# Moral Hazard Misconceptions: the Case of the Greenspan Put* 

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#### Abstract

Policy discussions on financial market regulation tend to assume that whenever a corrective policy can be put in place ex post to ameliorate the effects of a financial crisis, this policy entails negative side effects in terms of moral hazard ex ante. This paper shows that this is not a general theoretical prediction, focusing on the case of monetary policy. In particular, we show cases in which if the central bank does not intervene by monetary easing following a crisis, this creates an aggregate demand externality that makes borrowing ex ante inefficient. If instead the central bank follows an optimal discretionary monetary policy and intervenes to stabilize asset prices and real activity, the aggregate demand externality disappears reducing the need for ex ante intervention.


[^0]
## 1 Introduction

Many economists and commentators have remarked that the conduct of interest policy by the central bank can affect the incentives of the financial sector. In particular, a common complaint is that the so called "Greenspan put" encourages excessive leverage and risk taking by banks. The argument goes as follows. Suppose that when an adverse shock hits and drives down asset prices, the central bank intervenes systematically by lowering interest rates. This becomes an implicit commitment to prop up asset prices in times of distress and encourages banks and other financial players to borrow more and take on more risk ex ante. In turns, this increases the risk of systemic crises, with possible harmful repercussions on aggregate activity. Therefore, countercyclical interest rate policy may end up increasing, rather than reducing macroeconomic volatility. Countercyclical monetary policy generates a form of moral hazard, where financial firms do not receive help directly, but are subsidized indirectly by the central bank's low interest rate policy.

A number of recent papers have formalized this idea in different ways, including Lorenzoni (2001), Chari and Kehoe (2011), Diamond and Rajan (2012), Farhi and Tirole (2012). This paper attacks the problem from a different perspective. While the existing literature emphasizes the distortionary role of an overly active monetary policy, here we emphasize the distortions that arise when monetary policy is too passive. In other words, we show that the lack of countercyclical interventions, in the face of negative aggregate shocks, can also worsen the problem of excessive leverage in the financial sector. While the existing literature emphasizes a form of pecuniary externality, which travels through asset prices, this paper emphasizes the presence of an aggregate demand externality, which travels through the level of spending. In this respect, our model builds on the recent literature on aggregate demand externalities started by Farhi and Werning (2013), Korinek and Simsek (2016), and Schmitt-Grohé and Uribe (2106).

We present first a simple example in which a countercyclical interest rate policy can completely eliminate the overborrowing distortion. We consider first a monetary policy regime in which the interest rate is completely unresponsive to real activity and asset prices. In that regime, the equilibrium is constrained inefficient and there is overborrowing. Then we show that in our example the interest rate policy can be designed so as to completely eliminate both output volatility and asset price volatility. Such a policy does indeed induce higher borrowing ex ante. However, under this interest rate policy the overborrowing problem also disappears. So even though the level of borrowing goes up, the distance between the lassez faire level of borrowing and the socially (constrained) efficient level is actually reduced to zero.

Two closely related papers are Benigno et al. (2013) and Korinek and Jeanne (2016), who analyze the relative benefits of ex ante and ex post policies to deal with financial instability in different models.

Korinek and Jeanne (2016) look at a model in which the ex post policy is a form of bailout in which resources are transferred from lenders to borrowers. Proposition 6 in their papers shows that when more resources are available for an ex post bailout, this can reduce the need for ex ante macroprudential policy, that is ex ante and ex post policies can be substitutes rather than complements. That result is close in spirit to the message of this paper. However, their model is a purely real model with an exogenous interest rate pinned down by a storage technology, while here we focus on the use of interest rate policy as a tool to stabilize asset prices and deal with a crisis ex post.

Benigno et al. (2013) is closer to our model, in that the ex post intervention is also captured by monetary policy. There are two main differences between the two papers. First, our paper features an explicit role for asset prices, and thus can be used to discuss asset price stabilization by the central bank. Second, our paper focuses on a simpler model and on analytical results, while their analysis is mostly numerical, so we can identify how the result depends on the different roles of pecuniary externalities and aggregate demand externalities.

Finally, our paper is related to a classic debate on the benefits of asset price stabilization as a monetary policy objective. Bernanke and Gertler (1999) made the point that monetary policy should respond to asset price movements insofar as they affect aggregate demand. In this paper, we look indeed at monetary responses to asset prices that are motivated by aggregate demand management, and ask the question whether these responses ex post encourage instability and excess leverage ex ante.

## 2 The model

Consider a three period economy, with $t=1,2,3$. In periods 1 and 3 the economy is an endowment economy. In period 2 output is produced with a linear technology that uses only labor.

There are two groups of agents of equal size, labeled $A$ and $B$. The preferences of agent $A$ are represented by the utility function

$$
\begin{equation*}
E\left[c_{1}^{A}+u\left(c_{2}^{A}\right)-v\left(n^{A}\right)+u\left(c_{3}^{A}\right)\right] \tag{1}
\end{equation*}
$$

where $c_{t}^{A}$ denotes consumption, $n^{A}$ denotes labor effort in period $2, u$ is CRRA and $v$ is a
convex function. The preferences of agents $B$ are represented by the utility function

$$
\begin{equation*}
E\left[c_{1}^{B}+\beta u\left(c_{2}^{B}\right)+\beta^{2} u\left(c_{3}^{B}\right)\right] \tag{2}
\end{equation*}
$$

where $c_{t}^{B}$ denotes consumption and $0<\beta<1$. The assumption of linear utility for both agents in period 1 simplifies the welfare analysis, by making utility transferable ex ante.

Both agents receive a large endowment of the single consumption good in period 1. In period 2, agent $A$ receives labor income by supplying labor on a competitive labor market at the wage rate $w$.

There is a risky asset in fixed supply in the economy, which pays $\delta(s)$ units of consumption good in period 3. There is a discrete set of states of the world $s \in S$, with probability distribution $\pi(s)$. The state of the world $s$ is revealed in period 2 . The risky asset can only be held by agents $B$ in period 1 . In period 1, all agents trade a real non-state contingent bond that pays 1 unit of consumption goods in period 2. Assuming that only $B$ agents can hold the risky asset in period 1 and that they are more impatient than $A$ agents is a simple way of obtaining levered agents, whose balance sheets are exposed to shocks and affect aggregate spending in the economy. The channels captured in this simple model would extend to a richer model of intermediation, with levered intermediaries exposed to aggregate risk who make lending decisions that affect aggregate spending.

In period 2, there is a continuum of monopolistic firms on the interval $[0,1]$ that produce intermediate goods. Each intermediate good is produced with a linear technology that uses only labor $x_{j}=n_{j}$. The goods are then combined to produce consumption goods according to the usual Dixit-Stiglitz aggregator

$$
\left[\int_{0}^{1} x_{j}^{\frac{\epsilon-1}{\epsilon}} d j\right]^{\frac{\epsilon}{\epsilon-1}}
$$

with $\epsilon>1$. The firms selling the differentiated goods set the price of their good $p_{2 j}$ one period in advance, in period 1. The firms are fully owned by $A$ agents. The nominal price level in period $t$ is denoted by $p_{t}$.

Policy is captured by three instruments. First, the central bank sets the nominal interest rate $i_{t}$ in periods 1 and 2 . Second, the government sets a subsidy $\sigma$ on the production of intermediate goods. Third, we allow for a simple macroprudential policy that is a tax on borrowing at $t=1$.

### 2.1 Continuation equilibrium

Let us first characterize a continuation equilibrium, that is, an equilibrium in dates 2 and 3 , given the state variables inherited by the economy from period 1 . This characterization can be done independently of the policy regime. In the next section, we complete the equilibrium characterization, after introducing alternative policy regimes.

Let $D$ denote the real debt of the $B$ agents, which, by market clearing, is also equal to the bonds held by the $A$ agents. Since prices are pre-set one period in advance monetary policy can determine the real interest rate between periods 2 and 3 , which we denote by $r .{ }^{1}$ Since all price setters face the same problem, they will all choose the same nominal price $p_{j 2}=p_{2}$. To characterize a continuation equilibrium we can ignore nominal variables and focus on real variables. In our characterization of a continuation equilibrium, we ignore the optimality condition for labor supply. That condition will be used later to determine the equilibrium wage rate $w$ which enters the price setters's optimality condition. The optimality condition of price setters at time 1 will be discussed below.

The balance sheet of the $B$ agents entering period 2 is

$$
Q-D,
$$

where $Q$ is the (real) price of the risky asset which is just given by

$$
Q=\frac{\delta}{1+r}
$$

Given $D$ and $r$, a continuation equilibrium is then simply given by the quantities $c_{2}^{A}, c_{3}^{A}, c_{2}^{B}, c_{3}^{B}, n^{A}$ that satisfy: ${ }^{2}$

- optimality for the $A$ agents, given by the conditions ${ }^{3}$

$$
\begin{gathered}
u^{\prime}\left(c_{2}^{A}\right)=(1+r) u^{\prime}\left(c_{3}^{A}\right) \\
c_{2}^{A}+\frac{1}{1+r} c_{3}^{A}=n^{A}+D
\end{gathered}
$$

[^1]- optimality for the $B$ agents, given by the conditions

$$
\begin{aligned}
& u^{\prime}\left(c_{2}^{B}\right)=(1+r) \beta u^{\prime}\left(c_{3}^{B}\right), \\
& c_{2}^{B}+\frac{1}{1+r} c_{3}^{B}=\frac{1}{1+r} \delta-D
\end{aligned}
$$

- market clearing for the goods markets

$$
c_{2}^{A}+c_{2}^{B}=n^{A}, \quad c_{3}^{A}+c_{3}^{B}=\delta .
$$

We use $Y$ to denote output in period 2, which is equal to $n^{A}$. Then the definition of continuation equilibrium above defines a mapping $Y(D, r, s)$ which gives equilibrium output in period 2 as a function of the initial debt level and of the interest rate.

A complete characterization of the equilibrium requires three additional steps: determining how montary policy chooses the level of $r$ in each state of the world; ensuring that the price setters optimality condition is satisfied at $t=1$; determining the level of $D$ which arises in equilibrium at $t=1$. The solution of these steps depends on the monetary policy regime. In the next section we will consider different possible regimes and complete the equilibrium characterization in each regime.

## 3 Monetary policy regimes

In this section we characterize the equilibrium under three different policy regimes. For the moment, we assume that there is no macroprudential intervention in place at date 1, so the only policy tools available are the choice of the nominal interest rate and the choice of the subsidy $\sigma$. In the next sections, we will investigate the benefits of adding macroprudential policy under each monetary policy regime.

We first introduce two value functions which are useful in our equilibrium characterization. Let $V^{A}(b, D, r, s)$ denote the expected utility in state $s$ of an agent $A$ who enters period 2 with $b$ units of bonds in an economy in which all other $A$ agents hold $D$ units of bonds, all other $B$ agents have $D$ units of debt, and in which the real interest rate is $r$. Let $V^{B}(d, D, r, s)$ denote the analogous value for the $B$ agents, with $d$ denoting the individual level of debt. In equilibrium, we have $b=d=D$, but for the analysis it is convenient to separate individual decisions from aggregates.

It is useful to give explicit definitions for $V^{A}$ and $V^{B}$ :

$$
\begin{aligned}
V^{A}(b, D, r, s) & \equiv \max _{c_{2}^{A}, c_{3}^{A}} u\left(c_{2}^{A}\right)-v(Y(D, r, s))+u\left(c_{3}^{A}\right) \quad \text { s.t. } \quad c_{2}^{A}+\frac{1}{1+r} c_{3}^{A}=Y(D, r, s)+b, \\
V^{B}(d, D, r, s) & \equiv \max _{c_{2}^{B}, c_{3}^{B}} u\left(c_{2}^{B}\right)+\beta u\left(c_{3}^{B}\right) \quad \text { s.t. } \quad c_{2}^{B}+\frac{1}{1+r} c_{3}^{B}=\frac{1}{1+r} \delta(s)-d .
\end{aligned}
$$

Notice that we are letting $A$ agents choose consumption optimally but not labor supply. This comes from the fact that, as argued above, optimality for labor supply is not used in characterizing a continuation equilibrium.

Using the envelope theorem and using optimality conditions to replace the Lagrange multipliers on the two budget constraints with, respecively, $u^{\prime}\left(c_{2}^{A}\right)$ and $u^{\prime}\left(c_{2}^{B}\right)$, we obtain the following lemma.

Lemma 1. At $b=d=D$ the following conditions hold

$$
\begin{aligned}
\frac{\partial V^{A}}{\partial b} & =u^{\prime}\left(c_{2}^{A}\right) \\
\frac{\partial V^{B}}{\partial d} & =-u^{\prime}\left(c_{2}^{B}\right), \\
\frac{\partial V^{A}}{\partial D} & =\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial D} \\
\frac{\partial V^{B}}{\partial D} & =0 \\
\frac{\partial V^{A}}{\partial r} & =u^{\prime}\left(c_{2}^{A}\right)\left(Y+D-c_{2}^{A}\right) \frac{1}{1+r}+\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r} \\
\frac{\partial V^{B}}{\partial r} & =-u^{\prime}\left(c_{2}^{B}\right)\left(c_{2}^{B}+D\right) \frac{1}{1+r} .
\end{aligned}
$$

Throughout the paper we assume that policy aims to maximize the ex ante welfare criterion

$$
\begin{equation*}
E\left[V^{A}(D, D, r, s)+\beta V^{B}(D, D, r, s)\right] \tag{3}
\end{equation*}
$$

with side transfers taking place at date 0 to reallocate the welfare gains among $A$ and $B$ agents. Notice that the presence of heterogeneous discount factors introduces one possible source of time inconsistency in optimal policy. By focusing on the ex ante welfare criterion (3) we leave aside that source of time inconsistency (other interesting sources are present, as we shall see).

### 3.1 Rigid regime

The first regime features a rigid interest rate policy at $t=2$. Under this regime the timing is as follows. First, the equilibrium debt level $d=b=D$ is determined in the bond market at date $1 .{ }^{4}$ Next, at the beginning of period 2, before the realization of the state $s$, the central bank chooses a non-state-contingent interest rate $r=\bar{r}$ to maximize (3). After the realization of $s$, the central bank cannot revisit its choice of $r$. Agents choose their spending in period 2 and that determines output. Finally, in period 3, agents consume their endowment net of their final bonds positions. The subsidy $\sigma$ is set to ensure that the price setting condition of the producers at $t=1$ is satisfied. ${ }^{5}$ The crucial restriction in this regime is that the central bank cannot use the interest rate policy to mitigate the drop in the asset price $Q$ for low realizations of $\delta$. The idea is that by restricting its response the central bank hopes to reduce the incentive of $B$ agents to borrow in period 1. As we shall see, this is indeed what happens in equilibrium.

An equilibrium under this regime is given by a pair $D, \bar{r}$ that satisfies the following two conditions:

- optimal monetary policy, characterized by the first-order condition

$$
\begin{equation*}
E\left[\frac{\partial V^{A}(D, D, \bar{r}, s)}{\partial r}+\beta \frac{\partial V^{B}(D, D, \bar{r}, s)}{\partial r}\right]=0 \tag{4}
\end{equation*}
$$

- equilibrium in the bond market at date 1 , characterized by the condition

$$
\begin{equation*}
E\left[\frac{\partial V^{A}(D, D, \bar{r}, s)}{\partial b}\right]=\beta E\left[\frac{\partial V^{B}(D, D, \bar{r}, s)}{\partial d}\right] . \tag{5}
\end{equation*}
$$

Recall our timing assumption that the central bank chooses the interest rate $\bar{r}$ after the bonds market in period 1 has cleared. This means that the central bank takes $D$ as given when choosing $\bar{r}$. At the same time, consumers do not internalize the effect that the equilibrium level of $D$ has on the central bank's choice of $\bar{r}$, simply because they are atomistic. This explains why strategic considerations do not appear in conditions (4) and (5).

[^2]Using Lemma 1 condition (4) can be rewritten as

$$
\begin{equation*}
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r}\right]+E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right)\left(Y+D-c_{2}^{A}\right)\right] \frac{1}{1+r}=0 \tag{6}
\end{equation*}
$$

this condition shows that the central bank is balancing two effects of changing interest rates. The first effect is a standard new Keynesian effect: changing the interest rate affects equilibrium output and this increases or decreases welfare depending on the sign of $u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)$. The difference $u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)$ captures the welfare effect of a change in equilibrium output. With flexible prices the difference would always be zero, but in a new Keynesian environment this term reflects the presence of an efficiency loss which we call an "output gap". When $u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)>0$ we say there is a positive output gap, as increasing hours worked leads to a marginal social benefit in terms of additional output which is greater than the social marginal cost. ${ }^{6}$ The second effect is a pecuniary externality channel associated to incomplete markets: changing the interest rate re-allocates resources from borrowers to lenders and if the marginal utilities of borrowers and lenders are different ex post this can have welfare benefits. Here, given that $r$ is not allowed to be state contingent, the monetary authority chooses a level of the interest rate that balances these two welfare effects in expectation.

Also using Lemma 1 we can rewrite (5) as follows

$$
\begin{equation*}
E u^{\prime}\left(c_{2}^{A}\right)=\beta E u^{\prime}\left(c_{2}^{B}\right) \tag{7}
\end{equation*}
$$

which shows that debt level ex ante is chosen to equalize expected marginal utilities ex post.

Conditions (6) and (7) will be used to characterize the equilibrium in the rest of the paper.

### 3.2 Flexible regime

The second regime features a fully state contingent interest rate policy at $t=2$. The timing is as in the previous regime, but the monetary authority sets $r(s)$ optimally state by state. As we shall see, this flexible policy will mitigate the drop in asset prices by

[^3]reducing interest rates when the realization of $\delta$ is lower.
An equilibrium under this regime is given by a debt level $D$ and interest rates $\{r(s)\}_{s \in S}$ that satisfy the following two conditions:

- optimal monetary policy, characterized by the first-order condition

$$
\begin{equation*}
\frac{\partial V^{A}(D, D, r(s), s)}{\partial r}+\beta \frac{\partial V^{B}(D, D, r(s), s)}{\partial r}=0 \text { for all } s \in S ; \tag{8}
\end{equation*}
$$

- equilibrium in the bond market at date 1 , characterized by the condition

$$
\begin{equation*}
E\left[\frac{\partial V^{A}(D, D, r(s), s)}{\partial b}\right]=\beta E\left[\frac{\partial V^{B}(D, D, r(s), s)}{\partial d}\right] . \tag{9}
\end{equation*}
$$

As in the previous regime, Lemma 1 can be used to rewrite these conditions. In particular, now (6) holds state by state and takes the form

$$
\begin{equation*}
\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r}+\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right)\left(Y+D-c_{2}^{A}\right) \frac{1}{1+r}=0 \tag{10}
\end{equation*}
$$

Condition (9) takes the same form (7) as in the rigid regime.

### 3.3 Output gap targeting

The third regime we consider, is a regime in which the central bank aims to replicate the equilibrium that would arise if good prices were flexibly set at $t=2$, rather than pre-set a period in advance. We assume the subsidy $\sigma$ is set exactly at the level that offsets the monopolistic distortion, so we are looking at a regime in which the following condition holds state by state

$$
u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)=0
$$

The ex ante optimality condition that ensures equilibrium in the debt market at $t=1$ is the same as in the previous regimes

$$
E u^{\prime}\left(c_{2}^{A}\right)=\beta E u^{\prime}\left(c_{2}^{B}\right) .
$$

The previous conditions, together with the conditions characterizing a continuation equilibrium, are sufficient to pin down the equilibrium allocation under this regime. However, for the analysis in the next sections we need to be precise on the way in which this allocation is implemented. Namely, we assume that the central bank derives the real interest rates $r(s)$ that implement the flexible-price allocation just defined and commits at
date 1 to set those interest rates. Therefore this regime is different from the previous two regimes not only because of the specific objective the central bank is aiming for, but also for its commitment to future interest rates. As we shall see, these assumptions will deliver a sharp characterization of this regime in terms of the benefits of macroprudential policy.

## 4 More borrowing, less overborrowing

We now turn to our main two questions: How does the monetary regime influence the level of borrowing ex ante? How does the monetary regime influence the benefits of macroprudential intervention ex ante? In a simple example with log preferences, the answers to these two questions go in opposite directions, which is the main point of the paper. In this section, we present the log example and provide some intuition for it . In the next section, we go beyond the example to investigate some more general implications of our analysis.

Let us first briefly characterize a continuation equilibrium with log preferences. Consumer optimization yields

$$
c_{2}^{A}=\frac{1}{2}(Y+D),
$$

and

$$
c_{2}^{B}=\frac{1}{1+\beta}\left(\frac{\delta}{1+r}-D\right) .
$$

Combining these conditions with the goods market clearing condition we obtain

$$
Y=c_{2}^{A}+c_{2}^{B}=\frac{1}{2}(Y+D)+\frac{1}{1+\beta}\left(\frac{\delta}{1+r}-D\right)
$$

and we can derive equilibrium output

$$
Y=\frac{1}{1+\beta} \frac{\delta}{1+r}+\left(1-\frac{2}{1+\beta}\right) D
$$

Notice that due to the lower discount factor, the $B$ agents have a higher marginal propensities to consume than the $A$ agents (the MPC it is $1 /(1+\beta)$ for $B$ agents and $1 / 2$ for $A$ agents). This implies that an increase in $D$, which is a transfer from $B$ agents to $A$ agents, has a contractionary effect on output:

$$
\begin{equation*}
\frac{\partial Y}{\partial D}=-2\left(\frac{1}{1+\beta}-\frac{1}{2}\right)<0 \tag{11}
\end{equation*}
$$

Proposition 1. With log preferences, the flexible interest rate regime and the output gap targeting regime yield the same allocation and feature a higher level of borrowing $D$ in equilibrium than the rigid interest rate regime.

To get some intuition for this result, it is useful to first characterize the consumption of the borrowers under the two regimes. With log preferences the expression for consumption in period 2 is

$$
c_{2}^{B}=\frac{1}{1+\beta}\left(\frac{\delta}{1+r}-D\right),
$$

in both regimes. In the rigid regime, $r$ is constant and equal to $\bar{r}$. So the consumption of the $B$ agent is sensitive to the $\delta$ shock. In the flexible regime, instead, we can show that the optimal interest rate policy ex post entails a level of the gross interest rate $1+r$ which exactly offsets movements in $\delta$. In this example, the optimal discretionary policy achieves perfect stabilization of asset prices through interest rate policy, an especially stark example of the Greenspan put. Summing up, in the rigid regime, $B$ agents are exposed to the $\delta$ shock, while in the flexible regime they are perfectly insured by endogenous movements in $r$.

Turning now to the ex ante choice of $D$, we can see that the rigid regime makes it more costly to borrow ex ante, because the cost of repaying debt is higher in bad states of the world, in which $c_{2}^{B}$ is lower. In the flexible regime, instead, borrowers are perfectly insured and this induces them to take on more debt ex ante. From this positive description this would seem like a perfect example of the evil incentive effects of the Greenspan put. However, as we shall see in the next proposition, the normative conclusions are quite different.

To evaluate the benefits of macroprudential policy we look at the effect of a marginal change in $D$ near the equilibrium with a zero borrowing tax.

Proposition 2. With log preferences, in the rigid interest rate regime there is excessive borrowing ex ante, that is, social welfare can be increased by reducing $D$ :

$$
\frac{d}{d D} E\left[V^{A}(D, D, \bar{r}, s)+\beta V^{B}(D, D, \bar{r}, s)\right]<0
$$

In the flexible interest rate regime and in the output gap targeting regime the level of borrowing is socially efficient and

$$
\frac{d}{d D} E\left[V^{A}(D, D, r(s), s)+\beta V^{B}(D, D, r(s), s)\right]=0
$$

In order to build intuition for this result, let us first provide some derivations that
characterize the marginal social benefit of a change in $D$. Notice that using the results in Lemma 1 and the private optimality condition for debt

$$
E u^{\prime}\left(c_{2}^{A}\right)=\beta E u^{\prime}\left(c_{2}^{B}\right),
$$

the marginal welfare effects of a change in $D$ is equal to

$$
\begin{equation*}
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial D}\right] \tag{12}
\end{equation*}
$$

This is true in all monetary regimes considered. ${ }^{7}$
Let us now understand each term in this expression.
Notice that with $\log$ preferences $\partial Y / \partial D$ is constant and independent of the state of the world, of the real interest rate and of the monetary regime, so we can focus on establishing the sign of $E\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right]$.

In the rigid regime, it is possible to show that there is a positive output gap in the states in which $\delta$ is low and a negative output gap in the states in which $\delta$ is high. This is due to the fact that in that regime the risk $\delta$ is not insured and a reduction in $\delta$ reduces the wealth of the $B$ agents who have a relatively larger marginal propensity to consume. Therefore the sign of $E\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right]$ depends on whether the positive output gap wedge $u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)$ dominates the negative output gap wedge in good states. The proof of Proposition 2 in the Appendix shows that the first effect dominates. In economic terms, borrowing more ex ante exarcebates recessions in the bad states and dampens excessive booms in good states. The first effect dominates in welfare terms, so, overall, borrowing more ex ante is welfare detrimental.

Turning to the other two regimes, it turns out that in both the allocation is the same and the output gap is always zero, that is, $u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)=0$, state by state. Therefore, in the flexible regime there is no welfare gain from changing the level of borrowing ex ante, as the level of output is already at its socially efficient level in every state.

Summing up, using interest rate policy to fight a recession leads to higher borrowing ex ante, but this is not a symptom of inefficient borrowing ex ante. In fact, the ex post policy makes the ex ante borrowing higher but it eliminates the distance between equilibrium borrowing and its socially efficient level.

[^4]
## 5 Tradeoffs, complementarity and substitutability

In the case of $\log$ preferences the flexible interest rate regime achieves the same allocation as the zero output gap regime. To understand why that is the case, let us go back to the optimality condition for $r$ in the flexible regime

$$
\begin{equation*}
\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r}+\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right)\left(Y+D-c_{2}^{A}\right) \frac{1}{1+r}=0 \tag{13}
\end{equation*}
$$

As argued above, this condition includes an output gap term and an insurance term, as lowering interest rates increases output and, at the same time, reallocates resources between borrowers and lenders. A special feature of our log preference example is that the flexible regime is able to set both terms to zero at the same time. In other words, by choosing the state contingent interest rate $r$ the central bank can take care of both frictions present in our environment: rigid prices and lack of insurance against asset price movements for $B$ agents. This is clearly a knife-edge result which makes the result sharper but leaves us with some open questions. What happens when monetary policy ex posts faces a real trade off? Namely, when monetary policy has to choose between a traditional macro objective (stabilizing aggregate output) and an objective of financial stability (using the interest rate to redistribute in favor of $B$ agents hit by a negative shock)? Is macroprudential policy a useful tool when the monetary policy instrument is not enough to address both frictions? Is macroprudential still less needed when monetary policy is more flexible? In this section we address these questions, turning to the general case of CRRA preferences with a coefficient of relative risk aversion $\gamma \neq 1$.

### 5.1 Misalinged output and financial stability objectives

First, let us understand better the nature of the trade off faced by the monetary authority ex post. To do so it is useful to concentrate on the equilibrium in the output gap targeting regime, where, by definition, the first term in (13) is set to zero. When $\gamma<1$ we can show that the asset price $Q$, and thus the consumption of the $B$ agents $c_{2}^{B}$, is lower in the states when $\delta$ is lower, while the consumption of the $A$ agents goes in the opposite direction. Equilibrium in the bonds market at $t=1$ requires

$$
E\left[u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right]=0 .
$$

Therefore, we have

$$
u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)<0
$$

Figure 1: Output gap targeting and flexible regimes

Interest Rates


Output

in the low $\delta$ states and

$$
u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)>0
$$

in the high $\delta$ states. This means that, starting from pure output gap stabilization, the central bank's incentive is to further lower interest rates in the bad states and to further increase them in the good states. That is, the central bank is driven to stabilize asset prices over and above what is required to stabilize output. This happens because the insurance motive is still present at the zero output gap allocation, so moving one dollar from lenders to borrowers in the bad state means transferring it to agents that assign a relatively higher marginal value for that dollar. Since the $B$ agents are borrowers $\left(c_{2}^{B}=Y+D-c_{2}^{A}>0\right)$, reducing the interest rate achieves this transfer.

A numerical example illustrates this result. ${ }^{8}$ In Figure 1 we show how the interest rate $r$ responds to $\delta$ in the output gap targeting regime and in the flexible, discretionary optimal, regime. In the second case, the central bank uses the interest rate more aggressively in response to bad shocks.

[^5]
### 5.2 Implications for macroprudential policy: an example where more ex ante policy is needed

Let us turn now to macroprudential policy. As we did above, let us focus on the marginal benefit of changing $D$ starting from no macroprudential tax. This benefit is still equal to

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial D}\right],
$$

as derived above. In the output gap targeting regime this expression is immediately equal to zero. So in that regime there is never any benefit from imposing a macroprudential tax. In the flexible regime, on the other hand, this expression is in general different from zero once $\gamma \neq 1$. In the numerical example above the value of this expression is negative.

We derive two conclusions from this analysis. One is that once we move away from our knife-edge log case, optimal policy requires a combination of monetary tools and macroprudential tools. So montary policy is no longer a perfect substitute for macroprudential policy. The second conclusion, is that we finally have a result more in line with conventional wisdom on moral hazard effects. When we go from a regime of pure output gap targeting to a fully discretionary regime, the central bank increases its interest rate response, stabilizes asset prices more, and we have a stronger motive to impose a borrowing tax ex ante. Digging a bit more in the intuition, though, shows us how much this result relies on the central bank exceeding its output objectives. In the bad states of the world the central bank is choosing to overstimulate the economy, so we are reaching an allocation with $u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)<0$. In the good states of the world the opposite is true. Inducing agents to borrow less ex ante leads to more output ex post. This is actually bad in the bad state of the world where the economy is already overstimulated. So the benefit has to come from the good state of the world, in which output is below potential. So while it is interesting that our model can deliver a moral hazard result, the effect here relies on the fact that monetary policy is too recessionary in the good states of the world.

### 5.3 Implications for macroprudential policy: another example where less ex ante policy is needed

So far we focused on comparing our second and third regime. What happens when we go back to our first regime of completely rigid interest rates? In that case, the sign of the macroprudential policy can be derived analytically, extending the first part of Proposition 2.

Table 1: Macroprudential benefits in the rigid and flexible regimes

|  | Rigid regime | Flexible regime |
| :--- | :---: | :---: |
| Absolute level of overborrowing | 0.0449 | 0.0028 |
| Welfare increase due to macro prudential tools | $0.58 \%$ | $0.05 \%$ |
| Marginal benefit of increasing $D\left(\frac{\partial W}{\partial D}\right)$ | -0.120 | -0.004 |

Proposition 3. In the rigid interest rate regime there is always overborrowing

$$
\frac{d}{d D} E\left[V^{A}(D, D, \bar{r}, s)+\beta V^{B}(D, D, \bar{r}, s)\right]<0 .
$$

To compare the rigid regime to the flexible regime we go back to our numerical example. In Table 1 we report two measures of macroprudential intervention. We show the value of $D$ in the case of no borrowing tax and its value at the optimal tax and we show the value of the marginal benefit of increasing $D$ under no borrowing tax (which has been the focus of our analytical derivations so far). We know that both measures identify no need for macroprudential intervention in the output targeting regime. But what is more interesting is that the rigid regime requires larger macroprudential intervention relative to the flexible regime, thus extending our result of the previous section.

We can then give a more nuanced answer the question: does a more aggressive countercyclical monetary policy increases or decreases the need for ex ante regulation? The answer turns out to be sensitive not only to the parameters of the model but to our starting point in the policy space. If our starting point is a monetary policy that does not respond at all to negative shocks a more aggressive policy reduces the need for macroprudential policy, as it mitigates the aggregate demand externality. If our starting point is a monetary policy that is already responding strongly enough so as to eliminating the output gap, then a more aggressive policy increases the need for macroprudential policy.

These specific conclusions are of course in part driven by the special nature of our example. In all the numerical examples we have explored the benefit of macroprudential policy are lower in the flexible regime as compared to the rigid regime, but we do not know if that can be proved in general. However, the message that seems to be easy to generalize is that moral hazard effects are not necessarily associated to a more aggressive use of policy ex post and that they might or might not appear, depending on our starting point in the policy space.

## 6 Unconventional monetary policy

[To be completed]

## 7 Appendix

### 7.1 Characterization of continuation equilibrium

Here we provide a characterization of a continuation equilibrium. This characterization will be used in the proofs to follow. The utility function $u$ has the CRRA form

$$
u(c)=c^{1-\gamma}
$$

From consumer optimization, we obtain

$$
c_{2}^{A}=\frac{Y+D}{1+(1+r)^{\frac{1}{\gamma}-1}}
$$

and

$$
c_{2}^{B}=\frac{1}{1+\beta^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}-1}}\left(\frac{\delta}{1+r}-D\right)
$$

Aggregating and using goods market clearing we have

$$
Y=\frac{Y+D}{1+(1+r)^{\frac{1}{\gamma}-1}}+\frac{1}{1+\beta^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}-1}}\left(\frac{\delta}{1+r}-D\right)
$$

After rearranging and defining $\tilde{R}=(1+r)^{\frac{1-\gamma}{\gamma}}$ and $\tilde{\beta}=\beta^{\frac{1}{\gamma}}$, the continuation equilibrium quantities in period 2 can be written as follows:

$$
\begin{align*}
c_{2}^{A} & =\frac{1}{1+\tilde{R}}(Y+D),  \tag{14}\\
c_{2}^{B} & =\frac{1}{1+\tilde{\beta} \tilde{R}}\left(\frac{\delta}{1+r}-D\right),  \tag{15}\\
Y & =-\frac{1-\tilde{\beta}}{1+\tilde{\beta} \tilde{R}} D+\frac{1+\tilde{R}}{\tilde{R}(1+\tilde{\beta} \tilde{R})} \frac{\delta}{1+r} . \tag{16}
\end{align*}
$$

Taking derivatives of the last expression with respect to $D$ and and $r$ we obtain

$$
\begin{align*}
\frac{\partial Y}{\partial D} & =-\frac{1-\tilde{\beta}}{1+\tilde{\beta} \tilde{R}^{\prime}}  \tag{17}\\
\frac{\partial Y}{\partial r} & =\theta_{1}(r) D+\theta_{2}(r) \delta \tag{18}
\end{align*}
$$

where

$$
\begin{aligned}
& \theta_{1}(r)=\frac{1}{1+r} \frac{1-\gamma}{\gamma} \frac{(1-\tilde{\beta}) \tilde{\beta} \tilde{R}}{(1+\tilde{\beta} \tilde{R})^{2}} \\
& \theta_{2}(r)=-\frac{1}{(1+r)^{2}} \frac{1}{\tilde{R}(1+\tilde{\beta} \tilde{R})^{2}} \frac{1}{\gamma}[1+\tilde{\beta} \tilde{R}+(\gamma(1-\tilde{\beta})+\tilde{\beta}(1+\tilde{R})) \tilde{R}]
\end{aligned}
$$

Notice that $\theta_{2}(r)$ is negative, while the sign of $\theta_{1}(r)$ depends on whether $\gamma$ is greater or lower than 1.

With $\log$ preferences $(\gamma=1)$ the expressions above become:

$$
\begin{gathered}
c_{2}^{A}=\frac{1}{2}(Y+D), \quad c_{2}^{B}=\frac{1}{1+\beta}\left(\frac{\delta}{1+r}-D\right) \\
Y=-\frac{1-\beta}{1+\beta} D+\frac{2}{1+\beta} \frac{\delta}{1+r}
\end{gathered}
$$

### 7.2 Proofs for Section 4

Let us first characterize the equilibrium with flexible interest rate regime in the following lemma.

Lemma 2. With log preferences the flexible interest rate regime and the output gap targeting regime lead to the same allocation with a constant asset price

$$
Q=\frac{\delta}{1+r}=\bar{Q}
$$

and perfect insurance for the $B$ agents ( $c_{2}^{B}$ constant across states).
Proof. We guess and verify that an equilibrium with constant asset prices satisfies all equilibrium conditions in both regimes. Consumption levels in period 2 are

$$
c_{2}^{A}=\frac{1}{2}(Y+D)
$$

and

$$
c_{2}^{B}=\frac{1}{1+\beta}(\bar{Q}-D) .
$$

So we need to find values for $D$ and $\bar{Q}$ that satisfy ex ante equilibrium for $D$

$$
u^{\prime}\left(\frac{1}{2}(Y+D)\right)=\beta u^{\prime}\left(\frac{1}{1+\beta}(\bar{Q}-D)\right)
$$

and the zero output gap condition

$$
u^{\prime}\left(\frac{1}{2}(Y+D)\right)=v^{\prime}(Y)
$$

Finding explicit expressions for $D$ and $\bar{Q}$ requires specifying the function $v$, but in any case the solution will be unique. That the conditions for the output gap targeting regime are satisfied is immediate. We just need to check that also the optimal monetary policy condition (10) is satisfied. That is easy to check since both terms of that condition are equal to zero in all states.

### 7.2.1 Proof of Proposition 1

The aggregate level of debt in the economy is pinned down by the following optimality condition.

$$
\beta E\left[\left(c_{2}^{B}\right)^{-1}\right]-E\left[\left(c_{2}^{B}+D\right)^{-1}\right]=0 .
$$

Substituting in the expression for $c_{2}^{B}$ and rearrangin we have

$$
\beta E\left[\left(\frac{\delta}{1+r}-D\right)^{-1}\right]-E\left[\left(\frac{\delta}{1+r}+\beta D\right)^{-1}\right]=0
$$

Let $D^{F}$ denote the equilibrium level of aggregate debt in the flexible interest rate regime. In order to show the desired result, we shall show that in the rigid regime the expression above evaluated at the $D^{F}$ is positive. We shall first show that when $\frac{\delta}{1+r}$ varies but has the same mean as ths ratio in the flexible regime the expression above is positive. Let us use a change of variables. Define $\eta=\frac{\delta}{1+r}-D^{F}$, so that the aggregate level of debt satisfies

$$
E\left[\frac{\beta}{\eta}-\frac{1}{\eta+(1+\beta) D^{F}}\right]=0
$$

Notice that in the flexible interest rate regime equilibrium $\eta$ is constant at level $\bar{\eta}=$ $\frac{1+\beta}{1-\beta} \beta D$. Let the expression in the expectation be defined as $f(\eta)$. Taking first and second order derivatives of this function we have

$$
\begin{aligned}
f^{\prime}(\eta) & =-\left(\frac{\beta}{\eta^{2}}-\frac{1}{\left(\eta+(1+\beta) D^{F}\right)^{2}}\right) \\
f^{\prime \prime}(y) & =2\left(\frac{\beta}{\eta^{3}}-\frac{1}{\left(\eta+(1+\beta) D^{F}\right)^{3}}\right)
\end{aligned}
$$

The second derivative implies that this function is convex when $\eta \leq \frac{1+\beta}{1-\beta^{\frac{1}{3}}} \beta^{\frac{1}{3}} D^{F}$, and increasing when $\eta \geq \frac{1+\beta}{1-\beta^{\frac{1}{2}}} \beta^{\frac{1}{2}} D^{F}$. The latter also implies that the function is increasing in the concave region, $\eta \leq \frac{1+\beta}{1-\beta^{\frac{1}{3}}} \beta^{\frac{1}{3}} D^{F}$. Consider the following function

$$
g(\eta)= \begin{cases}f(\eta) & \text { if } \eta \leq \frac{1+\beta}{1-\beta^{\frac{1}{3}}} \beta^{\frac{1}{3}} D^{F} \\ f\left(\frac{1+\beta}{1-\beta^{\frac{1}{3}}} \beta^{\frac{1}{3}} D^{F}\right) & \text { if } \eta>\frac{1+\beta}{1-\beta^{\frac{1}{3}}} \beta^{\frac{1}{3}} D^{F}\end{cases}
$$

This function is convex, is pointwise lower than $f(\eta)$, and $g(\bar{\eta})=f(\bar{\eta})=0$. Therefore, we have that

$$
E\left[\frac{\beta}{\eta}-\frac{1}{\eta+(1+\beta) D^{F}}\right] \geq g(E(\eta)) \geq g(\bar{\eta})=0
$$

Let us define $\tilde{\eta}=\frac{\delta}{1+r^{R}}-D^{F}$. We shall now show that $\frac{\delta}{1+r^{R}} \leq E\left[\frac{\delta}{1+r^{F}}\right]$.
Lemma 3. In the rigid interest rate regim, the economy, on average, has a positive output gap. That is,

$$
E\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right]>0
$$

Proof. The proof is the first step of the proof of proposition 2. It is true for the general CRRA utility function.

### 7.3 Proof of Proposition 2

Step 1: Show that

$$
E\left[\left[u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right]\left(c_{2}^{B}+D\right)\right]>0
$$

Notice that given CRRA and a constant interest rate the ratio $c_{2}^{A} /(Y+D)$ is constant. Since $Y=c_{2}^{A}+c_{2}^{B}$, this implies that there is a constant $\xi$ such that

$$
c_{2}^{A}=\xi\left(c_{2}^{B}+D\right)
$$

We then want to evaluate

$$
E\left[\left[\left(\xi\left(c_{2}^{B}+D\right)\right)^{-\gamma}-\beta\left(c_{2}^{B}\right)^{-\gamma}\right]\left(c_{2}^{B}+D\right)\right] .
$$

Notice that there is a unique cutoff $\hat{c}_{2}^{B}$ such that

$$
\left(\xi\left(c_{2}^{B}+D\right)\right)^{-\gamma}-\beta\left(c_{2}^{B}\right)^{-\gamma} \gtrless 0
$$

iff $c_{2}^{B} \gtrless \hat{c}_{2}^{B}$. Therefore

$$
E\left[\left[\left(\xi\left(c_{2}^{B}+D\right)\right)^{-\gamma}-\beta\left(c_{2}^{B}\right)^{-\gamma}\right]\left(c_{2}^{B}-\hat{c}_{2}^{B}\right)\right]>0 .
$$

Moreover, from consumers optimality at $t=1$ we have

$$
E\left[\left[\left(\xi\left(c_{2}^{B}+D\right)\right)^{-\gamma}-\beta\left(c_{2}^{B}\right)^{-\gamma}\right]\right]=0
$$

Combining the last two equations we have

$$
E\left[\left[\left(\xi\left(c_{2}^{B}+D\right)\right)^{-\gamma}-\beta\left(c_{2}^{B}\right)^{-\gamma}\right]\left(c_{2}^{B}+D\right)\right]>0
$$

Step 2. From Step 1 and optimality of monetary policy we deduce that

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r}\right]<0
$$

With log preferences and a constant interest rate we deduce that

$$
\begin{equation*}
\frac{\partial Y}{\partial r}=-\Xi_{1} \frac{\delta}{(1+\bar{r})^{2}} \text { and } \frac{\partial Y}{\partial D}=-\Xi_{2} \tag{19}
\end{equation*}
$$

for some positive constant terms $\Xi_{1}, \Xi_{2}$. Therefore, we have the inequality

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \delta\right]>0
$$

The expression $u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)$ is a monotone decreasing function of $\delta$, so we have the chain of inequalities

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right)\right] E[\delta]>E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \delta\right]>0
$$

Using (19) we then conclude that

$$
E\left[\frac{\partial V^{A}}{\partial D}+\beta \frac{\partial V^{B}}{\partial D}\right]=E\left[\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right] \frac{\partial Y}{\partial D}\right]<0
$$

### 7.4 Proof of Proposition 3

Throughout the proof all prices and quantities are in the rigid regime equilibrium. We want to prove

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial D}\right]<0
$$

and equation (17) shows that $\partial Y / \partial D$ is negative and constant across states in the rigid regime. So we need to prove

$$
\begin{equation*}
E\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right]>0 \tag{20}
\end{equation*}
$$

Since interest rate $\bar{r}$ is chosen optimally, the following condition holds

$$
E\left[\frac{1}{1+r}\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right)\left(c_{2}^{B}+D\right)+\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r}\right]=0
$$

Using the optimal debt choice condition and the fact that $D$ and $r$ are independent of the state we can rearrange this into

$$
\begin{equation*}
\frac{1}{1+r} E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right) c_{2}^{B}\right]+E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r}\right]=0 \tag{21}
\end{equation*}
$$

Rewrite (14) as

$$
c_{2}^{A}=\frac{1}{1+\tilde{R}}\left(c_{2}^{A}+c_{2}^{B}+D\right)
$$

to get

$$
c_{2}^{A}=\frac{c_{2}^{B}+D}{\tilde{R}} .
$$

This implies

$$
u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)=\tilde{R}^{\gamma}\left(c_{2}^{B}+D\right)^{-\gamma}-\beta\left(c_{2}^{B}\right)^{-\gamma}
$$

This expression satisfies single crossing in $c_{2}^{B}$, that is, there exists a level $\hat{c}_{2}^{B}$ such that the expression is zero at $c_{2}^{B}=\hat{c}_{2}^{B}$ and is positive if and only if $c_{2}^{B}>\hat{c}_{2}^{B}$. This, together with $E\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right)=0$, implies that

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right) c_{2}^{B}\right]=E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-\beta u^{\prime}\left(c_{2}^{B}\right)\right)\left(c_{2}^{B}-\hat{c}_{2}^{B}\right)\right]>0 .
$$

Equation (21) then implies

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right) \frac{\partial Y}{\partial r}\right]<0
$$

Substituting in the derivative of output with respect to the interest rate (18), we obtain

$$
E\left[\left(u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right)\left(\theta_{1}(r) D+\theta_{2}(r) \delta\right)\right]<0
$$

Rearranging the expression above, using $\theta_{2}(r)<0$, we have

$$
\begin{equation*}
\frac{\theta_{1}(r) D+\theta_{2}(r) E(\delta)}{\theta_{2}(r)} E\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y)\right]>-\operatorname{cov}\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y), \delta\right] . \tag{22}
\end{equation*}
$$

Since $Y$ and $c_{2}^{A}$ are increasing in $\delta$, together with concavity of $u$ and convexity of $v$, we have that $\operatorname{cov}\left[u^{\prime}\left(c_{2}^{A}\right)-v^{\prime}(Y), \delta\right]<0$. Therefore, a necessary and sufficient condition for equation (20) to hold is

$$
\theta_{1}(r) D+\theta_{2}(r) E(\delta)<0
$$

Substituting in the expressions for $\theta_{1}(r)$ and $\theta_{2}(r)$ and rearranging we have

$$
\begin{equation*}
\left(\frac{1+\tilde{\beta} \tilde{R}}{\tilde{R}}+\gamma(1-\tilde{\beta})+\tilde{\beta}(1+\tilde{R})\right) E\left(\frac{\delta}{1+r}\right)>(1-\gamma)(1-\tilde{\beta}) \tilde{\beta} \tilde{R} D \tag{23}
\end{equation*}
$$

This holds immediately if $\gamma \geq 1$. Let us show it also holds for $\gamma<1$. Recall that $D$ is implicitly defined by the following condition

$$
\tilde{R} E\left[\left(c_{2}^{B}+D\right)^{-\gamma}\right]^{\frac{1}{\gamma}}=\tilde{\beta} E\left[\left(c_{2}^{B}\right)^{-\gamma}\right]^{\frac{1}{\gamma}} .
$$

Substituting in the expression for $c_{2}^{B}$ from equation (15) and rearranging we get

$$
\tilde{R} E\left[\left(\frac{\delta}{1+r}+\tilde{\beta} \tilde{R} D\right)^{-\gamma}\right]^{\frac{1}{\gamma}}=\tilde{\beta} E\left[\left(\frac{\delta}{1+r}-D\right)^{-\gamma}\right]^{\frac{1}{\gamma}}
$$

Notice this equation (looking at the right-hand side) implies that $D<\frac{\delta}{1+r}$ for any realization of $\delta$, so that

$$
D<E\left(\frac{\delta}{1+r}\right) .
$$

Therefore, a sufficient condition for equation (23) to hold when $\gamma<1$ is that

$$
\frac{1+\tilde{\beta} \tilde{R}}{\tilde{R}}+\gamma(1-\tilde{\beta})+\tilde{\beta}(1+\tilde{R})>(1-\gamma)(1-\tilde{\beta}) \tilde{\beta} \tilde{R}
$$

Rearranging this equation we have

$$
(1+\tilde{\beta} \tilde{R})\left[\frac{1}{\tilde{R}}+\gamma(1-\tilde{\beta})+\tilde{\beta}\right]>0
$$

which holds as all terms on the left-hand side are positive. This completes the argument.

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[^0]:    *Preliminary and incomplete. For very useful comments we thank Olivier Jeanne.

[^1]:    ${ }^{1}$ So we have $r=\left(1+i_{2}\right) p_{2} / p_{3}-1$.
    ${ }^{2}$ For ease of notation, whenever possible we omit denoting explicitly the dependence of continuation equilibrium variables on the state $s$.
    ${ }^{3}$ Here we are using the fact that both labor income and monopoly profits in period 2 go to $A$ agents, so the entire output $n^{A}$ appears in their budget constraint.

[^2]:    ${ }^{4}$ Since prices are flexible in period 1 the central bank can choose the nominal interest rate $i_{1}$ but has no power to affect the real interest rate between dates 1 and $2,\left(1+i_{1}\right) p_{1} / p_{2}$, which is determined at the level that clears the bond market.
    ${ }^{5}$ This optimality condition is

    $$
    E\left[Y\left((1+\sigma) u^{\prime}\left(c_{2}^{A}\right)-\frac{\epsilon}{\epsilon-1} v^{\prime}(Y)\right)\right]=0 .
    $$

    This is where we use the optimality condition for labor supply $w / p_{2} u^{\prime}\left(c_{2}^{A}\right)=v^{\prime}(Y)$ to determine the equilibrium wage rate $w$.

[^3]:    ${ }^{6}$ In defining this notion of output gap, we are thinking of the welfare benefits of producing more goods and assigning them to agent $A$. Given that there are two agents and imperfect insurance, the notion would be different if we looked at the welfare benefits of assigning the extra goods to agent $B$. This would change slightly the decomposition and interpretation of our two main forces, but of course it would change nothing in the substance of the analysis.

[^4]:    ${ }^{7}$ One may wonder why there is no term capturing the fact that changing $D$ ex ante will affect the choice of $r$ ex post. In the first two regimes, where monetary policy is chosen under discretion, this happens because the ex post choice of $r$ is optimal, so an envelope argument implies that we can ignore this effect. In the last regime, the output targeting regime, this happens because the central bank commits to $r$ ex ante and does not change it off equilibrium path if $D$ ex ante is changed. This is where the assumption of commitment in the third regime helps to simplify the analysis.

[^5]:    ${ }^{8} \mathrm{We}$ set $\beta=0.5$ and the coefficient of relative risk averstion to 0.5 . We let $v(Y)=Y^{2} / 2$, and assume $\delta \in\{0.7,1.3\}$ with equal probabilities.

