footnote{Equation 30b with $\Pi_{t-1} = \bar{\Pi} + 2$ and $\varepsilon_t = 0$, for $i = e$ and $i = c$ is negative and positive, respectively.} Accordingly, we have once again trading activity in the market. Due to the fact, though, that the absolute change in the net earnings variable is significantly smaller compared to the previous year, trading activity occurs at a lesser degree. Finally, in contrast to the previous period, extrapolators (conservatives) decrease (increase) their proportion of the fleet shares. The procedure described in this paragraph continues, at a decreasing rate, until the net earnings variable converges to its long-term mean. After this point, the economy has reached its steady state (as depicted by the solid black line in the graphs), and consequently, there is neither overvaluation, nor trading activity in the market.

We now examine our economy after a negative shock of equal magnitude. Once again, at $t = 0$ the model is in its steady state. Accordingly, there is no deviation between the agent-specific valuations of the vessel, and as a result, no trading activity is observed in the market. In year 1, we produce a negative $2$ million shock (i.e. a two standard-deviation shock). Regarding the behaviour of net earnings, this is the mirror image of the positive shock case. In particular, extrapolators expect the net earnings variable to revert to its long-term mean slower than conservatives, who in turn, also expect increased persistence compared to the actual one (Panel A of Figure 8).

Due to extrapolation of fundamentals, the agent-specific valuations of the risky asset are substantially lower compared to the fundamental one. As a result, also the equilibrium price of the vessel is noticeably below its fully rational value. While for extrapolators, both the equilibrium price and conservatives’ valuation are extremely above their valuation of the asset, for conservatives, the equilibrium price is below their subjective valuation. In turn, this implies that extrapolators (conservatives) consider the asset to be overvalued (undervalued) compared to the prevailing market price (Panel B of Figure 8). Consequently, extrapolators’ (conservatives’) demand for the 5-year old vessel is decreased (increased) compared to the steady state of the economy. Exactly the same principles hold for the valuation of the corresponding 6-year old vessel at $t = 1$. Therefore, also extrapolators’ (conservatives) demand for the 6-year old vessel is decreased (increased), compared to their demand for the respective 5-year old vessel one period before (Panel C of Figure 8).
Figure 8: Model-Implied Impulse Response Functions Following a Negative Shock.

Figure 8 displays model-implied impulse response functions following one negative $2 million shock to net earnings. Panels A-D correspond to the case of heterogeneous extrapolators, related to the simulation exercise. Panel A illustrates the actual evolution of net earnings, and the evolution perceived by each extrapolator type. Panel B shows the model-generated 5-year old vessel prices, and the agent-specific valuation. Panel C demonstrates the agent-specific share demands for the 5 and 6-year old vessels. Finally, Panel D, plots the trading activity in the market. The horizontal solid black line in each panel shows the steady state value of the corresponding variable.
In particular, at \( t = 1 \) extrapolators’ demands for both the 5 and 6-year old vessels are equal to zero. Namely, equations 14a and 18a suggest that for the parameter values incorporated in this section, the two exit points are higher than the corresponding net earnings variable at \( t = 2 \). Namely, \( \Pi_{t=1} = 1.1, \Pi_{5}^c = 2.44 \) and \( \Pi_{5}^e = 2.42 \). Therefore, both extrapolators’ exit from the market and as a result, 5 and 6-year old vessel prices are given by equations 14b and 18b, respectively. The degree of undervaluation in this case is estimated through (25). As an immediate effect of this rapid change in demand, there is significant trading activity in the second-hand market for vessels. In particular, extrapolators (conservatives) reduce (increase) their relative fractions of the risky asset (Panel D of Figure 8). Since the short-sale constraints are binding, however, trading activity is given by equation 33. As analysed extensively in the previous subsection, due to the chosen parametrisation, this value is significantly lower compared to the respective one in the positive shock case (equation 30e).

In year 2, net earnings increase, evolving towards their long-term mean. However, they are still below both exit thresholds, \( \Pi_{5}^c \) and \( \Pi_{5}^e \) (since \( \Pi_{t=2} = 1.94 \)). Therefore, extrapolators stay out of the market, and there is no trading activity during this date. In year 3, net earnings are slightly higher than \( \Pi_{5}^e \), but still below \( \Pi_{5}^c \) (\( \Pi_{t=2} = 2.43 \)). Hence, while extrapolators related to the 5-year old vessel stay out of the market, the ones related to the 6-year old asset enter. Accordingly, there is rather insignificant trading activity, given by equation 34. In the next year, though, activity becomes noticeably higher, since the demand for the 6-year old vessel is substantially higher than the zero demand for the 5-year old vessel in year 4. Consequently, trading activity at this period, which is again quantified through (34), increases significantly. In order to illustrate that, it suffices to compare equation 34 at \( t = 3 \) and \( t = 4 \). At \( t = 3 \), the net earnings variable is further below \( \bar{\Pi} \), compared to \( t = 4 \). Hence, \( \Pi_3 - \bar{\Pi} < \Pi_4 - \bar{\Pi} < 0 \). However, \( \Pi_5^c < 0 \), thus, \( \Pi_5^e (\Pi_3 - \bar{\Pi}) > \Pi_5^e (\Pi_4 - \bar{\Pi}) > 0 \). Since the second term in the absolute value in (34) is negative, the absolute value at \( t = 3 \) is smaller than the one in \( t = 4 \). Finally, from this point and onwards, both agents are present in the market, and trading activity is strictly decreasing with time, until the net earnings variable converges to its long-term mean, at which point it becomes zero.\(^{25}\)

In conclusion, in line with the theoretical predictions of the model and the sensitivity analysis, those impulse response functions further clarify the mechanism that creates positive correlation between net earnings and trading activity in our model. Namely, a high fraction of conservatives accompanied by a significant degree of heterogeneity in investors’ beliefs result in this positive relation.

\(^{25}\)In particular, trading activity is estimated through (30e). Accordingly, since net earnings changes, after a one-time shock, are a decreasing function of time, (30a) also decreases with \( t \). Similar to the positive shock case, we can use equation 30b in order to estimate precisely the agent-specific demand changes.
III.E. Expectations of Returns and Realised Returns

This subsection presents further results related to agent-specific expectations about future returns and the corresponding realised returns. In line with Section II, agent $i$’s one-period expected return from operating the vessel for the interval between its fifth and sixth year of economic life is given by

$$R_t^i \equiv R_{t \rightarrow t+1}^i = \frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t \equiv \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - P_{5,t}. \quad (40)$$

Since at each $t$ there is one market clearing price, the agent $i$’s expected return depends on the agent-specific beliefs and the current realisation of the net earnings variable. In particular, the numerator is, ceteris paribus, an increasing function of $\rho_i$. As a result, during prosperous market conditions, i.e. $\Pi_t > \bar{\Pi}$, extrapolators have higher expectations about future returns compared to conservatives, and vice versa. Equivalently, during prosperous market conditions extrapolators are more eager to invest compared to conservatives. The inverse is true during deteriorating market conditions.

Accordingly, it would be interesting to compare which investor’s expectations are on average closer to the realised returns in the market one period after. In order to perform this comparison, we define the agent-specific prediction discrepancy, $D_t^i$, as the absolute deviation between agent $i$’s expected return and the actual return. Mathematically

$$D_t^i = |R_t^i - R_t^a|, \quad (41)$$

where the actual returns, denoted by $R_t^a$, are estimated in the usual manner

$$R_t^a \equiv R_{t \rightarrow t+1}^a = \frac{P_{6,t+1} + \Pi_t - P_{5,t}}{P_{5,t}}. \quad (42)$$

In equation 42, $P_{5,t}$ and $P_{6,t+1}$ are estimated through Proposition and Corollary 1, respectively. Plugging (41) and (42) in equation 40 yields

$$D_t^i = \left|\frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t \equiv \bar{\Pi}) + 21\bar{\Pi} - X_5^i \sigma_\varepsilon^2 Q - \Pi_t - P_{6,t+1}\right| \frac{1}{P_{5,t}}. \quad (43)$$

Equivalently, the numerator is the absolute difference between the expected one-period income and the realised one. In the heterogeneous-agent economy, the discrepancy is generated due to two
factors. The first, more straightforward one, is the stochasticity of the net earnings variable. The second is the determination mechanism of the equilibrium market price. Since both agents neglect the strategy of the other type, there is further discrepancy between the actual and the expected returns. In particular, both agents’ valuation and in turn their investment strategy are based on the misbelief that the price of the vessel will revert to its fair value (fair according to their beliefs) within one period. If conservatives explicitly incorporated in their valuation the strategy of extrapolators, then on one hand they would be able to exploit the fact that they are holding relatively more correct beliefs, and on the other, they would always have more accurate returns expectations. Due to competition neglect, however, the equilibrium price depends on a complex weighted average of both agents’ beliefs. As a result, the equilibrium price apart from the net earnings process, heavily depends on the beliefs of the larger population fraction in the economy.

In order to further clarify this argument, we simulate 10,000 paths for the heterogeneous-agent economy, using three key parameterisations which differ only in the relative fraction of each agent-type. Namely, the agent-specific perceived persistence is the same as in the previous two subsections; that is, $\rho_c = 0.65$ and $\rho_e = 0.99$. We incorporate exactly the same procedure and basic parameter values as in subsection III.B. For each valid path we estimate the mean and standard deviation of each agent $i$’s expected return; the mean and standard deviation of the actual returns; the mean and standard deviation of the agent-specific prediction discrepancy. Accordingly, we estimate the mean value of each statistic across all valid paths.

In addition, we estimate the expected returns, $R_t^f$, the actual returns, and the discrepancy between the two, $D_t^f$, in the counterfactual benchmark rational economy. In this case, the expected return formula is simplified to

$$R_t^f \equiv R_{t-c+1}^f = \frac{Y_5^f \sigma_e^2 Q}{1 - \rho_f^{-21} (\Pi_t - \bar{\Pi}) + 21 \bar{\Pi} - [X_5^f + Y_5^f] \sigma_e^2 Q}.$$  \hspace{1cm} (43)

By construction, if no shock occurs between two consecutive periods, the rationally expected return is equal to the actual one. Furthermore, in the steady state equilibrium of the rational economy, the expected return, $R_t^*$, is

$$R_t^* = \frac{Y_5^f \sigma_e^2 Q}{21 \bar{\Pi} - [X^f + Y^f] \sigma_e^2 Q}.$$
For our parameter values, this is approximately equal to 0.1044. Therefore, we should expect that after a large number of simulations, both the actual and the expected returns in the rational economy will converge to this value.

Table 4 summarises the statistics of interest for the three different parameterisations of the heterogeneous-agent economy (Panels A-C) and the benchmark rational economy (Panel D). In addition, Figure 8 presents a probability density function of each agent’s prediction discrepancy, including the observations from all valid paths.

Table 4: Expected Returns, Realised Returns, and Discrepancy.

The table summarises the mean and standard deviation of the quantities of interest presented in the left column, for three different populations compositions. Panel A presents the case where conservatives constitute a very large fraction of the population. Panel B illustrates the case where each agent type constitutes half of the population. Panel C summarises the case where extrapolators constitute a very large fraction of the population. Finally, Panel D presents the corresponding results for the benchmark rational economy.

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<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
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<tr>
<td>Panel A: Expected Returns, Realised Returns and Discrepancy for ({0.95,0.65,0.9})</td>
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<td>Panel B: Expected Returns, Realised Returns and Discrepancy for ({0.5,0.65,0.9})</td>
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</table>

The results in Table 5 illustrate the previously analysed arguments. In particular, for a large fraction of conservatives, their average expectation discrepancy is extremely smaller than extrapolators’, that
Figure 9 presents probability density functions of agent-specific discrepancies between the expected and the realised returns. The incorporated parameterisation is $\mu = 0.95$, $\rho_e = 0.65$ and $\rho_e = 0.9$. Panel A illustrates the case for conservatives and Panel B for extrapolators.
is severe. Figure 9 depicts these results. In support of this finding, conservatives’ average expected return is closer to the average realised return than extrapolators’. Moreover, the standard deviations of both expected returns and discrepancy are particularly low, especially compared to extrapolators’ (Panel A). As mentioned above, conservatives’ discrepancy is mainly attributed to the stochasticity of the error term and their slight extrapolative expectations, and secondarily to the competition neglect, since extrapolators constitute a very small fraction of the population. The inverse is true for extrapolators. Following the same line of reasoning, the relatively small standard deviation of the actual return is mainly attributed to the volatility of cash flow shocks and secondarily to agents’ competition neglect and extrapolative expectations. Finally, note that in this case the model-generated average actual return in the market is significantly close to the actual average one-period return in our dataset, which is equal to 0.0902.

In contrast, when extrapolators hold the largest fraction of the population, both agent-specific discrepancies are severe (Panel C). In the case of conservatives, this discrepancy is mainly attributed to their competition neglect, since they constitute a very small fraction of the population, and secondarily to the stochasticity of the error term and their extrapolative expectations. In the case of extrapolators, the inverse is true. Accordingly, the severe standard deviation of actual returns is mainly attributed to agents’ extrapolative expectations and competition neglect, and secondarily to the stochasticity of net earnings. In contrast to the previous case, with this parameterisation, the model-generated average actual return in the market substantially deviates from the empirical value of the average one-period return. The case where each agent-type constitutes half of the populations lies somewhere in the middle of those two extreme scenario. Namely, conservatives’ expectations about future returns are closer to the actual returns because they hold more rational beliefs about the net earnings process compared to extrapolators. However, conservatives’ expectations are highly inaccurate since extrapolators’ fraction is significant.

Finally, Panel D of Table 8 shows that while the average expectations of rational investors converge to the average realised returns there still exists an average discrepancy between the two values. This relatively small discrepancy is solely attributed to the volatility of the cash flow shock. In line with this argument, the standard deviations of both the expected return and the discrepancy are substantially low. Regarding the former, we observe that expected returns have almost zero standard deviation; hence, in the benchmark rational economy, investors have essentially constant required returns.

III.F. Changes in Investor Wealth

The results in the previous subsection suggest that for a parameterisation of the model sufficient to reproduce the observed regularities, i.e. \( \{0.95, 0.65, 0.9\} \), extrapolators have both more volatile
expected returns and more inaccurate expectations compared to conservatives. Therefore, one should expect that the former will have significantly more skewed distribution of one-period wealth changes than the latter. In order to examine this prediction, we estimate agent $i$’s one-period change in wealth, $\Delta w_{t+1}^i$, through equation 4. Namely,

$$\Delta w_{t+1}^i = N_{s,t}^i (\Pi_t + P_{6,t+1} - P_{5,t})$$

(44)

that is, the one-period change in wealth of agent $i$ equals his time $t$ holdings of the risky asset multiplied by the realised net income at $t$. Figure 10 illustrates the probability density functions of both agents one-period changes in wealth. The most striking feature of these simulations is that extrapolators realise a zero change in one-period wealth with a probability approximately equal to 27.5%. This is an immediate consequence of the fact that extrapolators exit from the market rapidly as market conditions deteriorate. When they are present in the market, however, the substantial inaccuracy and volatility of their expectations regarding future returns result in very volatile one-period wealth changes. Accordingly, the probability distribution of their one-period wealth change is significantly skewed. Namely, the mean, standard deviation, skewness, and excess kurtosis of the distribution are equal to 2.29, 6.28, 1.19, and 6.9, respectively.

In contrast, as illustrated in the sensitivity analysis, conservatives are present effectively during the entire shipping market cycle, since their exit point is significantly high. In particular, we observe that conservatives change in wealth closely resembles a normal distribution with mean, standard deviation, skewness, and excess kurtosis equal to 2 and 2.94, 0.15, and -0.04 respectively. As analysed in the previous subsection, conservatives realised returns heavily depend on the stochasticity of the error term. In turn, the normally distributed error term combined with fact that these investors are always present in the market result in this probability density function. In conclusion, while extrapolators’ one-year strategy offers a 0.14 times higher mean wealth increase compared to conservatives’ one, it is more than two times as volatile as the latter. This fact is in line with our assumption that extrapolators are in general more risk tolerant compared to conservatives.

IV. Robustness

In this section we test the robustness of our model’s predictions by examining several alternative hypotheses regarding the characteristics of the investor population. Accordingly, we compare the corresponding simulation results to the empirical values and the results obtained from our basic setting in subsection III.B. The environment of our model and the respective empirical estimation were based on the assumption that the economy consists of conservative and extrapolative agents, holding heterogeneous beliefs regarding he cash flow process. Basically, this heterogeneity was modelled
Figure 10 presents probability density functions of agent-specific one-period changes in wealth. The incorporated parameterisation is $\mu = 0.95$, $\rho_e = 0.65$ and $\rho_e = 0.9$. Panel A illustrates the case for conservatives and Panel B for extrapolators.
through the condition $\rho_0 \leq \rho_c < \rho_e < 1$. The proposed framework, however, can be easily extended in order to incorporate alternative investor population specifications. Accordingly, in this section we examine five alternative hypotheses. We allow our economy to consist of (i) contrarians and fundamentalists, (ii) contrarians and extrapolators, (iii) fundamentalists, (iv) extrapolators and (v) contrarians.

We introduce contrarian investors, denoted by $x$, in a straightforward manner. Namely, we assume that they hold irrational beliefs regarding the net earnings process in the inverse way compared to extrapolators; that is, they overestimate the mean reversion of the net earnings variable. Equivalently, they assume that after a cash flow shock, net earnings will revert to their long-term mean significantly quicker than the actual stochastic process suggests. Accordingly, their perceived persistence of the net earning variable, $\rho_x$, lies in the interval $[0, \rho_0)$. As a result, during market peaks and troughs contrarians undervalue and overvalue vessels, respectively. Apart from this feature, contrarians behave exactly as the other agent types. In particular, they also neglect the future demand responses of the other types and they upgrade the perceived riskiness of the investment as they grow older. Therefore, Proposition 1 and Corollaries 1-5 can be directly extended in order to capture this alternative specification. Finally, in order to examine the predictions of our model for the three homogeneous settings, we simply set $\mu = 1$ and in turn define the respective perceived persistence of the agent.

Table 5 summarises the results obtained from these alternative hypotheses, for a variety of investor population characteristics. For reasons of brevity, we present only the statistics related to the main quantities of interest. Similar to subsection III.D., we examine the predictions of our model for different combinations of the three main parameters, $\{\mu, \rho_x, \rho_i\}$, across 10,000 simulated paths. Accordingly, we report the average value of each quantity across these paths. The basic value parameters are the ones summarised in Table 2. In addition, we have set contrarians coefficient of absolute risk aversion, $\alpha_x$, equal to 0.55; that is, we have assumed that this agent is more risk averse compared to fundamentalists, conservatives and extrapolators.

Evidently, the results in Table 5 suggest that these alternative hypotheses are not able to simultaneously match in a sufficient manner the empirical values. To begin with, in the heterogeneous-agent scenario (Panels B and C), we observe that the main effect of contrarians’ presence in the market is the attenuation of the vessel price volatility. This should be a priori expected, however, since as analysed extensively in the previous sections, price volatility is an increasing function of the perceived persistence. In order to illustrate this fact, consider the case of the homogeneous-contrarian economy. In line with equation 11, the volatility of contrarian prices equals $1.11 \sigma(\Pi_t)$, which for a given sequence of net earnings shocks is significantly lower that the volatility
of fundamental prices, $2.39\sigma(\Pi_t)$. Therefore, in terms of price volatility, contrarians have the opposite effect in the market compared to extrapolators; that is, they generate “lack of volatility”.

Following Corollary 4 and the sensitivity analysis in Section III, the agent with the lower (higher) perceived persistence exits from the market during prosperous (deteriorating) market conditions. Therefore, contrarians’ exit point will lie somewhere in the interval $(\bar{\Pi}, \infty)$, depending on the precise population characteristics. As a result, in an economy consisting of contrarians and fundamentalists (Panel B), the latter will be exiting form the market during market troughs. Incorporating this observation, it is straightforward to extend the analysis conducted in Section III in order to interpret the remaining results in Table 6. As a characteristic example, consider the contrarian-extrapolator economy in Panel B. In line with Section III, a very small fraction of extrapolators combined with a sufficient degree of heterogeneity beliefs results in low average trading activity and positive correlation between trading activity and net earnings. However, due to the large population fraction of contrarians the volatility of vessel prices is significantly reduced; namely, it has been approximately halved compared to the respective fundamental economy. Accordingly, the specifications that generate “excess volatility” cannot simultaneously approach the significant positive correlation between trading activity and net earnings.

Finally, in the homogeneous-agent economy (Panel D) there is no trading activity in the market, since the heterogeneity of beliefs is what motivates trading in our model. Regarding the price volatility statistic, the homogeneous extrapolative economy can generate significant “excess volatility”, which for a given sequence of cash flow shocks is equal to $[(1 - \rho_e^{z_1})(1 - \rho_f)]/[(1 - \rho_e)(1 - \rho_f^{z_1})]$. Therefore, by appropriately adjusting the value of the perceived persistence of extrapolators, it is straightforward to generate the observed “excess volatility” statistic. In contrast, as analysed above, since $\rho_x < \rho_f$ the homogeneous contrarian economy generates “lack of price volatility”.

In conclusion, we have illustrated that the incorporation of a variety of alternative hypotheses regarding the investor composition, cannot reproduce the results generated by the conservative-extrapolator economy and in turn cannot approach the corresponding stylised facts. Of course, there exists a variety of alternative model extensions that can be considered. As an example, it is straightforward to model the coexistence of contrarians, fundamentalists, and extrapolators in the economy. However, the results obtained from this extension are not expected to either improve the fit of the model regarding the main quantities of interest or to alter the economic interpretation of the results.

It is of utmost importance to note that this article provides a plausible explanation for a number of stylised facts observed in the second-hand market for vessels. In particular, in line with recent developments in the financial economics literature, we motivate and explain the existence of “excess
Table 5: Model predictions for the quantities of interest under alternative hypotheses.

This table summarises the heterogeneous-agent model's predictions for the quantities of interest presented in the first row. Table 2 summarises the basic model parameters. In addition, we have set $\alpha_x = 0.85$. Panel A presents the empirical values of these quantities for the period 1989-2014. Notice that trading activity data are available from 1995. Panels B-D report the average value of each quantity across 10,000 simulated paths, for a given parametrisation depending on the population characteristics. The second column refers to the mean of the 5-year old vessel prices; the third column refers to the ratio of the standard deviation of 5-year old vessel prices in the extrapolative heterogeneous-agent economy to the standard deviation of the fundamental value of the 5-year old asset, under the same net earnings sequence. Columns four, five, and six present the mean of the annual trading activity, estimated through (29), the correlation between net earnings and trading activity, and the correlation between trading activity and absolute one-year changes in the net earnings variable, respectively.

<table>
<thead>
<tr>
<th>Population Characteristics</th>
<th>Average Price</th>
<th>Price Volatility</th>
<th>Average Trading Activity</th>
<th>Correlation Between Activity and Net Earnings</th>
<th>Correlation Between Activity and Absolute Net Earnings Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Actual Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>22.86</td>
<td>1.34</td>
<td>0.06</td>
<td>0.53</td>
<td>0.65</td>
</tr>
<tr>
<td>Panel B: Contrarians and Fundamentalists</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>${0.1,0.1,0.58}$</td>
<td>22.93</td>
<td>0.51</td>
<td>0.04</td>
<td>0.07</td>
<td>0.99</td>
</tr>
<tr>
<td>${0.5,0.1,0.58}$</td>
<td>22.96</td>
<td>0.72</td>
<td>0.12</td>
<td>0.02</td>
<td>1.00</td>
</tr>
<tr>
<td>${0.95,0.1,0.58}$</td>
<td>23.00</td>
<td>0.97</td>
<td>0.02</td>
<td>-0.12</td>
<td>0.95</td>
</tr>
<tr>
<td>Panel C: Contrarians and Extrapolators</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>${0.1,0.1,0.9}$</td>
<td>23.97</td>
<td>3.42</td>
<td>0.16</td>
<td>-0.46</td>
<td>0.67</td>
</tr>
<tr>
<td>${0.5,0.1,0.9}$</td>
<td>24.55</td>
<td>2.09</td>
<td>0.29</td>
<td>-0.03</td>
<td>0.68</td>
</tr>
<tr>
<td>${0.95,0.1,0.9}$</td>
<td>22.99</td>
<td>0.53</td>
<td>0.06</td>
<td>0.41</td>
<td>0.77</td>
</tr>
<tr>
<td>${0.5,0.1,0.65}$</td>
<td>22.98</td>
<td>0.81</td>
<td>0.16</td>
<td>-0.01</td>
<td>0.99</td>
</tr>
<tr>
<td>Panel D: Homogeneous-Agent Economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_f = 0.58$</td>
<td>23.00</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_e = 0.9$</td>
<td>23.72</td>
<td>3.73</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_x = 0.1$</td>
<td>22.93</td>
<td>0.47</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
volatility” and trading activity in the market through a heterogeneous beliefs behavioural approach that lies on the intersection of microeconomics and asset pricing. To the best of our knowledge, this is the first structural microeconomic model in the literature aiming to explain the second-hand market for vessels. While there can be alternative explanations for trading activity (e.g. limits to arbitrage, portfolio diversification policies, other information frictions) or “excess price volatility”, the motivation provided has the advantage of simultaneously explaining these two key findings, and furthermore, the economic interpretation of the model and the respective results are plausible and in line with the nature of the shipping industry.

Finally, for robustness, we have also estimated empirically the proposed framework using monthly data on an annual frequency. Namely, we follow the procedure described in subsection III.B., after making the necessary adjustments to the corresponding dataset. The main predictions of the model are both qualitatively and quantitatively similar to the ones obtained with annual data. Namely, a large fraction of conservatives combined with significant heterogeneity of beliefs generates both “excess volatility” of prices, low average trading activity, and positive correlation between market conditions and net earnings. Therefore, our model’s predictions are robust, irrespective of the frequency of the dataset.

IV. Conclusion

In this article, we examine the market for second-hand vessels related to the Handysize dry bulk sector. In particular, our partial equilibrium framework simultaneously investigates the formation of vessel prices and trading activity in the market. For this purpose, we develop a behavioural asset-pricing model with microeconomic foundations. Our discrete time economy consists of two agent types, conservatives and maximalists, who differ only in the way in which they form expectations about future net earnings. Specifically, agents form their demand for the vessel by extrapolating current net earnings, and under(over)estimating the future demand responses of their competitors. As a result, prices fluctuate significantly more compared to the rational benchmark of the model. Moreover, while a model with homogeneous extrapolative beliefs can capture sufficiently well the observed price behaviour, it cannot account for the second-hand market activity. Accordingly, by incorporating a heterogeneous beliefs framework, we are able to capture the positive relation between trading activity and both current market conditions and absolute changes in market conditions. As a by-product, our model also accounts for the low liquidity that characterises this market.

Both theoretical predictions and empirical estimation of the model suggest that in order to simultaneously capture these stylised facts, conservatives must constitute a very large fraction of the
population, while shipping investors of both types must hold extrapolative expectations. Furthermore, after a significant number of simulations, our model’s predictions are consistent with additional stylised facts in the shipping literature, such as the forecasting power of the net earnings yield. In conclusion, the proposed general equilibrium framework provides a first step towards the modelling of the joint behaviour of net earnings, vessel prices and trading activity, which to the best of our knowledge, had never been examined from the perspective of a structural, behavioural economic model in the shipping literature before.

Appendix

A. Derivation of the Demand Functions for the 5-year Old Vessel

We begin by estimating the time $t$ demand function for the 5-year old vessel, for each agent type. Notice that for notational simplicity, the age index corresponding to the 5-year old vessel is dropped in these derivations. Therefore, $N^i_t$ and $P_t$ refer to the time $t$ agent $i$’s demand for and price of the 5-year old vessel, respectively. Accordingly, we have omitted the age index from all parameters and constants. Since vessels are real assets with limited economic lives, we can estimate this demand recursively. Specifically, at the terminal date, $T$, the price of the vessel has to be equal to the cash flow realised on that date. In reality, this cash flow is equal to the scrap price of the vessel. However, since this scrap price is substantially correlated to the net earnings variable, we impose the simplifying assumption that the scrap price is equal to the net earnings variable corresponding to period $T$.

Hence, $P_T = \Pi_T$. Using equation 5 of the main text, agent $i$’s objective at time $T - 1$ is

$$\max_{N^i_{T-1}} \mathbb{E}_{T-1} \left[ -e^{-\alpha(w^i_{T-1} + N^i_{T-1}(\Pi_{T-1} + P_T - P_{T-1}))} \right].$$

(A1)

Using the fact that $P_T = \Pi_T$, and accordingly, incorporating (2) of the main text, results in

$$\max_{N^i_{T-1}} -e^{-\alpha(w^i_{T-1} + N^i_{T-1}((1+\rho)i\Pi_{T-1} + (1-\rho)i\bar{\Pi} - P_{T-1}))} + \frac{\alpha^2 i^2}{2} \sigma^2 \varepsilon^2.$$  

(A2)

Hence, agent $i$’s first order condition yields

$$N^i_{T-1} = \frac{(1 + \rho)i\Pi_{T-1} + (1 - \rho)i\bar{\Pi} - P_{T-1}}{\alpha^2 i^2 \sigma^2 \varepsilon^2}.$$  

(A3)

26 Alternatively, it is straightforward to assume either a scrap value given by an AR(1) process where the long-term mean is equal to the average scrap value in our sample and the random (white noise) term is highly correlated to the error term in (1) and (2). Furthermore, we could also assume a zero terminal value of the asset.
The market clearing condition at $T - 1$, along with A3, suggest that

$$\Rightarrow P_{T-1} = (1 + \rho_i)\Pi_{T-1} + (1 - \rho_i)\Pi - \frac{\alpha^i\theta^i\sigma^2}{\mu^i}\left[Q - (1 - \mu^i)N_{T-1}^{-i}\right]. \quad (A4)$$

In a similar manner, at time $T - 2$, trader $i$’s objective is

$$\max_{N_{T-2}^i} \left\{-e^{-\alpha^i(w_{T-2}^i + N_{T-2}^i(\Pi_{T-2} - P_{T-2}))}E_{T-2}^i \left[e^{-\alpha^i(1+\rho_i)\Pi_{T-1} + (1+\rho_i)(1-\rho_i)\Pi - \alpha^i\theta^i\sigma^2 Q - P_{T-2}}\right] \right\}. \quad (A5)$$

Incorporating equation A4, the expectation in (A5) can be expressed as

$$e^{-\alpha^iN_{T-2}^i\left[(1+\rho_i)\Pi_{T-2} + (2+\rho_i)(1-\rho_i)\Pi - \alpha^i\theta^i\sigma^2 Q - P_{T-2}\right]}E_{T-2}^i \left[e^{-\alpha^i(1+\rho_i)\Pi_{T-1} + (1+\rho_i)(1-\rho_i)\Pi - \alpha^i\theta^i\sigma^2 Q - P_{T-2}}\right]. \quad (A6)$$

At this point, we assume that each agent is characterised by an additional form of bounded rationality in the following sense. Agent $i$, instead of taking into account the strategy of agent $-i$, that is, trying to forecast the evolution of $-i$’s demand, he makes the simplifying assumption that, in all future periods, $-i$ will just hold his per-capita fraction of the risky asset supply (Barberis et al, 2015b). Accordingly, he assumes that at each future period, the other agents’ demand share will be equal to the constant $\mu^{-1}Q$. Hence, accounting for this bias, and using equation 2 of the main text, the objective function (A5) is simplified to

$$\max_{N_{T-2}^i} \left\{-e^{-\alpha^i(w_{T-2}^i + N_{T-2}^i(\Pi_{T-2} + (2+\rho_i)(1-\rho_i)\Pi - \alpha^i\theta^i\sigma^2 Q - P_{T-2}))} \right\}. \quad (A7)$$

Therefore, agent $i$’s first order condition implies

$$N_{T-2}^i = \frac{(1 + \rho_i + \rho_i^2)\Pi_{T-2} + (2 + \rho_i)(1 - \rho_i)\Pi - \alpha^i\theta^i\sigma^2 Q - P_{T-2}}{\alpha^i\theta^i(1 + \rho_i)^2\sigma^2}. \quad (A8)$$

Similar to the previous two maximizations problems, agent $i$’s first order condition at time $T - 3$, yields

$$N_{T-3}^i = \frac{(1 + \rho_i + \rho_i^2 + \rho_i^3)\Pi_{T-3} + (3 + 2\rho_i + \rho_i^2)(1 - \rho_i)\Pi - \alpha^i\theta^i\sigma^2 [1 + (1 + \rho_i)^2]Q - P_{T-3}}{\alpha^i(1 + \rho_i + \rho_i^2)^2\theta^i\sigma^2}. \quad (A9)$$

Extending the pattern illustrated in equations A3, A8, and A9 up to 20 periods before the end of the vessels’ economic activity, and applying basic properties of geometric series, we obtain:
\[ N_t^i = \frac{1 - \rho_i^{21}}{1 - \rho_i} (\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - X^i \sigma_e^2 Q - P_t \]  
\[ Y_t = \frac{1 - \rho_i^{20}}{1 - \rho_i} \alpha_i \theta_i \]  
\( \pi_t \leq \bar{\Pi} + \frac{(\chi^m - X^c)\sigma_e^2 Q}{1 - \rho_m^{21}} - \frac{1 - \rho_c^{21}}{1 - \rho_c} \)  

B. Proof of Proposition

In order to prove the Proposition, it is convenient to define the aggregate demand at time \( t \) as
\[ N_t = \mu N_t^\xi + (1 - \mu) N_t^\xi', \]  
where \( N_t^\xi \) is given by equation 9. To begin with, we can directly observe that the lower the price of the vessel, the higher the value of aggregate demand. Vice versa, demand can be equal to zero for a sufficiently high value of the vessel price variable. Formally, aggregate demand is a continuous function of the vessel price, \( P_t \). Moreover, it is a strictly decreasing function of \( P_t \) (as a sum of strictly decreasing functions), with a minimum value of zero. Accordingly, since the market supply of vessels cannot be negative, there always exists a (positive) vessel price at which the aggregate demand for the risky asset at time \( t \) is equal to the aggregate supply of the vessel, \( Q \). Due to monotonicity of the aggregate demand function, this price is unique. We call this value “market clearing price” or “equilibrium price” of the vessel at each \( t \), and we denote it by \( P_t^\ast \).

Accordingly, we determine this equilibrium price by proceeding similar to Barberis et al (2015b). In particular, we begin by defining the price at which investor \( i \)'s short-sale constraint binds at time \( t \)
\[ \tilde{P}_t = \frac{1 - \rho_t^{21}}{1 - \rho_t} (\Pi_t - \bar{\Pi}) + 21\bar{\Pi} - X^i \sigma_e^2 Q. \]  

Since \( \frac{1 - \rho_t^{21}}{1 - \rho_t} \) is an increasing function of the perceived net earnings’ persistence, \( \rho_t \), when market conditions deteriorate, and specifically, when:

\[ \pi_t \leq \bar{\Pi} + \frac{(\chi^m - X^c)\sigma_e^2 Q}{1 - \rho_m^{21}} - \frac{1 - \rho_c^{21}}{1 - \rho_c} \]  

\( \text{As a sum of continuous functions. Notice that } \max(f(x), 0) \text{ is continuous for all continuous } f; \text{ and in our case, } f(P_t), \text{ which is given by plugging (18) in equation 6, is a continuous function of } P_t. \)
then $\tilde{P}_t^m \leq \tilde{P}_t^c$, that is, the cut-off price of maximalists is (equal to or) lower compared to the one of conservatisms. Vice versa, during a market upturn, and in particular when:

$$\Pi_t \geq \Pi + \frac{(X^m - X^c) \sigma_\varepsilon^2 Q}{1 - \rho_m^{21} - \frac{1 - \rho_c^{21}}{1 - \rho_c}}$$

then $\tilde{P}_t^c \leq \tilde{P}_t^m$; namely, the cut-off price of conservatisms is (equal to or) lower than the one of maximalists.

In order to simplify the illustration, we denote the highest and lowest cut-off prices at time $t$ by $\tilde{P}_t^1$ and $\tilde{P}_t^0$, respectively, so that $\tilde{P}_t^1 \geq \tilde{P}_t^0$. Furthermore, we define the aggregate demand when the price is equal to $\tilde{P}_t^i$ as $N_{\tilde{P}_t^i}$. Due to the fact that demand is strictly decreasing in vessel price,

$$\tilde{P}_t^1 \geq \tilde{P}_t^0 \implies 0 = N_{\tilde{P}_t^1} \leq N_{\tilde{P}_t^0}.$$

Accordingly, we have to distinguish between two scenarios. First, assume that $N_{\tilde{P}_t^0} < Q$, that is, the aggregate demand at the lowest cut-off price at time $t$ is lower than the market supply of vessels. Due to market clearing, however, total demand has to adjust in order to be equal to total supply at each point in time. Therefore, aggregate demand at time $t$, $N_t$, will increase, and accordingly, it will become higher than $N_{\tilde{P}_t^0}$. In order, though, for demand to increase, price has to decrease beyond $\tilde{P}_t^0$, which is the lowest cut-off price at this point in time (since aggregate demand is a strictly decreasing function of the price). In turn, this price decrease implies that the demand of the trader with the lowest cut-off price becomes positive as well. Hence, in this scenario, all traders in the market have strictly positive demand. Accordingly, substituting equation 8a in the market clearing condition, (6), and rearranging for $P_t$, we obtain the equilibrium price of the vessel

$$P_t^{c+m} = 21\Pi + \frac{\mu Y^c X^c + (1 - \mu) Y^c X^m + Y^c Y^m}{\mu Y^m + (1 - \mu) Y^c} \sigma_\varepsilon^2 Q.$$

This corresponds to equation 13b of the main text.
Second, assume that $N_{\tilde{P}_t^m} \leq Q \leq N_{\tilde{P}_t^1}$.

Due to the fact that aggregate demand is a strictly decreasing function of the price, it follows that the equilibrium price belongs in the interval defined by the lowest and the highest cut-off prices; that is, $\tilde{P}_t^0 \leq P_t^* \leq \tilde{P}_t^1$. Accordingly, in equilibrium, only the agents with the highest cut-off price will have strictly positive demand for the vessel. Intuitively, once again, due to the market clearing condition, the aggregate demand for the risky asset has to be equal to the aggregate supply. Therefore, in order for the aggregate demand to be equal to supply, the price has to be lower than $\tilde{P}_t^1$ and higher than $\tilde{P}_t^0$. When, however, price is lower than the highest cut-off price, the corresponding agents’ demand becomes positive; thus they are in the market. At the same time, though, the price while lower than the highest cut-off price, still remains higher than the lowest one. Therefore, the corresponding agent type has zero demand, and therefore, stays out of the market. In conclusion, in this second scenario, only one type of agent is active in the market. Which type is this, and therefore, the determination of the equilibrium price, depends on the prevailing market conditions.

Specifically, as mentioned above, in a market downturn (as defined by condition B2)

$$\tilde{P}_t^0 = \tilde{P}_t^m \leq \tilde{P}_t^c = \tilde{P}_t^1 \Leftrightarrow N_{\tilde{P}_t^m} \leq N_{\tilde{P}_t^c}.$$  

When market conditions sufficiently deteriorate, the demand of maximalists becomes zero, and only conservatives have strictly positive demand. Therefore, for $N_{\tilde{P}_t^m} = 0$, from equation 8a along with the market clearing condition (6), we obtain the equilibrium price of the vessel in the scenario where only conservatives hold the risky asset

$$P_t^* = 21 \bar{\Pi} \frac{1 - \rho_c}{1 - \rho_c} (\Pi_t - \bar{\Pi}) - \left[ X^c + \frac{Y^c}{\mu} \right] \sigma^2 \epsilon Q. \quad (B5)$$

This equation corresponds to (14b) of the main text. Furthermore, it is straightforward to find the critical point at which maximalists exit the market. Namely, at this point, the short-sale constraint of maximalists is binding; hence the equilibrium price of the market is given also by equation B1. Since, the equilibrium price at each $t$ is unique, by equating B1 with B2, we can obtain the value of the net earnings variable at which maximalists exit the market, $\Pi_{t}^m$. Accordingly,

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28 Obviously, since the aggregate supply of the risky asset cannot be negative, we cannot observe the scenario where $Q < N_{\tilde{P}_t^1}^c$. 
\[ \Pi^m = \bar{\Pi} + \frac{(X^m - X^c - \frac{Y^c}{\mu})\sigma^2_\xi Q}{\frac{1 - \rho_m^{21}}{1 - \rho_m} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} \]  
(B6)

Condition B6, corresponds to (14a) of the main text. As expected, since \( Y^c \) is positive, \( \Pi^m \) is lower than the value of the net earnings variable indicated by condition B2.

In a similar manner, during a market upturn (as defined by condition B3)

\[ \bar{P}_t^0 = \bar{P}_t^c \leq \bar{P}_t^m = \bar{P}_t^1 \iff N_{\tilde{P}_t^c} \leq N_{\tilde{P}_t^m}. \]

When market conditions significantly improve, the demand of fundamentalists becomes zero, and only maximalists have strictly positive demand. Therefore, for \( N_t^c = 0 \), from equation 8a along with the market clearing condition (6), we obtain the equilibrium price of the vessel in the scenario where only maximalists hold the risky asset

\[ P_{t^m}^* = 21\bar{\Pi} + \frac{1 - \rho_m^{21}}{1 - \rho_m} (\bar{\Pi} - \Pi) - \left[ X^m + \frac{Y^m}{1 - \mu} \right] \sigma^2_\xi Q. \]  
(B7)

This equation corresponds to (15b) of the main text.

Following the same line of reasoning (i.e. equating B1 with B7), the value of the net earnings variable at which conservatives exit the market, \( \Pi_t^c \), is

\[ \Pi^c = \bar{\Pi} + \frac{(X^m - X^c + \frac{Y^m}{1 - \mu})\sigma^2_\xi Q}{\frac{1 - \rho_m^{21}}{1 - \rho_m} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} \]  
(B8)

Condition B8, corresponds to (15a) of the main text. Since \( Y^c \) is positive, \( \Pi_t^c \) is higher than the value of the net earnings variable indicated by condition B3.

In conclusion, the necessary and sufficient conditions for agents to coexist in the market is

\[ \Pi^m = \bar{\Pi} + \frac{(X^m - X^c - \frac{Y^c}{\mu})\sigma^2_\xi Q}{\frac{1 - \rho_m^{21}}{1 - \rho_m} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} < \Pi_t < \bar{\Pi} + \frac{(X^m - X^c + \frac{Y^m}{1 - \mu})\sigma^2_\xi Q}{\frac{1 - \rho_m^{21}}{1 - \rho_m} - \frac{1 - \rho_c^{21}}{1 - \rho_c}} = \Pi^c. \]  
(B9)

Condition B9, corresponds to (13a) of the main text. Furthermore, for our parameter values, B9 implies that when \( \Pi_t = \bar{\Pi} \), both agents are present in the market.

\[ \square \]

C. Proof of Corollary 3
From the previously stated proposition and the definition of the “steady state” equilibrium, it is straightforward to derive equations 20a and 20b of the main text. Specifically, equation 9 combined with the fact that in the steady state $\Pi_t = \bar{\Pi}$, results in

$$N^i_t = \max \left\{ \frac{21\bar{\Pi} - X^i\sigma_e^2 Q - \bar{P}^*_t}{Y^i\sigma_e^2}, 0 \right\}. \quad (C1)$$

Since, however, in the steady state, both agents coexist in the market, type $i$’s time $t$ demand becomes

$$N^i_t = \frac{21\bar{\Pi} - X^i\sigma_e^2 Q - \bar{P}^*_t}{Y^i\sigma_e^2}. \quad (C2)$$

Substituting (C2) in the market clearing condition, we obtain

$$\bar{P}^*_t = 21\bar{\Pi} - \frac{\mu Y^m X^c + (1 - \mu)Y^c X^m + Y^c Y^m}{\mu Y^m + (1 - \mu)Y^c} \sigma_e^2 Q. \quad (C3)$$

Moreover, both types hold the risky asset in analogy to their fraction of the total population if and only if

$$N^i_t = \frac{21\bar{\Pi} - X^i\sigma_e^2 Q - \bar{P}^*_t}{Y^i\sigma_e^2} = Q \Leftrightarrow \bar{P}^*_t = 21\bar{\Pi} - (X^i + Y^i)\sigma_e^2 Q.$$

However, since the steady state equilibrium price is unique

$$X^c + Y^c = X^m + Y^m = X^f + Y^f, \quad (C4)$$

which corresponds to condition 20b of the main text. The above restriction implies that

$$X^i + Y^i = \frac{\mu Y^m X^c + (1 - \mu)Y^c X^m + Y^c Y^m}{\mu Y^m + (1 - \mu)Y^c}. \quad (C5)$$

Incorporating (C5) in (C3) yields equation 20a of the main text, which is a special case of (10) with $\Pi_t = \bar{\Pi}$. Finally, by construction, restrictions C4 ensure that, in the steady state, both agents are present in the market. Namely, the parenthesis and in turn the second term of the right hand side of condition B6 (B8) is negative (positive); hence, $\Pi^m_t < \bar{\Pi} < \Pi^f_t$. Following exactly the same procedure, we obtain the steady state equilibrium conditions for the 6-year old case. 

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References


