Interbank Trading in a Segmented OTC Market*

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Abstract

We present a core-periphery model of trading in the overnight interbank market. We model periods of crises as an increase in the number of peripheral banks that lose access to core dealers, resulting in segmentation between core and peripheral markets. Our model implies that such an increase in segmentation raises (i) the market power of periphery banks connected to the core, (ii) the dispersion of rates in the interbank market, and (iii) inefficient recourse to the central bank standing facilities. We argue that these implications are consistent with stylised facts about the interbank market and propose new predictions about trading in a segmented OTC market.

Keywords: OTC markets, Core-Periphery Networks, Segmentation, Interbank markets, Monetary Policy Implementation, Corridor System.

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1 Introduction

Overnight interbank markets play an important role in the economy. Indeed, they determine short term interest rates and ultimately the cost of funding at longer horizons for various economic agents. For this reason, they are the epicentre of monetary policy implementation (see, for instance, Bindseil (2005)).\(^1\) Moreover, the payoffs of many interest rates derivatives are pegged to overnight interbank rates.\(^2\) Understanding how rates are formed in this market and their volatility is therefore of considerable interest.

In this paper, we contribute to this effort by developing a new model of trading in an over-the-counter (OTC) interbank market with a “core-periphery” structure (see, for instance, Bech and Atalay (2010) and Craig and von Peter (2014)). The core is populated by a few large banks acting as dealers for smaller, peripheral, banks.\(^3\) Our analysis is motivated by a series of stylised facts about this markets.

[Insert Figure 1 about here]

Importantly, there is evidence that core and periphery markets are segmented to some extent in the sense that not all peripheral banks have access to core banks’ liquidity.\(^4\) Figure 1 illustrates this point by showing trading relationships in the euro area overnight interbank market in June 2008 (before the Lehman bankruptcy) and November 2011 (at the height of the sovereign debt

\(^{1}\)Modern central banks implement their monetary policy through steering the overnight interest rate. They typically rely on three instruments for this: (i) open market operations, (ii) minimum reserve requirements and (iii) standing facilities (described below). See, for instance, Garcia-de Andoain, Heider, Hoerova, and Manganelli (2016) for a description of how these instruments are used in the euro area.

\(^{2}\)For instance, the periodic floating payments of overnight indexed swaps (OIS) swaps are computed using unsecured interbank rates, such as the Fed Funds rate or the EONIA. The spread between the OIS and LIBOR rates is also considered as a measure of stress in money markets. Thus, understanding the way overnight rates are determined is key since it is a possible cause of fluctuations in the LIBOR-OIS spread.

\(^{3}\)The core-periphery structure can be found in a number of other OTC markets, e.g., for municipal bonds (Li and Schuerhoff (2014)), European sovereign bonds (Dunne, Hau, and Moore (2015)), Corporate bonds (DiMaggio, Kermani, and Song (2016)), Credit Default Swaps (Peltonen, Scheicher, and Vuillemez (2014)), or securitized assets (e.g., ABS and CDOs; see Hollifield, Neklyudov, and Spatt (2014)). Accordingly, our model could also be used to gain insights into these markets.

\(^{4}\)Cocco, Gomes, and Martins (2009), Brauning and Fecht (2012), and Afonso, Kovner, and Schoar (2011) show that the interbank network is generally very sparse, with most banks relying on a very limited set of counterparties.
crisis). This figure shows that (i) some peripheral banks never trade with core banks and (ii) the fraction of peripheral banks that do not trade with the core increased from between 2008 and 2011 (from 6.85% to 20.87%). In other words, the overnight interbank market is segmented (some peripheral banks only trade with other peripheral banks) and the level of segmentation has increased during the crisis.\footnote{The various crisis episodes since 2007 have had profound effects on the structure of interbank markets. For example, Afonso, Kovner, and Schoar (2011) show a decline in borrowing volumes and the number of counterparties directly following the Lehman bankruptcy. In turn, access to the Fed’s discount window increased significantly. Similarly, Garcia de Andoain, Hoffmann, and Manganelli (2014) document limited market access for banks from peripheral countries throughout the sovereign debt crisis.}

[Insert Figures 2 and 3 about here]

Figures 2 and 3 highlight two additional important stylised facts that motivate our analysis. First, Figure 2 shows that the distribution of interbank rates in November 2011 is more dispersed than in June 2008 and is bimodal, while it was unimodal in June 2008. Second, Figure 3 shows that the volume of transactions at the European Central Bank (ECB) “standing facilities” has considerably increased during the crisis.\footnote{Many central banks (e.g., the ECB, the Bank of England, the Bank of Canada, and to some extent the US Federal Reserve since October 2008) rely on a corridor system for steering short-term interest rates. This system allows banks that need liquidity on a given day to borrow from the central bank using its “lending facility” (known as the “discount window” in the U.S.) or to park liquidity using its “deposit facility”. The two associated interest rates are set by the central bank and define a corridor within which banks borrow and lend to each other bilaterally in the overnight interbank market.}

In particular, there is simultaneous resort to both facilities on the same day, that is, some banks obtain funding from the ECB at its lending rate while others park their excess reserves at its deposit rate (the interval between these rates is called the “corridor”). This observation is surprising because, if trading were frictionless, the short side of the market on a given day (e.g., banks with excess reserves when there is a deficit of reserves in aggregate) should not trade with the central bank.\footnote{For instance, if there is an aggregate deficit of reserves, a lender could offer a rate strictly within the corridor rates. This would make both the lender and the borrower better off than resorting to the central bank standing facilities.}

Our model can replicate these two facts based on the increased segmentation between core and peripheral banks in crisis times. As is standard in the literature (e.g., Poole (1968), Bech...
and Monnet (2015), or Afonso and Lagos (2015)), banks in our model are hit by liquidity shocks (random shocks to their reserves balances) every day. These shocks generate gains from trade between banks with a shortage of reserves ("borrowers") and banks with an excess of reserves ("lenders"), which can be realized in the interbank market. In contrast to the extant literature, we assume that this market comprises two types of banks: (i) core banks that serve as dealers for peripheral banks and (ii) peripheral banks that can bilaterally trade with each other or with core banks. Importantly, a fraction $\lambda$ of peripheral banks has no access to core banks. Thus, $\lambda$ measures the level of segmentation between the core and the peripheral markets. A bank that does not find a counterparty in the interbank market must eventually resort to the central bank’s standing facilities.

We first consider a centralized Walrasian market as a benchmark case. In this case, the equilibrium of our model is identical to that in Poole (1968). On a given day, all trades take place at the same rate, so that there is no rate dispersion for a given realization of banks’ reserves. Moreover, on a given day, banks either use the central bank’s lending facility or its deposit facility, but not both. Therefore, the benchmark model cannot explain the facts documented in Figures 2 and 3.

We then consider the more realistic case in which trading among peripheral banks take place in a decentralized OTC market. Specifically, peripheral banks meet bilaterally in sequence. One bank is chosen randomly and makes a take-it-or-leave-it offer to another bank. If the latter rejects the offer, it can itself make an offer to another bank, etc. If a bank’s offer is rejected, it can either contact a core bank (if connected to the core) or use the central bank’s standing facilities. Although stylized, this bargaining process captures two important features of the trading process in the interbank market: (i) banks’ strategically choose the offers they make or accept and (ii) the core market rate affects this choice because it influences the outside option of connected
peripheral banks and therefore their market power. We assume that the core market operates like a centralized interdealer market so that the core market rate is such that the net demand (demand minus supply) of liquidity is equal to zero. As this demand also reflects liquidity needs of peripheral banks that choose to trade in the core, the core rate depends on the equilibrium rates in the periphery market. Thus, the rates in both segments are jointly determined in equilibrium.

In the absence of segmentation (i.e., \( \lambda = 0 \)), the equilibrium is identical to that obtained in the benchmark case where all transactions take place at a single rate. The equilibrium outcome is different in the presence of market segmentation (\( \lambda > 0 \)) because of heterogeneity in peripheral banks’ market power. Intuitively, connected peripheral banks are able to obtain better terms of trade than unconnected ones because their outside option of trading with a core bank is more attractive than the outside option of trading with the central bank.\(^8\) For a given level of peripheral banks’ aggregate reserves (i.e., on a given day), this feature generates dispersion in the rates at which different pairs of peripheral banks trade together. For instance, when, in aggregate, peripheral banks are short of reserves, peripheral banks with excess reserves lend at a markup relative to core dealers’ rate and peripheral lenders connected to the core charge a higher markup than unconnected ones.\(^9\)

Thus, according to our model, the empirical distribution of daily rates observed in interbank markets over multiple days (Figure 2) should reflect (i) variations in core and peripheral banks’ aggregate reserves over time due to liquidity shocks and (ii) cross-sectional dispersion in the rates at which transactions take place among banks on a given day. While the first determinant is standard (see, for instance, Poole (1968) or Bech and Monnet (2015)), the second one arises

\(^{8}\text{Indeed, in equilibrium, the core market rate is always within the standing facilities’ rates.}\)

\(^{9}\text{A significant share of the empirical literature examines the dispersion in borrowing rates across different types of banks and relates it to variations in bargaining power (see, for instance, Furfine (2001), Ashcraft and Duffie (2007), Bech and Klee (2011), Allen, Chapman, Echenique, and Shum (2012)). For instance, Ashcraft and Duffie (2007) document that lenders with a higher trading activity charge higher interest rates to borrowers with little trading activity, in line with an important role of traders’ position within the interbank network and the associated trading opportunities. Similarly, they show that bank’s eagerness to trade (i.e. their relative level of reserves) is an important pricing factor.}\)
only when the core and periphery markets are segmented in our model \((\lambda > 0)\). When \(\lambda > 0\), the volatility of rates is higher than for \(\lambda = 0\) due to the additional dispersion in interbank rates across different trades on the same day. Moreover, with segmentation, the distribution of rates can be bimodal when the liquidity conditions in the core and periphery markets are sufficiently different. For example, consider the situation where core banks have a significant excess of reserves while peripheral banks have a significant shortage. In this case, interest rates in the core will be relatively low, while rates in the periphery will be high. With transactions clustering around the rates for the two different market segments, this gives rise to a bimodal distribution.

In equilibrium, when \(\lambda > 0\), some gains from trade between peripheral banks are not realized because one side (e.g., the lender) strategically asks for too much in the negotiation, in the hope of finding a better bargain with another match. This behavior increases the likelihood that peripheral banks need to recourse to the central bank’s standing facilities.

For instance, consider a day in which peripheral banks are short of reserves on aggregate. On such a day, some unconnected peripheral banks will necessarily use the lending facility since there is an excess of liquidity demand in the peripheral market. Now consider a peripheral lender on this day, i.e., a bank with an excess of liquidity. This bank’s market power is high since there are more borrowers than lenders. It can therefore optimally choose to make offers that only attract borrowers with the worst outside option (i.e., unconnected banks) to extract more surplus from borrowers. However, this strategic behavior increases the likelihood that the lender’s offer is rejected and that, therefore, it ultimately trades with a core bank. Thus, market power reduces liquidity provision in the peripheral market when it is needed while increasing the likelihood that core banks end up the day with excess reserves. The first effect increases the volume at the borrowing facility (relative to the case in which the lender would not behave strategically) while
the second one increases the chance that both facilities are used on the same day, as observed in crisis times (see Figure 5). We show that this inefficient outcome becomes more likely when the core and the periphery markets become more segmented.10

In sum, our model offers an explanation for the distribution of interbank rates during the crisis (Figure 2) and banks’ recourse to central banks’ standing facilities. Our theory implies that the distribution of rates in this market should be explained by (i) the distribution of daily shocks to banks’ aggregate reserves, (ii) the level of segmentation between core and peripheral banks (as measured by the fraction of peripheral banks that cannot access the core), and (iii) the ratio of core banks’ aggregate reserves to peripheral banks’ aggregate reserves (the distribution should be bimodal when this ratio is negative and small). The first implication is common to other theories of rate formation in the overnight interbank market while, to our knowledge, the two others are specific to our model. Hence, they offer a sharp way to test it.

Our model delivers a number of additional testable implications. For example, an increase in segmentation implies an increase in the number of trades taking place between two peripheral banks, and a decrease in the number of trades between peripheral and core banks. It also highlights that market segmentation can pose a challenge to monetary policy implementation because equilibrium strategies, and thus interest rates, do no longer depend on aggregate liquidity, but rather on the breakdown between core and periphery banks.

Our model contributes to two distinct strands of literature. First, it contributes to the growing literature on trading in overnight interbank markets. We differ from the existing theoretical literature (e.g., Afonso and Lagos (2015) and Bech and Monnet (2015)) in two respects. First, we show how the level of segmentation between peripheral and core banks in these markets affect

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10 This is consistent with the empirical findings in Allen, Chapman, Echenique, and Shum (2012) for the Canadian overnight interbank market. Indeed, they find that inefficient outcomes in this market are more frequent after the onset of the financial crisis (which we associate with an increase in segmentation between the periphery and the core) and are positively correlated with injections of cash balances by the Bank of Canada.
equilibrium outcomes (the distribution of interbank rates, their volatility, the trading volume at the standing facilities, trading efficiency etc.).\(^\text{11}\) Second, we explicitly allow peripheral banks to be strategic in the choice of their offers while the existing literature uses the Nash bargaining approach to determine the division of trading surpluses between lenders and borrowers. This difference in modeling choices is important because it implies that, in our setting, peripheral banks might not trade together even though gains from trade exist between these banks. Thus, equilibrium outcomes are not necessarily efficient. This feature is important for our results regarding the possibility of two sided recourse to central bank facilities and offers a starting point to analyze policy interventions in the design of interbank markets.\(^\text{12}\)

More broadly, our model contributes to the literature on price formation in over the counter (OTC) markets. Starting with Duffie, Garleanu, and Pedersen (2005), this literature emphasizes search frictions to understand prices in these markets. In this paper, we focus on a different type of friction: the extent to which smaller (“peripher”) traders (banks) are connected to larger (“core”) traders, and we study how this segmentation affects equilibrium outcomes. In this way, our model contributes to the nascent theoretical literature on price formation in OTC markets with a core-periphery structure (see, for instance, Neklyudov (2014) or Babus and Kondor (2015)). This literature is important given that, as previously noticed, this structure is prevalent in many OTC markets and that the empirical literature emphasizes that time-varying connections to the core affect price patterns observed in these markets (see, for instance, DiMaggio, Kermani, and Song (2016)).

\(^\text{11}\)Vari (2015) extends the Poole (1968)’s model when some banks are cut off from the interbank markets and can only borrow from the central banks. He uses his model to interpret the volume of trading at the ECB deposit facility and the evolution of the interbank rate during the financial crisis. In our case, we assume that some peripheral banks are cut off from the core market but can still trade together bilaterally, as observed empirically.

\(^\text{12}\)Acharya, Gromb, and Yorulmazer (2012) develop a model of interbank loans that also generates a suboptimal equilibrium due to the market power of surplus banks. Their model features two banks bargaining over a loan with a long maturity. In contrast, we study a market for overnight loans with a continuum of banks, and the inefficiency is due to the combination of incomplete information about other banks and OTC trading. Heider, Hoerova, and Holthausen (2015) show how counterparty risk, which is absent in our model, can also generate inefficient equilibria in interbank markets.
The paper is organized as follows. We describe the model in Section 2 and derive its equilibrium in Section 3. The testable implications of the model are discussed in ???. Section 5 concludes. Proofs of the main results are in the appendix.

2 A core-periphery model of OTC trading in the overnight interbank market

2.1 Assumptions

2.1.1 Market participants and timing

We consider an interbank market with three types of participants: (i) a continuum of “peripheral” banks, (ii) a continuum of “core” banks, and (iii) the central bank. In this market, core and peripheral banks lend or borrow money from each other to adjust their reserve balances at the central bank. We normalize the required level of reserves to zero. Banks with a deficit of reserves at the end of the day must borrow from the central bank, using its marginal lending facility, at rate \( R^P \) while banks with an excess of reserves can deposit them at the central bank, using its marginal deposit facility, at rate \( R^D \) (all rates are gross rates: one plus the rate of return). These rates are referred to as corridor rates.

As explained below, core banks act as dealers for peripheral banks and trade together in a centralized market. Peripheral banks can either trade bilaterally (over the counter) or contact a core bank. As is observed empirically (see the introduction), we assume that a fraction \( \lambda \) of peripheral banks are not connected to core banks. As \( \lambda \) increases, the interbank market becomes more segmented: fewer peripheral banks can trade with core banks. We refer to peripheral banks that are connected to the core as “connected” banks and to other peripheral banks as “unconnected” banks. The type of a peripheral bank is unknown to other peripheral banks. We
focus on the case $\lambda \leq \frac{1}{2}$ for simplicity.

Our model should be interpreted as modeling a day of trading in the interbank market. We assume that core and peripheral banks start every day with reserves equal to $\bar{z}^{co}$ and $\bar{z}^{pe}$ on average (superscripts $co$ and $pe$ refer to core and peripheral banks, respectively). Every day, core and peripheral banks’ aggregate reserves deviate from their average level due to day-specific random shocks.\(^\text{13}\) We denote the realization of these shocks on a given day by $\epsilon^{pe}$ for peripheral banks and $\epsilon^{co}$ for core banks. Both shocks have a normal distribution with mean zero and variance $\sigma_k^2$ for $k \in \{pe, co\}$. Thus, on a given day, the actual aggregate reserves of core and peripheral banks are respectively: $z^{co} = \bar{z}^{co} + \epsilon^{co}$ and $z^{pe} = \bar{z}^{pe} + \epsilon^{pe}$. We assume that $\epsilon^{pe}$ (and therefore $z^{pe}$) is known at the beginning of the day while $\epsilon^{co}$ is known only at the end of the day. The role of this assumption will become clear in Section 2.2. We denote banks’ aggregate reserves before the realization of $\epsilon^{pe}$ by $\bar{z} = \bar{z}^{co} + \bar{z}^{pe}$ and after the realization of $\epsilon^{pe}$ by $z = \bar{z} + \epsilon^{pe}$.

On each day, a fraction $\pi_l(z^{pe})$ of peripheral banks initially have a reserves balance equal to $l(|z^{pe}|) > 0$ while a fraction $\pi_b(z^{pe}) = 1 - \pi_l(z^{pe})$ of peripheral banks have a reserves balance of $-l(|z^{pe}|)$. We assume that:

$$\pi_l(z^{pe}) = \frac{z^{pe}}{2l(|z^{pe}|)} + \frac{1}{2},$$

so that the aggregate level of reserves for peripheral banks, i.e., $l(|z^{pe}|)(\pi_l(z^{pe}) - \pi_b(z^{pe}))$, is equal to $z^{pe}$. Moreover, $l(\cdot)$ is such that $\frac{|z^{pe}|}{l(|z^{pe}|)}$ is (i) increasing in $|z^{pe}|$, (ii) strictly lower than 1, and (iii) converges to 1 as $|z^{pe}|$ goes to infinity.\(^\text{14}\) Hence, $\pi_l$ increases with $z^{pe}$ and converges to 1 when $z^{pe}$ goes to $+\infty$. If $z^{pe} > 0$ then there are more lenders than borrowers in the periphery ($\pi_l(z^{pe}) > \pi_b(z^{pe})$) and vice versa if $z^{pe} < 0$. If $z^{pe} = 0$, the mass of lenders is just equal to the mass of borrowers. Finally, observe that $\pi_l(z^{pe}) = \pi_b(-z^{pe})$.

\(^\text{13}\)As in other models of overnight interbank markets (e.g., Poole (1968) or Becht and Monnet (2015)), these shocks capture the fact that interbank payments redistribute aggregate reserves balances among banks.

\(^\text{14}\)For instance, $l(|\theta|) = 1 + |\theta|$ is such that all these properties are satisfied.
The heterogeneity of reserves among banks generates gains from trade. Indeed, a bank with a deficit of reserves at the end of the day pays a cost of $R^P$ to the central bank, while a bank with a surplus of reserves earns $R^D$. Hence, a trade in which the latter lends its excess reserves to the former, at a rate within the corridor $(R^D, R^P)$, is mutually beneficial. The timing is the following:

- **Stage 1**: Trading takes place between peripheral banks.

- **Stage 2**: Connected peripheral banks that were not matched in the first stage or chose not to trade with a peripheral bank trade with core banks.

- **Stage 3**: Core banks are hit by the shock $\epsilon^{co}$. All banks with a surplus of reserves deposit them at rate $R^D$ at the central bank, while banks with a deficit of reserves borrow at rate $R^P$ from the central bank.

The trading protocol in each stage is described in detail in the next section. The possibility for (connected) peripheral banks to trade in the core market if they fail to trade with a peripheral bank captures the notion that trading with core dealers is an option of “last resort” for peripheral banks. This assumption is consistent with evidence on OTC markets with a core-periphery structure (see, for instance, Li and Schuerhoff (2014)).

### 2.1.2 Market structure

**The peripheral market.** Trading in the peripheral market is decentralized in the sense that peripheral banks trade bilaterally according to a process described in Figure 4.

[Insert Figure 4 about here]

Peripheral banks make their decisions sequentially. When its turn arrives, bank $j$ can either make a take-it-or-leave-it offer for lending or borrowing $l(|z^{pe}|)$ units of reserves to its successor,
$j+1$, at a given rate or accept the offer (if it exists) of its predecessor, $j-1$. A bank automatically rejects the offer of its predecessor if it has the same trading need (e.g., both banks are borrowers). Otherwise, it rejects this offer if it expects to obtain a better rate with another trading partner (see below).

If bank $j$ is a lender, its successor is a borrower with probability $\beta(z^{pe})$ or a lender with probability $(1 - \beta(z^{pe}))$. If bank $j$ is a borrower then its successor is a lender with probability $\alpha(z^{pe})$. If two peripheral banks trade together, they exit the market. If bank $j$ makes an offer and this offer is rejected then bank $j$ can trade in the core market in Stage 2 if it is connected to this market. Otherwise, it trades at the central bank standing facilities in Stage 3.\(^\text{16}\)

Thus, $\beta(z^{pe})$ and $\alpha(z^{pe})$ denote the transition probabilities from a lender to a borrower and from a borrower to a lender in the “chain” of peripheral banks. For consistency, these transition probabilities must be such that the stationary probability that bank $j$ is a lender (resp., borrower) is $\pi_l$ (resp., $\pi_b$). This imposes $\frac{\alpha(z^{pe})}{\beta(z^{pe})} = \frac{\pi_l(z^{pe})}{\pi_b(z^{pe})}$ and therefore $\alpha(0) = \beta(0)$. To avoid introducing an artificial friction due to sequential matching in the periphery, we further assume that $\alpha(0) = \beta(0) = 1$. That is, when there are as many lenders as borrowers, each lender is matched with a borrower with certainty.

A simple specification for $\alpha(z^{pe})$ and $\beta(z^{pe})$ that satisfies these requirements is:

\[
\begin{align*}
\alpha(z^{pe}) &= \min\left(1 - \frac{\pi_b(z^{pe})}{\pi_l(z^{pe})}, 1\right), \quad (2) \\
\beta(z^{pe}) &= \min\left(1 - \frac{\pi_l(z^{pe})}{\pi_l(z^{pe})}, 1\right). \quad (3)
\end{align*}
\]

\(^{15}\)For tractability, we assume that all trades between peripheral banks are for exactly $|\ell(\theta)|$ units of reserves so that if banks trade together, their excess reserves after trading are just equal to zero. This assumption is natural since, by assumption, peripheral banks have excess reserves of the same size ($|\ell(\theta)|$). Trading exactly this size allows peripheral banks to fully remove the risk of having an excess or deficit of reserves at date 4.

\(^{16}\)Trading with a core bank dominates, at least weakly, resorting to the central bank facility since $R^D \leq R^{co} \leq R^P$.\(^{11}\)
Observe that $\alpha(z_{pe})$ weakly increases with $z_{pe}$ while $\beta(z_{pe})$ weakly decreases with $z_{pe}$. Moreover, when there are more lenders than borrowers (i.e., $z_{pe} > 0$ so that $\pi_0(z_{pe}) < 1/2$ and $\pi_l(z_{pe}) > 1/2$) then $\alpha(z_{pe}) = 1$ while $\beta(z_{pe}) < 1$. In this case, borrowers are sure to be matched with a lender while lenders might not find a borrower in the peripheral market since, in the aggregate, peripheral banks have an excess of reserves. The opposite obtains when $z_{pe} < 0$. Thus, our model captures, in a simple way, the idea that finding a counterparty is more difficult for lenders (resp., borrowers) when, in aggregate, there is an excess (resp., deficit) of liquidity in the peripheral market.

The core market. The core market operates like a centralized Walrasian market (as in Poole (1968)). Namely, each bank submits a schedule that specifies its demand (possibly negative) of reserves at the core market rate $R_{co}$ and behaves competitively (i.e., take the core market rate as given). The equilibrium rate in the core market is such that this market clears. Thus, the core market is frictionless. This is similar to the assumption that interdealer trading is frictionless in other models of OTC markets (e.g., Duffie, Garleanu, and Pedersen (2005) or Lester, Rocheteau, and Weill (2015)).

2.2 Benchmark

As a benchmark, it is useful to consider the equilibrium that obtains if all banks trade only in the core market. Let $R_{co}^0 \in [R^D, R^P]$ be the core market rate in this case.\footnote{A rate outside this range cannot be obtained in equilibrium since it would lead to arbitrage opportunities and therefore an infinite supply or demand of reserves by banks. For instance, if $R_{co}^0 > R^P$, all banks can make infinite profits by lending at rate $R_{co}^0$ and borrowing eventually from the central bank. Thus, banks’ demand is infinitely negative and the interbank market does not clear.} We denote by $q_{i}^{co}(R_{co}^0)$ the net demand of reserves of core bank $i$ and by $q_{j}^{pe}(R_{co}^0)$ the net demand of reserves of peripheral bank $j$ at the core rate (i.e., their desired change in reserves relative to their starting reserves levels).

Core bank $i$’s reserve balance at the end of the day is $q_i(R_{co}^0) + z_{co}$, where $z_{co} = z_{co} + e_{co}$. The core market operates like a centralized Walrasian market (as in Poole (1968)). Namely, each bank submits a schedule that specifies its demand (possibly negative) of reserves at the core market rate $R_{co}$ and behaves competitively (i.e., take the core market rate as given). The equilibrium rate in the core market is such that this market clears. Thus, the core market is frictionless. This is similar to the assumption that interdealer trading is frictionless in other models of OTC markets (e.g., Duffie, Garleanu, and Pedersen (2005) or Lester, Rocheteau, and Weill (2015)).
Thus, core bank $i$’s expected profit after trading and before observing $\epsilon^{co}$ is:

$$\mathbb{E}(\Pi_i) = R^P \mathbb{E}_{\epsilon^{co}}\left[\min(q_i^{co}(R_0^{co}) + z^{co}, 0)\right] + R^D \mathbb{E}_{\epsilon^{co}}\left[\max(q_i^{co}(R_0^{co}) + z^{co}, 0)\right] - R_0^{co} q_i,$$

(4)

Core banks’ optimal demand for reserve balances in Stage 2, $q_i^{co}(R_0^{co})$, maximize their expected profit. Using eq.(4), the first order condition to this problem is:\textsuperscript{18}

$$R^P \Phi_{co}(-(q_i^{co} + \bar{z}^{co})) + R^D (1 - \Phi_{co}(-(q_i^{co} + \bar{z}^{co})) = R_0^{co},$$

(5)

where $\Phi_{co}(.)$ is the cumulative probability distribution of $\epsilon^{co}$ (the daily shock to core banks’ aggregate reserves). To understand this condition, observe that $\Phi_{co}(-(q_i^{co} + \bar{z}^{co}))$ is the probability that core bank $i$ is short of reserves at the end of the day given its reserve balance after trading $(q_i^{co} + \bar{z}^{co})$. In this case, it covers its deficit by borrowing at rate $R^P$, using the central bank’s lending facility. Otherwise, core bank $i$ ends the day with a surplus of reserves on which it earns the rate $R^D$ by using the central bank’s deposit facility. Hence, the L.H.S of Condition (5) is the rate at which core bank $i$ expects to trade with the central bank at the end of the day given its reserve balance at the end of Stage 2 $(q_i^{co} + \bar{z}^{co})$. Thus, it gives core bank $i$’s marginal valuation for reserves when its net demand in Stage 2 is $q_i^{co}$. Condition (5) states that core bank $i$’s net demand for reserves equalizes its marginal valuation for reserves to the rate in the core market.

We deduce from Condition (5) that the demand for reserves of a core bank is $q^{co}(R_0^{co}) = -(\Phi_{co}^{-1}(R_0^{co} - R^D) + \bar{z}^{co})$. It decreases with $R_0^{co}$. Indeed, as the core market rate increases, the expected cost of borrowing reserves in the core market (i.e., $R_0^{co} - R^D$) increases since any dollar of excess reserves at the end of the day returns only $R^D$.

Now consider a peripheral bank with an excess of reserves. Following the same reasoning as

\textsuperscript{18}The second order condition for a maximum is always satisfied because $R^D < R^P$. 
for a core bank, its expected profit after trading $q_j$ in Stage 2 is:

$$E(\Pi_j) = [R^D \max(q_j + l(|z^{pe}|), 0) + R^P \min(q_j + l(|z^{pe}|), 0)][q_j + l(|z^{pe}|)] - R_0^{co} q_j.$$  \hspace{1cm} (6)

As $R_0^{co} \in [R^D, R^P]$, it is immediate that peripheral banks’ optimal demand is equal to $q_j^{pe}(R_0^{co}) = -l(|z^{pe}|)$. That is, at rate $R_0^{co} \in [R^D, R^P]$, a peripheral bank with an excess of reserves supplies all its reserves (is a lender) to optimally end up with zero reserves at the end of the day.\footnote{Peripheral banks’ net demand is inelastic to the rate in the core market, in contrast to core banks, because they face no uncertainty on their reserves at the end of the day given their trading decision in Stage 2.} The same reasoning implies that the optimal demand for a peripheral bank with a shortage of reserves is $q_j^{pe}(R_0^{co}) = l(|z^{pe}|)$. Thus, the aggregate demand of all peripheral banks at a rate $R_0^{co} \in [R^D, R^P]$ is: $-\pi_l(z^{pe})l(|z^{pe}|) + \pi_b(z^{pe})l(|z^{pe}|) = -z^{pe}$.

In equilibrium, the core rate is such that banks’ net aggregate demand for reserves is zero (i.e., the aggregate demand for reserves is equal to the aggregate supply). That is, the interbank rate $R_0^{co}$ that clears the interbank market is such that:

$$q^{co*}(R_0^{co,*}) - z^{pe} = 0.$$  \hspace{1cm} (7)

Hence, using eq. (5), we obtain the following result:

**Proposition 1** (Benchmark). Let $z = \bar{z} + \epsilon^{pe}$ be banks’ aggregate reserves after the realization of $\epsilon^{pe}$. When trading among all banks is centralized, the equilibrium rate is:

$$R_0^{co} = \omega_0(z) R^D + (1 - \omega_0(z)) R^P,$$  \hspace{1cm} (8)

where $\omega_0(z) = (1 - \Phi_{co}(-z))$. It decreases with the amount of excess reserves of banks ($z$) and is equal to the mid-point of the corridor ($\frac{R^P + R^D}{2}$) if and only if $z = 0$.\footnote{Peripheral banks’ net demand is inelastic to the rate in the core market, in contrast to core banks, because they face no uncertainty on their reserves at the end of the day given their trading decision in Stage 2.}
Thus, when all banks trade in a centralized market, the equilibrium rate is a weighted average of the corridor rates and these weights are only determined by banks’ aggregate excess reserves \((z)\). In particular, when banks’ aggregate excess reserves \((z)\) increase, the equilibrium rate in the interbank market decreases. Thus, variations in the equilibrium rate from one day to the next are only due to idiosyncratic shocks to banks’ reserves \((\epsilon_{pe}^c)\) in the model or changes in their average levels of reserves \((e.g., \bar{z}^c)\). In this case, it is easily shown that the distribution of the equilibrium rate is uni-modal\(^{20}\). Moreover, in this case, only core banks eventually resort to the central bank and, more important, trading with the central bank either takes place at \(R^D\) when, at the end of the day, there is an excess of aggregate reserves \((i.e., z + \epsilon^{co} > 0)\), or at rate \(R^P\) otherwise. Thus, the benchmark model with centralized trading cannot explain the facts documented in Figures 2 and 3.

The outcome obtained in the benchmark case is identical to that predicted by Poole (1968). In fact, when all banks trade in the core market, our model is very close to that of Poole (1968). The only difference is that, in our model, some banks (the peripheral banks) know their liquidity shock before trading while in Poole (1968) all banks learn this shock after trading (like core banks do in our model). This feature implies that peripheral banks’ optimal trading strategy at any rate within the corridor is simply to transfer entirely their reserves to core banks. In this sense, core banks can be viewed as providing liquidity to peripheral banks since they absorb any excess reserves from the latter\(^{21}\).

Last, the analysis of the benchmark case shows why it is convenient to assume that core banks learn their liquidity shock, \(\epsilon^{co}\), after trading with peripheral banks. Indeed, if this shock were

\(^{20}\)The peak of the distribution is obtained for the rate obtained when \(\epsilon^{pe} = 0\).

\(^{21}\)In fact there is an analogy between the behavior of core banks in our model and dealers’ behavior in so called inventory models of dealership markets in market microstructure (see Foucault, Pagano, and Roell (2013), Ch.3 and 4). Indeed, by taking positions, core banks might end up the day with either an excess of reserves that they deposit at a low rate or a deficit of reserves that they must cover by borrowing at a high rate. This is similar to the risk of unwinding a long position at a low price (relative to the price at which the position was established) or a short position at a relatively high price for a dealer in the market for a risky asset.
known before trading, core banks’ demand for reserves would be inelastic at any rate within the corridor (like for peripheral banks) and therefore the equilibrium rate in the core market would be either $R^p$ or $R^d$ (except in the zero probability event in which $\pi^{co} + \pi^{pe} = 0$).\textsuperscript{22} For this reason, the literature on interbank markets usually assumes that banks trade without knowing perfectly the size of the shock affecting their reserves or, equivalently, allows for the possibility of a last shock after trading takes place (see, for instance, Poole (1968) and Bindseil (2005)).

3 Equilibrium

In this section, we solve for the equilibrium of the model backward. We first solve for the equilibrium rate in the core market (Section 3.1) and we then solve for equilibrium rates in the peripheral market (Section 3.2).

3.1 Equilibrium in the Core Market

In contrast to the benchmark case, aggregate excess reserves of peripheral banks that choose to trade in the core market are not necessarily equal to $z^{pe}$. Indeed, the masses of peripheral lenders and borrowers participating to the core market are endogenous: they depend on equilibrium conditions in the peripheral market. Thus, we denote the excess reserves of peripheral banks trading in the core by $\Delta(z^{pe}, \lambda)$. Its equilibrium value will be derived in the next section and in general, as shown in Lemma 3, $\Delta(z^{pe}, \lambda) \neq z^{pe}$, unless $\lambda = 0$ or $z^{pe} = 0$. Following exactly the same reasoning as in the benchmark case (see Section 2.2), we obtain that the core market rate, $R^{co}$, solves:

$$q^{co}(R^{co}) = \Delta(z^{pe}, \lambda),$$

\textsuperscript{22}This follows from eq.(8) by taking $\sigma_{co}^2$ to 0 since the case in which $\sigma_{co}^2 = 0$ is identical to the case in which core banks know their aggregate liquidity shock.
where \( q^{co}(R^{co}) = -(\Phi^{-1}_{co}(\frac{R^{co}-R^D}{R^{co}-R^P}) + z^{co}) \) is core banks’ demand for reserves. Hence, we obtain the following result.

**Proposition 2.** Let \( z^* = \Delta(z^{pe}, \lambda) + z^{co} \). The equilibrium rate in the core market is:

\[
R^{co*} = (1 - \Phi_{co}(-z^*))R^D + \Phi_{co}(-z^*)R^P. \tag{10}
\]

It decreases with core banks’ aggregate reserves, \( z^{co} \).

The equilibrium rate in the core market is in general not the Walrasian rate obtained if all banks participate to a centralized market, i.e., \( R^{co*} \neq R^{co*}_0 \) because \( \Delta(z^{pe}, \lambda) \neq z^{pe} \). As \( \Delta(z^{pe}, \lambda) \) is endogenous, to analyze the effect of \( \lambda \) or \( z^{pe} \) on the equilibrium rate in the core market, one must first solve for the equilibrium in the peripheral market, as we do next.

### 3.2 Equilibrium in the peripheral market

Now, we study the equilibrium in the peripheral market. Our first step is to formally define peripheral banks’ strategies and the equilibrium concept for the peripheral market. We focus on Markov strategies, i.e., the decision made by a bank is contingent on the offer it receives, but not on previous offers made by other banks. This is natural since there is no reporting of past trades in the OTC interbank market, which precludes strategies based on other traders’ past decisions.

Consider a connected borrower (\( C \)) who has just received an offer \( R_l \) from a lender. The borrower first decides whether to accept the offer or not. If he rejects the offer, he may either go to the core market, or make a new offer in the peripheral market. These decisions can be characterized by a triplet \( S^C_b(R_l) = \{d^C_b, q_b, \rho^C_b\} \), where: (i) \( d^C_b(.) \) is a function of \( R_l \) equal to 1 if the borrower accepts an offer at rate \( R_l \), and 0 otherwise; (ii) \( q_b \) is the probability that the borrower makes a new offer in the peripheral market, rather than go to the core market, if he does not accept a previous offer; (iii) \( \rho^C_b \) is a mixed strategy (with support \( \mathbb{R}^+ \)) over the rates offered
by the borrower in the latter case. A particular case is of course a pure strategy in which the borrower always offers the same rate with probability one. The case of an unconnected borrower is similar, except that an unconnected bank does not have access to the core market (so that \( q_b = 1 \) for an unconnected borrower). Hence, an unconnected borrower’s strategy is characterized by a pair \( S^U_b = \{ d^U_b, \rho^U_b \} \).

Symmetrically, a connected lender’s strategy is characterized by \( S^C_l(R_b) = \{ d^C_l, q_l, \rho^C_l \} \), where \( d^C_l(R_b) \) takes a value 1 if the lender accepts an offer at rate \( R_b \); \( q_l \) is the probability that if the lender does not accept a previous offer he makes a new offer in the peripheral market; and \( \rho^C_l \) is a mixed strategy profile over the interest rates the lender offer to the next bank. An unconnected lender’s strategy is described by \( S^U_l = \{ d^U_l, \rho^U_l \} \).

A bank’s strategy is contingent on its type but not on its counterparty’s type because banks do not know other banks’ type. This reflects the idea that banks do not know with certainty who are the counterparties of their own counterparties (as in Caballero and Simsek (2013)), e.g., whether they have access or not to core dealers.

Consider a peripheral lender who receives an offer \( R_b \) from a borrower (i.e., an offer to borrow cash from the lender at \( R_b \)). If the lender is connected, his expected payoff (or return) with strategy \( S^C_l \) is:

\[
W^C_l(R_b, S^C_l(R_b)) = d^C_l(R_b)R_b + (1 - d^C_l(R_b))[q_l\varphi_l(R_l)R_l + (1 - q_l\varphi_l(R_l))R_{co}^*],
\]

where \( R_l \) is the offer made by the lender if he rejects the borrower’s offer and contacts another peripheral bank; \( \varphi_l(R_l) \) is the likelihood that this offer is accepted. Indeed, if the lender accepts

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\( ^{23} \)We allow banks to play mixed strategies when they reject an offer in two ways: (i) if connected, they can mix between making a new offer to another peripheral bank or contacting a core dealer without making an offer to a peripheral bank and (ii) they can mix between various rates. It is useless to consider the possibility for banks to mix over the rejection and the acceptance of an offer since such equilibria cannot exist. Indeed, they require a bank to be indifferent between accepting or rejecting an offer. But if a bank mixes between these two possibilities, a bank making the offer can improve its rate by an infinitesimal amount to break the indifference. Thus, when a bank is indifferent between accepting or rejecting an offer, we assume that it accepts the offer with probability one.
the borrower’s offer, he gets a return $R_b$ on his loan. If instead he turns down this offer, with probability $q_l$ he makes a new offer at $R_l$, which is accepted with probability $\varphi_l(R_l)$. Alternatively, with probability $(1 - q_l \varphi_l(R_l))$, either the lender does not make a new offer to a peripheral bank, or his offer is rejected. In both cases, he obtains a return $R^{co\ast}$ by trading in the core market.

Similarly, if the lender is unconnected, his expected payoff is:

$$W^U_l(R_b, S^U_l(R_b)) = d^U_l(R_b)R_b + (1 - d^U_l(R_b))[\varphi_l(R_l)R_l + (1 - \varphi_l(R_l))R^D]$$

(12)

because if the unconnected lender makes an offer to another peripheral bank and this offer is rejected then the lender uses the central bank’s marginal deposit facility and earns $R^D$ (rather than $R^{co\ast}$).

Denoting $\varphi_b(R_b)$ the probability that an offer at rate $R_b$ is accepted by a lender, we obtain symmetric expressions for the expected payoff (i.e., expected funding cost) of a borrower who receives an offer $R_l$ from a lender:

$$W^C_b(R_l, S^C_b(R_l)) = -d^C_b(R_l)R_l + (1 - d^C_b(R_l))[-q_b\varphi_b(R_b)R_b - (1 - q_b\varphi_b(R_b))R^{co\ast}]$$

(13)

$$W^U_b(R_l, S^U_b(R_l)) = -d^U_b(R_l)R_l + (1 - d^U_b(R_l))[-\varphi_b(R_b)R_b - (1 - \varphi_b(R_b))R^P].$$

(14)

Definition 1. A Markov-perfect equilibrium of the peripheral market is a set of strategies $\Sigma^* = \{S^C_l, S^U_l, S^C_b, S^U_b\}$ such that (i) for all $R_b$, $S^i_b(R_b)$ maximizes the expected payoff, $W^i_l(R_b, S^i_l(R_b))$, of a lender of type $i \in \{C, U\}$ given that other banks behave according to $\Sigma^*$ and (ii) for all $R_l$, $S^i_b$ maximizes the expected payoff, $W^i_b(R_l, S^i_b(R_l))$, of a borrower of type $i \in \{C, U\}$ given that other banks behave according to $\Sigma^*$.

Henceforth, we denote by $V^i_l$ and $V^i_b$ the respective equilibrium continuation values of lenders and borrowers of type $i$, that is, the expected payoffs conditional on rejecting an of-
These payoffs are given by the terms in brackets in the expressions for $W_i(R_b, S^{*}_i(R_b))$ and $W_b(R_l, S^{*}_b(R_l))$. From now on, we assume $\lambda > 0$ and consider the case $\lambda = 0$ at the end of this section. The following observations are useful to characterize basic properties of any equilibrium of the peripheral market:

- **Observation 1.** A bank accepts an offer if and only if it is as good as the expected rate it could obtain by rejecting it. Hence, in equilibrium: $d_b(R_l) = 1$ if $R_l \leq -V^{*}_b$, and 0 otherwise. Symmetrically, $d_l(R_b) = 1$ if $R_b \geq V^{*}_l$, and 0 otherwise.

- **Observation 2.** Connected banks can always mimic unconnected banks, so their expected payoff must be at least as high as the one for unconnected banks. Hence, $V^{C*}_l \geq V^{U*}_l$ and $V^{C*}_b \geq V^{U*}_b$. It follows from the previous observation that a lender’s offer must optimally be either relatively low and equal to $R_l = -V^{C*}_b$ or relatively high and equal to $R_l = -V^{U*}_b$. The former offer is accepted by all borrowers while the latter is accepted by unconnected borrowers only. Symmetrically, borrowers’ offers in equilibrium are either high (i.e., $R_b = V^{C*}_l$) or low (i.e., $R_b = V^{U*}_l$).

- **Observation 3.** Connected banks can borrow or lend at rate $R^{cos}$ in the core market. Thus, $V^{C*}_l \geq R^{cos}$ and $V^{C*}_b \geq -R^{cos}$. From this we deduce that the optimal offers for connected lenders and borrowers are, respectively, $R_l = -V^{U*}_b$ and $R_b = V^{U*}_l$. As these offers are accepted only by unconnected banks, there is no transaction between two connected banks in equilibrium.

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24 Note that $V^{C*}_b$ and $V^{U*}_b$ are negative because they represent the cost of funding for borrowers.

25 For instance suppose that a lender chooses to make an offer $R_l$ that is strictly less than $-V^{C*}_b$. Using the first observation, we deduce that this offer is accepted by all borrowers. This would still be the case if the lender made an offer at $R_l + \epsilon$ such that $R_l + \epsilon \leq -V^{C*}_b$ and $\epsilon > 0$. As the latter makes the lender better off, an offer at $R_l$ that is strictly less than $-V^{C*}_b$ cannot be optimal for any lender. A similar reasoning implies that if the lender makes an offer in $(-V^{C*}_b, -V^{U*}_b)$ then he must optimally choose $-V^{U*}_b$.

26 Indeed, suppose that a high type lender makes an offer at $R_l = -V^{C*}_b$ instead of $R_l = -V^{U*}_b$. This offer is smaller than $R^{cos}$. Hence, in this case, $V^{C}_l$ for the lender is less than $R^{cos}$. This means that the lender’s offer cannot be optimal since he can always lend at least at rate $R^{cos}$ by trading with core banks.
Observation 4. It follows from the second and third observations that only unconnected banks might play a mixed strategy in equilibrium when they choose an offer and that if they mix, they choose between only two offers. Henceforth, we denote by $p_b^*$ the probability that an unconnected peripheral borrower offers a rate equal to $R_b = V_l^{C*}$ (rather than $V_l^{U*}$) and by $p_l^*$ the probability that an unconnected peripheral lender offers a rate equal to $R_l = -V_b^{C*}$ (rather than $-V_b^{U*}$).

The next lemma is a direct implication of these observations. We denote by $R_i^*$ a rate offered by a lender of type $i \in \{U, C\}$ in equilibrium and by $R_i^*$ a rate offered by a borrower of type $i$.

Lemma 1. An equilibrium of the peripheral market is fully characterized by the 4-tuple $(p_l^*, p_b^*, q_l^*, q_b^*)$.

The equilibrium behavior of banks in this market is as follows:

- A connected lender accepts any offer from a borrower at rate $R_b \geq V_l^{C*}$. If he does not accept an offer, with probability $q_l^*$ he makes an offer to another peripheral bank at rate $R_l^{C*} = -V_b^{U*}$, and with probability $(1 - q_l^*)$ he trades with a core bank (in Stage 2).

- An unconnected lender accepts any offer from a borrower at rate $R_b \geq V_l^{U*}$. If he does not accept an offer, he makes an offer at rate $R_l^{U*} = -V_b^{C*}$ with probability $p_l^*$, or an offer at rate $R_l^{U*} = -V_b^{U*}$ with probability $1 - p_l^*$.

- A connected borrower accepts any offer from a lender at rate $R_l \leq -V_b^{C*}$. If he does not accept an offer, with probability $q_b^*$ he makes an offer to another peripheral bank at rate $R_b^{C*} = V_l^{U*}$, and with probability $(1 - q_b^*)$ he trades with a core bank (in Stage 2).

- An unconnected borrower accepts any offer at rate $R_l \leq -V_b^{U*}$. If he does not accept an offer, he makes an offer at rate $R_l^{U*} = V_l^{C*}$ with probability $p_b^*$ or an offer at rate $R_l^{U*} = V_l^{U*}$ with probability $1 - p_b^*$.

To further characterize the equilibrium of the peripheral market, we must derive equilibrium rates and the probabilities $(p_l^*, p_b^*, q_l^*, q_b^*)$ with which these rates are offered by banks. For given
parameter values (i.e., $z^p_e, z^o, \lambda, R^D$ and $R^P$), there exists either a pure strategy equilibrium or a mixed strategy equilibrium. Moreover, if the equilibrium is in pure strategies, it is unique (see Proposition 3 below). The set of parameters for which mixed strategy equilibria obtain is typically “small” and economic intuitions in these equilibria are similar to those obtained in pure strategy equilibria. Thus, henceforth, we only focus on the pure strategy equilibrium for brevity. For completeness, we provide a characterization of the mixed strategy equilibria in the on-line appendix of this paper. It turns out that the pure strategy equilibrium goes through three different regimes:

- **Active Connected Banks (ACB):** $p^*_l = p^*_b = q^*_l = q^*_b = 1$. In this regime, unconnected banks make offers that are accepted by all banks with an opposite trading need (i.e., $p^*_l = p^*_b = 1$). Connected banks’ offers are only accepted by unconnected banks with an opposite trading need. When they reject an offer, connected banks make an offer to another peripheral bank ($q^*_l = q^*_b = 1$).

- **Inactive Connected Lenders (ICL):** $p^*_l = q^*_b = 1, p^*_b = q^*_l = 0$. In this regime, unconnected borrowers make offers that are only accepted by unconnected lenders ($p^*_b = 0$), while unconnected lenders make offers that are accepted by all borrowers ($p^*_l = 1$). Connected borrowers make offers that are accepted by unconnected lenders only and when they reject an offer, they always make an offer to another peripheral bank ($q^*_b = 1$). In contrast, connected lenders only trade in the core market ($q^*_l = 0$).

- **Inactive Connected Borrowers (ICB):** $p^*_b = q^*_l = 1, p^*_l = q^*_b = 0$. In this regime, unconnected lenders make offers that are accepted by unconnected borrowers only ($p^*_l = 0$), while unconnected borrowers make offers that are accepted by all lenders ($p^*_b = 1$). Connected lenders make offers that are accepted by unconnected borrowers only and when they reject an offer, they always make an offer to another peripheral bank ($q^*_b = 1$). In contrast, connected
borrowers only trade in the core market ($q^*_b = 0$).

In the ACB regime, all connected banks trade with a positive probability with unconnected banks. In contrast, in the ICL (resp., ICB) regime, connected lenders (resp., borrowers) exclusively trade in the core market. In this sense, they are “inactive” in the peripheral market. We show below (see Proposition 3) that the ICL or ICB regimes obtain when (i) core and peripheral banks have opposite liquidity needs (e.g., core banks have an excess of reserves while peripheral banks have a deficit of reserves) and (ii) core banks’ aggregate reserves ($\tau^c$) are large enough in absolute value, so that the core rate is either relatively small or relatively large. Otherwise, the ACB regime obtains. The derivation of this result, however, requires two intermediate results derived in Lemma 2 and 3. Let $\omega(\lambda, x) = \frac{\lambda(1-x)}{1-x\lambda(2-\lambda)}$.

Lemma 2. Suppose $\lambda > 0$. The ACB regime always obtains when $z^{pe} = 0$. When $z^{pe} \neq 0$, the ACB regime obtains if and only if:

$$ (1 - \omega(\lambda, \alpha))R^D + \omega(\lambda, \alpha)R^P < R^{cos} < \omega(\lambda, \beta)R^D + (1 - \omega(\lambda, \beta))R^P \quad (15) $$

while the ICL regime obtains if and only if $z^{pe} > 0$ and:

$$ R^{cos} > \omega(\lambda, \beta)R^D + (1 - \omega(\lambda, \beta))R^P. \quad (16) $$

and the ICB regime obtains if and only if $z^{pe} < 0$ and:

$$ R^{cos} < (1 - \omega(\lambda, \alpha))R^D + \omega(\lambda, \alpha)R^P. \quad (17) $$

When $z^{pe} < 0$, $\beta = 1$ while $\alpha < 1$. Hence, $\omega(\lambda, \beta) = 0$ while $0 < \omega(\lambda, \alpha) < 1/2$ because $\lambda < \frac{1}{2}$. Thus, the R.H.S of Condition (15) is satisfied and the ICL regime never obtains in
this case (Condition 16 cannot be satisfied). Indeed, peripheral banks are, in aggregate, short of liquidity. This shortage allows connected lenders to charge rates that are above the core rate. Thus, connected lenders prefer to trade with an unconnected peripheral borrower whenever possible (i.e., if they are matched with one). This explains why the ICL regime cannot obtain in this case. In contrast, connected borrowers cannot exert market power since there is a shortage of reserves for peripheral banks. However, to trade with a peripheral lender, they must obtain a rate that does not exceed the core market rate since this is their outside option. If the core market rate is large enough (i.e., if Condition (17) does not hold), trading at this rate is more profitable for unconnected lenders than offering a higher rate, which is accepted only by unconnected lenders. Indeed, with the latter, they increase the risk of getting their offer rejected and finally earning only the rate on the central bank deposit facility. In this case, the ACB regime obtains. If instead, the core market rate is small (i.e., if Condition 17 is satisfied) then unconnected lenders are better off taking this risk. This means that connected borrowers cannot find a counterparty in the peripheral market and they therefore trade with core dealers. In this case, the ICB regime obtains. The interpretation of Lemma 2 is symmetric when \( z^{pe} > 0 \).

The core market rate is endogenous and depends on \( z^{co} \) and \( z^{pe} \) through \( \Delta(z^{pe}, \lambda) \), the aggregate excess reserves of connected peripheral banks that did not find a counterparty in the peripheral market. Thus, to offer a full characterization of the equilibrium in the peripheral market (i.e., the mapping between the exogenous parameters and the equilibrium that is obtained), we must first compute \( \Delta(z^{pe}, \lambda) \). To this end, let \( \mu^c_{l}(z^{pe}, \lambda; p_l, q_b, p_b, q_l) \) and \( \mu^c_{b}(z^{pe}, \lambda; p_l, q_b, p_b, q_l) \), be, respectively, the masses of connected lenders and borrowers that, in equilibrium, trade in the core market. These masses are derived in closed-form in Appendix A.5. By definition, 

\[
\Delta(z^{pe}, \lambda; p_l, q_b, p_b, q_l) = l(z^{pe})[\mu^c_{l} - \mu^c_{b}],
\]

where we emphasize the fact that \( \Delta(z^{pe}, \lambda; p_l, q_b, p_b, q_l) \) depends on \( (p_l, q_b, p_b, q_l) \) and therefore, holding \( (z^{pe}, \lambda) \) constant, the regime of the pure strategy
Lemma 3. In the pure strategy equilibrium, the aggregate excess reserves, $\Delta(z^{pe}, \lambda; p_l, q_b, p_b, q_l)$, of connected peripheral banks that trade with core dealers are positive if $z^{pe} > 0$ and negative if $z^{pe} < 0$. It is equal to zero if $z^{pe} = 0$. In a given regime, it increases with $z^{pe}$. Moreover, holding $z^{pe}$ constant, it is higher in absolute value in the ICL or ICB regime than in the ACB regime, that is $\Delta(z^{pe}, \lambda; 1, 1, 0, 0) > \Delta(z^{pe}, \lambda; 0, 0, 1, 1)$ for $z^{pe} > 0$ and $\Delta(z^{pe}, \lambda; 0, 0, 1, 1) < \Delta(z^{pe}, \lambda; 1, 1, 1, 1)$ for $z^{pe} < 0$.

This result is intuitive. If $z^{pe} > 0$, there are more lenders than borrowers in the peripheral market. Thus, in this market, connected borrowers are more likely to find a counterparty than connected lenders. As a result, in aggregate, peripheral banks that trade with core banks are more likely to be lenders than borrowers, so that $\Delta(z^{pe}, \lambda) > 0$. For symmetric reasons, $\Delta(z^{pe}, \lambda) < 0$ if $z^{pe} < 0$. Proposition 2 implies that the rate in the core market, $R^{co}$, decreases with aggregate excess reserves of all banks in this market, $z^* = z^{co} + \Delta(z^{pe}, \lambda)$. Thus, Lemma 3 implies that the equilibrium rate in the core market decreases with $z^{pe}$, as in the benchmark case.

Using the expression for the equilibrium rate in the core market (given by eq.(10)), we can now rewrite the conditions of existence for the various regimes (given in Proposition 2) in terms of the exogenous parameters of the model only. To see this, suppose again that $z^{pe} < 0$. In this case, the ICL regime never obtains and the ICB regime obtains iff Condition (17) is satisfied. Using the fact that $R^{co} = R^D(1 - \Phi_{co}(-z^*)) + \Phi_{co}(-z^*)R^P$, we rewrite Condition (17) as:

$$z^{co} > \tilde{z}^{-}(z^{pe}, \lambda),$$

(18)

where $\tilde{z}^{-}(z^{pe}, \lambda) = - \left[ \Phi^{-1}_{co}(\omega(\lambda, \alpha)) + \Delta(z^{pe}, \lambda; 0, 0, 1, 1) \right]$ where $\tilde{z}^{-} > 0$ since $\omega(\lambda, \alpha) < 1/2$ and $\Delta(z^{pe}, \lambda; 0, 0, 1, 1) < 0$ for $z^{pe} < 0$. Using again the expression for $R^{co}$ and Condition (15), we
deduce that when \( z^{pe} < 0 \), the ACB regime obtains iff:

\[
\bar{z}^{-}(z^{pe}, \lambda) > 0 \text{ since } \omega(\lambda, \alpha) < 1/2 \text{ and } \Delta(z^{pe}, \lambda; 1, 1, 1, 1) < 0 \text{ for } z^{pe} < 0. \]

Moreover, using Lemma 3, we deduce that \( \hat{z}^{-}(z^{pe}, \lambda) > z^{-}(z^{pe}, \lambda) > 0. \)

When \( z^{pe} > 0 \), a symmetric reasoning implies that the ICL regime obtains iff \( \bar{z}^{+}(z^{pe}, \lambda) \) while the ACB regime obtains iff \( \bar{z}^{+}(z^{pe}, \lambda) \) where \( \hat{z}^{+}(z^{pe}, \lambda) = - [\Phi^{+1}(1 - \omega(\lambda, \beta)) + \Delta(z^{pe}, \lambda; 1, 1, 1, 1)] \) and \( z^{+}(z^{pe}, \lambda) = - [\Phi^{+1}(1 - \omega(\lambda, \beta)) + \Delta(z^{pe}, \lambda; 1, 1, 0, 0)] \). Moreover, \( \hat{z}^{+}(z^{pe}, \lambda) < z^{+}(z^{pe}, \lambda) < 0 \). Thus, we have obtained the following result.

**Proposition 3.** Suppose \( \lambda > 0 \) and let \( \Omega = (-\infty, \hat{z}^{+}(z^{pe}, \lambda))(z^{+}(z^{pe}, \lambda), z^{-}(z^{pe}, \lambda)) \cup(z^{-}(z^{pe}, \lambda), +\infty) \).

When \( \bar{z}^{co} \in \Omega \), there is a unique equilibrium in the peripheral market and in this equilibrium, peripheral banks follow pure strategies.

1. For \( z^{pe} > 0 \), the pure strategy equilibrium is in the ACB regime when \( \bar{z}^{co} > z^{+}(z^{pe}, \lambda) \) and in the ICL regime when \( \bar{z}^{co} < \hat{z}^{+}(z^{pe}, \lambda) \) where \( \hat{z}^{+}(z^{pe}, \lambda) < z^{+}(z^{pe}, \lambda) < 0 \).

2. For \( z^{pe} < 0 \), the pure strategy equilibrium is in the ACB regime when \( \bar{z}^{co} < z^{-}(z^{pe}, \lambda) \) and in the ICB regime when \( \bar{z}^{co} > \hat{z}^{-}(z^{pe}, \lambda) \) where \( \hat{z}^{-}(z^{pe}, \lambda) > z^{-}(z^{pe}, \lambda) > 0 \).

Figure 5 illustrates Proposition 3.\(^{27}\) It shows that when core and periphery banks’ aggregate reserves (\( z^{pe} \) and \( \bar{z}^{co} \)) have the same signs, the active connected banks (ACB) regime always obtains. Other regimes, in which connected lenders or borrowers exclusively participate to the core market, obtains if and only if (i) core banks’ aggregate reserves and peripheral banks’ aggregate reserves have opposite signs and (ii) the difference in their absolute size is large enough.

\(^{27}\) We set \( \ell(|\theta|) = 1 + |\theta|, \lambda = 0.4, \sigma^{co} = 1. \)
For instance, the ICB regime obtains when core banks have excess reserves on average \((\bar{z}^c > \bar{z}^- (z^{pe}, \lambda) > 0)\) while peripheral banks have a deficit of reserves \((z^{pe} < 0)\). Intuitively, in this case, the core rate tends to be low because core banks have an excess of reserves. As a result, connected borrowers are better off directly trading in the core market in equilibrium and unconnected lenders prefer to make offers that are attractive to unconnected borrowers only because attracting connected borrowers would require offering too low rates. These interactions illustrate how the equilibrium in the core and the peripheral market are *jointly* determined, even though these two markets are segmented.

[Insert Figure 5 about here]

The previous proposition does not describe how banks behave when \(z^{co} \in [\bar{z}^+, z^+]\) or when \(z^{co} \in [z^-, \bar{z}^-]\). In these cases, there is no equilibrium in pure strategies but mixed strategy equilibria exist. We provide a complete characterization of these equilibria in the on-line appendix of the paper. In Figure 5, the differences between \(\bar{z}^+\) and \(z^+\) or \(\bar{z}^-\) and \(z^-\) are so small that they cannot be visualized. We have checked through extensive numerical simulations that this is the case in general.

The next proposition completes the characterization of equilibria in the peripheral market by providing in closed-form the rates obtained in each possible equilibrium. Let
\[
\omega^U_l (\lambda, z^{pe}, z^{co}) = \frac{(1-\lambda)\Phi_{co}(-z^*)}{1-\beta(z^{pe})\lambda}, \quad \omega^C_l (\lambda, z^{pe}, z^{co}) = 1 - \frac{\alpha(z^{pe})(1-\lambda)(1-\Phi_{co}(-z^*))}{1-\alpha(z^{pe})\lambda}, \quad \omega^C_b (\lambda, z^{pe}, z^{co}) = 1 - \frac{\beta(z^{pe})(1-\lambda)\Phi_{co}(-z^*)}{1-\beta(z^{pe})\lambda},
\]
and
\[
\omega^U_b (\lambda, z^{pe}, z^{co}) = \frac{(1-\lambda)(1-\Phi_{co}(-z^*))}{1-\alpha(z^{pe})\lambda},
\]
where \(z^* = \Delta(z^{pe}, \lambda) + z^{co}\).

**Proposition 4.** Suppose \(\lambda > 0\). In equilibrium, connected lenders make offers at \(R^*_l(z^{pe}, \lambda, z^{co}) = (1-\omega^C_l)R^D + \omega^C_l R^P\) with probability \(q^*_l\) and connected borrowers make offers at \(R^*_b(z^{pe}, \lambda, z^{co}) = \omega^C_b R^D + (1-\omega^C_b)R^P\) with probability \(q^*_b\). Moreover, unconnected lenders make offers at \(R^*_u(z^{pe}, \lambda, z^{co}) = (1-\omega^U_l)R^D + \omega^U_l R^P\) with probability \(p^*_l\) and offers at rate \(R^*_l(z^{pe}, \lambda, z^{co})\) with probability \((1-p^*_l)\).
Similarly, unconnected borrowers make offers at $R_b^U(z^{pe}, \lambda, z^{co}) = \omega_b^U R^D + (1 - \omega_b^U) R^P$ with probability $p_b^*$ and offers at rate $R_b^{C*}(z^{pe}, \lambda, z^{co})$ with probability $(1 - p_b^*)$.

We conclude this section by considering the limit case in which $\lambda$ tends to 0.

**Proposition 5.** When $\lambda$ goes to zero, $z^+$ (resp., $z^-$) tends to $-\infty$ (resp., $+\infty$). Hence, the likelihood that an ACB equilibrium is obtained goes to one. Moreover, in equilibrium, the core market rate and unconnected periphery banks’ offers converge to the rate obtained when all banks trade in a centralized Walrasian market, i.e., $R^{co*}_0 = (1 - \Phi^{co}(-z_0^*)) R^D + \Phi^{co}(z_0^*) R^P$ where $z_0^* = z^{co} + z^{pe}$.

Thus, as one would expect, when more and more banks are connected to core banks, the rates at which transactions take place among peripheral banks become closer and closer to the rate at which trades would take place if all trading were centralized.

### 4 Implications

In this section, we derive testable implications of the model.

#### 4.1 Distribution of interest rates

We first show that our model can capture the changes in the distribution of interest rates that occurred during the Euro area sovereign debt crisis, as shown in Figure 2 and discussed in the introduction. To do so, we report, in Figure 8, the distribution of interest rates implied by our model for various values of the parameters. This distribution is obtained through numerical simulations by repeatedly drawing realizations for the shock to peripheral banks’ aggregate reserves, $\epsilon^{pe}$, and computing associated equilibrium rates.\(^{28}\)

\(^{28}\)More specifically, we draw randomly 100,000 realizations of the shock to peripheral banks’ aggregate reserves, $\epsilon^{pe}$, from a normal distribution with mean zero and standard deviation 0.5. For each draw, we compute equilibrium rates and record the likelihood that these rates are observed in equilibrium based on the stationary probability...
The distribution of interest rates in our model has two distinct determinants. First, on a given day (for a given \( z^{pe} \) and \( z^{co} \)), there is *cross-sectional dispersion* in rates because different types of banks trade at different interest rates. The rates and their relative occurrence are determined endogenously in equilibrium. Second, there is *time-series variation* because the same type of bank will trade at different rates on different days, depending on the realizations of \( z^{pe} \) and \( z^{co} \).

To understand the role of each source, we run simulations using four different parametrizations. Panel A of Figure 8 shows the distribution of interest rates for \( \lambda = 0 \) and \( z^{co} = z^{pe} = 0 \). In this benchmark case, all transactions take place at rate \( R^{co} \). There is no cross-sectional dispersion in rates, and thus all of the variation is due to the time-series dimension (random liquidity shocks), as in Poole (1968). As liquidity shocks follow a unimodal distribution, the distribution of interest rates is unimodal as well and peaks at the mid-point of the corridor.

In Panel B, we still assume that \( \lambda = 0 \), but we set \( z^{pe} = -2 \) and \( z^{co} = 3 \). In this case, all transactions still take place at the same rate \( R^{co} \). However, as aggregate reserves are positive (\( z^{pe} + z^{co} = 1 \)), \( R^{co} \) shifts to the left and is now below the mid-point of the corridor. The distribution of the rates however remains unimodal (it peaks at \( R^{D} + \Phi_{co}(-1)(R^{P} - R^{D}) \)).

In Panel C, we assume the same balanced liquidity conditions as in Panel A (\( z^{co} = z^{pe} = 0 \)), but we introduce segmentation by setting \( \lambda = 0.4 \) (that is, 40% of all peripheral banks do not have access to the core). This yields cross-sectional dispersion in interest rates. To illustrate this, we also plot the distributions for each possible rate (\( R^{C}_{l} \), \( R^{U}_{l} \), \( R^{C}_{b} \), \( R^{U}_{b} \)). Rates offered by lenders (borrowers) are skewed to the left (right). Note that Proposition 3 implies that under these liquidity conditions the equilibrium is always in the ACB regime. Accordingly, the resulting distribution of all rates (top left) remains fairly similar to that of the benchmark case.

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Further details and derivations are provided in Appendix A.5. We then divide the interval between \( R^{D} \) and \( R^{P} \) in 1,000 bins and report in Figure 8 the sum of all likelihoods for rates observed in each bin.
Finally, in Panel D we maintain $\lambda = 0.4$ and now set $\pi^{pe} = -2$ and $\pi^{co} = 3$. Under this parametrization, the equilibrium can be either in the ACB or ICB regime, depending on the realization of $\epsilon^{pe}$. The introduction of unbalanced liquidity conditions leads to a distribution of rates that differs significantly from the one obtained in the benchmark case. In particular, it becomes bimodal, as observed empirically during the European sovereign debt crisis (see Figure 2). The first mode is equal to the core rate obtained for $\epsilon^{pe} = 0$ and is below the mid-point of the corridor. This reflects the fact that (i) many transactions take place in the core market (because some connected banks fail to trade in the periphery) and (ii) excess liquidity in the core ($\pi^{co} = 3$) depresses $R^{co\ast}$. The second hump is above the mid-point of the corridor. It stems from the fact that, in aggregate, peripheral banks have a shortage of liquidity, which increases the prevailing interest rates. In particular, most transactions take place at rates offered by unconnected borrowers, $R^{U}_b$.

In sum, two ingredients are required to explain the pattern of Figure 2. First, segmentation between the core and the periphery ($\lambda > 0$) is a necessary condition for obtaining some cross-section dispersion. However, this is not sufficient. In addition, we require that liquidity conditions in both market segments are different. Otherwise, days on which the core rate is higher than rates in the periphery are roughly as likely as days on which we have the opposite, yielding a symmetric distribution with a peak in the center.

### 4.2 Cross-section of interest rates

Our model can generate a number of additional predictions. We first discuss the implications for the cross-section of interest rates on a given day, holding $\lambda$, $\pi^{pe}$, and $\pi^{co}$ constant. Suppose that a researcher has a dataset containing the interest rates of all interbank transactions over a given day. Our model implies that the cross-sectional variation of these interest rates should be explained by the different type of banks that are interacting in a given transaction. In particular,
Proposition 4 implies that the interest rates at which connected and unconnected banks transact should differ as follows.

**Corollary 1.** Consider transactions between periphery banks only. In the ACB regime, unconnected lenders receive lower rates than connected lenders and unconnected borrowers pay higher rates than connected borrowers. In the ICL regime, unconnected and connected borrowers trade at the same rates, while in the ICB regime, unconnected and connected lenders trade at the same rates.

While it is intuitive that connected banks trade at more favorable conditions than unconnected banks, this result depends on the type of equilibrium and the side of the market that has better terms of trade. For example, excess liquidity in the core implies a low core rate $R^{\text{core}}$, which is good for connected borrowers, but to the detriment of connected lenders. This suggests that an empirical analysis of interest rates across different types of banks should be conducted conditional on the liquidity imbalance between the core and the periphery.

One can also reformulate this prediction in terms of banks’ market power. Indeed, in each transaction between two peripheral banks, the total gains from trade are equal to $R^P - R^D$ and we can measure a bank’s market power by the fraction of this surplus it captures in equilibrium. It follows from Proposition 4 that this fraction is equal to $\omega^k_l$ (resp., $\omega^k_b$) for a lenders (resp., borrower) with type $k \in \{C, U\}$. Thus, Corollary 1 also holds for banks’ relative market power. Hence, the implications for interest rates and banks’ market power are identical, and one could test the prediction above by building implied estimates of high types and low types banks’ market power, for example using the same method as in Bech and Klee (2011). For instance, in the ACB regime, connected banks have more market power than unconnected banks while in the ICL regime unconnected and connected borrowers’ market power is identical.

All transactions within the core as well as trades between core and periphery banks take
place at rate $R^{co}$. Empirically, it should be easy to estimate this rate. Our model has a stark prediction about when bilateral transactions between two periphery banks occur at this rate.

**Corollary 2.** Some transactions between peripheral banks occur at the core rate, $R^{co}$, in the ACB regime, while this is never the case in the ICL and ICB regimes.

This corollary gives an important characterization of the different regimes, as equilibria differ with respect to the set of bilateral trades that occur at the core rate. Moreover, only connected banks can trade at $R^{co}$ in every equilibrium.

To understand this result, consider the case where $z^{pe} > 0$, so that borrowers have more bargaining power (the case $z^{pe} < 0$ is symmetric). In the ACB regime, liquidity conditions in both market segments are relatively similar, such that it is optimal for unconnected borrowers to compete with core dealers and offer to borrow at the core rate $R^{co}$ (connected lenders will not accept a lower rate). Accordingly, some unconnected lenders also enjoy trading at $R^{co}$ although they are not able to trade directly with the core. They reap this windfall gain because their type is private information. In the ICL equilibrium, this no longer happens because the core rate $R^{co}$ is too high so that unconnected borrowers are better off posting a rate below the core rate and trading only with unconnected lenders.

### 4.3 Effects of time-varying segmentation

We now turn to the implications of our model regarding shocks on $\lambda$. These implications could be tested using an event-study methodology around crisis periods, which correspond to sharp increases of $\lambda$ from a value close to zero to a positive number.

We first study the impact of $\lambda$ on the dispersion of interest rates. A standard measure of dispersion is the spread between the lowest and the highest interest rate, denoted $\text{Spread}_{HL}$.

\footnote{Another possible measure is the standard deviation of interest rates. However, we cannot compute this standard deviation in closed-form.}
Using Proposition 4, we obtain that in each possible equilibrium regime:

\[
Spread_{HL} = \begin{cases} 
\Phi_{co}(-z^*)(\frac{1-\beta}{1-\beta\lambda})(R^P - R^D) & \text{if } z^{pe} > 0, \\
\Phi_{co}(z^*)(\frac{1-\alpha}{1-\alpha\lambda})(R^P - R^D) & \text{if } z^{pe} < 0.
\end{cases}
\]  \hspace{1cm} (20)

Thus, the dispersion in rates is proportional to the width of the corridor \((R^P - R^D)\), which is natural because it ultimately determines the gains from trade between banks. The next corollary follows directly from the expression for \(Spread_{HL}\).

**Corollary 3.** In a given equilibrium regime, the high-low spread increases in the level of segmentation, \(\lambda\), between core and periphery banks. Moreover, an increase in core banks’ aggregate reserves, \(z^{co}\), raises the dispersion in rates when peripheral banks have an excess of liquidity \((z^{pe} > 0)\) while it decreases it otherwise.

Intuitively, an increase in \(\lambda\) gives more market power to connected banks. They are able to extract a larger surplus from making offers to unconnected banks with opposite trading needs, and unconnected banks have to offer them more attractive interest rates. This mechanism generates a larger difference between the highest and the lowest equilibrium interest rates.

Next, we study how segmentation, \(\lambda\), affects the trading volume between core and periphery banks (defined as the amount of reserves exchanged between periphery and core banks in equilibrium) and the trading volume between periphery banks (defined as the amount of reserves exchanged between periphery banks in equilibrium). We obtain the following result.

**Corollary 4.** When the segmentation between the core and the periphery markets increases, the trading volume between core and periphery banks decrease while the trading volume between periphery banks increase.

This result implies that the share of total trading volume in the interbank market accounted
by periphery-periphery trades increases as the interbank market becomes more segmented. To our knowledge, this prediction is new and has not yet been tested empirically. Frutos, Garcia-de-Andoain, Heider and Papsdorf (2016) find that the relative share of cross-border interbank transactions in the Euro area declined sharply following a) the Lehman bankruptcy and b) the escalation of the Euro area sovereign debt crisis in 2011 (see Abbassi, Bräuning, Fecht, and Peydró (2016). The previous corollary is consistent with this finding if one interprets cross-border banks that connect domestic banks to the international (cross-border) market as core banks, as in Cocco, Gomes and Martins (2009) and Craig and von Peter (2014).

We next turn to examining aggregate trading volume of trades in which periphery banks are involved (that is, trades between periphery banks only or trades between periphery and core banks). While connected banks always end up trading with another bank (either with an unconnected bank or with the core), unconnected banks are forced to resort to the central bank if their offers are not accepted. Hence, ultimately, the volume of trades involving at least one periphery bank is inversely related to the likelihood that periphery banks resort to the central bank. An increase in $\lambda$ has two opposite effects on the likelihood of this outcome. First, it increases the mass of unconnected banks and therefore the mass of periphery banks that might have to resort to the central bank. This effect decreases the volume of trades involving periphery banks. However, in the ICB and ICL regimes, an increase in $\lambda$ raises the probability for unconnected banks to trade together.$^{30}$ This effect, however, is only present in the ICB and ICL regimes.

**Corollary 5.** The volume of trades involving at least one periphery bank decreases in $\lambda$ in the

$^{30}$For instance, consider the following sequence of arrivals in the periphery market: (i) a connected lender, (ii) an unconnected borrower, (iii) an unconnected lender, and (iv) a connected lender. In the ACB regime, the connected lender trades with the unconnected borrower while the unconnected lender eventually does not trade (he makes an offer that is rejected by the last connected lender in the chain). In contrast in the ICL regime, the first connected lender trades with a core bank while the unconnected borrower makes an offer to the unconnected lender and this offer is accepted. Thus, the previous sequence of arrivals lead to more trades in the ICL regime than in the ACB regime. An increase in $\lambda$ raises the likelihood of sequences like the one we just considered. This explains why in the ICL regime, the effect of $\lambda$ on the volume of trades by periphery banks is ambiguous.
ACB regime, and can either decrease or increase in $\lambda$ in the ICL and ICB regimes.

The empirical evidence indeed suggests that market volumes (conditional on demand) have declined in crisis periods. For example, Afonso, Kovner and Schoar (2012) present evidence for lower trading volumes in the Fed Funds market following the Lehman bankruptcy (after controlling for borrower fixed effects). Similar Garcia-de-Andoain, Hoffmann, and Manganelli (2014) show how several crisis episodes forced banks to increasingly rely on central bank operations to cover their liquidity demand. Additional evidence is also presented in Bech and Monnet (2013).

Finally, we study how banks’ resort to the central bank is affected by segmentation. These quantities are empirically observable, and they also provide a measure of inefficiencies in the interbank market. To see this, let $T^P$ and $T^D$ be the respective amounts of reserves borrowed from the central bank and deposited with the central bank in Stage 4. Suppose $T^P \geq T^D > 0$. In this case, gains from trade are lost because an amount $T^D$ of reserves could be lent by banks who deposit reserves at the central bank to banks who borrow money from the central banks. Any such transaction at a rate in $(R^D, R^P)$ would make both parties better off. The case $T^D \geq T^P > 0$ is symmetric. Thus, any equilibrium outcome in which $\xi = \min(T^D, T^P) > 0$ is inefficient while equilibria for which $\xi = 0$ are efficient in the sense that they achieve all possible gains from trade between banks.31

**Corollary 6.** Holding $z^{pe}$ fixed, the equilibrium is efficient when $\lambda = 0$. When $\lambda > 0$, the equilibrium is inefficient ($\xi > 0$) if and only if the aggregate reserves held by core banks after trading with periphery banks (i.e., $z^* = \Delta(z^{pe}, \lambda) + z^{co}$) and periphery banks’ aggregate reserves (i.e., $z^{pe}$) have opposite signs.

Thus, our model implies that, on a given day, one might observe a use of both the deposit

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31 Allen, Chapman, Echenique, and Shum (2016) develop a notion of efficiency that is based on cooperative game-theory. More specifically, they examine whether interest rates negotiated in the interbank market satisfy certain equilibrium conditions. In contrast, our notion of inefficiency is based on banks failing to trade in the market despite the existence of mutually beneficial trading opportunities. Both concepts can be seen as complements.
and the borrowing facilities provided by the central bank (i.e., \( \xi > 0 \) for a given \( z^{pe} \)). A necessary condition for this to happen is that (i) the core and the peripheral market are segmented to some extent (\( \lambda > 0 \)) and (ii) core banks’ aggregate reserves have a sign opposite to that of peripheral banks (e.g., \( z^{pe} < 0 \) and \( \pi^{co} > 0 \)). Importantly, segmentation alone is not sufficient to induce inefficiency because the liquidity imbalance in the core and the periphery may go in the same direction. In this case, banks from both core and periphery will use the same central bank facility, and all gains from trade are realized. The first condition fits well with Figure 3, which shows that the simultaneous use of the ECB’s lending and borrowing facilities, within the same day, considerably increased during the crisis (which also coincides with an increase in \( \lambda \)). The second condition is an additional testable prediction of our model.

4.4 Effects of time-varying liquidity conditions

Finally, we consider the implications of our model regarding changes in \( z^{pe} \) and \( \bar{z}^{co} \). These implications are related to the variation of interest rates over time due to liquidity shocks, as well as changes in monetary policy. To analyze the impact of monetary policy, we write \( \bar{z}^{co} = m^{co} + \bar{a}^{co} \) and \( \bar{z}^{pe} = m^{pe} + \bar{a}^{pe} \) and we interpret \( m^{co} \) and \( m^{pe} \) as being the components of core and peripheral banks’ reserves that are controlled by the central bank through open market operations.\(^{32}\) These components can be positive (when the central bank injects liquidity) or negative (when the central bank withdraws liquidity).

We first study how banks’ aggregate reserves (\( \bar{z}^{co} \) and \( z^{pe} \)) affect peripheral banks’ market power (measured by \( \omega^k_l \) for a lender and \( \omega^k_b \)) for a borrower with type \( k \in \{C,U\} \)) and thus interest rates.

**Corollary 7.** In a given equilibrium regime, peripheral lenders’ market power decreases with \( z^{pe} \)

\(^{32}\)In practice, some central banks restrict the set of banks that are eligible for their monetary policy operations (see Kraenzlin and Nellen (2015)), which allows for differences in \( m^{co} \) and \( m^{pe} \). If the central bank cannot discriminate between core and periphery banks then \( m^{co}=m^{pe}=m/2 \).
and \( z^{co} \) while peripheral borrowers’ market power increases with \( z^{pe} \) and \( \bar{z}^{co} \). Thus, equilibrium rates in the peripheral market decrease with \( z^{pe} \) and \( \bar{z}^{co} \). When \( z^{pe} = 0 \) and \( \bar{z}^{co} = 0 \), lenders and borrowers have identical market power. In this case all transactions in the peripheral market and in the core market take place at the same rate, equal to \( R^{co*} \).

Thus, an injection of liquidity by the central bank (an increase in \( z^{pe} \) or \( \bar{z}^{co} \)) has the expected effect: it lowers lenders’ market power and therefore all rates in equilibrium. For instance, consider the effect of an increase in \( z^{pe} \) when \( z^{pe} > 0 \), so that \( \beta < 1 \) and \( \alpha = 1 \). This increase reduces lenders’ market power for several reasons. First, peripheral banks expect \( R^{co*} \) to be smaller. Hence connected lenders have a less attractive outside option. Second, lenders are less likely to find a trading partner in the peripheral market if they reject an offer (\( \beta \) weakly decreases with \( z^{pe} \)). Third, borrowers are more likely to find one if they reject an offer (\( \alpha \) weakly increases with \( z^{pe} \)). The first two effects allow borrowers to make less competitive offers while the third effect incentivizes lenders to make more competitive offers. Overall, borrowers can grab a larger fraction of gains from trade in equilibrium. This result is similar to Afonso and Lagos (2015), where the market power of lenders and borrowers is determined by the mass of banks with excess reserves relative to the mass of banks with a shortage of reserves.

Figures 6 and 7 illustrate Corollary 7. Figure 6 shows the evolution of lenders’ market power (Panel A) and borrowers’ market power (Panel B) for two different levels of core banks’ expected reserves: \( z^{co} = 0 \) (black lines) and \( z^{co} = 3 \) (red lines). Parameter values are such that the active connected banks regime always obtains. The dashed lines depict connected banks’ market power while the plain lines depict unconnected banks’ market power. Figure 7 illustrates this by plotting the equilibrium interest rates in the ACB equilibrium. The blacklines are the rates offered by lenders and the red lines are the rates offered by borrowers. Plain lines are the rates offered by connected banks while dashed lines are the rates offered by unconnected banks. The green line
is the rate in the core market.

While Corollary 7 shows that interest rates respond in a natural way to monetary policy in a given equilibrium regime, our model can also generate more counter-intuitive implications because changes in liquidity conditions, and thus monetary policy, affect the core and the periphery differently unless the market is perfectly integrated ($\lambda = 0$). This feature distinguishes our model from other frameworks.

**Corollary 8.** *In the absence of segmentation ($\lambda = 0$), equilibrium strategies and interest rates depend only on total liquidity injections by the central bank, $m^{co} + m^{pe}$, and not on its breakdown between periphery banks and core banks. This is no longer true when $\lambda > 0$. In particular, the effect of an increase in liquidity on interest rates depend on whether the liquidity increase affects core banks (e.g., $m^{co}$ increases) or periphery banks (e.g., $m^{pe}$) increases.*

A particularly interesting application of this corollary is to consider the case in which the central bank alters $m^{co}$, but not $m^{pe}$.\(^{33}\) If all banks are connected to each other via the core ($\lambda = 0$), it is irrelevant where central banks inject liquidity, and any change in $m^{co}$ will ultimately affect all banks in the same way. However, this is no longer the case when the market becomes segmented ($\lambda > 0$), because the flow of reserves between core and periphery is disrupted. In this setting, central bank liquidity injections can have unintended effects if they increase the imbalance between the core and the periphery, and thereby cause a regime shift from ACB to ICL or ICB.

**Corollary 9.** *Consider an increase in the average aggregate reserves of core banks (e.g., an increase in $m^{co}$) that shifts the equilibrium from the ACB to the ICB. This shift (weakly) increases the core rate, the rates at which all periphery banks trade and the average rate across all transac-

\(^{33}\)This could be the case because the central bank only relies on a specific set of counterparties for conducting open market operations. This is for example the case of the U.S. Federal Reserve. In contrast, the (reverse) open market operations conducted by the Eurosystem via its main refinancing operation are accessible to all Euro area banks.
tions when \( z^{pe} < 0 \). A decrease in the average aggregate reserves of core banks (e.g., a decrease in \( mco \)) that shifts the equilibrium from the ACB to the ICL has opposite effects when \( z^{pe} > 0 \).

This corollary has an important implication for the implementation of monetary policy. When the market is segmented, for some parameters a liquidity injection in the core (an increase in \( mco \)) can lead to an increase in all interest rates, including \( R^{cos} \), rather than a decrease. The reason for this counter-intuitive result is the strategic behavior of periphery banks, which is at the heart of our model. Indeed, when \( z^{pe} < 0 \), rates in the periphery are higher than in the core.

If liquidity is injected in the core so that the equilibrium regime switches from the ACB to the ICB regime, connected borrowers only trade with core banks. Due to this inflow of borrowers, the net supply of liquidity in the core market is lower, and the core rate increases. At the same time, in the periphery, unconnected lenders strategically choose to offer less competitive rates (rates that are accepted only by unconnected borrowers only). As a result, the injection of liquidity in core banks has the opposite effect of its intended goal: it raises rates in the interbank market rather than decreasing them.

5 Conclusion

Our model offers a theory of OTC trading in interbank markets in crisis times, that also nests the traditional model of Poole (1968) as a special, “normal times” case. We show that when some banks lose access to the group of “core” banks, liquidity imbalances between core and peripheral banks can give rise to inefficient equilibria and a significant dispersion of the rates at which banks trade with each other. For instance, if there is a shortage of liquidity in the periphery and an excess in the core, the lenders in the periphery try to exploit the fact that some borrowers cannot benefit from the low interest rate offered by core banks. As a result, some peripheral banks can trade at very high rates even if the market as a whole benefits from excess liquidity.
This segmentation between core and peripheral banks poses significant challenges to central banks. The central bank should fight the dispersion of interest rates, as it implies that the average rate no longer properly reflects the borrowing conditions of banks. This typically requires to move to a “floor system”, in which the central bank injects a lot of liquidity. However, this system can be efficient only if the central bank can allocate liquidity both to core and to peripheral banks, which can be challenging. Moreover, large liquidity injections can be costly, in which case the optimal policy trades off interest rate targeting with costs and uses both excess liquidity and the rates of the two standing facilities.

A Proofs

A.1 Proof of Proposition 1

The expression for the core market rate (eq.(8)) follows directly from the expression for core banks’ demand for reserves, \( q^{co}(R_{0}^{co}) \), and the clearing condition (eq. (7)). Moreover, using eq. (8), we obtain:

\[
\frac{\partial R^{co}}{\partial z^*} = -(R^P - R^D) \frac{\partial \Phi_{co}(-z)}{\partial z} < 0,
\]

since \( \Phi_{co}(x) \) increases with \( x \). Moreover, \( \Phi_{co}(0) = 1/2 \) since \( \Phi_{co}(. \) is the cumulative probability distribution of a normally distributed variable with mean 0. Thus, using eq. (8), \( R^{co} = (R^P + R^D)/2 \) iff \( z = 0 \).

A.2 Proof of Proposition 2

Identical to the proof of Proposition 1.

A.3 Proof of Lemma 1

Follows immediately from observations 1 to 4 that precede the lemma.
A.4 Proof of Lemma 2

ACB Regime. We start by deriving the conditions under which the active connected banks (ACB) regime obtains. In this equilibrium, a connected lender’s offer is accepted if and only if the lender is matched with an unconnected borrower. This event has probability $\lambda \beta$, so that:

$$V_{C}^{*} = \lambda \beta R_{C}^{*} + (1 - \lambda \beta) R_{co}^{*}. \quad (21)$$

Symmetrically, a connected borrower’s offer is accepted with probability $\lambda \alpha$. Therefore:

$$V_{C}^{*} = -\lambda \alpha R_{C}^{*} - (1 - \lambda \alpha) R_{co}^{*}. \quad (22)$$

Moreover, an unconnected lender’s offer is accepted if and only if the lender is matched with a borrower, of any type, which gives:

$$V_{U}^{*} = \beta R_{U}^{*} + (1 - \beta) R_{D}. \quad (23)$$

And, symmetrically:

$$V_{U}^{*} = -\alpha R_{U}^{*} - (1 - \alpha) R_{P}. \quad (24)$$

Furthermore, from Lemma 1, we know that $R_{C}^{*} = -V_{b}^{*}$, $R_{C}^{*} = V_{U}^{*}$, $R_{C}^{*} = -V_{b}^{*}$, and $R_{b}^{*} = V_{l}^{*}$.

Combining these conditions with eq.(21), (22), (23), and (24), we obtain a system of 4 equations with 4 unknowns ($R_{C}^{*}$, $R_{U}^{*}$, $R_{b}^{*}$, $R_{b}^{*}$). Solving this system, we obtain:

$$R_{b}^{*} = V_{l}^{*} = \frac{(1 - \beta \lambda) R_{co}^{*} + \beta \lambda (1 - \alpha) R_{P}}{1 - \alpha \beta \lambda}$$

$$R_{C}^{*} = V_{U}^{*} = \frac{(1 - \beta) R_{D} + \beta (1 - \alpha \lambda) R_{co}^{*}}{1 - \alpha \beta \lambda} \quad (25)$$

$$R_{l}^{*} = V_{b}^{*} = \frac{(1 - \alpha \lambda) R_{co}^{*} + \alpha \lambda (1 - \beta) R_{D}}{1 - \alpha \beta \lambda}$$

$$R_{l}^{*} = -V_{b}^{*} = \frac{(1 - \alpha) R_{P} + \alpha (1 - \beta \lambda) R_{co}^{*}}{1 - \alpha \beta \lambda}. \quad (26)$$
Equations (25) and (26) yield equilibrium rates in the ACB regime.

It remains to derive the conditions under which banks have no incentive to deviate from their equilibrium behavior in the ACB regime. First, observe that we have \( R^C_b \leq R^{co} \) and \( R^C_l \geq R^{co} \). Hence, \( q^*_l = q^*_b = 1 \) is optimal for connected banks. Second, we have \( R^U_b \leq R^P \) and \( R^U_l \geq R^D \) so that unconnected banks are better off trading with another peripheral bank than with the central bank.

Last, we need to check that \( p^*_l = p^*_b = 1 \) is optimal for unconnected banks. Consider an unconnected lender. In equilibrium, he obtains an expected profit of \( V^U_l \) with an offer at \( R^U_l = -V^C_b \). As explained in the text (see Observation 2), his most profitable deviation is to offer the same rate as a connected lender, i.e., \( R^C_l = -V^U_b \). This alternative offer is accepted only by unconnected borrowers, i.e., with probability \( \beta \lambda \). Thus, an unconnected lender is better off playing the equilibrium strategy if and only if:

\[
V^U_l > (1 - \beta \lambda) R^D - \beta \lambda V^U_b. \tag{27}
\]

Symmetrically, an unconnected borrower is better off playing the equilibrium strategy if and only if:

\[
V^U_b > -(1 - \alpha \lambda) R^P - \alpha \lambda V^U_l. \tag{28}
\]

Using (25) and (26), Conditions (27) and (28) can be rewritten as:

\[
\frac{R^{co} - R^D}{R^P - R^D} > \frac{\lambda(1 - \alpha)}{1 - \alpha \lambda(2 - \beta \lambda)} \tag{29}
\]

\[
\frac{R^P - R^{co}}{R^P - R^D} > \frac{\lambda(1 - \beta)}{1 - \beta \lambda(2 - \alpha \lambda)} \tag{30}
\]

When \( z^{pe} = 0, \beta = \alpha = 1 \). Hence, Conditions (29) and (30) are satisfied since \( R^D < R^{co} < R^P \).
Thus, in this case, the unique possibility in equilibrium is the ACB regime.

Now consider the case in which peripheral banks have an excess of reserves in the aggregate, i.e., \( z_{pe} > 0 \) (the case \( z_{pe} < 0 \) is symmetric). In this case, \( \alpha = 1 \) while \( \beta < 1 \). This means that a borrower who rejects an offer is sure to find a lender while a lender is not since there is an excess supply of reserves in the periphery. In this case, Condition (29) is always satisfied because \( \alpha = 1 \). Thus, unconnected lenders always make an offer that attract both types of peripheral borrowers. In contrast, Condition (30) might not (it requires \( R_{CO}^{*} \) to be close enough to \( R_{D}^{*} \) so that the L.H.S of (30) is close to one). Thus, when \( z_{pe} > 0 \), Condition (30) is necessary and sufficient for the equilibrium to be in the ACB regime. Symmetrically, when \( z_{pe} < 0 \), \( \beta = 1 \) and Condition (29) is necessary and sufficient for the equilibrium to be in the active connected banks regime. After straightforward manipulations, these two conditions can be shown to be equivalent to Condition () in Lemma 2.

We now derive conditions under which the other regimes are obtained in equilibrium.

**Inactive connected lenders (ICL) regime.** In this equilibrium, connected borrowers make offers that are accepted by unconnected lenders only, i.e., \( R_{b}^{C} = V_{U}^{*} \). Moreover, connected lenders reject offers from borrowers (whether unconnected or connected) and trade with a core bank with probability 1 \( (q_{l}^{*} = 1) \). Hence, in this equilibrium, we have:

\[
V_{l}^{C} = R_{CO}^{*} \tag{31}
\]

\[
V_{b}^{C} = -\lambda \alpha V_{l}^{U} - (1 - \lambda \alpha)R_{CO}^{*} \tag{32}
\]

In addition, in this equilibrium, unconnected lenders make offers that are accepted by all borrowers, i.e., \( R_{l}^{U} = -V_{b}^{C} \), while unconnected borrowers make offers that are accepted by unconnected
lenders only, i.e., \( R_b^U = V_l^U \). Thus, we have:

\[
V_l^U = -\beta V_b^C + (1 - \beta) R^D \\
V_b^U = -(1 - \alpha \lambda) R^P - \alpha \lambda V_l^U.
\] (33) (34)

Solving the previous system of equations for \( V_l^U, V_b^U, \) and \( V_b^C \), we obtain that in a ICL regime:

\[
R_b^U = R_b^C = V_l^U = \frac{(1 - \alpha \lambda) R^c^+ + (1 - \beta) R^D}{1 - \lambda \alpha}.
\] (35)

\[
V_b^U = -\left( \frac{\lambda \alpha \beta (1 - \alpha \lambda) R^c^+ + \alpha \lambda (1 - \beta) R^D}{1 - \lambda \alpha} \right) + (1 - \alpha \lambda) R^P
\] (36)

\[
R_l^U = -V_b^C = \frac{(1 - \alpha \lambda) R^c^+ + \alpha \lambda (1 - \beta) R^D}{1 - \lambda \alpha}.
\] (37)

\[ V_l^C = R^c^+ . \] (38)

We now check that no bank has an incentive to deviate from this equilibrium when \( \frac{R^P - R^c^+}{R^P - R^U} < \frac{\lambda (1 - \beta)}{1 - \lambda \beta (2 - \lambda)} \). First, from eq. (37), we deduce that \( V_b^C > -R^c^+ \), so that connected borrowers are better off making an offer at rate \( R_b^C \) to another peripheral bank when they reject an offer rather than contacting directly a core bank. This implies \( q_b^* = 1 \) is optimal for a connected borrower, as it should in the ICL regime.

In the ICL regime, unconnected borrowers offer rates that are accepted by unconnected lenders only. Their best deviation is to offer a rate \( V_l^C \) that is accepted by all types of lenders (see Observation 2 in the text). This deviation is not optimal when other banks behave as in the ICL regime iff:

\[
V_b^U > -(1 - \beta) R^P - \alpha V_l^C.
\] (39)

First, consider the case \( z^p \leq 0 \) so that \( \beta = 1 \). In this case, substituting \( V_b^U \) and \( V_l^C \) by their expressions in Condition (39), we observe that it requires \( R^c^+ \geq R^P \). This is impossible. Thus,
Condition (39) cannot be satisfied when \( z^{pe} < 0 \) and therefore in this case the ICL regime cannot obtain. Now consider the case \( z^{pe} > 0 \) so that \( \alpha = 1 \) and \( \beta < 1 \). In this case, substituting \( V^U_b \) and \( V^C_i \) by their expressions in Condition (39), we can rewrite this condition as:

\[
\frac{R^P - R^{co}}{R^P - R^D} < \frac{\lambda(1 - \beta)}{1 - \lambda \beta (2 - \lambda)}. \tag{40}
\]

Now consider the lenders. In the ICL regime, a connected lender directly contacts a core bank if it rejects an offer \((q^*_l = 0)\). Its best deviation (see Observation 2 in the text) is to make an offer at the largest rate that unconnected borrowers are willing to accept, i.e., \(-V^U_b\). This deviation cannot be optimal if \( R^{co} > -V^U_b \). Substituting \( V^U_b \), we find that when \( z^{pe} > 0 \), this condition is satisfied if (40) is satisfied. Thus, when \( z^{pe} > 0 \) (which, as just explained, is a necessary condition for the ICL regime) and Condition (40) holds, \( q^*_l = 0 \) is a best response for connected lenders.

Finally, unconnected lenders offer a rate that is accepted by all type of borrowers, i.e., a rate equal to \( R^U_i = -V^C_b \). Their best deviation is to offer a rate \(-V^U_b\) that is accepted by unconnected of borrowers only (see Observation 2 in the text). This deviation is not optimal iff:

\[
V^U_i > (1 - \beta \lambda) R^D - \beta \lambda V^U_b. \tag{41}
\]

When \( z^{pe} > 0 \), Condition (41) is equivalent to:

\[
\frac{R^P - R^{co}}{R^P - R^D} < \frac{1 - \lambda}{1 - \lambda \beta}, \tag{42}
\]

by substituting \( V^U_i \) and \( V^U_b \) by their expressions. When \( \lambda < 1/2 \), Condition (42) is satisfied if (40) is satisfied. We deduce that \( z^{pe} > 0 \) and Condition (40) are necessary and sufficient for obtaining the ICL regime. This condition is equivalent to Condition (16) in Lemma 2.

**Inactive connected borrowers (ICB) regime.** We can show that this equilibrium obtains
if and only if $z^{pe} < 0$ and:

$$\frac{R^{cos} - R^D}{R^P - R^D} < \frac{\lambda(1 - \alpha)}{1 - \lambda \alpha(2 - \lambda)},$$ \hspace{1cm} (43)

by proceeding exactly as we did for ICL regime. This condition is equivalent to Condition (17) in Lemma 2.

### A.5 Stationary measures

Each time a new peripheral bank arrives in the peripheral market, eight possible events can happen: (i) A connected lender makes an offer or goes to the core market (without making an offer to another peripheral bank); (ii) A connected lender accepts an offer; (iii) A unconnected lender makes an offer; (iv) A unconnected lender accepts an offer; (v) A connected borrower makes an offer or contacts a core bank (without making an offer to a peripheral bank); (vi) A connected borrower accepts an offer; (vii) An unconnected borrower makes an offer; (viii) An unconnected borrower accepts an offer.

We denote by $\mu^*_i$ the stationary probabilities of event $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$ in equilibrium and let $M^* = (\pi^*_{i,j})$ be the transition matrix from event $i$ to event $j$, where $\pi^*_{i,j}$ is the likelihood that event $i$ is followed by event $j$ given banks’ equilibrium actions and the arrival process for borrowers and lenders. Let $\mu^*$ be the (line) vector of the $\mu$s. It solves

$$\mu^* = \mu^* M^* \text{ and } 1(\mu^*)^t = 1,$$ \hspace{1cm} (44)

where $1 = (1, 1, ..., 1)$ and $(\mu^*)^t$ is the transpose of $\mu^*$.

First, consider the case $z^{pe} > 0$, so that we necessarily have $p^*_i = q^*_b = 1$ for any equilibrium
type. The equilibrium actions are such that:

\[
M^* = \begin{pmatrix}
A(\beta, \lambda) & C(\beta, \lambda, 1, q_t^*) \\
C(\alpha, \lambda, p_b^*, 1) & A(\alpha, \lambda)
\end{pmatrix},
\]

where \(A(x, \lambda)\) and \(C(x, \lambda, p, 1)\) are two matrixes defined as:

\[
A(x, \lambda) = (1 - x) \begin{pmatrix}
(1 - \lambda) & 0 & \lambda & 0 \\
(1 - \lambda) & 0 & \lambda & 0 \\
(1 - \lambda) & 0 & \lambda & 0 \\
(1 - \lambda) & 0 & \lambda & 0
\end{pmatrix},
\]

and

\[
C(x, \lambda, p, q) = x \begin{pmatrix}
(1 - \lambda) & 0 & \lambda(1 - q) & \lambda q \\
(1 - \lambda) & 0 & \lambda & 0 \\
(1 - \lambda)(1 - p) & (1 - \lambda)p & 0 & \lambda \\
(1 - \lambda) & 0 & \lambda & 0
\end{pmatrix}.
\]
Solving eq. (44), we obtain the stationary measures. Define:

\[\mu_{1a}(x, p, q) = (1 - \lambda)[1 - x\lambda(1 + (1 - \lambda)p)]/M(x, p, q)\]
\[\mu_{2a}(x, p, q) = x(1 - \lambda)^2\lambda p[1 - q(1 - x\lambda(1 - \lambda))]/M(x, p, q)\]
\[\mu_{3a}(x, p, q) = \lambda[1 - x + x\lambda(1 - \lambda)q(1 - p(1 - x(1 - \lambda)))]/M(x, p, q)\]
\[\mu_{4a}(x, p, q) = x\lambda(1 - \lambda)[1 - \lambda q(1 + xp(1 - \lambda)^2)]/M(x, p, q)\]
\[\mu_{1b}(x, p, q) = x(1 - \lambda)^2[1 - x\lambda(p + (1 - p)\lambda)]/M(x, p, q)\]
\[\mu_{2b}(x, p, q) = x\lambda(1 - \lambda)[1 - x + x\lambda(1 - \lambda)q(1 - p(1 - x(1 - \lambda)))]/M(x, p, q)\]
\[\mu_{3b}(x, p, q) = x\lambda(1 - \lambda)[1 - q(1 - x\lambda(1 - \lambda))]/M(x, p, q)\]
\[\mu_{4b}(x, p, q) = x\lambda[1 - x\lambda + (1 - \lambda)q(1 - x\lambda p)(1 - \lambda x(1 - \lambda))]/M(x, p, q)\]
\[M(x, p, q) = (1 + x)[1 + x\lambda(1 - p(1 - \lambda)(1 - \lambda x(1 - \lambda)))\]

When \(z^{pe} > 0\), we have \(\mu_i^* = \mu_{ia}(\beta, p^*_b, q^*_l)\) for \(i \leq 4\) and \(\mu_i^* = \mu_{ib}(\beta, p^*_b, q^*_l)\) for \(i > 4\). For \(z^{pe} < 0\), a symmetric reasoning yields \(\mu_i^* = \mu_{ib}(\alpha, p^*_l, q^*_b)\) for \(i \leq 4\) and \(\mu_i^* = \mu_{ia}(\alpha, p^*_l, q^*_b)\) for \(i > 4\).

**A.6 Proof of Lemma 3**

Suppose \(z^{pe} \geq 0\). We first compute the mass of connected lenders who trade with a core bank. A connected lender trades with a core bank when (i) she directly contacts a core bank upon arrival or (ii) she makes an offer to a peripheral bank and this offer is rejected. The likelihood of the first event is \(\mu_{1a}^* q^*_l \left(\frac{(1 - \beta) + \beta(1 - \lambda)}{\mu_{1a}(\beta, p^*_b, q^*_l)}\right)\) where \(\mu_{1a}^* = \mu_{1a}(\beta, p^*_b, q^*_l)\) (see Section A.5). Thus, the mass of connected lenders who eventually trade in the core market is: \(\mu^*_l \left(\frac{((1 - \beta) + \beta(1 - \lambda))q_l + (1 - q^*_l))}{\mu^*_1(1 - \beta \lambda q^*_l)}\) where \(\mu^*_1 = \mu_{1a}(\beta, p, q_l)\) (see Section A.5). Thus, using the definition
of $\Delta(z^{pe}, \lambda; p_t, q_b, p_b, q_l)$ and the expressions for $\mu_1^*$ and $\mu_5^*$ obtained in Section A.5, we deduce that, for a given equilibrium, we have:

$$\Delta(z^{pe}, \lambda; p_t, q_b, p_b, q_l) = \ell(|z^{pe}|)[\mu_1^*(1-\beta \lambda l) - \mu_5^*(1-\lambda)] = \ell(|z^{pe}|)\frac{N(\beta(z^{pe}), \lambda; p_t, q_b, p_b, q_l)}{D(\beta(z^{pe}), \lambda; p_t, q_b, p_b, q_l)},$$

where

$$N(\beta, \lambda; p_t, q_b, p_b, q_l) \equiv (1-\lambda)[1-\beta+\beta \lambda(1-\lambda)(1-p_b)-q_l]+q_l\beta^2[\lambda(1-p_b)(1-\lambda)^2+\lambda+p_b(1-\lambda)] > 0,$$

and

$$D(\beta, \lambda; p_t, q_b, p_b, q_l) \equiv (1+\beta)[1+\beta \lambda - \beta p_b q_l(1-\lambda)(1-\beta \lambda(1-\lambda))] > 0.$$

Moreover, $p_t = q_b = p_b = q_l = 1$ in the ACB regime while $p_t = q_b = 1$ and $p_b = q_l = 0$ in the ICL regime.

Hence, when $z^{pe} \geq 0$, $\Delta(z^{pe}, \lambda; p_t, q_b, p_b, q_l) \geq 0$. Observe that $N(\beta(z^{pe}), \lambda; p_t, q_b, p_b, q_l)$ decreases in $\beta$ and $D(\beta(z^{pe}), \lambda; p_t, q_b, p_b, q_l)$ increases in $\beta$. As $\beta$ decreases with $z^{pe}$, we deduce that the ratio $\frac{N(\beta(z^{pe}), \lambda; p_t, q_b, p_b, q_l)}{D(\beta(z^{pe}), \lambda; p_t, q_b, p_b, q_l)}$ increases with $z^{pe}$. As $l(|z^{pe}|)$ increases in $z^{pe}$ as well, we deduce that $\Delta(z^{pe}, \lambda; 1,1,1,1)$ (the ACB regime) and $\Delta(z^{pe}, \lambda; 1,1,0,0)$ (the ICL regime) increase in $z^{pe}$. Finally, simple but tedious calculations show that $\frac{N(\beta(z^{pe}), \lambda; 1,1,0,0)}{D(\beta(z^{pe}), \lambda; 1,1,0,0)} > \frac{N(\beta(z^{pe}), \lambda; 1,1,1,1)}{D(\beta(z^{pe}), \lambda; 1,1,1,1)}$. Thus, $\Delta(z^{pe}, \lambda; 1,1,1,1) < \Delta(z^{pe}, \lambda; 1,1,0,0)$. Finally when $z^{pe} = 0$, $\beta = 1$ and the regime is necessarily ACB. Thus $p_b = q_l = 1$ when $z^{pe} = 0$. Calculations yield $N(1, \lambda; 1,1,1,1) = 0$ and therefore $\Delta(0, \lambda; 1,1,1,1) = 0$. The reasoning is symmetric for $z^{pe} < 0$.

**A.7 Proof of Proposition 3**

The three regimes (ICL, ICB, and ACB) cover all possible equilibrium behaviors of peripheral banks in a pure strategy equilibrium (an implication of Lemma 1). When $\varpi^{co} \in \Omega$, only one
of these three regime can be obtained in a pure strategy equilibrium. Moreover, equilibrium
rates in each case are unique (see the proof of Lemma 2). In the internet appendix, we show
that when \( \zeta^\omega \not\in \Omega \) then there is no pure strategy equilibrium. We conclude that the pure
strategy equilibrium is unique. The rest of the proposition directly follows from the discussion
that precedes the proposition.

A.8 Proof of Proposition 4

The expressions for equilibrium rates follow from the expressions for the rates obtained in Lemma
2.

A.9 Proof of Proposition 5

Suppose \( z^{pe} > 0 \). In this case, the likelihood that the active connected banks regime is obtained
is equal to \( \Pr(\zeta^\omega > z^+(z^{pe}, \lambda)) \). By definition:

\[
z^+(z^{pe}, \lambda) = - \left[ \Phi^{-1}_{\zeta^\omega} (1 - \omega(\lambda, \beta)) + \Delta(z^{pe}, \lambda; 1, 1, 0, 0) \right]
\]

(46)

Now, we have \( \lim_{\lambda \to 0} \omega(\lambda, \beta) = 0 \). Thus, using eq.(46),

\[
\lim_{\lambda \to 0} z^+(z^{pe}, \lambda) = -\infty.
\]

Thus, the likelihood that the active connected banks regime obtains goes to one as \( \lambda \) goes to
zero. Moreover, for \( z^{pe} > 0 \), we deduce from Appendix A.5 that \( \lim_{\lambda \to 0} \mu^*_1 = (1 + \beta(z^{pe}))^{-1} \) and
\( \lim_{\lambda \to 0} \mu^*_5 = \beta(1 + \beta(z^{pe}))^{-1} \). Thus, using eq.(45), we deduce that:

\[
\lim_{\lambda \to 0} \Delta(z^{pe}, \lambda) = \ell(|z^{pe}|)(2\pi U - 1) = z^{pe} \text{ when } z^{pe} > 0.
\]
It follows that

$$\lim_{\lambda \to 0} R^{co} = R_0^{co}.$$ 

Finally, using the expressions for equilibrium rates in Proposition 4, we also obtain that

$$\lim_{\lambda \to 0} R_{bU} = \lim_{\lambda \to 0} R_{lU} = R_0^{co}.$$ 

A symmetric reasoning shows that the same findings obtain when $z^{pe} < 0$.

### A.10 Proof of Corollary 1

The proof directly follows from comparing the expressions of the interest rates derived in Proposition 4.

### A.11 Proof of Corollary 2

The proof directly follows from comparing the expressions of the interest rates derived in Proposition 4.

### A.12 Proof of Corollary 3

The proof follows from direct inspection of equation (20).

### A.13 Proof of Corollary 4

We focus on the case $z^{pe} > 0$ (the Proof for $z^{pe} < 0$ is symmetric). Trades between periphery banks and core banks occur whenever a connected bank finds its offer rejected. In the ACB regime, this quantity is given by $l(\theta)[\mu_{1a}(1 - \beta\xi) + \mu_{1b}(1 - \xi)]$, which is decreasing in $\lambda$. In the ICL, core-periphery volume is equal to $l(\theta)[\mu_{1a} + \mu_{1b}(1 - \lambda)]$, which is also decreasing in $\lambda$. 

51
Finally, comparing across equilibrium regimes, the difference between the two is

\[
[\mu_{1a}(1, 1)(1 - \beta \lambda) + \mu_{1b}(1, 1)(1 - \lambda)] - [\mu_{1a}(0, 0) + \mu_{1b}(0, 0)(1 - \lambda)],
\]

which is positive, that is there are more core-periphery trades in the ACB regime for identical parameters.

Note that all transactions among periphery banks necessarily involve at least one unconnected bank. The periphery-periphery volume can thus be written as \(l(\theta)[\lambda - \mu_{3a}(1 - \beta)]\) in the ACB regime, which is increasing in \(\lambda\). The same conclusion obtains in the ICL regime, the volume being \(l(\theta)[\lambda - \mu_{3a}(1 - \beta) - \mu_{3b}(1 - \lambda)]\). Finally, \(\mu_{3a}(0, 0) > \mu_{3a}(1, 1)\), so that the periphery-periphery volume is higher in the ICL regime than in the ACB regime.

A.14 Proof of Corollary 5

All periphery banks ultimately trade with another bank, except the low type periphery banks that ultimately trade with the central bank. Focusing on the case \(z^{pe} > 0\), total trading volume is thus inversely related to \(\mu_{3a}(1 - \beta)l(|\theta|)\) (ACB regime) or \([\mu_{3a}(1 - \beta) + \mu_{3b}(1 - \lambda)]l(|\theta|)\) (ICL regime). It can easily be verified that the first expression is increasing in \(\lambda\) (thus implying that trading volume is decreasing in \(\lambda\)), while the second expression can both increase and decrease in \(\lambda\).

A.15 Proof of Corollary 6

Consider the case \(z^{pe} > 0\). In the ACB regime we have \(T^P = -\min(0, z^*)\) and \(T^D = l(z^{pe})\mu^*_3(\beta, 1, 1)(1 - \beta) + \max(0, z^*)\) with \(z^* = \zeta^{op} + \Delta\) and in the ICL regime we have \(T^P = l(z^{pe})\mu^*_3(\beta, 0, 0)(1 - \lambda) - \min(0, z^*)\) and \(T^D = l(z^{pe})\mu^*_3(\beta, 0, 0)(1 - \beta) + \max(0, z^*)\). When \(\lambda = 0\), we have \(\mu^*_7(\beta, 0, 0) = \mu^*_3(\beta, 1, 1) = 0\). Thus, \(\xi = \min(T^P, T^D) = 0\). When \(\lambda > 0\), it is immediate that \(T^D > 0\) since
\( \mu_{3b}(\beta, p, q) > 0 \) and \( \mu_{3a}(\beta, p, q) > 0 \) (see Section A.5). Moreover, \( T^P > 0 \) if and only if \( z^* < 0 \).

The case \( z^{pe} < 0 \) is symmetric.

**A.16 Proof of Corollary 7**

The proof directly follows from comparing the expressions of the interest rates derived in Proposition 4.

**A.17 Proof of Corollary 8**

Follows from inspection of equilibrium strategies and interest rates.

**A.18 Proof of Corollary 9**

Focusing on the case \( z^{pe} > 0 \), moving from the ACB to the ICL regime increases \( \Delta \), which decreases \( R^{co} \). Considering the rates in the periphery, \( R^L_b \) changes from \( R^{co} \) to a weighted average of \( R^{co} \) and \( R^D \), with a lower value of \( R^{co} \), hence \( R^U_b \) decreases. \( R^U_l \) and \( R^C_b \) are the same weighted averages of \( R^D \) and \( R^{co} \) in both types of equilibria, so the lower value of \( R^{co} \) in the unbalanced equilibrium implies that both rates decrease. Finally, \( R^C_l \) is never offered in the unbalanced equilibrium. To compute the average interest rate, observe for \( z^{pe} > 0 \) the highest interest rate is \( R^{co} \) and the lowest one is \( R^C_b \) (connected borrowers have the highest market power), both in the ACB and in the ICL regime. In the ACB regime, we can rewrite interest rates as
\[ R_b^U = R_t^C = R^{co} = R^D + \Phi_{co}(-S_{co})(R^P - R^D) \]
\[ R_t^U = R^D + \Phi_{co}(-S_{co})\frac{(1 - \lambda)}{1 - \beta \lambda}(R^P - R^D) \]
\[ R_b^C = R^D + \Phi_{co}(-S_{co})\frac{\beta(1 - \lambda)}{1 - \beta \lambda}(R^P - R^D). \]

In the unbalanced equilibrium these expressions are the same, except that \( R_t^C \) doesn’t exist and \( R_b^U \) takes the same form as \( R_b^C \). In both cases, we thus observe three interest rates: \( R^{co}, R_t^U, R_b^C \), with \( R^{co} \geq R_t^U \geq R_b^C \). Since all three rates decrease in the unbalanced equilibrium, to prove that the average decreases it is enough to prove that the relative occurrence of the first two rates is lower in the unbalanced than in the balanced equilibrium. Denote \( \nu_b^{co} \) and \( \nu_u^{co} \) the proportion of transactions that occur at \( R^{co} \) in both types of equilibria, and \( \nu_b^{IL}, \nu_u^{IL} \) those that occur at \( R_t^U \). We have

\[ \nu_b^{co} = \frac{\mu_{1a}(1, 1) + \mu_{1b}(1, 1)(1 - \lambda) + \lambda \mu_{3b}(0, 0) + \beta \mu_{3a}(0, 0)}{\mu_{1a}(1, 1) + \mu_{1b}(1, 1) + \mu_{3b}(1, 1) + \beta \mu_{3a}(1, 1)} \]
\[ \nu_b^{IL} = \frac{\beta \mu_{3a}(1, 1)}{\mu_{1a}(1, 1) + \mu_{1b}(1, 1) + \mu_{3b}(1, 1) + \beta \mu_{3a}(1, 1)} \]
\[ \nu_u^{co} = \frac{\mu_{1a}(0, 0) + \mu_{1b}(0, 0)(1 - \lambda)}{\mu_{1a}(0, 0) + \mu_{1b}(0, 0) + \lambda \mu_{3b}(0, 0) + \beta \mu_{3a}(0, 0)} \]
\[ \nu_u^{IL} = \frac{\beta \mu_{3a}(0, 0)}{\mu_{1a}(0, 0) + \mu_{1b}(0, 0) + \lambda \mu_{3b}(0, 0) + \beta \mu_{3a}(0, 0)} \]

Computations then show that \( \nu_b^{co} < \nu_u^{co} \) and \( \nu_b^{IL} < \nu_u^{IL} \), thus completing the proof.

B Figures

54
Figure 1: Core-Periphery Structure of the Euro Area Interbank Market. The figure depicts trading relationships (aggregated at the group level) among 265 banks in the euro area overnight interbank market. Banks are classified in two groups: (i) core banks, defined as banks with at least 30 different counterparties, of which more than 1/3 are foreign, and (ii) peripheral banks (all remaining banks). For June 2008, we observe that core bank borrow and lend on 70.6% of all days on average while the corresponding figure for periphery banks is only 11.7%. This supports the notion that core banks act as dealers for other banks since they are often lending and selling on the same day. Each red line corresponds to a trading relationship between two peripheral banks (the line is thicker when the amount traded between these banks is larger) and each blue line corresponds to a trading relationship between a periphery bank and a core bank.
Figure 2: Distribution of interbank interest rates.

Figure 3: Recourse to the ECB’s standing facilities (June 2008 and November 2011).
Figure 4: Trading in the periphery. This figure depicts the trading process in the periphery market for a particular sequence of arrivals. Red circles designate a borrower while blue circles designate lenders. We denote by $R^i_j$, the offer made by a borrower of type $j \in \{H, L\}$ and by $R^l_j$, the offer made by a lender of type $j \in \{H, L\}$. A high type borrower (H1) makes an offer to borrow at rate $R^H_b$. Its counterparty is either another borrower (with probability $\alpha$) who then turns down the offer and makes a new one or a lender. With probability $\lambda$, this lender has no access to the core (is low type) and with probability $(1 - \lambda)$, it has access to the core. If a lender rejects H1’s offer then H1 will trade with a core dealer at date $t = 3$ because H1 has access to the core. On the figure, the lender rejects the offer if it has a high type because, as we will show, high types banks do no trade together in equilibrium.
Figure 5: Equilibrium type as a function of $z_{pe}$ and $z_{co}$.
Figure 6: Peripheral Banks’ Market Power. Parameters values: $\lambda = 0.3$, $\sigma_{co} = 5$. Red lines: $\bar{z}^{co} = 0$; Black line: $\bar{z}^{co} = 3$. In all cases the equilibrium is in the ACB regime. The dashed lines represent high type banks’ market power while plain lines represent low type banks’ market power.
Figure 7: Equilibrium interest rates, as a function of $z^{pe}$. Parameter values: $\lambda = 0.3$, $\sigma_{co} = 5$. Black lines: $\bar{z}^{co} = 0$; Red line: $\bar{z}^{co} = 3$. $R^P = 100$bps and $R^D = 50$bps. In all cases the equilibrium is in the ACB regime. The dashed lines represent rates offered by high type banks while the plain lines are rates offered by low type banks. The black lines are rates offered offered by borrowers while red lines are rates offered by lenders. The green line is the equilibrium rate in the core market.
Figure 8: Distribution of equilibrium interest rates.

Panel A: No segmentation. Balanced liquidity conditions. Parameters: $\lambda = 0$, $\bar{z}^{co} = \bar{z}^{pe} = 0$, $\sigma^{co} = 8$, $\sigma^{pe} = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. All banks trade at $R^{co}$.

Panel B: No segmentation. Excess liquidity in the core, shortage in the periphery. Parameters: $\lambda = 0$, $\bar{z}^{co} = 3$, $\bar{z}^{pe} = -2$, $\sigma^{co} = 8$, $\sigma^{pe} = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. All banks trade at $R^{co}$. 
Panel C: Segmentation. Balanced liquidity conditions.

Parameters: $\lambda = 0.4$, $\bar{\epsilon}_c = \bar{\epsilon}_p = 0$, $\sigma_c = 8$, $\sigma_p = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. We report the distribution of all interest rates pooled together, as well as the distribution of each type of rates.
Panel D: Segmentation. Excess liquidity in the core, shortage in the periphery.

Parameters: $\lambda = 0.4$, $z^{co} = 3$, $z^{pe} = -2$, $\sigma_{co} = 8$, $\sigma_{pe} = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. We report the distribution of all interest rates pooled together, as well as the distribution of each type of rates.
References


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64


