A Holistic Visual "Wheel of Relationships" in Production Theory

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Abstract: Production theory in advanced microeconomics comprises two intricate optimization problems: **cost minimization** and **profit maximization**. The cost minimization problem essentially involves deriving conditional factor demand functions and the cost function. The profit maximization problem involves deriving the output supply function, the profit function, and unconditional factor demand functions. Additionally, there are numerous mathematical equations in the setting of production theory which aid us in deriving each of the mentioned functions from another. These include Shephard's lemma, Hotelling's lemmas, direct and indirect mathematical relations, and other mathematical elements. All of these elements contribute to forming a neat wheel of relationships in production theory. However, these complexities and unseen, underlying linkages can bring about difficulties for both instructors to teach as well as students to learn the material. Much of this complexity has its root in the intricacies inherent in the mathematical theory of duality. The primary purpose of this poster is to illuminate these theoretical complexities through a holistic visual wheel of relationships in production theory in order to ease the teaching and learning of advanced production theory. More specifically, this poster introduces two instructional visuals which can be used as complementary teaching tools to demonstrate the connections between the profit-maximization and cost-minimization problems as well as other components of production theory in modern microeconomics. This visual clarification also allows for easier elaboration of three notions of optimality which must exist in any economic production system, i.e. **technical optimality**, **allocative optimality**, and **scale optimality**. The focus of the present poster is on the two extreme market structures: **perfect competition** and **monopoly**.

Symbols and Notations:

max: Maximize
min: Minimize
s.t.: Subject to
x: Vector of Input Quantities
W: Vector of Input Prices
y: Firm's Output Quantity (in general and also for a price-taking firm)
Y: Market Output Quantity (in general sense and also for a monopolistic firm)
C(W,x): Cost Relation (OR, the Amount of Cost)
y=f(x): Production Function
x^C(W,y): Vector of Conditional Factor Demands (conditional on output)
C(W,y): "The" Cost Function (OR, total cost function, i.e. TC)
AC: Average Cost Function
AVC: Average Variable Cost Function
MC: Marginal Cost Function
Zero-Profit Point: The point at which AC=MC
Shutdown Point: The point at which AVC=MC
$P_{Zero-Profit}$: The price corresponding to the Zero-Profit point
$P_{Shutdown}$: The price corresponding to the Shutdown point
P_{PC}: The optimal price for a perfectly competitive firm
P_{MON}: The optimal price for a monopolistic firm
Y_{ZP}: The output level corresponding to the Zero-Profit point
Y_{SD}: The output level corresponding to the Shutdown point
Y^*_{PC}: The optimal quantity for a perfectly competitive firm
y^*_{MON}: The optimal quantity for a monopolistic firm
π(P,W,y): Profit Relation for a perfectly competitive firm
π(P(Y),W,Y): Profit Relation for a monopolistic firm, which finally reduces to π(W,Y)
TR: Total Revenue which equals P.Y for a perfectly competitive firm and P(Y).Y for a monopolistic firm
AR: Average Revenue
MR: Marginal Revenue
y*(P,W): Output Schedule or Supply Function for a perfectly competitive firm
P: Output Price for a perfectly competitive firm
P(Y): Inverse Demand Function for the output produced by a monopolistic firm
Y*(W): Output quantity schedule in terms of input prices for a monopolistic firm
P*(W): Output price schedule in terms of input prices for a monopolistic firm
X^U(W,y): Unconditional Factor Demand for a perfectly competitive firm
X^U(W): Unconditional Factor Demand for a monopolistic firm
Π(P,W): "The" Profit Function for a perfectly competitive firm, in which everything is in terms of prices
Π(W): "The" Profit Function for a monopolistic firm

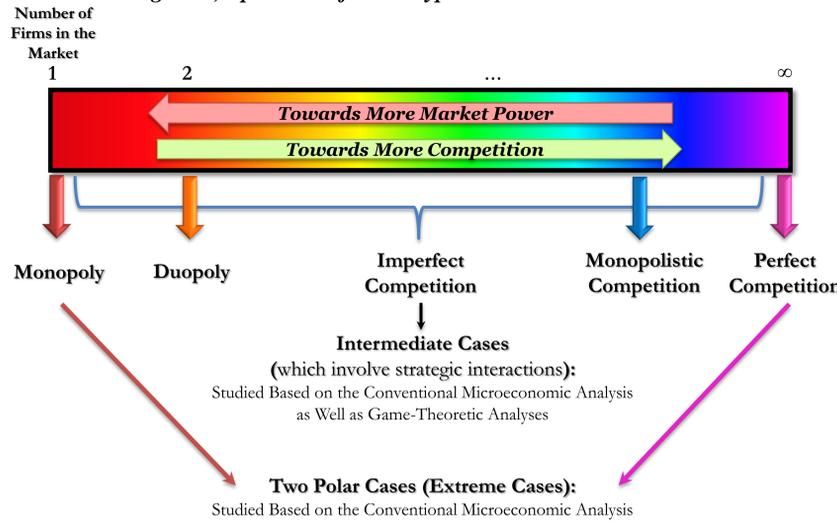Figure 1) Spectrum of Firm Types and Market Structures

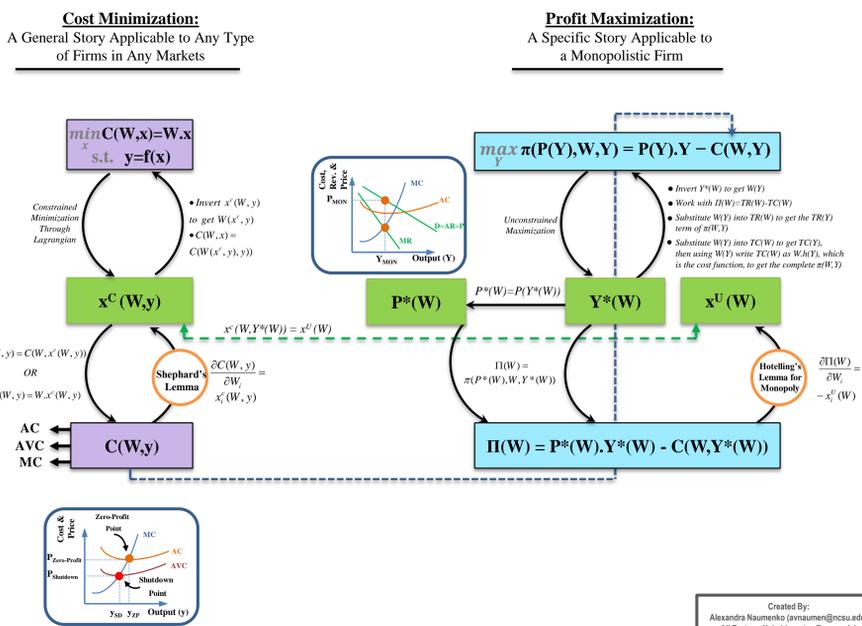Figure 2) Wheel of Relationships in Production Theory (Case of Monopoly)

Figure 3) Wheel of Relationships in Production Theory (Case of Perfect Competition)
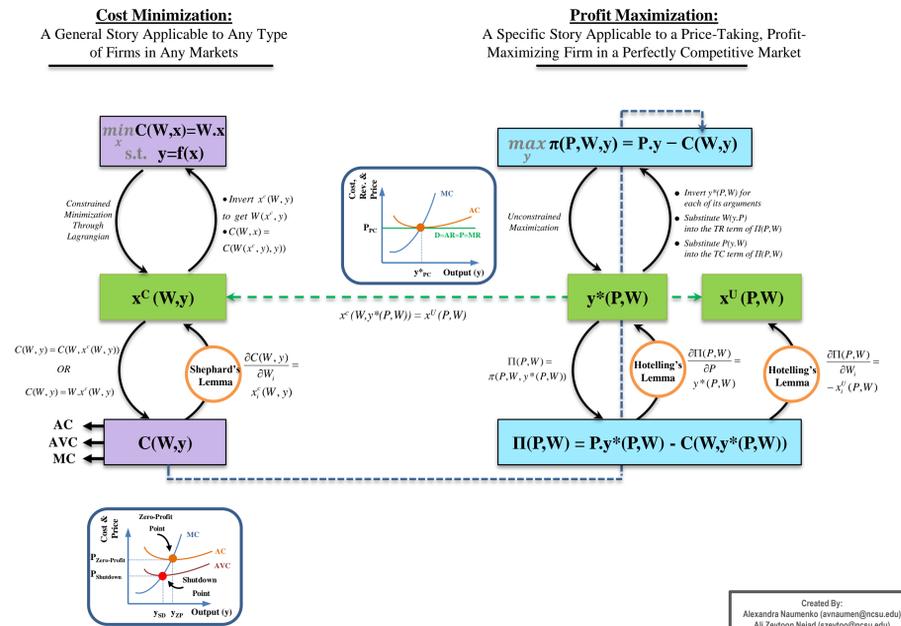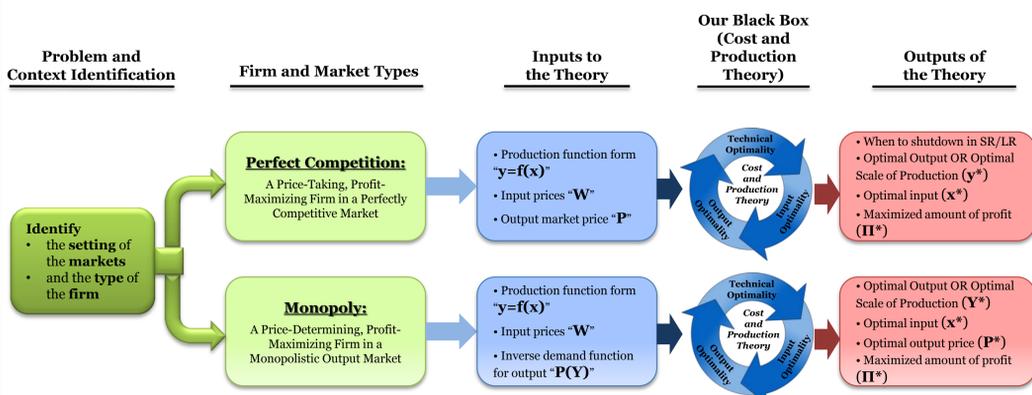

Figure 4) The Process of Determining the Potential Market Outcomes

Figure 5) Three Notions of Optimality in Production Theory