The Size of Symmetric Free-Trade Blocs and World Welfare

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Abstract

In a symmetric model of free-trade blocs, the seemingly innocuous normalization rule by setting the world price of a given good as the numéraire will cause asymmetry in the Nash equilibrium. This paper presents a new symmetric tariff game approach and a new price normalization rule that are logically required and consistent in order to examine the effects of changes in the size of free-trade blocs on world welfare. In a symmetric model, a trading bloc does not act on the actions of the rest of the world as a whole but to individual external blocs in a one-to-one fashion. The relative bargaining power between any two blocs is always evenly matched. We show that the optimal tariff schedule is monotonically decreasing in relative bloc size, and the world welfare unambiguously increases if the trading world becomes more integrated. As a result, the pessimal (lowest world welfare) bloc number is much larger than the ones obtained in the Krugman and Bond-Syropoulos models. Our results support Ohyama-Kemp-Wan-Shimomura's findings and strengthen the case for regionalism as a stepping stone (building block) toward a complete world economic integration. The paper thus provides a theoretical foundation for the benefits of economic integration.

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1. Introduction

Krugman's model of symmetric trading-bloc formation (Krugman 1989, 1991a, 1991b, 1993) has drawn a great deal of attention among trade theorists. His striking conclusion was that the world welfare reaches its lowest level when the number of trading blocs is three (his pessimal number). (See Fig. 1.) His model was reexamined by Bond and Syropoulos (1996) in a more general model and they found similar results, except that the pessimal number varies with countries' commodity endowments and the consumption elasticity of substitution in a CES world. Their pessimal number may lie between two and four, and could be even more. Toward the end of the 1980's, the world seemed to integrate into three blocs: Asia, Europe, and the Americas. Krugman's pessimal number seemed to be playing out in reality and somewhat alarmed the trade theorists.

In theory, Viner (1950) suggested that the world welfare may improve or deteriorate with the formation of customs unions (CUs). However, Vanek (1965), Ohyama (1972) and Kemp and Wan (1976) have shown that economic integration is unambiguously welfare improving if there are appropriate reallocations of gains to the losers.[1] Specifically, the Kemp-Wan proposition states that a CU can be Pareto non-inferior for every individual in every country in the world if it can preserve its *aggregate* trading volumes with all non-member countries after the formation of the CU and if some suitable compensation scheme is applied to the CU members. Ohyama (2002) and Panagariya and Krishna (2002) extended the Kemp and Wan proposition on CUs to the free trade associations (FTAs) by showing that an FTA whose members can adjust their tariffs to keep their *individual* volumes of trade with non-member countries at the pre-FTA levels can be Pareto non-inferior for the world economy. In addition, Kemp and Shimomura (2001) further extended the Kemp-Wan proposition to the case where trading blocs apply optimum tariffs.[2] Kemp (2007) further showed that, if the compensated excess demand functions of member countries are differentiable, then except in a singular case, there always exists a world welfare-enhancing FTA in which the pre-FTA *aggregate* trade vectors of member countries with non-member countries are preserved. In addition, a CU is more beneficial or less harmful to the world economy than a comparable but distinct FTA that is not a CU.

Bond et al. (2004) built a three-country model[3] together with Cobb–Douglas consumer utility function to determine how the formation of FTAs affects optimal tariffs and welfare. They found that, at constant rest-of-the-world (ROW) tariffs, the adoption of internal free trade induces the FTA members to reduce their external tariffs, which results in improving the ROW's terms of trade and welfare. They showed how FTAs may hamper the attainment of global free trade. Saggi (2006) demonstrated that when countries are symmetric, the formation of preferential trade agreements hinders multilateral trade liberalization. Under an FTA, the incentives of the members prove beneficial for multilateral tariff cooperation whereas under a CU, it is the incentives of members that thwart cooperation.[4] Kowalczyk and Riezman (2009) presented estimates of the terms-of-trade effects upon moving from a noncooperative tariff equilibrium to a global free trade world under perfect competition. They found the possibility of large adverse terms-of-trade effects that can outweigh any positive contribution from the consumption effects.

In general, the size of a trading union is related to the union power for levying common external tariffs on outside members. The purpose of this paper is to examine the effects of changes in the size of a free-trade

[1] Grinols (1981) discussed actual compensation schemes.

[2] A similar result was obtained by Goto and Hamada (1998) in a four-country model with a production sector.

[3] They examined an endowment model in which there are N regions and N goods and assumed that N regions are divided into three countries.

[4] Nonpreferential trade agreements (or open regionalism) have been discussed by Bergsten (1997), and Raimondos-Møller and Woodland (2006), and are adopted by the Asia-Pacific Economic Cooperation. Such agreements aim at promoting regional agreements as building blocs, rather than stumbling blocs, toward global trade liberalization

bloc on world welfare. we consider the symmetric framework studied by Krugman (1989) and Bond and Syropoulos (1996), and shed light on a new treatment associated with the normalization rule to derive the optimal tariff structure. To characterize the effects of the size of free-trade blocs, we introduce two key approaches. One is our game structure in tariff bargaining. Unlike Bond and Syropoulos (1996) that chooses the world price of good 1 as the numéraire,[5] we choose the logically required assumption in a symmetric model that a trading bloc does not act in response to the actions of the rest of the world but to the actions of individual trading blocs in a one-to-one fashion, even though all the trading blocs are symmetric to one another. Thus, the relative bargaining power of each bloc to another bloc is measured by one-to-one bargaining, not one to the rest-of-the-world bargaining. The other approach is our choice of commodity numéraire in deriving the elasticity of a foreign bloc's import demand for the home bloc's (composite) good. We choose each bloc's exportable at the world price as the numéraire instead of always choosing (as assumed by Bond and Syropoulos) the single home bloc's exportable good as the numéraire.

The central results of this paper are the derivation of the optimum tariff formula, its properties, and its implications for trade integration. We succeed in characterizing our optimal tariff schedule by showing that it is monotonically decreasing in the relative bloc size. This monotonicity result is not present in Bond and Syropoulos' result. Their optimum tariff rate may rise or fall as their "absolute" bloc size becomes larger.

Using the monotonicity property of our optimal tariff schedule, this paper draws some interesting implications of trade integration for world welfare. We show that the pessimal number of trading blocs that yields the lowest world welfare is much larger than the numbers found by Krugman and by Bond and Syropoulos. We also show that the U-shape property of the world welfare as a function of the relative bloc size (at a fixed optimal tariff rate) is an intrinsic property of the CES function. The central implication of our paper is that, as the world economic integration deepens in the sense of increasing the relative bloc size, the world welfare *unambiguously* increases. This in some sense provides a new theoretical foundation for the benefits of trade integration. It provides a parallel result to those obtained by Ohyama (1972), Kemp and Wan (1976) and Kemp and Shimomura (2001), and strengthens the case for regionalism as a stepping stone or building block toward ultimate world economic integration as argued by Krugman (1991b), Summers (1991) and Ethier (1998), among others.[6] In addition, we show that the case for integration is stronger if there is an increase in the similarity of goods (measured by their substitutability) and similarity of countries (measured by their endowment structures).

This paper is organized as follows. In Section 2., we introduce a basic framework to develop the autarkic and tariff-ridden trading equilibrium. In particular, we discuss the appropriate normalization rule for the present model and derive the optimum tariff formula. Most of the proofs of the propositions and remarks are provided in the appendices. The key differences between our results and those of Krugman's and Bond and Syropoulos's are discussed in Appendix C. In Section 3., we use the Newton-Raphson method to simulate the optimum tariff rate and the level of world welfare. In addition, we prove that our optimum tariff rate is always decreasing in relative bloc size, irrespective of the number of countries and the elasticity of

[5] Chang et al. (2016) proved that the normalization rule in Bond and Syropoulos (1996) of choosing the world price of good 1 as the numéraire causes asymmetry at the Nash equilibrium which contradicts their symmetric assumption when one derives other blocs' optimum tariffs from their viewpoints. They further show that Bond and Syropoulos's model only works for a two-trading-bloc case but not more-than-two-trading-bloc cases.

[6] Some economists have argued that regionalism can be a stumbling block toward multilateralism. See, for example, Bhagwati (1991, 1995) and Levy (1997). Kennan and Riezman (1990) showed that, under symmetric endowment and linear expenditure demand structure, if each country's endowment of its export good is not too large relative to the total world endowment, then any pair of countries can be better off than free trade by forming a customs union. Such possibility of gains may create a stumbling block toward free trade. Krueger (1999) maintains that there appears to be insufficient knowledge to judge whether preferential trade arrangements are building or stumbling blocks in the pursuit of multilateralism. For a recent survey of the theoretical and empirical literature on regionalism, see Freund and Ornelas (2010).

substitution. Finally, Section 4. provides concluding remarks.

2. Basic Framework

2.1. Tariff-Ridden Trading Equilibrium

Consider a world of N countries, indexed by $n = 1, ..., N$. Country n is endowed with Ω_{nn} units of an exportable good and Ω_{nk} units of $(N-1)$ importable goods, $k = 1, ..., N$, $k \neq n$, where $\Omega_{nn} > \Omega_{nk} \geq 0$. Let $C_{nk} \geq 0$ be country n's consumption of the good imported from country k, C_{nn} the consumption of its own exportable good, and $M_{nk} \equiv C_{nk} - \Omega_{nk}$ its import of good k. Consumers in each country have identical tastes, and country n's representative utility function is given by

$$U_n = K \times \left(\sum_{j=1}^{N} C_{nj}^{\theta} \right)^{1/\theta} = \left(\frac{1}{N} \sum_{j=1}^{N} C_{nj}^{\theta} \right)^{1/\theta}, \tag{1}$$

where $K = N^{-1/\theta}$ and $\theta \in (0, 1]$ is the preference parameter related to the elasticity of substitution σ by $\theta = (\sigma - 1)/\sigma$ where $\sigma > 1$.[7]

Let Q_k and P_{nk} be good k's nominal world price and nominal domestic price in country n, respectively. Assume that each country joins only one trading bloc and imposes no tariff against own bloc members while it adopts a uniform own bloc's tariff structure against all outside countries. Let B_n be *the set of countries* in bloc n to which country n belongs and let B be *the total number of symmetric blocs* in the world. Let τ_n be county n's ad valorem tariff rate on imports from outside countries[8] and $\tau_n > -1$. $\tau_n > 0$ is an import tariff and $\tau_n < 0$ is an import subsidy. Thus, $P_{nk} = (1 + \tau_n) Q_k$ for $k \notin B_n$ and $P_{nk} = Q_k$ for $k \in B_n$. The nominal value of total endowments of country n is

$$Y_n \equiv \sum_{k=1}^{N} P_{nk} \Omega_{nk}. \tag{2}$$

The aggregate consumer budget constraint in country n is

$$\sum_{k=1}^{N} P_{nk} C_{nk} \leq Y_n + R_n, \tag{3}$$

where R_n is tariff revenue:

$$R_n = \sum_{k \notin B_n}^{N} (P_{nk} - Q_k) M_{nk}. \tag{4}$$

The tariff revenue is redistributed in a lump-sum fashion to the consumers after they have made their initial consumption decisions; hence, only the government controls R_n via import tariffs.

Let $q_{nk} \equiv Q_k / Q_n$. If $k \in B_n$ then $q_{nk} = 1$; and if $k \notin B_n$, we define q_{nk} as simply q_n. q_n then is country

[7] Our specification of the utility function looks different from Krugman's and Bond and Syropoulos's specifications, but it does not change the structure of the model since we take the number of countries, or equivalently, varieties of goods N as given. Actually, we shall obtain the same indirect utility function as Krugman's and also Bond and Syropoulos's.

[8] If a more general model with asynchronous Nash tariff game is considered, then τ_n would have been τ_{nk} for $k \notin B_n$ if the trading world has not attained a symmetric equilibrium. For tractability, here we follow Krugman and Bond and Syropoulos by assuming the synchronous Nash tariff game so that the outcome of any two blocs' game is simultaneously replicated by all other bilateral blocs. This in effect amounts to all blocs move in tandem.

k's terms of trade vis-à-vis country n, and thus $1/q_n$ is n's terms of trade vis-à-vis country k. Then the indirect utility function, derived in Appendix 1.1., is given by

$$V_n = \frac{By_n}{1 + (B-1)(1+\tau_n)^{-\sigma} q_n^{1-\sigma}} \left(\frac{1 + (B-1)(1+\tau_n)^{1-\sigma} q_n^{1-\sigma}}{B} \right)^{\sigma/(\sigma-1)}, \tag{5}$$

where y_n is country n's total "normalized" real value of its endowments:

$$y_n \equiv \frac{1}{N} \sum_{k=1}^{N} q_{nk} \Omega_{nk}. \tag{6}$$

The indirect utility function in (5) has the following properties:

Remark 1 *Consider the symmetric equilibrium. If either $B = 1$ or free trade among all blocs in the world, then $V_n = y_n$; and if $B \to N$ and $N \to \infty$, then V_n approaches y_n.*

Proof. See Appendix 1.2.. ■

Remark 1 suggests the world welfare, proxied by the consumer's utility function, is a function of B and is U-shaped.

Note that B is bounded by the number of countries N. If we define $\tilde{\beta} = N/B$ as the absolute bloc size which would seem to be a natural definition, we cannot make comparisons among models having different N since $\tilde{\beta} \in [1,\ N]$. Thus, we choose to normalize $\tilde{\beta}$ by N, and define $\beta \equiv \tilde{\beta}/N$ as our "relative" bloc size. It represents each bloc's country share in the world. Notice that our β has the same value as Bond and Syropoulos's (1996) "absolute" bloc size $1/B$. Although the two bloc-size measures are mathematically identical, they are defined from different perspectives.

2.2. Normalization

Without loss of generality, we specify the endowment structure under symmetry as follows:

Assumption 1 *$\Omega_{nn} = A + \omega$ and $\Omega_{nk} = \omega$ for all n and $k \neq n$.*

This assumption is also assumed in Bond and Syropoulos (1996). The world endowment of good n is $\sum_{k=1}^{N} \Omega_{kn} = A + \omega N$, $n = 1, 2, ..., N$. Table 1 presents the world endowment matrix:

Table 1: Endowment Structure

	Goods				
Countries	1	2	$\cdots$	$N-1$	N
1	$A+\omega$	ω	$\cdots$	ω	ω
2	ω	$A+\omega$	$\ddots$	ω	ω
$\vdots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$	$\vdots$
$N-1$	ω	ω	$\cdots$	$A+\omega$	ω
N	ω	ω	$\cdots$	ω	$A+\omega$
Total	$A+\omega N$	$A+\omega N$	$\cdots$	$A+\omega N$	$A+\omega N$

Without loss of generality, the next assumption normalizes the world endowment of each good to N.

Assumption 2 $\alpha + \omega = 1$, *where* $\alpha \equiv A/N$.

This assumption implies $\omega < 1$. α measures dissimilarity of countries and ω their similarity. In a symmetric equilibrium, $q_n \equiv 1$ and the above two assumptions imply

$$y_n = \alpha + \omega = 1. \tag{7}$$

Applying (7) and omitting the subscripts, the normalized version of V_n at $q_n = 1$ is given by

$$V(\tau) = \frac{B}{1 + (B-1)(1+\tau)^{-\sigma}} \left(\frac{1 + (B-1)(1+\tau)^{1-\sigma}}{B} \right)^{\sigma/(\sigma-1)}. \tag{8}$$

This functional form is identical to the one obtained by Krugman and Bond and Syropoulos.

2.3. Autarkic Equilibrium

In our model, there are two kinds of autarky: one is country-level autarky and the other is bloc-level autarky. In the following argument, the word "autarkic" refers to the bloc-level autarky. Thus, in the "autarky" equilibrium, countries in the trading bloc freely trade their goods, but there is no trade beyond each trading bloc.

In the symmetric autarkic equilibrium, the bloc consumption ratios between any two goods are the same as the corresponding endowment ratios, and the consumption quantities are evenly divided among member countries. Therefore, as the number of countries in one bloc is N/B, the consumption of good k in country n is given by

$$C_{nk}^0 = \frac{\sum_{i \in B_n} \Omega_{ik}}{N/B}, \tag{9}$$

where $\sum_{i \in B_n} \Omega_{ik}$ is bloc n's total endowment of good k:

$$\sum_{i \in B_n} \Omega_{ik} = \begin{cases} \dfrac{N}{B}\omega + A & (k \in B_n) \\ \\ \dfrac{N}{B}\omega & (k \notin B_n) \end{cases}. \tag{10}$$

Thus, the autarkic welfare level when the world is divided into B symmetric trading blocs is

$$V_B^0 = \left(\frac{(\omega + \alpha B)^{(\sigma-1)/\sigma} + (B-1)\,\omega^{(\sigma-1)/\sigma}}{B} \right)^{\sigma/(\sigma-1)}. \tag{11}$$

If the whole world is a free-trade zone, then $B = 1$ and $V_B^0 = 1$. If a tariff-ridden trade is the rational outcome, we must have $V(\tau) \geq V_B^0$ (we confirm this later in numerical simulations). Note that the tariff rate τ^∞ yielding $V(\tau^\infty) \leq V_B^0$ is the prohibitive rate.

2.4. The Optimum Tariff Formula

Existence of Nash equilibrium in multiple-country models has been studied by Johnson (1953-54), Graaf (1949-50), Otani (1980), Kennan and Riezman (1990), Kose and Riezman (2000), Syropoulos (2002), and others. The aim here is to derive the optimum tariff formula in our symmetric model. Let ϵ_m be the elasticity of country m's import demand for country n's good where $m \notin B_n$.

Proposition 1 *In the symmetric multi-country model, country n's optimal tariff on its imports from country $m \notin B_n$ solves $\tau_n^{opt} = 1/(\epsilon_m - 1)$, where ϵ_m is a function of the tariff profile $(\tau_1^{opt}, \ldots, \tau_N^{opt})$ such that $\tau_n^{opt} = \tau^{opt}$ under symmetry.*

Proof. See Appendix B. ■

Proposition 2 *At the symmetric equilibrium, the optimum tariff solves the fixed point problem for $1 + \tau^{opt}$ in the following formula:*

$$1 + \tau^{opt} = \frac{\dfrac{\sigma (1 + \tau^{opt})^{\sigma} + B - 1}{(1 + \tau^{opt})^{\sigma} + B - 1} - \dfrac{\omega (B - 1)}{B}}{\dfrac{\sigma (1 + \tau^{opt})^{\sigma} + B - 1}{(1 + \tau^{opt})^{\sigma} + B - 1} + \omega \dfrac{(1 + \tau^{opt})^{\sigma}}{B} - 1}. \tag{12}$$

Proof. See Appendix 2.1.. ■

Remark 2 *$\sigma = 1$ and $\omega = 0$ imply $\tau^{opt} \to \infty$; $B \to N$ and $N \to \infty$ imply $\tau^{opt} \to \infty$.*

Proof. See Appendix 2.2.. ■

3. Numerical Simulations and Implications

In the following analysis, the optimum tariff rate is obtained numerically by solving the fixed-point problem for $1 + \tau^{opt}$ in (12) by the standard Newton-Raphson method. The obtained optimum tariff rate is then substituted into the indirect utility function (5) to generate the world welfare. In Appendix C, we apply the same algorithm to obtain the optimum tariff rate from Bond and Syropoulos's formula in order to verify our computation algorithm as well as to compare our results to theirs in the following sections.

3.1. Optimum Tariff Rate

In order to compare our model with Krugman's and Bond and Syropoulos's models, we first consider $\sigma = \{2, 4, 10\}$ for $\omega = \{0, 0.5, 0.8\}$ (Krugman's model corresponds to $\omega = 0$ in Bond and Syropoulos's model since Krugman assumes no endowment of importables). For the simulation, we focus on the relative bloc size $\beta \equiv 1/B$ instead of the number of blocs B, since β is between zero and one while B lies between 1 and N, where N is unbounded from above.

The relationships between the relative bloc size β and the optimum tariffs are shown in Fig. 2. The optimum tariffs computed from our formula (solid curves) and those from Bond and Syropoulos's formula (dashed curves) do not coincide with each other unless $B = 2$ ($\beta = 0.5$). (See Appendix C for the discussion of the key difference.)

The key difference between our formula and theirs is that our formula shows the optimum tariff schedule is monotonically decreasing in β in all cases; hence, the Nash tariff rate decreases as the world gets more integrated (i.e., as β becomes larger). Intuitively, the optimum tariff in a free-trade economy is zero.

By symmetry, the relative power of each trading bloc to any other trading bloc is always even. Thus, each country cannot be a small one even if the world is completely divided when no country belongs to a trading bloc, as the country can evenly affect the terms of trade of a trading partner. Our result that τ^{opt} being monotonically decreasing in β implies that the elasticity of import demand is increasing in β. As β approaches one, the welfare level approaches the one-world (free trade) welfare level because the weight of a single good in the utility function approaches zero. (See Section 3.2. for detail.) The U-shaped welfare level is thus not produced by imposing the optimum tariff but is the result of the symmetric property of the CES function. Our view of the symmetric world is thus in contrast to Krugman's perception that a completely divided world is the same as a small-open-economy model. In his view, such a country is against the rest of the world, but that would introduce an asymmetry, contrary to the symmetry assumption.

To characterize the optimum tariff by the similarity measures, Figs. 3 and 4 depict the iso-optimal tariff rates in the (β, σ) and (β, ω) spaces, respectively. The left charts are computed from Bond and Syropoulos's formula and the right ones are from ours. Let $\bar{\tau}(\beta, \sigma)$ and $\bar{\tau}(\beta, \omega)$ be the iso-optimal tariff loci in the (β, σ) and (β, ω) spaces, respectively. Fig. 3 shows that our $\bar{\tau}(\beta, \sigma)$ are always decreasing while Bond and Syropoulos's are not. Such result carries over to Fig. 4 for $\bar{\tau}(\beta, \omega)$. All charts in both figures show that in our model, for a given β, $\bar{\tau}$ decreases as σ or ω increases.

Suppose there is a sufficiently large number of countries and $B \to N$ and $N \to \infty$ so that $\beta \to 0$. In this case, the consumption of a country's exportable good has measure zero in the utility function and in the budget constraint. For example, the budget constraint is given as a continuum form when $N \to \infty$, so that we find

$$y_n \equiv \frac{1}{N} \sum_{k=1}^{N} q_{nk} \Omega_{nk} \quad \Rightarrow \quad y_n \equiv \int_0^N q_n(k) \, \Omega_n(k) \, dk, \tag{13}$$

where $q_{nk} \to q_n(k)$ and $\Omega_{nk} \to \Omega_n(k)$ for $N \to \infty$. Since only one good is endowed as an exportable good, the endowment of the exportable good is then computed as

$$\int_n^n q_n(k) \, \Omega_n(k) \, dk = 0. \tag{14}$$

The same thing happens in the utility function. Therefore, in the symmetric open economy, as $N \to \infty$, the utility level becomes $V_n \equiv U_n \to C_{nm}$. To consume such a quantity, the country needs to import $C_{nm} - \omega$ in exchange for exporting a "measure zero" amount. Thus, the country needs to impose an infinitely large tariff rate, and the elasticity of import demand is necessarily one.

Fig. 5 depicts the elasticity of import demand (ϵ), derived in (37), as a function of β with given various values of σ and ω under free trade and optimum Nash tariff. The dashed curves are computed from Bond and Syropoulos's formula and the solid ones are from ours. The thick curves indicate the values under Nash tariff and the thin ones are the values under free trade. In our formula, ϵ is increasing in β, and $\epsilon \to 1$ as $\beta \to 0$ (eventually, no trade at $\beta = 0$). However, in Bond and Syropoulos's formula, ϵ is either hump-shaped or decreasing in β, and $\epsilon > 1$ as $\beta \to 0$.

The next proposition provides our central result that τ^{opt} is monotonically decreasing in β, irrespective of the values of N, σ and other endowment parameters A and ω:

Proposition 3 *The optimum tariff rate decreases as the relative bloc size increases (or equivalently, as the number of symmetric trading blocs decreases), which we interpret as deepening the integration of the world trading system.*

Proof. See Appendix 2.3. ■

3.2. World Welfare Level

Here we show the relationship between the welfare level and the bloc size in Fig. 6. The "relative" bloc size is seen to extend to $\beta = 1$ in order to include the case of a unified world. Again the solid and dashed curves represent the welfare levels computed from our formula and Bond and Syropoulos's formula, respectively. The parameter values used are $\sigma = \{2, 4, 10\}$ and $\omega = \{0, 0.5, 0.8\}$. It is seen that there exist minima of world welfare levels for all parameter sets.

Next, by adding the balance of payment conditions (18) in Appendix 1.1. for all members in a bloc, we can derive the Marshall-Lerner condition for each trading bloc:

$$\left(N - \frac{N}{B}\right)\epsilon_n + \frac{N}{B}\chi_n > \left(N - \frac{N}{B}\right) \quad \Rightarrow \quad (1-\beta)\,\epsilon_n + \beta\chi_n > 1 - \beta, \tag{15}$$

where ϵ_n is the elasticity of import demand and χ_n is the elasticity of export supply of trading bloc n (as discussed in Appendix D, the country-based and bloc-based ϵ's are equal to each other). As (15) suggests, the Marshall-Lerner condition is violated when $\beta \to 0$ as $\epsilon_n \to 1$ (see (43)).[9] Thus, countries raise their tariff rates to reduce imports when β approaches 0. For the general case, we note the property about χ_n:

Remark 3 $\lim_{N\to 1}\chi_n = \tilde{\chi}_n \leq \chi_n$.

Proof. See Appendix D. ■

Remark 3 implies that a sufficient condition to satisfy the Marshall-Lerner condition (15) is $(1-\beta)\,\epsilon_n + \beta\tilde{\chi}_n > 1 - \beta$. Since $\beta = 0$ implies $\epsilon_n = 1$, we confirm that the Marshall-Lerner condition is satisfied for all $\beta > 0$, but is violated at $\beta = 0$, as is implied by (15). Furthermore, in the case where $\beta = 0$, our result in (44) shows $\tau^{opt} \mapsto \infty$, which does not indicate free trade, contrary to Krugman's perception.[10]

The original proposition advanced by Kemp and Wan (1976) shows that formation of a customs union is Pareto improving so long as there is an appropriate reallocation scheme among member countries and the customs union does not raise import tariffs against outside countries.[11] Kemp and Shimomura (2001) further show that the Kemp and Wan proposition carries over to the case in which all customs unions resort to the use of optimal tariffs. In our model, there is no compensation scheme to share the gains from formation of customs unions among members. Due to the assumption of symmetry, the compensation scheme is no longer meaningful in the present model. Our result that the optimum tariff rate is decreasing in β implies that the Nash tariff rate decreases as the world economy becomes more integrated. Our finding thus reinforces the Kemp-Wan-Shimomura result. However, this is not the case in Bond and Syropoulos (1996) since their optimum tariff rate may not decrease as β increases.

[9] In trade theory, a tariff equilibrium is a state in which each country imposes its Nash equilibrium rates. If the tariff equilibrium is unique, at least one country in an asymmetric case must be worse off than the free trade case, and all are worse off in a symmetric case. It is known that some trade is better than no trade. By moving away from an autarkic state resulting from prohibitive tariff rates, all nations in a symmetric case can gain from trade by lowering their tariffs so that some trades can take place. In our present model, as β goes to 0, the no integration equilibrium with an infinite number of countries is an unstable one and thus the Marshall-Lerner condition is violated.

[10] Note that in the case where $\beta = 1$, we have only one block in the world. By definition of a bloc case, the world economy is a free trade zone. Thus, our formula (44) should not be applied to this case as it needs, at least, two blocs.

[11] Konishi et al. (2009) showed that the world cannot reach a free trade equilibrium if trading blocs are in the core of cooperative customs union game of Kowalczyk and Sjöström (1994) and are practicing the Grinols compensation scheme (1981).

4. Conclusion

In this paper, we have examined the effects of changes in the size of a free-trade bloc in the multilateral trading system on the world welfare. By presenting a new measure of bloc size and a price normalization rule, we derive an optimum tariff structure in a discrete model. Our numerous simulation results produce insights on the welfare effects of trade integration.

We find that the optimum tariff schedule is unambiguously decreasing in relative bloc size and a completely divided world is not a unified world, contrary to Krugman's (1989) assertion. When the relative bloc size is infinitesimal, the elasticity of import demand is necessarily one, unlike Bond and Syropoulos's (1996) extended result. In a completely divided world, our result shows that the optimum tariff rate necessarily goes to infinity and the Marshall-Lerner condition must be violated.

In a non-symmetric world, the common result that the optimum tariff rate increases with the size of a country stems from the advantage of having a stronger terms of trade effect or from a stronger relative bargaining power in tariff games. In a symmetric model, however, the relative bargaining powers of any two countries are always evenly matched. Therefore, as the relative bloc size decreases—the number of countries in each bloc becomes smaller—countries need to raise tariff rates to keep the symmetric consumption pattern. In addition, as indicated in our result and in Bond and Syropoulos's, increases in the similarity of goods and similarity of countries—measured respectively by the elasticity of substitution and the endowments—unambiguously reduce the optimum tariff rate if the relative bloc size is given. We have shown that our iso-optimal tariff rates exhibit negative trade-offs between the relative bloc size and both similarity measures. This is not the case in Bond and Syropoulos' result. We have shown that the pessimal number of blocs that yields the lowest world welfare is much larger that those of Krugman's and Bond and Syropoulos's. In addition, the U-shape welfare function in relation to the relative bloc size is the natural outcome of assuming a CES utility function.

The central result of this paper is that the optimal tariff rate is monotonically decreasing in the relative bloc size. Thus, as trade integration deepens in the sense of increasing the relative bloc size, the world welfare unambiguously increases. This provides, in some sense, a new theoretical foundation for the benefit of economic integration. Such result parallels those of Kemp-Wan-Shimomura's as well as Ohyama-Panagariya-Krishna's and reinforces the case for regionalism as a stepping stone or building block toward the ultimate world economic integration. Finally, we have also shown that the case for integration is strengthened if there is an increase in the similarity of goods (measured by their substitutability) and in the similarity of countries (measured by their endowments).

Appendix

A Indirect Utility Function

1.1. Utility Maximization Problem

Consider the optimization problem of the consumers:

$$\underset{C_{nm}\ \forall\ m=1,2,\ldots,N}{\text{Maximize}}\ (1)\ \text{subject to}\ (3)\ \text{with given}\ Y_n\ \text{and}\ R_n. \tag{16}$$

This problem has an interior solution and its first-order conditions with respect to consumption yield

$$p_{nm} \equiv \frac{P_{nm}}{Q_n} = \left(\frac{C_{nm}}{C_{nn}}\right)^{\theta-1}, \quad \forall\, n, m = 1, ..., N. \tag{17}$$

Next, consider the problem of a country's government in bloc n maximizing the social welfare represented by its consumers' utility. The constraint on the government is the balance of trade condition given by

$$\sum_{k=1}^{N} Q_k C_{nk} = \sum_{k=1}^{N} Q_k \Omega_{nk}, \tag{18}$$

which is derived from (3) and (4). From (17), we have $C_{nm} = p_{nm}^{-\sigma} C_{nn}$ which, together with (18), yields

$$C_{nn} \sum_{k=1}^{N} q_{nk} p_{nk}^{-\sigma} = \sum_{k=1}^{N} q_{nk} \Omega_{nk}. \tag{19}$$

It follows that

$$C_{nn} = \frac{\sum_{k=1}^{N} q_{nk} \Omega_{nk}}{\sum_{k=1}^{N} q_{nk} p_{nk}^{-\sigma}}, \tag{20}$$

where $q_{nk} \equiv Q_k / Q_n$ is the relative price of good k in terms of the price of own exportable good n, and the denominator in (20) is

$$\sum_{k=1}^{N} q_{nk} p_{nk}^{-\sigma} = \frac{N}{B} + \left(N - \frac{N}{B}\right) (1 + \tau_n)^{-\sigma} q_n^{1-\sigma}, \tag{21}$$

where $q_n = q_{nk}$ for all $k \notin B_n$, and $q_n = 1$ for all $k \in B_n$. Thus, by (20) and (6), the demand functions in country n are

$$C_{nn} = \frac{B y_n}{1 + (B-1)(1+\tau_n)^{-\sigma} q_n^{1-\sigma}}, \tag{22}$$

$$C_{nm} = \frac{(1+\tau_n)^{-\sigma} q_n^{-\sigma} B y_n}{1 + (B-1)(1+\tau_n)^{-\sigma} q_n^{1-\sigma}}, \tag{23}$$

where $C_{nk} \equiv C_{nn}$ for all $k \in B_n$ and $C_{nk} \equiv C_{nm}$ for all $k \notin B_n$ by symmetry. Therefore, by substituting (17), (22) and (23) into the utility function (1), country n's indirect utility function is given by

$$V_n = \frac{B y_n}{1 + (B-1)(1+\tau_n)^{-\sigma} q_n^{1-\sigma}} \left(\frac{1 + (B-1)(1+\tau_n)^{1-\sigma} q_n^{1-\sigma}}{B}\right)^{\sigma/(\sigma-1)}. \tag{24}$$

1.2. Characterization of the Indirect Utility Function

In the case where $B = 1$ $(\beta = 1)$, it is straightforward to verify that $V_n = y_n$. In this case, all countries belong to a bloc and by definition of a trading bloc, $\tau_n = 0$ and $q_n = 1$ for all n.

Consider the case where $B \to N$ and $N \to \infty$. C_{nn} in (22) converges to $y_n / (1+\tau_n)^{-\sigma} q_n^{1-\sigma}$. From (17), $C_{nm} = p_{nm}^{-\sigma} C_{nn} = (1+\tau_n)^{-\sigma} q_n^{1-\sigma} C_{nn}$. As a result, C_{nm} in (23) approaches y_n:

$$\lim_{B\to N,\ N\to\infty} C_{nm} = y_n. \tag{25}$$

In addition, since the consumption shares of a bloc's exportable goods and imported goods are β and $1-\beta$, respectively, we find that U_n, same as V_n, also approaches y_n as $B \to N$ and $N \to \infty$.

B Optimum Tariff

We first derive the "optimum tariff formula" analogous to the well-known two-country model: $\tau^{opt} = 1/\left(\epsilon^* - 1\right)$, where τ^{opt} and ϵ^* are the home country's optimum tariff rate and a foreign country's elasticity of import demand. Our result shows that the analogous formula still holds in the multi-country model as suggested by Bond and Syropoulos (1996). We then derive the explicit expression of the elasticity of import demand.

Let $U_n = U\left(C_{n1}, \ldots, C_{nN}\right)$ be country n's direct utility function. After consumers' optimizing decision and noting that the relative price in terms of country n's own exportable good is a function of all tariff rates $(\tau_1, \ldots, \tau_N)$, we can express each demand function by $C_{nm} = C_{nm}\left[\tau_n, q_n\left(\tau_n, \tau_{-n}\right), y_n(\cdot)\right]$, where τ_n is the tariff rate imposed by country n in B_n on all other countries not in B_n, $\tau_{-n} = \{\tau_1, \ldots, \tau_N\} \setminus \{\tau_n\}$ is the set of all outside blocs' tariff rates and $y_n(\cdot) = y_n\left[q_n\left(\tau_n, \tau_{-n}\right)\right]$.[12] To simplify the expressions, we write $C_{nm}\left[\tau_n, q_n\left(\tau_n, \tau_{-n}\right), y_n(\cdot)\right] \equiv \tilde{C}_{nm}\left(\tau_n; \tau_{-n}\right)$.

Consider now a bloc's choice of optimal tariff rate. The objective of a customs union is to maximize the union's welfare. Due to the symmetry assumption, we can just use a representative member country's welfare for its objective. Bloc n chooses τ_n to maximize $U_n = U\left[\tilde{C}_{n1}\left(\tau_n; \tau_{-n}\right), \ldots, \tilde{C}_{nN}\left(\tau_n; \tau_{-n}\right)\right]$. The first-order partial derivative with respect to τ_n, given τ_{-n}, can be expressed as

$$\frac{\partial U_n}{\partial \tau_n} = \sum_{k=1}^{N} \frac{\partial U_n}{\partial C_{nk}} \frac{\partial \tilde{C}_{nk}}{\partial \tau_n}, \tag{26}$$

where $\partial U_n/\partial C_{nk} = \partial U_n/\partial \tilde{C}_{nk}$, and $\partial \tilde{C}_{nk}/\partial \tau_n$ is the derivative of $\tilde{C}_{nk}$ with respect to τ_n, holding τ_{-n} constant. Let n be the home country, n' a country in the same bloc, and m an outside country; that is, $n, n' \in B_n$ and $m \notin B_n$. Under symmetry, (26) becomes

$$\frac{\partial U_n}{\partial \tau_n} = \frac{\partial U_n}{\partial C_{nn}} \frac{\partial \tilde{C}_{nn}}{\partial \tau_n} + \left(\frac{N}{B} - 1\right) \frac{\partial U_n}{\partial C_{nn'}} \frac{\partial \tilde{C}_{nn'}}{\partial \tau_n} + \left(N - \frac{N}{B}\right) \frac{\partial U_n}{\partial C_{nm}} \frac{\partial \tilde{C}_{nm}}{\partial \tau_n}, \tag{27}$$

where $\partial U_n/\partial C_{nk}$ is

$$\frac{\partial U_n}{\partial C_{nk}} = C_{nk}^{\theta-1} \times \left(\sum_{j=1}^{N} C_{nj}^{\theta}\right)^{(1-\theta)/\theta}, \; k = 1, ..., N. \tag{28}$$

The term $\partial \tilde{C}_{nk}/\partial \tau_n$ in (26) consists of three components: the direct effect of the tariff rate, the terms of trade effect and the income effect:

$$\frac{\partial \tilde{C}_{nk}}{\partial \tau_n} = \frac{\partial C_{nk}}{\partial \tau_n} + \left(\frac{\partial C_{nk}}{\partial q_n} + \frac{\partial C_{nk}}{\partial y_n} \frac{dy_n}{dq_n}\right) \frac{\partial q_n}{\partial \tau_n}. \tag{29}$$

Since $M_{nk} \equiv C_{nk} - \Omega_{nk}$, we can similarly define $\tilde{M}_{nk}\left(\tau_n; \tau_{-n}\right) \equiv \tilde{C}_{nk}\left(\tau_n; \tau_{-n}\right) - \Omega_{nk}$. Using (27), (28),

[12] The expression of y_n as a function of $q_n = q_n\left(\tau_n, \tau_{-n}\right)$ follows from (6). Since each country has N goods but only one exportable good and there are no tariffs on all imports from member countries, by the symmetry assumption, all goods from within own bloc will have an identical price when measured in country n's own price. Thus, by Hicksian aggregation, we can treat all these intra-bloc goods as a composite good. The same inference applies to any outside bloc's goods. τ_{-n} here, therefore, is a scalar which is the common tariff rate applied by all own member countries on imports from outside blocs.

$C_{nn} = C_{nn'}$, and $(C_{nm}/C_{nn})^{\theta-1} = (1+\tau_n)\, q_n$, the first-order condition $\partial U_n/\partial \tau_n = 0$ is arranged as[13]

$$\underbrace{\frac{\partial \tilde{M}_{nn}}{\partial \tau_n} + \left(\frac{N}{B} - 1\right)\frac{\partial \tilde{M}_{nn'}}{\partial \tau_n}}_{\text{Home Trading Bloc}} + \underbrace{\left(N - \frac{N}{B}\right)\frac{\partial \tilde{M}_{nm}}{\partial \tau_n}(1+\tau_n)\, q_n}_{\text{Outside Trading Bloc}} = 0, \tag{30}$$

where $(1+\tau_n)\, q_n = (\partial U/\partial C_{nm}) / (\partial U/\partial C_{nn}) = (C_{nm}/C_{nn})^{\theta-1}$.

The balance-of-trade condition of country n is

$$\sum_{k=1}^{N} q_{nk} M_{nk} = 0,$$

which implies

$$\underbrace{\frac{\partial \tilde{M}_{nn}}{\partial \tau_n} + \left(\frac{N}{B} - 1\right)\frac{\partial \tilde{M}_{nn'}}{\partial \tau_n}}_{\text{Home Trading Bloc}} + \underbrace{\left(N - \frac{N}{B}\right)\left(\frac{\partial \tilde{M}_{nm}}{\partial \tau_n} q_n + \frac{\partial q_n}{\partial \tau_n} M_{nm}\right)}_{\text{Outside Trading Bloc}} = 0. \tag{31}$$

Then, from (30) and (31), for $B \neq 1$, the first-order condition is simplified as[14]

$$\tau_n q_n \frac{\partial \tilde{M}_{nm}}{\partial \tau_n} = M_{nm} \frac{\partial q_n}{\partial \tau_n}, \tag{32}$$

which is further manipulated as

$$-\frac{\tau_n}{M_{nm}} \frac{\partial \tilde{M}_{nm}}{\partial \tau_n} \times \tau_n = -\frac{\tau_n}{q_n} \frac{\partial q_n}{\partial \tau_n}.$$

Therefore, the optimum tariff is determined by $\tau_n = \eta_n/\rho_n$, where ρ_n and η_n are

$$\rho_n \equiv -\frac{\partial \tilde{M}_{nm}}{\partial \tau_n} \frac{\tau_n}{M_{nm}} \quad \text{and} \quad \eta_n \equiv -\frac{\partial q_n}{\partial \tau_n} \frac{\tau_n}{q_n}.$$

It should be noted that the bloc-based calculation of the elasticity of import demand is the same as the country-based calculation. This is because the outside trading blocs' terms in (30) and (31) are not altered by the change of the basis of calculation and the home bloc's terms in these equations also change similarly to yield (32).

Next, we use $1/\tau_n = \rho_n/\eta_n$ to derive $\epsilon_m - 1 = \rho_n/\eta_n$ by showing $\epsilon_m \eta_n \equiv \rho_n + \eta_n$. We then derive the optimum tariff in the present multi-country model and show that it is the same as the well-known formula: $\tau^{opt} = 1/(\epsilon_m - 1)$ in the two-country world. Let M_{mn} be foreign country m's import demand function for good n and $q_m \equiv Q_n/Q_m$ be the world relative price of M_{mn} from country m's viewpoint; hence, $q_n \equiv 1/q_m$ and $dq_n/q_n = -dq_m/q_m$. Using the abbreviated notation as similarly defined in (29), here with respect to the world relative price, we define ϵ_m as

$$\epsilon_m \equiv -\frac{dM_{mn}}{dp_m} \frac{p_m}{M_{mn}} \equiv -\frac{\partial M_{mn}}{\partial q_m} \frac{dq_m}{dp_m} \frac{(1+\tau_m)\, q_m}{M_{mn}} = -\frac{\partial M_{mn}}{\partial q_m} \frac{q_m}{M_{mn}},$$

[13] Here we divide the first order condition by $C_{nn}^{\theta-1}$ and the common term appearing in (28).

[14] Notice that $N - N/B$ is canceled out here.

where $p_m = (1+\tau_m) q_m$. Thus we have

$$\begin{aligned}
\epsilon_m \eta_n &= \left(-\frac{\partial \tilde{M}_{mn}}{\partial q_m}\frac{q_m}{M_{mn}}\right)\left(-\frac{\partial q_n}{\partial \tau_n}\frac{\tau_n}{q_n}\right) \\
&= \left(\frac{\partial q_n}{\partial q_m}\frac{q_m}{q_n}\right)\frac{\partial \tilde{M}_{mn}}{\partial q_n}\frac{\partial q_n}{\partial \tau_n}\frac{\tau_n}{M_{mn}} = -\frac{\partial \tilde{M}_{mn}}{\partial \tau_n}\frac{\tau_n}{M_{mn}}.
\end{aligned}$$

By the pairwise balance-of-trade condition, we must have $M_{mn} = q_n M_{nm}$. It follows that

$$\frac{\partial \tilde{M}_{mn}}{\partial \tau_n} = M_{nm}\frac{\partial q_n}{\partial \tau_n} + q_n \frac{\partial \tilde{M}_{nm}}{\partial \tau_n},$$

which is further expressed in elasticity terms:

$$-\frac{\partial \tilde{M}_{mn}}{\partial \tau_n}\frac{\tau_n}{M_{mn}} = -\frac{\partial q_n}{\partial \tau_n}\frac{\tau_n}{q_n} - \frac{\partial \tilde{M}_{nm}}{\partial \tau_n}\frac{\tau_n}{M_{nm}} = \eta_n + \rho_n.$$

Thus, $\epsilon_m - 1 = \rho_n/\eta_n = 1/\tau_n$, which yields the well-known formula:

$$\tau^{opt} = \frac{1}{\epsilon_m(\tau^{opt}) - 1} \quad \Leftrightarrow \quad 1+\tau^{opt} = \frac{\epsilon_m(\tau^{opt})}{\epsilon_m(\tau^{opt}) - 1}. \tag{33}$$

2.1. More on Optimum Tariff

By rearranging terms of (23), country n's import demand function of good m is given by

$$M_{nm} = \frac{By_n}{(1+\tau_n)^\sigma q_n^\sigma + (B-1) q_n} - \Omega_{nm}.$$

By symmetry, from the viewpoint of country $m \notin B_n$ (an outside country), its import demand for one of our bloc's product (say good n) is given by

$$M_{mn} = \frac{By_m}{(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m} - \Omega_{mn}, \tag{34}$$

where q_m, as already defined, is equal to Q_n/Q_m. Note that, as already discussed earlier, q_m is country n's terms of trade vis-à-vis country m, and thus $1/q_m$ is m's terms of trade vis-à-vis country n

From (34), we obtain

$$\begin{aligned}
&-\left(\frac{\partial M_{mn}}{\partial q_m} + \frac{\partial M_{mn}}{\partial y_m}\frac{dy_m}{dq_m}\right) \\
&= \left(\frac{\sigma(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m}{(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m} - \frac{dy_m}{dq_m}\frac{q_m}{y_m}\right)\frac{By_m/q_m}{(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m} \\
&= \left(\frac{\sigma(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m}{(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m} - \frac{dy_m}{dq_m}\frac{q_m}{y_m}\right)\frac{M_{mn} + \Omega_{mn}}{q_m},
\end{aligned} \tag{35}$$

where $n \notin B_m$.

Using $\epsilon_m = -q_m \partial M_{mn}/M_{mn}\partial q_m$ and $p_m = (1+\tau_m) q_m$, we find

$$\epsilon_m \equiv \left(\frac{\sigma(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m}{(1+\tau_m)^\sigma q_m^\sigma + (B-1) q_m} - \frac{dy_m}{dq_m}\frac{q_m}{y_m}\right)\frac{M_{mn} + \Omega_{mn}}{M_{mn}}, \tag{36}$$

where

$$\frac{dy_m}{dq_m} = \frac{d}{dq_m}\left(\frac{1}{N}\sum_{k=1}^{N} q_{mk}\Omega_{mk}\right) = \frac{\omega(B-1)}{B}$$

and

$$\begin{aligned}\frac{M_{mn}}{M_{mn}+\Omega_{mn}} &= 1-\Omega_{mn}\left(\frac{(1+\tau_m)^\sigma q_m^\sigma + (B-1)\,q_m}{By_m}\right)\\ &= 1-\Omega_{mn}\frac{(1+\tau_m)^\sigma + B-1}{B}.\end{aligned}$$

Therefore, ϵ_m is obtained as a function of τ_m:

$$\epsilon_m(\tau_m) \equiv \frac{\dfrac{\sigma(1+\tau_m)^\sigma + B-1}{(1+\tau_m)^\sigma + B-1} - \dfrac{\omega(B-1)}{B}}{1-\omega\dfrac{(1+\tau_m)^\sigma + B-1}{B}}. \tag{37}$$

In the symmetric Nash tariff (or optimum tariff) equilibrium, we find $\tau_n = \tau_m = \tau^{opt}$. The optimum tariff solves $1+\tau^{opt} = \epsilon_m(\tau^{opt})/\{\epsilon_m(\tau^{opt})-1\}$, as shown in (33), which becomes

$$\begin{aligned}1+\tau^{opt} &= \frac{\dfrac{\sigma(1+\tau^{opt})^\sigma + B-1}{(1+\tau^{opt})^\sigma + B-1} - \dfrac{\omega(B-1)}{B}}{\left(\dfrac{\sigma(1+\tau^{opt})^\sigma + B-1}{(1+\tau^{opt})^\sigma + B-1} - \dfrac{\omega(B-1)}{B}\right) - \left(1-\omega\dfrac{(1+\tau^{opt})^\sigma + B-1}{B}\right)}\\ &= \frac{\dfrac{\sigma(1+\tau^{opt})^\sigma + B-1}{(1+\tau^{opt})^\sigma + B-1} - \dfrac{\omega(B-1)}{B}}{\dfrac{\sigma(1+\tau^{opt})^\sigma + B-1}{(1+\tau^{opt})^\sigma + B-1} + \omega\dfrac{(1+\tau^{opt})^\sigma}{B} - 1}.\end{aligned} \tag{38}$$

2.2. Further Characterizations of the Optimum Tariff

First, consider $\sigma = 1$ (the Cobb-Douglas case) and $\omega = 0$. Let $\epsilon_m \to 1$ in (37). Then (12) becomes

$$1+\tau^{opt} = \frac{1}{1-1} \to \infty. \tag{39}$$

Therefore, $\sigma = 1$ and $\omega = 0$ imply $\tau^{opt} \to \infty$. This result is well known (see, for example, Kennan and Riezman (1988)).

Next, we consider the convergence of the utility function (1) for $B \to N$ and $N \to \infty$. Recall that $B \in [1, ..., N]$. In this case, the budget constraint of the representative consumer in country n is

$$C_{nn} + (N-1)\,p_{nm}C_{nm} \leq R_n + \Omega_{nn} + (N-1)\,p_{nm}\Omega_{nm},$$

where $R_n = \tau_n q_{nm}(N-1)\,M_{nm}$ and $p_{nm} = (1+\tau_n)\,q_{nm}$. This budget constraint is rearranged as

$$\frac{C_{nn}}{N} + \left(1-\frac{1}{N}\right)p_{nm}C_{nm} \leq \frac{R_n}{N} + \frac{\Omega_{nn}}{N} + \left(1-\frac{1}{N}\right)p_{nm}\Omega_{nm}. \tag{40}$$

Thus, if $N \to \infty$, we obtain $(1+\tau_n)\, q_{nm} C_{nm} \leq (1+\tau_n)\, q_{nm} \Omega_{nm}$, which is equivalent to

$$q_{nm} C_{nm} \leq q_{nm} \Omega_{nm}, \tag{41}$$

since $(1+\tau_n) > 0$.

When $N \to \infty$, the utility function converges to the linear form, such as $U_n = C_{nm}$ for any m. Then the utility maximization problem is simply the maximization of $U_n = C_{nm}$ subject to (41). Hence, the maximization requires the budget constraint be strictly equal:

$$q_{nm} C_{nm} = q_{nm} \Omega_{nm} \Leftrightarrow q_{nm} M_{nm} = 0, \tag{42}$$

which implies the volume of trade, M_{nm}, approaches zero (no trade) as $N \to \infty$. The elasticity of import demand of country n is then computed from (42) as

$$\lim_{N\to\infty} \epsilon_n \equiv \lim_{N\to\infty} -\frac{dM_{nm}}{dq_{nm}} \frac{q_{nm}}{M_{nm}} = 1. \tag{43}$$

By symmetry, we have $\epsilon_m = \epsilon_n = 1$. Note that we can directly verify the convergence of the right-hand-side of the optimum tariff formula in (12):

$$1 + \tau^{opt} = \lim_{B\to N,\ N\to\infty} \frac{1-\omega}{(1-\omega)-(1-\omega)} \to \infty. \tag{44}$$

Therefore, from the optimum tariff formula, we obtain $\tau^{opt} \to \infty$ and $M_{nm} \to 0$ as $\beta \to 0$.

2.3. Proof of Proposition 3

Let f and g be respectively defined by

$$\begin{aligned} f &\equiv h - \frac{\omega\,(B-1)}{B} > 0, \\ g &\equiv h + \omega \frac{(1+\tau)^{\sigma}}{B} - 1 > 0, \end{aligned}$$

where the inequalities follow from $\omega \geq 0$ and

$$h \equiv \frac{\sigma\,(1+\tau)^{\sigma} + B - 1}{(1+\tau)^{\sigma} + B - 1} > 1,$$

since $\sigma > 1$. (12) can now be written as

$$1 + \tau = \frac{f\,(\tau, B)}{g\,(\tau, B)} = J\,(\tau, B)\,. \tag{45}$$

At the equilibrium, $\tau^{opt} = \tau\,(B)$. It follows that

$$\frac{d\tau}{dB} = \frac{\partial J}{\partial \tau}\frac{d\tau}{dB} + \frac{\partial J}{\partial B} \Leftrightarrow \frac{d\tau}{dB} = \frac{\partial J/\partial B}{1-\partial J/\partial \tau}. \tag{46}$$

We have

$$\begin{aligned}\frac{\partial J}{\partial \tau} &= \frac{1}{g}\left[\frac{\partial f}{\partial \tau} - \frac{f}{g}\frac{\partial g}{\partial \tau}\right] = \frac{1}{g}\left[\frac{\partial f}{\partial \tau} - (1+\tau)\frac{\partial g}{\partial \tau}\right] \\ &= -\frac{1}{g}\left[\tau\frac{\partial h}{\partial \tau} + \frac{\sigma\omega}{B}(1+\tau)^{\sigma}\right] < 0. \end{aligned} \tag{47}$$

Since

$$\begin{aligned}\frac{\partial f}{\partial B} &= -\frac{(\sigma-1)\,1+\tau^{\sigma}}{\left[(1+\tau)^{\sigma}+B-1\right]^{2}} - \frac{\omega}{B^{2}} < 0, \\ \frac{\partial g}{\partial B} &= -\frac{(\sigma-1)(1+\tau)^{\sigma}}{\left[(1+\tau)^{\sigma}+B-1\right]^{2}} - \omega\frac{(1+\tau)^{\sigma}}{B^{2}} < 0,\end{aligned}$$

we find

$$\frac{-\frac{\partial g}{\partial B}\frac{B}{g}}{-\frac{\partial f}{\partial B}\frac{B}{f}} = \frac{\dfrac{\sigma-1}{(1+\tau)^{\sigma}+B-1} + \dfrac{\omega}{B^{2}}(1+\tau)^{\sigma}}{\dfrac{\sigma-1}{(1+\tau)^{\sigma}+B-1} + \dfrac{\omega}{B^{2}}} \times (1+\tau) > 1. \tag{48}$$

It follows that

$$\frac{\partial f}{\partial B}\frac{B}{f} > \frac{\partial g}{\partial B}\frac{B}{g}, \tag{49}$$

which implies $\partial J/\partial B > 0$. Inspection of (46) then confirms $d\tau/dB > 0$. The optimum tariff is increasing in B and is thus decreasing in β.

C Krugman-Bond-Syropoulos' Optimum Tariff

Krugman classifies the world into two groups: home and the rest of the world (see, for example, Krugman (1989)). Using the notation in Krugman's original study, the budget constraint of the rest of the world is given by $d^W + pm^W = y^W$, where d^W is the volume of the rest of world's consumption of its own products, m^W is rest of world's imports from our trading bloc, y^W is the volume of rest of world's output, and p is the price of our bloc's output relative to rest of world's output in the world (not internal) markets. If we do not take derivatives, choosing any good as a numéraire should not affect the equilibrium outcome. But when derivatives must be taken to derive the elasticities of foreign import demand, the use of the rest-of-the-world's budget constraint to derive the elasticity of import demand suppresses the individual country's import price effect on that country's imports. Normalization by the price of the home bloc's exportable becomes inappropriate. To correct such problem, we need to use each country's exportable price as the numéraire to compute its elasticity of import demand. Since the price elasticity is invariant to changes in the numéraire, we can then obtain the appropriate individual country's elasticity of import demand.

In addition to the normalization problem, Krugman considers $dy^W = 0$ to derive the elasticity of import demand. This is at odds with the fact that y^W includes p. Bond and Syropoulos (1996) tried to address the point by extending Krugman's model with nonnegative endowments of imported goods (i.e., $\omega \geq 0$ under symmetry). They derived their analogous optimum tariff formula, $\tau^{BS} = 1/\left(\epsilon^{BS} - 1\right)$, where ϵ^{BS} is the elasticity of import demand of the trading partners in their paper. Their optimum tariff, denoted by τ^{BS}, solves

$$\frac{1}{\tau^{BS}} = \left(1 + \frac{\left(1+\tau^{BS}\right)^{\sigma} - 2}{B}\right)\left(\frac{(\sigma-1)\,B}{B + \left(1+\tau^{BS}\right)^{\sigma} - 1} + \frac{\sigma B}{\alpha^{BS}B + 1 - \left(1+\tau^{BS}\right)^{\sigma}}\right), \tag{50}$$

where α^{BS} is defined as the measurement of comparative advantage: $\alpha^{BS} \equiv z/Nx$, $z \equiv A$, and $x \equiv \omega$ so that $\alpha^{BS} \equiv (1-\omega)/\omega$ in our notation. Our measure of the endowment structure (α and ω) ensures that the ranges of these indicators are bounded between 0 and 1. For example, if countries are not endowed with respective import goods ($\omega = 0$ and $\alpha^{BS} \to \infty$), then (50) approaches Krugman's (1989) formula as noted by Bond and Syropoulos (1996).

The critical difference between our approach and those of Krugman's and Bond and Syropoulos's is the treatment of the rest of the world and the income effect. According to Bond and Syropoulos (1996), the price of the home bloc's exports is defined as the numéraire. However, the elasticity of import demand of each trading bloc cannot be calculated from such normalization if there are more than two blocs. Note that all variables are symmetrized in equilibrium and the group reaction functions can still be derived in the symmetric model. Consequently, Krugman's and Bond and Syropoulos's calculations neglect to take account of the income effect generated by the change in tariff rates (or the terms of trade). Therefore, the specifications of Krugman's as well as Bond and Syropoulos's would be correct if there are only two trading blocs (i.e., $\beta = 0.5$) as indicated in Fig. 2.

Algebraically, for our tariff to be equal to their rates, we compare (12) and (50). Let $B = 2$ and we can obtain from (50) and (12):

$$1+\tau^i = \frac{-\omega + 2\sigma\left(\tau^i+1\right)^\sigma - \omega\left(\tau^i+1\right)^\sigma + 2}{\left(\tau^i+1\right)^\sigma\left(2\sigma+\omega+\omega\left(\tau^i+1\right)^\sigma - 2\right)},$$

where $i = BS$ and opt.

D The Elasticity of a Bloc's Export Supply

Letting $E_n \equiv \Omega_{nn} - (N/B)\, C_{nn}$ be the export supply of country n's exportable good to the other trading blocs, the corresponding elasticity of export supply χ_n is thus defined as

$$\chi_n = \frac{dE_n}{dp_n}\frac{p_n}{E_n} = \frac{dE_n}{dq_n}\frac{q_n}{E_n}, \tag{51}$$

where

$$\frac{dE_n}{dq_n} \equiv \frac{\partial E_n}{\partial q_n} + \frac{\partial E_n}{\partial y_n}\frac{dy_n}{dq_n}.$$

From (22), we have

$$E_n = \Omega_{nn} - \frac{N}{B}\frac{By_n}{\Phi}, \tag{52}$$

where $\Phi = 1 + (B-1)\left(1+\tau_n\right)^{-\sigma} q_n^{1-\sigma}$. Thus,

$$\frac{\partial E_n}{\partial q_n} = \frac{N}{B}\frac{(\sigma-1)(\Phi-1)}{\Phi^2}.$$

It follows that

$$\frac{N}{B}\left(\frac{\partial E_n}{\partial q_n} + \frac{\partial E_n}{\partial y_n}\frac{dy_n}{dq_n}\right) = \frac{N}{B}\left(\frac{(\sigma-1)(\Phi-1)}{\Phi} + \frac{q_n}{y_n}\frac{dy_n}{dq_n}\right)\frac{By_n/q_n}{\Phi}$$
$$= \left(\frac{(\sigma-1)(\Phi-1)}{\Phi} + \frac{q_n}{y_n}\frac{dy_n}{dq_n}\right)\frac{Ny_n/q_n}{\Phi}.$$

Thus, the elasticity of export supply is

$$\chi_n = \left(\frac{(\sigma - 1)(\Phi - 1)}{\Phi} + \frac{q_n}{y_n} \frac{dy_n}{dq_n} \right) \frac{y_n}{\Phi} \frac{N}{E_n}, \tag{53}$$

where

$$E_n = \alpha N + \omega - \frac{N y_n}{\Phi}, \ \frac{dy_n}{dq_n} = \frac{d}{dq_n} \left(\frac{1}{N} \sum_{k=1}^{N} q_{nk} \Omega_{nk} \right) = \frac{\omega (1 - B)}{B}.$$

In the symmetric equilibrium, therefore, the elasticity of export supply is given by

$$\chi_n = \left(\frac{(\sigma - 1)(B - 1)}{(1 + \tau_n)^{\sigma} + (B - 1)} + \frac{\omega (B - 1)}{B} \right) \left\{ \left(\alpha + \frac{\omega}{N} \right) \left\{ 1 + (B - 1)(1 + \tau_n)^{-\sigma} \right\} - 1 \right\}^{-1}. \tag{54}$$

Let $\tilde{\chi}_n = \lim_{N \to 1} \chi_n$. Then $\tilde{\chi}_n$ is the lower-bound of χ_n, where

$$\tilde{\chi}_n = (1 + \tau_n)^{\sigma} \left(\frac{\sigma - 1}{(1 + \tau_n)^{\sigma} + (B - 1)} + \frac{\omega}{B} \right). \tag{55}$$

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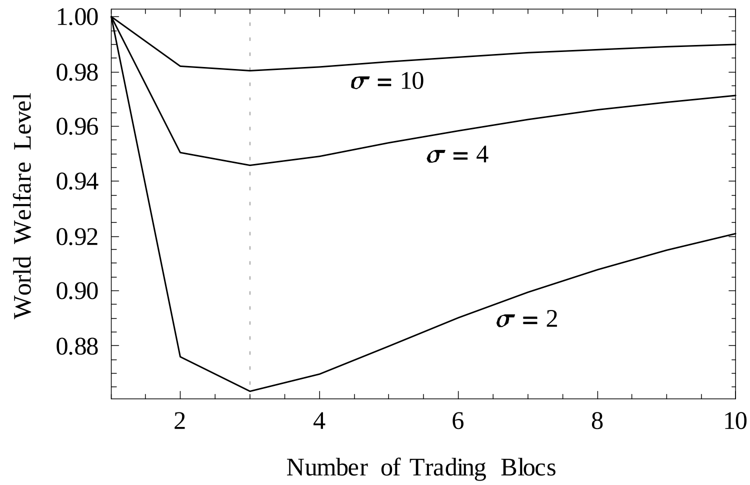

Figure 1: This chart is our reproduction of Krugman's model which also corresponds to Bond and Syropoulos's (1996) model in the exclusive endowment case. σ is the elasticity of substitution between goods. The world has the lowest welfare levels when it has three symmetric trading blocs as shown at the dotted grid line.

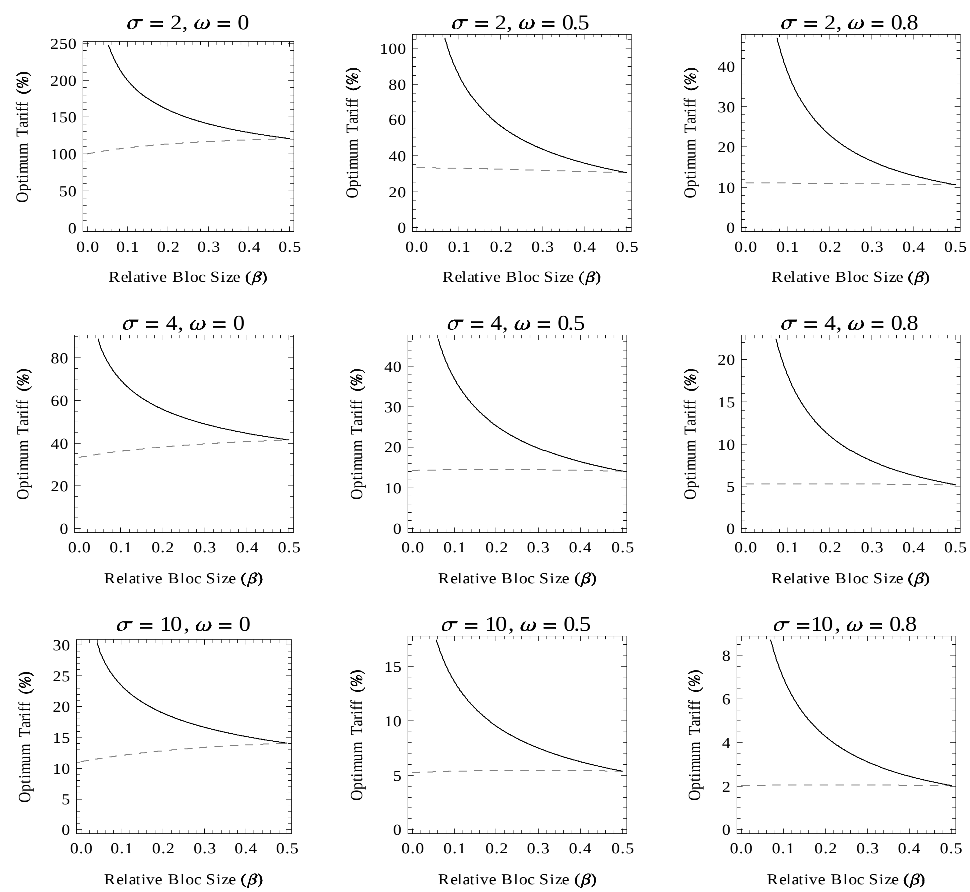

Figure 2: These charts show the computed optimum tariff rate as a function of β for various values of σ and ω. Solid curves are from our formula and dashed ones are from Bond and Syropoulos's. Note that if $\beta = 0.5$, the number of trading blocs is two.

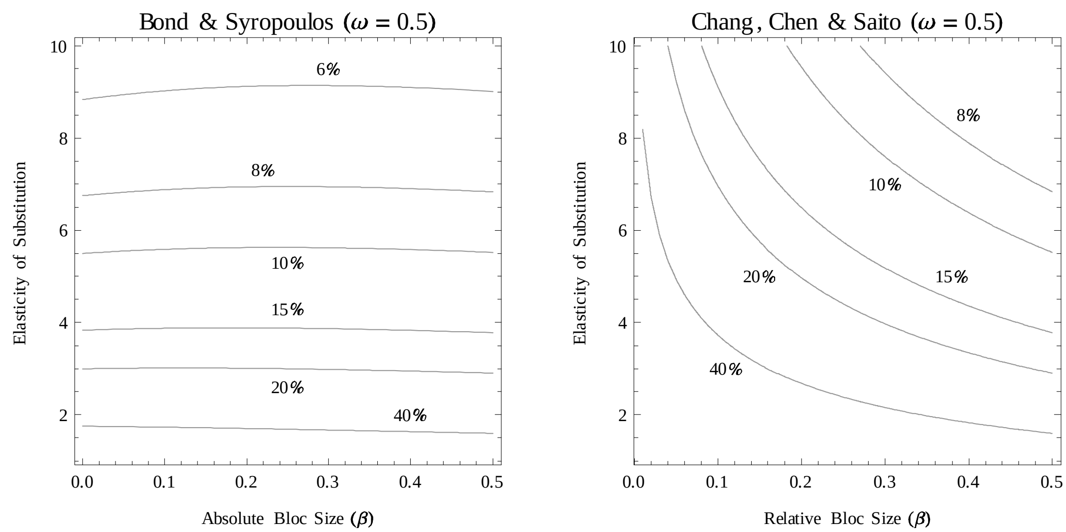

Figure 3: Both charts depict the iso-optimal tariff rate loci in the (β, σ) space. The left chart is calculated from Bond and Syropulos's formula and the right chart is from ours. Both charts are produced under $\omega = 0.5$.

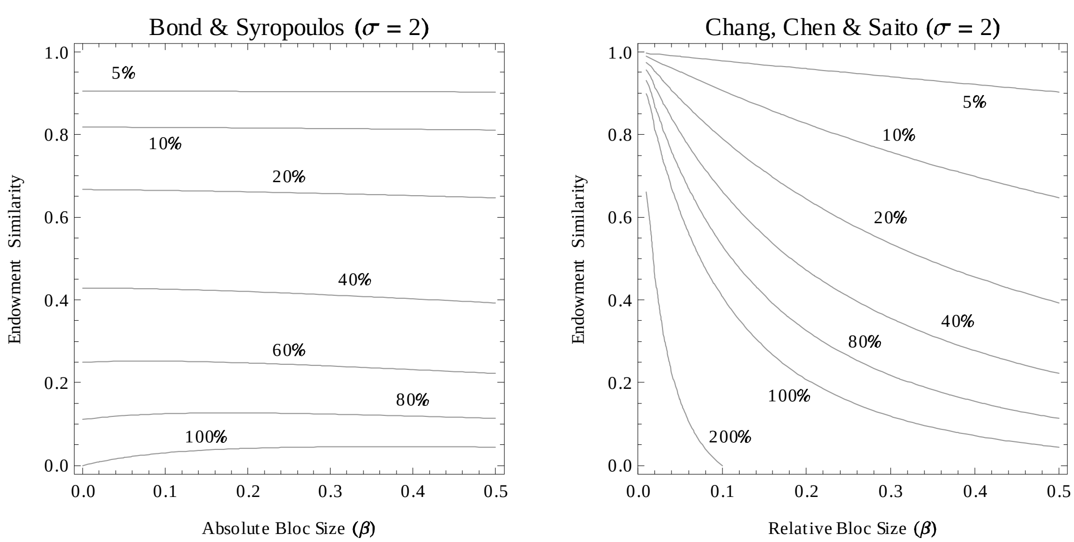

Figure 4: Both charts depict the iso-optimal tariff rate loci in the (β, ω) space. The left chart is calculated from Bond and Syropulos's formula and the right chart is from ours. Both charts are produced under $\sigma = 2$.

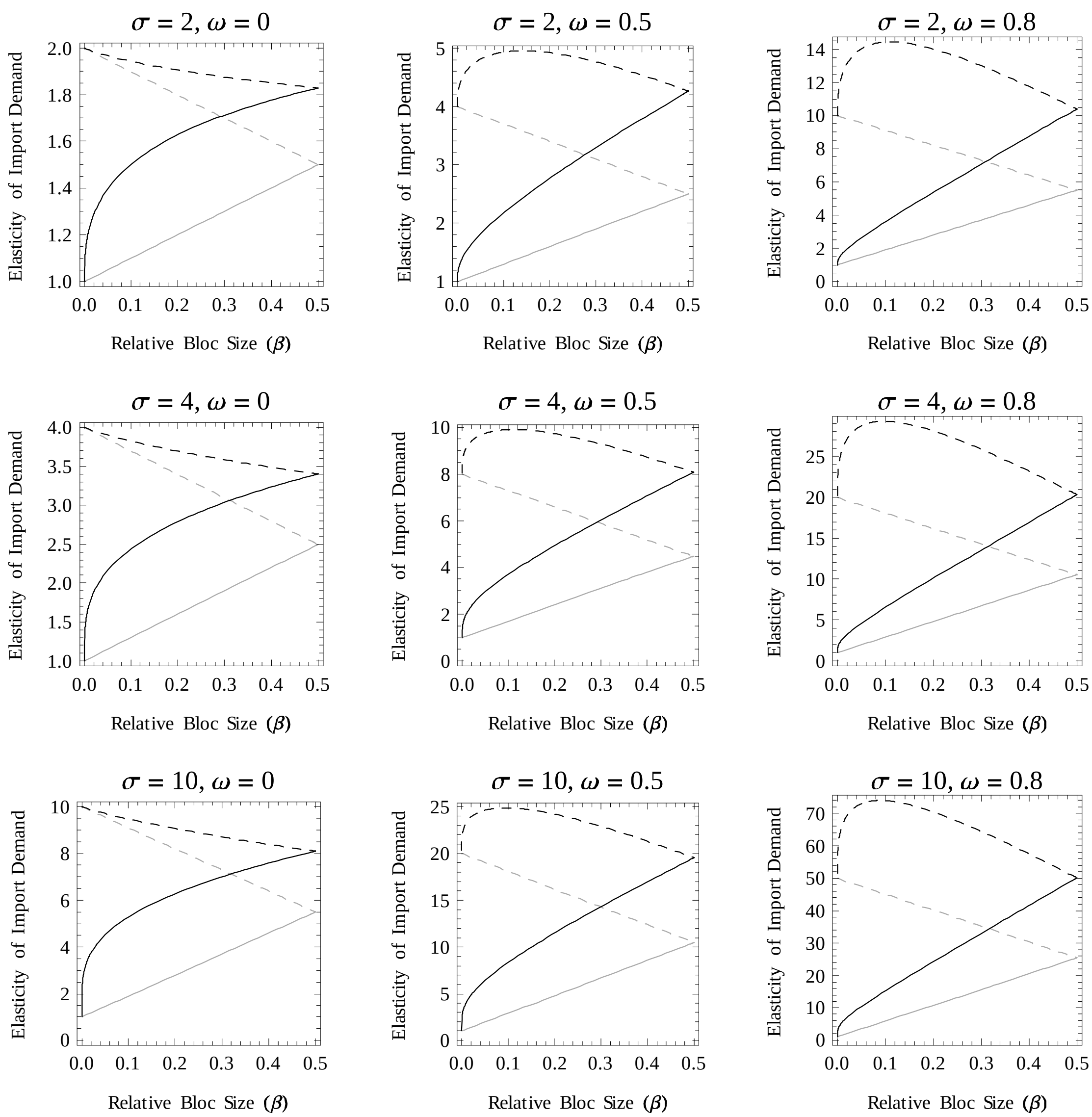

Figure 5: These charts show the relationship between the elasticity of import demand and the relative bloc size. The dashed curves indicate the values computed from Bond and Syropoulos's formula and the solid curves are from ours. In addition, the black curves indicate the values under Nash tariff and the gray ones indicate the values under free trade.

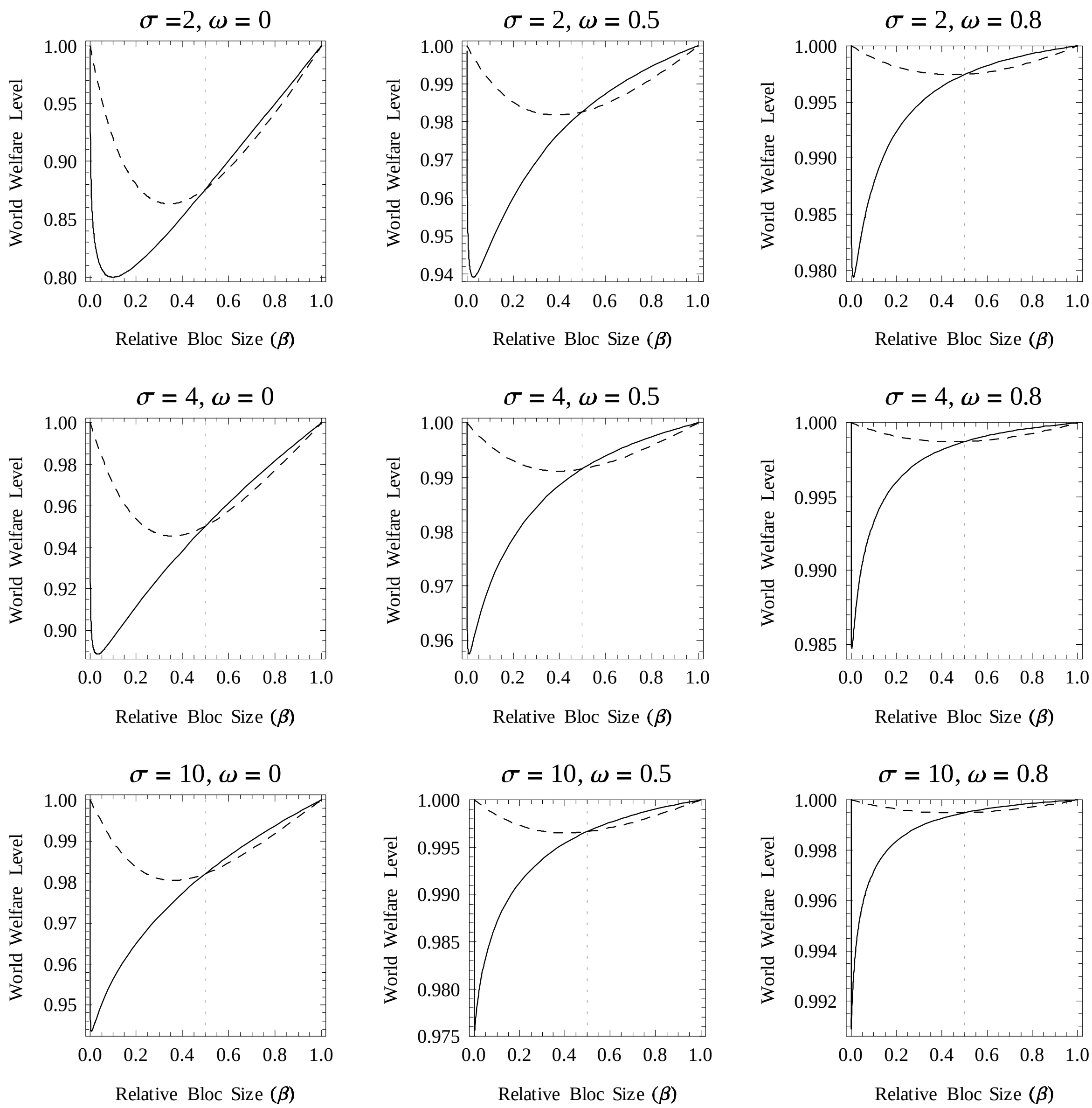

Figure 6: These charts show the relationship between the welfare level and relative bloc size for various values of σ and ω. Solid and dashed curves represent welfare levels computed from our formula and Bond and Syropoulos's formula, respectively.