Adverse Selection in the Irish Tontines of 1773, 1775, and 1777

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Abstract

We construct and analyze a new data set on the mortality experience of the nominees of the 1773, 1775, and 1777 Irish Tontines. The active participation of Genevan speculators in these Irish Tontines has been well documented. We use our new data to ascertain both the extent of their presence and the fact that the Genevan nominees were indeed significantly longer-lived than non-Genevan nominees—particularly so for the 50 nominees selected by a Genevan investment syndicate. We show that this enhanced longevity had only trivial consequences for the Irish government issuer but led to an investment return gap of approximately 7% between Genevan and non-Genevan nominees in tontine issue which had the largest Genevan presence. These findings have implications for recent proposals to introduce tontine-like features into contemporary retirement plans.

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1 Introduction

In his book King William’s Tontine, Moshe Milevsky (2015) argues that tontines may have an valuable role in financing public sector retirement systems. In contrast to a traditional life annuity, which pays a regular, guaranteed income stream to an annuitant for the remainder of his or her life, a tontine is a financial product that pays a regular, guaranteed sum to a pool of subscribers—a sum which is then divided up among the surviving members of that pool. Milevsky argues, in particular, that tontine-like features are potentially welfare-improving in retirement products because they can facilitate hedging against aggregate mortality risk.¹

These hedging benefits are most easily seen from the point of view of the party responsible for providing the regular income stream, henceforth assumed to be a central government. Consider a cohort of newly retired 65-year-old individuals “locking in” a pension for the remainder of their lives, and consider a government with a fixed sum of resources $R$ set aside to fund these pension payments. Suppose first that, as in the U.S. Social Security system, the government pays a pension in the form of a life annuity: it promises each individuals a fixed annual income for as long as that individual remains alive, with the magnitude of the annual income calibrated so that the expected discounted cost equals $R$. In this case, the government is fully exposed to aggregate mortality risk: if the cohort turns out to have significantly lower mortality than expected, then the cost of providing the pensions will greatly exceed the resources available.

Contrast that with the case of a pure tontine, where the government earmarks a regular annual payment to the entire cohort of individuals, and, each year, divides that annual payment up among the surviving members of that cohort. In this case, the government is minimally exposed to aggregate mortality risk. It is true that, if the cohort turns out to be longer-lived than expected, the government will be responsible for a longer stream of regular payments; but, in contrast to the traditional annuity, all of these extra payments will occur some forty or more years in the future, after the last member of the cohort was originally expected to decease. At a 5% discount rate, for example, even if death is “cured” and mortality drops to zero, the government is exposed to a maximum risk of less than 15% (i.e., $(1.05)^{-40}$) of the expected present discounted value.

As we discuss in Section 3, the public sector use of both tontines and traditional annuities was common in early modern Europe. Milevsky (2014), for example, examines King William’s tontine of 1693—the earliest use of public-sector life-contingent debt in Great Britain—which involved using both types of products to raise funds.

¹See also Milevsky and Salisbury (2015) and Forman and Sabin (2014).
We compile and analyze a new data set on the Irish Tontines of 1773, 1775, and 1777. These have been discussed in earlier literature, notably by Gautier (1951) and Jennings and Trout (1983), but with significantly less detailed data. Our data, compiled from several archival sources, contain a complete list of all lives nominated for these tontines, their age at the time of nomination, their domicile, and partial but—because of imperfect record-keeping—incomplete, information on their death dates.

As discussed in Gautier (1951) and Jennings and Trout (1983), Genevan bankers subscribed heavily to these tontines. These bankers took advantage of the fact that, like most life-contingent debts issued in early modern Europe, the owner of the tontine income stream did not have to have an insurable interest in the nominee on whose life the tontine payments were contingent. Indeed, as discussed in Cramer (1946), these Genevan bankers had reportedly mastered the art of picking particularly long-lived nominees through extensive participation as investors in the French life-contingent debt studied by Weir (1989) and Velde and Weir (1992).

We use our new data set to document several things. First, we show that Genevan nominees indeed had mortality rates significantly below those of the non-Genevan nominees, confirming some back-of-the envelope calculations in Jennings and Trout (1983) and Gautier (1951).

Second, we are able to compare the longevity of the 50 nominees identified in Gautier (from the records of a Genevan investment syndicate) with the other Genevan nominees—and find that the syndicate appears to have been markedly more skillful in selecting high-return nominees.

Third, our data allow us to decompose longevity enhancements into two components: a component attributable to the better age and gender “selection” by Genevan nominees, and a component attributable to selection of nominees with lower gender- and age-conditional mortality rates. We find that most or all of the enhanced longevity can be attributed to lower age- and gender-conditional mortality.

Fourth, we provide a quantitative assessment of the enhancement of financial returns earned by the nominators of Genevan nominees. We show that Genevan nominees earned, on average 7% more, in present value, than did the non-Genevan nominees (in the class 3 1777 tontine issue, where the Genevan presence was most notable). Importantly, however, the cost to the Irish government of raising funds was essentially unaffected by the presence of Genevan bankers.

These findings suggest that adverse selection was quantitatively important in this life-contingent debt-issue—mirroring Rothschild’s (2009) findings for the British Life Annuities of 1808-1850. Unlike Rothschild’s study however—which documented significant
gains by speculative investors in government-issued life annuities at the expense of the
British Government—the tontine structure of the Irish life-contingent debt successfully
insulated the issuing government from the costs of this selection; the costs of adverse se-
lection were borne within the nominee pool by the investors with less adversely-selected
nominees.

From a public policy perspective, then, this evidence indicates that—as suggested by
Milevsky (2015) and Milevsky and Salisbury (2015)—tontine-like features can, in practice,
indeed have significant mortality-risk-hedging benefits at the cohort level. It also high-
light a potential concern about such schemes: they may have non-trivial distributional
consequences if mortality rates are heterogeneous within a given cohort.

Our paper proceeds as follows. In Section 2, we use a simple analytical model to
clarify the important aggregate-mortality-hedging differences between tontines and life
annuities. In Section 3, we describe the Irish Tontines and the construction of our data
set. In Sections 4 and 5, we analyze these data. In the former, we test for the extent
and patterns of longevity enhancements among Genevan nominees using complete but
coarse mortality data. In the latter, we use an incomplete but richer data set to estimate
the financial implications of these longevity differences. Section 6 concludes.

2 A Simple Model of Tontines vs Life Annuities

This section uses a highly stylized model to illustrate the significant hedging benefits of
tontines versus traditional annuities.

Suppose there are many individuals $i$, each with a time-independent mortality hazard
$(1 - s_i)$. A government with a discount rate of $\delta = 1/(1 + r)$ is considering offering one
of two products. The first is a life annuity, which will pay a constant real income stream
of $y$ at a price normalized to 1. The second is a tontine, with shares again normalized to
a price of 1, which will pay out a total income of $Nz$ each year, where $N$ is the number
of individuals in the tontine pool. The government cannot distinguish individuals, and
believes that each has some time-independent mortality hazard $(1 - \hat{s})$. Given this belief,
it prices the two products in order to break even, in expectation:

$$y = \left( \sum_{t=1}^{\infty} \delta^t \hat{s}^t \right)^{-1}$$  (1)
Figure 1: Government losses (actuarially fair price - actual price)/(actuarially fair price) as a function of a uniform mortality reduction by a factor $\alpha$ for tontine pools of various sizes. Assumptions: baseline mortality $(1 - \hat{s}) = 1/40$, interest rate $r = 5\%$.

and

$$z = \left( \sum_{t=1}^{\infty} \delta^t \left( 1 - (1 - \hat{s}^t)^N \right) \right)^{-1}.$$  \hfill (2)

If the government’s belief about the survival hazard $\hat{s}$ is correct, then the government breaks even at these prices. If the true mortality hazard $s_i$ is lower than $\hat{s}$ for some or all of the nominees, the government will lose money in expectation: it charges a price 1 for the income stream $y$ or $z$, when, to break even, it should be charging a price $p$ greater than 1 (or, equivalently, offering a lower payout stream at a unit price).

Figure 1 plots the magnitude $(p - 1)/p$ of the government losses in the case of a uniform-across-nominee mortality reduction. Specifically, it considers the special case where $(1 - s_i) = (1 - \alpha)(1 - \hat{s})$ for some mortality reduction factor $\alpha \in [0, 1]$, and it plots the losses, as a function of $\alpha$, for tontine pools of various sizes $N$ (with $N = 1$ corresponding to an ordinary life annuity). The figure assumes an interest rate of $r = 5\%$, and a baseline survival hazard of $\hat{s} = 39/40$, corresponding to a life expectancy of 40 years.

Figure 1 shows that the government losses rise rapidly with the mortality reduction factor $\alpha$ for an annuity, with a 20% mortality reduction corresponding with an approximately 15% loss. Government losses are much smaller for tontines. For a tontine pool as
small as 5, losses barely exceed 5% even for a 90% mortality reduction. For a tontine pool with 30 members, government losses barely register at all. This is because, under our simple assumptions, an $N = 30$ tontine pool is effectively a perpetuity: there is a 99.63% chance that one of its members will live for 70 or more years; and with an interest rate of 5%, less than 3.3% of a perpetuity’s present value is attributable to the “beyond 70 years” tail.

Next, consider instead an uneven reduction in the mortality hazard. Specifically, suppose that half of the nominee population has mortality hazard $(1 - s_i) = (1 - \alpha)(1 - \hat{s})$ for some $\alpha \in [0, 1]$, while the other half has unreduced mortality $1 - s_i = 1 - \hat{s}$. Figure 2 plots the effects of this mortality on reduction for tontine pools of size $N = 10$ and $N = 30$. In particular, it plots both the associated government losses—which, per figure 1, are modest for all $\alpha$—as well as the expected present discounted value gains accruing to the higher-longevity types and the expected present discounted value losses borne by the types with un-reduced mortality. The figure clearly shows that the across-type effects swamp the effects on the government at all levels of $\alpha$: the primary effects of mortality changes in tontine pools are distributional.

We now turn to the data from the Irish Tontines to illustrate the extent to which these
same effects played out as a result of selection-on-lives among the Genevan bankers.

3 Historical Background and Data

The tontine was first proposed by Lorenzo Tonti, an expatriate Neopolitan who secured a position in the court of the French King in the 1650s (Hendriks, 1861). Tonti’s proposal was first carried out by King Louis XIV in 1689, and a total of 15 tontine issues were undertaken by the governments of Britain and France over the subsequent century (viz Weir (1989) who, along with Jennings and Trout (1982), is excellent an source for additional historical details).

This paper focuses exclusively on the Irish Tontines of 1773, 1775, and 1777. These tontines were the most successful of all of the tontine issues on the British isles. They raised a total of almost £1000—almost twice as much as the other four British tontine issues combined (Weir, 1989, table 3, page 105). Each of the Irish Tontine issues were were divided into three classes: Class 1 consisted of nominees over 40, class 2 of nominees between 20 and 39, and class 3 of nominees under 20. All allowed nominees of either gender. Each share cost £100, and multiple shares could be taken out on a given nominee’s life. For the 1773 and 1775 issues, each share carried a 6% dividend rate—with the exception of the 1773, class 1 issue, which carried a 6.5% dividend rate. In 1777, the dividend rate was 7.5%.

There was no requirement of an “insurable interest” for investing in one the Irish Tontines: As with most life-contingent debt issues in early modern Europe (viz Rothschild, 2009), there was no requirement for the party receiving the tontine payments to have had any relation whatsoever to the nominee upon whose life those payments were contingent. This made the tontine issue a perfect opportunity for speculative investments—most notably by those in the Genevan banking community who had recently been actively investing in French life-contingent debt (Cramer, 1946). Indeed, as discussed in Jennings and Trout (1983), the Genevan banking community was, in the early 1770s, actively developing and exploiting the so-called “Geneva formula” which involved an investor or syndicate purchasing life-contingent debt on multiple lives, and then pooling and securitizing the resulting income stream. There is both direct and anecdotal evidence that the Genevan community engaged in just such a scheme in at least the 1777 Irish Tontine.

Gautier (1951) discusses the presence of 50 nominees in the 1777 who were explicitly part of a syndicate. Jennings and Trout (1983) use some limited observations of nominee

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2For a first-hand account of the actions of this syndicate, see a letter therefrom on pp. 15-16 of the 1813 Report from the Committee on the Tontine Annuities of Ireland, 1773, &c
mortality to argue that these 50 nominees were significantly longer lived than the other nominees. They claim, in particular, that the 32 of these 50 nominees survived until 1837, as compared with only 461 out of 1091 overall. They also compare the nominees to (then) contemporary life tables, and argue they were longer lived than those tables would have suggested.

Owing to their limited access to data, neither Gautier (1951) nor Jennings and Trout (1982) attempt to further quantify the relative mortality of these nominees and the many other Genevan nominees. In particular, while both papers speculate about the high returns presumably earned by the Genevan nominees, neither has the necessary data to quantify these returns. Moreover, neither paper looks at the 1773 and 1775 tontines, which also had a non-zero Genevan presence.

We assemble a comprehensive data set using several archival primary sources in the National Archives in London which allows us to dig deeper into these issues.

Our first primary data source is a set of three “Nominee Books” documenting the details of the original lives nominated for the three waves of the tontine (Clements, 1775, 1777, 1779). These Nominee Books provide a detailed profile of the nominees at the time they were nominated for the tontine, notably the nominee names, their country of residence, their age, their gender, and the amount subscribed on their lives (number of £100 shares). Hand written notes of death dates were added to this book over time, as were forfeiture dates for the nominees who failed to claim payments for three consecutive year; since the financial gains from claiming dividends were substantial, these forfeitures likely to reflect unrecorded deaths in many cases, and, for the subsequent analysis, we treat a forfeiture in year $t$ as a death in year $t - 3$.

Unfortunately, the death and forfeiture records in the Nominee book are extremely incomplete: the majority of nominees have neither a death date nor a forfeiture date recorded.

Our second major source is a nearly-complete set of “payment books” documenting the period from December, 1802 until the termination of the Tontines. Each page of the payment books provides a given semi-annual payment. It reports, in particular, two key pieces of information: a list of newly deceased or forfeited shares, and the “extra” dividend paid out to surviving shares as a result of these and earlier deaths and forfeitures.3

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3The precise accounting works as follows. First, they compute a total “redundancy” in the accounts. This consists of any carried over redundancy, plus all of 6%, 6.5%, or 7.5% dividend payments that would have accrued to the shares which have already “fallen” through forfeiture or death. Second, the resulting redundancy is then divvied up evenly among the remaining shares to determine an “extra” payment that shareholders receive in addition to their initial 6%, 6.5%, or 7.5% payment. Typically, round numbers are used for this extra payment, which results in a modest remaining redundancy, which then carries over to the next semi-annual payment period.
The payment books are remarkably complete, but there are, nevertheless, three types of omission therefrom. First, the payment books only begin in 1802—when British Parliament took over administration from Irish Parliament. So data from the 1773-1802 period are lacking. Second, the payment book recordings for each class cease when the last nominee dies—so the death of the final nominee is not recorded. Third, there are two missing payment books: Christmas 1850 and Midsummer 1851. Across all 9 class-year issues, the payment books identify approximately 75% of the death (or forfeiture) dates of the lives initially nominated (a total of 2701 of the 3558).

We are able to partially fill in approximately one-forth of these missing data using other sources—most prominently the nominee books. Specifically, the nominee books can be used to identify the death ages of all of the last-to-die nominees missing from the end of the payment books and the death or forfeiture dates of a number of individuals who presumably appeared in the missing 1850 and 1851 books for several nominees from the 1773 class 3, 1775 class 2 and 3, and 1777 class 2 and 3 issues. They also contain a sub-set of death ages for those who died prior to 1802, but only in the 1777 class 1 (where the data are complete) and class 3 nominee books (which is about half-complete in the pre-payment-book data). So it is only for these two latter issues that we have any data on death ages prior to 1802 (with the exception of a small number of nominees reported in the first payment book who died a few years prior to 1802).

We also make modest use of three supplementary sources—largely as consistency checks. First, Gautier (1951) reports the death dates for all 50 of the Genevan syndicate nominees discussed above—one of which was otherwise unknown. The data from Gautier is otherwise consistent—within 1 year in all but one 2-year case—with the data in the nominee and payment books.

Second, the House of Commons issued a report in 1811 (“Report from the Committee on Irish Tontine Annuities,” 1811) which contains a comprehensive list of all nominees from all three tontine issues who were certified to be living in December, 1810. We used this 1810 data to corroborate the data from other sources, and we find virtually perfect agreement. Across all 9 class-year tontine issues, we identified only four nominees who were certified to be alive in 1810 and for whom we could not identify a death date from the payment or nominee books. These are presumably individuals who appear in one of the two missing payment book pages (1850.2 or 1851.1). We therefore attribute a death

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4A similar 1830 report contains a comprehensive list of all nominees who received their half-yearly tontine payment in early 1830. This is less useful, as a large number of subscribers appear not to have collected this payment by the time the report was constructed, but who remained in the pool, collecting payments thereafter. Presumably, these nominees collected their 1830 payment at a later date. Regardless, the 1830 report is not comprehensive enough to be useful.
date of 1851 to these four individuals.

Third and finally, Marie Antoinette, whose 1793 death date is well known, was among the nominees lacking a recorded death date. We manually add this death date.\(^5\)

Table 1 summarizes the sources of death dates and the missing data. It also reports the percentage of “deaths” which were recorded as forfeitures.

<table>
<thead>
<tr>
<th>Year:</th>
<th>1773</th>
<th>1775</th>
<th>1777</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Class:</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Payment Book</td>
<td>39</td>
<td>178</td>
<td>551</td>
<td>63</td>
</tr>
<tr>
<td>Other Source</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>84</td>
<td>230</td>
<td>705</td>
<td>115</td>
</tr>
<tr>
<td>% Missing</td>
<td>54</td>
<td>23</td>
<td>20</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 1: Sources of Death Data

We are particularly interested in comparing the longevity experience—and hence financial returns—of Genevan nominees with those of the non-Genevans. Insofar as the Genevan nominees had relatively greater longevity, we are further interested in knowing the extent to which these longevity enhancements can be attributed to better selection of ages and genders versus better selection of lives conditional on age and gender.

Tables 2-4 provide summary statistics on the numbers and basic demographics of the Genevan and non-Genevan nominees in the various tontine issues. Each table breaks out the number of nominees, the number of shares, and the gender and age breakdown by Genevan vs Other residential status at the time of nomination. In class 3 of the 1777 tontine, the table further breaks down the number by “Gautier” and “Other Genevan” status—since the 50 nominees identified in Gautier (1951) as being nominated by an investment syndicate represent a identifiably distinct group. This distinctness is clear from the demographics: the Gautier 50 were all female, and had a significantly lower age than did both the other Genevan nominees and the non-Genevan nominees, and the number of shares per nominee was significantly higher than any other group in any tontine issue. There is some suggestive evidence from table 3 that the Genevan nominees in class 3 of the 1775 issue were selected, at least partially, by investors using a similar strategy: they are predominantly female and much young as compared with non-Genevan nominees, but they only have about one share per nominee, and they are still a modest presence in

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\(^5\)There is also a set Register Books (Irish Tontine Leger Session, 1773 and 1775 and Irish Tontine Leger Session 1777) which record a subset of nominees, all of whom died or forfeited after the start of the payment book data. We used these to spot check some nominee and payment book data book data, but otherwise did not employ it.
the overall tontine pool. By contrast, the Genevan nominees in other classes and years—including the non-Gautier Genevans in 1777 class 3—are not predominantly female or are not younger than the non-Genevan nominees and had few shares per nominee.

In keeping with the fact that the posted annual dividend per share was independent of age class\(^6\), the vast majority of nominees in all three years were in class 3.

<table>
<thead>
<tr>
<th>Class 1</th>
<th>Class 2</th>
<th>Class 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominees</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Shares</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>% Female</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Mean Age (Female)</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Mean Age (Male)</td>
<td>Genevan</td>
<td>Other</td>
</tr>
</tbody>
</table>

Table 2: Summary Statistics: 1773 Irish Tontines

<table>
<thead>
<tr>
<th>Class 1</th>
<th>Class 2</th>
<th>Class 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominees</td>
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<td>Other</td>
</tr>
<tr>
<td>Shares</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>% Female</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Mean Age (Female)</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Mean Age (Male)</td>
<td>Genevan</td>
<td>Other</td>
</tr>
</tbody>
</table>

Table 3: Summary Statistics: 1775 Irish Tontines

<table>
<thead>
<tr>
<th>Class 1</th>
<th>Class 2</th>
<th>Class 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominees</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Shares</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>% Female</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Mean Age (Female)</td>
<td>Genevan</td>
<td>Other</td>
</tr>
<tr>
<td>Mean Age (Male)</td>
<td>Genevan</td>
<td>Other</td>
</tr>
</tbody>
</table>

Table 4: Summary Statistics: 1777 Irish Tontines

Figure 3 plots the distribution of ages for the three groups of nominees in the largest and most interesting of the tontines: the 1777 class 3 issue. The Gautier 50 are clearly carefully selected, in terms of demographics, with all ages roughly uniform between 1

\(^6\)The higher 6.5% dividend for the 1773 class 1 issue may have been implemented because some of the initial nominees died after purchasing shares but before the payments commenced.
and 5, and no nominees older than 7. In contrast, both the other Genevan nominees and the non-Genevan nominees have long tails extending into the upper ages of the class.

The demographic differences visible in Figure 3 clearly corroborate the anecdotal evidence in Gautier (1950) and others about the Genevan syndicate was actively selecting lives on the basis of observable characteristics (age and gender) in pursuit of high financial returns. Anecdotally (see Jennings and Trout (1983)), they were also actively selecting on the basis of unobservable (to us) characteristics. In the next two sections, we attempt to quantify the extent to which they were successful in selecting more favorable nominees, and the extent to which this success is attributable to selection on observable versus unobservable characteristics.
4 Non-parametric Tests for Differential Selection

This section examines the differential longevity of Genevan and non-Genevan nominees in the various tontine issues. We use all data for the 1777 class 1 issue, for which we have complete death data. For the 7 tontine issues for which we have no death data prior to the payment book data (that is, all but the 1777 class 1 and 3 issues), we make use of all available data. For the one class where we have partial but incomplete death data prior to the payment books, 1777 class 3, we take a conservative approach and throw out the data from before 1802. This censoring allows us to obviate any concerns about systematic differences in recording errors in the pre-1802 period.\footnote{We revisit and relax this conservative assumption in the next section.}

Throwing out the pre-1802 data for the 1777 class 3 means that for all 9 issues, each individual either has a known death year or else is known to have a death year prior to 1802. Any two individuals can thus be ranked in terms of the order of their death, with the tacit assumption that two individuals who died before 1802 are tied. Given these ordinal data, we can test for differences in longevity of sub-populations using a Wilcoxon rank-sum test statistic (viz Mann and Whitney (1947)).

We use the test statistic for three types of test: an unconditional test for longevity differences between two sub-groups (e.g., Genevan vs non-Genevan), a test for whether there are longevity differences between sub-groups attributable to different mixes of gender and age, and a test for whether there are longevity differences between sub-groups attributable to different by age- and gender-conditional longevities across sub-groups.

The three tests are based on the standard Wilcoxon rank-sum test statistic. First, we compute the standard Wilcoxon rank-sum test statistic in the baseline data; we refer to this as the baseline statistic. Second, we randomly permute the censored death-age data across all individuals 10,000 times and, for each draw, we recompute the Wilcoxon rank-sum test statistic; we refer to the resulting distribution of test statistics is the equal-mortality distribution. Third, we randomly permute the censored death-age data within each (nomination) age-gender bin and again recompute the Wilcoxon rank-sum test statistic; we refer to the resulting distribution as the conditionally-equal-mortality distribution.

The fraction of elements in the equal-mortality distribution which exceed the baseline statistic is reported in the first column of Tables 5 and 6. Each element is interpretable as the $p$-values associated with a one-tailed test of the hypothesis that the two groups have the same mortality (since under this hypothesis any of the permutations underlying the equal-mortality distribution is equally likely). These unconditional tests strongly that Genevan nominees were, in a number of sub-populations, longer-lived than the other
nominees. The evidence is particularly strong for the 1775 class 3, the 1773 class 1 and especially within the 1777 class 3, pools.\textsuperscript{8} Moreover, within the 1777 class 3 pool, the 50 syndicated nominees appear to have been long-lived even relative to the other Genevan nominees.

The probability that the test statistic for a randomly chosen draw from the equal-mortality distribution exceeds the test statistic for a randomly chosen draw from the conditionally-equal-mortality distribution is reported in the second column of Tables 5 and 6. These elements are interpretable as the $p$-values associated with a one-tailed test of the hypothesis that the two groups have the same longevity under the maintained hypothesis that the age-gender-conditional mortalities are definitely equal. To wit: under the maintained hypothesis of conditionally equal mortality, any of the conditional permutations are equally likely, and the $p$-value in column 2 is simply the average across all of these assumed-equally-likely data-sets of the $p$-values for each set. Less formally: these are the $p$-values associated with tests of whether differences across groups in the distribution of nomination ages and genders are contributing significantly to longevity differences across those groups. Since the $p$-values in this column are all large by conventional standards, there appears to be no strong evidence to support longevity differences being driven by the careful selection of ages and genders.

The third column of Tables 5 and 6 performs the complementary exercise, testing whether age- and gender-conditional differences in mortality contribute significantly to overall longevity differences between two groups. Specifically, it reports the fraction of the conditionally-equal-mortality distribution exceeds the baseline statistic. Unsurprisingly—in light of the first two columns—these are highly significant for the 1777 class 3 ton-tines: the overall longevity differences appear to be driven mostly by age- and gender-conditional longevity differences. That is, they appear to be driven more by selection of favorable lives for any given demographics rather than selection of favorable demographics. This is somewhat surprising in light of the emphasis placed on these demographic selection variables in the prior literature, notably Cramer (1945) and Jennings and Trout (1983).

4.1 Possible explanations

We view the preceding results as strongly indicative of the greater longevity of the Genevan nominees in general and the Genevan syndicate nominees in particular. A natural inter-

\textsuperscript{8}The 1773 class 1 pool is notable, as there were only two Genevan nominees in the pool—but those nominees were the third and 12th longest lived nominees among the 84 nominees.
Table 5: Wilcoxon Rank-Sum Tests, Genevan vs Non-Genevan, 1773 and 1775 tontines. Each cell contains the \( p \)-value associated with a (one-tailed) Wilcoxon Rank-Sum-based test of the hypothesis that Genevan and Non-Genevan nominees have the same longevity. Column 1 contains unconditional tests for differences in longevity. Column 2 tests for the presence of longevity differences driven by differences in nomination-age and gender composition of Genevan and Non-Genevan nominees. Column 3 tests for the presence of longevity differences driven by differences in nomination-age- and gender-conditional longevity differences. See text for details.

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>Across Age/Gender</th>
<th>Within Age/Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>0.01</td>
<td>0.46</td>
<td>0.05</td>
</tr>
<tr>
<td>Class 2</td>
<td>0.21</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>Class 3</td>
<td>0.52</td>
<td>0.60</td>
<td>0.37</td>
</tr>
<tr>
<td>Class 1</td>
<td>0.43</td>
<td>0.72</td>
<td>0.12</td>
</tr>
<tr>
<td>Class 2</td>
<td>0.09</td>
<td>0.05</td>
<td>0.84</td>
</tr>
<tr>
<td>Class 3</td>
<td>0.01</td>
<td>0.24</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 6: Wilcoxon Rank-Sum Tests 1777 tontines. Each cell contains the \( p \)-value associated with a (one-tailed) Wilcoxon Rank-Sum-based test of the hypothesis that two populations have the same longevity. See table 5 and text for details.

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>Across Age/Gender</th>
<th>Within Age/Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1 Genevan – Non-Genevan</td>
<td>0.20</td>
<td>0.58</td>
<td>0.11</td>
</tr>
<tr>
<td>Class 2 Genevan – Non-Genevan</td>
<td>0.07</td>
<td>0.19</td>
<td>0.41</td>
</tr>
<tr>
<td>Class 3 Gautier – Other-Genevan</td>
<td>0.00</td>
<td>0.21</td>
<td>0.03</td>
</tr>
<tr>
<td>Class 3 Genevan – Non-Genevan</td>
<td>0.00</td>
<td>0.16</td>
<td>0.01</td>
</tr>
<tr>
<td>Class 3 Gautier – Non-Genevan</td>
<td>0.00</td>
<td>0.11</td>
<td>0.01</td>
</tr>
<tr>
<td>Class 3 Other-Genevan – Non-Genevan</td>
<td>0.03</td>
<td>0.32</td>
<td>0.09</td>
</tr>
</tbody>
</table>

As is typically the case in testing for adverse selection, however, it is impossible to disentangle adverse selection from moral hazard (viz Chiappori and Salanie, 2000). There are at least two possible sources of moral hazard. First, the Genevan nominees could have been longer-lived because investors spent money and attention looking after their heath—as seems plausible given the discussion in Jennings and Trout (1983). Second, as discussed by Milevsky (2014) in the context of the King William Tontine of 1693, there could have been (differential) fraud, due, for example, to false certifications of “alive” status. Jennings and Trout (1983) indicate that at least the 50 nominees listed in Gautier were local celebrities in Geneva, which makes this latter interpretation somewhat less plausible in the Irish Tontines than in Milevsky’s context.
Assuming that differences indeed reflect selection rather than moral hazard effects, it is also natural to interpret these results as evidence that the Genevan nominators were better at selecting low-mortality nominees. This is consistent with the anecdotal evidence reported by Jennings and Trout and others that Genevans were skilled at identifying long-lived nominees. It is also consistent with a skill-free interpretation, however: perhaps Genevans were simply longer-lived, on average, than non-Genevans. These explanations are not mutually exclusive, and our data (and lack of good contemporary life-tables from Geneva and Ireland and Britain) cannot fully elucidate this. The fact that the Guatier nominees—who were known to be actively selected for longevity—were longer lived than other Genevan nominees strongly suggests that, at least for those nominees, skill at selecting long-lived nominees played a role.

5 Distributional Effects of Differential Selection

The previous section was deliberately conservative, in choosing to ignore all data from the incomplete pre-1802 period. In this section, we use the complete data set, together with a natural—but ultimately unverifiable—identification assumption to calculate the investment returns for the tontine investors and sub-groups thereof.

We perform two related exercises. First, we estimate the present-value returns per tontine share among three sub-groups of nominees: the non-syndicate Genevan nominees, the Non-Genevan nominees, and for the 1777 class 3 tontine, the syndicated “Gautier 50” nominees.

Second, we use a counterfactual simulation to estimate how much of this gap can be attributed to better age and gender selection among Genevan nominees, and how much is instead due to longer-lived Genevans conditional on age and gender.

5.1 Baseline non-parametric estimates of the returns gap

Computing the present value of the investment returns to Genevans and non-Genevans is straightforward for the one tontine issue for which we have complete death data: class 1, 1777. In that case, we first infer the number of nominees in a given pool who were alive in each year, and then use this to find the semi-annual dividend to each surviving nominee in each half-year. For each nominee, we can then compute the present value of that nominee’s realized payment stream (using a discount rate equal to the annual per-share dividend corresponding to that class, i.e., 7.5%). Finally, we compute a share-weighted
average across all Genevan nominees and all non-Genevan nominees to compute the differential average returns across groups. We compute standard errors via bootstrapping.

<table>
<thead>
<tr>
<th></th>
<th>Non-Genevan</th>
<th>All Genevan</th>
<th>p-value of gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imputed Dividends</td>
<td>0.9701</td>
<td>1.0150</td>
<td>0.3400</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.0099</td>
<td>0.0403</td>
<td>-</td>
</tr>
<tr>
<td>Payment Book Dividends</td>
<td>0.9733</td>
<td>1.0131</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7: Rates of return for Genevans and non-Genevans in the 1777 Class 1 Irish Tontine

The first line of Table 7 contains the baseline estimate (with standard errors in the second line). It indicates that the 28 Genevan nominees in the 1777 class 3 tontines earned a 4.5% higher rate of return than the non-Genevans (1.015-.9701), although it is not statistically significant. This is, of course, consistent with Table 6, which showed no evidence of greater longevity among the Genevan nominees in the 1777 class 1 tontine issue.

The dividends used for computing returns in the first line of Table 7 are inferred from the number of individuals alive at any given moment in time—that is, we compute the total semi-annual dividend as .0375 (i.e., 7.5%/2) times the of 205 initial shares subscribed, and then divide by the number of still-living shares. We do this imputation of dividends in spite of the fact that we observe the (very modestly different) actual dividends in the payment books from 1802 on. We do this because dividends in the bootstrapped samples need to be imputed (as realized dividends are, of course, sample-dependent). The third line of Table 7 verifies that using imputed dividends is sensible by computing returns—for the baseline sample only—with the payment-book derived dividends whenever they are available. The returns change only marginally, indicating that imputed dividends are close approximations to actual dividends.

Lacking death dates for a subset of the nominees complicates our task, and, for the 7 classes for which we have no death data prior to the payment books, renders it effectively impossible. But for the most interesting tontine issue—the 1777 class 3 issue—we have non-trivial, albeit partial, data from the nominee books prior to the payment book data, which we can use, under a natural identifying assumption, to compute returns as we do for class 1, 1777. Specifically, we assume that any two individuals with the same gender, age, and nomination sub-group (i.e., Gautier, Other Genevan, or Non-Genvan) also had the same probability of having their death data recorded. Under this assumption, the distribution of the recorded death dates within any (gender,age,group)-“bin” is a consis-

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9We return to the method used to compute the p-values in the final column of table 7 after discussing the 1777 class 3 nominees.

10This is not surprising since the method of computing actually dividends tracks our imputation up to some minor rounding. Indeed, big differences would suggest errors in the data set.
tent estimate of the distribution of death dates for the unrecorded death dates. We can therefore use these recorded dates to randomly “fill in” the unknown death dates.

While the validity of this identifying assumption is ultimately unverifiable, Figure 4 provides some evidence that it is plausible. It compares the distribution of death years for the pre-1802 non-payment book death observations to the theoretical distribution that would have obtained from the entire subset of the nominees who died prior to the commencement of the payment books. This theoretical distribution is computed by using the Carlisle life tables (Milne, 1815), a widely used, approximately contemporary life table derived from observations in Carlisle, England, between 1779 and 1787. If nominee deaths that occurred later in the 1779 to 1802 period were systematically more likely to be recorded than deaths in the mid 1780s, then Figure 4 would indicate an upward trend in the observed death dates relative to the predicted death dates. We see no such pattern. Indeed: the observed distribution is broadly consistent with the Carlisle predictions—except possibly during the first 4-5 years of the tontine issue, which is plausibly attributable to simple adverse selection (as nobody would nominate a currently-unhealthy life).\footnote{Moreover, note that even if there was relative under-recording in the early years, it would have to be differential between Genevan and non-Genevan populations to pose problems for our identifying assumption.}

Our imputation procedure for the 1777 class 3 issue runs as follows. First, we start with the original data set (or, when we bootstrap for standard errors, draw a bootstrapped sample of the entire original data set, which including all nominees with missing death dates). Second, for each nominee \( i \) within this sample with a missing death date, we identify all the set \( J \) of all other nominees who: (a) have the same age and gender, and are in the same group (e.g., non-Genevan, Gautier 50, or other Genevan); and (b) have a recorded death date prior to 1802 (from a non-payment book source). We then randomly select a member \( j \in J \) of this set and set \( i \)’s death date equal to \( j \)’s death date.\footnote{In a few cases, the set \( J \) is empty for Genevan nominees. In those cases, we instead choose from the set \( J' \) of non-Genevan nominees with the same age and gender. Very occasionally, bootstrapping leads to a situation where both \( J \) and \( J' \) are empty. In these cases, we randomly assign a death date from among all of the death dates in that time window. The results do not depend substantively on this imputation choice.} This gives us a data set with a complete set of death dates. We then employ this imputed-complete set of death dates just as we did with the actually-complete 1777 class 1 data to compute the time evolution of the number of nominees alive in each year of the tontine, and hence the per-share returns (using the per-share dividend rate relevant to that group).

The first line of Table 8 reports these average returns; bootstrapped standard errors are reported in the second line. Non-Genevans earned, on average, only 0.9708, while the syndicated Genevans earned, on average 1.0532 per pound—over an 8% advantage.
over the non-Genevans. The other (non-syndicated) Genevans earned essentially the same as the overall average—i.e., in between the non-Genevans and the Gautier 50—and Genevans overall earned about 7% greater returns than the non-Genevan nominees.

<table>
<thead>
<tr>
<th></th>
<th>Non-Genevan</th>
<th>All Genevan</th>
<th>Gautier 50</th>
<th>Non-Gautier Genevan</th>
<th>All Nominees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imputed Dividends</td>
<td>0.9708</td>
<td>1.0417</td>
<td>1.0532</td>
<td>0.9997</td>
<td>0.9988</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.0079</td>
<td>0.0119</td>
<td>0.0157</td>
<td>0.0230</td>
<td>0.0000</td>
</tr>
<tr>
<td>Equalized Mortality</td>
<td>0.9885</td>
<td>1.0144</td>
<td>1.0213</td>
<td>0.9891</td>
<td>0.9988</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.0087</td>
<td>0.0133</td>
<td>0.0181</td>
<td>0.0241</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 8: Unconditional and (age,gender)-conditional rates of return for various subpopulations of the 1777 class 3 Irish Tontine

The first two rows of last column of Table 8 starkly illustrate a key point raised in Section 2. Specifically, they indicate that the present value of the overall return was .9988 per pound consistent with the tontine being effectively a perpetuity from the point of view of
the class as a whole. An alternative interpretation of this 0.9988 is as the present-value cost per for the the Irish government to raise a unit of funds via the tontine (assuming their relevant discount rate is 7.5%). The vanishingly small standard deviation of this overall mean thus highlights the fact that the tontines were an extremely effective hedge against aggregate mortality risk for the government. Essentially all of the mortality risk from a tontine is born by the nominees themselves.

In fact, the 1777 class 3 tontine points to one particular risk faced by tontine participants: that some other nominees in the tontine will end up being dramatically longer-lived than expected and will there imposing a significant pecuniary externality on the other nominees. The return gaps between the syndicated nominees and the non-Genevans in the 1777 class 3—and other gaps—are indeed economically large. They also appear to be statistically significant.

Standard statistical tests based on asymptotic normality could confirm this statistical significance, but, for a more precise test—one which will extend more naturally to the tests we use in the next subsection for decomposing the gap into a “within” and “across” demographic component—we instead use an exact permutation-based test. Specifically, we first run a large number of “baseline” draws using the baseline data set but imputing the 96 missing death dates, as described above. Second, we run a large number of “equalized-mortality” draws which first mechanically equalize the longevity across all groups by randomly permuting all death observations (including the missing ones) across all nominees. We then compute the fraction of pairs of the two types of draws for which the return gap in the equalized-mortality draw exceeded the return gap in baseline draw. Under the null hypothesis of equal longevity, any one of the permutation-based draws is ex-ante equally likely, so this fraction is interpretable as the p-value associated with the observed sample under an equal-mortality null hypothesis.

These p-values are reported in the first column of Table 9. They indicate that the differences between Genevans in general and the syndicated Genevans in particular and the non-Genevans were highly statistically significant (p < .0001). The evidence for return differences between the non-syndicated Genevans non-Genevans (p = 0.058) and between the non-syndicated and syndicated Genevans (p = 0.105) is more equivocal.

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13 The last member died in 1871, 91 semi-annual payments after payments commenced. At an interest rate of 7.5%, the present value of the post 1871 “tail” of a perpetuity amounts to approximately 0.12% of its initial value.

14 The p-value reported in table 7 is computed using the same basic method, but because we have complete death data the baseline sample does not need to be simulated.
Table 9: Tests of significance for differences in rates of return for various sub-populations in the 1777 class 3 Irish Tontine. Elements are based on three independent simulated draws on the data: (i) a “baseline” draw which randomly imputes the missing 96 death years but otherwise maintains the baseline data set; (ii) a “equal-mortality” draw which randomly permutes death years across all observations and then imputes the missing 96 death years; (iii) a “conditionally-equal-mortality” simulation which randomly permutes death years within each (nomination age × gender) bin and then imputes the missing 96 death years. The first column reports the probability that the return gap from an equal-mortality draw exceeds the return gap from a baseline draw. The second column reports the probability that the return gap from an equal-mortality draw exceeds the return gap from a conditionally-equal-mortality draw. The third column reports the probability that the return gap from a conditionally-equal-mortality draw exceeds the return gap from a baseline draw.

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>Across-Age/Gender</th>
<th>Within Age/Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genevan – Non-Genevan</td>
<td>0.000</td>
<td>0.214</td>
<td>0.004</td>
</tr>
<tr>
<td>Gautier 50 – Non-Genevan</td>
<td>0.000</td>
<td>0.208</td>
<td>0.006</td>
</tr>
<tr>
<td>Other Genevan – Non-Genevan</td>
<td>0.056</td>
<td>0.471</td>
<td>0.087</td>
</tr>
<tr>
<td>Gautier 50 – Other Genevan</td>
<td>0.105</td>
<td>0.266</td>
<td>0.272</td>
</tr>
</tbody>
</table>

5.2 Decomposing the return gap

The observed differences in financial returns across age groups are potentially attributable to their different choices of ages and genders of nominees—as illustrated in Table 4 and Figure 3—i.e., to selection on “unused observables” (per Finkelstein and Poterba (2014)). Alternatively, they may reflect different longevity conditional on age and gender, i.e., on “unobservables”.

To tease out the extent to which of these two potential sources of lower mortality rates is driving the differential returns, we run a third set of simulated draws “in-between” the baseline draws and the equal-mortality draws described in the preceding sub-section. This intermediate set of draws first randomly permutes the death ages across all nominees within each (nomination-age × gender) bin. It then randomly imputes the 96 missing death ages. Under the maintained hypothesis that the (nomination age × gender)-conditional mortality rates are equal across all nominees, each of the permutations underlying these draws are equally likely. Comparing the distribution of return gaps across these conditionally-equal-mortality draws with the distribution of return gaps across the fully-equal-mortality draws provides a test of the equality of returns conditional on the maintained hypothesis of conditionally equal mortality.

The second column of Table 9 reports the probability that a random fully-equal-mortality draws exceeds a random conditionally-equal-mortality. This is interpretable as the $p$-
value associated with a test of equal-returns conditional on the maintained hypothesis of 
(nomination-age × gender)-conditionally equal mortality. The fact that none are signifi-
cant at conventional levels indicates that there is no strong evidence that return gaps are 
being driven by differences in the distributions of age and gender across different groups. 
This is again consistent with the evidence on longevity in Table 6 (viz the second column, 
which is built on the same basic permutation-based test).

The third column of Table 9 reports tests for “other part” of the decomposition of 
overall return gap—i.e., return gap differences attributable differences across groups in 
(nomination-age × gender)-conditional longevity differences. Specifically, it reports the 
probability that a random conditionally-equal-mortality draw exceeds a random baseline 
draw. If there were indeed no differences in longevity conditional on nomination age 
and gender, then the return gaps in the baseline draws would not be outliers within the 
distribution of conditionally-equal-mortality draws. The low $p$-values in the third col-
umn of Table 9 show that, contrariwise, the baseline draws are indeed outliers, strongly 
suggesting that return gaps are being driven by (nomination-age × gender)-conditional 
longevity differences across groups.

In summary, although we cannot rule out that the syndicated Genevan nominees may 
have benefited modestly from better age and gender selections, the data appear to be 
consistent with all of the unconditional return differences being attributable to the lower 
mortality conditional on age and gender. Selection effects for the 1777 class 3 nominees 
thus appear to be largely driven by differences in higher age- and gender-conditional 
longevity among Genevan nominees.

6 Conclusions

Our newly developed data set on the nominees of the 1773, the 1775, and, particularly, 
the 1777 Irish Tontines strongly supports earlier contentions that there was a substantial 
presence of savvy Genevan investors in these tontines. We show that the Genevan nom-
inees were longer lived than the other nominees, and, as a result earned investors on the 
order of 8% higher lifetime returns on the tontine than investors in non-Genevan nomi-
nees. The subset of Genevan nominees known to be nominated by a Genevan syndicate 
which was investing specifically to earn high returns appears to have earned particularly 
high returns. In contrast to the anecdotes in the earlier literature, we find that most of the 
return gap appears to be attributable to higher longevity conditional on age and gender as 
opposed to better selection of ages and genders.

Our case study of these tontines also illustrates a more general principle: tontines
are highly effective at hedging government exposure to adverse selection and moral hazard. A government issuing life annuities exposes itself to non-trivial losses if the annuitants are significantly longer lived—as the British government experienced in the early 19th century (Rothschild, 2009). But any adverse selection or moral hazard costs in a government-issued tontine context are born almost entirely by other participants in the tontine.

This observation has important implications for evaluating recent proposals to introduce a tontine element into retirement plans (Milevsky (2014)). On the one hand, implementing such a proposal would have a clear benefit as a hedge against aggregate mortality risk (owing either to adverse selection or medical advances, e.g.). On the other hand, differential mortality improvements across different groups, e.g., between low-income and high-income sub-populations, can expose particular sub-populations to significant extra risk, as compared with standard life-annuity based retirement systems. Distributional consequences should be an important consideration when incorporating tontine-like elements in retirement schemes—otherwise, some groups at risk of losing out to the modern-day-equivalent of the Genevan speculators in the Irish Tontines of the 1770s.

References


Clements, N. (1775): “A List of the Persons on Whose Lives the Sum of 265,000£was Subscribed,” In Irish Life Annuities 1773 & 1775 List of Nominees.

——— (1777): “A List of the Persons on Whose Lives the Sum of 175,000£was Subscribed,” In Irish Life Annuities 1773 & 1775 List of Nominees (National Archives).

——— (1779): “A List of the Persons on Whose Lives the Sum of 300,000£was Subscribed,” In Irish Life Annuities 1777 List of Nominees (National Archives).

Committee on the Tontine Annuities of Ireland (1775): “Irish Tontine Ledger Session 1773 and 1775,” A register of subscribers and nominees, executors, administrators, attorneys, and assignees. 1,2 and 3 classes.

——— (1777): “Irish Tontine Ledger Session 1777,” A register of subscribers and nominees, executors, administrators, attorneys, and assignees. 1,2 and 3 classes.
— (1811): “Report from Committee on Irish Tontine Annuities,” list of nominees certified to be living as of Dec 25, 1810.

— (1813): “Report on the Tontine Annuities of Ireland, 1773,” Discussion paper, The House of Commons of Ireland, reports on the problem of paying out sums when there should have been forfeitures.


Mann, H. B., and D. R. Whitney (1947): “On a test of whether one of two random variables is stochastically larger than the other,” The annals of mathematical statistics, pp. 50–60.


