Partition Obvious Preference and Mechanisms Design: Theory and Experiment

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Experimental Evidence & Theoretical Explanations

- **School Choice:**
  - Chen and Sönmez 2006
  - Fack et al. 2015
  - Hassidim et al. 2016

- **Matching Program**
  - Rees-Jones 2016

- **Public Goods**
  - Attiyeh 2000

- **Voting**
  - Esponda and Vespa 2014

- **Auctions**
  - Kagel et al. 1987
  - Kagel and Levin 1993
  - Harstad 2000
  - Garratt et al. 2011

- Standard Game Theory
  - Li 2016
  - Obviously Strategy-Proof

- Convergence in Obvious Dominant Strategy Mechanisms
The Implication Is More Fundamental

**Decision Theory** \(\xrightarrow{\text{Belief on what the other play think or do?}}\) **Game Theory**

- **Violations of fundamental axioms** \(\xrightarrow{}\) **Deviations from Dominant Strategies**
- **Compliance with weaker axioms** \(\xrightarrow{}\) **Convergence to Obviously Dominantly Strategy**
Alerted by An Critical Fact

- Dominant Strategy
  - Essential
  - Reasoning State-by-state
    - Not
      - Obviously Dominant Strategy
Reason by Partitioning the State Space

The coarser the more bounded rational

the Finest Partition → the Coarsest Partition

Can Reason Event-by-Event But Not State-by-State Within Each Event
Why different ways of partitioning matters?
An example with two states

The finest partition:

- **Problem 1**
  - State 1 | State 2
  - A       | 20     
  - B       | 25     

- **Problem 2**
  - State 1 | State 2
  - A       | 13     
  - B       | 20     

The Coarsest Partition:

- (State 1, State 2)
  - State 1 | State 2
  - A       | **20**  
  - B       | 25     

- (State 1, State 2)
  - State 1 | State 2
  - A       | 13     
  - B       | 20     
Why different ways of partitioning matters?
An example with four states

Problem 3

<table>
<thead>
<tr>
<th>Event</th>
<th>B₁</th>
<th>B₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>s₁ s₂</td>
<td>s₃ s₄</td>
</tr>
<tr>
<td>U</td>
<td>20 11</td>
<td>5 8</td>
</tr>
<tr>
<td>D</td>
<td>25 22</td>
<td>10 20</td>
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</table>

Problem 4

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<tr>
<th>Event</th>
<th>B₁</th>
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</table>
Reasoning by partitioning

Partition Obvious Preference

Subjective Expected Utility Theorem

The Finest Partition
The Notation

- $X$ be the set of deterministic outcomes
- $Z$ be the set of distributions over $X$ with finite supports
- Acts:

\[ f : \Omega \rightarrow Z \]

- A finite partition of $\Omega : \Sigma$.
- The range of $f$ given event $B : O^B(f)$
Partition Obvious Preference

Partition Obvious Monotonicity

For any $f, g \in F$, if for any $B \in \Sigma$, we have, for all $p \in O^B(f), q \in C^B(g), p \succeq q$, then $f \succeq g$;

In addition, if for a non-null event $B' \in \Sigma$, it is strictly satisfied then $f \succ g$. 
Mixed Acts

- **Mixed Act:** for any $f, h \in F, \alpha \in [0, 1]$ and $\omega \in \Omega$, 
  $[\alpha f + (1 - \alpha)h](\omega) \equiv \alpha f(\omega) + (1 - \alpha)h(\omega)$.

- **Partition Constant Act:** $F^c(\Sigma)$, constant act give each event of the partition.
Understanding Mixed Acts

Reasoning by the given partition, or a finer one?

Partition Constant Acts

A finer one

Other Acts
Partition Continuity and Independence

- **Partition Independence**: For any three acts $f, g, h \in F^c(\Sigma)$ and any $\alpha \in (0, 1]$, $f \succeq g$ implies that $\alpha f + (1 - \alpha)h \succeq \alpha g + (1 - \alpha)h$.

- **Partition Continuity**: For any action $g \in F$ and any two acts $f, h \in F^c(\Sigma)$ such that $f \succeq g \succeq h$, there are $\alpha, \beta \in (0, 1)$ such that $\alpha f + (1 - \alpha)h \succeq g \succeq \beta f + (1 - \beta)h$. 
Partition Obvious Preference

- Subjective Expected Utility
  - weak order
  - monotonicity
  - Independence
  - Continuity
  - Nondegeneracy

- Partition Obvious Preference
  - weak order
  - Partition obvious monotonicity
  - Partition independence
  - Partition continuity
  - Nondegeneracy

Equivalent when the partition is the finest
5 Axioms are satisfied if and only if there exists a nonconstant affine function $u : Z \rightarrow \mathbb{R}$, a probability function $P : \Sigma \rightarrow [0, 1]$ and a function $\alpha : \mathcal{F} \rightarrow [0, 1]$ such that $\succsim$ is represented by the preference functional $V : \mathcal{F} \rightarrow \mathbb{R}$ given by

$$V(f) = \sum_{k=1}^{n} V(f|B_k) P(B_k) \tag{1}$$

where,

$$V(f|B_k) = \alpha(f) \max_{p \in C^{B_k}(f)} u(p) + [1 - \alpha(f)] \min_{q \in C^{B_k}(f)} u(q). \tag{2}$$
Partition Obvious Preference

Partition Obvious Preference

The Coarsest Partition

Other Partitions

Subjective Expected Utility Theorem

The Finest Partition
It is often argued academically that no science can be more secure than its foundations, and that, if there is controversy about the foundations, there must be even greater controversy about the higher parts of the science.

-Savage, The foundation of statistics.
Decision Environment

- The Domain of uncertainty:
  The strategy of opponent, $S_{-i}$ & moves of nature, $\Omega_N$

- The subjective state space:
  $\Omega_i = S_{-i} \times \Omega_N$
A strategy is partition dominant if it is an obviously dominant strategy in all events of the partition.
A Proposition: a strategy is partition dominant if and only if any partition obvious preference prefers it to any deviating strategy at any information set.
Implications for Mechanism Design

- A second best choice
  Especially when the state space becomes larger

- Manipulate the Partition
  The Choice of Presentation matters
  Not necessarily framing but bounded rationality

- Why Dynamic Mechanism
  Help people who reason in coarser partitions
A Laboratory Experiment: In progress

- A pair of games: a variation of Random Serial dictatorship
- A pair of individual decision tasks

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<thead>
<tr>
<th>Treatment A</th>
<th>Dominant Strategy</th>
<th>Obvious DS?</th>
<th>Partition DS?</th>
<th>Error Rate?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>High</td>
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</table>

<table>
<thead>
<tr>
<th>Treatment B</th>
<th>Dominant Strategy</th>
<th>Obvious DS?</th>
<th>Partition DS?</th>
<th>Error Rate?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Low</td>
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Future Research: Theoretical Work

- Necessary & Sufficient Condition for Implementation in Partition Dominant Strategy
- Endogenize Partitions & Learning Dynamics
- An Equilibrium Concept: Partition Obvious Equilibrium

The diagram shows the relationship between the finest partition, the coarsest partition, and the ex post equilibrium in relation to the partition obvious equilibrium.
Future Research: Experimental Work

▶ Manipulations of Partitions: An eye-tracking study (with James Chen)
▶ Pay for non-instrumental information (solo work)
The Psychological and the Bounded Rational

- Joys of winning
  Harrison 1989
- Reason-based Choice
  Shafir et al. 1993
- Spite Motives
  Morgan 2003
- Partition Obvious Preference
- The Magic Thinking of Tempt-fate
  Fudenberg and Levine 1998
Thank you!

All rational people are rational in the same way, all irrational people are irrational in different ways.

-Schmeidler

(a variant of Tolstoy’s original version)