# Jointly Optimal Taxes for Different Sources of Income 

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#### Abstract

We analyze a setting in which the government can impose different tax schedules on distinct types of income, such as wage, capital or self-employment income. Compared to standard optimal income tax formulas, optimal schedular income tax rates additionally depend on cross-elasticities between tax bases capturing fiscal externalities. We take an empirical approach and calculate income source specific optimal tax rates for Germany using rich panel data from administrative tax records. First, we provide evidence that responses to taxes differ significantly by income source. Second, we calculate marginal social welfare weights implicit in the German personal income tax schedule accounting for differences in the responsiveness across income types. We show that average welfare weights differ significantly between income sources. Using these estimates, we calculate optimal linear income tax rates for Germany. We find that optimal tax rates are significantly lower for wage income than for income from self-employment and capital. We discuss how our estimates vary with the size of fiscal externalities across tax bases.


JEL Classificiation: H21, H24, H26
Keywords: Optimal Taxation, Income Sources, Multidimensional Heterogeneity, Marginal Social Welfare Weights, Administrative Data

[^0]
## 1 Introduction

The optimal income tax literature following the seminal paper by Mirrlees (1971) has provided a variety of models incorporating socio-economic characteristics besides income in the optimal tax schedule following the "tagging idea" of Akerlof (1978). Potential criteria for the application of different tax schedules on distinguishable groups of taxpayers have been analyzed in various contexts such as age (Blomquist and Micheletto 2008; Bastani et al. 2013; Weinzierl 2011; Best and Kleven 2013), height (Mankiw and Weinzierl 2010), gender (Cremer et al. 2010; Alesina et al. 2011) or marital status (Boskin and Sheshinski 1983; Kleven et al. 2009). The rationale behind levying different taxes from taxpayers with different socio-economic characteristics can be based on two conditions: (i) The characteristics of the considered types of taxpayers have to be observable and immutable to a sufficient degree. (ii) The different types have to vary in their responsiveness to taxes such that the government can exploit the differential in the efficiency costs of taxation, or the welfare weights of the types differ to make redistribution across groups desirable.

In this paper, we derive a model in which the government taxes different sources of income on separate schedules and simulate it for the case of Germany. ${ }^{1}$ In light of the discussed requirements this attempt seems to be promising. First, the source of income is easy to observe for the government. In fact in most actual tax systems taxpayers have to assign the reported income to different categories when filing taxes such as wage income or capital gains. ${ }^{2}$ Second, it is a well documented observation that the responsiveness of reported income to taxation differs considerably across different types of income. In particular self-employed workers have a higher elasticity with respect to the net-of-tax rate than wage earners (Saez 2010; Kleven and Schultz 2014), indicating that the efficiency costs of taxation are higher in the first group. Third, the distribution of different income sources varies along the level of taxable income. In Figures 1a-b we display for Germany the fraction of taxpayers according to their main source of income (capital, wage, or self-employment) as a function of taxable income. While the major part of taxpayers below $€ 40,000$ primarily receives wage income, the fractions of self-employed and capital income earners surge dramatically after $€ 40,000$, and stabilize at a higher level. Figures $1 \mathrm{c}-\mathrm{d}$ show the average fractions of wage, capital and self-employment income as a function of

[^1]taxable income. While at the bottom of the income distribution wage income is the predominant income source, income from self-employment gains importance at a higher level of taxable income.

In light of this, the desirability of conditioning taxes on legally defined and distinguishable sources of income seems to be high. The literature on the taxation of different sources of income under the condition of joint optimality is relatively scarce. ${ }^{3}$ Rothschild and Scheuer (2013) and Scheuer (2014) provide models of optimal income taxes for entrepreneurs in economies where individuals can have different skills in different occupations. Ooghe and Peichl (2014) study the optimal (linear) taxation of different characteristics (which could be different types of income) in the case unobserved ability and unobserved preference heterogeneity. A major challenge in modeling jointly optimal taxes is to incorporate cross-effects between tax bases which is not considered in the standard optimal income tax model. However, inter- and intra-temporal income shifting (Auerbach and Slemrod 1997; Slemrod 1998; Gordon and Slemrod 2002; Kreiner et al. 2013; Harju and Matikka 2015; Kreiner et al. 2014) has been identified as an important component of the elasticity of taxable income (Saez et al. 2012). Piketty et al. (2014) provide a formula for the optimal top tax rate incorporating fiscal externalities due to income shifting.

In the first part of the paper, we set up an optimal tax model in the spirit of Diamond (1998) and Saez (2001) to derive jointly optimal tax schedules for different income sources taking into account fiscal externalities occurring through differentials in tax rates. In the second part, we empirically calculate optimal linear tax rates using administrative tax return data for Germany. To do so, we first estimate heterogenous elasticities for single income types, such as wage or capital income, with respect to the net-of-tax rate. Second, we calculate the distribution of welfare weights inherent in the current German income tax schedule. We then use our estimates of income type specific elasticities and welfare weights to simulate jointly optimal linearized taxes for wage, capital and self-employment income. We find that for reasonable values for the responsiveness of different income sources the optimal tax rate for wage income is much lower than for capital or self-employed income.

The remainder of this paper is structured as follows. In section 2 we derive and characterize the conditions for jointly optimal income taxes for different types of income. Section 3 explains the institutional background of the German tax law as well as the data used in this paper. In section 4 we present estimates of income type specific elasticities with respect to the net-of-tax rate. Section 5 presents an empirical investigation of (income type specific) marginal social welfare weights for the case of Germany before we turn to the simulation results in section 6.

[^2]We summarize the findings and conclude in section 7 .

## 2 Jointly Optimal Income Taxes for Different Sources of Income

We analyze jointly optimal linear income taxes for different sources of income in a model in the spirit of Diamond (1998) and Saez (2001). We use a sufficient statistics approach, in which behavioral responses are captured in terms of elasticities of tax bases with respect to the net-of-tax rates. Our model takes into account fiscal externalities occurring through differentials in tax rates across tax bases.

Tax units. Suppose there are $n$ distinguishable sources of income on which the government can levy a tax. A tax unit earns income $\hat{z}_{i}$ of type $i$ and reports an amount of $z_{i}$ to the government which is taxed according to the tax rate $\tau_{i}$. The difference between earned income $\hat{z}_{i}$ and reported income $z_{i}$ represents the amount of income shifted from or to tax base $i .{ }^{4}$ Tax payers are heterogenous in their consumption preferences and ability to generate and shift income. A tax unit's traits are captured by the vector $\mathbf{k}$ which is distributed across the population according to $F(\mathbf{k})$. Utility of an tax unit of type $\mathbf{k} \in \mathbf{K}$ reads

$$
U(\mathbf{k})=u^{\mathbf{k}}\left(c ; \hat{z}_{1}, \ldots, \hat{z}_{n} ; z_{1}, \ldots, z_{n}\right)
$$

where consumption $c=\sum_{i=1}^{n} z_{i}-\tau_{i} z_{i}$ and $\frac{\partial U}{\partial c}>0$ and $\frac{\partial U}{\partial z_{i}}<0$. We further assume that income shifting is costly, and thus $\frac{\partial U}{\partial z_{i}}<0$ if $z_{i}>\hat{z}_{i}$ and $\frac{\partial U}{\partial z_{i}}>0$ if $z_{i}<\hat{z}_{i}$. We denote the joint distribution of reported income by $H\left(z_{1}, \ldots, z_{n}\right)$ and the marginal distribution of income type $i$ by $H_{i}\left(z_{i}\right)$.
Behavioral responses of reported income to marginal tax rates are captured by the elasticity of tax base $j$ with respect to the net-of-tax rate of tax base $i$ defined by $\zeta_{j i}=\frac{\partial z_{j}}{\partial\left(1-\tau_{i}\right)} \frac{\left(1-\tau_{i}\right)}{z_{j}}$. For simplicity we assume that income effects and extensive margin responses are zero. We call $\zeta_{i i}$ the own-elasticity of tax base $i$ and $\zeta_{j i}$ the cross-elasticity of tax base $j$ with respect to the net-of-tax rate of tax base $i$. We quantify the interdependency of tax bases according to the parameter $\beta_{j i}=-\frac{\partial z_{j}}{\partial\left(1-\tau_{i}\right)} / \frac{\partial z_{i}}{\partial\left(1-\tau_{i}\right)}$ which can be interpreted as the share of the cross effect on tax base $j$ in the total effect on tax base $i$ due to a change in the net-of-tax rate for tax base $i$. In principle, the sign of this parameter is ambiguous: if income sources are substitutes (complements), an increase in earned income of type $i$ is accompanied by a fall (rise) in income of type $j$, which results in a positive (negative) $\beta_{j i}$. Furthermore, we have to consider that tax differentials induce taxpayers to shift parts of their reported income across tax bases as a form of semi-legal tax avoidance. Therefore, an increase in the net of tax rate of tax base $i$ may draw

[^3]income from another tax base $j$ to tax base $i$ resulting in a positive $\beta_{j i}$. Empirical evidence on this parameter is scarce. Existing work by Kleven and Schultz (2014), Pirttilä and Selin (2011), Jacob (2014) and Mortenson (2014) provide evidence on positive cross-effects between tax bases indicating that the empirically relevant assumption on cross effects is $\beta_{j i}>0$.

The government's problem. The government maximizes social welfare according to a social welfare function $S(\cdot)$ facing an exogenous revenue requirement of $E$. The maximization problem reads

$$
\begin{align*}
& \max _{\left\{\tau_{i}(\cdot)\right\}_{i}} \int_{\mathbf{k} \in \mathbf{K}} S(U(\mathbf{k})) d F(\mathbf{k})  \tag{1}\\
& \text { s.t. } \quad \int_{\mathbf{k}} \sum_{i=1}^{n} \tau_{i} z_{i}(\mathbf{k}) d F(\mathbf{k}) \geq E .
\end{align*}
$$

The marginal social welfare weight for a tax payer of type $\mathbf{k}$ is given by $S^{\prime}(U(\mathbf{k})) U_{c}^{\prime}(\mathbf{k}) / \lambda$, where $\lambda$ is the multiplier on the government's budget constraint. We define the income type specific average marginal social welfare weight as a function of $z_{i}$ as $g_{i}\left(z_{i}\right)=\int_{\mathbf{k} \in \mathbf{K}} S^{\prime}(U(\mathbf{k})) U_{c}^{\prime}(\mathbf{k}) d F\left(\mathbf{k} \mid z_{i}\right) / \lambda$. Intuitively, $g_{i}\left(z_{i}\right)$ measures the average value of giving one dollar to a person with $z_{i}$ in terms of public funds.
The following result characterizes the optimal linear tax system:
Optimal Linear Tax Rates. The optimality condition for the tax vector $\boldsymbol{\tau}=\left(\tau_{1}, \ldots, \tau_{n}\right)^{\prime}$ in a linear income tax system is given by:

$$
\left(\begin{array}{c}
\mathbf{m}_{\mathbf{1}} \\
\vdots \\
\mathbf{m}_{\mathbf{i}} \\
\vdots \\
\mathbf{m}_{\mathbf{n}}
\end{array}\right) \times \boldsymbol{\tau}=\left(\begin{array}{c}
\left(1-g_{1}\right) \\
\vdots \\
\left(1-g_{i}\right) \\
\vdots \\
\left(1-g_{n}\right)
\end{array}\right)
$$

where

$$
\begin{aligned}
\mathbf{m}_{\mathbf{i}} & =\left(-\beta_{1 i} \zeta_{i i}, \ldots,-\beta_{i-1 i} \zeta_{i i},\left(1+\zeta_{i i}-g_{i}\right),-\beta_{i+1 i} \zeta_{i i}, \ldots,-\beta_{n i} \zeta_{i i}\right) \\
\beta_{j i} & =-\frac{\partial z_{j}}{\partial\left(1-\tau_{i}\right)} / \frac{\partial z_{i}}{\partial\left(1-\tau_{i}\right)} \\
g_{i} & =\int z_{i} g_{i}\left(z_{i}\right) d H_{i}\left(z_{i}\right), \text { with } Z_{i}=\int z_{i} d H_{i}\left(z_{i}\right)
\end{aligned}
$$

## Derviation:

Setting up the Lagrangian, we obtain

$$
\begin{aligned}
L= & \int_{\mathbf{k} \in \mathbf{K}} S\left(u^{\mathbf{k}}\left(\left(1-\tau_{1}\right) z_{1}+\cdots+\left(1-\tau_{n}\right) z_{n} ; \hat{z}_{1}, \ldots, \hat{z}_{n} ; z_{1}, \ldots, z_{n}\right)\right) d F(\mathbf{k}) \\
& +\lambda\left(\int_{\mathbf{k} \in \mathbf{K}} \tau_{1} z_{1}+\cdots+\tau_{n} z_{n} d F(\mathbf{k})-E\right) .
\end{aligned}
$$

Applying the envelope theorem on the individual utility maximization problem the first-order condition of the government's optimization problem for tax base $i$ reads

$$
\begin{aligned}
\frac{\partial L}{\partial\left(1-\tau_{i}\right)}= & \int_{\mathbf{k} \in \mathbf{K}} z_{i}(\mathbf{k}) S^{\prime}(U(\mathbf{k})) U^{\prime}(\mathbf{k}) d F(\mathbf{k}) \\
& +\lambda\left(\int_{\mathbf{k} \in \mathbf{K}} \tau_{1} \frac{\partial z_{1}(\mathbf{k})}{\partial\left(1-\tau_{i}\right)}+\cdots+\tau_{n} \frac{\partial z_{n}(\mathbf{k})}{\partial\left(1-\tau_{i}\right)}-z_{i}(\mathbf{k}) d F(\mathbf{k})\right)=0 .
\end{aligned}
$$

Making use of the definitions of the own elasticity $\zeta_{i i}=\frac{\partial z_{i}}{\partial\left(1-\tau_{i}\right)} \frac{\left(1-\tau_{i}\right)}{z_{i}}$ and the share of crossresponses $\beta_{j i}=-\frac{\partial z_{j}}{\partial\left(1-\tau_{i}\right)} / \frac{\partial z_{i}}{\partial\left(1-\tau_{i}\right)}$ which we assume are constant, it follows

$$
\begin{aligned}
& \left(1-\tau_{i}\right) \cdot \int_{\mathbf{k} \in \mathbf{K}} z_{i}(\mathbf{k}) S^{\prime}(U(\mathbf{k})) U^{\prime}(\mathbf{k}) d F(\mathbf{k}) / \lambda \\
- & \left(\int_{\mathbf{k} \in \mathbf{K}} \tau_{1} \beta_{1 i} \zeta_{i i} z_{i}(\mathbf{k})+\cdots+\tau_{n} \beta_{n i} \zeta_{i i} z_{i}(\mathbf{k})+\left(1-\tau_{i}\right) z_{i}(\mathbf{k}) d F(\mathbf{k})\right)=0 .
\end{aligned}
$$

Denoting total income of type $i$ by $Z_{i}=\int_{\mathbf{k} \in \mathbf{K}} z_{i}(\mathbf{k}) d F(\mathbf{k})=\int z_{i} d H_{i}\left(z_{i}\right)$ and introducing the average welfare weight of income type $i$ as $g_{i}=\int_{\mathbf{k} \in \mathbf{K}} z_{i}(\mathbf{k}) S^{\prime}(U(\mathbf{k})) U^{\prime}(\mathbf{k}) d F(\mathbf{k}) /\left(\lambda Z_{i}\right)=$ $\int z_{i} \int_{\mathbf{k} \in \mathbf{K}} S^{\prime}(U(\mathbf{k})) U^{\prime}(\mathbf{k}) d F\left(\mathbf{k} \mid z_{i}\right) /\left(\lambda Z_{i}\right) d H_{i}\left(z_{i}\right)=\int z_{i} g_{i}\left(z_{i}\right) d H_{i}\left(z_{i}\right)$, we obtain

$$
-\beta_{1 i} \zeta_{i i} \tau_{1}-\cdots-\beta_{i-1 i} \zeta_{i i} \tau_{i-1}+\left(1+\zeta_{i i}-g_{i}\right) \tau_{i}-\beta_{i+1 i} \zeta_{i i} \tau_{i+1}-\cdots-\beta_{n i} \zeta_{i i} \tau_{n}=1-g_{i}
$$

and arrive at the proposition since this condition has to hold in an optimal linear tax equilibrium $\forall i$. The optimal tax equilibrium consists of a system of equation capturing fiscal externalities arising from tax differentials. In the plausible case where income types are substitutes and hence $\beta_{j i}>0$, optimal tax rates turn out to be higher than without considering cross-effects. In contrast, if income types are complements and hence $\beta_{j i}<0$, optimal tax rates turn out to be lower than without considering cross-effects. If there are no cross responses between tax bases and hence $\beta_{j i}=0 \forall j \neq i$, the standard formula for the optimal linear tax rate is nested in the system of equations as $\tau_{i}=\frac{1-g_{i}}{1+\zeta_{i i}-g_{i}}$.
For illustrative purposes, Figures 2 a-f show comparative statics for optimal linear tax rates for the case of three separate tax bases with different elasticities. We assume own-elasticities of 0.5 for income type $1,0.75$ for income type 2 , and 0.25 for income type 3 .

Figures 2a-b plot optimal tax rates as functions of the total share of cross responses where we
assume the same level of cross-responses for each pair of tax bases $(i, j)$, or formally $\beta_{i j}=\beta$. Figure 2a assumes income type specific average welfare weights of 0 , which corresponds to the revenue maximizing case, Figure 2 b assumes income specific average welfare weights of 0.7. Optimal tax rates increase with higher levels of cross responses and approach one when behavioral responses are fully offset by fiscal externalities. The differential in tax rates decreases with the strength of cross responses.
In Figures 2c-d, we relax the assumption of uniform cross-effects between tax bases. We plot optimal tax rates for the special case where fiscal externalities only take place between tax bases 1 and 2. Intuitively, optimal tax rates for income base 1 and 2 are increasing with the level of cross-elasticities while the optimal tax rate for income base 3 remains unaffected.
Figures 2 e -f show comparative statics for the own-elasticities holding the share of cross responses constant. We assume that cross-responses between tax bases are uniform and aggregate to 0.4. Compared to the case with zero cross-responses (dashed lines), optimal tax rates decrease less due to a rise in own-elasticities when cross-responses are accounted for.

## 3 Institutional Background and Data

### 3.1 The personal income tax in Germany

All individuals in Germany are subject to personal income taxation. ${ }^{5}$ The first step is to determine a tax unit's broad gross income from different sources and to allocate it to the seven forms of income the German tax law distinguishes between: income from agriculture and forestry, (non-corporate) business income, entrepreneurial income, salaries and wages from employment, investment income, rental income, and other income (including, for example, pensions, annuities and certain capital gains). ${ }^{6}$ For our empirical analysis, we group income from those seven sources into three categories: labor, capital and self-employment income. Labor income consists of salaries and wages from employment, self-employment income comprises income from agriculture and forestry, business income as well as entrepreneurial income, whereas capital income comprises investment income, rental income, and other income.

Second, for each type of income, the tax law allows for certain income-related expenses (Werbungskosten). In principle, all expenses that are necessary to obtain, maintain or preserve the income from a source are deductible. These include, for instance, commuting costs, expenses for work materials or costs of training. For non-itemizing taxpayers, there is an allowance for labor earnings ( $€ 920$ in 2008) and capital income ( $€ 750$ in 2008). The sum of broad gross

[^4]income minus income-related expenses per income source yields the adjusted gross income. As a third step, deductions, including expenses for investment in human capital, child care costs, donations to charity or political parties and church tax payments, are taken into account and subtracted from adjusted gross income yielding taxable income.

Finally, the income tax is calculated by applying the rate schedule to taxable income. In contrast to most other countries who use a bracket system with constant marginal tax rates within a bracket, Germany uses a formula (which is quadratic in income) to compute the tax liability. ${ }^{7}$ As a consequence, marginal tax rates increase linearly in income (up to an top marginal tax rate of $42 \%$ ). The formula for the years 2007 and 2008 is defined as follows ${ }^{8}$ :

$$
T= \begin{cases}0 & \text { if } T I \leq 7,664 \\ \left(883.74 \frac{T I-7,664}{10,000}+1,500\right) \frac{T I-7,664}{10,000} & \text { if } 7,664<T I \leq 12,739 \\ \left(228.74 \frac{T I-12,739}{10,000}+2,397\right) \frac{T I-12,739}{10,000}+989 & \text { if } 12,739<T I \leq 52,151 \\ 0.42 T I-7,914 & \text { if } 52,151<T I \leq 250,000 \\ 0.45 T I-15,414 & \text { if } T I>250,000\end{cases}
$$

In addition to the personal income tax, households additionally pay the "Solidaritätszuschlag", a tax supplement originally introduced to finance the German reunification. During the period of interest, 2000-2008, the supplement amounts to $5.5 \%$ of the income tax liability.

### 3.2 Reforms 2001-2008

Figure 3 shows the marginal tax rate schedule for the years 2001-03, 2004 and 2005-08. Taxpayers with a high taxable income and those with a taxable income slightly exceeding the basic tax allowance experienced the largest marginal tax rate cuts. Between 2000 and 2005, a major reform of the German personal income tax took place. The basic tax allowance was increased in several steps from $€ 6902$ in 2000 to $€ 7664$ (2004-2008) with $€ 7206$ in 2001 and $€ 7235$ in $2002 / 03$. The lowest marginal tax rate decreased from $22.9 \%$ in 2000 to $15 \%$ (2005-2008) with $19.9 \%(2001-03)$ and $16 \%(2004)$ in between. The top marginal tax rate was reduced from $51 \%$ in 2000 to $42 \%$ in 2005 with $48.5 \%$ (2001-03) and $45 \%$ (2004) in between. The threshold where the top marginal tax rate kicks in was reduced from $€ 58,643$ in 2000 to $€ 52,151$ in 2004 with values of $€ 55,007$ (2001-03) in between. In 2007, an additional tax bracket at the top (for taxable income above $€ 250,000$ ) was introduced with a top marginal tax rate of $45 \%$. Tax rates in the medium range of the schedule were lowered as well.

[^5]
### 3.3 Data

Data set: We use the German Taxpayer Panel, which is an administrative data set collected by German tax authorities, provided and administered by the German Federal Statistical Office (Kriete-Dodds and Vorgrimler 2007). The unit of observation is the taxpayer, i.e., either a single individual or a couple filing jointly. The panel covers all German tax units in the period 2001 to 2008. We have access to a $5 \%$ random sample of the Taxpayer Panel and employ the respective weights provided by the Statistical Office. The dataset contains all information necessary to calculate a taxpayer's annual income tax, this includes basic socio-demographic characteristics such as birth date, gender, family status, number of children as well as detailed information on income sources and tax base parameters such as work-related expenses and (claimed and realized) deductions.

Sample Selection and Summary Statistics: We consider all taxpayers who are taxed individually, i.e. singles. [NOTE: In the next version of the paper, we will conduct a separate analysis of singles and couples.] Table 1 shows summary statistics for the key variable of the analysis. The sample consists of $2,253,691$ million individuals with mean taxable income of $€ 17,854$. Wage income is the predominant income source with roughly $70 \%$ percent of individuals reporting a non-zero amount and mean of $€ 18,552$. Capital income accounts only for a small share with a mean of $€ 6,659$ and is least unequally distributed. Income from selfemployment is most unequally distributed with a $99 \%$ percentile of $€ 213,052$, which is more than twice as high as the corresponding value for wage income and four times for capital income.

## 4 Empirical Estimation of Income Type Specific Elasticities

### 4.1 Empirical model

This section describes the empirical model and outlines our identification strategy to estimate income type specific elasticities. We follow the ETI literature and especially Doerrenberg et al. (2015) to estimate the effect of the net-of-tax rate on taxable income employing panel-regression models following Gruber and Saez (2002) and Weber (2014). ${ }^{9}$ For taxpayer $i$ in year $t$, we regress the change in our left-hand side variable of interest (either taxable or gross income), $\Delta W_{i, t}$, on the change in the marginal net-of-tax rate, $\Delta(1-\tau)$. The operator $\Delta$ indicates the difference between year $t$ and base-year $t-k$. In the baseline, we estimate the model given below in 2-year differences:

$$
\begin{equation*}
\Delta \ln W_{i, t}=\epsilon_{W} \Delta \ln \left(1-\tau_{i, t}\right)+f\left(G I_{i, t-k}\right)+\phi \mathbf{X}_{i, t}+\gamma_{t}+\eta_{i, t}, \tag{2}
\end{equation*}
$$

[^6]where $f\left(G I_{i, t-k}\right)$ is a function of individual base-year gross income, $\mathbf{X}_{i, t}$ a vector containing standard demographic variables (dummies for joint filing / marital status, number of children, age, and West- vs. East-Germany), $\gamma_{t}$ a set of year fixed effects and $\eta_{i, t}$ an individual error term. ${ }^{10}$ The coefficient of interest, $\epsilon_{W}$, can be interpreted as an elasticity since the outcome measure, $W_{i, t}$, and the net-of-tax rate, ( $1-\tau_{i, t}$ ), enter the regression in logs.
We follow standard practice in the literature to address potential threats to identification (Saez et al. 2012). First, we use panel data and estimate the model in differences to wipe out timeinvariant individual confounders. Second, we account for mean reversion and secular trends in income inequality by controlling for gross income (Auten and Carroll 1999). Since the literature shows that ETI estimates are fairly sensitive to the way of controlling for income, we report results for a variety of different income controls. In our main estimations, we follow the idea of Kopczuk (2005) - recently applied in Kleven and Schultz (2014) - and include 10-piece splines in logged $t-k-1$ income as well as 10-piece splines in the difference between logged income in $t-k$ and logged income in $t-k-1$. These two income controls serve the purpose of controlling for transitory income - through the difference between base-year income and its lag - as well as permanent income - through lagged base-year income. ${ }^{11}$
Third, we have to account for the mechanical relationship between our left-hand-side variables and the net-of-tax rate. An increase in income automatically changes the net-of-tax rate because, in progressive systems, higher incomes are taxed at higher marginal tax rates. The same reasoning applies when tax deductions are used on the left-hand side of the equation: higher deduction claims reduce taxable income and therefore also affect the tax rate. This mechanical relationship between the left-hand side variables and $\left(1-\tau_{i, t}\right)$ requires to find an instrument for the net-of-tax rate that is unrelated to the error term in the above regression model. Following Gruber and Saez (2002), most studies in the literature use an instrument which is based on predicted changes in tax rates that are solely due to legislative tax reforms (e.g., Chetty et al. 2011; Kleven and Schultz 2014). The net-of-tax rate in year $t$ is instrumented with the "synthetic" net-of-tax rate that is constructed by applying the tax schedule in year $t$ to income in base-year $t-k$. As a result, the synthetic instrument intents to capture statutory tax rate changes caused by reforms while it abstracts from mechanical tax-rate changes in progressive tax systems that are due to changing income (or deductions).
However, there is a growing concern in the literature that this synthetic net-of-tax rate is not sufficiently exogenous. In fact, it depends on base-year income and shocks to base-year income are part of the error term in the regression equation - despite flexible ways of controlling for

[^7]base-year income (Blomquist and Selin 2010; Weber 2014). To overcome this potential threat of endogeneity, we employ an instrument that was proposed by Weber (2014), and which we denote $\left(1-\tau_{i, t}^{\text {synth }}\right)$. Instead of making the instrument a function of base-year income, the synthetic instrument is a function of lagged base-year income. That is, the instrument we use is constructed by applying the tax schedule of year $t$ to income in the year before the base-year, $t-k-1$. Weber (2014) shows that such an instrument is more exogenous to the error term than an instrument that is simply based on base-year income, and therefore reduces the correlation between the error term and the instrument. ${ }^{12}$
Fourth, mechanical effects induced by simultaneous tax-rate and tax-base reforms may have important implications for the definition and construction of variables for our analysis. To circumvent this complication, the literature uses the broadest definition of the tax base (see Saez et al. 2012). We follow this approach in our paper.

### 4.2 Elasticities Results

This section presents regression-based evidence using changes in tax rates induced by all tax reforms between 2001 and 2008 for identification. We estimate regression model (2) using twostage least squares and cluster standard errors on the individual level. First-stage regressions (not shown) of $\Delta \ln \left(1-\tau_{i, t}\right)$ on $\Delta \ln \left(1-\tau_{i, t}^{\text {synth }}\right)$ are strong with large $F$-statistics exceeding at least 400 in all our estimations.
Figures 6 and 7 in the Appendix provide graphical evidence of the first stage, as well as the reduced-form regressions following the exposition in Weber (2014) and Doerrenberg et al. (2015). The graphs plot fourth-order polynomial regressions, based on regression model (2) for 2-year differences, excluding income controls and other control variables. Figure 6 visualizes the first stage; as expected, the instrument and the variable of interest, the net-of-tax rate, are positively related. As depicted in Figure 7, we observe a mild positive relationship between the instrument and taxable income. This reflects the positive elasticities that we estimate using the full model.
Table 2 displays the regression estimates. We report the effect of two-year net-of-tax rate changes for taxable income using the Gruber and Saez (2002) and Weber (2014) approaches. We report estimates for the full sample, by income source and by income quintile. The baseline estimates for the ETI for the full sample are 0.30 using the Gruber-Saez estimator and 0.35 using the Weber approach. [Note that the results are a bit smaller than those in Doerrenberg et al. (2015) as we only use single individuals whereas they use both singles and couples for their analysis.] Differentiating by income source, we find small (and insignificant) elasticities od 0.15-

[^8]0.22 for capital income, larger (and significant) elasticities of $0.29-0.32$ for wage income and of 0.33-0.43 for self-employment income. Furthermore, elasticities are (somewhat) increasing with income - the highest elasticities are found for the fourth quintile whereas elasticities for the lowest quintile are actually negative. ${ }^{13}$

## 5 Empirical Estimation of Marginal Social Welfare Weights

### 5.1 Derivation

We derive and estimate marginal social welfare weights implicit in the German tax system similar to Lockwood and Weinzierl (2015) for the US and Zoutman et al. (2015) for the Netherlands. ${ }^{14}$ We decompose the elasticity of taxable income into income type specific elasticities assuming constant elasticities for each type independent of the income level. This implicitly endogenizes the aggregate elasticity with respect to the amount of taxable income since shares of income types change along the income distribution (see Figures 1c-d).

Consider a social planner with a standard welfare function $S(\cdot)$ introducing a non-linear tax schedule $T(z)$ to maximize social welfare. We assume that she is restricted to levy the same tax amount $T(z)$ from every taxpayer with a level of taxable income of $z$. Therefore, the tax schedule solely depends on total taxable income $z$ and ignores the levels of single income sources $z_{i}$, as it pertained to German tax law until 2008. Since mechanical effects through tax changes do not depend on the responsiveness of single income sources, the mechanical effect of a marginal increase $d \tau$ in a small band $z+d z$ is the same as in Saez (2001).
However, the social planner will take into account the differentials in the elasticities between different types of income. For a taxpayer with total income of $z$ the average behavioral response in reported income of type $i$ is given by $\left(\int_{0}^{z} h_{i}\left(z_{i}^{\prime} \mid z\right) z_{i}^{\prime} \zeta_{i i} d z_{i}^{\prime}\right) /(1-\tau(z))$, and it follows that the elasticity effect is given by

$$
-d \tau d z \sum_{i=1}^{n}\left(\int_{0}^{z} h_{i}\left(z_{i}^{\prime} \mid z\right) z_{i}^{\prime} \zeta_{i i} d z_{i}^{\prime}\right) h(z) \frac{\tau(z)}{1-\tau(z)}
$$

Optimality implies that the mechanical, welfare, and the elasticity effect offset each other, and thus

[^9]$$
\int_{z}^{\infty} g\left(z^{\prime}\right) d H\left(z^{\prime}\right)=1-H(z)-\sum_{i=1}^{n}\left(\int_{0}^{z} h_{i}\left(z_{i}^{\prime} \mid z\right) z_{i}^{\prime} \zeta_{i i} d z_{i}^{\prime}\right) h(z) \frac{\tau(z)}{1-\tau(z)}
$$

Taking the derivative with respect to $z$ yields

$$
g(z)=-\frac{1}{h(z)} \frac{d}{d z}\left(1-H(z)-\sum_{i=1}^{n}\left(\int_{0}^{z} h_{i}\left(z_{i}^{\prime} \mid z\right) z_{i}^{\prime} \zeta_{i i} d z_{i}^{\prime}\right) h(z) \frac{\tau(z)}{1-\tau(z)}\right)
$$

### 5.2 Empirical results

Figures 4a-c display estimates of the marginal social welfare weights implicit in the German tax law as a function of the fractiles in the income distribution. We present estimates for the years 2001, 2004, and 2007 which correspond to the three different stages of the German income tax reform during the 2000s (see Figure 3). The estimation sample consists of individuals with a level of taxable income higher than the tax exempt amount. We distinguish between income type specific elasticities obtained from the Gruber and Saez (2002) and Weber (2014) approach (see section 4).

In all regimes the distribution of marginal social welfare weights seems qualitatively similar, independent of the underlying elasticity assumptions. Overall, weights seem to increase until the fifth decile and are decreasing after. Interestingly, the decrease of marginal social welfare weights in the upper part of the income distribution is not monotonic. Instead, welfare weights exhibit a slight increase around the eighth decile followed by a sharp decrease making the estimates close to zero. Interestingly, our estimates indicate that marginal social welfare weights are smallest around the $97 \%$ percentile, followed by a sizeable increase for higher incomes.

Comparing the estimates for 2001, 2004, and 2007, we find qualitative similarity of marginal social welfare weights across tax regimes. However, reducing the progressivity of the tax system seems to have had a quantitative impact. Welfare weights for the bottom $50 \%$ decreased slightly from 2001 to 2007 in contrast to the substantial increase at the top of the income distribution. The strongest change can be found for the top $10 \%$ percent of the income distribution. Aggregating the effects, we can compute the development of average welfare weights weighted by taxable income.

Marginal social welfare weights for Gruber and Saez (2002) and Weber (2014) elasticities seem qualitatively similar. However, the higher elasticities obtained from the Weber (2014) estimation strategy yield a higher variation in the distribution of marginal social welfare weights. Intuitively, stronger behavioral responses to taxation induce higher efficiency costs of taxes.

Holding the tax system constant, higher elasticities implicitly increase the social planner's value for redistribution and yield more dispersed marginal social welfare weights along the income distribution. As a counterfactual, if behavioral responses to taxation were absent, marginal social welfare weights would always equal one independent of the level of taxable income.

Table 3 displays income-type specific average welfare weights which are weighted with a taxpayer's amount of the particular income type. In summary, we observe highest average welfare weights for wage income, while moderate levels for capital income. Income from selfemployment exhibits relatively low average welfare weights. This is due to the comparatively high elasticity of self-employment income in conjunction with its concentration in the very top of the income distribution. We furthermore, observe that the increase in average social welfare weights from 2001 to 2007 is entirely driven by income from self-employment and in particular capital. In contrast, average welfare weights for wage income seem constant over the entire time period. In intuitive terms, the drastic change in marginal social welfare weights in the top of the income distribution predominantly benefited self-employment and capital income due to its concentration in the top.

## 6 Optimal Tax Rates

We calculate optimal linear tax rates for Germany using our estimates for income type specific elasticities and implied average welfare weights in 2001, 2004 and 2007. By using estimates of implied average welfare weights of the previous section, this approach implicitly linearizes the tax system. However, it allows for differential taxation of different income sources. As before, we distinguish between income type specific elasticities obtained from the Gruber and Saez (2002) and Weber (2014) estimation strategies for income from wages, capital, and self-employment. In our calculations, we make two assumptions on the cross-elasticities between tax bases. First, for each tax base $j$ the share of cross-responses in the total income type-specific elasticity is the same. Second, all income sources $i \neq j$ account for the same proportion in the cross-elasticities for tax base $j$. Formally, both conditions can be expressed as $\beta_{j i}=\beta \forall i, j, i \neq j$.

Figures 5a-f show optimal linear tax rates as functions of the level of cross-elasticities between tax bases. Focusing on the results using Gruber and Saez (2002) elasticity estimates in Panels a,c and e, we find highest optimal tax rates for capital income, and lowest for wage income using the 2001 estimates. Despite the high elasticity for income from self-employment, the low average welfare weight induces a much higher optimal tax rate for this income source compared wage income. Increasing the proportion of cross-elasticities yields higher optimal tax rates due to lower efficiency costs of taxation. Furthermore, we find that the changes of implied average welfare weights from 2001 to 2007 decreased optimal linear tax rates for income from self-
employment and particularly capital but not for wages. Intuitively, reducing the progressivity of the tax system leads to an increase in the welfare weights for capital and self-employment income. In turn, this diminishes the gap between optimal linear tax rates in our case since differences in implied average welfare weights are attenuated.

Estimates employing Weber (2014) elasticity estimates in Panel b, d and f tend to provide lower values for optimal tax rates for income from self-employment and capital but not for wage income. Intuitively, when using the Weber (2014) instead of the Gruber and Saez (2002) estimation strategy, elasticity estimates for self-employment and capital income increase more strongly relative to those for wage income. Furthermore, we find that the substantial increase in the average welfare weight for capital income from 2001 to 2007 decreased the optimal tax rate below the level for income from self-employment.

## 7 Conclusion

When behavioral responses and welfare impacts through taxation differ across income sources, a global income tax system is suboptimal. Instead, a social planner may assign different tax rates to individual types of income in order to balance the tradeoff between efficiency and redistribution among different income sources. In fact, many countries assign differential tax schedules to different income concepts. In the US as well as Germany capital gains are taxed at a lower rate than income from wages and self-employment. Scandinavian countries, including Denmark, Sweden, Norway and Finland, apply a dual income tax system with a flat tax on all capital income and a progressive schedule for other income sources.

Despite the empirical relevance, there is little theoretical and empirical work on the optimal differential taxation of different income sources under joint optimality. In this study we approach this problem providing a theoretical model of jointly optimal income taxes for different sources of income. We incorporate fiscal externalities due to cross-effects between tax bases. These can stem from semi-legal income shifting as the predominant form, but also include cross-effects arising from substitutability and complementarity of different income types. We show that optimal linear tax rates are increasing and convex in the proportion of cross-responses in the total income type specific elasticity with respect to the net of tax rate.

In the empirical part of this paper, we calibrate optimal linear taxes for different sources of income for the case of Germany. To do this, we first provide evidence that behavioral responses to taxation differ strongly across income sources. Using the Gruber and Saez (2002) estimation strategy, we find a higher elasticities of .325 for income from self-employment with respect to the net-of-tax rate. For capital income we find a lower level of .146 . Wage income
exhibits moderate responsiveness in a magnitude of .293. Using Weber (2014) instruments, we obtain qualitatively similar but higher estimates. In a second step, we calculate marginal social welfare weights implied in the German tax code. We document that marginal social welfare weights are roughly constant for the bottom $60 \%$ of taxpayers and are decreasing for higher incomes. Implied average welfare weights are highest for wage income and lowest for self-employment income. Furthermore, we provide evidence that the German tax reforms from 2001 to 2007 increased average welfare weights for income from self-employment and capital while not for wage income. We use our estimates of income type specific elasticities and average welfare weights to calculate optimal linear tax rates for income from wages, selfemployment, and capital. This procedure implicitly linearizes the tax system but allows for a differential treatment of single income sources. Absent any cross-effects between tax bases, we find substantially lower optimal tax rates for wage income relative to capital income and in particular income from self-employment. Optimal tax rates increase and converge with higher levels of fiscal externalities.

Figure 1: Distribution of Income Sources


Notes: Figures 1a-d provide graphical evidence on the distribution of income from wages, self-employment, and capital in Germany as of 2007. Figures 1a-b display the fraction of taxpayers according to their main source of income as a function of the level of taxable income. For this purpose the main income source is defined as the one with the highest level. The figures are obtained from local polynomial regressions with Epanechnikov kernel of the fraction of taxpayers according to their reported main income source on taxable income. Figures 1c-d report the average fractions of income from wages, self-employment, and capital as functions of the level of taxable income. The figures are obtained from local polynomial regressions with Epanechnikov kernel of the fraction of a taxpayer's type of income on taxable income.

## Figure 2: Optimal Linear Income Tax Rates Dependent on the Level of CrossElasticities, Own-Elasticities and Average Welfare Weights



Notes: Figures 2a-f display comparative statics of optimal linear tax rates depending on the level of cross-elasticities, own-elasticities and average welfare weights. Assumed own-elasticities are 0.5 (income type 1 ), 0.75 (income type 2 ), 0.25 (income type 3). Figures 2a-d display optimal linear income tax rates as functions of the share of the cross-elasticity in the total elasticity of each income type for different average welfare weights. Given a certain share $\beta$, the own-elasticity for each income-type can be decomposed in a real response of size $1-\beta$ and a fiscal externality of size $\beta$. Figures $2 \mathrm{a}-\mathrm{b}$ assume that each income type exhibits the same $\beta$ and accounts for the same proportion in the cross-elasticity for the other income types. Figures $2 \mathrm{c}-\mathrm{d}$ assume that cross-responses occur only between tax bases 1 and 2. Figures $2 \mathrm{e}-\mathrm{f}$ display optimal linear income tax rates as functions of the own-elasticity for different average welfare weights. Solid lines display optimal tax rates assuming cross-responses in a magnitude of .4 of the level of own-elasticities. (All income sources are assumed to account for the same proportion in the cross-elasticities.) Dashed lines display optimal tax rates assuming no cross-responses.

Figure 3: Marginal Tax Rates


Notes: Figure 3 shows the marginal tax rate as a function of taxable income for Germany from 2001 to 2008. Additional to the income tax the government levies a surcharge of $5.5 \%$ of the tax liability. Therefore, the effective marginal tax rate is given by $1.055 \cdot \tau$.

Figure 4: Marginal Social Welfare Weights: Elasticity Decomposition by Income Type


Notes: Figures 4a-c display marginal social welfare weights implied by the German tax system as functions of the taxable income distribution for 2001, 2004, and 2007. The figures consider two scenarios for income-type specific elasticities estimated by the Gruber-Saez and Weber approach (see figure notes). Dashed gray lines indicate the effective marginal tax rate for a given fractile in the taxable income distribution. Dotted gray lines represent the density of taxable income.

Figure 5: Optimal Linear Tax Rates: By Income Type


Notes: Figures 5a-f display optimal linear tax rates for income from wages, self-employment, and capital. The figures use income-type specific average welfare weights as implied by the German tax system as of 2001, 2004 , and 2007. Furthermore, the figures consider two scenarios for income-type specific elasticities estimated by the Gruber-Saez and Weber approach (see figure notes).

Table 1: Summary Statistics

|  | Taxable <br> Income | Wage <br> Income | Capital <br> Income | Income from <br> Self-Employment |
| :--- | :--- | :--- | :--- | :--- |
| Mean | 17854 | 18552 | 6589 | 20559 |
| p25 | 3460 | 3528 | 542 | 474 |
| p50 | 11736 | 13873 | 3216 | 5741 |
| p75 | 25541 | 27889 | 9531 | 20000 |
| p90 | 38083 | 40563 | 15102 | 46415 |
| p95 | 48831 | 50478 | 21693 | 75863 |
| p99 | 90170 | 82283 | 56243 | 213052 |
| N | 2253691 | 1509876 | 802892 | 728996 |

Notes: Table 1 reports summary statistics for the key variables in our analysis. Distribution parameters are weighted with the sampling weights provided by the German Statistical Office. All statistics are conditional on whether a non-missing amount was reported.

Table 2: ETIs

|  | Gruber-Saez |  |  |
| :--- | :---: | :--- | :---: |
| all | $0.299^{* * *}$ | Weber |  |
|  | $(0.0195)$ | $(0.0243)$ |  |
| by income source |  |  |  |
| wage | $0.293^{* * *}$ | $0.320^{* * *}$ |  |
|  | $(0.0203)$ | $(0.0250)$ |  |
| self | $0.325^{* * *}$ | $0.434^{* * *}$ |  |
|  | $(0.0313)$ | $(0.0380)$ |  |
| capital | 0.146 | $0.223^{*}$ |  |
|  | $(0.105)$ | $(0.120)$ |  |
| by income quantile |  |  |  |
| Q1 | $-0.172^{* * *}$ |  |  |
|  | $(0.0500)$ | $-0.133^{* *}$ |  |
| Q2 | $-0.0896^{* *}$ | $(0.0641)$ |  |
|  | $(0.0361)$ | $(0.0457$ |  |
| Q3 | $0.132^{* * *}$ | $0.171^{* * *}$ |  |
|  | $(0.0315)$ | $(0.0398)$ |  |
| Q4 | $0.709^{* * *}$ | $0.798^{* * *}$ |  |
|  | $(0.0306)$ | $(0.0380)$ |  |
| Q5 | $0.268^{* * *}$ | $0.265^{* * *}$ |  |
|  | $(0.0193)$ | $(0.0236)$ |  |
| No. obs. | $1,241,029$ |  |  |

Notes: Table 2 reports elasticities of taxable income estimated with the Gruber and Saez (2002) and Weber (2014) approaches.

Table 3: Income-Type Specific Average Welfare Weights

| Year | Gruber and Saez (2002) Elasiticities |  |  | Weber (2014) Elasiticities |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Wage | Self- <br> Employment | Capital | Wage | Self- <br> Employment | Capital |
| 2001 | . 878 | . 643 | . 769 | . 863 | . 579 | . 728 |
| 2004 | . 876 | . 689 | . 839 | . 861 | . 634 | . 812 |
| 2007 | . 883 | . 730 | . 917 | . 868 | . 679 | . 902 |

$\overline{\text { Notes: }}$ Table 3 reports income-type specific average welfare weights weighted with the income of the particular type. We report estimates for income from wages, self-employment, and capital for elasticities estimated with the Gruber and Saez (2002) and Weber (2014) approaches.

## Appendix A: Comparative Statics in Cross-Elasticities

For illustrative purposes consider the case of $n=2$ tax bases. The optimality condition for the optimal tax vector $\left(\tau_{1}^{*}, \tau_{2}^{*}\right)$ read

$$
\begin{aligned}
\tau_{1}^{*}\left(1+\zeta_{11}-g_{1}\right)-\tau_{2}^{*} \beta_{21} \zeta_{11} & =\left(1-g_{1}\right) \\
-\tau_{1}^{*} \beta_{12} \zeta_{22}+\tau_{2}^{*}\left(1+\zeta_{22}-g_{2}\right) & =\left(1-g_{2}\right) .
\end{aligned}
$$

W.l.o.g. we can solve for the optimal $\tau_{1}^{*}$ and analyze its comparative statics in $\beta_{21}$ (the magnitude of the fiscal externality of tax base 1 on tax base 2 ) as well as $\beta_{12}$ (the magnitude of the fiscal externality of tax base 2 on tax base 1 ). The optimal $\tau_{1}^{*}$ reads

$$
\tau_{1}^{*}=\frac{\left(1-g_{1}\right)\left(1+\zeta_{22}-g_{2}\right)+\left(1-g_{2}\right) \beta_{21} \zeta_{11}}{\left(1+\zeta_{11}-g_{1}\right)\left(1+\zeta_{22}-g_{2}\right)-\beta_{12} \zeta_{22} \beta_{21} \zeta_{11}} .
$$

Now define $f_{1}:=\left(1-g_{1}\right)\left(1+\zeta_{22}-g_{2}\right), f_{2}:=\left(1-g_{2}\right) \zeta_{11}, f_{3}:=\left(1+\zeta_{11}-g_{1}\right)\left(1+\zeta_{22}-g_{2}\right)$, $f_{4}:=\zeta_{22} \zeta_{11}$ and assume that $\forall i, f_{i}>0$. The ladder condition always holds if the social planner assigns a positive average welfare weight to each tax base and the own-elasticities for each tax base are positive. We also assume that $\beta_{j i} \geq 0 \forall j \neq i$.

Comparative statics in $\beta_{21}$ :

$$
\begin{aligned}
\frac{\partial \tau_{1}^{*}}{\partial \beta_{21}} & =\frac{f_{2} f_{3}+f_{4} \beta_{12} f_{1}}{\left(f_{3}-f_{4} \beta_{12} \beta_{21}\right)^{2}}>0 \\
\frac{\partial^{2} \tau_{1}^{*}}{\partial \beta_{21}^{2}} & =\frac{2 f_{4} \beta_{12}\left(f_{3}-f_{4} \beta_{12} \beta_{21}\right)\left(f_{2} f_{3}+f_{4} \beta_{12} f_{1}\right)}{\left(f_{3}-f_{4} \beta_{12} \beta_{21}\right)^{4}} \geq 0
\end{aligned}
$$

In intuitive terms, the optimal tax rate for a given tax base is increasing and convex in the share of the fiscal externality of this tax base on the other tax base.

Comparative statics in $\beta_{12}$ :

$$
\begin{aligned}
\frac{\partial \tau_{1}^{*}}{\partial \beta_{12}} & =\frac{f_{4} \beta_{21}\left(f_{1}+f_{2} \beta_{21}\right)}{\left(f_{3}-f_{4} \beta_{21} \beta_{12}\right)^{2}} \geq 0 \\
\frac{\partial^{2} \tau_{1}^{*}}{\partial \beta_{12}^{2}} & =\frac{2 f_{4}^{2} \beta_{21}^{2}\left(f_{3}-f_{4} \beta_{21} \beta_{12}\right)\left(f_{1}+f_{2} \beta_{21}\right)}{\left(f_{3}-f_{4} \beta_{21} \beta_{12}\right)^{4}} \geq 0
\end{aligned}
$$

In intuitive terms, the optimal tax rate for a given tax base is increasing and convex in the share of the fiscal externality of the other tax base on this tax base.

In the last step we can analyze the comparative statics in $\beta_{21}$ and $\beta_{12}$ jointly by assuming $\beta_{j i}=\beta \forall j \neq i$. This condition implies that the cross-elasticity is independent of the considered tax bases.

Comparative statics in $\beta$ :

$$
\begin{aligned}
\frac{\partial \tau_{1}^{*}}{\partial \beta} & =\frac{f_{2} f_{3}+2 \beta f_{4} f_{1}+\beta^{2} f_{4} f_{2}}{\left(f_{3}-f_{4} \beta^{2}\right)^{2}}>0 \\
\frac{\partial^{2} \tau_{1}^{*}}{\partial \beta^{2}} & =\frac{\left(2 f_{4} f_{1}+2 \beta f_{4} f_{2}\right)\left(f_{3}-f_{4} \beta^{2}\right)^{2}+4 f_{4} \beta\left(f_{3}-f_{4} \beta^{2}\right)\left(f_{2} f_{3}+2 \beta f_{4} f_{1}+\beta^{2} f_{4} f_{2}\right)}{\left(f_{3}-f_{4} \beta^{2}\right)^{4}}>0
\end{aligned}
$$

In intuitive terms, the optimal tax rate for a given tax base is increasing and convex in the share of the fiscal externality between tax bases.

## Appendix B: Additional Tables \& Figures

Figure 6: First-stage


Notes: Graphical evidence of the first-stage for specification (I) of Panel B in Table ETI-reg (2-year differences). German tax return data for 2001-2008. Graphs are based on a $5 \%$ sample of the universe of German taxpayers. The figure plots a fourth-order local polynomial regression of the change in the log marginal net-of-tax rate on the changes in the predicted log marginal net-of-tax rate. No control variables included. The dashed lines are $95 \%$ confidence intervals. The graphical illustration is based on Weber (2014).

Figure 7: Reduced form: taxable income


Notes: Graphical evidence of the first-stage for specification (III) of Panel B in Table ETI-reg (2-year differences). German tax return data for 2001-2008. Graphs are based on a $5 \%$ sample of the universe of German taxpayers. The figure plots a fourth-order local polynomial regression of the change in log aggregated taxable income on the changes in the predicted log marginal net-of-tax rate. No control variables included. The dashed lines are $95 \%$ confidence intervals. The graphical illustration is based on Weber (2014).

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[^1]:    ${ }^{1}$ We consider the separate taxation of different income sources such as wage or capital income. Another special case of this exercise is the optimal taxation of couples (Mirrlees 1972; Boskin and Sheshinski 1983 for initial contributions; Cremer et al. 2003; Cremer et al. 2012; Schroyen 2003; Brett 2007; Kleven et al. 2009 and Immervoll et al. 2011 for a recent state-of-the-art) with different tax schedules for men and women (gender based taxation; Cremer et al. 2010; Alesina et al. 2011).
    ${ }^{2}$ Most countries' tax systems follow the Haig-Simons standard of comprehensive income taxation by defining a single measure of taxable income as the sum of incomes from all sources to which a single rate schedule is applied. Notable exceptions are dual income tax systems (mostly Nordic countries and since 2009 also Germany). In such a schedular tax system, capital income is taxed at a low flat marginal rate whereas labor income is taxed at a progressive schedule (see Boadway 2004 for an overview of dual income tax systems).

[^2]:    ${ }^{3}$ Of course, a long literature on the (separate) optimal taxation of labor income (see, e.g., Piketty and Saez 2013 for a recent survey) and capital income (see, e.g., Kopczuk 2013) exists.

[^3]:    ${ }^{4}$ Following the standard model in the literature, we assume that taxpayers cannot evade taxes. We therefore require $\sum_{i=1}^{n} z_{i}=\sum_{i=1}^{n} \hat{z}_{i}$.

[^4]:    ${ }^{5}$ This section is based on Doerrenberg et al. (2015) who provide a more detailed description of the German income tax system.
    ${ }^{6}$ The following types of income are tax exempt: payments from health insurance, accident insurance and insurance for disability and old age, welfare benefits and scholarships.

[^5]:    ${ }^{7}$ The reason for using such a formula for the German tax schedule instead of tax brackets was "to avoid bunching at kink points" (see, e.g., Riebesell 1922, Chapter 5).
    ${ }^{8}$ For married taxpayers filing jointly, the tax is twice the amount of applying the formula to half of the married couple's joint taxable income: $T\left(T I_{1}, T I_{2}\right)=2 \cdot T\left(\frac{T I_{1}+T I_{2}}{2}\right)$.

[^6]:    ${ }^{9}$ To be precise, we use exactly the same sample ans estimation code as Doerrenberg et al. (2015) for our baseline estimate.

[^7]:    ${ }^{10}$ Note that in our empirical specification we abstract from estimating income effects as this is common in the literature (Saez, Slemrod, and Giertz 2012). See, e.g., Blomquist and Selin (2010) for a study allowing for income effects.
    ${ }^{11}$ Doerrenberg et al. (2015) show that gross income grew differently across income deciles over the period of our data sample, suggesting that heterogeneous income trends have to be addressed in our context. They also show that results are robust to including only the 10-piece splines in logged $t-k-1$ income as well as to using the 10 -piece splines in logged $t-k-1$ income as an instrument for the $t-k$ income splines.

[^8]:    ${ }^{12}$ In the presence of heterogeneous income trends, Weber (2014) proposes to include lagged income splines directly into the estimation model or to use them as an instrument for base-year income controls. While we use the Kopczuk (2005)-type splines (which include controls for lagged base-year income) in our baseline regressions, we results are robust to using lagged splines either directly or as instruments (see Doerrenberg et al. 2015).

[^9]:    ${ }^{13}$ Note that there is also large heterogeneity of elasticities by income source and income quintile [not shown]: for instance the largest elasticity in the fifth quintile is for capital income, while wage income is driving the elasticity in the fourth quintile.
    ${ }^{14}$ See Bourguignon and Spadaro (2012) and Bargain et al. (2014a,b) for further applications of the optimal tax inversion procedure.

