Paying Bonuses During Crisis: Dynamic Compensation under Uncertainty Shocks and Limited Commitment*

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Abstract

This paper studies optimal dynamic compensation where firms face volatility shocks but have only limited ability to commit to long-term contracts. I analyze a continuous-time dynamic principal-agent model with private effort and regime switching in cash flow volatility. In high volatility times, limited-commitment firms are forced to expedite payments to managers because sufficient deferred compensation is no longer credible; meanwhile, contract termination becomes more likely. In contrast, full-commitment firms defer compensation even more in high volatility times. This relationship between payment timing and expected contract length sheds light on empirical observations of compensation and volatility.

JEL Classification: D82, D86, G34, M12, M52

Key Words: Uncertainty shock, Dynamic compensation, Limited commitment

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1 Introduction

Contemporary firms, especially those in the financial industries, rely heavily on incentive pay, which constitutes the majority of overall firm cost as well as total employee compensation. While incentive pay is directly motivated by the presence of uncertainty, most extant research on managerial compensation has so far focused on its relationship with profitability rather than with uncertainty. Moreover, while a growing body of research identifies uncertainty shocks as the key contributing factor to financial crises and business cycles,[1] how compensation and uncertainty *dynamically* evolve with each other is relatively less addressed.

The current paper fills this gap by studying how uncertainty dynamically affects compensation through a continuous time principal-agent model. Due to moral hazard, the optimal contract incentivizes the manager by promising delayed cash bonuses after his performance exceeds some benchmark. When a shock increases uncertainty, the manager's performance becomes a noisier signal and his effort harder to gauge. I show that, absent of any other contracting friction, the optimal contract *raises* the performance hurdle, prescribes a *back-loaded* compensation, and the manager is *less likely* to receive cash payments in the high uncertainty state.

This benchmark theoretical prediction, however, does not reconcile with recent empirical findings on managerial compensation and risk. In cross-sectional studies, Peters and Wagner (2014) show that higher industry-level equity volatility is strongly associated with *higher* managerial compensation. Cheng et al. (2015) find a similar relationship for firms in the financial sector; moreover, the higher compensation in riskier firms mainly take the form of incentive pay such as cash bonuses.[2] The financial sector's behavior in the recent financial crisis is also at odds with the benchmark result. Despite being in a period of heightened

[1]See for instance Bloom (2009), Di Tella (2013), He and Krishnamurthy (2013), Atkeson et al. (2014), Brunnermeier and Sannikov (2014).

[2]Frydman and Jenter (2010) show that performance insensitive compensation (e.g. salary) makes up less than 20% of top manager's total compensation.

uncertainty, some firms actually *front-loaded* their employees' compensation and expedited payments.[3] Overall, huge losses of company wealth notwithstanding, many bankers and executives still received substantial compensation in the form of cash bonuses during the financial crisis,[4] which unsurprisingly raised much public attention and scholarly debate over the efficiency of the current compensation structure.

I argue that the discrepancy between what the benchmark model predicts and what is actually observed for compensation and uncertainty stems from an overlooked but critical market friction. Namely, the benchmark result is predicated on firms' ability to fully commit to long-term incentive contracts, which is not always consistent with the nature of such contracts. More specifically, to provide correct incentives to managers, firms must commit in two ways: commit to making payments when due, and commit to retaining managers until their performance is sufficiently poor. The latter type of commitment is generally infeasible in practice given the prevalence of at-will employment. Under US labor laws, firms can fire employees without having to establish just cause or give warning. Firms can also liquidate anytime, after which they are no longer liable for any future compensation promised to employees.

In light of these observations, I re-examine the implication of uncertainty on compensation *when firms have ability to unilaterally terminate contracts*, which I refer to as principal's *limited commitment*. Limited commitment restricts firms' ability to use deferred compensation as incentive. Moreover, the concern over firms' commitment is greatest when uncertainty is high and firm value low. I find, therefore, that firms with limited commitment are forced to *lower* their performance hurdles, make *front-loaded* compensation, and managers are *more*

[3] For example, Bank of America announced in 2009 that it would accelerate the vesting period of their employees' stocks from 2012 to 2011, claiming that such action is "critical for retention of talent" during volatile times. Deloitte reported instances of financial firms lowering performance hurdles during the crisis so employees would still be entitled to bonuses. See *Forbes* Insight (2009) and *Management Today* (2010).

[4] Wall Street Compensation–'No Clear Rhyme or Reason'. *Wall Street Journal* (2009). Moreover, Kaplan and Rauh (2010) and Philippon and Reshef (2012) show that the high level incentive payments are not exclusive to a handful of managerial elites only.

likely to receive cash payments in the period immediately after entering the high uncertainty state. These key results are robust to introducing real costs associated with default and to allowing contract renegotiation.

Different from comparative statics analysis, I model uncertainty shocks through stochastic regime switching between low and high volatility states. Under regime switching, firms optimally allocate deferred compensation to managers until the marginal value before the uncertainty shock is equal to the marginal value after the shock. These important dynamics are absent from simple comparative statics, which implicitly hold managerial deferred compensation constant when comparing different volatility levels. Moreover, the dynamics of regime switching reveal that cash payments should not be confused with "reward". In fact, under limited commitment, the lifetime present value managers derive from a contract that results in more immediate payments during financial crises is lower, due to a higher probability of termination.

This paper contributes in several ways to the literatures on contract theory, managerial compensation, and corporate governance. On the modeling side, this is to my knowledge the first paper to jointly consider agency, limited commitment, and regime switching. It adds to the growing literature of continuous-time dynamic agency models, such as DeMarzo and Sannikov (2006), Biais et al. (2007), Sannikov (2008), He (2009), Biais et al. (2010), and Zhu (2013), and is the closest to DeMarzo and Sannikov (2006) and Zhu (2013) in modeling the cash flow process and the moral hazard problem.[5] The regime switching technique for continuous time models is adopted from Hoffmann and Pfeil (2010), Piskorski and Tchistyi (2010), Narita (2011), Li (2012), and DeMarzo et al. (2012). Different from these models which focus on stochastic regimes of profitability, this paper studies regimes of cash flow

[5] It thus differs from most of the long-standing limited commitment literature which focuses on the optimal risk sharing contract between a risk-neutral principal and a risk-averse agent. In such an environment, termination is never part of an optimal compensation scheme. By contrast, I study a setting where a risk-neutral agent is protected by limited liability. In this environment, incentives are optimally provided in part through the threat of contract termination. This is a critical feature for my study, given the high frequency of managerial turnover in financial firms.

volatility. Although some of the results are replicable with stochastic profitability, volatility is more suitable for analyzing incentive compensation, which is the predominant component of compensation for modern firms. More discussions about the unqiue predictions from stochastic volatility can be found on the Internet Appendix.

2 Model

In this section I describe the model. I start with the standard principal-agent environment with volatility regime switching representing the normal and crisis states, and solve for the optimal contract. Then I discuss the effect of considering principals limited commitment and the different optimal contract implied.

2.1 Basic Environment

Time is continuous. A principal, representing a firm, must hire an agent, representing a manager, to run a project. Both the principal and the agent are risk neutral. The cash flow Y_t of the project follows

$$dY_t = \mu(e_t)dt + \sigma_t dZ_t \ ,$$

where Z_t is a standard Brownian motion; $\mu(e_t)$ is the expected cash flow rate depending on the agent's effort.

The agent controls the cash flow growth rate by choosing a binary effort level $e_t \in \{\overline{e}, \underline{e}\}$, representing "working" and "shirking", respectively. I assume $\mu(\overline{e}) = \mu$ and $\mu(\underline{e}) = \mu - C$ where $C > 0$, that is, shirking results in lower expected cash flow. However, the agent enjoys a private benefit λC whenever he shirks, where $\lambda \in (0, 1]$. Effort is private to the agent: the principal can observe Y_t but not e_t. The principal discounts future cash flows by r and the agent by $\gamma > r$, so the agent is more impatient.[6]

[6]The asymmetry of discount rates is a standard, essential requirement for a non-trivial incentive compat-

The volatility of the cash flow, σ_t, is stochastic. For simplicity I assume σ_t takes two values: σ_l or σ_h, representing "normal" and "crisis" times respectively, with $\sigma_l < \sigma_h$. If the current state is s, in any given time interval $(t, t+dt)$, the transition probability to the other state $\widehat{s}$ is $\pi_s dt$. In the remainder of this paper, I further simplify the model by assuming that $\pi_h = 0$, so the state h is absorbing. This means the economy starts with low volatility σ_l and experiences a one-time transition into the high volatility state with probability $\pi_h dt$ within any time interval dt. I will refer to this one-time change in volatility as the "uncertainty shock" to the economy.[7].

For now, I assume that the principal can commit to any contract once it is signed, but the agent is protected by limited liability with an outside option whose value R is normalized to 0. The principal has an outside option L which she receives whenever the contract is terminated. Both the principal and the agent take no further action after the contract termination, which eliminates reputation concerns. Finally, I assume the principal always prefers to induce the agent to work, which is true as long as C, the cost of shirking, is high enough.

2.2 Optimal Contract with Full Commitment

I now solve the optimal contract with principal having full power of commitment. Let $\mathscr{F}_t$ be the filtration generated by the cash flow history. A contract which specifies a compensation process $\{I_t\}_{t\geq 0}$ from the principal to the agent, a termination time τ, and a recommended

ible contract to exist in this type of model. However, once I impose the principal's commitment constraint, this additional constraint leads to the existence of an optimal contract even for the case where $r=\gamma$, which I describe at the end of this section

[7] Most of the analytical results do carry through when I allow the states to be recurring, i.e. when $\pi_h > 0$. Discussion of the optimal contract under recurring states is given in the appendix. I also assume that π_l is a small number to ensure that states l and h have their proper definitions. If π_l is too large, the value function (derived later) in the low volatility state converges to the value function in the high volatility state. To keep them sufficiently distant, π_l must be small enough. See the Appendix for more a detailed discussion.

effort process e_t defines the agent's continuation utility W_t:

$$W_t = E\left[\int_t^\tau e^{-\gamma(s-t)}\left(dI_s + \lambda C \mathbb{1}_{\{e_t=\underline{e}\}} dt\right) | \mathscr{F}_t\right] .$$

where $\mathbb{1}_{\{e_t=\underline{e}\}}$ is an indicator function that takes value 1 if $e_t = \underline{e}$ and zero other wise. W_t simply measures the present value of all expected future payments.

Similarly, the contract defines the principal's valuation of the project V_t which is the expectation of total future cash flow minus the payment to the agent plus the liquidation value when the contract is terminated.

$$V_t = E\left[\int_t^\tau e^{-r(s-t)}\left(dY_t - dI_s\right) + e^{-r(\tau-t)} L | \mathscr{F}_t\right] ,$$

Let $V(W_t)$ denote the principal's value function, which represents the highest valuation the principal can achieve, or the *firm value*, as a function of the agent continuation utility. There are two value functions $V_s(W_t)$, one for each state $s \in (l, h)$, that satisfy the principal's Hamiltonian-Jacobian-Bellman (HJB) equation:

$$\begin{aligned} rV_s(W_t) \quad = \quad & \max_{\beta_t \geq \lambda, \delta_s} \mu + \left(\gamma W_t - \pi_t \delta_s(W_t)\right) V_s'(W_t) + \frac{1}{2}\beta_t^2 \sigma_s^2 V_s''(W_t) \\ & + \pi_s \left(V_{\widehat{s}}(W_t + \delta_s(W_t)) - V_s(W_t)\right) . \end{aligned} \tag{1}$$

The principal chooses β_t and $\delta_s(W_t)$ optimally. β_t can be interpreted as the *pay-for-performance sensitivity* of the contract, which is incentive compatible as long as $\beta_t \geq \lambda$. That is, the agent must be given sufficient "skin-in-the-game" to work. In the Appendix I show that the value function $V_s(W_t)$ is globally concave, which implies the optimal contract provides minimal pay-for-performance sensitivity needed for incentive compatibility. That is, $\beta_t = \lambda$. The variable $\delta_s(W_t)$ denotes the discontinuous adjustment in the agent's continuation utility at the time of regime switching. Such adjustment exists because the shadow value of the agent's continuation utility is different for the principal in different states. The principal can promise future compensation conditional on the state of the economy and substitute immediate payments with more future payments if the value of the agent's continuation utility is higher in one state. Therefore, the choice of $\delta_s(W_t)$ is determined by matching the first order derivatives of the principal's value functions before and after the regime switching, that is

$$V'_{\hat{s}}(W_t + \delta_s(W_t)) = V'_s(W_t), \text{ if } W + \delta_s > R \; , \tag{2}$$

$$\delta_s(W_t) = R - W_t, \text{ otherwise} \; , \tag{3}$$

In other words, the principal optimally deploys the agent's continuation utility until its marginal value to the principal is equalized across states. In the case where the first order derivatives cannot be matched for any δ_s such that $W + \delta_s > R$, the contract is simply terminated.[8]

To sum up, the optimal contract can be characterized by the following proposition:

Proposition 1. *The optimal contract under volatility regime switching with full commitment defines a pair of value functions $V_s(W)$ and payment boundaries $\overline{W}_s$, $s \in \{l, h\}$ such that*

$$\begin{aligned} rV_s(W) &= \mu + (\gamma W - \pi_s \delta_s(W)) V'_s(W) + \frac{1}{2}\lambda^2 \sigma_s^2 V''_s(W) \\ &\quad + \pi_s \left(V_{\hat{s}}(W + \delta_s(W)) - V_s(W) \right) \; , \end{aligned} \tag{4}$$

subject to boundary conditions $V_s(R) = L$; $V'_s(\overline{W}_s) = -1$; and $V''_s(\overline{W}_s) = 0$. $V_s(W)$ is concave, continuous, and twice differentiable when $W \in [R, \overline{W}_s]$. $\delta_s(W)$ is determined by (2) and (3).

[8] It is worth noting that the discontinuity in the agent's continuation utility δ has non-trivial solutions even when the transition probability π_l approaches zero, that is when the pair of value functions $V_s(W)$ converge to two independent functions with different values of variance. The effect of π_l on determining δ is small when π_l is close to zero because $V_s(W)$ moves relatively little. This implies analyses of δ can be made almost independently of π_l for small π_l which greatly simplifies the mathematics.

Under the optimal contract, the value function in each state has two boundaries: a termination boundary R and payment boundaries $\overline{W}_s$. The contract must be terminated when $W_t = R$ due the agent's limited liability, therefore $V_s(R) = L$ ("value matching"). The agent receives instant cash payment of size $W_t - \overline{W}_s$ once W_t exceeds $\overline{W}_s$, thus $V_s'(\overline{W}_s) = -1$ ("smooth pasting"). This compensation scheme resembles cash bonuses that managers in practice receive for good performance, where $\overline{W}_s$ represents the "performance hurdle" managers must clear before getting paid. Finally, the payment boundaries are optimally chosen by the principal, which leads to the "super contact" condition $V_s''(\overline{W}_s) = 0$.

The optimal contract is associated with welfare losses due to moral hazard. The efficient allocation calls for the principal and the agent to split the maximal surplus generated by running the project permanently; that is, $V_s(W) + W = \mu/r$ for both $s \in l, h$. However, when moral hazard is present, incentive compatibility requires delaying payments to the agent, who is more impatient than the principal. Substituting boundary conditions for $\overline{W}_s$ into the principal's HJB equation yields $rV_s(\overline{W}_s) + \gamma\overline{W}_s = \mu$, which I refer to as the "second best" frontier, a critical boundary when considering the optimal contract with principal's limited commitment.

A numerical example of the principal's value functions are illustrated in Figure 1. Firm value is always lower in the high volatility state for any given level of W except at the termination boundary, which leads to the following conclusion:

Corollary 1. *$V_h(W) < V_l(W)$ for every $W > R$, and $\overline{W_h} > \overline{W_l}$. That is, the payment boundary under high volatility is higher for the full commitment contract.*

Intuitively, since cash flow serves as a signal for the principal to infer the agent's private effort, a more noisy signal increases the likelihood of contract termination which is a necessary but costly action for the principal to provide proper incentives. That is why the regime switching from the low to the high volatility state is referred to as a negative shock in this paper. More importantly, Corollary 1 states that the principal defers payments to the

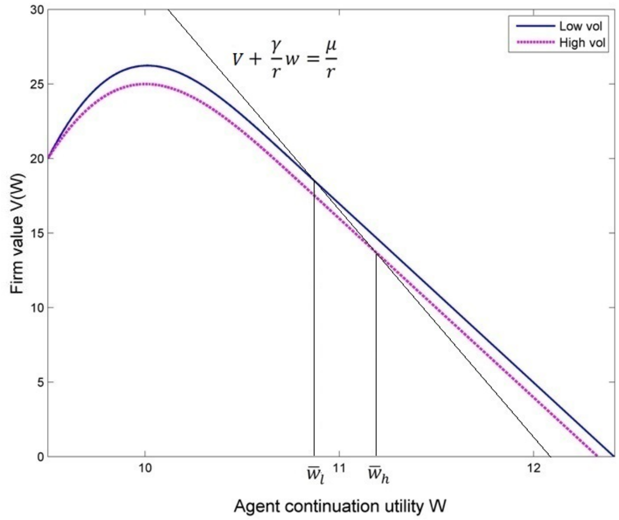


Figure 1 – Value Functions for the Optimal Contracts: Full Commitment
This figure plots firm value functions under regime switching and full commitment on the principal's side. Parameter values are $L = 20$, $R = 0$, $\gamma = 0.04$, $r = 0.02$, $\mu = 1$, $\lambda = 0.1$, $\sigma_l = 5.9$, $\sigma_h = 6.5$, $\pi_l = 0.001$, $\pi_h = 0$.

agent when volatility is high under the full commitment contract. Since the cost of providing incentives to the agent is the possibility of early termination after sufficiently poor performance, it is higher when volatility is higher, as rising uncertainty of cash flows increases the likelihood of sufficiently poor performance and the subsequent early termination. The principal adjusts the contract optimally by giving the agent more financial slack. Here financial slack, defined as $\overline{W}_s - R$, measures how much loss the principal is willing to take before terminating the agent's contract. Greater flexibility to the agent regarding his performance lowers the possibility of costly early termination and is thus optimal for the principal under higher volatility.

2.3 Optimal Contract with Limited Commitment

So far, the structure of the optimal contract characterized in Proposition 1 relies on the principal's commitment to all future payments once the contract is signed. However, before the agent's continuation utility W hits the payment boundary $\overline{W}$, the agent is not actually paid. His continuation utility measures the present value of the total amount of payment he expects to receive in the future, only if the principal honors the contract. Just as the agent is tempted to quit his job when W approaches his reservation utility R, the principal will likewise be tempted to exercise her outside option, which in this model is liquidating the project and receiving L, if the firm value V drops below the liquidation value before W reaches the payment boundary. If enforcement is not perfect and commitment becomes a binding constraint before the cash payment boundary is reached, the dynamics of the optimal contract will consequently be different.

To consider this impact, I assume that the principal can terminate the agent's contract anytime. As discussed earlier, this assumption of limited commitment on the part of the principal is more realistic, as firms generally are free to fire managers or liquidate projects at any time in practice. Once the contract is terminated, I assume both parties will receive the value of their outside options: L for the principal and R for the agent. This assumption sets this model apart from the other models in the relational contract literature in that termination time is the only aspect of the contract to which the principal cannot commit. Conditional on the continuation of the contract, the principal can still commit to all payments once the payment boundary is reached, suggesting the existence of long-term contracts although subject to a participation constraint from the principal.[9]

Given the limited commitment constraint I introduce here a heuristic approach that derives the optimal contract under principal's limited commitment by separating the commitment constraint from moral hazard, the other contractual friction in the model. First, suppose there is only one volatility state, and the agent's effort is observable to the princi-

[9] The assumption that firms can default on W anytime is a simple yet without loss of generality characterization of the important commitment constraint that is both prevalent in practice and commonly used in studies. See, for instance, Levin (2003), Berk et al. (2010), Rampini and Viswanathan (2013), Ai and Li (2014). The assumption that the principal receives her outside option L and the agent receives R no matter who breaches the contract is made here to illustrate the implications of limited commitment without losing tractability and can easily be relaxed, which I do in Section 4

pal such that the only contractual constraints are the principal's limited commitment and the agent's limited liability. Limited commitment implies a participation constraint for the principal:

$$V_t \geq L \, . \tag{5}$$

Combined with the agent's participation constraint $W_t \geq R$, they define a payoff space $\{(W,V)|W \geq R, V \geq L\}$ where, if the continuation value delivered by a contract falls into the space, the contract will not be terminated, i.e. the contract is *self-enforcing*[10].

Given the self-enforcing contracting space, consider now adding moral hazard. Analysis from the previous section applies. The principal's value function must lie in the area bounded by $W \geq R$, $V \geq L$ and the "second best" frontier $rV + \gamma W = \mu$. Intuitively, the boundary condition of the payment boundary $\overline{W}$ depends on whether the value function crosses the "second best" frontier or the self-enforcing border $V(W) = L$ first. If the value function meets $rV(W) + \gamma W = \mu$ first, it immediately follows $V''(\overline{W}) = 0$ and the principal's commitment constraint is not binding. In contrast, if the value function reaches $V(W) = L$ first, then the payment boundary is no longer optimally chosen, and the "super contact" condition is replaced with a physical boundary condition $V(\overline{W}) = L$. The reason why only firm value at the payment boundary turns out to matter under limited commitment is the combination of a concave value function, $V(R) = L$ on the left boundary, and $\overline{W}$ as a reflecting right boundary.

Let variables with a superscript L represent variables in the limited commitment environment, the following proposition summarizes the optimal contract. A formal verification theorem of the optimality of this contract is provided in the appendix.

[10] One may be worried that a limited commitment principal should not be able to "commit" to terminating the contract immediately when the agent's participation constraint binds either. The principal can "reset" the contract once W reaches R, for instance through renegotiation. I address this point the later section, and derive explicitly the renegotiation-proof contract in the Appendix to demonstrate that this concern does not affect the validity of the main results. At this point it is useful to simply assume the principal can also commit not to renegotiate the contract.

Proposition 2. *The optimal contract under volatility regime switching with limited commitment on the principal's side defines a pair of value functions $V_s^L(W)$ and payment boundaries $\overline{W}_s^L$, $s \in \{l, h\}$such that $V_s^L(W)$ satisfies the same system of ODE (4) and boundary conditions $V_s^L(R) = L$; $V_s^{L\prime}(\overline{W}_s^L) = -1$, and*

$$V_s^{L''}(\overline{W}_s^L) = 0, \text{ if } V_s^L(\overline{W}_s^L) \geq L \ ,$$

$$V_s^L(\overline{W}_s^L) = L, \text{ otherwise} \ .$$

The boundary conditions specified in this Proposition imply that under limited commitment, the optimal contract takes three different forms depending on whether the principal's participation constraint (5) is binding at the payment boundary in each state: first, if (5) is not binding for either $s = l$ or $s = h$, this contract is simply identical to the full-comment contract and the principal's commit power does not matter. Secondly, the limited commitment constraint is binding in the high volatility state but not the low volatility state. Third, the constraint is binding in both states.[11]

Of the three types of contracts, the first type is obviously the least interesting since it is identical to the contract with full commitment. The second type can resemble contracts of either the first or the third type, depending on parameter values. I leave the details of this type to the Appendix. The third type of contract produces the most distinct implications for the dynamics of compensation between the full commitment and the limited commitment case. In the remainder of this paper, I will concentrate discussion on this type only. That is, unless stated otherwise, I assume the parameter space is such that under full commitment, $V_s(\overline{W}) < L$ for both $s = l$ and $s = h$[12].

[11]It is impossible for the constraint to be binding in the low volatility state but not in the high volatility state, since firm value is always lower when volatility is higher. See the Appendix for details.

[12]The exact space of parameters satisfying such condition is difficult to characterize. However, $\overline{W}$ is larger and $V(\overline{W})$ smaller whenever γ is closer to r, holding other parameters constant. This implies if the principal and the agent have similar patience level, there is a potentially large parameter space in which the limited commitment constraint will be binding in both states once it is imposed.

A numerical examples of the optimal contract under limited commitment is shown in Figure 2, using the same parameters as Figure 1 does. Note that in Figure 1, firm value in both states is below the liquidation value L and therefore payment boundaries in neither state can be sustained without principal's full commitment. Figure 2 thus represent the case in which the limited commitment constraint binds in both state.

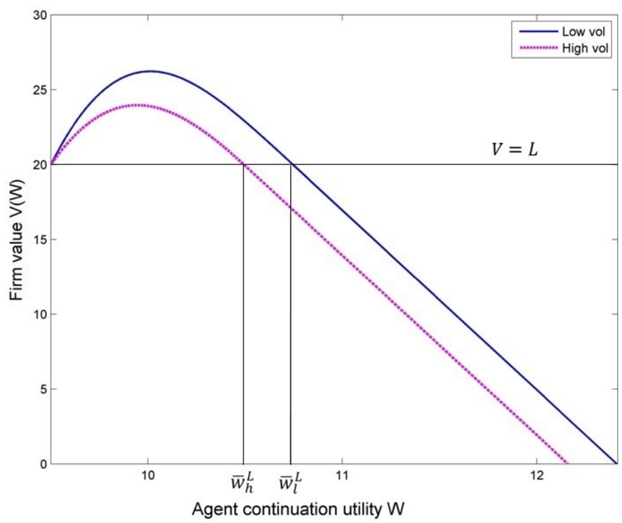


Figure 2 – Value Functions for the Optimal Contracts: Limited Commitment
This figure plots firm value functions under regime switching and limited commitment on the principal's side. Parameter values are the same as in Figure 1

Comparing Figure 2 to Figure 1, the contrast between optimal contracts under different commitment assumptions is apparent. The most crucial comparison is the position of the payment boundary, which is summarized as the following:

Corollary 2. *If* $V_s(\overline{W}) < L$ *and* $V_s^L(\overline{W}^L) = L$ *for both* $s = l$ *and* $s = h$*, then* $\overline{W}_h^L < \overline{W}_l^L$*. That is, the payment boundary under high volatility is lower for the limited commitment contract.*

Corollary 2 states that while the principal raises the performance hurdle when volatility is high under the full commitment contract, under limited-commitment, performance hurdle is lower. Intuitively, limited commitment implies that the principal's participation constraint $V_s^L(W) \geq L$ must be satisfied at any given time. In other words, the contract must guarantee firm value of at least L, which restricts the amount of future cash flow generated by continuing the project that can be used as compensation to the agent. When uncertainty becomes higher, the total value of the project is lower. A principal lacking the ability to commit to future payments when firm value is too low is forced to lower the performance hurdle because the principal can now credibly promise less compensation in the future. The relative positions of the payment boundaries under each volatility state determine the timing of the cash payment to the agent, the expected length of the contract, as well as the concavity of the principal's value function, all of which are essential in studying the compensation structure in the next section.

Limited commitment also expands the space of parameters in which the optimal contract exists: the case when $r = \gamma$. In the full commitment model, $r = \gamma$ means the principal can costlessly delay payments to the agent. The payment boundary is therefore infinity, and the optimal contract does not exist. In contrast, the limited commitment constraint puts a physical bound on the payment boundary such that the payment boundary is where the limited commitment constraint binds.

3 Implications on Dynamic Compensation

In this section I derive the implications of contracts under different level of commitment on compensation. I present these implications first through numerical examples from simulations and then through formal analytics.

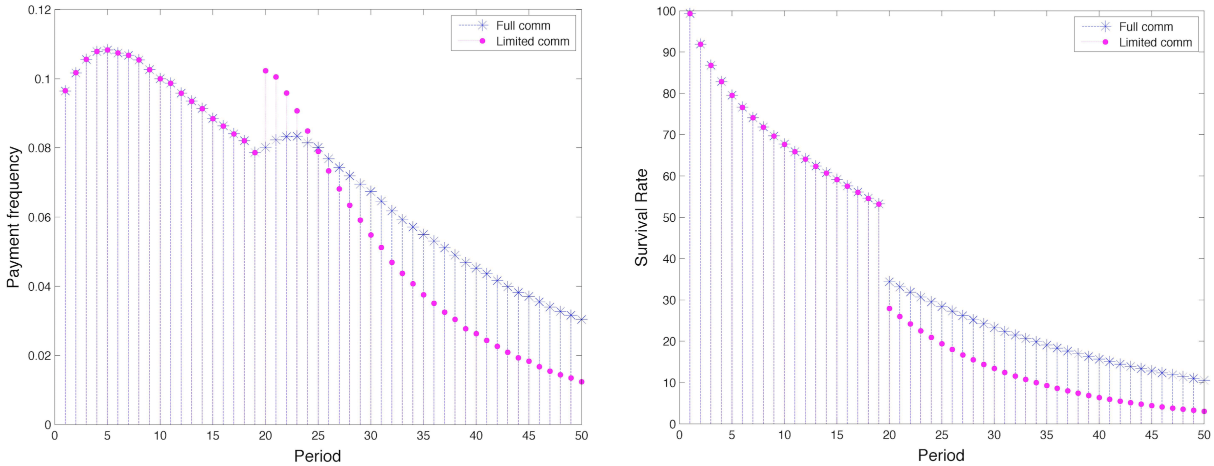
3.1 Numerical Illustration

The optimal contracts under full and limited commitment differ in how the payment boundary is determined. Their implications for compensation thus also differ, as the agent only receives payments in the form of cash bonuses once his continuation utility W exceeds the payment boundary. In this section I show how considering the optimal contract under limited commitment generates conclusions different from those with full commitment.

I begin the analysis with numerical simulations, in order to provide a transparent view of contract dynamics. In the simulations I segment the continuous-time model into discrete time intervals. The economy starts with low volatility, and the agent's initial wealth W_0 is drawn uniformly from the interval $(R, \overline{W}_l)$. I simulate N different paths of cash flows, and for each path, allow the state to switch to σ_h following a poisson arrival process, representing the transition into the crisis time. For each period before and after the uncertainty shock, I calculate the frequency of cash payments by taking the average number of recorded payments among all firms still surviving after the crisis. I repeat this simulation procedure for both the full commitment and limited commitment contract.

The results from simulation are shown in Figure 3, with $N = 1,000,000$, and the volatility shock occurs is indicated as period 20. Panel A plots the frequency of payments, while Panel B plots the fraction of active projects (managers) at each given time. Both full and limited commitment contracts are shown, using exactly the same parameter values. I choose the parameters such that once the limited commitment constraint is imposed, it will be binding in both low and high volatility states.

Two observations emerge from the frequency of cash bonuses shown in Figure 3. On the one hand, under the limited commitment contract, the frequency of payments in the few periods immediately after the uncertainty shock is much higher than the frequency under the full commitment contract and the frequency in the low volatility state. On the other hand, payment frequency under the limited commitment contract quickly diminishes to zero due to a higher rate of contract termination, while it is much more persistent under the full



Panel A. Frequency of Payments

Panel B. Survival Rate

Figure 3 – Simulation Results

This figure plots the frequency of cash compensation (bonuses) and the fraction of active projects from simulating 1,000,000 paths of cash flows. Period 20 corresponds to the volatility shock (from σ_l to σ_h). Parameter values are the same as those in Figure 3.

commitment contract. Both observations can be formalized using mathematical concepts in stochastic calculus and are rigorously proven in the next subsection. Here I offer readers with a general interest a heuristic derivation and an intuitive explanation of the mechanism behind these results.

Frequency of cash payments can be rationalized when considered jointly with the likelihood of contract termination. When uncertainty is higher, firms with full commitment power optimally set higher bonus hurdles so managers are able to build large continuation utility, reducing the likelihood of early contract termination. In contrast, without full commitment power, large deferred payments are no longer credible. The higher the cash flow uncertainty, the lower the value from running the firm and the more likely it is for firms to terminate managers' contracts before any bonuses are realized. Managers thus have to be compensated with bonuses early for the increased likelihood of turnover. Put differently, during crisis when firm value is low, firms have to make higher payments to retain their managers, who are more worried about loosing their jobs in the near future. A similar argument applies to the comparison between normal and crisis times for the limited commitment contract, under which the capacity of credibly deferring payments is correlated with firm value in each state.

It is important to clarify here that, despite the above description of immediate payment as a result of shorter expected tenure, the two are not fruit and tree to each other but rather two sides of the same coin. Both compensation and contract length are endogenously determined by the dynamics of the state variable W. In the high volatility state, the dynamics of the agent's continuation utility are given by $dW_t = \gamma W_t - dI_t + \lambda dZ_t$. Payment dI_t reduces W_t and thus increases the likelihood of termination. As shorter length stimulates more front-loaded contracts, a front-loaded contract also implies more aggressive managerial replacement following negative performance.

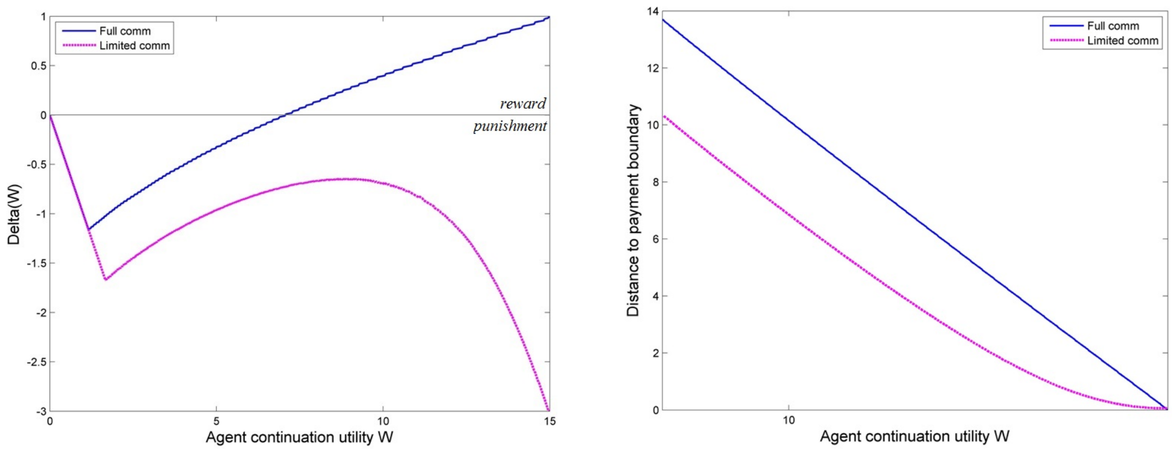
I also calculate the average size of cash payments for each period before and after the uncertainty shock, which produces a pattern almost identical to that observed in Panel A of Figure 3. This is not surprising given that conditional on receiving payments, the size of payments depends only on the variance σ which is a constant once the state is fixed.

3.2 Formal Analysis of the Regime Switching Model

While numerical simulations provide intuitive and transparent stories, I now formally state and prove the results. In addition to being mathematically rigorous, the formal argument also provides new insights into some important and controversial topics in the research on executive compensation.

3.2.1 Adjustment in the Agent's Wealth

The first step in formally showing the result of compensation in high volatility times is to derive the adjustment of the agent's continuation utility $\delta(W)$. The size of $\delta(W)$ takes different values depending on the different payment boundaries specified according to the type of the contract. Figure 4 Panel A illustrates the change in $\delta_l(W)$ and $\delta_l^L(W)$ as functions of the agent's wealth W before the state transition.



Panel A. Size of $\delta_l(W)$ Panel B. Distance to the Payment Boundary

Figure 4 – Allocation of Agent's Continuation Utility
This figure plots the size of δ_l (left panel) and $\overline{W}_h - W_{t+}$ (right panel), the distance between agent's continuation utility and the payment boundary after the uncertainty shock.

For both types of contracts, the region to the left of the kink in δ represents the region in which contracts are terminated once the uncertainty shock arrives, and the agent's entire promised future utility is wiped out. However, the full commitment and limited commitment contracts behave very differently thereafter: while $\delta_l(W)$ for the full commitment contract increases in W continues to grow until it becomes positive, $\delta_l^L(W)$ is hump-shaped, and remains negative all the way to the payment boundary for the limited commitment case. The reason for this difference is that under limited commitment as the agent's wealth increase, the limited commitment constraint is more likely to be binding under higher volatility, when firm value is lower and the capacity of the principal making promises of future payments is limited. Thus, the marginal value of the agent's wealth is always less in the high volatility state.

Observations from Figure 4 can be formally summarized in the following proposition:

Proposition 3. *There exist cut-off levels of the agent's continuation utility $\widehat{W}$ such that if $W_{t^-} > \widehat{W}$:*

$$\delta_l(W_{t^-}) > 0$$

$$\delta_l^L(W_{t^-}) < 0$$

Panel B of Figure 4 illustrates the distance to the payment boundary after the volatility shock. While the payment boundary of the limited commitment contract is lower, the agent's adjusted continuation utility after the uncertainty shock is also closer to the payment boundary. The following Corollary formally summarizes this observation:

Corollary 3. *Let W_{t^-} be the agent's continuation utility before the uncertainty shock, and $W_{t^+} \equiv W_{t^-} + \delta_l(W_{t^-})$ and $W_{t^+}^L \equiv W_{t^-} + \delta_l^L(W_{t^-})$ be the agent's continuation utility after the uncertainty shock under full and limited commitment contract, respectively, then*

$$\overline{W}_h^L - W_{t^+}^L < \overline{W}_h - W_{t^+}, \textit{if } W_{t^-} > \widehat{W}\ .$$

This conclusion plays a leading role in the analysis of compensation later, as being close to the payment boundary implies a larger probability of receiving more cash payments in the near future. At the same time, a lower payment boundary suggests a higher likelihood of contract termination following a series of poor performances. This trade-off between immediate cash payments and likelihood of termination is the central mechanism behind the dynamics of compensation.

After establishing the direction of $\delta(W)$ and the position of W_{t^+} relative to the payment boundary, I can formalize the observations from the simulation example using standard methods in stochastic calculus. The argument is presented in the next subsection

3.2.2 Analytical Characterization

Here I characterize the dynamics of compensation following uncertainty shocks. Given any W_{t^+}, the agent's continuation utility after the volatility increase, the goal is to characterize the distribution of the agent's wealth after a certain amount of time elapses. Given the property of Brownian motions, it is equivalent to characterize the amount of payment by the frequency of payment (as shown in Figure 3), which is the approach adopted in this section.

Following Cox and Miller (1977), given the dynamics of W, the transition density function $f(t, W; W_{t^+})$ for a process starts with W_{t^+} and satisfies the Kolmogorov forward equation:

$$\frac{\partial}{\partial t} f(t, W; W_{t^+}) = \frac{1}{2} \frac{\partial^2}{\partial W^2} \left[\lambda^2 \sigma_h^2 f(t, W; W_{t^+})\right] - \frac{\partial}{\partial W} \left[\gamma W f(t, W; W_{t^+})\right]$$

subject to boundary conditions:

$$f(t, R; W_{t^+}) = 0$$

$$\frac{1}{2} \frac{\partial}{\partial W} \left[\lambda^2 \sigma_h^2 f(t, W; W_{t^+})\right] |_{W=\overline{W}_h} - \gamma \overline{W}_h f(t, \overline{W}_h; W_{t^+}) = 0$$

Unfortunately, this partial differential equation is generally intractable. However, when γ is small, the dynamics of W can be approximated by a standard Brownian motion with one absorbing boundary R and one reflecting boundary $\overline{W}_h$, whose transition density has an explicit form[13]. Details on the approximation and the derivation of the transition density are shown in the Appendix by virtue of the method developed in Ward and Glynn (2003)

After obtaining the transition density, I can measure the likelihood of cash payments given a certain time period T after the shock using the concept of local time in stochastic processes. Given a time period T and initial point W_{t^+}, define local time $\mathcal{J}$

$$\mathcal{J}_h(T; W_{t^+}) = \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} \int_0^T \mathbb{1}_{\left\{\overline{W}_h - \varepsilon < W_t < \overline{W}_h + \varepsilon\right\}} dt | W_0 = W_{t^+}$$

[13]The assumption $\gamma > r$ is still needed for the benchmark full commitment contract to exist. It is not necessary, though, for the limited commitment contract. See the end of Section 2. for the discussion

where $\mathbb{1}_{\{.\}}$ is the indicator function. This local time is a random variable that measures the amount of time W spends in the neighborhood of the payment boundary. Since being at the payment boundary implies cash payments, this can be interpreted as the frequency of payments an agent with initial wealth W_{t^+} receives within time T after the economy enters crisis mode.

The definition of $\mathcal{J}_h$ allows the following proposition:

Proposition 4. *Assume γ is small. There exists $\widehat{T}$ and $\widehat{W}_{t^-}$ such that if $T < \widehat{T}$:*

$$E^L \left[\mathcal{J}_h(T; W_{t^-})\right] > E\left[\mathcal{J}_h(T; W_{t^-})\right]$$

$$E^L \left[\mathcal{J}_h(T; W_{t^-})\right] > E^L\left[\mathcal{J}_l(T; W_{t^-})\right]$$

for all $W_{t^-} > \widehat{W}_{t^-}$, where E^L represents expectation under the limited commitment contract

Proposition 4 is a formal characterization of the observations obtained from Figure 3. Despite the mathematical complexity, its basic intuition is quite simple: first, compare the limited and full commitment contract, Corollary 3 shows W is closer to the payment boundary after the uncertainty shock under the limited commitment contract. When γ is small, the process of W behaves similarly to a standard Brownian motion and thus spends more time at the payment boundary whenever the starting point is closer to the boundary. In more intuitive terms, the agent should expect more frequent payments in the near future if his cumulative performance is closer to the target bonus hurdle set by his contract. The similar argument applies to the comparison between the limited commitment contract in low and high volatility states: W_{t^+} is closer to $\overline{W}_h^L$ than W_{t^-} is to $\overline{W}_l^L$.

Why does Proposition 4 hold only when T is small? This is because while W_{t^+} is closer to the payment boundary after the shock under the full commitment contract, it is also closer to the termination boundary because the agent is overall punished according to Proposition 3. As T increases, the likelihood of contract termination rises faster for the limited commitment contract. That is, agents now operate under tighter financial slack. The longer into a crisis, the more likely is termination, as the possibility of realizing a series of losses becomes more real. The conclusion in Proposition 4 thus holds only for T small enough, when the probability of termination is negligible. As shown by the numerical simulations, this pertains to the second observation that cash payment vanishes very quickly under the high volatility state under limited commitment. The notion of termination likelihood can be formally described using the concept of stopping time, as the next proposition shows:

Proposition 5. *Define* $\tau_s = \inf\left\{t : W_t = R | \overline{W}_s\right\}$ *as the termination time given payment threshold* $\overline{W}_s$ *then:*

$$E^L(\tau_h) < E^L(\tau_l)$$

$$E^L(\tau_h) < E(\tau_h)$$

When the commitment constraint is binding, the agent's expected termination time is shorter under high volatility

Intuitively, given the absorbing boundary R and reflecting boundary $\overline{W}_h$, a process with initial value W_{t^+} is in expectation stopped earlier whenever W_{t^+} is closer to R and $\overline{W}_h$ is smaller. The limited commitment contract satisfies both conditions. Further, it should be noted that this proposition does not require the assumption of a small γ, as the expected speed of growth for W is lower when W_{t^+} is lower, which the limited commitment contract again satisfies. Nevertheless the proof of Proposition 5 still imposes the restriction on γ for the sole purpose of analytical tractability.

The results of this subsection imply that the recipients of crisis time bonuses are those who perform relatively well before the crisis. Proposition 4 states that more frequent cash compensation is conditional on the agent's wealth before the shock W_{t^-} surpassing a certain threshold, and higher W_{t^-} represents better before-shock performance. Those who perform relatively poorly ex ante are no longer around after the crises as a result of either replacement or firm liquidation. Combined with Proposition 3, this suggests that those who produce the largest profits before the crisis are being criticized the most for receiving bonuses during the crisis. One should keep in mind, however, that the huge loss of firm wealth is primarily due to the risky aggregate environment and, despite receiving bonuses for a short period into the crisis, managers are being harmed overall.

4 Discussions

In this section I address two questions one might have regarding the approach: (1) Is the limited commitment assumption too extreme? (2) How do state varying outside options L and R affect the result?

4.1 Alternative Commitment Mechanism

Economic research has long recognized that firms do not possess full commitment power over labor contracts.[14] Recent research shows that firms can also default on labor contracts because of limited or costly access to financial markets.[15] Furthermore, most of these studies also assume that firms, in addition to not being able to commit to intertemporal compensation, also cannot commit to terminal payments such as severance pay.[16] In practice, severance pay, especially for incentive purposes, is rarely paid in the event of firm liquidation. It is difficult to specify severance pay in contracts such that it is fully contingent. The execution

[14] There are many studies on risk sharing in labor contracts that emphasize the lack of commitment from the firm side, for example Thomas and Worrall (1988) Abreu et al. (1990), Ray (2002), Berk et al. (2010), Grochulski and Zhang (2011), and Miao and Zhang (2014). More generally, the relational contract literature, such as Atkeson (1991), Levin (2003), Grochulski and Zhang (2013), and Opp and Zhu (2015), has studied the lack of commitment from both contracting parties

[15] For example, Ellul et al. (2014) and Palacios and Stomper (2014).

[16] For instance, Berk et al. (2010), Ai and Li (2014), and Bolton et al. (2015). Even when severance pay is included in managerial contracts, as long as it is not fully guaranteed, limited commitment remains and the argument of this paper still applies

of severance pay is also subject to changes in the external enforcement environment.[17]

Furthermore, any implementation mechanisms of terminal pay is likely not immune to the limited commitment problem. Imagine if firms could set up for the agent an escrow account of balance W, and allow the agent to withdraw the balance in the event of contract termination. This mechanism for implementing the terminal payment, however, must necessarily give the principal access to the escrow account in order to update W according to the agent's performance and at the same time forbid the agent access to the account to prevent balance manipulation. This is essentially giving firms the control right to that account and asking firms to "commit" to paying the agent out of the balance of that account. This is effectively no different than giving firms the control right to cash flows, which is what this paper describes. The same difficulty of commitment remains, as it is intrinsically difficult to prevent the principal from drawing down the balance of any account to which she has control right before she defaults.

The assumption that the principal can default with impunity can also be easily relaxed. In the Appendix I derive a reneogiation-proof contract, which would address the concern of complete default but does not change the qualitative results of the paper. Furthermore, even if firms can credibly pledge payments to a degree when terminating the agent's contract, in other words some but not all of W is lost at the time of contract termination, for example due to the cost of legal procedure, the same concern of limited commitment still applies.

Finally, using a novel approach, I also provide a justification for the limited commitment constraint based on the security implementation of optimal contracts. Details of such implementation can be found in the Internet Appendix. The implementation involves standard securities such as equity and debt and hence potential tension between their holders. Following an uncertainty increase, the face value of long-term debt must decline, implying the redemption of debt which entails a wealth transfer from equity holders. Under high volatil-

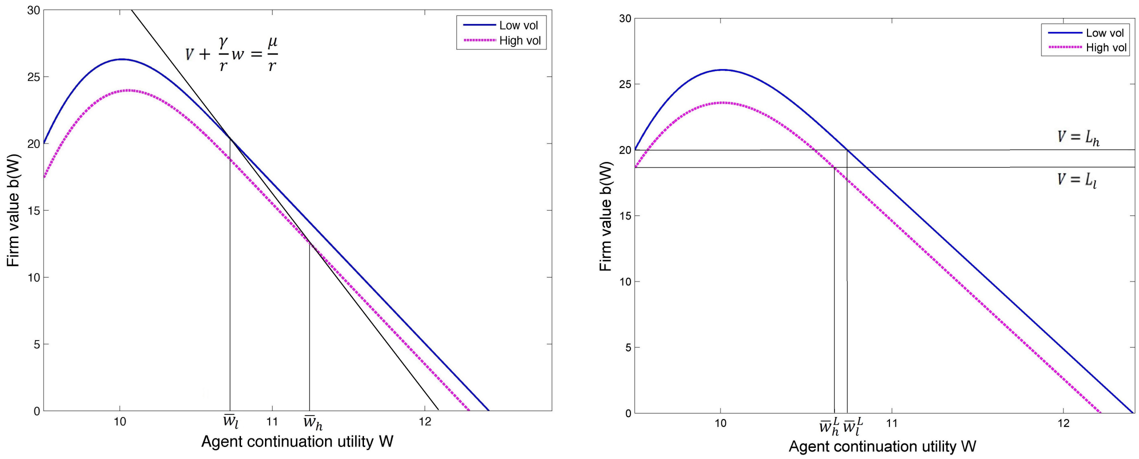
[17]Two recent cases: during the bankruptcies of Hostess Brands Inc. and Hawker Beechcraft Corp., the US Justice Department blocked proposals to grant extra bonuses to the executives of those companies.

ity, firm value is low, in which case equity holders may find it optimal to default rather than recall the debt. Therefore, in the implementation of the contract, assuming that the principal has full ability to commit to the agent is equivalent to assuming that equity holders have full ability to commit to maintaining a certain capital structure. The latter assumption is largely unrealistic as equity holders usually can default on debt without considering the effect on the firm as a whole.

4.2 Outside Options

In the baseline model, the assumption that both the principal and agent receive constant outside value whenever the contract terminates is critical to obtaining a closed-form solution. It is not so, however, to establishing the key findings of this paper regarding the difference in payment frequency and termination likelihood under the full versus limited commitment contract. There are many ways to endogeinize firm's outside option L and therefore make it state-contingent. I discuss only one straightforward case here and show how the main results still hold. Suppose firms can, at any time, replace the incumbent manager with a new manager, by paying a search cost C^R. The contract with the new manager will specify an initial W which maximizes firm value in that state. That is, $L_s = V_s^*(W) - C^R$. Figure 5 illustrates the value functions and payment boundaries under this assumption.

In Figure 5, under both full commitment and limited commitment, the liquidation value is lower in the high volatility state, because $V_h^*(W)$ is lower. However, the payment boundaries still resemble the baseline case shown in Figure 1 and Figure 2 where it is higher in the high volatility state under full commitment, and lower in the high volatility state under limited commitment. Intuitively, a lower value implies a lower value for the principal's outside option, thus reducing the tightness of the commitment constraint imposed on the contract. Therefore, whether more frequent cash bonuses are paid in the high volatility state simply depends on which dominates–the decrease in L or the increase in σ, an interesting task for



Panel A. Full Commitment　　　　Panel B. Limited Commitment

Figure 5 – Endogenous State-varying Liquidation Value

This figure plots the value functions with endogenous L defined by $L_s = V_s^*(W) - C^R$. The full commitment case is shown in the left panel. The limited commitment case is shown in the right panel. Parameters are the same as used in Figure 1 and Figure 2, and $C^R = 6$

future empirical work.

Finally, it is also unclear whether outside option value for sure decreases in crisis times. In the above case, the search cost C^R can also be made state contingent, reflecting the likely different difficulty in finding a replacement manager in good versus bad times. How it is different, however, is not immediately clear. In bad times, more managers are laid off. The search for a replacement could therefore be less difficult due to a larger managerial talent pool; but it could also be more difficult due to greater information asymmetry regarding a potential manager's true ability). The degree of variation in the outside options of managers and firms under different volatility levels remains to be revealed by future empirical work. Large variation in volatility relative to small changes in outside options would produce results consistent with those of the present model.

5 Conclusion

This paper studies the optimal compensation contract under the twin assumptions of limited commitment of the principal and regime switching of cash flow volatility. It has been argued that a sudden and dramatic increase in market uncertainty is the most critical feature of financial crises. When uncertainty is high, investment becomes more risky and the value of continuing operations correspondingly low. Principals without full commitment cannot credibly pledge sufficient amounts of future payments and must therefore provide agents with more immediate compensation.

Results in this paper introduce a caveat to the popular perception that the high compensation observed in the financial crisis is a sign of managerial are entrenched and suboptimal contracts. In the aftermath of the recent financial crisis many economists and politicians blamed the current managerial compensation scheme for not aligning managerial incentives with long-term investor benefit. Consequently, policy recommendations to propagate the use of delayed payment as a solution to that problem were suggested.[18] However, the effectiveness of such recommendation hinges on the credibility that future payment promised to the executives will be delivered at full value. Without taking into account firms' commitment ability, policies intended to align managers' incentives with those of investors could actually backfire. First, during times of financial uncertainty, following the recommendation to defer compensation and restrict retention payments would actually do more harm than good as it would undermine managerial incentives. Secondly, if executives believe that when firms are in distress, investors will withdraw by selling their shares, then the value of their stock holdings is less the longer they have to wait to cash them out. As a result executives may require even higher and more immediate compensation at the time of distress.

The theoretical results of this paper generate testable empirical hypotheses: conditional

[18]For instance, the Troubled Asset Relief Program (TARP) limits the ability of executives of TARP firms to cash out their restricted stock until the government is repaid in full. See TARP Standards for Compensation and Corporate Governance, 74 Fed. Reg. 28,394, 28,410 (June 15, 2009)

on negative uncertainty shocks, commitment-constrained firms make larger immediate payments and have higher managerial turnover relative to unconstrained firms. While the empirical literature on corporate governance generally takes low total compensation and high pay-for-performance sensitivity as indicative of good governance, this paper shows the importance of considering these under the context of market uncertainty. Total compensation and pay-for-performance sensitivity are sensible proxies for firm governance only when firms have no commitment issue, which may not always be true, especially during spells of high volatility. Altogether, these are areas in which this model can be fruitfully applied.

Appendix A. Proofs

Proof of Proposition 1 and 2:

Much of the proof regarding the volatility regime switching can be inferred from Hoffmann and Pfeil (2010), and Piskorski and Tchistyi (2010). Therefore I combine the proof of Proposition 1 and 2 and focus on the case of limited commitment, beginning with the single state case.

The optimality condition for a single state, full commitment environment is identical to the baseline model in DeMarzo and Sannikov (2006). Now consider the limited commitment case. Define the social benefit function as $F(W) = W + V(W)$, which satisfies

$$F''(\overline{W}) = \frac{rF(\overline{W}) + (\gamma - r)\overline{W} - \mu}{\frac{1}{2}\lambda^2\sigma_s^2} \, .$$

When the principal's participation constraint is binding, $F^L(\overline{W}) = L + \overline{W}^L$ implying

$$F^{L''}(\overline{W}^L) = \frac{rL + \gamma\overline{W}^L - \mu}{\frac{1}{2}\lambda^2\sigma_s^2} \, .$$

Suppose $F^{L''}(\overline{W}^L) > 0$, that is, $rL + \gamma\overline{W}^L > \mu$, this implies that $V^{L''}(\overline{W}^L) > 0$. Since $V^L(\overline{W}^L) = L$, $rV^L(\overline{W}) + \gamma\overline{W}^L > \mu$. Compare this result to the case of full commitment, where $rV^(\overline{W}) + \gamma\overline{W} = \mu$. If $\overline{W}^L < \overline{W}$, since $rV(W) + \gamma W < \mu$ for all $W < \overline{W}$, it must be that $rV(\overline{W}^L) + \gamma\overline{W}^L < \mu$, which implies $V(\overline{W}^L) < V^L(\overline{W}^L)$. However this is a contradiction since $V(W) \geq V^L(W)$ for every W. If, on the other hand, $\overline{W}^L > \overline{W}$, but $V^L(\overline{W}^L) = L$ and $V(\overline{W}) < L$, which implies that $V^L(\overline{W}^L) > V(\overline{W}^L)$ again, contradiction. Therefore $F^{L''}(W) < 0$ in the neighbourhood of $\overline{W}^L$.

The rest of the argument about F^L being also concave besides the neighbourhood of $\overline{W}^L$ follows the standard argument. The proof also implies immediately that $rV^L(W) + \gamma W \leq \mu$ for all W if the boundary condition $V^{L''}(\overline{W}^L) = L$ is true. This conclusion is used in the following verification theorem.

Verification Theorem: for any incentive compatible contract, define an auxiliary gain process G as

$$G_t = \int_0^t e^{-rs}(dY_s - dI_s) + e^{-rt}V(W_t) \, ,$$

where W_t evolves according to dW_t. By Ito's lemma

$$e^{rt}G_t = \left(\mu + \gamma W_t V'(W_t) + \frac{1}{2}\beta_t^2\sigma^2 V''(W_t) - rV(W_t)\right) dt - (1+V'(W_t))dI_t + (1+\beta_t V(W_t))\sigma dZ_t \ . \tag{6}$$

The first two terms are negative and therefore G_t is a supermartingale. Now evaluating the principal's payoff for this contract

$$E\left[\int_0^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L\right] =$$

$$E\left(G_{t\hat{}\tau}\right) + e^{-r\tau}E\left[1_{\{t\leq\tau\}}\left(E_t\left(\int_t^\tau e^{-r(s-t)}(dY_s - dI_s) + e^{-r(\tau-t)}L\right) - V(W_t)\right)\right] \ .$$

First, $E\left(G_{t\hat{}\tau}\right) \leq G_0$ since G_t is a supermartingale. Then, $E_t\left(\int_t^\tau e^{-rs}(dY_s - dI_s) + e^{-rt}L\right) \leq \frac{\mu}{r} - W_t$, since by the argument above, $rV(W) + \gamma W \leq \mu$ for all W . Letting $t \to \infty$ implies that

$$E\left(\int_0^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L\right) \leq V_0(W) \ .$$

Moving on to the two volatility states case: since the high volatility state is assumed a absorbing state, the value function in such state follows directly from the analysis above, while the optimality conditions for the low volatility state can be proved in a very similar manner. Differentiate the corresponding social benefit function with respect to W, substituting in the boundary conditions and evaluating the equation at the payment boundary $\overline{W}_L$ implies

$$F_l'''(\overline{W}_l) = \frac{(\gamma - r) + \left(\gamma\overline{W}_l - \pi_l\delta_l(\overline{W}_L)\right)F_l''(\overline{W}_l)}{\frac{1}{2}\lambda^2\sigma_l^2} \ ,$$

where $F_l''(W_l)$ is given by

$$F_l''(\overline{W}_l) = \frac{rF_l(\overline{W}_l) + (\gamma - r)\overline{W}_l - \mu + \pi_l\left(F_h(\overline{W}_l + \delta_l(\overline{W}_l)) - F_l(\overline{W}_l)\right)}{\frac{1}{2}\lambda^2\sigma_l^2} \ .$$

Piskorski and Tchistyi (2010) show the optimality conditions for the full commitment case. Under limited commitment, if the commitment constraint is not binding in the low volatility state, the proof is identical to theirs. Now suppose it is binding, which implies that it must also be binding in the high volatility state. Given the fact that $V_l^{L\prime}(\overline{W}_l^L) = V_h^{L\prime}(\overline{W}_h^L) = -1$, the slope matching procedure that pins down δ implies $\delta_l^L(\overline{W}_l^L) = \overline{W}_h^L - \overline{W}_l^L$, Given that $rF_l^L(\overline{W}_l^L) + \gamma\overline{W}_l^L < \mu$ from Corollary 1, if $\overline{W}_h^L < \overline{W}_l^L$, then $\delta_l^L(\overline{W}_l^L) < 0$, and $F_l^{L\prime\prime\prime}(\overline{W}_l^L) > 0$. If $\overline{W}_h^L > \overline{W}_l^L$, then $\gamma\overline{W}_l^L - \pi_l\delta_l^L(\overline{W}_l^L) > 0$ as $\pi_l < \frac{\gamma\overline{W}_l^L}{\delta_l^L(\overline{W}_l^L)}$. Since

$\delta_l^L(\overline{W}_l^L) < \overline{W}_h^L < \frac{\mu - rL}{\gamma}$, $\gamma\overline{W}_l^L - \pi_l\delta_l^L(\overline{W}_l^L) > 0$ as long as $\pi_l < \frac{\overline{W}_l^L}{\mu - rL}$. Note that for a non trivial contract, $\overline{W}_l^L > R = 0$, there is always π_l small enough such that $\pi_l < \frac{R}{\mu - rL}$ is satisfied. The subsequent verification is similar to that used in Piskorski and Tchistyi (2010) thus is omitted here.$\square$

Proof of Corollary 1 and 2:

The relationship between $V(W)$ and $V^L(W)$ is fairly straightforward: if $V^L(W) > V(W)$ instead, then $V(W)$ cannot be the optimal value function for the principal since the contracting space with the commitment constraint is a strict subset of the contracting space without the constraint.The relationship between $\overline{W}$ and $\overline{W}^L$ follows that $rV(\overline{W}) + \gamma\overline{W} = \mu$ and $rV(\overline{W}^L) + \gamma\overline{W}^L \leq \mu$ and the inequality is strict whenever $V(\overline{W}) < L\square$

From Corollary 1 and Proposition 2, $V_h(W) < V_l(W)$ and $V_h^L(W) < V_l^L(W)$for all $W > R$. For the full commitment contract, $V_l''(\overline{W}_l) = V_h''(\overline{W}_h) = 0$ implies $rV_s(\overline{W}_s) + \gamma\overline{W}_s$ for $s = l, h$, then Corollary 1 implies $\overline{W}_l < \overline{W}_h$. For the limited commitment contract, $V_l^L(\overline{W}_l^L) = V_h^L(\overline{W}_h^L) = L$ and $V_s'(W) < 0$ near the payment boundary implies $\overline{W}_l^L > \overline{W}_h^L\square$

Proof of Proposition 3:

This proposition is proved in two steps. First, I show that both V_l' and V_h' are convex functions at the payment boundary. This conclusion utilizes the concavity of the value function which is true for both full commitment and limited commitment so only the former is shown. Differentiate the principal's HJB equation with respect to W, and substituting in $V_h'(W + \delta_l(W)) = V_h'(W)$, a condition that is always satisfied around the neighbourhood of the payment boundary because $V_s'(\overline{W}) = -1$ regardless of state and contract type. This yields

$$rV_s'(W) = (\gamma W - \pi_s\delta_s(W))\, V_s''(W) + \frac{1}{2}\lambda^2\sigma_s^2 V_s'''(W) + (\gamma - \pi_s\delta_s'(W))\, V_s'(W)\ .$$

Evaluating this at the payment boundary in the high volatility state yields

$$V_h'''(\overline{W}_h) = \frac{(\gamma - r) - \gamma\overline{W}_h V_h''(\overline{W}_h)}{\frac{1}{2}\lambda^2\sigma_h^2} > 0\ ,$$

since $V_h''(\overline{W}_h) \leq 0$. Similarly,

$$V_l'''(\overline{W}_l) = \frac{(\gamma - r) - \gamma\overline{W}_l V_l''(\overline{W}_l) + \pi_l X(\overline{W}_l)}{\frac{1}{2}\lambda^2\sigma_l^2}\ ,$$

where

$$X(\overline{W}_l) = \delta_l'(\overline{W}_l)V_l'(\overline{W}_l) + \delta_l(\overline{W}_l)V_l''(\overline{W}_l) \ .$$

Letting $\pi_L \to 0$ yields

$$V_l'''(\overline{W}_l) = \frac{(\gamma - r) - \gamma\overline{W}_l V_l''(\overline{W}_l)}{\frac{1}{2}\lambda^2\sigma_l^2} > 0 \ .$$

Therefore $V_l'''(\overline{W}_l) > 0$ for small enough π_l, and both V_l' and V_h' are convex functions at the payment boundary. The same argument applies for the limited commitment contract.

The second step compares the variation of δ near the payment boundary. Consider the full commitment contract: $V_h(W) < V_l(W)$ and $V_h(R) = V_l(R) = L$ implies $V_h'(R) < V_l'(R)$. Since $V_h'(\overline{W}_h) = V_l'(\overline{W}_l) = -1$, there must exist $\widehat{W}$ such that $V_h'(\widehat{W}) = V_l'(\widehat{W})$. Moreover, $\overline{W}_h > \overline{W}_l$ and, because V_s' are convex functions, there is a unique $\widehat{W}$ after which $\delta_l(W) > 0$ for all $W > \widehat{W}$.

For the limited commitment contract, $V_h^{L\prime}(R) < V_l^{L\prime}(R)$, $V_h^{L\prime}(\overline{W}_h^L) = V_l^{L\prime}(\overline{W}_l^L) = -1$ and $\overline{W}_h^L < \overline{W}_l^L$ by Corollary 2 implies there exists $\widehat{W}^L$ such that $\delta_l^L(W) > 0$ for all $W > \widehat{W}^L$ Let $\widehat{W}$ be the largest between the two cut-offs for the full and limited commitment contract, and note that $\widehat{W} < \overline{W}_l^L$ since $\delta_l^L(\overline{W}_l^L) < 0$ and $\delta_l(\overline{W}_l) > 0$ proves this proposition.□

Proof of Corollary 3:

Define $\Delta(W) = \left(\overline{W}_h - (W + \delta_l(W))\right) - \left(\overline{W}_l - W\right)$ as the difference between the distances to the payment boundary before and after the uncertainty shock for the full commitment contract, and $\Delta^L(W)$ as the same distance but for the limited commitment contract. Then $\Delta^L(W) - \Delta(W) = \left(\overline{W}_h^L - \overline{W}_h\right) - \left(\overline{W}_l^L - \overline{W}_l\right) - \left(\delta_l^L(W) - \delta_l(W)\right)$. For small π_l, $\overline{W}_l^L - \overline{W}_l$ is small. Therefore $\Delta^L(W) - \Delta(W) < 0$ as long as $\overline{W}_h^L - \delta_l^L(W) < \overline{W}_h - \delta_L(W)$. Notice that $\overline{W}_h - \overline{W}_h^L = \delta_L(\overline{W}_l) - \delta_l^L(\overline{W}_l^L)$, and $\delta_l'(\overline{W}_l) > 0$ while $\delta_l^{L\prime}(\overline{W}_l^L) < 0$ by Proposition 3. Therefore $\delta_l(W) - \delta_l^L(W) < \delta_l(\overline{W}_l) - \delta_l^L(\overline{W}_l^L) = \overline{W}_h - \overline{W}_h^L$ for any $W > \widehat{W}$. That is, $\Delta^L(W) - \Delta(W) < 0$ for all $W > \widehat{W}$.□

Proof of Proposition 4:

Following Cox and Miller (1977), the transition density of the process W in the high variance state given initial value W_{t^+} follows the Kolmogorov forward equation:

$$\frac{\partial}{\partial t} f(t, W; W_{t^+}) = \frac{1}{2}\frac{\partial^2}{\partial W^2}\left[\lambda^2\sigma_h^2 f(t, W; W_{t^+})\right] - \frac{\partial}{\partial W}\left[\gamma W f(t, W; W_{t^+})\right] \ ,$$

subject to the boundary conditions

$$f(t,0;W_{t^+}) = 0$$

$$\frac{1}{2}\frac{\partial}{\partial W}\left[\lambda^2\sigma_h^2 f(t,W;W_{t^+})\right]|_{W=\overline{W}_h} - \gamma\overline{W}_h f(t,\overline{W}_h;W_{t^+}) = 0\ ,$$

where f is a density function conditional on $W_{t^+} = W$.

Define $\sigma^2 = \lambda^2\sigma_h^2$ as the overall variance of the W process. Let f_γ be the solution to this boundary value problem for a particular γ. According to Ward and Glynn (2003), when γ is closer to zero, f_γ can be approximated by

$$f_\gamma(t,W;W_{t^+}) = k(\gamma)g(t,W;W_{t^+}) + o(\gamma)\ , \tag{7}$$

where $k(\gamma) = \left(1 - \frac{\gamma}{2\sigma^2}W_{t^+}^2 + \frac{\gamma}{2\sigma^2}W^2 + \frac{\gamma}{2}t\right)$ and g is the corresponding transition density function for the same process but with $\gamma = 0$.

Now the problem becomes a Brownian motion between an absorbing and a reflecting barrier. In particular, $g(t,W;W_{t^+})$ satisfies the differential equation:

$$\frac{\partial}{\partial t}g(t,W;W_{t^+}) = \frac{1}{2}\frac{\partial^2}{\partial W^2}\left[\sigma^2 g(t,W;W_{t^+})\right]\ ,$$

subject to boundary conditions $g(t,R;W_{t^+}) = 0$, $\frac{1}{2}\sigma^2\frac{\partial}{\partial W}[g(t,W;W_{t^+})]|_{W=\overline{W}_h} = 0$.

The solution to this problem has been derived by Schwarz (1992) as

$$g(W,t) = \sum_{n=1}^{\infty} A_n \exp\left(-\alpha_n^2\frac{1}{2}\sigma^2 t\right)\cos\left(\alpha_n W\right)\ ,$$

where $\alpha_n = \frac{(2n-1)\pi}{2\overline{W}_h}$ and $A_n = \frac{\cos\left(\alpha_n W_{t^+}\right)}{\overline{W}_h}$.

Substituting this into the approximation function (7) yields $f(W,t)$ which can be used in the definition of the expected local time at the payment boundary

$$E\left[L_h(T;W_{t^+})\right] = \lim_{\varepsilon\to 0}\frac{1}{2\varepsilon}\int_0^T dt\int_{\overline{W}_h-\varepsilon}^{\overline{W}_h+\varepsilon} f(t,W;W_{t^+})dW$$

Fixed some $W_{t^+} < \overline{W}_h^L$, Let

$$E[\mathcal{J}_h(T;W_{t^+})] \equiv E[\mathcal{J}_h(T;W_{t^+})|\overline{W}_h]$$

be the expected local time given the full commitment value functions and payment boundaries, and define

$$E^L[\mathcal{J}_h(T;W_{t^+})] \equiv E[\mathcal{J}_h(T;W_{t^+})|\overline{W}_h^L]$$

be the expectation of local time at the payment boundary under the limited commitment contract. First, $\frac{\partial}{\partial T}E[\mathcal{J}_h(T;W_{t^+})]|_{T=0} > 0$, that is, the expected time spend at one point is longer whenever the time interval is longer, in particular when the time interval expands a small amount from 0. Secondly, such derivative is larger for smaller $\overline{W}_h$ because for a fixed W, $f(W,t)$ is decreasing in $\overline{W}_h$. The effect of expanding the time interval is bigger, the shorter distance between W_{t^+} and the reflecting boundary is. Note that in the case of $\sigma \gg \gamma$, the approximation adjustment term $h(\gamma)$ is close to one if W and W_{t^+} are near each other, this implies the most precise approximation is around the payment boundary, exactly the target of the analysis given here.

From Corollary 2, $\overline{W}_h > \overline{W}_h^L$, and $E^L[\mathcal{J}_h(0;W_{t^+})] = E[\mathcal{J}_h(0;W_{t^+})] = 0$ implies

$$E^L\left[\mathcal{J}_h(T;W_{t^+})\right] > E\left[\mathcal{J}_h(T;W_{t^+})\right]\ ,\ as\ T \to 0\ .$$

The expected local time grows faster for closer reflecting boundary near $T = 0$. Also

$$E^L\left[\mathcal{J}_h(T;W_{t^+})\right] < E\left[\mathcal{J}_h(T;W_{t^+})\right]\ ,\ as\ T \to \infty\ ,$$

which implies there is some $\widehat{T}$ such that

$$E^L\left[\mathcal{J}_h(\widehat{T};W_{t^+})\right] = E\left[\mathcal{J}_h(\widehat{T};W_{t^+})\right]\ ,$$

and

$$E^L\left[\mathcal{J}_h(T;W_{t^+})\right] > E\left[\mathcal{J}_h(T;W_{t^+})\right]\ ,for\ all\ 0 < T < \widehat{T}\ .$$

Finally, notice that given $\overline{W}_h$, $E\left[L_h(T;W_{t^+})\right]$ is decreasing in W_{t^+}, that is, the further W_{t^+} is from the reflecting barrier, the less time it spends there within a certain time. Therefore $E^L\left[\mathcal{J}_h(T;W_{t^+}^L)\right] > E\left[\mathcal{J}_h(T;W_{t^+})\right]$ as long as $\overline{W}_h^L - W_{t^+}^L < \overline{W}_h - W_{t^+}$. By Corollary 3 $\overline{W}_l^L - W_{t^+}^L < \overline{W}_h - W_{t^+}$, if $W_{t^-} > \widehat{W}$, therefore $E^L\left[\mathcal{J}_h(T;W_{t^-})\right] > E\left[\mathcal{J}_h(T;W_{t^-})\right]$ for all $0 < T < \widehat{T}$ as long as $W_{t^-} > \widehat{W}\square$

Proof of Proposition 5:

Consider the process of W in the high volatility state with initial position W_{t^+}. Let N be the number of times W reaches the reflecting boundary $\overline{W}_h$ before it is stopped. Then

$$E\left[\tau\right] = \sum_{i=0}^{\infty} E\left[\tau, N = i\right]\ .$$

First consider $N \geq 1$, if W reaches $\overline{W}_h$ at least once before it is stopped, then starting from $\overline{W}_h$, the expected stopping time is smaller whenever $\overline{W}_h - R$ is a shorter interval. Next consider the case $M = 0$, the expected stopping time is smaller whenever W_{t^+} is closer to R. Finally, the average speed of growth of W, γW, is slower for smaller W. From Corollary 2 and 3 it can be concluded that $E^L[\tau_h] < E[\tau_h]$because $\overline{W}_h^L < \overline{W}_h$ and $W_{t^+}^L < W_{t^+}$ for the same W_{t^-}.

Same comparison can be conducted between $E^L[\tau_h]$ and $E^L[\tau_l]$. The expected stopping time is smaller when $\overline{W}$ and the initial W is closer to R, and when σ is larger.

The exact value of $E[\tau]$ is difficult to compute due to the irregular process W follows. However, when γ is small, the same approximation method used in the proof of Proposition 4 can be applied here as well. The problem thus becomes a standard absorbing time question for a Brownian motion between an absorbing and a reflecting barrier, whose solution is given by Cox and Miller (1977) as

$$E[\tau] = \frac{W_{t^+}(2\overline{W}_h - W_{t^+})}{\sigma^2}$$

This solution confirms that $E[\tau]$ is positively related to $\overline{W}_h$ and W_{t^+} while negatively related to σ. Since $\overline{W}_h^L < \overline{W}_l^L < \overline{W}_h$, $W_{t^+}^L < W_{t^-}^L < W_{t^+}$, and $\sigma_h > \sigma_l$, $E^L[\tau_h]$ must be the smallest compare to $E[\tau_h]$ and $E^L[\tau_l]$ $\square$

Appendix B. Recurring States

In the main body of the paper I assume that the transition probability from high to low uncertainty state π_h is zero, that is the crisis state is absorbing. This assumption greatly simplifies the verification of the optimality of the contract, but is unnecessarily for the results of this paper to hold. In this appendix I provide a full characterization of the optimal contract when I relax such assumption. That is, when $\pi_h > 0$ and the economy switches between normal and crisis times stochastically. Li (2012) establishes the technical details required to study this case. The following proposition summarizes the result:

Proposition 6. *Suppose $\pi_l > 0$ and $\pi_h > 0$. Let N_t be the total number of state transitions at time t. The agent's continuation utility W_t follows*

$$dW_t = \gamma W_t - dI_t + \lambda(dY_t - \mu dt) + \delta_t(dN_t - \pi_t dt);. \tag{8}$$

The optimal contract is a pair of value functions $V_s(W)$ and payment boundaries $\overline{W}_s$,

$s \in \{l, h\}$ such that

$$\begin{aligned} rV_s(W) &= \mu + (\gamma W - \pi_s \delta_s(W)) V_s'(W) + \frac{1}{2}\lambda^2 \sigma_s^2 V_s''(W) \\ &\quad + \pi_s \left(V_{\widehat{s}}(W + \delta_s(W)) - V_s(W)\right) , \end{aligned} \tag{9}$$

subject to boundary conditions $V_s(R) = L$; $V_s'(\overline{W}_s) = -1$; and

$$V_s''(\overline{W}_s) = 0 ,$$

where $\delta_s(W)$ follows (2) and (3). If the principal has only limited commitment, the optimal contract is a pair of value functions $V_s^L(W)$ and payment boundaries $\overline{W}_s^L$, $s \in \{l, h\}$, such that $V_s^L(W)$ satisfies the same system of ODE (9) and boundary conditions $V_s^L(R) = L$; $V_s^{L\prime}(\overline{W}_s^L) = -1$, and

$$V_s^{L''}(\overline{W}_s^L) = 0, \textit{ if } V_s^L(\overline{W}_s^L) \geq L ,$$

$$V_s^L(\overline{W}_s^L) = L, \textit{ otherwise} .$$

Proof: The proof builds on iteration procedure described in Li (2012). I therefore only sketch the argument here in the interest of space. Consider first the case of full commitment. Applying the martingale method of Sannikov (2008), the agent's continuation utility follows (8). Ito's lemma implies that the principal's HJB equation satisfies (1). Let $\widetilde{V}_s(W)$ be a solution to (1). The concavity of $\widetilde{V}_s(W)$ can be shown using the method similar to Proposition 1. Take $\widetilde{V}_l(W)$ as given, define an auxiliary value function U_h^S as the payoff assuming the principal ceases to provide any incentive to the agent in the high volatility state until the next volatility shock arrives. The concavity of $\widetilde{V}_s(W)$ implies that $\widetilde{V}_h(W) > U_h^S$. Apply the similar argument to $\widetilde{V}_l(W)$ but take $\widetilde{V}_h(W)$ as given, Li (2012) shows that the procedure converges to a pair of function $V_s(W)$ satisfying equation (9). Finally, the same procedure also applied to the limited commitment contract as long as $V_s(W)$ remains concave, which is shown in Proposition 2 by replacing the $V_h(W)$ with $\widetilde{V}_h(W)$ in its proof.□

The optimal contract characterized under recurring state is qualitatively identical to the one summarized in section 2 under a one-time shock. In fact, principal's value functions of the contract under recurring states converge to value functions under a one-time shock when $\pi_h \to 0$. Given π_s are assumed to be small numbers the case of a one-time shock provides a good approximation for the general case of recurring states and does not lose any important result.

All the remaining results discussed in the main body regarding the position of payment boundaries, the frequency of cash payment and expected termination time are preserved in the recurring state contract, as long as the parameters are that once the limited commitment constraint is imposed, it is binding in both states. The discussion of "pay-for-luck" can be expanded to not only negative shocks but also positive shocks. The following result can be inferred from Proposition 3: Under the full commitment contract, managers whose accumulated performance is well enough receive less utility when volatility decreases; meanwhile managers under the limited commitment contract receive higher utility. The conclusion for limited commitment contract is consistent with empirical findings of "pay-for-luck" which further reinforce the importance of taking firms' commitment ability into account when understanding compensation under shocks.

Appendix C. Contracts with Limited Commitment Binding in One State Only

Section 2 introduced three types of contracts based on when the limited commitment constraint is binding. While the main text focuses on the third types, here I also provide some discussions of the second type: the contract where the limited commitment constraint is binding only in the high volatility state. In general, this type of contract can behave like contracts with either full commitment or those with limited commitment but the commitment constraint is binding in both states, depending on the parameter value σ in each state.

The main goal of this appendix is to establish conditions under which the main Propositions in Section 3 are still valid for the optimal contract when the limited commitment constraint is imposed. The proofs of Proposition 4 reveal that the key variable driving the dynamics of compensation is the distance between W_{t^+} and payment boundary $\overline{W}_h$. This leads to the conjecture that the dynamics of compensation of the type of contract discussed in this appendix section will be similar to the dynamics of the limited commitment contracts described in Section 3 as long as when the commitment constraint is imposed, the agent's continuation utility W_{t^+} is closer to the payment boundary $\overline{W}_h^L$ compared to the full commitment case. Due to the implicit form of the value function, it is analytically difficult to characterize the exact range of parameters under which this conjecture is true. Nevertheless the following proposition gives one sufficient condition for it.

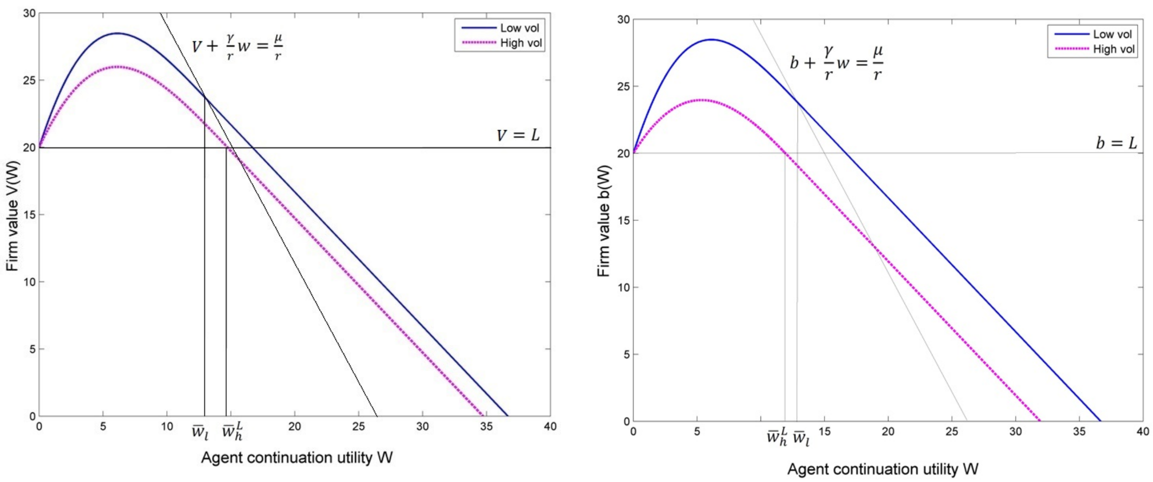
Proposition 7. *If the commitment constraint is binding only in the high variance state, then there exist $\widehat{W}$ such that $W_{t^+}^L - \overline{W}_h^L < W_{t^+} - \overline{W}_h$ for all $W_{t^-} > \widehat{W}$ as long as $\overline{W}_h < \overline{W}_l$.*

Proof: Similar to the proof in Section 3, consider the full commitment contract first. $V_h(W) < V_L(W)$ implies $\overline{W}_h > \overline{W}_l$ and $V_h'(R) < V_l'(R)$. Since $V_h'(\overline{W}_h) = V_l'(\overline{W}_l) = -1$, there must exist $\widehat{W}$ such that $V_h'(\widehat{W}) = V_l'(\widehat{W})$ and $\delta_l(W) > 0$ for all $W > \widehat{W}$.

Next, if the limited commitment constraint is imposed and binding, $V_h^{L\prime}(R) < V_l^{L\prime}(R)$, $V_h^{L\prime}(\overline{W}_h^L) = V_l^{L\prime}(\overline{W}_l^L) = -1$. If $\overline{W}_h^L < \overline{W}_l^L$, then there exists $\widehat{W}^L$ such that $\delta_h^L(W) > 0$ for all $W > \widehat{W}$. Let $\widehat{W} = \max\left\{\widehat{W}, \widehat{W}^L\right\}$, then $\delta_l'(\overline{W}_l) > 0$ while $\delta_l^{L\prime}(\overline{W}_l^L) < 0$ for all $W > \widehat{W}$. Finally, define $\Delta^L(W_{t^-}) - \Delta(W_{t^-}) = \left(\overline{W}_h^L - \overline{W}_h\right) - \left(\overline{W}_l^L - \overline{W}_l\right) - \left(\delta_l^L(W_{t^-}) - \delta_l(W_{t^-})\right)$.

For small π_l, $\overline{W}_l^L - \overline{W}_l$ is small. Notice that $\overline{W}_h - \overline{W}_h^L = \delta_l(\overline{W}_l) - \delta_l^L(\overline{W}_l^L)$, $\delta_l'(\overline{W}_l) > 0$ and $\delta_l^{L\prime}(\overline{W}_l^L) < 0$ implies $\delta_l = (W_{t^-}) - \delta_l^L(W_{t^-}) < \delta_l = (\overline{W}_l) - \delta_l^L(\overline{W}_l^L) = \overline{W}_h = -\overline{W}_h^L$ for any $W_{t^-} > \widehat{W}$. Therefore, $\Delta^L(W_{t^-}) - \Delta(W_{t^-}) < 0$ for all $W_{t^-} > \widehat{W}$.□

Given the sufficient condition above, the rest of the analysis follows exactly the one shown in the main text. Figure 6 demonstrate the difference between two levels of volatility in the high volatility state. For the same level of σ_l, the relative position of $\overline{W}_l$ and $\overline{W}_h^L$ are similar to the full commitment case when σ_h is moderate, but converge to the case in which the commitment constraint is binding in both states when σ_h becomes high enough.



Panel A. Low σ_h Panel B. High σ_h

Figure 6 – Contracts with the Commitment Constraint Binding in One State Only

This figure plots firm value functions when the limited commitment constraint is binding only in the high volatility state. Parameter values are the same as those in Figure 1 and Figure 2 except $\sigma_l = 5$ and $\sigma_h = 6$ for the left panel, and $\sigma_h = 6.5$ for the right panel

The finding of this section greatly expands the domain of contracts to which Propositions 4 and 5 can apply. Large bonuses in crisis times could be possible if the abrupt volatility increase is substantial enough that many firms that operate smoothly during normal times suddenly become constrained in the amount they can credibly pledge to pay their managers in the long-run. The greater increase of market risks during the crises, the more severe is this concern. Future research that calibrates or empirically investigates the real scope of this commitment constraint will be helpful in determining the proportion of firms that are subject to limited commitment contracts and firms whose dynamics of bonuses follow the predictions in this paper.

Appendix D. Renegotiation-Proof Contracts

In this paper the principal terminates the manager's contract whenever $W = R$, which may be suspect under limited commitment. However, this assumption is not required to deliver the qualitative results of this paper. One way to consider the principals lack of commitment when W approaches R is renegotiation. In this Appendix, I derive the renegotiation-proof contract and show that the main results carry through. Moreover, despite the principal having only limited power of commitment, renegotiation-proofness is not a necessary feature of the resulting equilibrium contract. In the limited commitment literature, [19] the off-equilibrium strategy for a defaulting party is usually autarky or complete exclusion from re-entering any contracting relationship. In practice, modifying the contract, via renegotiation or replacing the incumbent agent, is usually costly. Allowing for costly contract modification preserves the key predictions of the model, as explained further in the following paragraphs.

A renegotiation-proof contract requires the slope of the principal's value function to be non-positive. Such condition is ruled out in the main context of this paper because $V^L(R) = V^L(\overline{W}^L) = L$ when the limited commitment constraint binds, hence a non-trivial contract must have a region where the principal's valuation is increasing in the agent's continuation value W. To allow renegotiation-proof contracts I modify the assumption about the principal's commitment ability. I assume now that the principal will only withdraw the investment when firm values is below zero. This assumption is similar to the one made by Ai and Li (2014) and Bolton et al. (2015). The corresponding constraint on the payment boundary is now:

$$V(\overline{W}) \geq 0 \, .$$

The dynamics of the agent's continuation value under renegotiation-proof contracts follow

$$dW_t = \gamma W_t - dI_t + \lambda(dY_t - \mu dt) + \delta_t(dN_t - \pi_t dt) + dP_t \, .$$

The new term dP_t defines a reflecting termination boundary $\underline{W}$ which satisfies the boundary condition $V(\underline{W}) = L$ and $V'(\underline{W}) = 0$. Termination is stochastic at this boundary, with probability $\frac{dP_t}{\underline{W}-R}$to account for the extra term on the agent's continuation value and keep the contract incentive compatible.

For the main results of this paper to carry through, it is sufficient to prove the following proposition:

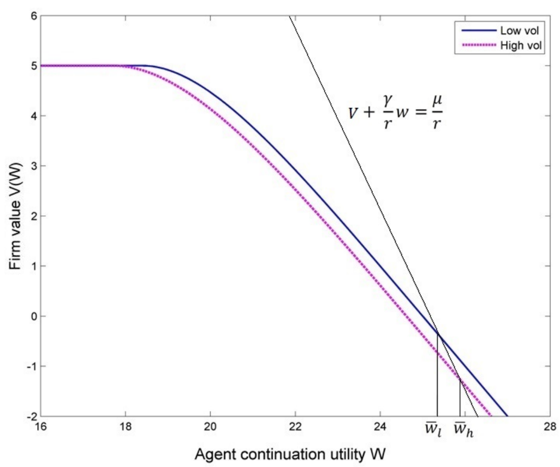
Proposition 8. *Under the renegotiation-proof contract, $\overline{W}_h > \overline{W}_l$ under full commitment and $\overline{W}_h^L < \overline{W}_l^L$ when the constraint is binding in both states.*

Proof: Clearly, Corollary 1 and 2 still apply to renegotiation-proof contracts. Therefore

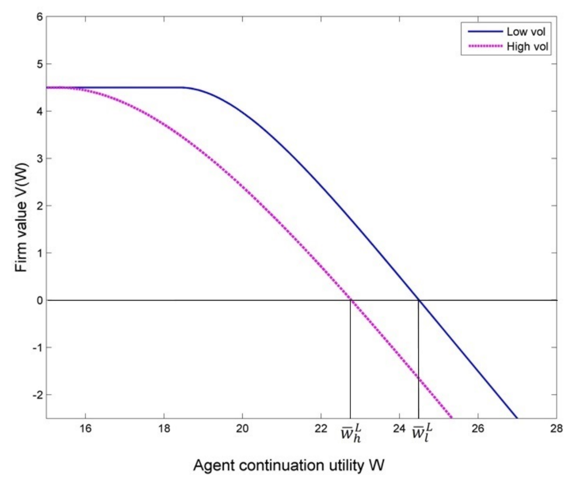
[19]For instance, Thomas and Worrall (1988) Abreu et al. (1990), Ray (2002), Levin (2003), and Miao and Zhang (2014)

$\overline{W}_h^L < \overline{W}_l^L$ since $V_h(W) < V_l(W)$ and $V(\overline{W}) = 0$ if the commitment constraint is binding in both states. Without the constraint the contract is a standard continuous-time dynamic contract with regime switching, and the argument of the boundary positions can be found in Hoffmann and Pfeil (2010)□

Given the relative positions of the payment boundaries for the full commitment and limited commitment contracts, one can easily see that a statement similar to Corollary 3 can be made here as well. Figure 7 shows the value functions for the renegotiation-proof contracts, where both the endogenous renegotiation boundaries as well as the payment boundaries are displayed. The graphs confirms Propositions regarding the relative positions of payment boundaries for both the full and limited commitment contracts. Such conclusions leads to the same dynamics of bonuses payment described in Section 3 and the details are thus omitted here.



Panel A. Full Commitment



Panel B. Limited Commitment

Figure 7 – Renegotiation-Proof Contracts

Last but not least, renegotiation-proofness is not a necessary feature for the contract to be optimal despite limited commitment. The principal is still able to rule out further renegotiation since the only action she cannot commit to is not to withdraw when firm value is negative. In particular, the principal can commit to the random termination schedule described above, which is crucial in keeping the manager's incentive properly. Further, the assumption of investors withdrawing their investment when firm value drops below zero replaces the earlier assumption of liquidation at any time, and therefore the value of the firm at the termination boundary is still the liquidation value since it is determined by the agent's effective limited liability constraint and the principal is able to commit to termination once that boundary is reached.

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