Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy

Wenxin Du, Carolin E. Pflueger, and Jesse Schreger¹

First draft: September 2015

This draft: December 2016

¹Du: Federal Reserve Board, 20th and C Street NW, Washington, D.C. 20551. Email wenxin.du@frb.gov. Pflueger: University of British Columbia, Vancouver BC V6T 1Z2, Canada. Email carolin.pflueger@sauder.ubc.ca. Schreger: Harvard Business School and NBER, Morgan Hall, Soldiers Field Road, Boston, MA 02163. Email: jschreger@hbs.edu.

We are grateful to Mark Aguiar, Daniel Andrei, Adrien Auclert, John Campbell, Lorenzo Garlappi, Joshua Gottlieb, Juan Carlos Hatchondo (discussant), Oleg Itskhoki, Thomas Mertens, Vincenzo Quadrini (discussant), Julio Rotemberg, Rosen Valchev (discussant), Adrien Verdelhan, Jenny Tang, and seminar participants at AEA 2016, UCLA Anderson, Columbia University, Stanford GSB, MIT Sloan, the 8th Macro-Society Meeting, NBER Summer Institute, UC Santa Barbara, the San Francisco Federal Reserve, the Federal Reserve Bank of Chicago, the Bank for International Settlements, and the University of British Columbia for helpful comments. Jiri Knesl and Sandra Ramirez provided excellent research assistance. Pflueger thanks MIT Sloan and Stanford GSB for their hospitality and UBC for research funding while working on this research. Schreger thanks the Princeton Economics Department for their hospitality during the research process and the Harvard Business School Division of Research for financial support. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System. All errors are our own.
Abstract

Nominal debt provides consumption-smoothing benefits if it can be inflated away during recessions. However, we document empirically that countries with more procyclical inflation, where nominal debt provides less consumption-smoothing, issue more local-currency debt. We propose that the effect of monetary policy credibility on nominal bond risk premia is an important determinant of the currency composition of sovereign debt. In our model, low credibility governments inflate during recessions, generating excessively countercyclical inflation in addition to the standard inflationary bias. With countercyclical inflation, investors require risk premia on nominal debt, making nominal debt issuance costly for low credibility governments. We provide empirical support for this mechanism, showing that countries with higher nominal bond-stock betas have significantly larger nominal bond risk premia and borrow less in local currency.
1 Introduction

Over the past decade, the market for emerging market government debt has undergone a remarkable transformation. In the 1980s and 1990s, most emerging market sovereigns and several developed country governments relied heavily on foreign currency (FC) in their foreign borrowing. This left borrowers vulnerable to currency fluctuations and financial crises (Eichengreen and Hausmann, 2005). Since the Asian Financial Crisis, local currency (LC) government bond issuance has grown rapidly. It now constitutes an important asset class for international investors and more than half of external debt issued by major emerging market sovereigns (Du and Schreger, 2015). However the shift towards local currency government bonds has been highly uneven across markets, raising the question of what drives these cross-country differences.

This paper takes an asset pricing perspective to understand cross-country differences in sovereign debt portfolio choice. The standard approach to optimal government finance implies that governments should smooth the costs of taxation across states of the world (Barro, 1979). If the costs of taxation rise during recessions, due to high marginal consumption utility or distortionary taxes, government should issue debt that requires lower repayments in recessions than expansions. Applying this argument to nominal local currency debt, a key benefit of local currency debt is that it can provide debt relief at just the right time, provided that inflation reduces the real debt burden in recessions (Bohn, 1990a,b; Barro, 1997; Lustig et al., 2008). However, we find empirically that countries where nominal local currency debt provides little or no flexibility during adverse states of the world, issue the most nominal debt.

Our primary proxy for the hedging properties of local currency debt is the regression beta of local currency bond returns with respect to stock market returns. A positive bond-stock
beta indicates that local currency bonds’ expected real cash flows decline in stock market
downturns and hence provide fiscal flexibility to the issuer. Figure 1 summarizes the key
stylized fact that countries with the lowest local currency bond betas have the highest local
currency debt shares. Even more puzzlingly, a substantial fraction of the most prolific local
currency debt issuers, including both developed and emerging markets, have negative bond-
stock betas, so local currency debt provides no hedging benefits or is even risky from these
issuers’ perspective. This is the opposite of what we would expect if governments issue local
currency debt to take advantage of its fiscal hedging properties.

We show that positive bond-stock betas coincide with countercyclical inflation, or neg-
avative inflation-output betas. This finding is important, because it indicates that inflation
expectations are a key driver of the hedging properties of local currency bonds, which de-
preciate when inflation expectations increase. We also show that local equity excess returns
have betas with respect to US equity excess returns that are statistically indistinguishable
from one, making it plausible that global investors require a risk premium for holding bonds
that depreciate during periods of high local marginal utility.

The finding that countries with more countercyclical local currency bond returns rely
more on nominal local currency debt is highly robust. Results look similar if we use the
local currency debt share in all central government debt, which is most closely related to
a central government’s active issuance decisions, or the local currency debt share held by
foreigners, which plausibly generates an especially strong ex post incentive to inflate. It also
robust to using the cyclicality of realized or expected inflation with respect to output and to
using bond betas that control for default risk, real exchange rate cyclicality, or exclude the

2We show average local currency debt shares in central government debt and the estimated slope coefficient
of local currency government bond returns against local stock market returns for the period 2005-2014 for a
sample of 30 emerging and developed countries. For details see Section 2.
financial crisis.

What explains this apparently puzzling relation? We demonstrate that it is the equilibrium outcome when monetary policy credibility drives the cyclicality of inflation and risk-averse investors require a risk premium to hold local currency bonds in countries with positive inflation cyclicality. In the model, the government communicates a contingent plan for future inflation, but with a given probability it may revert to a myopic policy (Kydland and Prescott, 1977; Barro and Gordon, 1983; Rogoff, 1985). When commitment fails, the government uses inflation to reduce the real burden of local currency debt. The incentive to inflate is more pronounced during low output states, when marginal utility is highest. Crucially, debt is priced by risk-averse lenders, whose stochastic discount factor (SDF) is correlated with domestic output.

The key insight of the model is that when governments with imperfect credibility borrow in nominal terms from risk-averse lenders, they not only have a classic inflationary bias, but also lack the ability to commit to a degree of state-contingency on the debt. With risk-averse lenders, a government’s temptation to generate excessively countercyclical inflation leads lenders to charge an inflation risk premium. This lowers average borrower consumption. But a government with full commitment that borrows from risk-averse lenders can lower the risk premium it pays on LC debt. It achieves this by committing to an inflation process that keeps LC bond payouts relatively stable during recessions, when investors’ marginal utility is high, thereby increasing the insurance value of its LC debt to international investors. In contrast, a government lacking commitment cannot credibly promise to restrict itself to such a limited amount of state-contingency and therefore pays a higher-than-optimal risk premium. In equilibrium, governments that obtain little or no consumption-smoothing from issuing nominal debt (those with more procyclical inflation) issue the most nominal debt,
and those that could obtain the most consumption-smoothing from issuing nominal debt (those with more countercyclical inflation) issue the least.

Significantly, in our model limited commitment alone cannot resolve the positive relationship between local currency debt shares and inflation cyclicality. The intuition is that without risk premia, high credibility issuers optimally commit to using inflation only in bad states of the world, thereby smoothing tax distortions over states of the world and generating countercyclical inflation. As a result, the relation between local currency debt shares and inflation cyclicality is flat or downward-sloping, in contrast to the data. In our model, it is only the interaction of imperfect commitment and risk-averse lenders that can explain the empirical patterns.

Finally, we present empirical evidence on the connection between local currency bond risk premia and bond return cyclicality, monetary policy credibility, and local currency debt issuance. First, we show that higher local currency bond-stock betas are associated with significantly higher local currency bond risk premia, supporting the model mechanism, whereby investors require a premium for holding local currency bonds that tend to depreciate during downturns. Second, we provide direct evidence for the model mechanism by relating local currency bond-stock betas and local currency bond risk premia to two de-facto measures of monetary policy credibility, based on official central bank inflation targets and newspaper text analysis. Third, we show empirical evidence that local currency debt shares are strongly negatively correlated with local currency bond risk premia. Decomposing local currency bond risk premia into a world consumption asset pricing model (CAPM) component and a residual or alpha, we find that the world CAPM component accounts for the majority of the downward-sloping relation between LC debt shares and risk premia. Finally, we show that changes in inflation forecast cyclicality, proxying for bond risks during periods when
many issuers did not have LC bond price data, from the 1990s to the 2000s have a positive relation with changes in LC debt issuance, providing evidence that the bond risks channel of monetary policy credibility can also help us understand the substantial changes in LC debt issuance since the 1990s.

The notion that limited inflation commitment constrains nominal debt issuance has a long-standing tradition in economics, going back at least to Kydland and Prescott (1977) and Lucas and Stokey (1983). The continued relevance of this question is emphasized by Patrick Bolton’s recent AFA presidential address, which analyzes sovereign debt finance within a corporate finance framing (see Bolton (2016) and references therein). By contrast, we study the asset pricing implications of limited monetary policy commitment, thereby contributing to the literature on optimal debt management with nominal and inflation-indexed debt (Bohn (1988); Calvo and Guidotti (1993); Barro (1997); Alfaro and Kanczuk (2010); Díaz-Giménez et al. (2008)), and to the contemporaneous and complementary work by Ottonello and Perez (2016) and Engel and Park (2016). In our framework, a low credibility issuer finds it expensive to borrow through local currency debt, because local currency bond yields reflect not only high inflation expectations, as would be standard in the previous literature, but also higher risk premia. A low credibility issuer hence expects to pay more in real terms on LC debt debt, even after taking into account that LC debt will be partly inflated away. We contribute both empirically – by documenting the relation between inflation cyclicality and local currency debt shares in a cross-section of countries – and theoretically – by proposing that investor risk aversion interacted with limited monetary policy credibility can explain this new stylized fact. Broner et al. (2013) consider a sovereign’s optimal debt maturity

---

3Engel and Park (2016) study the currency composition of debt with optimal contracts and endogenous default when investors are risk-neutral. Ottonello and Perez (2016) present a quantitative model that generates predictions for the business cycle properties of local currency debt issuance.
choice in the presence of risk-averse investors, but take the correlation between bond returns and investors’ stochastic discount factor to be exogenous. We add to that by explaining bond return cyclicality and risks as an endogenous outcome of monetary policy credibility and matching cross-country evidence of bond return cyclicality. This paper is also related to a recent literature on inflation commitment and debt limits when the debt denomination is exogenous (Jeanne, 2005; Araujo et al., 2013; Aguiar et al., 2014; Chernov et al., 2015; Sunder-Plassmann, 2014; Bacchetta et al., 2015; Du and Schreger, 2015; Corsetti and Dedola, 2015) and the large literature on government debt and inflation (Sargent and Wallace, 1981; Leeper, 1991; Sims, 1994; Woodford, 1995; Cochrane, 2001; Davig et al., 2011; Niemann et al., 2013), but differs in that it considers the optimal portfolio choice between local currency and foreign currency debt issuance.

We contribute to the international asset pricing literature along two dimensions. First, we argue that risk premia matter for sovereign debt portfolio choice. Second, we provide a channel for why LC debt of low credibility countries comoves with international investors’ stochastic discount factor and hence requires a risk premium. Similarly to Hassan (2016) and Hassan et al. (2016) we argue that international government bond yields reflect the insurance value for investors, even though the source of comovement that we focus on – namely monetary policy credibility – is different from the sources they emphasize. In our model, comovement with international fundamentals is priced, consistent with empirical evidence in Harvey (1991); Karolyi and Stulz (2003); Lewis (2011); Borri and Verdelhan (2011); Lustig et al. (2011); David et al. (2016); Della Corte et al. (2016) among others.

Finally, we contribute to a recent literature on time-varying bond risks (Baele et al., 2010; David and Veronesi, 2013; Campbell et al., 2014; Ermolov, 2015; Campbell et al., 2015), that is primarily focused on the US and the UK. Vegh and Vuletin (2012) also emphasize the
evolution and cross-country heterogeneity in the cyclicality of monetary policy, but do not study implications for sovereign debt portfolios. Poterba and Rotemberg (1990) examine the correlation between taxes and inflation under both commitment and no-commitment in five major developed countries, but do not consider the interaction with the currency composition of government debt. We do not take a stand on the interest rate policy needed to implement the optimal inflation process, which is studied in Campbell et al. (2015).

The structure of the paper is as follows. In Section 2, we present new stylized facts on the relation between the cyclicality of local currency bond risk and shares of local currency debt in sovereign portfolios. In Sections 3 and 4 we lay out the model, provide analytical intuition for the key mechanisms, and calibrate the model to demonstrate that it can replicate the observed patterns of the currency composition of sovereign debt and inflation cyclicality. Section 5 tests additional model implications for local currency debt issuance and risk premia. Section 6 concludes.

2 Empirical Evidence

In this section, we demonstrate the robust empirical relation that countries with more counter-cyclical inflation have lower local currency debt shares. This evidence is robust to proxying for inflation cyclicality with the inverse of local currency bond cyclicality. It is also highly robust to excluding the financial crisis and to using alternative measures of local currency debt shares, such as the share in all central government debt, the share only in long-term debt, or the share held by foreigners. Our evidence is based on as large a cross-section of countries as permitted by the availability of LC debt data, including eleven developed markets (Australia, Canada, Denmark, Germany, Japan, New Zealand, Norway,
Sweden, Switzerland, United States and United Kingdom) and 19 emerging markets (Brazil, Chile, China, Colombia, Czech Republic, Hungary, Indonesia, Israel, Malaysia, Mexico, Peru, Philippines, Poland, Russia, Singapore, South Africa, South Korea, Thailand and Turkey).

\section{Nominal Bond Risks: Bond-Stock Beta}

Asset markets incorporate investors’ forward-looking information at much higher frequency than surveys and can therefore provide valuable proxies for inflation cyclicality that are potentially less subject to measurement error and more robust given the relatively short time series. Local currency bond-stock betas serve as an asset market based proxy of inflation cyclicality. If stock returns are procyclical, we expect bond-stock betas to be inversely related to the cyclicality of inflation expectations.

We denote the log yield on a nominal LC $n$-year bond as $y_{nt}^{LC}$, where $y_{nt} = \log(1 + Y_{nt}^{LC})$. The log holding period return on the bond is given by

$$r_{n,t+\Delta t}^{LC} \approx \tau_{n}y_{nt}^{LC} - (\tau_{n} - \Delta t)y_{n-1,t+\Delta t}^{LC},$$

where $\tau_{n} = \frac{1-(1+Y_{nt}^{LC})^{-n}}{1-(1+Y_{nt}^{LC})^{-1}}$ is the duration of a bond selling at par (Campbell et al. (1997)). We approximate $y_{n-\Delta t,t+\Delta t}^{LC}$ by $y_{n,t+\Delta t}^{LC}$ for the quarterly holding period. We let $y_{1t}^{LC}$ denote

\footnote{For LC bond yields, we use primarily Bloomberg fair value (BFV) curves. BFV curves are estimated using individual LC sovereign bond prices traded in secondary markets. Since sufficient numbers of bonds spanning different maturities are needed for yield curve estimation, the availability of the BFV curve is a good indicator for the overall development of the LC nominal bond market. Countries such as Argentina, Uruguay and Venezuela only have a handful of fixed-rate bonds and hence do not have a BFV curve. As for most emerging markets in our sample BFV curves are available starting in the mid-2000s, we focus on the period 2005-2014 to maintain a balanced panel. To measure inflation risk and the perceived cyclicality of inflation, we use realized inflation from Haver and inflation forecasts from Consensus Economics, respectively. Finally, we measure the share of local currency debt in total sovereign debt portfolios with data from BIS Debt Securities Statistics, OECD Central Government Debt Statistics, and several individual central banks. All results winsorize the highest and lowest observation to ensure that results are not driven by outliers.}
the three-month T-bill yield and then the excess return on LC bonds over the short rate is given by

\[ x_{r_{\text{LC}}} = r_{n,t+\Delta t}^{\text{LC}} - y_{1t}^{\text{LC}}. \]

From a dollar investor’s perspective, we can rewrite the excess return as

\[ x_{r_{\text{LC}}} = [r_{n,t+\Delta t}^{\text{LC}} - (y_{1t}^{\text{LC}} - y_{1t}^{\text{US}})] - y_{1t}^{\text{US}}. \]

The dollar investor can hedge away the currency risk of the holding period \( \Delta t \) by going long a US T-bill and shorting a LC T-bill with the same market value as the LC bond. By doing so, any movement in the spot exchange rate of the LC has the same offsetting first-order impact on the bond position and the local T-bill position and hence cancels out. After hedging currency risk for the holding period, the dollar investor bears duration risk of the LC bond.

We define the local equity excess returns as the log return on local benchmark equity over the three-month LC Treasury bill:

\[ x_{r_{m,t+\Delta t}} = (p_{t+\Delta t}^m - p_t^m) - y_{1t}^{\text{LC}}, \]

where \( p_t^m \) denotes the log benchmark equity return index at time \( t \). Country subscripts are suppressed to keep the notation concise. We then compute the local bond-stock beta \( b(\text{bond}, \text{stock}) \) by regressing LC bond excess returns \( x_{r_{t+\Delta t}}^{\text{LC}} \) on local equity excess returns \( x_{r_{t+\Delta t}}^m \):

\[ x_{r_{t+\Delta t}}^{\text{LC}} = b_0 + b(\text{bond}, \text{stock}) \times x_{r_{t+\Delta t}}^m + \epsilon_t. \]  

(1)

Bond-stock betas measure the risk exposure of LC bond returns on local equity returns.
2.2 Cyclicality of Inflation Expectations: Inflation-Output Forecast Beta

We construct a new measure for the procyclicality of inflation expectations by regressing the change in the CPI inflation rate predicted by forecasters on the change in their predicted real GDP growth rate. Each month, professional forecasters surveyed by Consensus Economics forecast inflation and GDP growth for the current and next calendar year. We pool all revisions for 2006 through 2013 (so that the forecasts were all made post-2005), and run the country-by-country regression

\[ \Delta \tilde{\pi}_t = b_0 + b(\tilde{\pi}, \tilde{gdp}_t) \times \Delta \tilde{gdp}_t + \epsilon_t, \]  

where \( t \) indicates the date of the forecast revision. The revisions to inflation forecasts (\( \Delta \tilde{\pi}_t \)) and GDP growth forecasts (\( \Delta \tilde{gdp}_t \)) are percentage changes of forecasts made three months before and proxy for shocks to investors’ inflation and output expectations. The coefficient \( b(\tilde{\pi}, \tilde{gdp}_t) \) measures the cyclicality of inflation expectations and is the coefficient of interest.

Because forecasts are made for calendar years, the forecast horizon can potentially vary. Consensus forecasts the annual inflation rate up to two years in advance. This means that in January 2008, the forecast of calendar year 2008 inflation is effectively 11 months ahead and the forecast of calendar year 2009 is 23 months. We focus on revisions to the two-year forecast (13-23 months ahead) to minimize variation in the forecast horizon.
2.3 Cyclicality of Realized Inflation: Realized Inflation-Output Beta

While investors’ beliefs about inflation cyclicality enter into government debt prices and hence sovereign debt portfolio choice, it is useful to verify that the composition of debt portfolios also lines up with the cyclicality of realized inflation and output. We compute the realized inflation-output beta by regressing the change in the inflation rate on the change in the industrial production growth rate:

\[ \Delta \pi_t = b_0 + b(\pi, IP) \Delta IP_t + \epsilon_t, \]  

(3)

where \( \Delta \pi_t \) is the 12-month change in the year-over-year inflation rate and \( \Delta IP_t \) is the 12-month change in the year-over-year industrial production growth rate. The coefficient \( b(\pi, IP) \) measures the realized inflation cyclicality with respect to output. We obtain the seasonally adjusted consumer price index and the industrial production index from Haver between 2005 and 2014.

2.4 Local Currency Debt Shares

For developed countries, we construct the share of local currency debt based on the OECD Central Government Debt Statistics and supplement this data with hand-collected statistics from individual central banks.\(^5\) Central banks typically directly report the instrument composition of debt securities outstanding issued by the central government.

For emerging markets, we measure the share of local currency debt in sovereign debt

\(^5\)The OECD Central Bank Debt Statistics was discontinued in 2010. We collected the statistics between 2010-2014 from individual central banks.
portfolios using the BIS Debt Securities Statistics, supplemented with statistics from individual central banks. Table 16C of the Debt Securities Statistics reports the instrument composition for outstanding domestic bonds and notes issued by the central government \( (D_{\text{dom}}^t) \) starting in 1995. Table 12E of the Debt Securities Statistics reports total international debt securities outstanding issued by the general government \( (D_{\text{int}}^t) \). For emerging markets, as the vast majority of international sovereign debt is denominated in foreign currency, and local governments rarely tap international debt markets, \( D_{\text{int}}^t \) offers a good proxy for central government foreign currency debt outstanding. Data for developed countries are from individual central banks or the OECD. The share of local currency debt is computed as the ratio of the fixed-coupon domestic sovereign debt outstanding \( (D_{\text{int}}^t) \) over the sum of domestic and international government debt:

\[
s_t = \frac{D_{\text{dom}, \text{fix}}^t}{D_{\text{dom}}^t + D_{\text{int}}^t}.
\]

Inflation-linked debt, floating-coupon debt and FC debt are all treated as real liabilities. In our baseline results, we do not distinguish between foreign and domestically-owned debt, but we provide evidence in Appendix B that empirical results are similar for foreign-owned debt.

### 2.5 Summary Statistics

Table 1 reports summary statistics for inflation, inflation expectations, local currency bond yields, bond-stocks betas, inflation-output forecast betas, realized inflation-output betas, local equity-S&P betas, and local currency debt shares. Emerging market realized inflation is 2.4 percentage points higher and survey-based expected inflation is 2.0 percentage points
higher than in developed markets. In addition, expected inflation and realized inflation are less procyclical in emerging markets than in developed countries.

For local currency bonds, five-year local currency yields are 3.4 percentage points higher in emerging markets than in developed markets. Nominal bond returns are countercyclical in developed markets, as evident from negative bond-stock betas. By contrast, local currency bond returns are procyclical in emerging markets. Finally, developed markets borrow almost entirely with local currency debt, while the local currency debt share in emerging market averages only 60%.

Importantly, column (7) shows that the beta of local stock returns with respect to US S&P 500 stock returns, estimated as the slope coefficient of regressing local log excess equity returns onto US log equity excess returns, is one on average for both developed and emerging economies in our sample. If local stock return variation proxies for variation in the local stochastic discount factor and US equity returns reflect variation in international investors’ stochastic discount factor, a local-US stock beta close to one implies that assets that co-move with the local stochastic discount factor also co-move with the international investor’s stochastic discount factor and hence are risky for international investors. This evidence is also consistent with the evidence in David et al. (2016), who argue that emerging market stock returns have large betas with respect to the world equity portfolio and consequently comovement with local stock markets carries a risk premium in international markets. In particular, this implies that if the domestic government inflates away its local currency debt in states of high local marginal utility of consumption, local currency debt tends to depreciate in real terms in bad states of the world for international investors’, making it a risky investment.
2.6 Relation between Nominal Risk Betas and Sovereign Debt Portfolios

Figure 2 adds to the evidence in Figure 1, showing that patterns are similar if we measure bond return cyclicality with respect to US instead of local stock returns and if we replace bond return cyclicality with inflation cyclicality. The inverse of LC bond betas should proxy for the cyclicality of inflation expectations, if higher inflation expectations depress LC bond prices and stock returns fall during recessions. Panels A and B of Figure 2 confirm this intuition. Emerging markets tend to have lower local currency debt shares and more negative realized and expected inflation betas, as would be the case if they inflate during recessions. This finding is important, because it indicates that inflation and output dynamics are key to understanding the cross-country patterns of LC bond risks.

Panel C of Figure 2 shows LC bond betas with respect to US S&P returns, which is constructed analogously using LC bond betas with respect to S&P excess returns. For instance, the Brazilian bond-stock beta is estimated as the slope coefficient of LC bond excess returns with respect to Brazilian stock excess returns as in (1), while the Brazilian bond-S&P beta is the estimated slope coefficient of LC bond excess returns with respect to US S&P excess returns. Panel C shows a striking correlation between bond-stock betas and bond-S&P across countries. Given this result, it is unsurprising that the relation between LC debt shares and bond-S&P betas in Panel D is downward sloping, similar to the results for bond-stock betas in Figure 1. Taken together, 2 Panels C and D indicates that LC bonds that provide the best hedge for the issuer are also riskiest for an international investor. This finding is important, because it gives us further indication that it is reasonable to think of international investors as risk-averse over LC bonds that lose value in real terms during bad states of the world for local consumers, consistent with our modeling assumption that
domestic and international investors price bonds with correlated stochastic discount factors.

Table 2 shows cross-sectional regressions of local currency debt shares on measures of inflation cyclicality. The first three columns show that all nominal risk betas are significantly correlated with LC debt shares. A 0.16 increase in the bond-stock beta, corresponding to the average difference between emerging and developed markets, is associated with an 18 percentage point reduction in the LC debt share. Columns (4) and (5) show that the relation is robust to controlling for mean log GDP per capita and exchange rate regimes as classified by Reinhart and Rogoff (2004).

The relationship between the local currency debt share and nominal risk betas is robust to using long-term debt, excluding the financial crisis, adjusting for default risk, and using only externally held government debt. The robust result for the local currency debt share in long-term debt is important, because Missale and Blanchard (1994) argue that shorter debt maturity reduces the incentive to inflate away debt. The recent increase in emerging market LC debt issuance has been accompanied by a surge in importance of this asset class among international investors. Consistent with this, our findings are also robust to using (a proxy of) the LC debt share held in foreigners’ portfolios instead of the LC debt share in all government debt, thereby ruling out the possibility that high LC debt shares simply reflect high LC debt shares held domestically, which might lead to less inflationary incentives than LC debt held by foreigners. The finding that cross-sectional variation in LC debt shares is driven by LC debt held by foreigners is important, because it corresponds most closely to our modeling assumption that debt is held by foreign investors. It also further strengthens the puzzle, because a high LC debt share held by foreigners should be especially useful for smoothing domestic consumption with state-contingent inflation. Detailed results are available in Appendix A.
3 Model

This section describes the model and presents analytic solutions for the debt portfolio and inflation policies. The model has two periods. In period 1, the government chooses the government debt portfolio consisting of LC and FC debt, and communicates a desired inflation policy. In period 2, output is realized and debt is repaid. Building on the loose commitment mechanism of Debortoli and Nunes (2010), the parameter \( p \) captures credibility in the model and determines how easily the government can commit to an inflation policy. With probability \( p \), the government implements the previously communicated inflation plan. With probability \( 1 - p \), the no-commitment state in period 2 is realized and the government re-optimizes myopically. We show in Appendix B that the model can be embedded in a simple dynamic setup.

The government objective is standard, reflecting domestic agents’ power utility over consumption and a quadratic inflation cost. We assume that investor marginal utility is correlated with domestic marginal utility, consistent with the empirical evidence of high local-US stock betas in Table 1 and the close correspondence between bond-stock and bond-S&P betas in Figure 2 Panel C. We follow the portfolio choice literature (Campbell and Viceira (2002); Devereux and Sutherland (2011)) in using a log-quadratic expansion for log debt portfolio returns and ex-ante expected utility, but a log-linear expansion for the budget constraint in the period 2 no-commitment state. This solution method captures the dependence of portfolio choice on ex-ante hedging motives, but maintains tractability and allows the intuition behind the results to be clearly seen. It has been found to be highly accurate in many standard portfolio choice applications and we verify its numerical accuracy by comparing the analytic solution with an exact numerical solution in Appendix B.4.
3.1 Government Objective

We use lower-case letters to denote logs. The government’s loss function combines quadratic loss in log inflation $π_2$ and power utility over consumption:

$$L_2 = απ_2^2 - \frac{C^1_2}{1−γ}. \quad (4)$$

We do not take a stand on the source of inflation costs. A quadratic inflation cost of the form (6) may arise from price-setting frictions leading to production misallocation as in New Keynesian models (see Woodford (2003)). Period 2 output is log-normally distributed:

$$X_2 = \bar{\bar{X}} \exp(\frac{x_2}{\bar{\bar{X}}}), x_2 \sim N\left(0, σ^2_x\right). \quad (5)$$

Domestic consumption equals output minus real debt repayments to foreign bond holders $D_2$, generating an incentive for the government to reduce real debt repayments:

$$C_2 = X_2 - D_2. \quad (6)$$

One interpretation of (6) is that the government only cares about the consumption of non-bond holders, who could either be foreigners or domestic agents to whom the government attaches little weight for political economy reasons. We normalize steady-state period 2 consumption to one, so $α$ captures the cost of inflation distortions in units of period 2 consumption.\(^6\) Formally, we require that $\bar{\bar{X}} = 1 + \bar{\bar{D}}$.

\(^6\)Allowing period 2 steady-state consumption different from one would scale the loss function (6) by a constant, leaving the analysis unchanged.
3.2 Investors

Financial markets are integrated in the sense that all assets are priced by the same international investor. However, markets are segmented from the domestic borrower’s point of view, who has access only to LC and FC debt borrowing and cannot go long bonds. Inflation in the investor’s home currency is assumed to be zero for simplicity. International consumption and domestic consumption can differ if international agents prefer a different consumption bundle from domestic agents.

The international investor is risk-averse over world output $x_2^*$, which is log-normally distributed with standard deviation $\sigma^*$. We model the international investor’s stochastic discount factor (SDF) in reduced form with risk aversion coefficient $\theta$, similarly to Arellano and Ramanarayanan (2012), with local output $x_2$ loading onto world output:

\[
m_2^* = \log \beta - \theta x_2^* - \frac{1}{2} \theta^2 (\sigma^*)^2, \tag{7}
\]
\[
x_2 = \lambda x_2^* + \eta_2. \tag{8}
\]

Here, $\eta_2 \sim N(0, \sigma_\eta^2)$ is an idiosyncratic shock uncorrelated with world output. The SDF (7) captures risk-neutral investors as a special case when $\theta = 0$. If investor risk aversion $\theta$ is greater than zero and global and local output are positively correlated ($\lambda > 0$), the SDF (7) implies that investors’ and the domestic consumer’s marginal utility of consumption are positively correlated, or that bad states of the world for the domestic consumer also tend to be bad states of the world for the investor.

We interpret the stochastic discount factor (7) through (8) broadly, potentially reflecting several channels. First, if international investors are risk-averse over international consump-
tion and output, and international output is correlated with domestic output, this may give rise to a correlation between international and domestic marginal utility. We document a high correlation in output growth across countries, lending credence to this channel. In our sample, the average correlation between emerging market output growth and U.S. output growth is equal to 58%. Second, it is plausible that these correlations are a lower bound for the degree of international comovement in stochastic discount factors (Brandt et al., 2006). We find that the average correlation between emerging market stock returns and US stock returns is even higher at 70%, as would be the case if stochastic discount factors co-vary more than output. Interpreting (7) more broadly, highly correlated consumption growth (Colacito and Croce, 2011; Lewis and Liu, 2015), correlated discount rate news (Borri and Verdelhan, 2011; Viceira et al., 2016), or correlated risk premia (Longstaff et al., 2011) may further drive up the cross-country correlations between stochastic discount factors and stock returns. Moreover, correlations between international equity markets may increase during downturns, increasing their impact on risk premia (Ang and Bekaert, 2002). A different way to motivate an SDF of the form (7) and to generate the main channel in our model, would be if bond investors are domestic and hence risk-averse over domestic output, but the government has an incentive to expropriate bond holders, because ex-post it is more efficient to use the inflation tax rather than taxes that distort incentives and lead to deadweight costs, such income or sales taxes.7

The role of real exchange rate shocks in the model is to capture a principal cost of FC borrowing of Eichengreen and Hausmann (2005), namely that FC debt exposes issuers to sudden increases in the real cost of debt service. This cost incentivizes otherwise unconstrained borrowers, such as the US, to borrow in local currency. We normalize the real

7Appendix B.7 develops such a model extension, shows that the analytic solutions are of the same form as in our benchmark model, and illustrates that our channel remains quantitatively important.
exchange rate in period 1 to one. The period 2 real exchange rate (in units of international
goods per domestic goods) is given by

\[ \exp\left(\varepsilon_2 - \frac{1}{2}\sigma^2_{\varepsilon}\right), \quad \varepsilon_2 \sim N\left(0,\sigma^2_{\varepsilon}\right), \quad (9) \]

where \(\varepsilon_2\) is uncorrelated with all other shocks and realized after the government has chosen
inflation, so monetary policy takes effect more slowly than exchange rate shocks. In modeling
the real exchange rate, we face a tension arising from the well-known Backus-Smith puzzle.
While under complete international financial markets the real exchange rate of a country is
predicted to depreciate when consumption is high, empirically real exchange rates are close to
being uncorrelated with real economic fundamentals or even depreciate when consumption is
low (Backus and Smith (1993)). We choose the reduced-form specification (9) to be consistent
with the empirical evidence, so as to generate empirically relevant bond risk predictions. We
keep the specification of exchange rates quite general, but one possible driver of exchange
rates could be shocks to the risk bearing capacity of financial intermediaries, as in Maggiori
and Gabaix (2015). One important implication of (9) is that domestic inflation tends to
devalue local currency bonds for international agents and drives local currency bond risks,
consistent with the empirical evidence in Panels A and B of Figure 2.\footnote{However, our assumptions are considerably weaker than perfect purchasing power parity (PPP), since we need it to hold only on average and not state-by-state and at horizons corresponding to bond maturities, which in our data average 5 years. Given that there is some evidence that real exchange rates tend to depreciate in recessions, Appendix B.6 solves an extended model with this feature and finds that model implications are substantially unchanged.}

We can now price three different bonds: a) a foreign currency bond, which pays one unit
of real international consumption; b) a nominal LC bond, which delivers \(\exp(-\pi_2)\) units
of real domestic consumption; b) a real LC bond, which delivers one unit of real domestic
consumption. Bond prices are given by (see Appendix B for details):

\[ q_{FC}^{1} = \beta, \quad (10) \]
\[ q_{LC, real}^{1} = \beta, \quad (11) \]
\[ q_{LC}^{1} = E \left[ \exp \left( \log \beta - \phi x_{2} - \frac{1}{2} \phi^{2} \sigma_{x}^{2} \right) \exp(-\pi_{2}) \right], \quad (12) \]
\[ \phi = \theta \lambda \left( \frac{\sigma^{*}}{\sigma_{x}} \right)^{2}. \quad (13) \]

LC bonds are priced as if the international investor had effective risk aversion \( \phi \) over local output \( x_{2} \). Expression (13) shows that the international investor is effectively more risk-averse over local output if risk aversion \( \theta \) is high or if the local output loading onto world output \( \lambda \) is high. The ratio of the variances enters, because if local output is more volatile than world output, world output moves less than one-for-one with local output, so international investors appear less risk-averse over local output variation. The real exchange rate does not enter into the pricing of real and nominal LC bonds, because in expectation one unit of real domestic consumption buys one unit of real international consumption and exchange rate shocks are uncorrelated with all other shocks. Finally, we denote one-period log bond yields by

\[ y_{LC}^{1} = -\log q_{LC}^{1}, \quad y_{FC}^{1} = -\log q_{FC}^{1}. \]

We assume that domestic equity is a claim on domestic output and is priced by the same international investor, giving the equity risk premium faced by the international investor as

\[ E_{1} (r_{2}^{e}) + \frac{1}{2} Var_{1} (r_{2}^{e}) - y_{FC}^{1} = \theta Cov_{1} (x_{2}^{*}, x_{2}) = \phi \sigma_{x}^{2}. \quad (14) \]
Equity is in zero supply to financial investors, thereby not entering into domestic consumption. The expression for the equity premium will be useful in Section 4 to calibrate the magnitude of risk premia.

We abstract from the risk of outright sovereign default. Under the assumption of simultaneous default, which Du and Schreger (2016) and Jeanneret and Souissi (2016) show is empirically plausible, LC and FC debt by the same issuer bears the same default risk premium. Even then, issuing FC debt may be costly if it precludes the option to use inflation to avoid outright default. In the current framework, exchange rate volatility is the main driver making FC debt issuance costly, so adding such an additional cost of FC debt would act similarly to increasing the exchange rate volatility. For an analysis of the choice of the currency denomination of sovereign debt with strategic default, see Engel and Park (2016).

3.3 Budget Constraint

To focus on the portfolio choice component of the government’s decision, we assume that the government must raise a fixed amount $V$. The government chooses face values $D^{FC}$ and $D^{LC}$ to satisfy the budget constraint:

$$D^{FC}_1 q^{FC}_1 + D^{LC}_1 q^{LC}_1 = V.$$  (15)

Here, we do not explicitly allow the government to issue inflation-indexed LC debt. In contrast to the hypothetical real LC bond considered in the previous Section, in practice inflation-indexed bond issuance appears to be costly. Inflation-indexed bond issuance can be costly for reasons analogous to those for foreign currency debt, if indexation is imperfect, either because the inflation index does not correspond perfectly to the domestic borrower’s consumption basket, or because indexation occurs with lags. In addition, empirical evidence from the US suggests that inflation-indexed debt requires a substantial liquidity premium (Pflueger and Viceira (2016)). For this reason, in our empirical analysis we combine inflation-indexed and foreign currency debt to capture inflation-insulated debt issuance.
Let $s$ denote the share of nominal LC bonds in the government’s portfolio:

$$s_1 = \frac{q_1^{LC} D_1^{LC}}{V}. \quad (16)$$

We define the debt portfolio log return in excess of the log return on a domestic consumption risk-free bond:\footnote{Taking the expectation over $\varepsilon_2$, the average cost in terms of domestic consumption of repaying a unit face value FC bond is greater than one. While the mean exchange rate is one, the mean inverse exchange rate is not equal to one due to Jensen’s inequality. To purchase one unit of international consumption, the domestic borrower expects to give up more than one unit of real domestic consumption, because he has to average over states with different exchange rates. This divergence between the expected return on risk-free real FC and LC bonds is also known as Siegel’s paradox (Siegel (1972), Karolyi and Stulz (2003)).}

$$x_{r2}^d = \log \left( \frac{D_1^{FC} \exp \left( -\varepsilon_2 + \frac{1}{2} \sigma_x^2 \right) + D_1^{LC} \exp \left( -\pi_2 \right) }{\beta^{1-V}} \right). \quad (17)$$

### 3.4 Log-Quadratic Expansion for Loss Function

This section derives a log-quadratic expansion of the government loss function, which provides intuition and is used for the log-linear analytic solution. In contrast, the numerical solutions do not rely on the log-quadratic expansion, instead using the exact expressions in Sections 3.1 through 3.3. We obtain the following second-order expressions for local currency bond prices and risk premia:

$$q_1^{LC} \approx \beta \exp \left( -E_1 \pi_2 + \frac{1}{2} Var_1 \pi_2 + \phi Cov_1 (x_2, \pi_2) \right), \quad (18)$$

$$y_1^{LC} - E_1 \pi_2 + \frac{1}{2} Var_1 \pi_2 - y_1^{FC} = -\phi Cov_1 (x_2, \pi_2). \quad (19)$$

The output-inflation covariance $Cov_1 (x_2, \pi_2)$ enters as a risk premium term. Intuitively, a positive output-inflation covariance means that the issuer does not inflate during bad times,
making LC bonds safe from investors’ point of view and increasing the value to investors.

Note that provided that $\pi_2$ is a function of local output $x_2$, (19) also equals the possibly more familiar expression in terms global output $-\theta \text{Cov}_1(x^*_2, \pi_2)$. We expand bond portfolio excess returns log-quadratically following Campbell and Viceira (2002):

$$xr^*_2 + \frac{1}{2} (xr^*_d)^2 \approx (1 - s_1) \left( \varepsilon_2 + \frac{1}{2} (\varepsilon^2_2 + \sigma^2_\varepsilon) \right)$$

$$+ s_1 \left( -(\pi_2 - E_1 \pi_2) + \frac{1}{2} ((\pi_2 - E_1 \pi_2)^2 - \text{Var}_1 \pi_2) - \phi \text{Cov}_1(x^*_2, \pi_2) \right)$$

Substituting back into a log-quadratic expansion of the loss function, taking expectations, and ignoring policy independent terms gives the expected loss:

$$E_1L_2 = \alpha E_1 \pi^2_2 + s_1 \bar{D} (\gamma - \phi) \text{Cov}_1(x^*_2, \pi_2) + \frac{\gamma}{2} s_1^2 \bar{D}^2 \text{Var}_1 \pi_2$$

$$+ \frac{\gamma}{2} \bar{D}^2 (1 - s_1)^2 \sigma^2_\varepsilon + \bar{D} (1 - s_1) \sigma^2_\varepsilon.$$

(21)

Here, we define $\bar{D} = \beta^{-1} V$. We divide the expected loss into four terms. The first term “Inflation Cost” is simply the expected welfare cost of inflation. The second term “Hedging - Nominal Risk Premium” is new and is the focus of our analysis. This term captures the welfare benefits and costs of the state contingency of local currency debt. There are two opposing forces: the welfare benefit of domestic consumption smoothing from a positive inflation-output covariance is counteracted by the risk premium that can be earned by selling insurance to risk-averse investors. If $\gamma > \phi$, the model formalizes the intuition from the introduction, where a government inflates in bad times in order smooth consumption, and the benefits of doing so outweigh the risk-premium that needs to be paid.
for this insurance. In contrast, if $\phi > \gamma$, the benefit to the government from selling insurance to foreign investors outweighs the desire to smooth domestic consumption. In this case, the loss function decreases with the inflation-output covariance, because a government that inflates during good times and deflates during bad times earns a risk premium from risk-averse investors, thereby raising average domestic consumption. To preview our results, one of the most important considerations in solving this problem is to understand when the government can credibly promise a less negative or even positive inflation-output covariance. As long as the investor has non-zero risk aversion ($\phi > 0$), the government wants to limit the tendency to inflate during bad states of the world ex-ante, but may deviate ex-post.

The final two terms capture losses from consumption volatility induced by the volatility in debt repayments. The volatility of debt repayments enters into expected domestic consumption utility, because domestic consumers have a non-diversified, non-zero debt position, and consumption utility is concave. The third term, “Volatility Nom. Debt” captures the utility losses from consumption volatility caused by the fact that inflation volatility induces movements in the real amount repaid on local currency debt. If the country has no local currency debt ($s_1 = 0$) this effect disappears. The final term “Volatility+Convexity Real Debt” captures losses from borrowing in foreign currency induced by fluctuations in the exchange rate and disappears if the country has no FC debt ($s_1 = 1$). Exchange rate volatility lowers expected consumption through a convexity effect and induces variation in domestic real consumption, which is costly due to utility curvature. In the same way that inflation volatility induces fluctuations in consumption by inducing volatility in local currency debt repayments, so do exchange rate fluctuations through their effect on real debt repayments on foreign currency debt. In addition, foreign currency debt is costly because the expected inverse exchange rate is greater than one over the expected exchange rate.
3.5 Analytic Solution

This section solves the model analytically. Throughout the analytic solution, we keep only first-, and second-order terms of $\bar{D}$ in the loss function. This approximation is justified if the debt-to-GDP ratio is small and clarifies the intuition of the results. While we use a second-order expansion to solve for the government’s period 1 debt portfolio choice and commitment inflation policy, we rely on a first-order expansion to obtain the myopic period 2 government’s optimal inflation policy. This choice simplifies the analytic solution and provides sharper intuition. We restrict ourselves to solutions where local inflation is a function of local output. While there may be interesting implications from considering differential inflation loadings onto global and local shocks, complex monetary policy rules may be hard to verify and enforce. Our solution captures the important drivers in a world, where monetary policy can only commit to a simple rule that depends only on one variable, namely local output. For solution details see Appendix B.

3.5.1 Inflation Policy Functions

The government follows different inflation policies, depending on whether the commitment state or the no-commitment state is realized in period 2. If the no-commitment state is realized, which occurs with probability $1 - p$, the government myopically re-optimizes the period 2 loss function, taking as given any quantities that were determined in period 1. The no-commitment government in particular ignores the effect of its policy on bond prices, LC bond risk premia, and sovereign debt portfolio choice. The no-commitment inflation rule depends on the aggregate state $x_2$, but not on the exchange rate shock, which is realized after inflation:
\[ \pi_{2}^{nc} = \frac{s_1 \bar{D}}{2\alpha} - \gamma \frac{s_1 \bar{D}}{2\alpha} x_2. \]  \hspace{1cm} (22)

The first term in (22) captures the standard inflation bias of a myopic government. The bias increases with the amount of LC debt \( s_1 \bar{D} \) and decreases in the real marginal cost of inflation \( \alpha \). The second term captures inflation cyclicality, showing that the incentive to inflate is greatest during recessions, when output is low and the marginal utility of consumption is high for domestic consumers. The degree of countercyclicality depends on \( \gamma \frac{s_1 \bar{D}}{2\alpha} \). This term is intuitive, because \( \gamma \) is the curvature of the domestic agents’ consumption utility and determines how much the marginal utility of consumption increases in low-consumption states. The amount of local currency debt \( s_1 \bar{D} \) and the cost of generating inflation \( \alpha \) enter similarly as for the inflation level.

The inflation rule in the commitment state takes into account effects on risk premia:

\[ \pi_{2}^{c} = (\phi - \gamma) \frac{s_1 \bar{D}}{2\alpha} x_2. \]  \hspace{1cm} (23)

The commitment inflation rule (23) exhibits no inflationary bias on average. Inflation-cyclicality, as captured by the inflation-output slope coefficient also changes and has a new non-negative term \( \phi \). The slope coefficient in (23) is positive and the government wants to commit to procyclical inflation if and only if investors have higher effective risk aversion than the government, because government debt has hedging value to investors and sells at a premium.
3.5.2 Inflation Moments and LC Debt Share

Analogously to our empirical analysis, we define the inflation-output beta as the slope from regressing period 2 log inflation $\pi_2$ onto period 2 log output $x_2$. The mean, variance, and inflation-output beta for period 2 inflation then equal:

\[
E_1(\pi_2) = (1 - p)\frac{s_1\bar{D}}{2\alpha}, \quad (24)
\]

\[
Var_1(\pi_2) = \left(\frac{s_1\bar{D}}{2\alpha}\right)^2 \left(p(1 - p) + (\gamma^2 - p\phi(2\gamma - \phi))\sigma_x^2\right), \quad (25)
\]

\[
Beta(\pi_2, x_2) = \frac{(p\phi - \gamma) s_1\bar{D}}{2\alpha}, \quad (26)
\]

We can gain intuition by considering two special cases with zero credibility ($p = 0$) and full credibility ($p = 1$). With $p = 0$, the government has no ability to commit and the inflation-output beta reduces to $Beta(\pi_2, x_2) = -\frac{\gamma s_1\bar{D}}{2\alpha}$. A government without commitment is always tempted to inflate during recessions, leading to countercyclical inflation and a negative inflation-output beta.

With full credibility ($p = 1$), the inflation-output beta becomes $Beta(\pi_2, x_2) = \frac{(\phi - \gamma) s_1\bar{D}}{2\alpha}$, which is greater than the inflation-output beta with zero commitment as long as effective investor risk aversion $\phi$ is positive. In particular, when $\phi = \gamma$ the full credibility government’s inflation-output beta is zero and inflation is constant. More generally, provided that $\phi > 0$, (26) increases with credibility $p$. While it is well understood that a lack of credibility can lead to an inflationary bias, our contribution is to show that a lack of credibility also affects sovereign debt portfolio choice through an inflation cyclicity channel.

Substituting (24) through (26) into the expected loss function (21) and taking the first-
order condition with respect to the LC debt share $s_1$ gives:

$$s_1 = \frac{2\alpha \left[\gamma + 1/\bar{D}\right] \sigma^2_\varepsilon}{(1 - p) \left(1 + \phi^2 \sigma^2_x\right) - (\phi - \gamma)^2 \sigma^2_x + 2\alpha \gamma \sigma^2_\varepsilon}.$$  \hspace{1cm} (27)

### 3.5.3 Comparative Statics

From (27), we derive the comparative static for the local currency debt share with respect to credibility:

$$\frac{ds_1}{dp} = s_1^2 \frac{1 + \phi^2 \sigma^2_x}{2\alpha \left[\gamma + 1/\bar{D}\right] \sigma^2_\varepsilon} > 0.$$ \hspace{1cm} (28)

Provided that the LC debt share $s_1$ is at an interior solution, it increases with credibility. As credibility increases, the government faces smaller risk premia for issuing local currency debt. Moreover, the probability of inefficiently high inflation for a government with local currency debt declines. Both of these factors reinforce each other to increase the local currency debt share for high credibility governments.

Next, we explore the model implications for the relation between inflation cyclicality and LC debt shares. Combining (26) and (28) gives the total derivative:

$$\frac{dBeta(\pi_2, x_2)}{ds_1} = \frac{\partial Beta(\pi_2, x_2)}{\partial s_1} + \frac{\partial Beta(\pi_2, x_2)}{\partial p} \frac{ds_1}{dp}$$

$$= \frac{(p\phi - \gamma) D}{2\alpha} + \frac{\phi \bar{D} \left[\gamma + 1/\bar{D}\right] \sigma^2_\varepsilon}{s_1 \left(1 + \phi^2 \sigma^2_x\right)}$$ \hspace{1cm} (29)

Our main stylized empirical fact, that finds that LC debt shares are positively related to inflation-output betas, predicts $\frac{dBeta(\pi_2, x_2)}{ds_1} > 0$. The model inflation-output beta varies with the LC debt share $s_1$ through two channels. First, the direct effect of a higher LC debt share
is to increase both the consumption-smoothing benefits of countercyclical inflation and the amount of real consumption that can be gained from making LC debt safe for investors. Through this channel, the effect of increasing the LC debt share $s_1$ is proportional to the inflation-output beta (26). The first term in (29) is negative if the government is more risk-averse with respect to domestic output than than investors ($\gamma > \phi$), or if credibility $p$ is low. In this case, in order to generate a positive relation between inflation-output betas and LC debt shares as in the data, the second term would be to be sufficiently positive to outweigh the direct effect.

Second, the equilibrium relation between inflation-output betas and LC debt shares reflects the effect of credibility on both variables. Expression (26) shows that the inflation-output beta increases with credibility (strictly, if $\phi > 0$), because with higher credibility we need to put a higher weight on the stable inflation policy. Since the LC debt share also increases with credibility, variation in credibility induces a non-negative relation between LC debt shares and inflation-output betas. This second channel is larger if effective investor risk aversion $\phi$ is high. The reason is that a high credibility government has a stronger incentive to limit inflation state-contingency, when risk premia are large.

The case $\phi = 0$ illustrates that limited commitment alone cannot plausibly generate the upward-sloping relation between inflation-output betas and LC debt shares in the data. Risk-averse investors, therefore, are essential to matching the downward-sloping empirical relation between inflation-output betas and LC debt shares. In the absence of risk premia, a high credibility government optimally follows a countercyclical inflation policy that generates inflation only in bad states of the world to smooth consumption, generating a negative inflation-output beta (26) in contrast with the empirical evidence that countries with high LC bond shares have zero or even positive inflation-output betas and zero or negative bond-stock
betas. Moreover, (29) is negative, so the model predicts a downward-sloping relation between LC debt shares and inflation-output betas. Intuitively, because domestic consumption is far from perfectly hedged, as appears plausible empirically, the marginal benefit from further consumption hedging to the government is high. Consequently, a higher LC debt share increases the benefit of each additional percentage point of inflation, increasing the incentive to vary inflation countercyclically over the business cycle.\textsuperscript{11}

4 Calibrating the Model

In this section, we calibrate the model to examine whether the forces discussed in Section 3 can quantitatively replicate the empirical patterns and to assess the numerical accuracy of the analytic solution. The analytic solution helps us select parameter values without an expensive grid search. We use global solution methods to solve for the full non-linear solution (i.e. not the analytic solution).\textsuperscript{12} Table 3 reports calibration parameters and Table 4 compares model and empirical moments.

We solve the model for two calibrations that differ only in terms of credibility \( p \). The high credibility calibration uses \( p_H = 1 \), corresponding to full credibility, while the low credibility calibration has \( p_L < 1 \). We choose the low credibility calibration to target the difference

\textsuperscript{11}Only if domestic consumption is close to perfectly hedged, which appears less empirically plausible, can the model generate an upward-sloping relation between LC debt shares and inflation-output betas, because then the marginal benefit of inflation variation decreases with the amount of LC debt outstanding. However, with close to perfect domestic consumption hedging, the model predicts negative inflation-output betas and positive bond-stock betas for LC debt issuers, in contrast to the data. Formally, we capture limited consumption-smoothing in the analytic solution by taking an expansion with \( \bar{D}^3 \) small. For a solution that keeps third-order terms in \( D \), see Appendix B.

\textsuperscript{12}We minimize the Euler equation error for the inflation policy function in the no-commitment state over the no-commitment policy function. We then minimize the loss function over the commitment policy function and the local currency debt share. Both commitment- and no-commitment policy functions for log inflation are quadratic in log output. For details and a sensitivity analysis of model moments to individual parameters see Appendix B.
in empirical moments between emerging markets and developed markets, reported in the leftmost column of Table 4.

We set the government’s borrowing need to 13% of GDP, corresponding to the average share of external sovereign debt in emerging markets. We set exchange rate volatility to $\sigma_\varepsilon = 14\%$ to match the median annual volatility of emerging market exchange rate returns since 1990. A substantial cost of borrowing in foreign currency implies that the share of local currency debt falls relatively slowly with respect to $p$ in equilibrium, ensuring that even low credibility countries have some local currency debt.

With (22) and (24), we have that $E_1\pi_{2,L} = (1 - p_L)E_1\pi_{2,L}^{nc}$. Identifying $E_1\pi_{2,L}$ with average emerging market survey inflation in excess of developed market survey inflation and $E_1\pi_{2,L}$ with maximum emerging market survey inflation in excess of average developed market survey inflation pins down $p_L = 1 - \frac{2.00\%}{6.07\%} = 0.67$. We calibrate the inflation cost to match average emerging market survey inflation in excess of developed market survey inflation of 2.0%. With (24) we obtain:

$$\alpha = \frac{(1 - p_L)s_{1,L}D}{2E_1\pi_{2,L}} = \frac{0.33 \times 0.5 \times 0.13}{2 \times 0.02} = 0.5.$$

We explore model implications for a wide range of values for $\phi$. We set $\phi = \gamma$ for our benchmark calibration. The benchmark case of equal government and effective investor risk aversion has appealing implications. It implies that a full credibility issuer chooses an all LC debt portfolio and perfect inflation targeting, with no inflation variability, similarly to developed countries in our sample.\textsuperscript{13} We choose government and effective investor risk aversion ($\gamma$ and $\phi$) to match the empirical difference in inflation-output betas of $-0.21$. We

\textsuperscript{13}In our sample, the mean beta of local equity returns on US equity returns is .97 and the mean beta of local GDP growth on US GDP growth .86. Therefore, assuming equal risk aversion ($\gamma = \theta$) between the government and investors, the benchmark of $\gamma \approx \phi$ is natural.
substitute into (26):

$$Beta_L (\pi_2, x_2) - Beta_H (\pi_2, x_2) = -\frac{\gamma \bar{D}_{s1,L}}{2\alpha} (1 - p_L) = -\gamma \times \left[ \frac{0.13 \times 0.5}{2 \times 0.5} \times 0.33 \right] .$$

indicating that we need risk aversion on the order of $\gamma = 10$ to match the empirical difference in inflation-output betas across emerging and developed markets. While a risk aversion parameter of 10 is high, it is at the upper end of values considered plausible by Mehra and Prescott (1985).

Finally, high output volatility $\sigma_y = 8\%$ is needed to generate a plausible level for the equity premium. While this volatility is higher than emerging market output volatility in our sample, a higher volatility may be priced into asset markets if emerging markets are subject to crashes and crises. We do not attempt to explain the equity volatility puzzle (Shiller, 1981; LeRoy and Porter, 1981), which can be resolved if consumption and dividend growth contain a time-varying long-run component (e.g., Bansal and Yaron (2004)) or if preferences induce persistent fluctuations in risk premia (e.g., Campbell and Cochrane (1999)).

Table 4 shows that the calibration matches the empirical moments quite well. We obtain average low commitment inflation of around 3% and maximum no-commitment inflation of 8%. The inflation-output beta for the low credibility calibration is -0.27 compared to a high credibility beta of 0, matching the difference in betas in the data. The small difference between the global and analytic solutions reassures us that our approximations capture the main forces at play.
4.1 Policy Functions

Figure 3 contrasts government policy functions for inflation and real debt repayments as functions of log output. The top two panels show log inflation (left) and the conditional expected real debt portfolio excess excess return (right), averaged across commitment and no-commitment states. Blue solid lines correspond to low credibility and red dashed lines correspond to high credibility. All policy functions in Figure 3 use numerical solution methods.

The left panels of Figure 3 illustrate the inflation policy function features discussed in Section 3.5. The top left panel shows that the low credibility government implements a state-contingent inflation policy function, that is higher on average than for the high credibility government, and especially so during low output states. The middle and lower panels of Figure 3 decompose the differences between high and low credibility governments across commitment- and no-commitment states. In the commitment state, the low credibility government sets inflation to zero similarly to the high credibility government. In the no-commitment state, the low credibility government inflates away its local currency debt and chooses especially high inflation in low output states. The low credibility government reaches the no-commitment state with positive probability $1 - p_L > 0$, while the high credibility government reaches it with probability 0, so the average inflation profile for the low credibility government is higher and more countercyclical.

The right panels of Figure 3 show real debt portfolio excess returns, which are related to inflation by taking the expectation of (17) with respect to $\varepsilon_2$. The top right panel shows that countercyclical inflation translates into procyclical real debt repayments for the low credibility country. This is intuitive, because surprise inflation lowers real debt repayments on LC debt in low output states.
Even in the commitment state, credibility affects real excess returns of the sovereign bond portfolio, even though inflation in this state is close to zero. Credibility enters because ex-ante LC bond prices reflect non-zero inflation expectations and inflation risk premia, which can raise the cost of repaying LC debt ex-post. The low credibility government’s real debt repayments are highest in the commitment state, because this is a state of surprisingly low inflation relative to ex-ante investor expectations. With high average inflation expectations, the low credibility government has to issue a large face value of LC debt to raise a given amount of real resources, so in a state of low realized inflation real debt repayments are high. In the no-commitment state, real debt portfolio excess returns are close to zero on average, reflecting higher average inflation, and lowest in recessions, when inflation is high.

The analytic solution provides intuition regarding the average level of debt portfolio excess returns in the top right panel of Figure 3. Taking expectations of (20) over \( \varepsilon_2 \) and \( \pi_2(x_2) \) and substituting \( \phi = \gamma \) into (26), the analytic average debt portfolio excess return equals:

\[
E_1 \left[ x_t r^d_t + \frac{1}{2} \text{Var}_1 x_t r^d_t \right] = (1 - s_1) \sigma^2_\varepsilon + (1 - p) \frac{s_1^2 D}{2\alpha} \gamma^2 \sigma^2_x. \tag{32}
\]

The average debt portfolio excess return equals the FC debt share times the expected excess return required on FC debt plus the LC debt share times the LC bond risk premium. Investors understand that LC bonds issued by a high credibility government provide better hedging and require lower returns in excess of the real risk-free rate. This drives home a key insight of the model, namely that low credibility countries have an incentive to inflate away their local currency debt during states of the world that investors also value most, which leads those governments to pay more in expectation on their debt portfolios. Importantly,
the average inflationary bias does not enter in (32) and does not lead to higher debt repayments and lower consumption on average. The reason is that bond prices adjust one-for-one with expected inflation and only the comovement between inflation and investor marginal utility commands a risk premium.

We use the approximate analytic expression (32) for a simple back-of-the-envelope calculation. In the top right panel of Figure 3, the average gap in debt portfolio excess returns between low and high credibility governments is 1.75 percentage points. With LC bond risk premia of approximately

$$RP_L = -\phi \times Beta(\pi_2, x_2)_L \times \sigma^2_x = 1.73$$

percentage points, about

$$s_L \times RP_L = 0.54 \times 1.73 = 0.93$$

percentage points of the gap in average excess returns is due to local currency bond risk premia, with the remainder due to real exchange rate volatility and the expected excess return on foreign currency debt.

4.2 Comparative Statics

In this section, we analyze how local currency debt issuance, inflation, inflation-output betas, and local currency risk premia vary with credibility and investor risk aversion.

4.2.1 Credibility

Figure 4 shows that changes in credibility, or the probability of honoring the previously announced inflation plan, can explain substantial differences along key dimensions. An increase in credibility makes it less likely that the government will be tempted to inflate away the debt, leading to lower inflation expectations. A low credibility government is especially tempted to inflate away the debt during recessions, generating an upward-sloping relation between inflation-output betas and credibility. Risk-averse international investors require a return premium for holding local currency bonds that lose value precisely when
marginal utility is high, driving up local currency risk premia for low credibility governments. Finally, low credibility governments issue a smaller share of local currency debt, to constrain themselves from inflating in low output states, thereby reducing the real costs of inflation and risk premia.

4.2.2 Investor Risk Aversion

Figure 5 shows that model predictions vary substantially with investor risk aversion. In the case with risk neutral investors ($\phi = 0$), investors charge no risk premium for inflation-output covariances. In this case, the low credibility government has a high LC debt share, generates high inflation, and a strongly negative inflation-output beta. In fact, low and high credibility governments generate almost identical inflation-output betas, indicating clearly that this case cannot explain the cross-country variation in inflation cyclicality in the data.

While the benchmark calibration in Tables 3 and 4 replicates the empirical fact that inflation-output betas are greater in developed than emerging markets and generates zero inflation-output betas for high-credibility issuers, the model can easily generate even positive inflation-output betas if investors are effectively more risk-averse than the government ($\phi = 12$). With highly risk-averse investors, it is the government that sells insurance to the global investor by issuing LC debt, similarly to the setting considered in Farhi and Maggiore (2016), rather than the risk-neutral investor insuring the government by buying it. Higher investor risk aversion than government risk aversion could be due to political economy reasons, that induce the government to not fully adjust for risk. For instance, the risk of losing elections may lead to a divergence between private and government incentives especially during low output states, much as in Aguiar and Amador (2011), where a lower discount factor driven by political economy forces can engender a bias toward more debt.
5 Testing Additional Empirical Implications

The model presented in the previous two sections highlights the importance of monetary policy credibility for the level and cyclicality of local currency risk and sovereign debt portfolios. This section tests additional model predictions and provides direct evidence for our proposed mechanism. We provide evidence for the following three predictions: First, we predict that countries with positive bond-stock betas have higher LC bond risk premia. Second, we predict that low credibility countries have higher LC bond risk premia. Third, we predict an inverse relation between LC debt shares and LC bond risk premia.

5.1 Empirical Drivers of Risk Premia

In the model, bond risk premia act as an important channel linking monetary policy credibility, bond return cyclicality, and sovereign debt portfolios. We measure ex-ante risk premia for our cross-section of countries to correspond to the left-hand-side of (19):

\[ \bar{RP} = y_{LC} - \bar{\pi} + \frac{1}{2} Var(\pi) - \left( y_{US} - \bar{\pi}_{US} + \frac{1}{2} Var(\pi)_{US} \right). \]  

(33)

A bar indicates the mean from 2005-2014. Intuitively, (33) removes average local inflation from local currency bond yields to isolate the risk premium component. Unlike in the model, we correct for the fact that US inflation is non-zero. In Appendix A, we show that results are quantitatively and qualitatively robust to adjusting LC bond yields for default risk using synthetic default-free local currency bonds as in Du and Schreger (2016).14

In the model, bond risk premia are driven by their comovement with international fun-

14Due to our short sample, ex-post bond risk premia, measured as realized excess returns, are extremely noisy. We therefore prefer ex-ante measures, corresponding to those that governments see when making issuance decisions.
While comovement with international fundamentals is unlikely to explain all cross-sectional variation in LC bond risk premia, showing a qualitatively and quantitatively significant relation between bond risk premia and bond return comovements with US stock returns will provide important evidence for our proposed channel. We decompose each country’s risk-premium into two components by estimating the following regression:

$$\overline{RP}_i = \alpha + \kappa b(bond, S&P)_i + \varepsilon_i.$$  \hspace{1cm} (34)

Here, US stock returns proxy for world stock returns if the US equity market is well integrated with the rest of the world.

Column (2) of Table 5 estimates regression (34) and finds a statistically significant and quantitatively meaningful estimate for $\kappa$. A one unit increase in the bond-S&P beta is associated with an increase in the risk premium of ten percentage points in annualized units, which is the same order of magnitude as the US equity premium. The bond-S&P beta not only carries an economically and statistically significant price of risk, it also explains a substantial portion of cross-sectional variation in LC bond risk premia, with an R-squared of more than 40%. The estimated slope coefficient is similar in column (1), where we use the beta with respect to the local stock market instead of the S&P, supporting the notion that LC bonds that are the best hedges for the issuer tend to require the highest risk premia. In Appendix A.6, we show that the key risk premium relation in Table 5 column (2) remains highly statistically significant when using generalized method of moments to account for the fact that bond-S&P betas are estimated.

Next, we interact the bond-stock beta with the local-S&P beta, as a proxy for the co-movement between local and global SDFs. Column (3) shows that results are unchanged, indicating that the co-movement between local and global SDFs is sufficiently large and
consistent across countries that local inflation cyclicality indeed drives the cross-section of LC bond risk premia, as in the model. Column (4) further addresses concerns that cross-sectional differences in the local-global loadings might directly drive differences in LC bond risk premia. We regress risk premia onto the local-global beta $b(\text{stock}, S&P)$ directly, which does not enter significantly and has no explanatory power for LC bond risk premia.

Table 5 also provides evidence on the link between bond risk premia and monetary policy credibility using two de-facto measures that we construct. We prefer de-facto measures of central bank credibility to de-jure ones because recent measures of de-jure central bank independence have been found to be uncorrelated with average inflation (Crowe and Meade, 2007). Using Financial Times articles over the period 1995-2015, we construct the correlation between the key words “debt” and “inflation” for each country as a proxy for inverse inflation credibility. The intuition is that if inflation is solely determined by the central bank and debt is determined by the fiscal authority, these topics should be discussed separately, and the correlation should be low. On the other hand, if inflation and debt are determined by the same central government, we would expect newspaper articles to discuss both jointly, and the correlation should be high. We count the number of articles containing both keywords and the country name and divide them by the geometric average of the articles that contain one of the keywords combined with the country name. Consistent with the model, column (4) of Table 5 shows that this de-facto monetary policy credibility measure is strongly correlated with risk premia, with an $R^2$ of 42.4%.

Column (5) uses the gap between announced inflation targets and survey expectations to measure inverse inflation credibility. If credibility is low, we expect survey inflation to exceed announced inflation targets. We define the “Credibility Gap” as the greater of the average difference between the central bank inflation target and survey inflation expectations and
zero. Over the past decade, on average, the emerging markets in the sample have a mean credibility gap of 0.6 percent, whereas the developed markets in the sample have a mean credibility gap of 0.1 percent. Column (5) suggests that a 0.5 percentage point increase in the credibility gap, corresponding to the average difference between emerging and developed countries, is associated with a two percentage point increase in LC bond risk premia, which is economically large and in line with model predictions.

5.2 Evidence on Bond Risk Premia and Debt Portfolio Choice

Next, we turn to the model prediction that LC debt shares are negatively related to LC bond risk premia, and in particular to the component of LC bond risk premia that derives from bond return comovements with the international investor’s SDF. Consistent with this prediction, Table 6 shows a negative and statistically significant relation between LC debt shares on the left-hand-side and LC bond risk premia on the right-hand-side. LC bond risk premia explain a substantial 45% of variation in LC debt shares. A 2.4 percentage point increase in LC bond risk premia, roughly the average difference between emerging and developed countries, is associated with a (2.4 \times 8.6 =) 21 percentage point decrease in the LC debt share. Next, we decompose the risk premium into a world CAPM component – the component explained by the bond-S&P beta – and the alpha with respect to the US S&P:

\[ RP_{CAPM,i} = \hat{\kappa} b(bond, S&P)_i, \ a_{CAPM,i} = \overline{RP}_i - RP_{CAPM,i}. \]  

(35)

where \( \hat{\kappa} \) is the slope coefficient estimated in Table 5, column (2). The estimated alpha \( a_{CAPM} \) may reflect measurement error of the CAPM risk premium, for instance if the S&P is an imperfect proxy for the world portfolio, or pricing errors on the part of investors, so we
would expect LC debt shares to decreases with both $R_{CAPM}$ and $a_{CAPM}$. Table 6 column (2) supports the notion that sovereign issuers reduce LC issuance in response to higher LC bond risk premia, and that the riskiness of LC bonds for US investors, as proxied by the bond-S&P beta, accounts for a substantial portion the downward-sloping relation between LC debt issuance and LC risk premia. Columns (2) and (3) show that while both components of the risk premium contribute significantly to the explanatory power of risk premia for local currency debt shares, our proxy for the CAPM component enters with a larger coefficient and explains more than half the R-squared in column (1). Columns (4) through (6) show that the relation between risk premia and LC debt shares is robust to controlling for size, foreign exchange rate regime, and inflation.

5.3 Changes in LC Debt Issuance

One of the most striking developments in international bond markets over the past two decades is how many countries have gone from having very little LC debt during the 1990s to substantial LC debt shares in the 2000s. We now show evidence that the bond risks channel of monetary policy credibility can help us understand changes in LC debt issuance. Our analysis is constrained by the fact that our main proxy for the hedging properties of LC bonds – the bond-stock beta – can only be constructed if LC debt is actually available, which for many countries in our sample was not the case during the 1990s. We therefore rely on decade-by-decade estimates of inflation forecast betas to measure the risks of hypothetical LC bond risks over time. Figure 6 shows that the strongest increases in inflation-output forecast betas were accompanied by the most marked increases in LC debt shares, supporting the notion that the bond risks channel of monetary policy credibility not only explains level differences in LC debt shares across countries, but also changes since the 1990s. This evidence also
shows that our main stylized empirical fact holds in changes, thereby controlling for omitted variables that are constant at the country level, such as natural resource endowments.\textsuperscript{15}

6 Conclusion

This paper argues that differences in monetary policy credibility, combined with investors that require a risk premium for holding positive-beta bonds, explain the relation between sovereign debt portfolios and government bond risks across countries. We document that sovereigns whose local currency bonds tend to depreciate during recessions and hence provide the borrower with consumption-smoothing benefits, issue little local currency debt. We explain this stylized fact with a model where risk-averse investors charge a premium for holding local currency bonds that lose value during recessions, thereby making local currency debt expensive for low credibility governments and driving them towards foreign currency debt issuance. Importantly, both limited commitment on the issuer’s part and investor risk aversion are necessary to match the empirical evidence. The key contribution of the paper is to demonstrate how the interaction of lender risk aversion and monetary credibility can explain why countries with positive bond-stock betas, that would seemingly achieve most consumption-smoothing from issuing local currency debt, have the lowest local currency debt share. Our simple framework gives rise to a number of testable predictions on inflation, inflation-cyclicality, sovereign debt portfolios, and proxies of effective monetary policy credibility, which we verify in the data.

\textsuperscript{15}We compute inflation-output forecast betas and LC debt shares separately for the decades 1995-2004 and 2004-2015 for 20 countries. The rest of 10 sample countries are excluded due to missing forecasting data for the 1995-2004 period.
References


Aguiar, Mark, Manuel Amador, Emmanuel Farhi, and Gita Gopinath, 2014, Coordination and Crisis in Monetary Unions, NBER Working Paper wp20277.


Borri, Nicola, and Adrien Verdelhan, 2011, Sovereign risk premia, Working Paper, LUISS Guido Carli University and MIT.


Della Corte, Pasquale, Steven Riddiough, and Lucio Sarno, 2016, Currency premia and global...
Favilukis, Jack, Lorenzo Garlappi, and Sajjad Neamati, 2015, The Carry Trade and Uncovered Interest Parity When Markets are Incomplete, *Available at SSRN*.


Figure 1: Local Currency Debt Shares and Bond Betas

Note: This figure shows the share of local currency debt as a fraction of central government debt (in %) over the period 2005-2014. Bond-stock betas are estimated as the slope coefficient of quarterly local currency bond log excess returns onto local stock market log excess returns over the same time period

\[ x_{i,t+\Delta t}^{LC} = b_0 + b_{(bond, stock)} \times x_{i,t+\Delta t}^{m} + \epsilon_t. \]

Three-letter codes indicate currencies. Emerging markets are shown in red and developed markets in green. The highest and lowest observation are winsorized.
Figure 2: Local Currency Debt Shares, Inflation Betas, and Bond-S&P Betas

Note: Panels (A), (B), and (D) plot the share of local currency debt in the sovereign debt portfolio on the y-axis against expected inflation-output betas, realized inflation-output betas, and the beta of local currency bond returns with S&P returns, respectively. Panel (C) shows bond betas against local stock returns on the y-axis against bond-US S&P betas on the x-axis. Developed markets are denoted by green dots and emerging markets are denoted by red dots. The three-letter currency code is used to label countries. The highest and lowest observation are winsorized. More details on variable definitions can be found in section 2.
Figure 3: Policy Functions

Note: Blue solid indicates the low credibility calibration, while red dashed indicates the high credibility calibration. Left panels show log inflation. Right panels show real debt portfolio excess returns in percent, following equation (17). The y-axis shows log output in percent deviations from the steady-state. “Average” refers to the weighted average across commitment and no-commitment states, where the weights are given by credibility $p$. 
Figure 4: Varying Credibility

Note: This figure shows average inflation, the inflation-output beta, local currency bond risk premia, and the local currency debt share while varying credibility $p$. All other parameters are held constant at values shown in Table 3.
Figure 5: Varying Effective Investor Risk Aversion

Note: This figure shows average log inflation, the inflation-output beta, local currency bond risk premia, and the local currency debt share against effective investor risk aversion $\phi$ for low credibility (blue solid) and high credibility (red dashed) calibrations. All other parameters are held constant at values shown in Table 3.
Figure 6: Changes 1995-2004 vs. 2005-2014

Note: This figure shows decade-over-decade changes in the inflation forecast beta on the x-axis and changes in LC debt shares on the y-axis, where changes are from 1995-2004 to 2005-2014. The highest and lowest observation are winsorized.
Table 1: Summary Statistics for Developed and Emerging Markets (2005-2014)

<table>
<thead>
<tr>
<th></th>
<th>(1) $\pi$</th>
<th>(2) Survey $\pi$</th>
<th>(3) $b(\tilde{\pi}, \tilde{\text{gdp}})$</th>
<th>(4) $y^{LC}$</th>
<th>(5) $b(\pi, IP)$</th>
<th>(6) $b(\text{bond},\text{stock})$</th>
<th>(7) $b(\text{stock}, \text{S&amp;P})$</th>
<th>(8) $s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Developed Markets ($N = 11$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.70</td>
<td>1.83</td>
<td>0.42</td>
<td>2.62</td>
<td>0.05</td>
<td>-0.10</td>
<td>0.95</td>
<td>89.27</td>
</tr>
<tr>
<td>S.d.</td>
<td>0.81</td>
<td>0.64</td>
<td>0.15</td>
<td>1.24</td>
<td>0.06</td>
<td>0.04</td>
<td>0.21</td>
<td>11.23</td>
</tr>
<tr>
<td>Max</td>
<td>2.68</td>
<td>2.68</td>
<td>0.71</td>
<td>4.87</td>
<td>0.15</td>
<td>-0.03</td>
<td>1.34</td>
<td>100.00</td>
</tr>
<tr>
<td>Min</td>
<td>0.26</td>
<td>0.32</td>
<td>0.24</td>
<td>0.61</td>
<td>-0.04</td>
<td>-0.18</td>
<td>0.60</td>
<td>65.85</td>
</tr>
<tr>
<td>(B) Emerging Markets ($N = 19$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>4.09</td>
<td>3.83</td>
<td>0.20</td>
<td>6.01</td>
<td>-0.02</td>
<td>0.06</td>
<td>1.00</td>
<td>63.11</td>
</tr>
<tr>
<td>S.d.</td>
<td>2.05</td>
<td>1.66</td>
<td>0.32</td>
<td>2.91</td>
<td>0.15</td>
<td>0.12</td>
<td>0.27</td>
<td>25.58</td>
</tr>
<tr>
<td>Max</td>
<td>9.07</td>
<td>7.90</td>
<td>1.07</td>
<td>12.33</td>
<td>0.35</td>
<td>0.32</td>
<td>1.55</td>
<td>100.00</td>
</tr>
<tr>
<td>Min</td>
<td>2.05</td>
<td>2.06</td>
<td>-0.25</td>
<td>1.67</td>
<td>-0.50</td>
<td>-0.07</td>
<td>0.58</td>
<td>11.97</td>
</tr>
<tr>
<td>(C) Full Sample ($N = 30$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>3.21</td>
<td>3.10</td>
<td>0.28</td>
<td>4.77</td>
<td>0.01</td>
<td>0.01</td>
<td>0.98</td>
<td>72.70</td>
</tr>
<tr>
<td>S.d.</td>
<td>2.05</td>
<td>1.68</td>
<td>0.28</td>
<td>2.92</td>
<td>0.13</td>
<td>0.13</td>
<td>0.24</td>
<td>24.78</td>
</tr>
<tr>
<td>Max</td>
<td>9.07</td>
<td>7.90</td>
<td>1.07</td>
<td>12.33</td>
<td>0.35</td>
<td>0.32</td>
<td>1.55</td>
<td>100.00</td>
</tr>
<tr>
<td>Min</td>
<td>0.26</td>
<td>0.32</td>
<td>-0.25</td>
<td>0.61</td>
<td>-0.50</td>
<td>-0.18</td>
<td>0.58</td>
<td>11.97</td>
</tr>
<tr>
<td>(D) Mean Difference between Emerging and Developed Markets</td>
<td>-2.391***</td>
<td>-2.004***</td>
<td>0.215**</td>
<td>-3.388***</td>
<td>0.0736*</td>
<td>-0.160***</td>
<td>-0.04</td>
<td>26.16***</td>
</tr>
</tbody>
</table>

Note: This table reports summary statistics for the cross-sectional mean of seven variables for developed and emerging market groups. The variables include (1) $\pi$, realized inflation (%), (2) Survey $\pi$, survey inflation (%), (3) $y^{LC}$, five-year local currency LC bond yield, (4) $b(\tilde{\pi}, \tilde{\text{gdp}})$, inflation-output forecast beta, (5) $b(\pi, IP)$, realized inflation-output beta, (5), (6) $b(\text{bond},\text{stock})$, bond-stock beta, (7) $b(\text{stock}, \text{S&P})$ beta of local log equity excess returns with respect to log US S&P 500 excess returns, and (8) $s$, percentage share of local currency debt in total sovereign debt portfolios. Panel (A) reports results for developed markets. Panel (B) reports results for emerging markets. Panel (C) reports results for the pooled sample. Panel (D) tests the mean difference between developed and emerging markets. Robust standard errors are reported in parentheses. Significance levels are denoted by *** $p<0.01$, ** $p<0.05$, * $p<0.1$. 

**Note**: The table includes various financial statistics for developed and emerging markets across different years. It shows the mean, standard deviation, maximum, and minimum values for several economic indicators. The indicators include realized inflation, survey inflation, five-year local currency bond yield, inflation-output forecast beta, realized inflation-output beta, bond-stock beta, and the beta of local equity returns relative to the US S&P 500. The table also provides the mean difference between developed and emerging markets along with their respective robust standard errors.
Table 2: Cross-Sectional Regression of Local Currency Debt Shares on Nominal Risk Betas

<table>
<thead>
<tr>
<th>Local Currency Debt Share</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b(bond, stock) )</td>
<td>-116.3***</td>
<td>-106.1**</td>
<td>-106.3**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(21.40)</td>
<td>(34.19)</td>
<td>(31.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b(\bar{\pi}, \tilde{gdp}) )</td>
<td></td>
<td>57.63***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8.986)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b(\pi, IP) )</td>
<td></td>
<td>126.6***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(31.84)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(GDP)</td>
<td></td>
<td>1.579</td>
<td>1.092</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.280)</td>
<td>(3.815)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FX Regime</td>
<td></td>
<td>-1.412</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.998)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commodity Share</td>
<td></td>
<td>-0.172</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.179)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>74.14***</td>
<td>57.24***</td>
<td>72.00***</td>
<td>59.08</td>
<td>71.80*</td>
</tr>
<tr>
<td></td>
<td>(3.460)</td>
<td>(4.857)</td>
<td>(3.993)</td>
<td>(42.05)</td>
<td>(39.48)</td>
</tr>
<tr>
<td>Observations</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.367</td>
<td>0.392</td>
<td>0.156</td>
<td>0.370</td>
<td>0.401</td>
</tr>
</tbody>
</table>

Note: This table shows the cross-country regression results of the local currency debt share, \( s \) (between 0 and 1), on measures of inflation cyclicalilty. The independent variables in the first three columns are the bond-stock beta \( (b(bond, stock)) \), the inflation forecast beta \( (b(\bar{\pi}, \tilde{gdp})) \) and the realized inflation- output beta \( (b(\pi, IP)) \), respectively. In Column (4), we control for the mean log per capita GDP level between 2005 and 2014, log(GDP). In Column (5), we control for the average exchange rate classification used in Reinhart and Rogoff (2004), FX regime. More details on variable definitions can be found in section 2. The top and bottom observation are winsorized. Robust standard errors are used in all regressions with the significance level indicated by *** \( p<0.01 \), ** \( p<0.05 \), * \( p<0.1 \).
Table 3: Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Low Credibility</th>
<th>High Credibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credibility</td>
<td>$p$</td>
<td>0.67</td>
</tr>
<tr>
<td>Inflation Cost</td>
<td>$\alpha$</td>
<td>0.50</td>
</tr>
<tr>
<td>Output Vol.</td>
<td>$\sigma_x$</td>
<td>0.08</td>
</tr>
<tr>
<td>Government Risk Aversion</td>
<td>$\gamma$</td>
<td>10</td>
</tr>
<tr>
<td>Effective Investor Risk Aversion</td>
<td>$\phi$</td>
<td>10</td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>$\bar{D}$</td>
<td>0.13</td>
</tr>
<tr>
<td>Exchange Rate Vol.</td>
<td>$\sigma_\varepsilon$</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Note: All parameters are in annualized natural units.

Table 4: Empirical and Model Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Emerging-Developed</td>
<td>Low Credibility</td>
</tr>
<tr>
<td>Average Inflation</td>
<td>2.00</td>
<td>2.99</td>
</tr>
<tr>
<td>No-Commitment Inflation</td>
<td>6.07</td>
<td>8.48</td>
</tr>
<tr>
<td>Inflation Beta</td>
<td>-0.21</td>
<td>-0.27</td>
</tr>
<tr>
<td>LC Debt Share</td>
<td>0.63</td>
<td>0.54</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.35</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Note: All moments are in annualized natural units. The empirical moment for average inflation is the difference between average survey inflation for emerging and developed markets in Table 1. The empirical inflation-output beta is computed as the difference between average expected inflation-output betas in emerging and developed markets. The empirical no-commitment inflation is computed as the difference between maximum emerging market survey inflation and average developed market survey inflation in Table 1. The equity risk premium is the average local equity excess return in our sample. All model moments are computed using global solution methods.
Table 5: Empirical Drivers of Bond Risk Premia

<table>
<thead>
<tr>
<th>LC Bond Risk Premium</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RP</td>
<td>RP</td>
<td>RP</td>
<td>RP</td>
<td>RP</td>
<td>RP</td>
</tr>
<tr>
<td>$b(\text{bond, stock})$</td>
<td>15.30***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.117)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b(\text{bond, S&amp;P})$</td>
<td></td>
<td>11.36***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.194)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b(\text{bond, stock}) \times b(\text{stock, S&amp;P})$</td>
<td></td>
<td></td>
<td>14.60***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.805)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b(\text{stock, S&amp;P})$</td>
<td></td>
<td></td>
<td></td>
<td>1.041</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.553)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>News Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>36.08***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(9.524)</td>
<td></td>
</tr>
<tr>
<td>Credibility Gap</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.637***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.989)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.773***</td>
<td>2.151***</td>
<td>1.661***</td>
<td>0.856</td>
<td>-5.559***</td>
<td>0.508</td>
</tr>
<tr>
<td></td>
<td>(0.264)</td>
<td>(0.449)</td>
<td>(0.248)</td>
<td>(1.383)</td>
<td>(1.735)</td>
<td>(0.392)</td>
</tr>
<tr>
<td>Observations</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>22</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.610</td>
<td>0.302</td>
<td>0.616</td>
<td>0.010</td>
<td>0.472</td>
<td>0.323</td>
</tr>
</tbody>
</table>

Note: This table regresses the empirical risk premium proxy (33) on bond-stock betas and measures of monetary policy credibility. $b(\text{bond, stock})$ is the beta of LC bond excess returns with respect to the local stock market. $b(\text{bond, S&P})$ is the beta of LC bond returns with respect to US S&P returns. $b(\text{bond, stock}) \times b(\text{stock, S&P})$ is the interaction of bond-local stock return betas and the beta of local on US equity returns. $b(\text{stock, S&P})$ is the beta of local on US equity returns. “News Count” is the correlation of the keywords “debt” and “inflation” in Financial Times articles 1996-2015 from ProQuest Historical Newspapers. We compute the correlation as the number of articles mentioning both “debt” and “inflation” divided by the geometric average of articles that mention either “debt” or “inflation”. We require articles to also mention the country name. The inflation credibility gap is measured as the mean difference between the survey inflation expectations from Consensus Economics and the announced inflation target since 2005. The top and bottom observation are winsorized. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.
Table 6: Local Currency Debt Share and Bond Risk Premia

<table>
<thead>
<tr>
<th>Local Currency Debt Share</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Premium</td>
<td>-6.581***</td>
<td>-5.841***</td>
<td>-5.481***</td>
<td>-5.725***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.871)</td>
<td>(0.915)</td>
<td>(0.939)</td>
<td>(1.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RP_{CAPM}$</td>
<td>-8.937***</td>
<td>-8.937***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.600)</td>
<td>(1.886)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{CAPM}$</td>
<td>-5.562***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.985)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (GDP)</td>
<td>2.985</td>
<td>2.777</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.939)</td>
<td>(3.392)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FX Regime</td>
<td>1.975</td>
<td>1.984</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.103)</td>
<td>(3.151)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commodity Share</td>
<td>-0.144</td>
<td>-0.137</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.171)</td>
<td>(0.197)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Inflation</td>
<td></td>
<td></td>
<td>-1.943*</td>
<td>-0.303</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.092)</td>
<td>(1.791)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>85.70***</td>
<td>70.88***</td>
<td>82.85***</td>
<td>53.80*</td>
<td>89.88***</td>
<td>56.25</td>
</tr>
<tr>
<td></td>
<td>(3.796)</td>
<td>(3.739)</td>
<td>(3.574)</td>
<td>(3.131)</td>
<td>(5.271)</td>
<td>(37.55)</td>
</tr>
<tr>
<td>Observations</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.451</td>
<td>0.251</td>
<td>0.476</td>
<td>0.498</td>
<td>0.468</td>
<td>0.498</td>
</tr>
</tbody>
</table>

Note: This table regresses the average local currency debt share onto our empirical risk premium proxy, defined in Equation (33). $RP_{CAPM}$ is the risk premium component explained by the bond-S&P beta and $a_{CAPM}$ is the corresponding alpha, as defined in (35). The FX Regime is from Reinhart and Rogoff (2004). The top and bottom observation are winsorized. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.
Appendix - For Online Publication Only

This online appendix consists of Section A “Empirical Robustness” and Section B “Model Appendix”.

A Empirical Robustness

A.1 Long-Term Debt

The cross-sectional relationship between local currency risk betas and local currency debt shares is robust to measuring the local currency debt share only in long-term debt, as shown in Figure A.1. We obtain face values and issuance dates for all historical individual sovereign bond issuances from Bloomberg for 14 emerging markets and estimate the long-term local currency debt share as the outstanding amount of LC debt with 5 or more years remaining to maturity relative to all outstanding debt with 5 or more years remaining to maturity.

Figure A.1: Local Currency Debt Share in Long-Term Debt vs. Bond-Stock Beta

Note: This figure plots the bond-stock beta on the x-axis and the share of LC debt in all outstanding long-term debt on the y-axis. Long-term debt is defined as having a remaining time to maturity of five or more years. The share of LC debt in long-term debt is estimated from individual bond issuance data from Bloomberg.

A.2 TIC Data

In this section, we demonstrate that our results are robust to examining external debt separately, rather than all central government debt as in the main paper. The primary reason for not doing so is that it would reduce country coverage. We approximate foreign-owned debt by looking only at the debt owned by US domiciled investors. US investors report their security level holdings as part
Figure A.2: Nominal Share in External Debt
of the Treasury International Capital (TIC) data. Here, we calculate the LC debt share in the US portfolio by dividing the total value of government debt owned by US investors in the borrowing country’s currency by the total amount of that country’s sovereign debt owned by US investors. Figure A.2 shows that the negative relation between bond-stock betas and LC debt shares is robust to using this alternative measure of LC debt shares.

A.3 LC vs. FC Bond Betas

So far, we have assumed that real exchange rates are uncorrelated with all other shocks. This is clearly a simplification. However, what matters for the domestic borrower’s choice between LC and FC debt is the relative hedging properties and the relative risk premia of these two types of debt. If real exchange rate cyclicality is similar across countries, inflation cyclicality and LC bond return cyclicality are the relevant margins for understanding cross-sectional differences in LC debt shares. Figure A.3 shows the relation between LC debt shares on the x-axis against LC bond betas in excess of FC bond betas. We see that countries with lower LC debt shares have higher LC bond betas in excess of FC betas, confirming the empirical evidence in Figure 1.

Figure A.3: LC minus FC Bond-Stock Betas

Note: This figure shows the difference between unhedged LC bond betas and FC betas for dollar investors on the x-axis against local currency debt shares on the y-axis.
Note: This figure shows LC bond-stock betas excluding the period 2008-2009 on the x-axis and LC bond-stock betas for the full sample (including 2008-2009) on the y-axis. The highest and lowest observation are winsorized.

### A.4 Robustness to Excluding the Financial Crisis

One important period in the middle of our sample is the financial crisis of 2008-2009. While this period marked an important recession for the US and many other countries, we show in this section that our main empirical results are not driven by the financial crisis.

Figure A.4 shows our baseline LC bond-stock beta on the y-axis against a LC bond-stock beta excluding the financial crisis period on the x-axis. We see that bond-stock betas are extremely similar when excluding the financial crisis, indicating that our key bond cyclicality measure is not driven by a small number of observations. Figure A.5 shows that our main stylized fact in Figure 1 remains unchanged if we exclude the crisis period in our construction of LC bond betas.

### A.5 Default-Adjusted Bond Risk Premia

In order to adjust for default risk, we construct a synthetic default free nominal bond yield. We follow Du and Schreger (2016) by combining a US Treasury with a fixed-for-free cross currency...
Figure A.5: Local Currency Debt Shares and Bond Betas Excluding 2008-2009

Note: This figure differs from Figure 1 only in that it excludes 2008-2009 from the computation of LC bond betas on the x-axis. The highest and lowest observation are winsorized.
Figure A.6: Local Currency Debt Shares and Default-Adjusted Bond Betas

Note: This figure differs from Figure 1 only in that it excludes 2008-2009 from the computation of LC bond betas on the x-axis. The highest and lowest observation are winsorized.
swap to create a synthetic default free local bond. In the absence of financial market frictions and sovereign default risk, we would expect \( y^{LC} = y^{LC^*} \). Figure A.6 plots the LC debt share against default-adjusted bond-stock betas, which are computed by replacing LC bond yields by synthetic default-free LC bond yields in the computation of LC bond returns. The strong similarity with Figure 1 shows that our main empirical finding is robust to adjusting for the default component of LC bond returns.

We now show that results in section 5 are robust to using default-adjusted bond risk premia. We define an alternative measure of the risk premium that removes sovereign default risk from the nominal bond yield.

\[
\bar{RP}_{alt} = y^{LC^*} - \bar{\pi} + \frac{1}{2} Var\pi - \left( y^{US} - \bar{\pi}^{US} + \frac{1}{2} Var\pi^{US} \right)
\]

\[
= \left( y^{US} + ccs \right) - \bar{\pi} + \frac{1}{2} Var\pi - \left( y^{US} - \bar{\pi}^{US} + \frac{1}{2} Var\pi^{US} \right),
\]

where \( ccs \) denotes the fixed-for-free cross currency swap rate. Tables A.1 and A.2 replicate the empirical results in Section 5 using default-adjusted risk premia and find the results qualitatively unchanged.
<table>
<thead>
<tr>
<th>Table A.1: Empirical Drivers of Default-Adjusted Bond Risk Premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC Bond Risk Premium</td>
</tr>
<tr>
<td>b(bond, stock)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>b(bond, S&amp;P)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>b(bond, stock) x b(stock, S&amp;P)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>b(stock, S&amp;P)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>News Correlation</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Credibility Gap</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
</tbody>
</table>

Note: This table shows that Table 5 is robust to using the default-adjusted LC bond risk premium. \( b(bond, stock) \) is the beta of LC bond excess returns with respect to the local stock market. \( b(bond, S&P) \) is the beta of LC bond returns with respect to US S&P returns. \( b(bond, stock) \times b(stock, S&P) \) is the interaction of bond-local stock return betas and the beta of local on US equity returns. \( b(stock, S&P) \) is the beta of local on US equity returns. “News Count” is the correlation of the keywords “debt” and “inflation” in Financial Times articles 1996-2015 from ProQuest Historical Newspapers. We compute the correlation as the number of articles mentioning both “debt” and “inflation” divided by the geometric average of articles that mention either “debt” or “inflation”. We require articles to also mention the country name. The inflation credibility gap is measured as the mean difference between the survey inflation expectations from Consensus Economics and the announced inflation target since 2005. Robust standard errors are used in all regressions with the significance level indicated by *** \( p<0.01 \), ** \( p<0.05 \), * \( p<0.1 \).
Table A.2: Local Currency Debt Share and Default-Adjusted Bond Risk Premia

<table>
<thead>
<tr>
<th>Local Currency Debt Share</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-8.417***</td>
<td>-6.825**</td>
<td>-6.076**</td>
<td>-6.732*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.215)</td>
<td>(1.897)</td>
<td>(2.574)</td>
<td>(2.732)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RP_{CAPM}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-13.42***</td>
<td>-10.94***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.339)</td>
<td>(3.309)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{CAPM}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-5.654***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.248)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (GDP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.952</td>
<td>5.834</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.510)</td>
<td>(4.027)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FX Regime</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.601</td>
<td>2.645</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.646)</td>
<td>(3.786)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commodity Share</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.001</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.212)</td>
<td>(0.222)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-3.020</td>
<td>-0.166</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.568)</td>
<td>(2.854)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>81.90***</td>
<td>69.18***</td>
<td>82.98***</td>
<td>15.64</td>
<td>88.52***</td>
<td>17.07</td>
</tr>
<tr>
<td></td>
<td>(3.841)</td>
<td>(4.005)</td>
<td>(4.534)</td>
<td>(36.24)</td>
<td>(7.297)</td>
<td>(44.58)</td>
</tr>
<tr>
<td>Observations</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.404</td>
<td>0.257</td>
<td>0.461</td>
<td>0.479</td>
<td>0.426</td>
<td>0.479</td>
</tr>
</tbody>
</table>

Note: This table shows that Table 6 is robust to using a default-adjusted risk premium on the right-hand-side. This table regresses the average local currency debt share onto the alternative empirical risk premium proxy, defined in Equation (36). $RP_{CAPM}$ is the risk premium component explained by the bond-S&P beta and $a_{CAPM}$ is the corresponding alpha, corresponding to the fitted value and residual of Table A.1 column (2). The FX Regime is from Reinhart and Rogoff (2004). Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.
A.6 GMM Estimation

In this subsection, we re-estimate the key risk premium regression in Table 5 column (2) using GMM. This estimation accounts for the fact that betas are not known but are generated. To distinguish them from the first-stage parameters, we denote the second stage intercept and slope coefficient by $a$ and $b$. The first-stage intercept and slope coefficient for country $i$ are denoted by $\alpha_i$ and $\beta_i$.

We define the GMM moments as follows

$$g_{i,t} = \begin{cases} RP_i - a - b\beta_i & \text{for } 1 \leq i \leq N \\ (RP_i - a - b\beta_i) \beta_i & \text{for } N + 1 \leq i \leq 2N \\ x_{t}^{LC,i} - \alpha_i - \beta_i x_{t}^{m,USD} & \text{for } 2N + 1 \leq i \leq 3N \\ (x_{t}^{LC,i} - \alpha_i - \beta_i x_{t}^{m,USD}) x_{t}^{m,USD} & \text{for } 3N + 1 \leq i \leq 4N \end{cases}$$

(37)

The first $2N$ moment conditions are for the cross-sectional regression in the second stage. Moment conditions $2N + 1$ through $4N$ are for the first-stage regressions. We estimate $2N + 2$ moments using $4N$ moment conditions by setting a linear combination of the average moments to zero. We choose the weighting matrix $A = [2N + 2 \times 4N]$ such that GMM point estimates are identical to our two-stage procedure. For this, we set

$$A = \begin{bmatrix} 1_{1 \times N} & 0_{1 \times N} & 0_{1 \times 2N} \\ 0_{1 \times N} & 1_{1 \times N} & 0_{1 \times 2N} \\ 0_{2N \times N} & 0_{2N \times N} & I_{2N} \end{bmatrix}.$$  

(38)

Here $0_{M \times P}$ and $1_{M \times P}$ define block matrices of all zeros and ones with size $[M \times P]$, respectively. We use $I_{2N}$ to denote the identity matrix of size $2N$. The GMM estimator then is defined by setting

$$A \times \frac{1}{T} \sum_{t=1}^{T} g_t = 0.$$  

(39)

The GMM variance-covariance matrix of estimates is obtained as

$$V = T^{-1} (AD)^{-1} ASA'(AD)^{-1},$$

(40)

where $D = [4N,2N+2]$ is the sample average of the derivative of $g$ with respect to the parameter vector $\theta = [a,b,\alpha',\beta']$:

$$D = \begin{bmatrix} -1_{N \times 1} & -\beta & 0_{N \times N} & -bI_N \\ -\beta & -\beta^2 & 0_{N \times N} & -2b \times \text{diag}(\beta) \\ 0_{N \times 1} & 0_{N \times 1} & -I_N & -I_N \sum_{t=1}^{T} x_{t}^{m,USD} T^{-1} \\ 0_{N \times 1} & 0_{N \times 1} & -I_N \sum_{t=1}^{T} x_{t}^{m,USD} T^{-1} & -I_N \sum_{t=1}^{T} (x_{t}^{m,USD})^2 T^{-1} \end{bmatrix}. $$  

(41)
Since moments 1 through $2N$ are already effectively time averages and hence constant across $t$, we estimate the first $2N$ elements of the spectral density matrix from the cross-section. The spectral density for moments $2N + 1$ through $4N$ is estimates from the time series under the assumption of no serial correlation in return residuals:

$$
\hat{S} = \begin{bmatrix}
\mathcal{I}_N \hat{s}_1 & \mathcal{I}_N \hat{s}_{12} & 0_{N \times 2N} \\
\mathcal{I}_N \hat{s}_{12} & \mathcal{I}_N \hat{s}_2 & 0_{N \times 2N} \\
0_{N \times N} & 0_{N \times N} & T^{-1} \sum_{t=1}^T \left( \tilde{g}_t \tilde{g}_t' + \sum_{i=1}^m \left( 1 - \frac{i}{m+1} \right) \left( \tilde{g}_t \tilde{g}_{t-i} + \tilde{g}_{t+i} \tilde{g}_t' \right) \right)
\end{bmatrix}.
$$

(42)

Here

$$
\hat{s}_1 = \frac{T}{N-2} \sum_{t=1}^T \sum_{i=1}^N g_{i,t}^2,
$$

(43)

$$
\hat{s}_2 = \frac{T}{N-2} \sum_{t=1}^T \sum_{i=N+1}^{2N} g_{i,t}^2,
$$

(44)

$$
\hat{s}_{12} = \frac{T}{N-2} \sum_{t=1}^T \sum_{i=1}^N g_{i,t} g_{i+N,t},
$$

(45)

and $\tilde{g}_t$ refers to the vector containing elements $g_{2N+1,t}$ through $g_{4N,t}$. The bottom $2N \times 2N$ block of $\hat{S}$ is estimated with a Newey-West kernel with $m = 120$ lags to adjust for the fact that we use daily data on quarterly overlapping returns. The GMM standard errors of the second-stage coefficients are then given by the first two elements of

$$
SE(\hat{a}) = \sqrt{V(1,1)},
$$

(46)

$$
SE(\hat{b}) = \sqrt{V(2,2)}.
$$

(47)

The point estimates and standard errors for the second-stage parameters are reported in table A.3. Column (1) starts by replicating Table 5, but in contrast to Table 5 we use data that has not been winsorized. Comparing shows that winsorizing has no qualitative or quantitative effect on the results. Column (2) in Table A.3 reports results from the GMM procedure outlined above, taking into account that bond betas must be generated. The point estimates are identical to column (1) and the standard errors are only slightly larger without affecting statistical significance, as one would expect if bond betas are precisely estimated. Taken together, Table 5 suggests that bond betas are sufficiently precisely estimated over our sample period that generated regressors do not affect our benchmark results.
Table A.3: GMM: Bond Risk Premia onto Bond-S&P Betas

<table>
<thead>
<tr>
<th>LC Bond Risk Premium</th>
<th>OLS</th>
<th>GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b(\text{bond, S&amp;P})$</td>
<td>10.835**</td>
<td>10.835***</td>
</tr>
<tr>
<td></td>
<td>(3.948)</td>
<td>(4.067)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.114***</td>
<td>2.114***</td>
</tr>
<tr>
<td></td>
<td>(0.458)</td>
<td>(0.472)</td>
</tr>
</tbody>
</table>

Observations 30     30

Note: This table estimates the slope of bond risk premia (33) on bond-S&P betas. The specification in column (1) is analogous to Table 5 column (2) but uses non-winsorized data. Column (2) is estimated using the GMM procedure outlined above to account for generated regressors. Significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.
B Model Appendix

The Model Appendix is structured as follows:

- Subsection B.1 derives the expression for LC bond prices (12) and discusses the expression for approximate LC bond risk premia (19).
- Subsection B.2 provides details for the analytic model solution.
- Subsection B.3 provides details for the numerical model solution.
- Subsection B.4 analyzes the accuracy of the analytic solution.
- Subsection B.5 analyzes the sensitivity of calibration moments to moving one parameter at a time.
- Subsection B.6 describes and solves analytically an extended model, where the real exchange rate is allowed to depreciate during recessions.
- Subsection B.7 describes, solves analytically, and provides a simple calibration for an extended model, where debt is held domestically instead of by foreign investors.
- Subsection B.8 embeds our two-period model in a simple dynamic model.

B.1 Solving for Bond Prices

We assume that local output $x_2$ and world output $x^*$ are jointly log-normally distributed. We write local output as a component that is perfectly correlated with world output plus an idiosyncratic shock $\eta_2 \perp x^*_2$:

\[ x_2 = \lambda x^*_2 + \eta_2, \quad (48) \]
\[ \eta_2 \sim N(0, \sigma^2_\eta). \quad (49) \]

Here, $\lambda$ is the regression coefficient of local output onto world output:

\[ \lambda = \frac{Cov(x_2, x^*_2)}{Var(x^*_2)}. \quad (50) \]

Conversely, we can also write world output as a component that is perfectly correlated with local output plus a shock that is uncorrelated with local output $\eta^*_2 \perp x_2$:

\[ x^*_2 = \lambda^* x_2 + \eta^*_2. \quad (51) \]
Here, $\lambda^*$ is the regression coefficient of world output onto local output:

$$
\lambda^* = \frac{\text{Cov}(x^*_2, x^*_2)}{\text{Var}(x^*_2)}, \quad (52)
$$

$$
= \frac{\lambda (\sigma^*)^2}{(\sigma_x)^2}. \quad (53)
$$

One way of seeing that this is the correct expression for $\lambda^*$ is by noting that (48) and (53) imply

$$
\text{Cov}(x^*_2 - \lambda^* x^*_2, x^*_2) = \text{Cov}(x^*_2, \lambda x^*_2 + \eta^*_2) + \text{Cov}(-\lambda^* x^*_2, x^*_2), \quad (54)
$$

$$
= \lambda (\sigma^*)^2 - \lambda^* \sigma^2_x, \quad (55)
$$

$$
= 0, \quad (56)
$$

so by definition

$$
\eta^*_2 = x^*_2 - \lambda^* x^*_2 \quad (57)
$$

is uncorrelated with $x^*_2$.

### B.2 Analytic Model Solution

In this section, we solve the model analytically. Throughout the appendix we use the notation

$$
\mathcal{L} = E_1 L_2. \quad (58)
$$

We will also drop time 1 subscripts for the LC debt share and simply write it as $s$ for conciseness.

#### B.2.1 Log-Quadratic Loss Function

Using a log-quadratic expansion of the form

$$
\exp(z) - 1 = z + \frac{1}{2} z^2
$$

the loss function (4) becomes (ignoring constants)

$$
L_2 \approx \alpha \pi^2_2 - \left(c^2_2 + \frac{1}{2} c^2_2 \right) + \frac{\gamma}{2} c^2_2. \quad (59)
$$

Defining $\bar{D} = \beta^{-1} V$, we expand consumption in terms of output and the excess return on the debt

73
We expand bond portfolio excess returns similarly to Campbell and Viceira (2002):

\[
\begin{align*}
    x r^d_2 + \frac{1}{2} (x r_d)^2 & \approx \exp(x r^d_2) - 1,
    \\
    & \approx (1 - s_1) \left( \exp \left( \varepsilon_2 + \frac{1}{2} \sigma_\varepsilon^2 \right) - 1 \right) + s_1 \left( \frac{\beta}{q_{LC}} \exp(-\pi_2) - 1 \right),
    \\
    & = (1 - s_1) \left( \varepsilon_2 + \frac{1}{2} (\varepsilon_2^2 + \sigma_\varepsilon^2) \right) + s_1 \left( - (\pi_2 - E_1 \pi_2) + \frac{1}{2} (\pi_2 - E_1 \pi_2)^2 - Var_1 \pi_2 \right) - \phi Cov_1(x_2, \pi_2)
\end{align*}
\]

Substituting back into the loss function (59), ignoring policy independent terms, and taking expectations over \( x_2, \pi_2, \) and \( \varepsilon_2 \) gives the expected loss (21).

**B.2.2 FC Bonds**

The log pricing kernel for pricing assets in international consumption units is

\[
m^*_2 = \log \beta - \theta x^*_2 - \frac{1}{2} \theta^2 (\sigma^*)^2.
\]

The price of a FC bond hence is

\[
q_{1}^{FC} = \beta E_1 \left[ \exp \left( - \theta x^*_2 - \frac{1}{2} \theta^2 (\sigma^*)^2 \right) \right],
\]

\[
= \beta.
\]

**B.2.3 LC Bonds**

Now, let’s consider a bond, whose payoff is \( f(x_2) \) in domestic consumption units. If \( f(x_2) = 1 \), we are considering a real LC bond. If \( f(x_2) = \exp(-\pi_2(x_2)) \), we are considering a nominal LC bond.

The log pricing kernel for pricing assets in local consumption units is

\[
m_2 = \log \beta - \theta x^*_2 - \frac{1}{2} \theta^2 (\sigma^*)^2 + \varepsilon_2 - \frac{1}{2} \sigma_\varepsilon^2.
\]
The price of the bond is

\[ q_{1}^{LC,f} = \beta E_1 \left[ \exp \left( -\theta x_2^* - \frac{1}{2} \theta^2 (\sigma^*)^2 + \varepsilon_2 - \frac{1}{2} \sigma_2^2 \right) f(x_2) \right], \quad (70) \]

\[ q_{1}^{LC,f} = \beta E_1 \left[ \exp \left( -\theta \lambda^* x_2 - \theta^2 (\sigma^*)^2 - \frac{1}{2} \sigma_2^2 \varepsilon_2 \right) f(x_2) \right]. \quad (71) \]

Since \( \eta_2^* \) is independent of \( x_2 \), and \( \varepsilon_2 \) is uncorrelated with all other shocks, the expression for \( q_{1}^{f} \) simplifies to

\[ q_{1}^{LC,f} = \beta E_1 \left[ \exp \left( -\theta \lambda^* x_2 \right) f(x_2) \right]. \quad (72) \]

But with \( \eta_2^* \perp x_2 \) we have that:

\[ (\sigma^*)^2 = (\lambda^* \sigma_x)^2 + (\sigma_\eta^*)^2. \quad (73) \]

The bond price (72) hence simplifies further to

\[ q_{1}^{LC,f} = \beta E_1 \left[ \exp \left( -\theta \lambda^* x_2 - \frac{1}{2} \theta^2 (\lambda^*)^2 \sigma_x^2 \right) f(x_2) \right]. \quad (74) \]

Defining effective risk aversion as

\[ \phi = \theta \lambda^*, \quad (75) \]

\[ = \theta \lambda (\sigma^*)^2, \quad (76) \]

the price of a nominal LC bond then can be written as

\[ q_{1}^{LC} = \beta E_1 \left[ \exp \left( -\phi x_2 - \frac{1}{2} \phi^2 \sigma_x^2 \right) \right] \exp(-\pi_2) \]. \quad (77) \]

The price of a real LC bond then equals

\[ q_{1}^{LC,real} = \beta E_1 \left[ \exp \left( -\phi x_2 - \frac{1}{2} \phi^2 \sigma_x^2 \right) \right], \quad (78) \]

\[ = \beta. \quad (79) \]
B.2.4 Approximate Risk Premia

Approximate risk premia are

\[ y_1^{LC} - E_1 \pi_2 + \frac{1}{2} \text{Var}_1 \pi_2 - y_1^{\text{FC}} \approx -\theta \text{Cov}\left(x_2^*, \pi_2 - \varepsilon_2\right), \]

\[ = -\theta \text{Cov}\left(x_2^*, \pi_2\right), \]

\[ = \theta \lambda^* \text{Cov}\left(x_2, \pi_2\right), \]

\[ = \theta \lambda^* \sigma_x^2 \text{Beta}\left(\pi_2, x_2\right), \]

\[ = \theta \left(\sigma^*\right)^2 \lambda \text{Beta}\left(\pi_2, x_2\right) \] (84)

The parameters \( \theta \) and \( \sigma^* \) describe the international investor’s fundamental preferences and are the same for every country. However, \( \lambda \) (the local-global beta) and \( \text{Beta}\left(x_2, \pi_2\right) \) can differ across countries. In a cross-section of countries, LC bond risk premia should therefore be proportional to the product \( \lambda \times \text{Beta}\left(x_2, \pi_2\right) \).

Of course, the expressions for risk premia (82), (83), and (84) are equivalent and all are correct. (84) is more useful for comparing risk premia across countries, because \( \theta \) and \( \sigma^* \) are the same for every country. However, (77) expresses the bond price as the expectation over a function of local output and directly implies (82). For solving the issuing country’s portfolio choice problem, it is useful to express bond prices and risk premia in terms of the local output-inflation covariance and the local output variance, because all other costs and benefits of the issuer scale with \( \sigma_x^2 \). Using expression (83) effectively allows us to factor \( \sigma_x^2 \) out of the optimal portfolio choice problem and makes the solution independent of \( \sigma_x \) (apart from how it enters into \( \phi = \theta \times \lambda^* \)).

B.2.5 Full Commitment

We first consider the special case with \( p = 1 \). We assume without loss of generality that the government follows an inflation rule that is a linear function of second period output \( x_2 \):

\[ \pi_2 = \bar{\pi} + \delta x_2. \] (85)

We can then substitute this policy function into the quadratic loss function (21) and simplify:

\[ L^{p=1} = \alpha \left( \bar{\pi}^2 + \delta^2 \sigma_x^2 \right) + s \bar{D} (\gamma - \phi) \delta \sigma_x^2 + \frac{\gamma}{2} \bar{D}^2 s^2 \delta^2 \sigma_x^2 + \frac{\gamma}{2} \bar{D}^2 \sigma_x^2 \left(1 - s\right)^2 + \bar{D} \sigma_x^2 (1 - s). \] (86)

Expression (86) shows that it is optimal to set expected inflation to zero \( \bar{\pi} = 0 \). The first-order-condition for \( \delta \) becomes

\[ \delta = \frac{(\phi - \gamma) s \bar{D}}{2 \alpha + \gamma s^2 \bar{D}^2} \] (87)
The mean, variance, and output beta for period 2 inflation hence become

\begin{align*}
E_1 \pi_2 &= 0, \quad (88) \\
Var_1 \pi_2 &= \frac{(\phi - \gamma)^2 s^2 \bar{D}^2 \sigma_x^2}{(2\alpha + \gamma s^2 \bar{D}^2)^2}, \quad (89) \\
Beta(\pi_2, y_2) &= \frac{(\phi - \gamma) s \bar{D}}{2\alpha + \gamma s^2 \bar{D}^2}. \quad (90)
\end{align*}

Now, we express \( \frac{\bar{D}}{2\alpha + \gamma s^2 \bar{D}^2} \) as a Taylor expansion in \( \bar{D} \) and keep only terms that are second-order or lower in \( \bar{D} \). Intuitively, this corresponds to looking at the limit where \( \bar{D} \) goes to zero sufficiently slowly that \( \bar{D} \) and \( \bar{D}^2 \) have a substantial effects on equilibrium quantities, but \( \bar{D}^3 \) does not. In our calibration \( \bar{D}=0.13 \), so \( \bar{D}^2 =0.02 \) and \( \bar{D}^3 = 0.0022 \), and we are plausibly in a situation where \( \bar{D}^3 \) is small relative to average inflation and bond risk premia. The Taylor expansion

\[ \frac{\bar{D}}{1+x} = 1 - x + x^2 + O(x^3) \]

gives

\begin{align*}
\bar{D} &= \frac{\bar{D}}{2\alpha + \gamma s^2 \bar{D}^2} (1 - \gamma s^2 \bar{D}^2) + O(\bar{D}^3), \quad (91) \\
&= \frac{\bar{D}}{2\alpha} + O(\bar{D}^3). \quad (92)
\end{align*}

Dropping terms that are third-order in \( \bar{D} \) gives expressions (24) through (26) for the special case \( p = 1 \).

**B.2.6 No Commitment**

Next, we consider the special case with \( p = 0 \). We derive the analytic solution for inflation policy in the no-commitment state using the following first-order expansion for log debt portfolio excess returns:

\[ x\pi_2^d \approx (1-s) \varepsilon_2 - s(\pi_2 - E_1 \pi_2). \quad (93) \]

While we can solve for the no-commitment policy while retaining the second-order relation between debt portfolio returns and log inflation, a first-order solution gives more intuitive analytic solutions. No-commitment inflation is independent of real interest rate shocks, which are not known at the time of choosing inflation, and of any terms that are constant ex-post, such as risk premia. Substituting (93) into (21), the period 2 government hence chooses \( \pi_2^{nc} \) to minimize

\[ L_2^{nc} = \alpha \pi_2^2 - s \bar{D}(\pi_2 - E_1 \pi_2) + \frac{\gamma}{2} (x_2 + s \bar{D}(\pi_2 - E_1 \pi_2))^2. \quad (94) \]

The first-order condition for inflation becomes

\[ \pi_2 = \frac{s \bar{D}(1-\gamma x_2)}{2\alpha + \gamma s^2 \bar{D}^2} + \frac{\gamma s^2 \bar{D}^2}{2\alpha + \gamma s^2 \bar{D}^2} E_1 \pi_2. \quad (95) \]
We can now use (95) to solve for the expectation, variance, and output beta of period 2 inflation

\[ E_1 \pi_2 = \frac{s \bar{D}}{2\alpha}, \]  
\[ Var_1 \pi_2 = \frac{\gamma^2 \sigma_x^2}{(2\alpha + \gamma s^2 \bar{D}^2)^2} \bar{D}^2 s^2, \]  
\[ Beta(\pi_2, x_2) = \frac{-\gamma}{2\alpha + \gamma s^2 \bar{D}^2} s \bar{D}. \] 

(96) (97) (98)

Substituting (96) into (95) and dropping third-order terms in \( \bar{D} \) gives the inflation policy function (22). Dropping third-order \( \bar{D} \) terms in (96) through (98) gives expressions (24) through (26) for the special case \( p = 0 \).

While in the main paper we drop higher-order terms in \( \bar{D} \), effectively focusing on the empirically relevant case where \( \bar{D} \) is small, expression (98) has interesting information about higher-order terms in \( \bar{D} \). In particular it clarifies why the potential countervailing force alluded to in section 3.5.3 is dominated. (98) shows that \( Beta(\pi_2, x_2) \) initially decreases in the amount of LC debt \( s \bar{D} \), but for large values of \( s \bar{D} \) it increases in \( s \bar{D} \). The case of small \( s \bar{D} \) is similar to the limiting case considered in section 3, where an increase in the LC debt share leads to more countercyclical inflation.

The intuition for the large \( s \bar{D} \) case is that when the amount of LC debt is large, domestic consumption is close to perfectly hedged. An increase in LC debt therefore reduces the amount of inflation variation that is required to obtain perfect consumption hedging and makes inflation-output betas less negative. Since consumption in emerging markets appears far from perfectly hedged and external Debt/GDP is small, we focus on the case with small \( s \bar{D} \) throughout the paper.

**B.2.7 Partial Commitment**

Now, we consider the case for general \( 0 \leq p \leq 1 \). As before, inflation in the commitment state optimally follows a rule

\[ \pi^c_2 = \bar{\pi} + \delta y_2. \]  

(99)

We next derive the optimal expressions for \( \bar{\pi} \) and \( \delta \). In the no-commitment state, the government chooses inflation myopically with first-order condition

\[ \pi^{nc}_2 = \frac{s \bar{D} (1 - \gamma x_2)}{2\alpha + \gamma s^2 \bar{D}^2} + \frac{\gamma s^2 \bar{D}^2}{2\alpha + \gamma s^2 \bar{D}^2} (p \bar{\pi} + (1 - p) E_1 \pi^{nc}_2). \]  

(100)

Expected no-commitment inflation then equals

\[ E_1 \pi^{nc}_2 = \frac{s \bar{D}}{2\alpha + p \gamma s^2 \bar{D}^2} + \frac{\gamma s^2 \bar{D}^2}{2\alpha + p \gamma s^2 \bar{D}^2} p \bar{\pi}. \]  

(101)
Unconditional expected inflation equals

\[ E_1\pi_2 = p\bar{\pi} + (1-p)E_1\pi_2^{nc}, \quad (102) \]

\[ = (1-p)\frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} + \frac{2\alpha + \gamma s^2\bar{D}^2}{2\alpha + p\gamma s^2\bar{D}^2}p\bar{\pi}. \quad (103) \]

The variance and covariance of period 2 inflation then become

\[ Var_1\pi_2 = p(1-p)\left(\frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} - \frac{2\alpha}{2\alpha + p\gamma s^2\bar{D}^2}\right)^2 + p\delta^2\sigma_x^2 + (1-p)\frac{\gamma^2\sigma_x^2}{(2\alpha + \gamma s^2\bar{D}^2)s^2\bar{D}^2}, \quad (104) \]

\[ Cov_1(x_2, \pi_2) = p\delta\sigma_x^2 - (1-p)\frac{\gamma\sigma_x^2}{2\alpha + \gamma s^2\bar{D}^2}s\bar{D}. \quad (105) \]

The inflation sensitivity in the commitment state \( \delta \) enters into the variance and covariance exactly as before, but scaled by the commitment probability \( p \). It hence follows that the optimal commitment sensitivity \( \delta \) takes the form

\[ \delta = -\frac{(\gamma - \phi)\bar{D}s}{2\alpha + \gamma D^2s^2}. \quad (106) \]

Average commitment inflation \( \bar{\pi} \) is chosen to minimize

\[ \alpha \left( (1-p)\frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} + \frac{2\alpha + \gamma s^2\bar{D}^2}{2\alpha + p\gamma s^2\bar{D}^2}p\bar{\pi} \right)^2 + \left( \alpha + \frac{\gamma}{2}\bar{D}^2s^2 \right)p(1-p)\left( \frac{s\bar{D}}{2\alpha + p\gamma s^2\bar{D}^2} - \frac{2\alpha}{2\alpha + p\gamma s^2\bar{D}^2} \right)^2. \quad (107) \]

Dividing by common factors, optimal \( \bar{\pi} \) minimizes

\[ \alpha \left( (1-p)s\bar{D} + (2\alpha + \gamma s^2\bar{D}^2)p\bar{\pi} \right)^2 + \left( \alpha + \frac{\gamma}{2}\bar{D}^2s^2 \right)p(1-p)\left( s\bar{D} - 2\alpha\bar{\pi} \right)^2. \quad (108) \]

Taking the first-order condition of (108) with respect to \( \bar{\pi} \) then shows that inflation in the commitment state equals

\[ \bar{\pi} = 0. \quad (109) \]

With the two inflation policy functions, we can then solve for closed-form expressions for the expectation, variance, and output beta for period 2 inflation, taking the local currency debt share
s as given:

\[ E_1 (\pi_2) = (1 - p) \frac{s \dot{D}}{2 \alpha + p \gamma s^2 \bar{D}^2}; \]

\[ Var_1 (\pi_2) = p(1 - p) \left( \frac{s \dot{D}}{2 \alpha + p \gamma s^2 \bar{D}^2} \right)^2 + \frac{\gamma^2 - p \phi (2 \gamma - \phi)}{(2 \alpha + \gamma s^2 \bar{D}^2)^2} \bar{D}^2 s^2 \sigma_x^2; \]

\[ Beta (\pi_2, y_2) = \frac{(p \phi - \gamma) \bar{D} s}{2 \alpha + \gamma s^2 \bar{D}^2}; \]

Substituting in the inflation policy functions, the expected loss function simplifies to

\[
\mathcal{L} = \frac{1 - p}{2} \frac{s^2 \bar{D}^2}{2 \alpha + p \gamma s^2 \bar{D}^2} + \frac{1}{2} \frac{-(\gamma - \phi)^2 + (1 - p) \phi^2}{2 \alpha + \gamma s^2 \bar{D}^2} \bar{D}^2 s^2 \sigma_x^2 + \frac{\gamma \bar{D}^2 \sigma_x^2}{2} (1 - s)^2 \\
+ \bar{D} \sigma_x^2 (1 - s). \]

(113)

Dropping third-order terms in \( \bar{D} \) gives

\[
\mathcal{L} = (1 - p) \frac{s^2 \bar{D}^2}{4 \alpha} - \frac{(\phi - \gamma)^2}{4 \alpha} s^2 \bar{D}^2 \sigma_x^2 + \frac{1 - p}{4 \alpha} \frac{\phi^2}{s^2 \bar{D}^2} \sigma_x^2 \\
+ \frac{\gamma \bar{D}^2 \sigma_x^2}{2} (1 - s)^2 + \bar{D} \sigma_x^2 (1 - s). \]

(114)

and expressions (24) through (26). Taking the first-order condition of (114) with respect to \( s \) gives

\[ s_1 = \frac{2 \alpha \left[ \gamma + 1/\bar{D} \right] \sigma_x^2}{(1 - p) (1 + \phi^2 \sigma_x^2) - (\phi - \gamma)^2 \sigma_x^2 + 2 \alpha \gamma \sigma_x^2}; \]

(115)

From (115), we derive the comparative static for the local currency debt share with respect to credibility:

\[ \frac{ds_1}{dp} = s_1 \frac{1 + \phi^2 \sigma_x^2}{2 \alpha \left[ \gamma + 1/\bar{D} \right] \sigma_x^2}, \]

(116)

\[ > 0. \]

**B.3 Numerical Solution**

We solve the model numerically using global projection methods. To reduce the dimensionality of the optimization problem, we use the following numerical steps.

1. Starting from the analytic solution, we choose the no-commitment policy function to minimize the expected Euler equation error while holding constant the local currency debt share and the commitment policy function.
2. We choose the commitment policy function to minimize the expected loss function, while holding constant the local currency debt share and the no-commitment policy function.

3. We alternate steps 1 and 2 until the maximum absolute change in both policy functions is less than $10^{-12}$. This gives the loss function at a given local currency debt share.

4. We optimize over the local currency debt share to minimize the expected loss function, where for every single value of $s$ we repeat steps 1. through 3. to evaluate the loss function.

**B.3.1 Functional Form**

We solve for commitment- and no-commitment inflation policies of the form

$$
\pi_{2}^{nc} = b_1(s) + b_2(s)x_2 + b_3(s)x_2^2, \\
\pi_{2}^{c} = c_1(s) + c_2(s)x_2 + c_3(s)x_2^2,
$$

(117)

(118)

where all coefficients may depend on the local currency debt share $s$. We start our optimization routine at the analytic solution, that is

$$
b_1 = \frac{s\bar{D}}{2\alpha}, \\
b_2 = \frac{-s\bar{D}\gamma}{2\alpha}, \\
b_3 = 0, \\
c_1 = c_2 = c_3 = 0.
$$

(119)

(120)

(121)

(122)

The starting value for the local currency debt share is as given in Table 3 in the main paper.

For given no-commitment and commitment policy functions and a given local currency debt share, we compute local currency bond prices numerically as

$$
\frac{q_{1}^{LC}}{\beta} = (1 - p)E_1 \left[ \exp \left(-\phi x_2 - \frac{1}{2} \phi^2 \sigma^2 \right) \exp(-\pi_{2}^{nc}) \right] \\
+ pE_1 \left[ \exp \left(-\phi x_2 - \frac{1}{2} \phi^2 \sigma^2 \right) \exp(-\pi_{2}^{c}) \right].
$$

(123)

We evaluate the expectation in (123) using Gauss-Legendre quadrature with 30 node points, truncating the interval at -6 and +6 standard deviations.

**B.3.2 No Commitment Policy Function**

We choose the vector $(b_1, b_2, b_3)$ to minimize the expected squared Euler equation error

$$
\text{Error} = 2\alpha \pi_{2}^{nc} - E_{\varepsilon_2} \left( C_{2}^{nc,-\gamma} \right) \bar{D}s \frac{\beta}{q_{1}^{LC}} \exp(-\pi_{2}^{nc}),
$$

(124)
where we evaluate no-commitment consumption numerically according to
\[
C_{2}^{nc} = \bar{X} \exp(x_2/\bar{X}) - \bar{D} \left( (1-s) \exp \left( \frac{\varepsilon_2 + \frac{1}{2} \sigma^2}{\bar{X}} \right) + s \frac{\beta}{q_1} \exp(-\pi_{2}^{nc}) \right),
\] (125)
We evaluate the expectation $E[\text{Error}^2]$ again using Gauss-Legendre quadrature with 30 nodes and truncating the interval at -6 and +6 standard deviations.

**B.3.3 Commitment Policy Function**

For a given LC debt share and a given no-commitment policy function, the commitment inflation policy function minimizes
\[
E_1 [\alpha \pi_{2}^2] - pE_1 \left[ \frac{C_{2}^{c,1-\gamma}}{1-\gamma} \right] - (1-p)E_1 \left[ \frac{C_{2}^{nc,1-\gamma}}{1-\gamma} \right],
\] (126)
where we evaluate commitment consumption numerically
\[
C_{2}^{c} = \bar{X} \exp(x_2/\bar{X}) - \bar{D} \left( (1-s) \exp \left( \frac{\varepsilon_2 + \frac{1}{2} \sigma^2}{\bar{X}} \right) + s \frac{\beta}{q_1} \exp(-\pi_{2}^c) \right),
\] (127)
and no-commitment consumption and the ratio of bond prices are given by (125) and (123). All expectations are again evaluated numerically using Gauss-Legendre quadrature using the same grid points as before.

**B.3.4 Numerical Model Moments**

We use Gauss-Legendre quadrature to evaluate inflation moments for the numerical solution according to
\[
E_1 \pi_{2} = pE_1 \pi_{2}^c + (1-p)E_1 \pi_{2}^{nc},
\] (128)
\[
\text{Var}_1 \pi_{2} = pE_1 \left( (\pi_{2}^c)^2 \right) + (1-p)E_1 \left( (\pi_{2}^{nc})^2 \right) - (E_{1} \pi_{2}^2)^2,
\] (129)
\[
\text{Beta}(\pi_{2},x_2) = (pE_1 ((\pi_{2}^c - E_1 \pi_{2}) x_2) + (1-p)E_1 ((\pi_{2}^{nc} - E_1 \pi_{2}) x_2)) / \sigma^2.
\] (130)

We obtain the local currency bond risk premium as
\[
RP^{LC} = \log(q_1^{FC}) - \log(q_1^{LC}) - E_1 \pi_{2} + \frac{1}{2} \text{Var}_1 \pi_{2}.
\] (131)

**B.4 Accuracy of Analytic Solution**

Table A.2 shows model moments for the analytic model solution. The analytic solution is remarkably accurate, with all model moments broadly similar to Table 4.
Figure A.1 compares analytic (blue solid) and numerical (red dashed) policy functions while holding the local currency debt share constant at its numerical solution value of 0.54. Given the simplicity of the analytic solution, it is very accurate. The main difference is that the numerical policy function is more convex, bounding no-commitment inflation away from zero in high-output states.

Finally, Figures B.8 and B.9 reproduce the comparative statics with respect to credibility \( p \) and risk aversion \( \phi \) using the analytic solution. The strong similarity with Figures 4 and 5 indicates that the analytic model solution is indeed valuable for understanding the drivers of local currency bond issuance, both qualitatively and even quantitatively.

Table B.4: Analytic Solution for Model Moments

<table>
<thead>
<tr>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Emerging-Developed</td>
</tr>
<tr>
<td>Average Inflation</td>
<td>2.00</td>
</tr>
<tr>
<td>Average No-Commitment Inflation</td>
<td>6.07</td>
</tr>
<tr>
<td>Inflation Beta</td>
<td>-0.21</td>
</tr>
<tr>
<td>LC Debt Share</td>
<td>0.63</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.35</td>
</tr>
</tbody>
</table>

Note: All moments are in natural annual units. The difference between this table and Table 4 in the main paper is that this table uses the analytic models solution instead of global solution methods.
Figure B.7: Numerical Policy Functions

Note: The top panel shows log inflation (in percent) against log output in percent deviations from the steady-state in the commitment state. The bottom panel shows log inflation in the no-commitment state. The solid blue lines show the analytic solutions, while the red dashed lines show numerical solutions using projection methods.
Figure B.8: Analytic Solution: Varying Credibility

Note: This figure differs from Figure 4 only in that it shows the analytic solution instead of the numerical model solution.
Note: This figure varies from Figure 5 only in that it uses the analytic solution instead of the numerical model solution. This figure shows average log inflation, the inflation-output beta, local currency bond risk premia, and the local currency debt share against effective investor risk aversion $\phi$ for low credibility (blue solid) and high credibility (red dashed) calibrations. All other parameters are held constant at values shown in Table 3.
B.5 Calibration Sensitivity

Table B.5 shows how the model properties change as we change one calibration parameter at a time. Column (1) reproduces the benchmark calibration for reference. Columns (2) and (3) show the effect of setting effective investor risk aversion to 0 and 12 respectively. For $\phi = 0$, both high and low credibility governments hit the constraint of a 100% local currency debt share and therefore the inflation-output betas are almost equal. For $\phi = 12$, the model generates reasonable local currency debt shares, a positive inflation-output beta for the high credibility government, and a negative inflation-output beta for the low credibility government, similarly to the data. Increasing the inflation cost $\alpha$ compresses inflation-output betas towards zero, but does not change the model properties otherwise. Choosing a lower output volatility leaves inflation-output betas and local currency debt shares largely unchanged, but reduces the equity risk premium. Column (6) shows that reducing both investor and government risk aversion to 5 compresses the variation in inflation-output betas and risk premia across low and high credibility governments, but preserves the signs. Column (7) shows that, on the other hand, increasing the debt-to-GDP ratio increases the gap between high credibility and low credibility inflation-output betas. Finally, column (8) shows that reducing the exchange rate volatility to $\sigma_\varepsilon = 0.11$ reduces the local currency debt share for the low credibility government, but still generates an economically meaningful negative inflation-output beta.
### Table B.5: Calibration Sensitivity

<table>
<thead>
<tr>
<th>Low Credibility</th>
<th>(1) Benchmark</th>
<th>(2) $\phi = 0$</th>
<th>(3) $\phi = 12$</th>
<th>(4) $\alpha = 1$</th>
<th>(5) $\sigma_x = 0.06$</th>
<th>(6) $\gamma = \phi = 5$</th>
<th>(7) $\bar{D} = 0.18$</th>
<th>(8) $\sigma_e = 0.11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Inflation</td>
<td>2.99</td>
<td>4.60</td>
<td>2.76</td>
<td>2.21</td>
<td>2.90</td>
<td>2.58</td>
<td>4.61</td>
<td>1.91</td>
</tr>
<tr>
<td>No-Commitment Inflation</td>
<td>8.48</td>
<td>12.84</td>
<td>7.89</td>
<td>6.25</td>
<td>8.25</td>
<td>7.63</td>
<td>12.48</td>
<td>5.62</td>
</tr>
<tr>
<td>Inflation Beta</td>
<td>-0.27</td>
<td>-1.29</td>
<td>-0.13</td>
<td>-0.20</td>
<td>-0.29</td>
<td>-0.13</td>
<td>-0.53</td>
<td>-0.18</td>
</tr>
<tr>
<td>LC Debt Share</td>
<td>0.54</td>
<td>1.00</td>
<td>0.51</td>
<td>0.79</td>
<td>0.66</td>
<td>0.61</td>
<td>0.40</td>
<td>0.34</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.26</td>
<td>0.00</td>
<td>7.51</td>
<td>6.26</td>
<td>3.19</td>
<td>3.13</td>
<td>6.26</td>
<td>6.26</td>
</tr>
</tbody>
</table>

| High Credibility              |               |                |                 |                   |                        |                          |                     |
|-------------------------------|---------------|----------------|-----------------|-------------------|------------------------|--------------------------|----------------------|----------------------|
| Average Inflation             | 0.00          | 0.00           | 0.00            | 0.00              | 0.00                   | 0.00                     | 0.00                 | 0.00                 |
| No-Commitment Inflation       | 12.01         | 11.96          | 11.53           | 6.98              | 11.21                  | 11.34                    | 20.49                | 12.07                |
| Inflation Beta                | -0.01         | -1.34          | 0.25            | -0.01             | 0.00                   | 0.00                     | -0.12                | -0.01                |
| LC Debt Share                 | 0.99          | 1.00           | 1.00            | 0.99              | 1.00                   | 1.00                     | 1.00                 | 1.00                 |
| Equity Risk Premium           | 6.26          | 0.00           | 7.51            | 6.26              | 3.19                   | 3.13                     | 6.26                 | 6.26                 |

Note: This table varies one parameter at a time as indicated in each column. All other parameters are as in Table 3.
B.6 Model Extension: Real Exchange Rate Correlations

There is some empirical evidence that a country’s currency loses value in real terms during recessions. We can extend the model to account for this by modeling the real exchange rate as correlated with local output

\[ \varepsilon_2 = \rho x_2 + \nu_2, \]  

where \( \rho \) would in general be expected to be positive and \( \nu_2 \) is assumed to be uncorrelated with \( x_2 \) and \( x_2^* \). A positive value for \( \rho \) means that the local currency depreciates during recessions. We take \( \rho \) as given and outside the government’s control.

B.6.1 Quantitative Implications

To preview the results of this model extension, we find that for a plausible value of \( \rho \) the quantitative model implications are almost unchanged relative to the benchmark model. Since we have found the analytic model solution to be quite accurate in the benchmark case (see Table B.4), we use the analytic model solution for the comparison in this subsection. We set \( \rho = 0.4 \), corresponding to the empirical value reported in Favilukis et al. (2015). All other parameter values are as in Table 3. Table B.6 compares the analytic solution for the benchmark model \((\rho = 0, \text{columns (1) and (2)})\) and the model with correlated exchange rates \((\rho = 0.4, \text{columns (3) and (4)})\). We can see that an empirically plausible real exchange rate correlation leaves the model implications qualitatively and even quantitatively unchanged.

Table B.6: Model Extension – Correlated Real Exchange Rate: Empirical and Model Moments

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Model</th>
<th>Correlated Exchange Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low ( p )</td>
<td>High ( p )</td>
</tr>
<tr>
<td>Average Inflation</td>
<td>2.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Average No-Commitment Inflation</td>
<td>6.15</td>
<td>13.00</td>
</tr>
<tr>
<td>Inflation Beta</td>
<td>-0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>LC Debt Share</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
<td>6.25</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Note: All parameters are as in Table 3, except that we allow real exchange rates to be correlated with output \((\rho = 0.4)\) for columns (3) and (4). All moments are in annualized natural units. All model moments are computed using the analytic solution.
B.6.2 Bond Prices

The international investor prices contingent claims on real domestic consumption using the stochastic discount factor

\[ m_2 = m_2^* + \varepsilon_2 - \frac{1}{2} \sigma_{\varepsilon}^2. \]  
(133)

The price of a FC bond is exactly as before and equals

\[ q_{1}^{FC} = E_1[exp(m_2^*)], \]  
(134)

\[ = \beta. \]  
(135)

The price of a LC bond now equals

\[ q_{1}^{LC} = E_1[exp(m_2 - \pi_2)], \]  
(136)

\[ = \beta E_1 \left[ exp \left( -\phi x_2 + \rho x_2 - \frac{1}{2} \phi^2 \sigma_x^2 + \frac{1}{2} \rho^2 \sigma_x^2 - \pi_2 \right) \right]. \]  
(137)

\[ = \beta \exp \left( -\rho \phi \sigma_x^2 \right) E_1 \left[ \exp \left( -\left( \phi - \rho \right) x_2 - \frac{1}{2} \left( \phi - \rho \right)^2 \sigma_x^2 \right) \exp \left( -\pi_2 \right) \right]. \]  
(138)

The new parameter \( \rho \) enters in two ways into LC bond prices. First, if the real exchange rate depreciates in bad times \((\rho > 0)\), this makes investing into LC bonds risky, driving down the LC bond price. This first effect disappears if international investors are risk-neutral \((\phi = 0)\), but is independent of the state contingency of inflation. Second, \( \rho \) affects how inflation state-contingency enters into nominal local currency bond prices. This second effect is present even in the case when investors are risk-neutral \((\phi = 0)\). Even if investors are risk-neutral over international consumption goods, the marginal utility they derive from one unit of local goods varies and this variation is given by the real exchange rate. With \( \rho > 0 \), one unit of local consumption is less valuable for an investor who has preferences over international goods during recessions, so investors price bonds as if their marginal utility over local goods is lower during recessions. This drives a wedge between risk premia for real and nominal LC bonds.

Now, we can solve the extended model analogously to before. The price of a real local currency bond is

\[ q_{1}^{LC, real} = E_1[exp(m_2)], \]  
(139)

\[ = \beta \exp \left( -\rho \phi \sigma_x^2 \right). \]  
(140)

The debt portfolio log return in excess of the log return on a local currency real risk free bond is

\[ x_{r_{2}d} = \log \left( \frac{D_{1}^{FC} \exp \left( -\varepsilon_2 + \frac{1}{2} \sigma_{\varepsilon}^2 \right) + D_{1}^{LC} \exp \left( -\pi_2 \right)}{\beta^{-1} \exp \left( \rho \phi \sigma_x^2 \right) V} \right). \]  
(141)
Defining
\[ \bar{D} = \beta^{-1} \exp(\rho \sigma^2_x) V, \]  
(142) we can write period-2 consumption as
\[ C_2 = X_2 - \bar{D} \exp(xr_2^d). \]
(143)

### B.6.3 Log-Quadratic Expansion for Loss Function

We obtain the following second-order expansion for local currency bond prices
\[ q_{1}^{LC} \approx \beta \exp \left( -\rho \phi \sigma^2_x - E_1 \pi_2 + \frac{1}{2} \text{Var}_1 \pi_2 + (\phi - \rho) \text{Cov}_1 (x, \pi_2) \right). \]
(144) The approximate risk premium on LC bonds becomes
\[ y_{1}^{LC} - E_1 \pi_2 + \frac{1}{2} \text{Var}_1 \pi_2 - y_{1}^{FC} = -(\phi - \rho) \text{Cov}_1 (x, \pi_2) - \rho \phi \sigma^2_x. \]
(145)

We expand portfolio excess returns similarly to Campbell and Viceira (2002)
\[ xr_2^d + \frac{1}{2} \left( xr_2^d \right)^2 \approx \exp (xr_2^d) - 1, \]
(146)
\[ \approx (1 - s_1) \left( -\varepsilon_2 + \frac{1}{2} (\varepsilon_2^2 + \sigma^2_x) - \rho \phi \sigma^2_x \right) \]
(147)
\[ + s_1 \left( -(\pi_2 - E_1 \pi_2) + \frac{1}{2} ((\pi_2 - E_1 \pi_2)^2 - \text{Var}_1 \pi_2) - (\phi - \rho) \text{Cov}_1 (x, \pi_2) \right) \]

Substituting into the log-quadratic expansion for the loss function, taking expectations, and dropping policy-independent terms gives
\[ E_1 L_2 = \alpha E_1 \pi_2^2 + \bar{D} E_1 \left[ \frac{1}{2} \left( xr_2^d \right)^2 \right] \]
\[ + \left[ \frac{\gamma}{2} E_1 \left[ -2 D x_2 x_2^d + \bar{D}^2 \left( x_2^d \right)^2 \right] \right], \]
(148)
\[ = \alpha E_1 \pi_2^2 + \bar{D} \left[ (1 - s_1) \left( \sigma^2_x - \rho \phi \sigma^2_x \right) - s_1 (\phi - \rho) \text{Cov}_1 (x_2, \pi_2) \right] \]
(149)
\[ + \frac{\gamma}{2} \left[ 2 \bar{D} \left( (1 - s_1) \rho \sigma^2_x + s_1 \text{Cov}_1 (\pi_2, x_2) \right) + \bar{D}^2 \left( (1 - s_1)^2 \sigma^2_x + s_1^2 \text{Var}_1 \pi_2 \right) \right], \]
(150)
\[ = \alpha E_1 \pi_2^2 + s_1 \bar{D} (\gamma - (\phi - \rho)) \text{Cov}_1 (x_2, \pi_2) + \bar{D} (1 - s_1) (\gamma - \phi) \rho \sigma^2_x \]
\[ + \left( \bar{D} (1 - s_1) + \frac{\gamma}{2} \bar{D}^2 (1 - s_1)^2 \right) \sigma^2_x + \frac{\gamma}{2} \bar{D}^2 s_1^2 \text{Var}_1 \pi_2 \]
(151)

The expected loss function (151) is analogous to (21) in the benchmark model. The new parameter \( \rho \) enters in two instances. First, it increases the cost of FC debt issuance, thereby further contributing...
to unconstrained governments’ incentives to borrow in LC, acting effectively like an increase in exchange rate volatility in the benchmark model. The intuition is that if the real exchange rate depreciates during bad times, it is more likely that the real cost of servicing FC debt spikes during recessions, when marginal utility of consumption is already high. Second, a positive value for $\rho$ alters how the inflation-output covariance enters into the expected loss function. However, the inflation-output covariance continues to enter into (151) in a way that is qualitatively similar to the benchmark model and is also quantitatively similar if the real exchange rate-output loading is smaller than investor risk aversion (e.g. if $\rho \leq 1$ and $\phi = 10$).

\section*{B.6.4 Inflation Policy Functions}

The policy function in the no-commitment state is identical to before and equals

$$\pi_{2}^{nc} = \frac{s_{1}D}{2\alpha} - \gamma \frac{s_{1}D}{2\alpha} x_{2}. \quad (152)$$

The inflation policy function in the commitment state is analogous with effective investor risk-aversion $\phi$ replaced by $\phi - \rho$.

$$\pi_{2}^{c} = (\phi - \rho - \gamma) \frac{s_{1}D}{2\alpha} x_{2}. \quad (153)$$

The inflation-output beta is the weighted average of inflation-output betas in the commitment and no-commitment states

$$Beta(\pi_{2}, x_{2}) = (p(\phi - \rho) - \gamma) \frac{s_{1}D}{2\alpha} \quad (154)$$

The expected loss function after dropping third-order terms in $D$ becomes

$$E_{1}L_{2} = (1 - p) \frac{s^{2}D^{2}}{4\alpha} - \frac{(\phi - \rho - \gamma)^{2}}{4\alpha} s^{2}D^{2}\sigma_{x}^{2} + (1 - p) \frac{(\phi - \rho)^{2}}{4\alpha} s^{2}D^{2}\sigma_{x}^{2}$$

$$+ \frac{\gamma D^{2}(\sigma_{\varepsilon}^{2} + (\gamma - \phi)\rho\sigma_{x}^{2})}{2}(1 - s)^{2} + D(\sigma_{\varepsilon}^{2} + (\gamma - \phi)\rho\sigma_{x}^{2})(1 - s). \quad (155)$$

Taking the first-order condition of (155) with respect to $s_{1}$ gives

$$s_{1} = \frac{2\alpha \left[ \gamma + 1/D \right] (\sigma_{\varepsilon}^{2} + (\gamma - \phi)\rho\sigma_{x}^{2})}{(1 - p)(1 + (\phi - \rho)^{2}\sigma_{x}^{2}) - (\phi - \rho - \gamma)^{2}\sigma_{x}^{2} + 2\alpha\gamma(\sigma_{\varepsilon}^{2} + (\gamma - \phi)\rho\sigma_{x}^{2})}. \quad (156)$$

Equation (156) shows that the LC debt share increases with credibility, just as before. We only have to replace effective investor risk aversion by $\phi - \rho$ and the variance of exchange rate shocks by $\sigma_{\varepsilon}^{2} + (\gamma - \phi)\rho\sigma_{x}^{2}$. If $\rho > 0$, the government behaves as if exchange rates were more volatile and FC debt issuance hence costlier. Moreover, it behaves as if the international investor’s marginal utility varies less with local output, compared to a world where exchange rate shocks are uncorrelated with local fundamentals.
B.6.5 Comparative Statics

The comparative static of the LC debt share with respect to credibility becomes

\[ \frac{ds_1}{dp} = s_1^2 \frac{1 + (\phi - \rho)^2 \sigma^2_x}{2\alpha \left[ \gamma + 1/D \right] \left( \sigma^2_x + (\gamma - \phi) \rho \sigma^2_x \right)}, \]  

(157)  

\[ > 0. \]

To understand how the inflation-output beta changes with the LC debt share, we then derive the comparative static

\[ \frac{dBeta(\pi_2, x_2)}{ds} = \frac{\partial Beta(\pi_2, x_2)}{\partial s} + \frac{\partial Beta(\pi_2, x_2)}{\partial p} \frac{ds}{dp}, \]

(158)  

\[ = \frac{p(\phi - \rho) - \gamma}{2\alpha} \frac{D}{s} + \frac{(\phi - \rho) \ D \left( \gamma + 1/D \right) \left( \sigma^2_x + (\gamma - \phi) \rho \sigma^2_x \right)}{1 + (\phi - \rho)^2 \sigma^2_x}. \]

(159)  

Equation (159) is analogous to (29) in the benchmark model and continues to have the same main terms, except the expressions are now somewhat more complicated. We can again consider the case with risk neutral investors \((\phi = 0)\), in which case the comparative static (159) becomes

\[ \frac{dBeta(\pi_2, x_2)}{ds} = - (\rho p + \gamma) \frac{D}{2\alpha} - \rho \ \frac{D \left( \gamma + 1/D \right) \left( \sigma^2_x + (\gamma - \phi) \rho \sigma^2_x \right)}{1 + \rho^2 \sigma^2_x}, \]

(160)  

\[ < 0. \]

Expression (161) shows that when investors are risk-neutral, the model continues to have the counterfactual implication that in equilibrium the inflation-output beta should rise with LC debt shares. The conclusion that we need need risk aversion to explain our main empirical stylized fact therefore continues to hold in the extended model. The first term in (160) is negative, because with a real exchange rate that depreciates in bad times risk neutral investors want LC bonds that pay out during good times – after all this is when the local currency has real value in terms of international consumption goods. The government hence has an even stronger incentive to inflate in bad states of the world and this incentive increases with the LC debt share. The equilibrium effect with risk-neutral international investors \((\phi = 0)\) is also unambiguously negative and hence further contributes to the downward-sloping model-implied relation between inflation-output betas and LC debt shares if investors are risk neutral.

B.7 Model Extension: Domestic Debt

This section solves a model extension, where debt is held domestically and the government does not have access to lump-sum taxation. We show that the functional form of the analytic solution and quantitative model implications are very similar to the benchmark model. The model in this section has three key features that distinguish it from the benchmark model and make it a model...
of domestic debt: First, debt repayments no longer decrease domestic consumption one-for-one, but instead real taxes lead to quadratic deadweight losses as in the classic framework of Barro (1979); Second, investors and the government both care about the same consumption good; Third, investors and the government have the same coefficient of risk aversion.

B.7.1 Quantitative Implications

To preview the results of the domestic model, we start by showing its quantitative implications. We compare a calibrated version of the domestic debt model with the benchmark model. For the domestic debt model, we choose a debt-to-GDP ratio of \( \bar{D} = 1 \), corresponding to the high level of overall debt-to-GDP ratios for many countries in our sample, especially once pensions and social security are included, and a small tax distortion parameter \( \tau = 0.065 \). A tax distortion parameter of \( \tau = 0.065 \) implies that the log-linear approximate marginal benefit of reducing the real face value of debt by one percentage point is the same as in the benchmark model and equal to \( 2 \times \tau \times \bar{D} = 0.13 \) percentage points. All other parameters are set as in the benchmark model. Since we have found that the analytic solution is accurate for the benchmark model (see Table B.4), we use the analytic model solution to compare the two models. Overall we find that analytic model moments in Table B.7 are similar to Table 4, indicating that the bond risks channel of monetary policy credibility remains quantitatively important in the domestic debt setting.

<table>
<thead>
<tr>
<th>Table B.7: Domestic Model Extension: Empirical and Model Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark Model</strong></td>
</tr>
<tr>
<td>Average Inflation</td>
</tr>
<tr>
<td>Average No-Commitment Inflation</td>
</tr>
<tr>
<td>Inflation Beta</td>
</tr>
<tr>
<td>LC Debt Share</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
</tr>
<tr>
<td><strong>Domestic Debt Model</strong></td>
</tr>
<tr>
<td>Average Inflation</td>
</tr>
<tr>
<td>Average No-Commitment Inflation</td>
</tr>
<tr>
<td>Inflation Beta</td>
</tr>
<tr>
<td>LC Debt Share</td>
</tr>
<tr>
<td>Equity Risk Premium</td>
</tr>
</tbody>
</table>

Note: This table compares the benchmark model (columns (1) and (2)) to the domestic debt model (columns (3) and (4)). The domestic debt model assumes a debt-to-GDP ratio of \( \bar{D} = 1 \) and tax distortion parameter \( \tau = 0.065 \). All other parameters are as in Table 3. All moments are in annualized natural units and are computed using analytic model solutions.

B.7.2 Domestic Debt Framework

We now assume that the government does not have access to lump sum taxes. Instead, we assume that taxes are distortionary, so a debt repayment of \( D_2 \) has a deadweight cost of \( \tau D_2^2 \), as in Barro (1979). For instance, this functional form for deadweight costs could arise if the government only has access to labor taxes, which distorts labor supply. If raising taxes leads to costly distortion, the government has an incentive to use the “inflation tax” to reduce the real debt burden and hence...
real taxes. The government resource constraint becomes

\[ C_2 = X_2 - \tau D_2^2, \]  

(162)

implying that zero taxes have no deadweight costs, but increasing tax burdens have increasing marginal cost. The government’s loss function is given similarly to before

\[ L_2 = \alpha \pi_2^2 - \frac{C_2^{1-\gamma}}{1-\gamma}. \]  

(163)

For simplicity, we continue to assume that bonds are priced by an investor whose log stochastic discount factor is perfectly correlated with the exogenous component of local output with risk aversion \( \gamma \) :

\[ m_2 = \log(\beta) - \gamma x_2 - \frac{\gamma^2}{2} \sigma_2^2 \sigma_x^2. \]  

(164)

Abstracting from terms in the stochastic discount factor that directly depend on debt repayments keeps the solution tractable, while capturing risk premia arising from comovement with local economic conditions.

LC bonds have real payoffs per face value of \( \exp(-\pi_2) \) and FC bond have real payoffs of \( \exp(-\varepsilon_2 + \frac{1}{2} \sigma_\varepsilon^2) \), so LC and FC bond prices become

\[ q^{FC}_{t} = E_t \left[ \exp \left( m_2 - \varepsilon_2 + \frac{1}{2} \sigma_\varepsilon^2 \right) \right], \]  

(165)

\[ = \beta \exp(\sigma_\varepsilon^2), \]  

(166)

and

\[ q^{LC}_{t} = E_t \left[ \exp \left( \log(\beta) - \gamma x_2 - \frac{1}{2} \gamma^2 \sigma_x^2 \right) \exp(-\pi_2) \right]. \]  

(167)

Note that now since investors are domestic, we obtain a new \( \exp(\sigma_\varepsilon^2) \) term in the FC bond price (165). This obtains because investors and the government now care about the same consumption bundle, whereas in the benchmark model the real exchange rate drives a wedge between the price of the investor’s consumption bundle and the domestic consumption bundle. We again derive a log-quadratic approximation to the expected loss function. Expanding consumption log-quadratically
in terms of output and the excess return on the debt portfolio

\[ c_2 + \frac{1}{2} c_2^2 \approx C_2 - 1, \]
\[ \approx \bar{X} \exp (x_2/\bar{X}) - \tau \bar{D}^2 \left( \exp(x_r^d) \right)^2 - 1, \]
\[ = \bar{X} \left( \exp (x_2/\bar{X}) - 1 \right) - \tau \bar{D}^2 \left[ \left( \exp(x_r^d) - 1 \right)^2 + 2 \exp(x_r^d) - 2 \right] + \bar{X} - \tau \bar{D}^2 - 1, \]
\[ \approx x_2 + \frac{1}{2} \frac{x_2^2}{\bar{X}} - 2\tau \bar{D}^2 \left[ x_r^d + \left( x_r^d \right)^2 \right] + \bar{X} - \tau \bar{D}^2 - 1. \]  

We again make the normalization that steady-state consumption in period 2 equals one on average, giving that \( \bar{X} = 1 + \tau \bar{D}^2 \). The log-quadratic expansion for consumption then becomes

\[ c_2 + \frac{1}{2} c_2^2 \approx x_2 + \frac{1}{2} \frac{x_2^2}{\bar{X}} - 2\tau \bar{D}^2 \left[ x_r^d + \left( x_r^d \right)^2 \right]. \]

The second-order approximation to the log bond portfolio excess return is

\[ x_r^d + \frac{1}{2} \left( x_r^d \right)^2 = (1 - s) \left( \varepsilon_2 + \frac{1}{2} (\varepsilon_2^2 - \sigma_\varepsilon^2) \right) \]
\[ + s \left( -(\pi_2 - E_1 \pi_2) + \frac{1}{2} \left( (\pi_2 - E_1 \pi_2)^2 - Var_1 \pi_2 \right) - \gamma Cov(x_2, \pi_2) \right) \]

Substituting back into the loss function for \( x_r^d + \frac{1}{2} \left( x_r^d \right)^2 \), and ignoring policy independent terms gives

\[ L_2 \approx \alpha \pi_2^2 - \left( c_2 + \frac{1}{2} c_2^2 \right) + \frac{\gamma}{2} c_2^2, \]
\[ \approx \alpha \pi_2^2 - \left( x_2 + \frac{1}{2} \frac{x_2^2}{\bar{X}} - 2\tau \bar{D}^2 \left[ x_r^d + \left( x_r^d \right)^2 \right] \right) + \frac{\gamma}{2} \left( x_2 - 2\tau \bar{D}^2 x_r^d \right)^2, \]
\[ \approx \alpha \pi_2^2 + 2\tau \bar{D}^2 \left[ x_r^d + \frac{1}{2} \left( x_r^d \right)^2 \right] + \tau \bar{D}^2 \left( x_r^d \right)^2 + \frac{\gamma}{2} \left( -4\tau \bar{D}^2 x_r^d x_2 + 4\tau^2 \bar{D}^4 \left( x_r^d \right)^2 \right), \]
\[ \approx \alpha \pi_2^2 \]
\[ + 2\tau \bar{D}^2 \left[ (1 - s) \left( \varepsilon_2 + \frac{1}{2} (\varepsilon_2^2 - \sigma_\varepsilon^2) \right) \right] \]
\[ + s \left( -(\pi_2 - E_1 \pi_2) + \frac{1}{2} \left( (\pi_2 - E_1 \pi_2)^2 - Var_1 \pi_2 \right) - \gamma Cov_1 (x_2, \pi_2) \right) \]
\[ + (\tau \bar{D}^2 + 2\gamma \tau \bar{D}^4) \left( (1 - s)^2 \varepsilon_2^2 - 2(1 - s)s \varepsilon_2 (\pi_2 - E_1 \pi_2) + s \left( (\pi_2 - E_1 \pi_2) \right)^2 \right) \]
\[ - 2\gamma \tau \bar{D}^2 x_2 ((1 - s) \varepsilon_2 - s (\pi_2 - E_1 \pi_2)). \]

Debt repayments now are only costly to the extent that they induce deadweight losses, so they
appear pre-multiplied by $2\tau \bar{D}^2$. Taking expectations gives the expected loss function

$$E_1 L_2 = \alpha E_1 \pi_2^2 + 2\tau \bar{D}^2 \times s (\gamma - \gamma) \text{Cov}_1(\pi_2, x_2)$$

Inflation Cost

$$+ (\tau \bar{D}^2 + 2\gamma \tau \bar{D}^4) s^2 \text{Var}_1 \pi_2 + (\tau \bar{D}^2 + 2\gamma \tau \bar{D}^4) (1 - s)^2 \sigma_x^2$$

Volatility LC Debt

Volatility FC Debt

The expected loss function (176) takes exactly the same functional form as in the main text. Since the government has the same risk aversion over consumption as local consumers (who are now also the bond investors), the consumption hedging and nominal risk premium terms cancel ex-ante, exactly as in our calibration in the main paper, and in contrast to a model with risk neutral investors. Risk-aversion matters, because it generates an incentive for counter-cyclical inflation if the no-commitment state is realized in period 2. In addition, both inflation volatility and exchange rate volatility are now costly both because of utility curvature and because the government wants to smooth tax distortions across states of the world.

B.7.3 Full Commitment Solution

Under full commitment, it is clear that the government minimizes $E_1 \pi_2^2$, so inflation is constant at zero.

B.7.4 No Commitment Solution

Not taking into account the effect on expectations or risk premia ex post, the government in the no commitment state minimizes the no commitment loss function

$$L^{nc} = \alpha \pi_2^2 - 2\tau \bar{D}^2 s (\pi_2 - E_1 \pi_2) + (\tau \bar{D}^2 + 2\gamma \tau \bar{D}^4) s^2 (\pi_2 - E_1 \pi_2)^2$$

$+ 2\gamma \tau \bar{D}^2 x_2 s (\pi_2 - E_1 \pi_2)$

(177)

The first-order condition for no-commitment inflation becomes

$$\pi_2^{nc} = \frac{2\tau \bar{D}^2 s}{2\alpha + 2(\tau \bar{D}^2 + 2\gamma \tau \bar{D}^4) s^2} (1 - \gamma x_2) + \frac{2(\tau \bar{D}^2 + 2\gamma \tau \bar{D}^4) s^2}{2\alpha + 2(\tau \bar{D}^2 + 2\gamma \tau \bar{D}^4) s^2} E_1 \pi_2.$$  

(178)

Unconditional expected inflation equals

$$E_1 \pi_2 = \frac{\tau \bar{D}^2}{\alpha} s.$$  

(179)
Inflation variance and the inflation beta equal

\[ Var_1 \pi_2 = \gamma^2 \left( \frac{2 \tau \bar{D}^2 s}{2\alpha + 2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2} \right)^2 \sigma_x^2, \]  \hfill (180)

\[ Beta_1(\pi_2, x_2) = -\gamma \frac{2 \tau \bar{D}^2 s}{2\alpha + 2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2}. \]  \hfill (181)

\section*{B.7.5 Partial Commitment Solution}

We now solve the case \(0 \leq p \leq 1\) and show that the solution has analogous properties to the benchmark model in the main paper. We solve for inflation in the commitment state as following a rule

\[ \pi_2^c = \bar{\pi} + \delta x_2. \]  \hfill (182)

We next derive the expressions for optimal \(\bar{\pi}\) and \(\delta\). In the no-commitment state, the government chooses inflation myopaically according to the first-order condition

\[ \pi_2^{nc} = \frac{2\tau \bar{D}^2 s_1(1 - \gamma x_2)}{2\alpha + 2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2} + \frac{2(\tau \bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2}{2\alpha + 2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2} (p\bar{\pi} + (1 - p)E_1\pi_2^{nc}). \]  \hfill (183)

Expected no-commitment inflation then equals

\[ E_1\pi_2^{nc} = \frac{2\tau \bar{D}^2 s}{2\alpha + 2ps^2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2} + \frac{2ps^2(\tau \bar{D}^2 + 2\gamma^2 \bar{D}^4)}{2\alpha + 2ps^2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2} \bar{\pi}. \]  \hfill (184)

Unconditional expected inflation equals

\[ E_1\pi_2 = p\bar{\pi} + (1 - p)E_1\pi_2^{nc}, \]  \hfill (185)

The variance and covariance of period 2 inflation then become

\[ Var_1 \pi_2 = p(1 - p) \left( \frac{s \times 2\tau \bar{D}^2}{2\alpha + 2ps^2(\bar{D}^2 + 2\gamma^2 \bar{D}^4)} - \frac{2\alpha}{2\alpha + 2ps^2(\bar{D}^2 + 2\gamma^2 \bar{D}^4)} \bar{\pi} \right)^2 \]

\[ + p\delta^2 \sigma_x^2 + (1 - p) \frac{\gamma^2 \sigma_x^2}{(2\alpha + 2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2)^2} s^2 (2\tau \bar{D}^2)^2, \]  \hfill (186)

\[ Cov_1(x_2, \pi_2) = p\delta \sigma_x^2 - (1 - p) \frac{\gamma \sigma_x^2}{2\alpha + 2(\bar{D}^2 + 2\gamma^2 \bar{D}^4) s^2} \times 2\tau \bar{D}^2 s \]  \hfill (187)
Inflation sensitivity in the commitment state $\delta$ enters into inflation variances and covariances exactly as with full commitment, scaled by the commitment probability $p$. It hence follows that the optimal commitment inflation sensitivity is

$$\delta = 0.$$  \hspace{2cm} (188)

Average commitment inflation $\bar{\pi}$ is chosen to minimize

$$\alpha E_1 \pi_2^2 + \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) s^2 \text{Var}_1 \pi_2,$$

$$= \alpha (E_1 \pi_2)^2 + \left( \alpha + \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) s^2 \right) \text{Var}_1 \pi_2,$$  \hspace{2cm} (189)

$$\text{Drop terms ind. of } \pi$$

$$= \frac{\alpha (1 - p) \times 2 \tau \bar{D}^2 s}{2 \alpha + 2 ps^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right)} + \frac{2 \alpha + 2 s^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) p \bar{\pi}}{2 \alpha + 2 ps^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right)} p \bar{\pi} \times$$

$$+ \left( \alpha + \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) s^2 \right) \times$$

$$p(1 - p) \left( \frac{s \times 2 \tau \bar{D}^2}{2 \alpha + 2 ps^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right)} - \frac{2 \alpha}{2 \alpha + 2 ps^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) \bar{\pi}} \right)^2.$$  \hspace{2cm} (190)

Dividing by common factors, optimal $\bar{\pi}$ minimizes

$$\alpha \left( (1 - p) s \times 2 \tau \bar{D}^2 + (2 \alpha + 2 s^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) p \bar{\pi}) \right)^2$$

$$+ \left( \alpha + \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) s^2 \right) p(1 - p) \left( s \times 2 \tau \bar{D}^2 - 2 \alpha \bar{\pi} \right)^2.$$  \hspace{2cm} (191)

Taking the first-order condition of (191) shows that optimal $\bar{\pi}$ satisfies

$$0 = 4 \alpha \left( \alpha + s^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) \right) p \times \left( (1 - p) s \times 2 \tau \bar{D}^2 + (2 \alpha + 2 s^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) p \bar{\pi}) \right.$$  

$$- 4 \alpha \left( \alpha + s^2 \left( \tau \bar{D}^2 + 2 \gamma \tau^2 \bar{D}^4 \right) \right) p(1 - p) \left( s \times 2 \tau \bar{D}^2 - 2 \alpha \bar{\pi} \right),$$  \hspace{2cm} (192)

which is equivalent to

$$\bar{\pi} = 0.$$  \hspace{2cm} (193)

With the two inflation policy functions in the commitment and no-commitment states, we can express expected inflation, inflation variance, and the inflation-output beta, taking the local currency

99
debt share as given:

\[
E_1 \pi_2 = \frac{(1 - p) \times 2\tau \bar{D}^2 s}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} 
\]

\[
Var_1 \pi_2 = p(1 - p) \left( \frac{s \times 2\tau \bar{D}^2}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} \right)^2 
\]

\[
+ (1 - p) \frac{\gamma^2 \sigma_z^2}{(2\alpha + 2 (\tau D^2 + 2\gamma \tau^2 D^4) s^2)} s^2 (2\tau \bar{D}^2)^2, 
\]

\[
Beta_1 (\pi_2, x_2) = - (1 - p) \frac{\gamma}{2\alpha + 2 (\tau D^2 + 2\gamma \tau^2 D^4) s^2} \times 2\tau \bar{D}^2 s 
\]

B.7.6 First-Order Condition for LC Debt Share

Substituting in the expressions for inflation moments, the expected loss function simplifies to

\[
E_1 L_2 = \alpha \left( \frac{(1 - p) \times 2\tau \bar{D}^2 s}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} \right)^2 
\]

\[
+ (\alpha + (\tau \bar{D}^2 + 2\gamma \tau^2 \bar{D}^4) s^2) \left( p(1 - p) \left( \frac{s \times 2\tau \bar{D}^2}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} \right)^2 
\]

\[
+ (1 - p) \frac{\gamma^2 \sigma_z^2}{(2\alpha + 2 (\tau D^2 + 2\gamma \tau^2 D^4) s^2)} s^2 (2\tau \bar{D}^2)^2 
\]

\[
+ (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) (1 - s)^2 \sigma_z^2, 
\]

\[
= (1 - p) \frac{2\tau \bar{D}^2 s}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} 
\]

\[
+ \frac{1 - p}{2} \frac{\gamma^2 \sigma_z^2}{(2\alpha + 2 (\tau D^2 + 2\gamma \tau^2 D^4) s^2)} s^2 (2\tau \bar{D}^2)^2 
\]

\[
+ (\tau \bar{D}^2 + 2\gamma \tau^2 D^4) (1 - s)^2 \sigma_z^2. 
\]

Equivalently, the government minimizes the scaled expected loss function

\[
E_1 L_2^{scaled} = \frac{E_1 L_2}{\tau D^2}, 
\]

\[
= (1 - p) \frac{4s^2 \tau \bar{D}^2}{2\alpha + 2ps^2 (\tau D^2 + 2\gamma \tau^2 D^4)} 
\]

\[
+ \frac{1 - p}{2} \frac{\gamma^2 \sigma_z^2}{(2\alpha + 2 (\tau D^2 + 2\gamma \tau^2 D^4) s^2)} 4s^2 \tau \bar{D}^2 
\]

\[
+ (1 + 2\gamma \tau D^2) (1 - s)^2 \sigma_z^2. 
\]
Now, in contrast to externally-held debt, $\bar{D}$ is plausibly large for domestically held debt. However, the deadweight cost from taxation is plausibly on the order of magnitude of 5 to 10 percent, in which case we can treat second-order terms in $\tau$ as small. Dropping second-order terms in $\tau$ leads to the simplified and scaled expected loss function

$$E_1L_{2\text{scaled}} = (1-p)\frac{2s^2\tau\bar{D}^2}{\alpha} + (1-p)\frac{\gamma^2\sigma_z^2}{\alpha} s^2s^2\tau\bar{D}^2 + (1+2\gamma\tau\bar{D}^2)(1-s)^2\sigma_z^2$$

(201)

### B.7.7 Solution Summary

Similarly, dropping second-order terms in $\tau$ gives the following expressions for expected inflation and the inflation-output beta:

$$E_1\pi_2 = \frac{(1-p)\tau\bar{D}^2s}{\alpha},$$

(202)

$$Beta_1(\pi_2, x_2) = -(1-p)\frac{\gamma\tau\bar{D}^2s}{\alpha}.$$  

(203)

As in the main model, the inflation output beta increases with credibility $p$ and decreases with the LC debt share $2$, similarly to equation (26) in the main paper.

Taking the first-order condition with respect to $s$ of (201) gives

$$\frac{ds}{dp} = \frac{s^2}{2\alpha} \frac{2 + \gamma^2\sigma_z^2}{\left(1/(2\tau\bar{D}^2) + \gamma\right)\sigma_z^2}.$$  

(204)

which is clearly increasing in credibility $p$, with derivative

$$\frac{ds}{dp} = \frac{s^2}{2\alpha (1/(2\tau\bar{D}^2) + \gamma)\sigma_z^2}.$$  

(205)

The total derivative of the inflation-output beta with respect to the LC debt share is hence given by

$$\frac{dBeta(\pi_2, x_2)}{ds} = \frac{\partial Beta(\pi_2, x_2)}{\partial s} + \frac{\partial Beta(\pi_2, x_2)}{\partial p} \frac{1}{ds/dp},$$  

(206)

$$= -(1-p)\frac{\gamma\tau\bar{D}^2}{\alpha} + \frac{\gamma\tau\bar{D}^2s}{\alpha} \frac{2 \alpha (1/(2\tau\bar{D}^2) + \gamma)\sigma_z^2}{s^2 + \gamma^2\sigma_z^2}.$$  

(207)

Expression (207) is analogous to equation (29) in the main paper. As in the benchmark model, the direct effect generates a downward-sloping relation between LC debt shares and inflation-output
betas. This is counteracted by the indirect effect, which generates an upward-sloping relation between LC debt shares and inflation-output betas. In particular, we again need the equilibrium effect of LC debt shares increasing with credibility to generate an upward-sloping relation between inflation-output betas and the LC debt share.

B.8 Extension: Dynamic Model

This section specifies a simple dynamic extension of the two-period model, where solutions take the same form as in the two-period model. This extension illustrates that the solutions to the two-period model can be interpreted more broadly as the outcome of a dynamic setup. We assume that log output growth is i.i.d. and log-normal:

\[ \Delta x_t = \mu + \eta_t, \]  
\[ \eta_t \overset{i.i.d.}{\sim} N(0, \sigma_x^2). \]

The government minimizes the expected discounted sum of single-period losses:

\[ W_0 = E_0 \sum_{t=0}^{\infty} \delta^t L_t, \]

where the period-\( t \) loss equals:

\[ L_t = \alpha \pi_t^2 - \left( \frac{C_t}{X_{t-1}} \right)^{1-\gamma}. \]

The period \( t \) loss function can be interpreted as a combination of quadratic inflation costs, as in the two-period model, and power utility over consumption. We introduce external multiplicative habit similarly to Abel (1990). Introducing habit formation implies that the government faces a meaningful trade-off between the marginal cost of inflation and the marginal benefit of consumption along the growth path. Without habit, either the inflation loss component or the consumption utility component would dominate the loss function as \( t \to \infty \) and there would no longer be a trade-off between consumption and inflation. We specify habit in terms of lagged output instead of lagged consumption, because otherwise the government would have an incentive to lower aggregate consumption to reduce future habit. Specifying habit in terms of lagged output therefore is the natural extension of external habits to a government, whose policies can affect aggregate consumption but not aggregate output.

The international investors uses the following real log SDF to price contingent claims on real international consumption goods:

\[ m_t^* = \log \delta - \phi \Delta x_t - \frac{1}{2} \phi^2 \sigma_x^2. \]

We denote the real exchange rate (in units of FC in terms of LC) by \( \mathcal{E}_t \). We assume that the real
exchange rate follows a martingale, with the change in the log exchange rate given by:

\[ \Delta e_t = \varepsilon_t - \frac{1}{2} \sigma^2, \]  

(213)

\[ \varepsilon_t \overset{i.i.d}{\sim} N \left(0, \sigma^2 \right). \]  

(214)

The specification (214) implies that there is no predictability in real exchange rate returns, thereby precluding any incentive to time the real exchange rate by issuing FC debt. The international investor hence uses the following log SDF to price contingent claims on real domestic consumption goods:

\[ m_t = \log \delta - \phi \Delta x_t - \frac{1}{2} \phi^2 \sigma^2 + \varepsilon_t - \frac{1}{2} \sigma^2, \]  

(215)

Prices for FC and LC bonds are then given by

\[ q_{t-1}^{FC} = E_t \left[ \exp(m_t^*) \right], \]  

(216)

\[ = \delta \exp(-\phi \mu). \]  

(217)

From now on, we use \( \beta = \delta \exp(-\phi \mu) \) to denote the growth-adjusted discount rate. The prices for nominal and real LC bonds equal:

\[ q_{t-1}^{LC} = E_{t-1} \left[ \exp(m_t) \exp(-\pi_t) \right], \]  

(218)

\[ = \beta E_{t-1} \left[ \exp(-\phi \eta_t - \frac{1}{2} \phi^2 \sigma^2_x) \exp(-\pi_t) \right], \]  

(219)

\[ q_{t-1}^{LC, real} = E_{t-1} \left[ \exp(m_t) \right], \]  

(220)

\[ = \beta. \]  

(221)

The exchange rate shock \( \varepsilon_t \) drops out of the expressions for real and nominal LC bond prices, because it is assumed to be uncorrelated with all other shocks.

At the end of period \( t - 1 \), after consumption and inflation are realized, the government must raise a fixed amount that is proportional to output \( X_{t-1}V \) by issuing nominal LC and FC debt. If the government raises face values \( D_{t-1}^{LC} \) and \( D_{t-1}^{FC} \) of LC and FC debt, the budget constraint becomes:

\[ \frac{q_{t-1}^{FC}}{\xi_{t-1}} D_{t-1}^{FC} + q_{t-1}^{LC} D_{t-1}^{LC} = X_{t-1}V. \]  

(222)

The FC debt price enters into the budget constraint (222) divided by the real period \( t - 1 \) exchange rate, because \( q_{t-1}^{FC} \) units of international goods translate into \( \frac{q_{t-1}^{FC}}{\xi_{t-1}} \) units of domestic consumption
goods. We define the period $t - 1$ local currency debt share as:

$$s_{t-1} = \frac{q_{t-1}^L D_{t-1}^L}{X_{t-1} V}.$$  \hspace{1cm} (223)

The real domestic resources required to repay the debt portfolio at time $t$ are:

$$\frac{D_{t-1}^{FC}}{\mathcal{E}_t} + D_{t-1}^{LC} \exp(-\pi_t).$$  \hspace{1cm} (224)

The FC face value (224) appears divided by the period-$t$ exchange rate $\mathcal{E}_t$, because the domestic government must give up $\frac{1}{\mathcal{E}_t}$ units of domestic goods to repay one unit of foreign goods. Similarly to the two-period model, we define the debt portfolio log return in real domestic goods in excess of the log real LC bond as

$$x_{rt}^d = \log \left( \frac{D_{t-1}^{FC}/\mathcal{E}_t + D_{t-1}^{LC} \exp(-\pi_t)}{\beta^{-1} X_{t-1} V} \right),$$  \hspace{1cm} (225)

$$= \log \left( \frac{1}{\mathcal{E}_t} + s \frac{\beta}{q_{t-1}^L} \exp(-\pi_t) \right),$$  \hspace{1cm} (226)

$$= \log \left( 1 - s \frac{\exp\left(-\varepsilon_t \frac{1}{2} \sigma_t^2\right) + s \frac{\beta}{q_{t-1}^L} \exp(-\pi_t)}{\frac{1}{\mathcal{E}_t} + \frac{\beta}{q_{t-1}^L} \exp(-\pi_t)} \right).$$  \hspace{1cm} (227)
B.8.1 Dynamic Model Solution

Now, we guess and verify a particular solution. We guess that the period \( t \) objective function can be written as

\[
W_t = \alpha \pi_t^2 - \frac{C_t^{1-\gamma}}{1-\gamma} + t.i.p,
\]

(228)

where “t.i.p” denotes terms that are independent of period \( t \) policy.

We verify that there is a solution of this form by induction and show that it implies a constant LC debt share. Assume that the solution (228) holds from time \( t \) onwards. In period \( t \), consumption equals

\[
C_t = \exp (X_{t-1}) \left( \exp(\mu + \eta_t) - V \beta^{-1} \exp(x_{t-1}^d) \right), \tag{229}
\]

so the period \( t \) government minimizes

\[
W_t = \alpha \pi_t^2 - \frac{\left( \exp(\mu + \eta_t) - V \beta^{-1} \exp(x_{t-1}^d) \right)^{1-\gamma}}{1-\gamma} + t.i.p.
\]

(230)

We define a scaled value function

\[
\tilde{W}_t = \frac{W_t}{(\exp(\mu) - V \beta^{-1})^{1-\gamma}},
\]

(231)

\[
= \tilde{\alpha} \pi_t^2 - \left( \tilde{X} \exp(\tilde{\eta}_t) / \tilde{X} - \tilde{D} \exp(x_{t-1}^d) \right)^{1-\gamma} + t.i.p.,
\]

(232)

where

\[
\tilde{\alpha} = \frac{\alpha}{(\exp(\mu) - V \beta^{-1})^{1-\gamma}},
\]

(233)

\[
\tilde{X} = \frac{\exp(\mu)}{(\exp(\mu) - V \beta^{-1})^{1-\gamma}},
\]

(234)

\[
\tilde{D} = \frac{V \beta^{-1}}{\exp(\mu) - V \beta^{-1}},
\]

(235)

\[
\tilde{\eta}_t = \tilde{X} \eta_t.
\]

(236)

By definition, \( 1 + \tilde{D} = \tilde{X} \) and therefore the objective function (232) takes the same form as in the two-period model. At the end of period \( t-1 \), the government’s problem hence is to choose a LC debt share \( s_{t-1} \) and a commitment inflation policy \( \pi_t^c(\tilde{\eta}_t) \) to minimize (232), subject to the pricing relation (218), the debt portfolio return (227), and that the no-commitment inflation policy \( \pi_t^{nc} \) minimizes \( E(\tilde{W}_t | \tilde{\eta}_t) \). This is exactly the same problem as in the two-period model, so the solutions for the inflation policy and LC debt shares are also the same. In particular, the \( t-1 \) LC debt share and period \( t \) inflation policy functions do not depend on period \( t-1 \) consumption or inflation. It
follows that $W_{t-1}$ can be written in the form (228). q.e.d.