Relative Prices, Non-Homothetic Preferences, and Product Quality*

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Abstract

This paper develops a novel theory of price level differences across countries. In a model characterized by a quality hierarchy in demand, we illustrate that GDP per capita growth induces continuous demand shifts toward goods of higher quality. These demand shifts in turn lead to structural change in the form of labor reallocations along the product quality dimension. As higher quality goods are more difficult to produce they command higher prices. The demand shifts, thus, result in increasing price levels when countries experience rising levels of economic development.

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1 Introduction

Leading theories of structural change have collectively featured both supply- and demand-side mechanisms of price level growth.\(^1\) Production-side theories emphasize sectoral differences in productivity growth and capital intensities, which are ultimately manifested in differential sectoral price trends (Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008)). On the other hand, demand-side theories highlight the role of differences in income elasticities of demand across sectors, or, put differently, non-homothetic preferences (Kongsamut et al. (2001), Foellmi and Zweimüller (2008)). Indeed, recent studies have sought to model both channels under one framework (Boppart (2014), Herrendorf et al. (2014), Comin et al. (2015)). Nevertheless, the international macroeconomics literature has predominantly relied on the former explanation when examining cross-country aggregate price discrepancies of the kind displayed in Figure 1 (Balassa (1964), Samuelson (1964), Berka et al. (2014)), despite mixed evidence for this mechanism.\(^2\) Our paper focuses on the demand side and aims to provide a novel theory of the real exchange rate that builds on the strong empirical support for Engel’s law.\(^3\)

In a small open-economy model characterized by a quality hierarchy in the demand structure, we demonstrate that GDP per capita growth induces ongoing demand shifts toward more expensive higher quality goods. Such structural change in expenditure shares across goods in turn leads to higher average product quality in consumption and a rise in the aggregate price level of the economy. Thus, countries experiencing higher relative levels of economic development observe relatively appreciated real exchange rates. In contrast to the mainstream supply-side arguments of Balassa (1964), Samuelson (1964), and Baumol (1967)\(^4\) our explanation of price level discrepancies across countries reverts back to the ideas of Neary (1988), Dornbusch (1985), and Bergstrand (1991) who, among others, note that consumers’ demands shift as income changes. As a complementary explanation to the standard-supply side theory, our model builds on this mechanism by introducing vertical differentiation in goods and a class of preferences that generate non-homotheticity in demand along the product quality dimension.\(^5\)

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1The process of structural transformation typically refers to the large scale sectoral reallocations of labor, capital, output and expenditures in an economy as it develops over time (Kuznets (1973), Maddison (1980), Timmer et al. (2014), Comin et al. (2015)).

2On the mixed evidence, see for example Chinn (2012), Rogoff (1996), Chinn and Johnston (1997), and Chong et al. (2012).

3On the latter, see Houthakker and Taylor (1970) and Browning (2008) for instance. Engel (1857) finds that poorer families spend a higher proportion of their income on primary goods (i.e. food). The absolute amount spent on primary goods increases with income, but the percentage change in this spending is less than that of income.

4Even though, Balassa (1964) and Samuelson (1964) precede Baumol (1967), the explanation of a relative price movement engendered by a productivity growth differential is often attributed to Baumol. Apart from the “Balassa-Samuelson theory” or “Baumol effect”, there are alternative, partially related, but less prominent supply-side explanations of price level differences across countries. Kravis (1984) and Bhagwati (1984) argue that richer countries are more labor intensive which drives up the price level. Lipsey and Swedenborg (1997) argue that richer countries protect their tradable industries more which drives up the price level. Burstein et al. (2003) and MacDonald and Ricci (2001) emphasize the importance of the distribution sector in determining the real exchange rate.

5In this paper, “vertically differentiated” product varieties are goods that differ by quality (see Feenstra (2010) or Schott (2004) for example).
Relative to the standard neoclassical setup, our model differs along two lines. First, on the production side, there is a variety of goods that are vertically differentiated. Goods of higher quality require more resources in production, thus commanding higher prices. Second, on the demand side, consuming fewer higher-quality goods is equivalent to consuming a larger amount of lower-quality goods. The key element in our theory that induces shifting from lower- to higher-quality goods is the utility specification. Specifically, we introduce time-varying consumption weights that depend on the state of technology. As technology improves, higher-quality goods receive higher weights in consumption, while lower-quality goods receive lower weights in consumption. As a result, demand continuously shifts from lower- to higher-quality goods along the growth path.

The shifts in consumer demand in our model result in ongoing labor reallocations across sectors. Countries with higher levels of development allocate more resources to the production of higher-quality goods. These goods are more difficult to produce than lower-quality goods, requiring more resources, and thus commanding higher prices. Therefore, assuming that purchasing power parity holds only for goods of the same quality, countries exhibiting higher levels of development must also exhibit higher price levels. In terms of growth rates, countries with relatively high GDP per capita growth experience faster shifts toward higher quality levels, while countries with relatively low GDP per capita growth experience slower shifts toward higher quality levels. Hence, faster-growing countries experience appreciating real exchange rates, while slower-growing countries experience depreciating real exchange rates as seen in the data.

The model that we construct in this paper is based on the robust empirical support for Engel’s law (see e.g. Houthakker (1957)). Our framework is closely related to models in the literature.
on structural change that emphasize the role of demand shifts in reconciling factor reallocations with balanced growth.\(^6\) In terms of the mechanism, it is closest to Laitner (2000), Caselli and Coleman (2001), Gollin et al. (2002), Greenwood and Uyßal (2005), Matsuyama (2002) and Foellmi and Zweimüller (2008) where hierarchical demand structures lead to the entry of new goods. Our study differs from these papers in that it introduces a quality hierarchy of goods that generates the relative price effect needed to explain cross-country differences in price levels. In what follows, we develop our model in Section 2, before concluding in Section 3.

2 Theoretical Model

Our analysis considers a small open economy model. In this economy, goods vary along the quality dimension where good \(n \in \mathbb{Z} \equiv \{..., -2, -1, 0, 1, 2, ...\}\) is produced in the perfectly competitive sector \(n\). Meanwhile, the representative consumer derives greater utility from higher quality than lower quality goods, and supplies labor inelastically at the aggregate level.\(^7\)

2.1 Technology

The output of good \(n, y_{n,t}\), is generated according to the production function

\[ y_{n,t} = \Gamma^n A_t l_{n,t} \]  

(1)

where \(\Gamma \in (0, 1)\) is a parameter that governs how quality changes across goods with \((1/\Gamma)^n\) denoting the time invariant quality of good \(n\), \(A_t\) is the level of productivity common to all goods at time \(t\), and \(l_{n,t}\) is the amount of labor employed in industry \(n\) at time \(t\). Equation (1) indicates that a higher quality good, given by a higher \(n\), is more difficult to produce than a lower quality good, given by a lower \(n\), at any point in time. Productivity grows at an exogenously-given rate \(g > 0\).

\(^6\)We note that the Balassa-Samuelson model is difficult to reconcile with standard growth theory for two reasons. First, Balassa-Samuelson is difficult to reconcile with the neoclassical growth theory which states that productivity growth is labor augmenting, i.e. that productivity growth is higher in labor intensive industries such as services, see e.g. Uzawa (1961). It is therefore difficult to reconcile with the evidence on balanced growth as the literature on structural change shows, see e.g. Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). Second, it is hard to square with endogenous growth theory which suggests that technical change is directed toward scarce goods and factors. A relative price effect as the one that Balassa-Samuelson predicts can therefore hardly persist in the long run, see e.g. Acemoglu (2002). Our theory provides an explanation of price level differences across countries that is consistent with the long-term features of the economy studied in the growth literature. Furthermore, we note that the Balassa-Samuelson model is also difficult to reconcile with standard trade theory. As tradable and non-tradable goods are complements, this model implies that the world trade to GDP ratio declines over time. Real world data rather show the opposite. Our theory provides an explanation of price level differences across countries that is not at odds with the evidence from the trade literature.

\(^7\)We stress in advance that the conclusions of our real exchange rate theory follow directly from two rather innocent assumptions: 1) higher quality goods require more resources in production and thus have higher relative prices, and 2) richer consumers consume more higher quality goods, implying a positive relation between income levels and the average quality of goods in consumption. These assumptions are empirically plausible and thus render our model quite robust. Our results also emerge in a parameter-free version of the theory.
such that $A_t = A_0(1 + g)^t$ where $A_0 = (1 + g)^a$ and $a \in \mathbb{N}^+$. To maintain a tractable analysis, we impose the parameter constraint $\Gamma^n = 1/(1 + g)^a$. The representative firm in sector $n$ maximizes profits, $\pi_{n,t}$, given by

$$\pi_{n,t} = p_{n,t}y_{n,t} - w_{n,t}l_{n,t}$$

where $p_{n,t}$ and $w_{n,t}$ denote the price of the good and the return to labor respectively in that sector at time $t$. Optimization with respect to $l_{n,t}$ leads to the first-order condition

$$w_{n,t} = p_{n,t}\Gamma^n A_t.$$  

### 2.2 Preferences

Output $y_{n,t}$ can be used for consumption $c_{n,t}$. In welfare terms, consuming goods of higher quality offers more utility than consuming goods of lower quality. Formally, quality-adjusted consumption is given by $C_Q^n = \frac{1}{\Gamma} c_{n,t}$. The representative consumer has a comprehensive consumption index

$$C_t^Q = \left[ \sum_{n \in \mathbb{Z}} \gamma_{n,t}(C_Q^n)^{\theta-1} \right]^{\frac{1}{\theta}}$$

where $\gamma_{n,t}$ is the time-varying weight of good $n$ in total consumption with $\sum_{n \in \mathbb{Z}} \gamma_{n,t} = 1$ $\forall t$, and $\theta > 1$ is the goods elasticity of substitution. The consumer derives instantaneous utility $U$ from $C_t^Q$. The change in the weight of good $n$ in aggregate consumption is driven by the level of economic development. In particular, we define $\gamma_{n,t} = \left( e^{-|t+a-n|} \right) / z$ where $z = 1 + 2\sum_{i=1}^{\infty} e^{-i}$ ensures that the weights across goods sum to unity. Intuitively, this definition implies that demand is continuously shifting toward higher quality goods in a growing economy. Maximization of equation (4) subject to the usual expenditure constraint

$$E = \sum_{n \in \mathbb{Z}} P_Q^n C_Q^n$$

yields the relative demand for quality-adjusted goods

$$\frac{C_{n,t}^Q}{C_{-n,t}^Q} = \frac{\gamma_{n,t}}{\gamma_{-n,t}} \left( \frac{P_Q^n}{P_Q^{-n,t}} \right)^{\theta}$$

where $P_Q^{-n,t}$ and $P_Q^n$ denote the quality-adjusted prices of goods $-n$ and $n$ respectively. Given that the weights on goods depend on the level of development, and hence income implicitly, consumer preferences are non-homothetic. The relation between quality-adjusted and quality-unadjusted prices is straightforward as $P_Q^n C_Q^n = p_n c_n$. Specifically, it is given by

$$P_{n,t}^Q \left( \frac{1}{\Gamma} \right)^n = p_n.$$
Therefore, the relative demand for quality-unadjusted goods is given by

\[
\frac{c_{n,t}}{c_{n,t}} = \frac{\gamma_{n,t}}{\gamma_{n,t}} \left[ \frac{p_{n,t}}{p_{-n,t}} \right]^{\theta} \left[ \frac{\Gamma^n}{\Gamma^{-n}} \right]^{1-\theta}.
\] (8)

The consumer maximizes the present discounted value of lifetime utility from consumption

\[
U_0 = \sum_{t=0}^{\infty} \beta^t \left[ C^Q_t \right]^{1-\phi} \frac{1}{1-\phi}
\] (9)

subject to the standard budget constraint

\[
P_t^Q C_t^Q + \zeta_t B_{t+1} - B_t = \sum_{n \in \mathbb{Z}} w_{n,t} l_{n,t}
\] (10)

where \( \beta \) is the subjective discount factor, \( \phi \) is a parameter governing the intertemporal elasticity of substitution, \( B_t \) denotes the holdings of a risk-free international bond, and \( \zeta_t = 1/(1 + r_B^t) \) is the time \( t \) price of a unit of bond holdings in period \( t+1 \). The usual transversality condition is assumed to hold. The consumer supplies labor inelastically at the aggregate level with \( L_t = 1 \). Labor market clearing implies that the sum of labor allocated across industries equals the aggregate labor supply, namely

\[
\sum_{n \in \mathbb{Z}} l_{n,t} = L_t = 1
\] (11)

### 2.3 Equilibrium

A dynamic competitive equilibrium is described by a time path of prices \( \{P^Q_{n,t}, P^Q_t, w_{n,t}, p_{n,t}, \zeta_t\}_{t=0}^{\infty} \) and quantities \( \{C^Q_{n,t}, C^Q_t, c_{n,t}, l_{n,t}, y_{n,t}, B_t\}_{t=0}^{\infty} \) and \( n \in \mathbb{Z} \), given a level of productivity \( \{A_t\}_{t=0}^{\infty} \) and labor supply \( \{L_t\}_{t=0}^{\infty} \), which is consistent with firm and household optimization, perfect competition, resource constraints, and market clearing conditions.

### 2.4 Relative Prices

Combining the first-order conditions derived with respect to \( l_{n,t} \) and \( l_{-n,t} \) from the consumer’s intertemporal problem yields the equation \( w_n = w_{-n} \). Using the firm’s optimality condition in equation (3), wage equalization across sectors results in

\[
\frac{p_{n,t}}{p_{-n,t}} = \frac{\Gamma^{-n}}{\Gamma^n}
\] (12)

which implies that a good with a higher index \( n \), a higher-quality good that is more difficult to produce, commands a higher relative price.
Proposition 1. A higher-quality good commands a higher relative price. ■

Next, we derive a proposition that highlights the continuous reallocations of labor along the growth path i.e. structural change. We first normalize the quality-unadjusted price of good 1 to unity; \( p_1 = 1 \). Letting \(-n = 1\), it then follows from equation (12) that the price of good \( n \) is given by \( p_n = \Gamma^{1-n} \). Further assume that trade is balanced along the growth path in each sector so that \( y_n = c_n \). \(^8\) Substituting \( p_n = \Gamma^{1-n} \), \( c_n = y_n \), and equation (1) into equation (8) leads to the relative labor allocation

\[
\frac{l_{n,t}}{l_{n+1,t}} = \frac{\gamma_{n,t}}{\gamma_{n+1,t}}.
\]

Subsequently, we can employ equation (13) to derive a labor-allocation-weighted average quality index given by

\[
\sum_{n \in \mathbb{Z}} \frac{l_{n,t}}{L_t} n = \sum_{n \in \mathbb{Z}} \gamma_{n,t} n.
\]

This weighted average index provides an indication of the level of the quality of goods consumed at time \( t \). We can thus use this index to infer how average product quality in consumption evolves over time. In particular, as demand shifts toward higher-quality goods, average quality increases. Formally,

\[
\sum_{n \in \mathbb{Z}} \gamma_{n,t} n < \sum_{n \in \mathbb{Z}} \gamma_{n,t+1} n.
\]

To see that this inequality holds along the growth path, let us assume that good \( n \) is the good where \( a + t = n \). Then, \( \sum_{n \in \mathbb{Z}} \gamma_{n,t} n = \frac{1+e^{-1}}{1-e^{-1}} \frac{n}{2} \) and \( \sum_{n \in \mathbb{Z}} \gamma_{n,t+1} n = \frac{1+e^{-1}}{1-e^{-1}} \frac{n+1}{2} \), thus verifying equation (15).

Proposition 2. Labor continuously reallocates toward higher-quality firms over time. ■

Now consider two small open economies, Home (\( H \)) and Foreign (\( F \)), that have identical labor supplies but different levels of development consistent with \( A^H_0 > A^F_0 \). We define the weighted average price level of each country \( j = \{ H, F \} \) as

\[
P_j^t = \sum_{n \in \mathbb{Z}} \frac{p_{n,t}^j y_{n,t}^j}{L_t^j} = \sum_{n \in \mathbb{Z}} \frac{l_{n,t}^j}{L_t^j} p_{n,t}^j = \sum_{n \in \mathbb{Z}} \gamma_{n,t}^j \Gamma^{1-n}.
\]

Following the proof of proposition 2, the country with the higher level of development must have the higher aggregate price level. That is,

\(^8\)Hypothetically, with different goods there could be incentives for specialization in the production of higher- and lower-quality goods. But since both factor proportions as well as the quality of each good is assumed to be identical across countries, the gains from specialization are zero.
\[ \sum_{n \in \mathbb{Z}} \gamma_n^F \Gamma^{1-n} < \sum_{n \in \mathbb{Z}} \gamma_n^H \Gamma^{1-n}. \] (17)

To see this inequality let us assume that good \( n \) is the good where \( a^F + t = n \). Foreign’s price level is

\[ P_t^F = \sum_{n \in \mathbb{Z}} \gamma_n^F \Gamma^{1-n} = \Gamma^{1-n} \frac{\sum_{i=0}^{\infty} e^{-i \Gamma^{-i}}}{z} + \frac{\sum_{i=1}^{\infty} e^{-i \Gamma^i}}{z} = \frac{\Gamma^{1-n}}{z} \left( \frac{1}{1 - \frac{\Gamma}{e}} + \frac{\Gamma}{1 - \frac{\Gamma}{e}} \right). \] (18)

Since \( A_0^H > A_0^F \), let us also assume that good \( n + b \) is the good where \( a^H + t = n + b \) with \( b \in \mathbb{N}^* \). Home’s price level is then given by

\[ P_t^H = \sum_{n \in \mathbb{Z}} \gamma_n^H \Gamma^{1-(n+b)} = \Gamma^{1-(n+b)} \frac{\sum_{i=0}^{\infty} e^{-i \Gamma^{-i}}}{z} + \frac{\sum_{i=1}^{\infty} e^{-i \Gamma^i}}{z} = \frac{\Gamma^{1-(n+b)}}{z} \left( \frac{1}{1 - \frac{\Gamma}{e}} + \frac{\Gamma}{1 - \frac{\Gamma}{e}} \right). \] (19)

As \( \Gamma^{1-(n+b)} > \Gamma^{1-n} \), the price level of Home must be higher than the price level of Foreign, namely,

\[ P_t^H > P_t^F \] (20)

where the constraint \( e \Gamma > 1 \) is assumed to hold.

**Proposition 3.** An economy with a higher level of development produces a higher-quality good and therefore has a higher price level. \( \blacksquare \)

It is now relatively easy to demonstrate that Home’s real exchange rate is above unity and, thus, that purchasing power parity does not hold at the aggregate level. Let \( \xi_t \) denote the nominal exchange rate defined as national currency per unit of foreign currency at time \( t \). We assume that goods of identical quality can be traded at the same price internationally. Formally,

\[ \frac{1}{\xi_t} \frac{P_{n,t}^H}{P_{n,t}^F} = 1 \] (21)

where \( \xi_t = \frac{P_{n,t}^H}{P_{n,t}^F} \). Noting the normalization \( p_j^i = 1 \) \( \forall j = \{H, F\} \) so that \( \xi_t = 1 \), and continuing with the assumption \( A_0^H > A_0^F \), Home’s aggregate real exchange rate is

\[ q_t^H = \frac{1}{q_t^F} = \frac{1}{\xi_t} \frac{P_t^H}{P_t^F} = \frac{P_t^H}{P_t^F} > 1. \] (22)

Intuitively, the preference of richer consumers for higher-quality goods induces ongoing shifts toward goods of increasing price and quality. These shifts in turn imply that faster-growing countries display higher aggregate price level growth, and thus real exchange rate appreciation.
Proposition 4. An economy with a higher relative level of development has a higher real exchange rate.

3 Conclusion

Despite mixed empirical evidence on the mechanism, the standard supply-side theory of Balassa (1964), Samuelson (1964), and Baumol (1967) has dominated the international macroeconomics literature in explaining cross-country price level differences. Building on the strong empirical support for Engel’s law, our paper offers a novel demand-side approach to explaining real exchange rate dynamics.

We develop an open-economy model characterized by a quality hierarchy in demand, and show that GDP per capita growth induces ongoing demand shifts toward more expensive higher quality goods. The expenditure share changes, in turn, result in continuous reallocations of labor toward higher-quality goods, which tend to be more difficult to produce, thus commanding higher prices. Since higher-quality goods command higher prices, the shift in expenditure shares raises the aggregate price level of the economy. As we demonstrate, this implies that countries experiencing higher relative levels of economic development observe relatively appreciated real exchange rates.
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