Growth, Housing and Global Imbalances

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Abstract

In the two decades leading to the Great Recession, the United States experienced current account deficits and rising house prices while Japan, one of its major trading partner, experienced surpluses and falling house prices. During the same time period China, also a major trading partner of the United States, had large current account surpluses together with rapid house price growth. We develop a two-country life-cycle model in which each economy is populated by agents living for three periods (young, middle-aged, and old); young households need to borrow to buy houses but can do so up to a maximum Loan-to-Value (LTV) ratio. We allow for asymmetries across countries in terms of productivity growth and the tightness of the borrowing constraint. We calibrate the model to the United States and China and show that temporarily higher productivity in China and financial integration replicate the pattern of current account balances and growth of house prices observed by the two economies. Likewise, a temporary productivity slowdown in Japan leads to surpluses and falling house prices in Japan and deficits and rising house prices in the United States. Growth differentials among trading partners are key to the relationship between the interest rate, the current account, house prices and the real exchange rate. Moreover, we find that house prices act as an amplification mechanism behind global imbalances.

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*Link to the latest version.
1 Introduction

The period between the mid 1990s and the Great Recession has been characterized by global imbalances, namely the emergence of large and persistent current account deficits in some countries financed by large and persistent current account surpluses in other countries.\(^1\) At the same time, countries with current account deficits experienced large housing booms followed by crushing busts after the crisis.\(^2\) A large body of economic literature has emphasized the causal relationship between the global saving glut and the housing bubble in the United States – see for example Bernanke et al. (2005) – or the availability of cheap funding for Greece, Ireland, Spain and Portugal after joining the euro zone and their large house market swings relative to stable or even falling house prices in Germany, the surplus country. The common theme in this literature is an (almost) exogenous fall in the world real interest rate; the focus is on the behavior of the housing market in deficit countries.

The empirical evidence, however, is less clear-cut. Starting in 1990 until the Great Recession, the U.S. economy experienced large and persistent current account deficits, as shown in Figure 1, together with rising house prices, which are depicted in Figure 2. Over the same period Japan and China, two of the largest trade partners of the United States, run large current account surpluses – see Figure 1 – accompanied by decreasing house prices in Japan but rapidly growing house prices in China. Rising house prices are not necessarily accompanied by current account deficits and current account surpluses do not imply falling house prices.

Along the balanced-growth path, the rent-to-price ratio should be constant. Figure 3 shows the rent-to-price ratio in the United States, Japan and China. Between 1993 and 2007 the U.S. rent-to-price fell while the Japanese one rose considerably. The rent-to-price ratio in China displays a downward pattern starting in 2000. When interpreting housing market data for China, it is important to keep in mind that the Housing Reform started under Deng Xiaoping in 1988 and was gradually implemented over the subsequent decade. According to Yang and Chen (2014), house prices and rents were regulated and it was only starting in 1998 that these prices became primarily driven by market forces.

Japan and China are important trade partners of the United States. Figure 4 reports that these countries account for almost a third of the total trade of the United States. Two further facts are worth noticing. During the period under consideration, as we report in Figure 4, Japan started as the larger U.S. trading partner among the two countries, but China overtook Japan in 2002. This is to say that the U.S. economy was shifting its trade pattern from Japan,

\(^1\) See Obstfeld and Rogoff (2009), Gourinchas and Rey (2013) and the literature cited in these works.

\(^2\) See for example André (2010).
Figure 1: Current Account (% GDP) in the United States (USA), Japan (JPN) and China (CN). Constructed using data from Lane and Milesi-Ferretti (2007).

Figure 2: Real House Prices in the United States (USA), Japan (JPN) and China (CN). Constructed using data from the OECD, the Economist and Lane and Milesi-Ferretti (2007).
Figure 3: Rent-to-Price Ratio in the United States (USA), Japan (JPN) and China (CN). Constructed using data from the OECD and National Statistics Offices.

Figure 4: Trade weights of the US for Japan (JPN) and China (CN). Data from the BIS.
a financially developed economy, to China, an economy with a substantially lower degree of financial development – see Figure 6. Second, the United States were shifting trade weight from Japan, an economy in recession, to China, a fast-growing economy – see Figure 7.

![Graph of World Ex-post Real Interest Rate](image)

Figure 5: World Ex-post Real Interest Rate. Constructed using data from the OECD.

At last, the world real interest rate fell from 1990 to 2010 – see Figure 5.

Our paper argues that asymmetries across trading partners are fundamental to explain the observed joint behavior of global imbalances and house prices. To this end, we develop a two-country life-cycle (OLG) model in which each economy is populated by agents living for three periods (young, middle-aged, and old). Young households need to borrow to buy houses but can do so up to a maximum Loan-to-Value (LTV) ratio. Our model generates a joint behavior of house prices and current account balances in accordance with the empirical evidence presented above. This dynamics is explained by asymmetries across countries in growth rates, initial capital levels, and the level of financial development.

We carry out two experiments with our model. In the first experiment, we focus on the United States and China. As in Coeurdacier et al. (2015), we assume that China has temporarily higher productivity growth but faces tighter borrowing constraints than the United States. When the two countries become financially integrated, current account deficits arise in the slower-growing economy and surpluses in the fast-growing; the model also predicts an acceleration in the growth of house price in the United States.
Figure 6: Indices of financial markets heterogeneity in the US (USA), Japan (JPN) and China (CN). The index is from Abiad et al. (2008).

Figure 7: Real GDP growth rate in the US (USA), Japan (JPN) and China (CN).
A key mechanism for our results in the first experiment, as first found by Coeurdacier et al. (2015), is that a fall in the world interest rate leads to a fall in saving in the United States but an increase in China. In the financially developed country (the United States), the reduction in the interest rate leads to an increase in borrowing by young agents; in the more constrained economy (China), borrowing is tightly limited by the constraint and the increase in saving by middle-aged dominates. This different effect on saving across countries is due to differences in the borrowing limit and in productivity growth. The latter is responsible for rapid wage growth in China and the related income effect. The OLG structure of the model is essential in generating heterogeneous responses in cohort saving, which lead to current account imbalances.

The novel contribution of our paper is to link patterns of current account imbalances and house price developments. In our model the fall in the world interest rate stemming from financial integration is consistent with growth in house prices both in the United States and China. The United States experiences an acceleration in house price growth driven by the increase in borrowing and housing demand by young households; high productivity growth and capital scarcity lead to high growth of house prices in China – higher than in the long-run equilibrium. Moreover, we run the counterfactual experiment of fixing house prices in each economy at its autarky level and find an amplification effect of house prices on current account imbalances.

In the second experiment, we focus on the United States and Japan. The two countries are financially integrated and have a similar level of financial development when Japan experiences an unexpected recession due to a temporary reduction in productivity. The model predicts decreasing housing prices and current account surpluses in the country in recession and current account deficits and an acceleration in the growth of house prices in its trading partner. This result is driven by openness and the recession in Japan. The recession decreases the interest rate by less than it would be the case if Japan were a closed economy, as the world interest rate is a weighted average of the autarky rates in the two economies. Lower productivity in Japan coupled with financial integration leads to capital outflows toward the United States and a current account surplus in Japan. Borrowing by young households increases both in the United States and in Japan due to lower interest rate. House price growth accelerates in the United States driven by higher housing demand by young agents and rental investment. Housing price developments in Japan are explained by the negative income effect of lower growth in wages, which reduces housing demand by young. An additional factor depressing house prices in Japan is rental investment, which falls in order to raise its rate of return since
the interest rate falls less than productivity.

The main driving force behind our results is growth rate differentials across trading partners. Hence, house prices in the United States are affected by growth developments in its trading partners. Our counterfactual exercises suggest that differences in the marginal productivity of capital due to productivity differentials and/or transitional dynamics are the main determinant of the joint behavior of the current account and house prices. After financial integration, differences in financial development between the United States and China cannot by themselves replicate the qualitative behavior of the data. In the case of the United States and Japan, the recession in Japan is the only asymmetry between the two countries and the model is able to replicate the negative correlation of current account balances and house prices growth in both economies.

If growth differential is main factor contributing to our results, the OLG structure plays a key role to generate the desired correlation between saving and current account. Starting from a balanced current account, a standard frictionless representative agent model would predict current account deficits for China and Japan. Borrowing constraints are important to push youth saving upwards in China and, in general, contribute quantitatively to match national saving and the current account surplus in China.

Previous literature on the relationship between current account balances and house prices focuses primarily on the U.S. economy. A number of contributions are based on two-country models. Punzi (2013) develops a model following Iacoviello (2005) that differentiates between patient and impatient households in the domestic economy, and analyzes the business cycle properties after exogenous shocks to technology, housing preference and the LTV ratio. Ferrero (2015) also builds on Iacoviello (2005) and studies the response of the model to monetary policy and LTV-ratio shocks. These papers focus on domestic factors as the main drivers behind the joint behavior of the current account and house prices in the domestic economy. Our emphasis on foreign factors is novel; moreover we find them to be important determinants for the evolution of U.S. house prices.

Gete (2015) analyzes the effect of changes in house price expectations, U.S. population size and the LTV; it also studies the effects of an exogenous decrease in the interest rate as a proxy for a shock to the foreign discount factor. Our paper extends this analysis to a setup where growth asymmetries generate predictions consistent with the empirical evidence without the need of exogenous shocks to the interest rate. Favilukis et al. (2015) find no effect of international capital flows into U.S. safe assets on housing prices. They find that the reduction in the risk-free rate prompted by the flow of international capital into the U.S.
economy is compensated by the increase in the housing risk premia because of the reallocation in the domestic agents’ portfolio prompted by the foreign purchases of safe assets.

Small open economy models in the literature focus on the U.S. economy and study shocks to the world interest rate and the down-payment required to buy a house. Adam et al. (2012) use Bayesian learning in a small open economy model with collateral constraints in order to broaden the effect of an exogenous decrease in the international interest rate. Kiyotaki et al. (2011) study how interest rate and LTV shocks in a small open economy model with growth determine house prices. There is no emphasis on the evolution of the current account and the interest rate is exogenous. Franjo (2015) and Arslan (2014) study the effect of an exogenous decrease in the international interest rate and in the down-payment required to buy a house in a small open economy life-cycle model. Garriga et al. (2012) develop a representative agent model in which relaxed financial conditions and a decrease in the international interest rate generate housing prices dynamics. We build a two-country model with an endogenous interest rate and emphasize foreign factors as the driving force behind the joint behavior of current account balances and house prices.

This paper also contributes to the literature on global imbalances. Benhima (2013) proposes an investment approach by assuming that investment is depressed in developing countries due to the existence of financial frictions. Other contributions to this literature, closer to our approach, analyze the effects of financial frictions on savings. Caballero et al. (2008) and Mendoza et al. (2009) study, under different assumptions, the coexistence of low interest rates and global imbalances in the world economy. They stress how a large amount of savings coming from emerging economies during the 90s depressed international interest rates. Caballero et al. (2008) model the inability of emerging economies to supply risky assets, and Mendoza et al. (2009) analyze the demand for risky assets in developing economies when there are incomplete financial markets and uninsurable risk. Bacchetta and Benhima (2015) focus on the lack of liquidity for firms to carry out financial investment. Also, Coeurdacier et al. (2015) studies the importance of credit constraints and growth rate differentials in the global economy. Our work builds on Coeurdacier et al. (2015) and links global imbalances and dynamics in the world interest rate to house price developments. The existence of housing makes our borrowing constraint endogenous as an amplification mechanism and our model has also implications for the real exchange rate rate.

The rest of the paper is organized as follows. The model economy is presented in Section 2. In Section 3 we explain our computational experiment for the case of the United States and China; while the experiment for the United States and Japan is developed in Section
Section 5 discusses the implications of our model for the determination of the real exchange rate. Section 6 concludes.

2 Model

We build a two-country model. Each country is populated by overlapping generations of individuals who live for three periods: young (y), middle-aged (m), and old (o). Young and middle-aged individuals inelastically supply one unit of labor; old individuals are retired. We assume that labor is not mobile across countries while capital is. There are two sectors in each country: consumption and capital goods sector, and housing sector. We consider the consumption and capital goods sector as a tradable sector while housing is a non-tradable sector. The countries differ in two dimensions: the degree of financial development and the growth rate of productivity. Financial development is captured by a borrowing limit, which we model as the maximum loan-to-value ratio at which houses can be purchased. Life-time income is bell-shaped so that young individuals are constrained by the borrowing limit. Both countries have the same preferences and technology.

2.1 Production

2.1.1 Production: Consumption and Capital Goods

We define $K_i^t$ and $L_i^t$ as the amount of capital and labor, respectively, employed in the production of consumption and capital goods at time $t$ in country $i$. Firms choose labor $L_i^t$ and capital $K_i^t$ that maximize their profits. Output by this sector is produced using a Cobb-Douglas production technology:

$$Y_i^t = (Z_i^t L_i^t)^{1-\alpha} (K_i^t)^{\alpha},$$

where $0 < \alpha < 1$ is the capital share and $Z_i^t$ is country-specific productivity at time $t$ in country $i$.

We define the total labor employed in production, $L_i^t$, as the sum of labor supplied by the young ($L_{y,t}^i$) and the middle-aged ($L_{m,t}^i$) individuals:

$$L_i^t = \varepsilon L_{y,t}^i + L_{m,t}^i,$$

where $\varepsilon < 1$ stands for the relative productivity of young workers.

Capital follows the standard law of motion:
\[ K^{i}_{t+1} = (1 - \delta) K^{i}_t + I^i_t, \]

where \( \delta \) is the rate at which capital depreciates, and \( I^i_t \) is the investment in capital at time \( t \) in country \( i \).

Productivity and population grow over time at rates \( g_{Z,t}^i \) and \( g_{L,t}^i \), respectively, such that:

\[ Z^{i}_{t+1} = (1 + g_{Z,t}^i) Z^i_t \quad \text{and} \quad L^{i}_{t+1} = (1 + g_{L,t}^i) L^i_t. \]

Firms in the consumption and capital goods sector maximize their profits. Factors markets are competitive, so that \( w^i_t \) and \( r^k_{t} \) are, respectively, the marginal product of labor and capital at time \( t \) in country \( i \).

### 2.1.2 Production: Housing

We assume that there is a fixed supply of housing each period, \( H^i_t \), such that:

\[ H^i_t = N^i_t, \]

where \( N^i_t \) is the stock of residential land at time \( t \) in country \( i \). We also assume that the stock of land grows over time at the same rate as the population, \( g_{L,t}^i \), so that:

\[ N^i_{t+1} = (1 + g_{L,t}^i) N^i_t. \]

### 2.2 Households

There are 3 stages in the life cycle of each individual: \( j \in \{y, m, o\} \). Like capital, housing must be purchased one period before use. We do not model housing tenure considerations. We assume that young households start with no housing stock and must therefore rent a house in order to get housing services; middle-aged and old individuals instead own a house.

Let \( c^i_{j,t} \) denote non-housing consumption of individuals at age \( j \), in country \( i \) in period \( t \); \( f^i_{y,t} \) is rental spending of the young at time \( t \); and \( h^i_{j,t} \) is house purchase by individual of age \( j \in \{m, o\} \) that provides housing services in period \( t + 1 \).

The lifetime utility of an individual in country \( i \) at time \( t \) is:

\[ U^i_t = u(c^i_{y,t}, f^i_{y,t}) + \beta u(c^i_{m,t+1}, h^i_{m,t+1}) + \beta^2 u(c^i_{o,t+2}, h^i_{o,t+2}) \]

where \( \beta \) is the rate at which individuals discount the future.

We assume that the utility function has the following functional form:
\[
  u(c, x) = \frac{\left( (c_{j,t}^i)^\gamma (x_{j,t}^i)^{1-\gamma} \right)^{1-\sigma}}{1 - \sigma},
\]

where \( x \) denotes housing services from renting or owning a house, the parameter \( \gamma \) is the share of non-housing consumption in total expenditure, and \( \sigma \) is the coefficient of relative risk aversion.

Individuals may hold two types of assets in this model: housing or deposits at financial institutions. Let \( a_{j,t}^i \) denote deposits at financial institutions (claims to capital and rental housing stock) by individual \( j \) in country \( i \) at time \( t \).

We assume there is a minimum downpayment requirement an individual must pay to buy a house; this requirement is denoted by \( (1 - \theta^i) \). The remaining balance on the house is financed by borrowing from financial institutions. Hence \( \theta^i \) denotes the maximum loan-to-value ratio and it captures the degree of financial development. More financially developed countries are characterized by higher \( \theta^i \)’s.

Young households born in country \( i \) in period \( t \) solve:

\[
  \max U_t^i = u(c_{y,t}^i, f_{y,t}^i) + \beta u(c_{m,t+1}^i, h_{m,t+1}^i) + \beta^2 u(c_{a,t+2}^i, h_{a,t+2}^i)
\]

s.t.:

\[
\begin{align*}
  c_{y,t}^i + r_t^i f_{y,t}^i + q_t^i h_{o,t+1}^i + a_{y,t+1}^i &= w_t^i \varepsilon, \\
  c_{m,t+1}^i + q_{t+1}^i h_{m,t+2}^i + a_{m,t+2}^i &= w_{t+1}^i + q_{t+1}^i h_{m,t+1}^i + a_{y,t+1}^i (1 + r_{t+1}^i), \\
  c_{o,t+2}^i &= q_{t+2}^i h_{o,t+2}^i + a_{m,t+2}^i (1 + r_{t+2})^i, \\
  a_{y,t+1}^i &= -\theta^i q_t^i h_{m,t+1}^i, \quad a_{m,t+2}^i \geq -\theta q_{t+1}^i h_{o,t+2}^i.
\end{align*}
\]

where \( r_t^i \) is the rental price at time \( t \) in country \( i \); \( q_t^i \) is the housing price at time \( t \) in country \( i \); and \( r_t^i \) is the rate of return on deposits at time \( t \) in country \( i \).

When individuals are young they receive a fraction \( \varepsilon \) of the competitive wage, where \( \varepsilon \) stands for the productivity of young relative to middle-age workers. Young individuals decide consumption of non-durable goods \( c_{y,t}^i \), housing services from renting \( f_{y,t}^i \), housing purchases \( h_{m,t+1}^i \) to deliver housing services when middle-aged and saving. We define net worth as the sum of deposits \( a \) plus the market value of the owned house \( qh \) that is carried to the next period. When individuals are young it is optimal for them to borrow up to the limit

\[
a_{y,t+1}^i = -\theta^i q_t^i h_{m,t+1}^i.
\]

Middle-aged individuals receive the competitive wage and the return from the net worth they decided to carry over when young. For the generation born at time \( t \) this includes the
repayment of the loan $a_{y,t+1}^i (1 + r_{t+1}^i)$ and the housing value $q_{t+1}^i h_{m,t+1}^i$. Then, they decide non-housing consumption $c_{m,t+1}^i$ and net worth for the next period, $a_{m,t+2}^i$ and $h_{o,t+2}^i$. Middle-age individuals get housing services from the amount of housing they purchased when young.

Old individuals get housing services from the amount of housing purchased when middle-aged and get the return on net worth they carry from the previous period. They consume all their resources.

### 2.3 Financial Intermediaries

Financial intermediaries (“mutual funds”) receive deposits from and make loans to households and decide how to allocate the resources between capital ($K_{t+1}^i$) and rental housing stock ($H_{t+1}^i$). Formally, financial intermediaries solve the following problem:

$$\max_{K_{t+1}^i, H_{t+1}^i} \left( (1 + r_{k,t+1}^i) K_{t+1}^i + (r_{f,t+1}^i + q_{t+1}^i) H_{t+1}^i \right)$$

s.t. $K_{t+1}^i + q_{t+1}^i H_{t+1}^i = A_{t+1}^i$,

where

$$A_{t+1}^i = L_{y,t}^i a_{y,t+1}^i + L_{m,t}^i a_{m,t+1}^i.$$  

$A_{t+1}^i$ is the total amount of deposits net of loans held at financial institutions of country $i$ at the end of period $t$.

Defining the portfolio shares invested in capital and rental housing as $\theta_{k,t+1}^i = \frac{K_{t+1}^i}{A_{t+1}^i}$ and $\theta_{f,t+1}^i = \frac{q_{t+1}^i H_{t+1}^i}{A_{t+1}^i}$, we can restate this problem as:

$$\max_{\theta_{k,t+1}^i, \theta_{f,t+1}^i} \left( (1 + r_{k,t+1}^i) \theta_{k,t+1}^i + R_{t+1}^i \theta_{f,t+1}^i \right)$$

s.t. $\theta_{k,t+1}^i + \theta_{f,t+1}^i = 1$,

where $R_{t+1}^i = \frac{r_{f,t+1}^i + q_{t+1}^i}{q_{t+1}^i}$. For this problem to have an interior solution, we must have $(1 + r_{k,t+1}^i) = R_{t+1}^i$. We also define the rate of return on household savings as $1 + r_t^i \equiv (1 + r_{k,t+1}^i) \theta_{k,t+1}^i + R_{t+1}^i \theta_{f,t+1}^i$.

### 2.4 Government

The government in each country owns the new residential land and sells it on the market at price $q_t^i$. The government in country $i$ at time $t$ uses this revenue to finance wasteful spending, $G_{t+1}^i$. 

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\[ G^i_t = q^i_t (N^i_{t+1} - N^i_t). \]

### 2.5 Autarky vs. Integrated Equilibrium

In the autarkic equilibrium, the clearing market condition in the asset market is given by:

\[ L^i_{y,t} a^i_{y,t+1} + L^i_{m,t} a^i_{m,t+1} = K^i_{t+1} + q^i_t H^{f,i}_{t+1}. \]

In words, the amount of net deposits held by individuals at financial institutions in each country equals the capital stock plus the market value of the housing stock rented in each country.

However, in the integrated equilibrium, asset market clears in the global economy:

\[ \sum_i (L^i_{y,t} a^i_{y,t+1} + L^i_{m,t} a^i_{m,t+1}) = \sum_i (K^i_{t+1} + q^i_t H^{f,i}_{t+1}). \]

Clearing conditions in other markets are the same in both equilibria given that labor is not mobile across countries and housing is a non-tradable good. Thus, market clearing conditions are:

(a) Housing market:

\[ L^i_{y,t} h^i_{m,t+1} + L^i_{m,t} h^i_{o,t+1} + H^{f,i}_{t+1} = N^i_{t+1}. \]

(b) Rental market:

\[ L^i_{y,t} f^i_{y,t} = H^{f,i}_{t}. \]

(c) Labor market:

\[ \varepsilon L^i_{y,t} + L^i_{m,t} = L^i_t. \]

(d) Goods market:

\[ Y^i_t = C^i_t + I^i_t + G^i_t + TB^i_t, \]

where \( TB^i_t \) is the trade balance at time \( t \) in country \( i \). Note that, \( \sum_i (TB^i_t) = 0 \) in the integrated equilibrium; and \( TB^i_t = 0, \forall i \), in the autarkic equilibrium.
3 The United States and China

We are interested in the joint behavior of the current account and housing prices in the global economy and how this behavior is affected by differences in growth and financial development. To this end, we run two experiments. In the first experiment we focus on the United States and China; initially the two countries are in financial autarky and then they become financially integrated. We analyze the dynamics of house prices and the current at and after the two countries open up to trade.

We model China as a country with lower financial development but higher growth relative to the United States. During this period China became one of the main trading partners of the United States – U.S. trade weight for China increased from 0.05 to 0.173, making China the second U.S. trading partner in 2007, as shown in Figure 4. In this first experiment we study the implications for the current account and housing prices of financial integration and the role played by asymmetries in growth and in the level of financial development.

In this experiment we label the United States as country $H$ and China as country $L$; we assume that the two economies are initially in autarky and they unexpectedly open to trade. We identify China as an economy with lower financial development ($\theta^L < \theta^H$) and faster productivity growth ($g^L_Z > g^H_Z$) relative to the United States. As in Coeurdacier et al. (2015), we further assume that, before openness, the Chinese economy is scarce in capital and therefore in a transition to its autarkic steady state.

3.1 Calibration

Each period of time in the model is equivalent to 20 years. We use a three-period version of the model mainly for qualitative purposes and therefore assume differences in growth rates ($g^i_{Z,t}$) and in the degree of financial development ($\theta^i$), but keep all other parameters equal across countries.

We need to calibrate a number of parameters and exogenous processes in our model: the parameters associated with preferences ($\beta, \gamma, \sigma$), the downpayment requirement ($\theta^i$) and the productivity growth ($g^i_{Z,t}$) for each country $i$, the parameter associated with technology ($\alpha$), the population growth rate ($g_L$), the depreciation rate ($\delta$), and the relative productivity of young workers ($\varepsilon$).

We choose standard values for parameters common to United States and China and, later on, Japan. The discount factor is set to be $\beta = 0.97$ on an annual basis; the intertemporal elasticity of substitution $\sigma$ is set equal to 2; the depreciation rate of capital $\delta$ is calibrated at
0.1 per year; the share of non-housing consumption in the utility function is \( \gamma = 0.65 \); the capital share is set at \( \alpha = 0.28 \); and the relative productivity of young workers is \( \varepsilon = 0.33 \).

As for the country-specific parameters, we assume \( \theta^H = 0.8 \), which is equivalent to a loan-to-value ratio of 80\% in the US economy. For China, the loan-to-value ratio on new mortgages was capped at 0.8 starting in 2001. According to Bingxi and Lijuan (2007), residential mortgage loans increased almost 150-fold between 1997 and 2007 and mortgage loans to GDP rose from 0.2\% to 10.9\% of GDP over the same period; the average downpayment on active mortgages in large Chinese cities was 37.4\% in 2007. Although we have no direct data before 1997, the evidence suggests that loan-to-value ratios were extremely low until the mid 1990s and grew rapidly after that.\(^3\) We interpret this evidence as suggesting that market financing of housing purchases was very limited in China in 1990 and then it gradually improved. For this reason we set \( \theta^L \) equal to 0.02, as Coeurdacier et al. (2015). As long as \( \theta^L < \theta^H \), a higher value for \( \theta^L \) does not change our results qualitatively.

China is in transition to its autarkic steady state when financial integration occurs. We need to calibrate the initial level of capital stock of the Chinese economy, namely the level of capital in 1970. From the Penn World Tables in Feenstra et al. (2015) we compute the output of China relative to the United States, which is equal to 20\% in 1970, 32\% in 1990, and 85\% in 2010. We also need to specify the path of the productivity growth in China and the United States. We assume that U.S. productivity growth, \( g^H \), remains constant over time at 1.5\% while Chinese productivity growth \( g^L \) is 4.5\% between 1970 and 2010 and 1.5\% afterwards. These figures are in line with productivity growth rates from the Penn World Tables.

### 3.2 Results

The results of this experiment are plotted in Figures 8 and 9. Each period of time in the model is equivalent to 20 years. Period \( t = -1 \) in the graphs correspond to the autarkic equilibrium of each economy. The period of time between \( t = -1 \) and \( t = 0 \) corresponds to the period between 1970 and 1990, and the period between \( t = 0 \) and \( t = 1 \) corresponds to the period from 1990 to 2010. At period \( t = 0 \) the two countries become unexpectedly integrated; country \( L \) has faster technological growth than \( H \) (\( g^L > g^H \)) for periods \( t = -1, 0, 1 \); starting in period \( t = 2 \), which corresponds to the year 2030, country \( L \)'s productivity growth rate becomes the same as in country \( H \).

Figures 8 and 9 reports the path of several macroeconomic variables for the United States

\(^3\)For further evidence that housing financing options did not exist until the early 2000s, see Yang and Chen (2014).
Figure 8: The United States and China: Financial Integration
(solid blue) and China (dashed red); the vertical line in period $t = 0$ is financial integration. All variables are divided by the productivity level and therefore displayed in efficiency units. Under autarky ($t = -1$) China has higher marginal productivity of capital and interest rate $r$ due to the fact that technological growth is higher relative to the United States and capital is scarce. At time $t = 0$ the two countries open up to trade and capital flows from the United States to China until the marginal productivity of capital is equal in the two countries. Hence, the interest rate goes up in the United States but falls in China – the world interest rate is between the two autarky rates; investment as a percentage of GDP increases in China and falls in the United States when openness occurs and capital (displayed in efficiency units in the graph) also moves in opposite direction in the two locations. A capital inflow implies a current account deficit: from $t = -1$ to 0 China experiences a current account deficit and the United States a surplus.

The increase in the interest rate in $t = 0$ in the United States plays an important role in the following dynamics. U.S. young individuals borrow less ($a_y$ increases) and buy fewer houses; financial intermediaries reduce investment in rental housing because of higher returns on capital offered overseas. As a result the growth rate of house prices falls in the United States. Since house prices are low, middle-aged individuals purchase houses. These agents are not subject to a borrowing constraint and their purchases of a durable good are driven by arbitrage – when the price is below its steady-state level, they purchase as emphasized by Barsky et al. (2007). This is to say that the house price dynamics is explained by the behavior of borrowing-constrained agents, who react to an increase in the real interest rate.

The interest rate falls in China in $t = 0$ and, more importantly, it falls more than under autarky. Along the transition in autarky, capital in China grows faster than total labor productivity, thereby reducing the autarky interest rate. Once China opens up, capital flows in from the United States, thereby reducing the marginal productivity more than under autarky. Another way to see this is the following. In the integrated economy, the world interest rate is the weighted average of the autarky rate of the United States and China, where the former country contributes a larger weight due to its larger size. Since the U.S. autarky rate is lower than China’s one, $r$ is lower for China under integration in period $t = 0$. The reduction in the interest in China leads to a large increase in rental investment that raises house prices. Young agents would like to increase their borrowing but are constrained by a tight borrowing limit; moreover the increase in house prices is so pronounced that housing demand falls (both $h_m$ and $h_o$ fall).

The period between $t = 0$ and 1 in the model corresponds to the years between 1990 and
Figure 9: The United States and China: Financial Integration
2010. It is in this period that large global imbalances emerged, the world real interest rate fell and the United States experienced fast growth in house prices – see Figure 1 and 5. Our model predicts a fall in the real interest rate between $t = 0$ and 1. Several factors drive the world interest rate. High productivity growth raises the marginal productivity of capital in China; on the other hand, the capital inflow at $t = 0$ reduces it. The latter effect dominates the former. Since China is growing at a faster rate than the United States and its relative output growing, the world interest rate reflects productivity developments in China in an increasing manner and $r$ falls. The falling interest rates lead to an increase in investment and a fall in saving in the United States that generate a current account deficit.

Because of lower interest rates, young individuals borrow more in the United States at $t = 1$ ($a_y$ falls) and buy more houses ($h_m$ goes up). The growth rate of house prices accelerates in the United States. Wages fall at $t = 1$ in the United States due to the reduced stock of domestic capital after financial integration; this reduces labor income of working individuals. Middle-aged agents must also pay high interest payments on the debt taken in period 0. This negative income shock together with high house prices leads to a cut in consumption and housing demand.

At $t = 1$ the interest rate in China continues to fall but at a smaller rate than in the previous period. Rental housing investment falls, slowing down the rate of growth of house prices. The behavior of Chinese economy in this period is driven by important income effects. The young and middle-aged experience a positive income effect stemming from a large increase in wages following integration. As a result, their housing demand increases as well as saving by the middle-aged households. Old households suffer a negative income effect as they sell a smaller housing stock ($h_o$ falls at $t = 0$) at relatively low house prices; as a result, old-age consumption falls. These forces contribute to a large increase in aggregate saving and a current account surplus.

The behavior of the interest rate is key to explain the rent-to-house-price ratio. When the interest rate falls, rental housing investment becomes more attractive. As rental investment $H^f$ goes up, house prices grow while future rent falls. Hence, a fall in real interest rates is accompanied by a reduction in the rent-to-house-price ratio.

In our model housing prices grow at the same rate as technology along the balanced growth path. Since China has higher technological growth until period $t = 1$, its house price growth is also higher until $t = 1$. In the long run housing prices grow at the same rate in the United States and China. The dynamics of house prices at the beginning of our experiment is affected by three factors. First, the borrowing constraint on house purchases. This constraint affects
house purchases \( h_m \), which in turn is the main driver of house prices in the United States. Second, growth and capital differentials in an integrated world. The interest rate changes driven by these differentials cause current account imbalances. Third, the OLG structure. Interest rate changes and temporary growth differentials can be important determinants of national saving and current account if we allow for heterogeneity across cohorts.

### 3.3 Main Determinants

![Graphs of various economic indicators over time](image)

Figure 10: The US and China. Financial Integration (Growth differentials only)

There are three asymmetries between the Chinese and the U.S. economy in our model: higher temporary productivity growth; less financial development; capital scarcity. We are
interested in understanding the contribution of each of this asymmetry to our results. To this end, we run three counterfactual experiments and consider one asymmetry at the time.

The first counterfactual experiment analyzes the predictions of the model after openness when there are only temporary productivity differentials while both economies are equally financially developed and there is no capital scarcity in China. The results of this first experiment are plotted in Figure 10. The predictions of our benchmark model are maintained, thereby suggesting that growth differential is a fundamental factor behind our findings. Equal financial development means that Chinese young individuals can borrow more; as a result national saving is lower and the stock of capital in the long run is lower. House prices grow at a lower rate in this experiment. House prices grow at the same rate as capital; capital scarcity in China therefore raises housing price growth.

In the second counterfactual experiment we assume that the two countries have different levels of financial development but there is no capital scarcity in China and no temporary productivity differentials relative to the United States. The results of this experiment are plotted in Figure 11. The current account, saving, investment, and house prices all move in the opposite direction relative to the benchmark. Lower productivity and higher initial capital reduce the autarky interest rate in China relative to our benchmark scenario. Moreover, China is still financially less developed; this implies that young households can borrow very little, so that saving is higher and the marginal productivity of capital lower relative to the United States. All these factors contribute to higher marginal productivity of capital and autarky rate in the United States relative to China. When the two countries become financially integrated, capital moves from China to the United States and the interest rate falls in the United States. The world interest rate remains flat after 1990 and housing price growth falls between \( t = 0 \) and 1 in the United States. All these results are highly counterfactual. Hence, different levels of financial development cannot explain alone the path of house prices and current account imbalances described earlier.

In the last counterfactual experiment China is capital scarce but both economies have the same level of financial development and identical productivity growth rate (1.5% on an annual basis and in all periods). The results of this experiment are plotted in Figure 12. The qualitative behavior of the current account, saving, investment, and house price growth is similar to that of our benchmark model; the quantitative effects, however, are far smaller. Capital scarcity in China makes the initial marginal productivity of capital higher relative to the United States; hence, the interest rate goes up for the U.S. economy at \( t = 0 \). In China, capital increases at a faster rate than technology throughout the transition; this implies a
Figure 11: The US and China. Financial Integration (Financial Development differentials only)
Figure 12: The US and China. Financial Integration (Capital Scarcity differentials only)
downward path for the world interest rate after integration. Since financial development is lower in China, this guarantees an increase in aggregate saving driven by the middle aged and the old. The key mechanisms are therefore still in place; the absence of productivity growth differential attenuates both effects, thereby reducing the quantitative impact.

We also run the counterfactual experiment of fixing house prices in each economy at its autarky level (the steady state house price level for the United States and, in the case of China, the house price level before financial integration). In this experiment house prices are kept constant while supply is flexible and meets demand. We find that house prices amplify current account imbalances by 10% both in China and in the United States.

4 The United States and Japan

We are interested to understand the joint behavior of the current account and housing prices and how this behavior is affected by differences in growth when trading partners have the same level of financial development. To this end, we calibrate our model the United States and Japan, which we assume to be already financially integrated, and we analyze the effects of an unanticipated recession in the Japanese economy.

4.1 Computational Experiment

This experiment focuses on the Japanese recession that started in 1990. At this time, Japan and the United States were already financially integrated and Japan was one of the main trade partners of the U.S. economy as shown in Figure 4. The Japanese economy, like the U.S. one, is characterized by a high level of financial development as shown in Figure 6. More specifically, we assume that the two economies (\(H\) for the United States, and \(L\) for Japan) are already open to trade when the \(L\)-economy suffers an unexpected and temporary recession \((g_{Z,0}^L < g_{Z,-1}^L)\). This is to say, we assume that productivity growth falls in Japan between \(t = 0\) and \(1\) in the model, which corresponds to the period between 1990 and 2010. We keep the same assumptions for the U.S. economy as in the previous experiment and model Japan as equally financial developed \((\theta^L = \theta^H)\) as the United States.

4.2 Calibration

In our model, we need to calibrate the parameters associated with preferences \((\beta, \gamma, \sigma)\), the down-payment requirement \((\theta^i)\) for each country \(i\), and parameters associated with technology
(\alpha), as well as the population growth rate \((g_L)\), the depreciation rate \((\delta)\), and the relative productivity of young workers \((\varepsilon)\). We also need series for productivity growth rates \((g^i_{Z,t})\) in each country \(i\) and for each \(t\). Since our aim is to illustrate the importance of differences in growth rates \((g^i_{Z,t})\), we keep all other parameters equal across countries.

We choose standard values for parameters common to all countries. The discount factor is set to be \(\beta = 0.97\) on an annual basis; the intertemporal elasticity of substitution is \(\sigma = 2\); the depreciation rate of the capital stock is \(\delta = 0.1\) per year; the share of non-housing consumption in the utility function is \(\gamma = 0.65\); the capital share is set at \(\alpha = 0.28\); and the relative productivity of young workers is \(\varepsilon = 0.33\).

We assume that, before the recession, the Japanese economy is at the same balanced growth path as the U.S. economy and the current account balance is equal to zero in both countries. This is consistent with the evidence shown in Figure 1 for the year 1990. We need to specify the path of the productivity growth rate in Japan, \(\{g^L_{Z,t}\}_{t=1}^T\), where \(T\) is the period in which both economies reach the new integrated steady-state equilibrium; we assume that U.S. productivity growth, \(g^H_Z\), remains constant over time. We assume that both economies have an equal level of financial development, \(\theta^H = \theta^L\), as shown in Figure 6; we set the LTV ratio \(\theta\) to 0.8.

We use the Penn World Tables and calibrate productivity growth rate to match a relative output of Japan of 40% in 1990, and 30% in 2010; this implies an annual rate of productivity growth of 0.015% between the two dates. We assume that the long-run productivity growth rate (in annual terms) is the same in the United States and Japan and equal to 1.5% and that the United States grow at that rate throughout the experiment.

4.3 Results

Each period of time in the model is equivalent to 20 years. Period \(t = -1\) in the graphs correspond to the equilibrium where both economies are integrated with zero current account and productivity growth equal to its long-run level, namely the two economies are on their balanced growth paths. The period of time between \(t = -1\) and \(t = 0\) corresponds to the period between 1990 and 2010. At period \(t = 0\) the \(L\) economy experiences an unexpected temporary recession; at this time, country \(L\) starts growing at a slower growth rate \((g^L_Z < g^H_Z)\) for one period. From \(t = 1\) on the growth of Japan goes back to the long-term one.

Figures 13 and 14 show the results of this experiment. At \(t = -1\) the United States and Japan are at the same steady state since the two economies have identical financial depth and growth rate. At \(t = -1\), before the recession takes place, the two economies are already
Figure 13: The US and Japan. Recession in Japan
financially integrated and therefore have the same interest rate.

All decisions from $t = 0$ onward are taken knowing the path of productivity. During the unexpected recession in Japan ($t = 0$), the marginal productivity of capital is lower in Japan relative to the United States. Capital at $t = 0$ was decided and installed at $t = -1$; it takes one period to change the capital stock. Since the level of productivity will be lower at $t = 1$ in Japan relative to the United States and the two economies are already open, capital flows from Japan to the United States in period $t = 0$. This increases investment in the United States and decreases it in Japan. The world real interest rate is the average productivity of capital in the two countries, hence $r$ falls in period $t = 0$. The factor that determines the interest rate evolution is the lower productivity growth in Japan that decreases the marginal productivity of capital thus decreasing the international interest rate.

Because of lower interest rates, young agents buy more houses in the United States ($h^H_m$ goes up at $t = 0$) and borrow more ($a^H_y$ goes down at $t = 0$). However, in Japan wages go down due to the fall in productivity. Thus, young households take advantage of lower interest rate and borrow more ($a^L_y$ goes down at $t = 0$); they buy fewer houses in Japan ($h^L_m$ goes down at $t = 0$) and use the increase in loans in order to smooth consumption.

Temporary lower productivity in Japan generates a capital inflow in the United States that reduces its marginal productivity of capital. Financial intermediaries increase rental housing investment, which reduces the rental rate of return so as to equalize the lower marginal productivity of capital. This implies that the rental price in the United States ($r^{f,H}$) must go down as well. The rental price is the future return from rental investment. By increasing rental housing stock ($H^{f,H}$), $r^{f,H}$ goes down (next period) and house prices ($q^H$) goes up (this period). The increase in the stock of rental housing in the United States ($H^{f,H}$) decided at $t = 0$ implies an increase in services from renting for young ($f^H_y$ goes up).

Since Japan is an open economy, the interest rate is between the U.S. and Japanese autarky interest rates, which means that capital in Japan decreases more than if it were a closed economy. This also entails that the decrease in the interest rate is smaller than the decrease in productivity in Japan. From the non-arbitrage condition of financial intermediaries one can see that the total future return from rental, $R^{f,L}$ needs to increase. In fact, the rental price $r^{f,L}$ increases. Financial intermediaries in Japan reduce the rental housing stock at $t = 0$, thereby raising $r^{f,L}$ (next period) and putting downward pressure on house prices – $q^L$ goes down (this period). Hence $H^{f,L}$ goes down at $t = 0$ as does $f^L_y$.

In the United States, house price growth accelerates at $t = 0$. There are two forces behind this increase. First, the increase in rental housing investment discussed earlier; second, an
Figure 14: The US and Japan. Recession in Japan
increase in demand by young households ($h_m^H$ goes up). Elevated house prices cause housing demand by middle-age individuals to fall ($h_o^H$ goes down at $t = 0$). House prices go down in Japan at $t = 0$. The drivers are the decrease in demand by young agents ($h_m^L$ goes down) stemming from a fall in wages; and the reduction in rental housing investment discussed earlier. The only group that increases housing demand in Japan are the middle-aged ($h_o^L$ goes up at $t = 0$). Middle-age individuals are consumption smoothers and arbitrageurs, as they are not subject to a borrowing constraint. If house prices are below steady state level, they will purchase more housing all else equal; vice versa, if the price of houses is above steady state, they will purchase less.\footnote{See Barsky et al. (2007) for the intuition.}

As said earlier, $a_y^H$ goes down in the U.S. economy at $t = 0$ because young takes advantage of lower interest rates. Middle-aged have two effects: first, a negative income effect because they must pay back the debt taken when young at the steady-state interest rate, which is higher than the current interest rate; and second, a negative substitution effect coming with lower interest rate, which makes them save less. Middle-aged can also save by buying houses but since $q^H$ goes up at $t = 0$, they end up reducing $h_o^H$. As a result, middle-aged raise their saving. Aggregate saving is the sum of more borrowing by young and more saving by middle-aged; as a result, aggregate saving ($S^H$) does not change much in period $t = 0$ in the United States.

In Japan, youth borrowing increases ($a_y^L$ goes down) at $t = 0$ due to low wage income and low interest rates. Complementarity between $c_m^L$ and $h_m^L$ induces middle-aged households to decrease consumption ($c_m^L$ and then increase $h_o^L$ and $a_m^L$. Even though $r$ is lower, middle-aged prefer to save by purchasing houses this gives them utility and current prices are low. Higher saving by middle-aged is less than compensated by lower saving of young. As a result national saving ($S^L$) increases at $t = 0$ in Japan.

At period $t = 0$, the saving rate in the United States remains almost constant while investment increases, thus generating a current account deficit. Also, housing prices in the United States grow at a rate higher than along the balanced growth path. The model therefore predicts that, as a result of a temporary recession in the Japanese economy, housing price growth accelerates and a current account deficit emerges in the U.S. economy, which is in accordance with the empirical evidence.

The saving rate in Japan between period $t = -1$ and $t = 0$ increases, driven by an increase in deposits by the middle-aged group. During this period, Japan runs a current account surplus and experiences a negative housing prices growth rate, as shown in the data. After an
unanticipated and temporary recession in the Japanese economy, house prices increase in the economy running a current account deficit and decrease in the one running a current account surplus.

4.3.1 Open Economy Vs. Closed Economy: the effect on Housing Prices

To highlight the importance of openness in our previous results, we carry out a counterfactual experiment and show how the same recession would impact housing prices if Japan had been a closed economy. Figure 15 displays the dynamics of a subset of variables for the open economy (dashed red line labeled “L-OPEN”), which is the same as before, and for the closed economy (solid red line labeled “L-CLOSED”) after the same unexpected recession.

There are important differences in the behavior of housing prices, the interest rate, and the rent-to-price ratio. In the open economy, the interest rate is the weighted average of the autarky interest rates. Thus, a recession in closed-economy Japan decreases the interest rate by more relative to the open-economy case. In the closed economy house prices grow more than productivity. Figure 15 displays the rate of growth of house prices ($q_L$ (unnormalized) Growth Rate) as well as house prices normalized by productivity $\hat{q}$, namely $q_L/Z_L$. Intuitively, in the closed economy the rental return must fall more relative to the open economy because the reduction in the interest rate is larger. This is to say that financial intermediaries raise rental housing investment by more in the closed economy, thereby causing house prices to go up ($q_L$ positive and $\hat{q}$ goes up at $t = 0$) and the next period rental price to go down ($r^f/q$ falls at $t = 0$). Hence, house price growth is positive in closed-economy Japan and the rent-to-price ratio falls in $t = 0$. This is not the case in the open economy, where the productivity slowdown can be alleviated by a capital outflow. In the closed economy houses are a more attractive asset than in the open economy.

The rent-to-price ratio moves in opposite directions in the closed economy relative to the open one. As data show, the behavior of the rent-to-price ratio from the 90’s to 2010 is consistent with our open-economy case.

5 Implications for the Real Exchange Rate

In our model economy the non-tradable sector is the housing sector. In order to compute the real exchange rate of our economy we proceed as in the data and use our rental price as the rent of primary residence and owners’ equivalent rent. This is to say that the price of non-tradable goods in our economy is the rental price. We assume that the law of one price
Figure 15: The US and Japan. Recession in Japan (Open versus Closed Economy)
holds for the tradable goods. Given our utility function, the relevant expression for the real exchange rate in country $i$ relative to country $j$ at time $t$, $RER_{t}^{i/j}$, becomes:

$$RER_{t}^{i/j} = \left( \frac{r_{f,i}}{r_{f,j}} \right)^{1-\gamma}$$

where $1 - \gamma$ is the share of housing services in the utility function.

Figure 16 shows the real exchange rate in China and Japan relative to the United States, computed as the consumer price index of China and Japan relative to the consumer price index in the United States in terms of the foreign currency. While in Japan there is a clear depreciation of the Yen in China there is not a clear pattern along the period. However, after 2000, the year in which China became a more important trade partner of the US, there is a change in the trend and the Renminbi starts to appreciate relative to the US Dollar. We are aware that China has been intervening the exchange market by buying dollars to keep its nominal exchange rate artificially depreciated and this distorts the evolution of the real exchange rate. Then, to better compare our model to the Chinese data we compute the counterpart of the real exchange rate in our model for China by using the data for the rental price in the US and China and plug it into our equation for the real exchange rate. What we get is plotted in Figure 17. In this case the data shows that this constructed measure of the real exchange rate in China has appreciated along the period.

In Figures 18 and 19 we plot the prediction of our model for the bilateral real exchange rate of the United States relative to China and Japan. Our model predicts the depreciation of the Yen and also the appreciation of our constructed measure of real exchange rate in China.

## 6 Conclusion

In this paper we develop a two-country model with housing to study the joint behavior of housing prices and the current account. Our goal is to analyze the effects of financial integration and asymmetries in productivity developments, capital levels, and the degree of financial development. We focus on the period from 1990 to the Great Recession; during this time span China and Japan were important trade partners of the U.S. economy.

Our model is able to qualitatively replicate the pattern of house price growth and current account balances of the United States and China by assuming that the latter has temporarily faster productivity growth, low initial capital and a less developed financial market than the United States. Among these features, the differential in productivity growth plays by far the most important role, as it generates falling real interest rates and strong income effects in
Figure 16: Bilateral Real Exchange Rate. Data from IFS.

Figure 17: Constructed Bilateral Real Exchange Rate (China/US). Data: National Statistics.
China. Our model can also replicate the current account surpluses and falling house prices in Japan vis-a-vis the deficits and rapidly growing house prices in the United States. Once again, the driving force in these dynamics is a temporary productivity differential, namely a recession in Japan. Moreover, the implications of our model for the real exchange rate are in line with the data.

We speculate that our qualitative results will not be affected by extending the model to more countries. Allowing the United States to trade contemporaneously with Japan and China is likely to amplify its current account deficit and further accelerate the growth of house prices. At the same time, allowing China to trade with Japan, another financially developed country with a lower autarky interest rate and temporarily lower productivity growth, will not change the effects on China.

Extending the model to more periods is unlikely to change our results. For the United States - China case, the experiment with growth differential only and equal degrees of financial development delivered qualitatively identical results. We speculate that extending the model to a longer life cycle and reducing the proportion of the population subject to the borrowing constraint will not change our findings.

For simplicity we have set many parameter values equal across countries. Bringing the calibration closer to the data will help us matching the empirical evidence more closely from a quantitative point of view. This is a task that we leave to future extensions.
References


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Appendix A: Normalization

De-trending

Let \( \hat{x}_t = \frac{\ddot{x}_t}{Z_t} \) denote the “detrended” variable for any \( x_t \) (for example, \( \hat{\dot{w}}_t = \frac{w_t}{Z_t} \)). Also, let \( \tilde{\beta}_1^t = \beta(1 + g_{Z,t})^{\gamma(1-\sigma)} \), and \( \tilde{\beta}_2^t = \beta^2((1 + g_{Z,t})(1 + g_{Z,t+1}))^{\gamma(1-\sigma)} \).

Reformulation of the household problem

We can re-write household optimization problem as:

\[
\max \left( \hat{c}_y^{\gamma} f_{y,t}^{1-\gamma} \right)^{1-\sigma} + \tilde{\beta}_1^t \left( \hat{c}_m^{\gamma} h_{m,t+1}^{1-\gamma} \right)^{1-\sigma} + \tilde{\beta}_2^t \left( \hat{c}_o^{\gamma} h_{o,t+2}^{1-\gamma} \right)^{1-\sigma}
\]

s.t. \( \hat{c}_{y,t} + \hat{r}_t^{f} f_{y,t} + \hat{q}_{h_{m,t+1}} + \hat{a}_{y,t+1}(1 + g_{Z,t+1}) = \hat{\dot{w}}_t \varepsilon, \)

\( \hat{c}_{m,t+1} + \hat{q}_{h_{o,t+2}} + \hat{a}_{m,t+2}(1 + g_{Z,t+2}) = \hat{\dot{w}}_{t+1} + \hat{q}_{h_{m,t+1}} + \hat{a}_{y,t+1} (1 + r_{t+1}), \)

\( \hat{c}_{o,t+2} = \hat{q}_{h_{o,t+2}} + \hat{a}_{m,t+2}(1 + r_{t+2}), \)

\( \hat{a}_{y,t+1} \geq -\theta \hat{q}_{h_{m,t+1}} \frac{1}{1 + g_{Z,t+1}}, \quad \hat{a}_{m,t+2} \geq -\theta \hat{q}_{h_{o,t+2}} \frac{1}{1 + g_{Z,t+2}}. \)

We need the “t” subscripts only for the transition periods, and we can drop them when looking at the BGP solution (where all the relevant variables will remain constant).

The no-arbitrage condition for the financial intermediaries becomes:

\[ 1 + \hat{r}_t^k = \frac{(\hat{r}_t^{f} + \hat{q}_{t+1})(1 + g_{Z,t+1})}{\hat{q}_t}. \]

First-order and market-clearing conditions

Let \( k_t = \frac{K_t}{Z_t L_t} \). Assuming (which we need to check) that borrowing limit is binding for the young, but not binding for the middle-aged, equilibrium is described by the following system of optimality and market-clearing conditions:

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\[ [\hat{c}_{y,t}] : \gamma \left( \frac{f_{y,t}}{\hat{c}_{y,t}} \right)^{1-\gamma} (\hat{c}_{y,t} f_{y,t}^{1-\gamma})^{-\sigma} = \lambda_{y,t}, \]
\[ [f_{y,t}] : (1 - \gamma) \left( \frac{\hat{c}_{y,t}}{\hat{f}_{y,t}} \right)^{\gamma} (\hat{c}_{y,t} f_{y,t}^{1-\gamma})^{-\sigma} = \lambda_{y,t} \hat{r}_{t}^{f}, \]
\[ [\hat{c}_{m,t+1}] : \hat{\beta}_{t}^{1} \gamma \left( \frac{h_{m,t+1}}{\hat{c}_{m,t+1}} \right)^{1-\gamma} (\hat{c}_{m,t+1} h_{m,t+1}^{1-\gamma})^{-\sigma} = \hat{\beta}_{t}^{1} \lambda_{m,t+1}, \]
\[ [\hat{c}_{o,t+2}] : \hat{\beta}_{t}^{2} \gamma \left( \frac{h_{o,t+2}}{\hat{c}_{o,t+2}} \right)^{1-\gamma} (\hat{c}_{o,t+2} h_{o,t+2}^{1-\gamma})^{-\sigma} = \hat{\beta}_{t}^{2} \lambda_{o,t+2}, \]
\[ [\hat{a}_{y,t+1}] : \hat{\beta}_{t}^{1} \lambda_{m,t+1} (1 + r_{t+1}) + \mu_{y,t} = \lambda_{y,t} (1 + g_{Z,t+1}), \]
\[ [\hat{m}_{m,t+1}] : \hat{\beta}_{t}^{1} \mu_{y,t} + \hat{m}_{m,t+1} (1 + r_{t+1}) = \lambda_{y,t} \hat{q}_{t}, \]
\[ [\hat{h}_{o,t+2}] : \hat{\beta}_{t}^{2} \mu_{y,t} + \hat{h}_{o,t+2} (1 + r_{t+2}) = \lambda_{y,t} \hat{q}_{t}, \]
\[ [\hat{B}_{y,t}] : \hat{c}_{y,t} + \hat{r}_{t}^{d} f_{y,t} + \hat{q}_{t} h_{m,t+1} + \hat{a}_{y,t+1} (1 + g_{Z,t+1}) = \hat{w}_{t} \varepsilon, \]
\[ [\hat{B}_{m,t+1}] : \hat{c}_{m,t+1} + \hat{q}_{t+1} h_{m,t+2} + \hat{m}_{m,t+1} (1 + g_{Z,t+2}) = \hat{w}_{t+1} + \hat{q}_{t+1} h_{m,t+1} + \hat{a}_{y,t+1} (1 + r_{t+1}), \]
\[ [\hat{B}_{o,t+2}] : \hat{c}_{o,t+2} = \hat{q}_{t+2} h_{o,t+2} + \hat{a}_{m,t+2} (1 + r_{t+2}), \]
\[ [\hat{B}_{L_{y,t}}] : \hat{a}_{y,t+1} = -\hat{q}_{t} h_{m,t+1} \left( \frac{1}{1 + g_{Z,t+1}} \right) \]
\[ [MC_{K_{y}}] : \phi_{y} \hat{a}_{y,t+1} + \phi_{m} \hat{m}_{m,t+1} = (1 + g_{L}) \left( k_{t+1} + \frac{\hat{q}_{t}}{1 + g_{Z,t+1}} \phi_{y} f_{y,t+1} \right), \]
\[ [MC_{H_{y}}] : \phi_{y} h_{m,t+1} + \phi_{m} h_{o,t+1} + \phi_{y} (1 + g_{L}) f_{y,t+1} = (1 + g_{L}) \bar{n}, \]
\[ [FOC_{L_{t}}] : \hat{w}_{t} = (1 - \alpha) k_{t}^{\alpha}, \]
\[ [FOC_{K_{t}}] : \hat{r}_{t}^{k} = \alpha k_{t}^{\alpha - 1} - \delta, \]
\[ [NoArb_{t}] : (1 + r_{t}^{k}) \left( \frac{\hat{r}_{t+1}^{f} + \hat{q}_{t+1}(1 + g_{Z,t+1})}{\hat{q}_{t}} \right) \]

This gives us 17 equations in 17 unknowns:

\[ \hat{c}_{y,t}, \hat{c}_{m,t+1}, \hat{c}_{o,t+2}, f_{y,t}, h_{m,t+1}, h_{o,t+2}, \hat{a}_{y,t}, \hat{a}_{m,t+1}, \lambda_{y,t}, \lambda_{m,t+1}, \lambda_{o,t+2}, \hat{w}_{t}, \hat{r}_{t}^{f}, \hat{r}_{t}^{k}, \hat{q}_{t}, \mu_{y,t}, k_{t+1}. \]

**Integrated (open trade) equilibrium**

Market-clearing condition for capital (savings) changes to:
With growth

Let \( \Gamma_t^H = \frac{Z_t^H(\ell^H L_{m,t} + L_{m,t}^H)}{\sum_{i=H} Z_t^i(\ell^i L_{y,t} + L_{y,t}^i)} \) be the share of “effective labor” in country \( H \) to the total world effective labor in period \( t \). Then the above market-clearing condition for savings can be re-written as:

\[
\Gamma_t^H (\varphi_y^H \hat{a}_{y,t+1} + \varphi_m^H \hat{a}_{m,t+1}(1 + g_{Z,t+1})) + (1 - \Gamma_t^H) (\varphi_y^L \hat{a}_{y,t+1}(1 + g_{Z,t+1}) + \varphi_m^L \hat{a}_{m,t+1}(1 + g_{Z,t+1})) \\
= \Gamma_t^H (1 + g_L) (\hat{k}_{t+1} + \hat{q}_t^H \varphi_y f_t^H) + (1 - \Gamma_t^H) (1 + g_L) (\hat{k}_{t+1} + \hat{q}_t^L \varphi_y f_t^L)
\]

since \( \hat{k}_{t+1} = \hat{k}_{t+1} \) with open capital market, in which we must have \( r_{t+1}^k = r_{t+1}^L \).

**Appendix B: Definitions**

The **net foreign asset position** at the end of period \( t \) is defined as:

\[
NFA_{t+1} = L_{y,t} a_{y,t+1} + L_{m,t} a_{m,t+1} - K_{t+1} - q_t H_{t+1}^f
\]

or in terms of GDP:

\[
\frac{NFA_{t+1}}{Y_{t+1}} = \frac{L_{y,t} a_{y,t+1}}{Z_{t+1} L_{t+1} k_{t+1}^{\alpha}} + \frac{L_{m,t} a_{m,t+1}}{Z_{t+1} L_{t+1} k_{t+1}^{\alpha}} - \frac{K_{t+1}}{Z_{t+1} L_{t+1} k_{t+1}^{\alpha}} - \frac{q_t H_{t+1}^f}{Z_{t+1} L_{t+1} k_{t+1}^{\alpha}} = \\
\frac{1}{(1 + g_L)(1 + g_{Z,t+1})k_{t+1}^{\alpha}} ((1 + g_{Z,t+1}) \varphi_y \hat{a}_{y,t+1} + (1 + g_{Z,t+1}) \varphi_m \hat{a}_{m,t+1} - \\
(1 + g_L)(1 + g_{Z,t+1})k_{t+1} + (1 + g_L)q_t \varphi_y f_{y,t+1})
\]

The **current account position** in period \( t \) is defined as:

\[
CA_t = NFA_{t+1} - NFA_t
\]

or in terms of GDP:

\[
\frac{CA_t}{Y_t} = \frac{NFA_{t+1}}{Y_{t+1}} - \frac{NFA_t}{Y_t} = \frac{NFA_{t+1} Y_{t+1}}{Y_{t+1} Y_t} - \frac{NFA_t}{Y_t} = 
\]
\[(1 + g_{Z,t+1})(1 + g_L) \left( \frac{k_{t+1}}{k_t} \right)^\alpha \frac{NA_t}{Y_{t+1}} - \frac{NA_t}{Y_t} \]

**Investment** in period \( t \) is given by:

\[ I_t \equiv K_{t+1} - (1 - \delta)K_t, \]

or in terms of GDP:

\[ \frac{I_t}{Y_t} = \frac{Y_{t+1} K_{t+1} - (1 - \delta) K_t}{Y_t} = \frac{Z_{t+1} L_{t+1} k^{\alpha}_{t+1}}{Z_t L_t k^{\alpha}_t} k^{1-\alpha}_{t+1} - (1 - \delta) k^{1-\alpha}_t = \]

\[ = (1 + g_{Z,t+1})(1 + g_L) \left( \frac{k_{t+1}}{k_t} \right)^\alpha k^{1-\alpha}_{t+1} - (1 - \delta) k^{1-\alpha}_t \]

**Consumption** in period \( t \) is given by:

\[ C_t = L_{y,t} c_{y,t} + L_{m,t} c_{m,t} + L_{o,t} c_{o,t}, \]

or in terms of GDP:

\[ \frac{C_t}{Y_t} = \frac{L_{y,t}}{L_t} c_{y,t} \frac{1}{k^{\alpha}_t} + \frac{L_{m,t}}{L_t} c_{m,t} \frac{1}{k^{\alpha}_t} + \frac{L_{o,t}}{L_t} c_{o,t} \frac{1}{k^{\alpha}_t} = \]

\[ = \frac{1}{k^{\alpha}_t} \left( \varphi_y \hat{c}_{y,t} + \varphi_m \hat{c}_{m,t} + \frac{\varphi_m}{(1 + g_L)} \hat{c}_{o,t} \right) \]

**Aggregate savings** in period \( t \) is given by:

\[ S_t = Y_t + r_t NFA_t - C_t, \]

or in terms of GDP:

\[ \frac{S_t}{Y_t} = 1 + r_t \frac{NFA_t}{Y_t} - \frac{C_t}{Y_t} \]

**Relative output of the more constrained economy** in period \( t \) is given by:

\[ \frac{Y_t^L}{Y_t^H} = \frac{Z_t^L L_t^L (k_t^L)^\alpha}{Z_t^H L_t^H (k_t^H)^\alpha} = \frac{\sum_{i=H}^L (k_t^L)^\alpha}{\sum_{i=H}^L (k_t^H)^\alpha} \frac{(k_t^L)^\alpha}{(k_t^H)^\alpha} = 1 - \frac{\Gamma_t^H (k_t^L)^\alpha}{\Gamma_t^H (k_t^H)^\alpha} \]

**The level of saving of the young** in period \( t \) is given by:

\[ S_{y,t} = L_{y,t} \left( w_t \varepsilon - c_{y,t} - r_t^f f_{y,t} \right), \]

or in terms of GDP:
or in terms of GDP:

\[
S_{m,t} = L_{m,t} \left( w_t + r_t a_{y,t} - c_{m,t} \right),
\]

or in terms of GDP:

\[
S_{o,t} = L_{o,t} \left( r_t a_{m,t} - c_{o,t} \right),
\]

Aggregate savings in period \( t \) by aggregating individuals savings is given by:

\[
S_t = S_{y,t} + S_{m,t} + S_{o,t} = L_{y,t} w_t \varepsilon + L_{m,t} w_t + L_{m,t} r_t a_{y,t} + L_{o,t} r_t a_{m,t} - (L_{y,t} c_{y,t} + L_{m,t} c_{m,t} + L_{o,t} c_{o,t}) - L_{y,t} r_t^f f_{y,t} = \]

\[
= w_t \left( L_{y,t} \varepsilon + L_{m,t} \right) + r_t \left( L_{m,t} a_{y,t} + L_{o,t} a_{m,t} \right) - C_t - r_t^f L_{y,t} f_{y,t} = \]

\[
= w_t L_t + r_t \left( L_{y,t-1} a_{y,t} + L_{m,t-1} a_{m,t} \right) - C_t - r_t^f L_{y,t} f_{y,t} = \]

\[
= w_t L_t + r_t K_t + r_t NFA_t + \left( r_t q_{t-1} - r_t^f \right) L_{y,t} f_{y,t} - C_t = \]

\[
= Y_t - \delta K_t + r_t NFA_t + (q_t - q_{t-1}) L_{y,t} f_{y,t} - C_t,
\]

or in terms of GDP:

\[
\frac{S_t}{Y_t} = 1 + r_t \frac{NFA_t}{Y_t} - C_t \frac{Y_t}{Y_t} - \delta K_t \frac{Y_t}{Y_t} + (q_t - q_{t-1}) \frac{L_{y,t} f_{y,t}}{Y_t} = \]

\[
= 1 + r_t \frac{NFA_t}{Y_t} - C_t \frac{Y_t}{Y_t} - \delta K_t \frac{Y_t}{Z_t L_t k_t^\alpha} + (q_t - q_{t-1}) \frac{L_{y,t} f_{y,t}}{Z_t L_t k_t^\alpha} = \]

\[
= \text{Aggregate Savings in terms of GDP}
\]

\[
= \text{Aggregate Savings in terms of GDP}
\]

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\[
= 1 + r_t \left( \frac{NFA_t}{Y_t} - \frac{C_t}{Y_t} \right) - \delta k_t^{1-\alpha} + \left( \frac{(q_t - q_{t-1})}{Z_t} \right) \frac{L_{y,t} f_{y,t}}{k_t^{\alpha}} = \\
\text{Aggregate Savings in terms of GDP}
\]

\[
= 1 + r_t \left( \frac{NFA_t}{Y_t} - \frac{C_t}{Y_t} \right) - \delta k_t^{1-\alpha} + \left( \frac{\hat{q}_t - q_{t-1}}{(1 + g_{z,t-1})} \right) \frac{\varphi y_{f_{y,t}}}{k_t^{\alpha}} = \\
\text{Aggregate Savings in terms of GDP}
\]