A Credit Cycle Model of Bank Loans and Corporate Bonds: A Bank Capital View

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Abstract

This paper investigates macroeconomic effects of bank regulation in a continuous-time macro-finance framework with both bond-financing and bank-financing. Risky firms appreciate bank credit because banks are efficient at liquidating assets for troubled firms. However, risky firms must pay the risk premium for banks' exposure to aggregate risks. Our framework captures the feature that the cost of bank-financing endogenously fluctuates and the fact that bond-financing is less volatile and cyclical than bank-financing. We show that if bank regulation is to raise the marginal funding cost for banks, these financial intermediaries transfer most of the incremental cost to their borrowers. Hence, risky firms switch to bond-financing due to the raised loan rate. Although financial stability improves, the average productivity declines. The impact of bank regulation on economic growth is unclear because more bond-financing leads to more inefficient liquidation. Overall, bank regulation benefits the real sector and hurts financial intermediaries.

Keywords: bank capital, bank-financing, bond-financing, and credit cycles

1 Introduction

To fully examine the real impact of bank regulation, we should not ignore the fact that corporate bonds are as important as bank loans for firms to raise external funds. More importantly, the two types of financial instruments display very different dynamic behaviors

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in the business cycle. Based on the U.S. aggregate level data from 1953 to 2012, Becker and Ivashina (2014) observe that bank-financing is more volatile and cyclical than bond-financing and corporate debt is less affected by recessions than bank loans are. Nevertheless, when the economy is far away from its steady state such as the situation in the 2007-09 financial crisis, the credit loss caused by the shrinking of bank credit is made up by the surge of bond-financing to some degree, as Adrian et al. (2012) document for the 2007-09 financial crisis.

In this paper, we present a continuous-time macro-finance framework with a productive expert sector, a less productive household sector, and an explicit banking sector. This framework captures both the heterogeneity of band-financing and bond-financing *in the long run* (documented in Becker and Ivashina (2014)) and the substitution of bond credit for bank credit *in crises* (highlighted in Adrian et al. (2012)). Unlike many papers that assume exogenous financial shocks hitting the real sector, the financial shock, i.e., the cost of bank-financing in our framework fluctuates endogenously across business cycles because the financial health of the intermediary sector endogenously determine the cost of bank loans.

In our framework, firms can obtain external funds from receiving loans from banks and/or issuing corporate debt directly. Firms differ in the level of their riskiness. Once a negative signal about a risky firm's fundamental occurs, creditors of the firm would like to liquidate its assets to protect their investments from the firm's opportunistic behaviors. On the one hand, banks are more efficient than bondholders in terms of liquidating firms' assets (Bolton and Freixas, 2000); on the other hand, firms need to pay banks the risk premium for the aggregate risk that banks are exposed to. Therefore, it is not difficult to see that safe firms tend to issue corporate bonds rather than bank loans since the likelihood that safe firms would have to face costly liquidation is tiny. This is consistent with empirical findings in Rauh and Sufi (2010).

When the banking sector is well capitalized, it channels more funds from creditors to firms. This in turn improves the aggregate productivity of the economy and boosts asset prices. However, when an adverse aggregate shock hits the economy, both bank capital and productive experts' net worth decline disproportionately due to the use of leverage. As a result, the supply of bank loans shrinks, experts' holdings of assets declines, the aggregate productivity deteriorates, and asset prices decline. The depreciation of asset prices in turn hurts balance sheets of both banks and experts, which in turn lowers the supply of bank loans and experts' holdings of assets further. Therefore, the financial intermediary sector amplifies the effect of the initial aggregate shock. We name the impact of this amplification on asset prices as endogenous risk.

Since there is an explicit banking sector in addition to a productive expert sector, unlike standard continuous-time macro-finance frameworks (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012) our framework features two endogenous state variables: wealth shares of both experts and bankers. Experts' wealth share is an endogenous state variable because it affects the average productivity of the economy as well as the likelihood and the magnitude of asset fire-sales.

The share of bank capital in total wealth is also a key endogenous state variable that drives the credit cycle of the economy. When the share of bank capital is thin in the economy, the supply of bank loans is relatively small and thus the interest rate on bank loans is high. Therefore, when a negative shock hits the economy, the fall in bank capital increases banks' leverage and the risk premium that banks request for each unit of credit they lend out. The cost of raising bank credit endogenously fluctuates because it depends on the risk premium, which in turn relies on the financial health of the banking sector. In economic booms when the banking sector is financially sound, the cost of bank-financing is relatively low.

The result that the fluctuation in the cost of bank-financing is endogenous is in contrast with many other related papers such as De Fiore and Uhlig (2011), De Fiore and Uhlig (2015), and Crouzet (2014). Since these papers model the surge in the cost of bank financing as an exogenous shock, they could not have rich characterizations of dynamics of bank-financing and bond-financing as what we capture in our paper. One exception is Rampini and Viswanathan (2015), which also endogenize the cost of financial intermediation. But, this paper does not address the substitution between bank credit and bond credit.

Across credit cycles, risky firms tend to choose bank-financing when the banking sector is well capitalized and bank loans are relatively less expensive. However, safe firms always stick to bond-financing because high liquidation cost associated with bond-financing has little effect on their borrowing costs.

Bank-financing is pro-cyclical in our model. During economic upturns when the financial condition of the banking sector improves, it is relatively cheap to raise bank credit. Thus, more firms choose bank-financing, and these firms also take high leverage because of low endogenous risks in addition to cheap bank credit.

Bond-financing is less volatile than bank-financing as a result of two opposing effects. At the extensive margin, less firms choose to issue corporate debts during economic booms when bank loans are relatively cheap. Nevertheless, at the intensive margin, firm that still raise bond credit would like to issue more corporate bonds because of low endogenous risks in economic booms. In financial crises, however, more firms issue corporate bonds because of the rising cost of bank loans and firms also take high leverage due to high returns of holding assets caused by low asset prices. Therefore, we observe that the rise in bond credit in crises can make up the loss caused by the decline in the supply of bank loans.

Three features of our framework make it an ideal laboratory for examining the impact of bank regulation on the entire economy. First of all, our framework takes into account both bond-financing and bank-financing. Hence, our captures alternative financing method that typical firms have, which is especially important for firms when the cost of bank-financing increases. Secondly, there are both safe and risky firms in our framework, who correspond to firms with high and low credit ratings. It is a stylized fact that risky firms rely more on bank-financing than safe firms. With both types of firm in the framework, we can clearly observe how bank regulation affects the real sector or which part of the real sector. Lastly, the framework has an explicit banking sector. Elenev et al. (2016) also propose a framework with a financially constrained banking sector. However, firms cannot directly access bond-financing in their framework.

In this paper, we consider a type of bank regulation: reserve requirement. Assuming its low return, the reserve requirement essentially increases the funding cost for banks. Our framework highlights that when the government tightens the reserve requirement, banks transfer almost all the increase in the funding cost it faces to their loan borrowers. As a consequence, risky firms who are main loan borrowers switch to more costly bond-financing. Nevertheless, risky firms' overall leverage still falls. Our model shows that tightening reserve requirement benefits the entire economy in terms of mitigating the risk and significance of asset fire-sales and thus securing financial stability.

The influence of tightening reserve requirement on the real side of the economy is mixed. First of all, the average productivity of the economy falls as risky firms managed by productive experts find it hard to raise external funds from banks. Second, the impact of reserve requirement on economic growth is unclear. On the one hand, when risky firms use more bond-financing costly liquidation of assets increases, which undermines the accumulation of assets in the economy. On the other hand, as risky firms downsize safe firms hold more assets of the economy and less costly liquidation occurs in the economy. It is unclear to tell which of the two effects dominate.

A surprising consequence of tightening bank regulation is that banks' leverage actually increases when facing tight reserve requirement. The underlying reason is the following. Considering that banks can transfer most of the increase in its funding cost to its borrowers, the decreased risk of asset fire-sale and thus endogenous risk increases their incentives to take high leverage. This result highlights that lowering banks' leverage should not be the only target of bank regulation at least from the macro-prudential perspective.

The structure of the rest of the paper follows. Section 2 describes the set-up of the model and defines the equilibrium. In Section 3, we characterize the optimal choice of individual agents and the Markov equilibrium that this paper focuses on. Section 4 illustrates key properties of the Markov equilibrium with numerical examples. In Section 5, we explore macroeconomic implication of bank regulation in our framework. Lastly, Section 6 concludes the paper.

2 Model

In this section, we build a macro-finance model, in which firms can either directly issue corporate debt or raise credit via financial intermediaries. The economy is infinite-horizon, continuous-time, and has two types of goods: perishable final goods and durable physical capital goods. Final goods serve as the numéraire.

Three groups of agents populate in the economy: experts, bankers, and households. All agents have the same logarithmic preferences and time discount factor ρ. No of them accepts negative consumption. Bankers specialize in financial intermediation; both experts and households hold physical capital goods and produce final goods.

2.1 Technology

In each period, an expert can produce ak_t units of final goods with k_t units of physical capital. Households, who are less productive, also have a linear production function $y_t = a_h k_t$, where $a_h < a$. Both experts and households can convert $\iota_t k_t$ units of final goods into $k_t \Phi(\iota_t)$ units of physical capital, where

$$\Phi(\iota_t) = \frac{\log(\iota_t \phi + 1)}{\phi}.$$

Thus, there is technological illiquidity on the production side. Physical capital in the possession of experts depreciates at rate δ and, in normal households' hands, physical capital depreciates at rate δ_h.

Exogenous *aggregate* shocks are driven by a standard Brownian motion $\{Z_t, t \geq 0\}$. In the absence of any idiosyncratic shock, physical capital managed by an expert evolves according to

$$\mathrm{d}k_t = (\Phi(\iota_t) - \delta)k_t \mathrm{d}t + \sigma k_t \mathrm{d}Z_t. \tag{1}$$

Similarly, physical capital managed by normal households follows

$$\mathrm{d}k_t = (\Phi(\iota_t) - \delta_h)k_t \mathrm{d}t + \sigma k_t \mathrm{d}Z_t.$$

In the beginning of each period, an expert becomes a *safe* one with probability α or a *risky* one with probability $1 - \alpha$. Whether an expert becomes risky within a period is independent across the time. Within a period, an adverse public signal may occur to a risky firm (a firm that a risky expert manages) at probability λ after the firm has made its investment, production, and financing decisions. The adverse signal implies that the quality of a firm's assets is under question and the firm owner can take advantage of its creditors because they have less inside information. Naturally, risky experts establish an infinitely number of firms to diversify this idiosyncratic risk. Safe firms do not experience such adverse signals.

2.2 Corporate Debt, Bank Loan, and Liquidation

A firm can raise credit either from issuing corporate debt or from a bank. In addition, we assume that no firm can issue outside equity and that all firms have limited liability.

Both corporate bonds and bank loans are collateralized contingent debt. Collateralized borrowing implies that if a firm raises L units of capital from creditors it must put down physical capital worth of L as collateral. If an adverse signal occurs to a risky firm at the interim stage, we assume that creditors of the firm always find it optimal to seize the collateral and liquidate physical capital.[1]

Bondholders are assumed to be less efficient than banks in terms of liquidating physical capital. This is because it is harder and more time-consuming to achieve a collective decision for a number of bondholders during the liquidation process than it is for a single bank. In particular, we assume that the depreciation rate of physical capital rises to $\kappa^d + \delta$ if bondholders seize the collateral and the depreciation rate becomes $\kappa + \delta$ if banks liquidate the collateral, where $\kappa < \kappa^d$.

For simplicity, we assume that there is a passive mutual fund that serves the intermediary in the corporate bond market. The fund charges its borrowers the risk-free rate plus the expected loss due to costly liquidation and promises the risk-free rate r_t to its investors. Any loss or profit that the mutual fund has is shared by all households (including experts) in proportion to their net worth. Thus, the unit cost of bond-financing is $r_t + \lambda\kappa^d$ for a risky firm.

Similar to the mutual fund, banks raise funds from households and promise the risk-free rate r_t. Unlike the passive mutual fund, bank lending involves a intermediation cost τ for each dollar lent to a firm. In addition, banks will ask for a risk premium because their equity capital is exposed to the aggregate risk. Overall, the unit borrowing cost for bank-financing

[1] The micro-foundation for creditors' optimal decision is the following. Given the adverse signal, the quality of collateral is questionable and becomes unclear. As a result, it becomes easier for the firm owner to steal the collateral and nothing could be left to creditors. Therefore, the optimal decision for creditors is to seize the collateral given the negative signal.

is $r_t^\lambda + \lambda\kappa$, where effects of both the intermediation cost and the risk premium are factored into r_t^λ.

No liquidation is involved if a firm is self-financed.

2.3 An Expert's Problem

We conjecture that the equilibrium price of physical capital follows

$$\mathrm{d}q_t = \mu_t^q q_t \mathrm{d}t + \sigma_t^q q_t \mathrm{d}Z_t,$$

then the rate of return from holding physical capital for an expert in the absence of any shock is

$$R_t \mathrm{d}t \equiv \Big(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\Big)\mathrm{d}t.$$

Since costly liquidation does not happen to a safe expert, she raises external funds only through bond-financing and thus her dynamic budget constraint is

$$\frac{\mathrm{d}w_t}{w_t} = R_t \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t + b_t^0(R_t \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t - r_t \mathrm{d}t) - \frac{c_t}{w_t}\mathrm{d}t, \tag{2}$$

where b_t^0 is the bond-to-equity ratio. Without loss of generality, we drop the loss or benefit that the expert takes from the mutual fund.

A risky expert will choose the financing method for his firms: corporate debt, bank loans, or self-financing. Since all of the expert's firms are identical prior to the realization of the liquidity shock, financing decisions of all firms managed by the expert are the same. Thus, the debt-to-equity ratio of these firms is also the same, which is exactly the expert's debt-to-net-worth ratio. The law of motion for the risky expert's net worth is

$$\begin{aligned}\frac{\mathrm{d}w_t}{w_t} = {} & R_t \mathrm{d}t + (\sigma + \sigma_t^q)\mathrm{d}Z_t + b_t^\lambda\Big(\big(R_t - \lambda\kappa^d - r_t\big)\mathrm{d}t + \big(1 - \lambda\big)(\sigma + \sigma_t^q)\mathrm{d}Z_t\Big) \\ & + l_t\Big(\big(R_t - \lambda\kappa - r_t^\lambda\big)\mathrm{d}t + \big(1 - \lambda\big)(\sigma + \sigma_t^q)\mathrm{d}Z_t\Big) - \frac{c_t}{w_t}\mathrm{d}t\end{aligned} \tag{3}$$

where b_t^λ is firms' bond-to-equity ratio and l_t firm's loan-to-equity ratio. By the Law of Large Numbers, the adverse signal at the interim stage implies that creditors seizes λ proportion of the expert's physical capital. As a result, the risky expert partially unloads his exposure to the aggregate risk, $\lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t$.

Taking $\{q_t, r_t, r_t^\lambda, t \geq 0\}$ as given, an expert chooses $\{c_t, b_t^0, b_t^\lambda, l_t, t \geq 0\}$ to maximize her life-time expected utility

$$E_0\left[\int_0^\infty e^{-\rho t}\ln(c_t)\,\mathrm{d}t\right], \tag{4}$$

given that his net worth evolves in each period according to either equation (2) or (3) depending on her type in a period.

2.4 A Banker's Problem

A banker's net worth n_t evolves according to

$$\frac{\mathrm{d}n_t}{n_t} = x_t\big(r_t^\lambda \mathrm{d}t + \lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t\big) + (1-x_t)(r_t + \tau\mathbf{1}_{x_t>1})\mathrm{d}t - \frac{c_t}{n_t}\mathrm{d}t \tag{5}$$

,where x_t denotes the loan-to-equity ratio for the bank. This ratio is greater than 1 when the bank intermediates funds from households to experts. Hence, when $x_t > 1$, the bank absorbs deposits and the intermediation cost applies; when $x_t \leq 1$, the bank saves some of its equity capital in the mutual fund. The banker is exposed to the aggregate risk $x_t^\lambda\lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t$ because she takes over and resell the physical capital that backs her lending. Taking $\{q_t, r_t, r_t^\lambda, t \geq 0\}$ as given, a banker chooses $\{c_t, x_t^\lambda, t \geq 0\}$ to maximize her life-time expected utility

$$E_0\left[\int_0^\infty e^{-\rho t}\ln(c_t)\right] \tag{6}$$

subject to the dynamic budget constraint (5).

2.5 A Household's Problem

The rate of return from holding physical capital for a household in the absence of any shock is

$$R_t^h \mathrm{d}t \equiv \Big(\frac{a_h-\iota_t}{q_t} + \Phi(\iota_t) - \delta_h + \mu_t^q + \sigma\sigma_t^q\Big)\mathrm{d}t.$$

, which is similar to the corresponding term for experts R_t. The law of motion for a household's net worth w_t^h is

$$\frac{\mathrm{d}w_t^h}{w_t^h} = x_t^h(R_t^h \mathrm{d}t + (\sigma+\sigma_t^q)\mathrm{d}Z_t) + (1-x_t^h)r_t\mathrm{d}t - \frac{c_t}{w_t^h}\mathrm{d}t, \tag{7}$$

, where x_t^h is the portfolio weight of physical capital. Taking $\{q_t, r_t, t \geq 0\}$ as given, a households maximize his life-time expected utility

$$E_0\left[\int_0^\infty e^{-\rho t}\ln(c_t)\right] \tag{8}$$

by choosing $\{c_t, x_t^h, t \geq 0\}$ that satisfy the dynamic budget constraint (7).

2.6 Equilibrium

The aggregate shock $\{Z_t, t \geq 0\}$ drives the evolution of the economy. $\mathbf{I} = [0, 1)$ denotes the set of experts, $\mathbf{J} = [1, 2)$ the set of bankers, and $\mathbf{H} = [2, 3]$ the set of households. Given the idiosyncratic shock in period t, $\mathbf{I}_t^s$ is the set of safe experts in period t and $\mathbf{I}_t^r$ the set of risky experts.

Definition 1 *Given the initial endowments of physical capital $\left\{k_0^i, k_0^j, k_0^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}$ to experts, bankers, and households such that*

$$\int_0^1 k_0^i \mathrm{d}i + \int_1^2 k_0^j \mathrm{d}j + \int_2^3 k_0^h \mathrm{d}h = K_0,$$

an equilibrium is defined by a set of stochastic processes adapted to the filtration generated by $\{Z_t\}_{t=0}^\infty$: the price of physical capital $\{q_t\}_{t=0}^\infty$, risk-free rate $\{r_t\}_{t=0}^\infty$, the interest rate of bank loan $\left\{r_t^\lambda\right\}_{t=0}^\infty$, wealth $\left\{W_t^i, N_t^j, W_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^\infty$, investment decisions $\left\{\iota_t^i, \iota_t^h, i \in \mathbf{I}, h \in \mathbf{H}\right\}_{t=0}^\infty$, asset holding decisions $\left\{x_t^h, h \in \mathbf{I}_t^h\right\}_{t=0}^\infty$ of households, corporate debt financing decisions $\left\{b_t^{i,0}, b_t^{i,\lambda}, i \in \mathbf{I}_t\right\}_{t=0}^\infty$ of experts, bank financing decisions $\{l_t^i, i \in \mathbf{I}_t^r\}_{t=0}^\infty$ of risky experts, bank lending $\left\{x_t^{\lambda,j}, j \in \mathbf{J}\right\}_{t=0}^\infty$ and consumption $\left\{c_t^i, c_t^j, c_t^h, i \in \mathbf{I}, j \in \mathbf{J}, h \in \mathbf{H}\right\}_{t=0}^\infty$; such that

1. *$W_0^i = k_0^i q_0$, $N_0^j = k_0^j q_0$, and $W_0^h = k_0^h q_0$ for $i \in \mathbf{I}$, $j \in \mathbf{J}$, and $h \in \mathbf{H}$;*
2. *each expert, banker, and household solve for their problems given prices;*
3. *markets for final goods and physical capital clear, that is,*

$$\begin{aligned}\int_0^3 c_t^i \mathrm{d}i \quad + \quad & \frac{1}{q_t}\int_1^2 \tau n_t^j\big(x^{\lambda,j} - 1\big)\mathbf{1}_{x^{\lambda,j}>1}\mathrm{d}j = \frac{1}{q_t}\int_2^3 (a^h - \iota_t^h) w_t^h x_t^h \mathrm{d}h + \\ & \frac{1}{q_t}\int_{i\in\mathbf{I}_t^s}\left(a - \iota_t^i\right) w_t^i(1 + b_t^{i,0})\mathrm{d}i + \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r}\left(a - \iota_t^i\right) w_t^i(1 + b_t^{i,\lambda} + l_t^i)\mathrm{d}i\end{aligned}$$

for the market of final goods, and

$$\frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} w_t^i(1+b_t^{i,0})\mathrm{d}i + \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} w_t^i(1+b_t^{i,\lambda}+l_t^i)\mathrm{d}i + \frac{1}{q_t}\int_2^3 w_t^h x_t^h \mathrm{d}h = K_t$$

for the market of physical capital goods, where K_t evolves according to

$$\begin{aligned}\frac{\mathrm{d}K_t}{\mathrm{d}t} &= \frac{1}{q_t}\int_2^3 \left(\Phi(\iota_t^h)-\delta^h\right) w_t^h x_t^h \mathrm{d}h + \frac{1}{q_t}\int_{i\in\mathbf{I}_t^s} \left(\Phi(\iota_t^i)-\delta\right) w_t^i(1+b_t^{i,0})\mathrm{d}i \\ &+ \frac{1}{q_t}\int_{i\in\mathbf{I}_t^r} \left(\Phi(\iota_t^i)-\delta\right) w_t^i(1+b_t^{i,\lambda}+l_t^i) - \lambda\kappa^d w_t^i b_t^i - \lambda\kappa w_t^i l_t^i \,\mathrm{d}i\end{aligned}$$

4. the bank loan market clears

$$\int_{i\in\mathbf{I}_t^r} w_t^i l_t^i \mathrm{d}i = \int_1^2 n_t^j x_t^{\lambda,j}\mathrm{d}j.$$

The credit market for corporate debt clears automatically by Walras' Law.

3 Solving for the Equilibrium

Both experts' net worth and bank capital are important for the equilibrium. We expect that the price of physical capital declines as the share of both experts' net worth bank capital shrinks due to adverse exogenous shocks.

To solve for the equilibrium, we first derive first-order conditions with respect to optimal decisions of experts, bankers, and households; secondly, we solve for the law of motion for endogenous state variables, wealth shares of different groups of agents, based on market clearing conditions as well as first-order conditions; lastly, we use first-order conditions and state variables' law of motion to define partial differential equations that endogenous variables such as the price of physical capital satisfy.

3.1 Households' Optimal Choices

Households have logarithmic preferences. In the following discussion, we will take advantage of two well-known properties with respect to logarithmic preferences in the continuous-time setting: 1) a household's consumption c_t is ρ proportion of her wealth w_t^h in the same period, i.e.,

$$c_t = \rho w_t^h; \tag{9}$$

2) a household's portfolio weight on a risky investment is such that the Sharpe ratio of the risky investment equals the percentage volatility of her wealth.

A household's investment rate ι_t always maximizes $\Phi(\iota_t) - \iota_t/q_t$. The first-order condition implies that

$$\Phi'(\iota_t) = \frac{1}{q_t}, \tag{10}$$

which defines the optimal investment as a function of the price of physical capital $\iota(q_i)$.

Given the second property discussed above, it is straightforward to derive a household's optimal portfolio weight on the physical capital x_t^h, which satisfies [2]

$$x_t^h = \frac{\max\{R_t^h - r_t, 0\}}{(\sigma + \sigma_t^q)^2}. \tag{11}$$

3.2 Experts' Portfolio Choices

According to the second property highlighted above, it is straightforward to characterize a safe expert's optimal bond-to-equity ratio[3]

$$b_t^0 = \frac{\max\{R_t - r_t - (\sigma + \sigma_t^q)^2, 0\}}{(\sigma + \sigma_t^q)^2}. \tag{12}$$

For a risky expert, both bond-to-equity ratio b_t^λ and loan-to-equity ratio l_t affect the percentage volatility of her wealth $(1 + (1-\lambda)b_t^\lambda + (1-\lambda)l_t)(\sigma + \sigma_t^q)$. Hence, optimal b_t^λ and l_t must satisfy

$$\begin{aligned}
\frac{R - \lambda\kappa^d - r_t}{(1-\lambda)(\sigma + \sigma_t^q)} &= (1 + (1-\lambda)b_t^\lambda + (1-\lambda)l_t)(\sigma + \sigma_t^q) \\
\frac{R - \lambda\kappa - r_t^\lambda}{(1-\lambda)(\sigma + \sigma_t^q)} &= (1 + (1-\lambda)b_t^\lambda + (1-\lambda)l_t)(\sigma + \sigma_t^q),
\end{aligned}$$

if $b_t^\lambda > 0$ and $l_t > 0$. Nevertheless, if the spread between the loan rate and the risk-free rate $r_t^\lambda - r_t$ is sufficiently small, it is possible that bank-financing strictly dominates bond-financing since $\kappa^d > \kappa$ and thus $b_t^\lambda = 0$. Therefore, first-order conditions for optimal b_t^λ and l_t are

[2]Given that $R_t^h > r_t$, Sharpe ratio is $(R_t^h - r_t)/(\sigma + \sigma_t^q)$. The percentage volatility of the household's wealth is $x_t^h(\sigma + \sigma_t^q)$. Hence the optimal x_t^h is such that $x_t^h(\sigma + \sigma_t^q) = (R_t^h - r_t)/(\sigma + \sigma_t^q)$.

[3]In this case, Sharpe ratio is $(R_t - r_t)/(\sigma + \sigma_t^q)$. The percentage volatility of the safe expert's wealth is $(1 + b_t^0)(\sigma + \sigma_t^q)$.

$$\frac{R-\lambda\kappa^d-r_t}{(1-\lambda)(\sigma+\sigma_t^q)} \leq (1+(1-\lambda)b_t^\lambda+(1-\lambda)l_t)(\sigma+\sigma_t^q), \text{with equality if } b_t^\lambda>0; \quad (13)$$

$$\frac{R-\lambda\kappa-r_t^\lambda}{(1-\lambda)(\sigma+\sigma_t^q)} \leq (1+(1-\lambda)b_t^\lambda+(1-\lambda)l_t)(\sigma+\sigma_t^q), \text{with equality if } l_t>0. \quad (14)$$

When the cost of bond-financing equals the cost of bank-financing, i.e., $\lambda\kappa^d+r_t=\lambda\kappa+r_t^\lambda$, individual risky experts are indifferent between bond-financing and bank-financing and their portfolio choices are indeterminate. Without loss of generality, we assume that portfolio weights of both bond-financing and bank-financing, b_t^λ and l_t, are the same across all risky experts.

3.3 Banker's Optimal Choices

A banker's optimal portfolio weight on loans satisfies

$$x_t=\frac{r_t^\lambda-\tau\mathbf{1}_{x_t>1}-r_t}{\lambda^2(\sigma+\sigma_t^q)^2}. \quad (15)$$

Loan rate r_t^λ relies on the intermediation cost τ, banks' exposure to aggregate risk $\lambda(\sigma+\sigma_t^q)$, and banks' leverage x_t. The financing cost of bank loans for firms fluctuates endogenously not just because the price volatility of physical capital changes over time but also because banks' leverage varies across business cycles.

3.4 Market Clearing

Let W_t denote the total wealth that experts have in period t and N_t the total bank capital. Hence, the total bank loans issued in equilibrium denoted by x_tN_t satisfies

$$x_tN_t=(1-\alpha)W_tl_t. \quad (16)$$

The demand for final goods consists of consumption, intermediation costs, and investments. The aggregate consumption of households is ρq_tK_t. The total intermediation cost is $\tau x_tN_t\mathbf{1}_{x_t>1}$. Therefore, the market clearing condition with respect to final goods is

$$\begin{aligned}\rho q_tK_t+\tau N_t(x_t-1)\mathbf{1}_{x_t>1} &= \frac{q_tK_t-W_t-N_t}{q_t}(a_h-\iota_t)x_t^h\\ &+\alpha\frac{W_t}{q_t}(a-\iota_t)(1+b_t^0)+(1-\alpha)\frac{W_t}{q_t}(a-\iota_t)(1+b_t^\lambda+l_t) \quad (17)\end{aligned}$$

Finally, the market for physical capital clears if

$$\frac{q_t K_t - W_t - N_t}{q_t} x_t^h + \alpha \frac{W_t}{q_t}(1 + b_t^0) + (1 - \alpha)\frac{W_t}{q_t}(1 + b_t^\lambda + l_t) = K_t. \tag{18}$$

3.5 Wealth Distribution

Wealth shares of both experts and bankers matter for the equilibrium. Two endogenous state variables that characterize the dynamics of the economy are experts' wealth share $\omega_t = {}^{W_t}/_{(q_t K_t)}$ and bankers' wealth share $\eta_t = {}^{N_t}/_{(q_t K_t)}$.

The decline of experts' wealth share naturally leads to the fall of average productivity since financial markets are imperfect and households are less productive. If bankers' wealth share declines, then the supply of bank loans shrinks and the interest rate on bank loans rises, which, in turn, also lowers the aggregate productivity of the economy the increased cost of raising external finance for experts.

Given dynamic budget constraints of individual experts and bankers, it is straightforward to derive laws of motion for both W_t and N_t

$$\begin{aligned}
\frac{\mathrm{d}W_t}{W_t} &= \Big(R_t + \alpha b_t^0 (R_t - r_t) + (1-\alpha) b_t^\lambda (R_t - \lambda\kappa^d - r_t) + (1-\alpha) l_t (R_t - \lambda\kappa - r_t^\lambda) - \frac{c_t}{W_t}\Big)\mathrm{d}t \\
&\quad + \Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda)\Big)(\sigma + \sigma_t^q)\mathrm{d}Z_t \qquad (19) \\
\frac{\mathrm{d}N_t}{N_t} &= \Big(x_t r_t^\lambda + (1 - x_t)(r_t + \tau \mathbf{1}_{x_t > 1}) - \frac{c_t}{N_t}\Big)\mathrm{d}t + x_t \lambda (\sigma + \sigma_t^q)\mathrm{d}Z_t \qquad (20)
\end{aligned}$$

Dynamics of state variables in equilibrium also depend on the law of motion of the aggregate physical capital, which is

$$\begin{aligned}
&\frac{\mathrm{d}K_t}{K_t} = \mu_t^K \mathrm{d}t + \sigma \mathrm{d}Z_t, \text{where} \qquad (21) \\
&\mu_t^K \equiv \Phi(\iota_t) - \delta - (1 - \omega_t - \eta_t) x_t (\delta - \delta^h) - (1-\alpha)\omega_t \lambda (b_t^\lambda \kappa^d + l_t \kappa).
\end{aligned}$$

Given laws of motion of W_t, N_t, q_t, and K_t, we can apply Ito's Lemma to derive laws of motion for ω_t and η_t in equilibrium, which are summarized in the following lemma.

Lemma 1 *In equilibrium, experts' wealth share ω_t evolves according to*

$$\frac{\mathrm{d}\omega_t}{\omega_t} = \mu_t^\omega \mathrm{d}t + \sigma_t^\omega \mathrm{d}Z_t, \tag{22}$$

, where

$$\mu_t^\omega = R_t - \mu_t^q - \mu_t^K - \sigma\sigma_t^q + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t - \lambda\kappa^d - r_t^\lambda) + (1-\alpha)l_t(R_t - \lambda\kappa - r_t^\lambda)$$
$$- \big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda)\big)(\sigma + \sigma_t^q)^2 - \rho$$
$$\sigma_t^\omega = \big(\alpha b_t^0 + (1-\alpha)b_t^\lambda(1-\lambda) + (1-\alpha)l_t(1-\lambda)\big)(\sigma + \sigma_t^q).$$

And, the state variable η_t evolves according to

$$\frac{\mathrm{d}\eta_t}{\eta} = \mu_t^\eta \mathrm{d}t + \sigma_t^\eta \mathrm{d}Z_t, \tag{23}$$

where

$$\mu_t^\eta = \lambda x_t(\lambda x_t - 1)(\sigma + \sigma_t^q)^2 + r_t + \tau \mathbf{1}_{x_t > 1} - \mu_t^q - \mu_t^K - \sigma\sigma_t^q + (\sigma + \sigma^q)^2 - \rho$$
$$\sigma_t^\eta = (\lambda x_t - 1)(\sigma + \sigma_t^q)$$

The proof of Lemma 1 is in appendix.

3.6 Markov Equilibrium

Like other continuous-time macro-finance models (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2012), our framework also has the property of scale-invariance with respect to total physical capital K_t. Thus, we will focus on the equilibrium that is Markov in state variables ω_t and η_t. In the Markov equilibrium, dynamics of endogenous variables such as q_t can be characterized by laws of motion of ω_t and η_t and functions $q(\omega, \eta)$.

To solve for full dynamics of the economy, we derive a partial differential equations with respect to $q(\omega, \eta)$. The partial differential equation as well as its boundary conditions originates from equilibrium conditions and Ito's formula with $q(\omega, \eta)$. Ito's lemma with respect to the volatility of the price of physical capital implies that

$$q_t\sigma_t^q = q_\omega(\omega_t, \eta_t)\omega_t\sigma_t^\omega + q_\eta(\omega_t, \eta_t)\eta_t\sigma_t^\eta. \tag{24}$$

Given (q, ω, η), we can solve the equilibrium and derive all endogenous choice variables $(c, b^0, b^\lambda, l, x, x^h)$ and endogenous price variables $(r, r^\lambda, \mu^q, \sigma^q)$.[4] Therefore, volatility terms of two state variables $(\sigma^\eta, \sigma^\omega)$ are also known. Hence, equation (24) is a well-defined partial differential equation with respect to $q(\omega, \eta)$.

[4] At this stage given only (q, ω, η), we can only solve for $r - \mu^q$. However, it does not cause any problem for solving for $q(\omega, \eta)$.

In addition to the differential equation, we need boundary conditions to solve for $q(\omega, \eta)$. There are three boundary conditions that correspond to three boundaries for the domain of $q(\omega, \eta)$: $\{(\omega, \eta) : \omega = 0, 0 \leq \eta \leq 1\}$, $\{(\omega, \eta) : 0 \leq \omega \leq 1, \eta = 0\}$, and $\{(\omega, \eta) : 0 \leq \omega \leq 1, 0 \leq \eta \leq 1, \omega + \eta = 1\}$. When $\eta = 0$, no intermediary sector exists in the economy and differential equation (24) reduces to an ordinary differential equation with respect to $q(\omega, 0)$. It is straightforward to numerically solve for $q(\omega, 0)$. When $\omega = 0$, only households can hold physical capital in the economy and bankers will lend their equity capital to households without any intermediation. Thus, market clearing conditions for final goods and physical capital goods (17) and (18) imply that $q(0, \eta) = q_{min}$, where q_{min} satisfies

$$\rho q_{min} = a_h - \iota\left(q_{min}\right).$$

When $\omega + \eta = 1$, only experts hold physical capital and no intermediation is involved either. Hence, $q(\omega, 1 - \omega) = q_{max}$, where q_{max} satisfies

$$\rho q_{max} = a - \iota\left(q_{max}\right).$$

4 Results

In this section, we discuss main results of the model with numerical examples. The choice of parameter values is $\rho = 3\%$, $a = 0.275$, $a^h = 0$, $\delta = -0.05$, $\delta^h = 0$, $\phi = 5$, $\tau = 0.5\%$, $\alpha = 0.2$, $\lambda = 0.5$, $\kappa^d = 0.4$, $\kappa = 0.2$, and $\sigma = 0.1$.

4.1 Price and the Misallocation of Physical Capital

The misallocation of physical capital exists because productive experts cannot issue outside equity and the use of leverage expose them to the risk that their net worth could be completely wiped out. Therefore, when experts' wealth share is arbitrarily close to zero experts only hold a small fraction of physical capital in the economy (Figure 2) and the price of physical capital converges to its lower bound $q_{min} = 0.8696$ (Figure 1). Given the same level of experts' wealth share, the price of physical capital declines as bankers' wealth share drops (Figure 1). This is because the supply of bank loans becomes smaller if the banking sector is less capitalized. Hence, risky experts find it harder to raise external capital and the misallocation of physical capital becomes more severe (Figure 2).

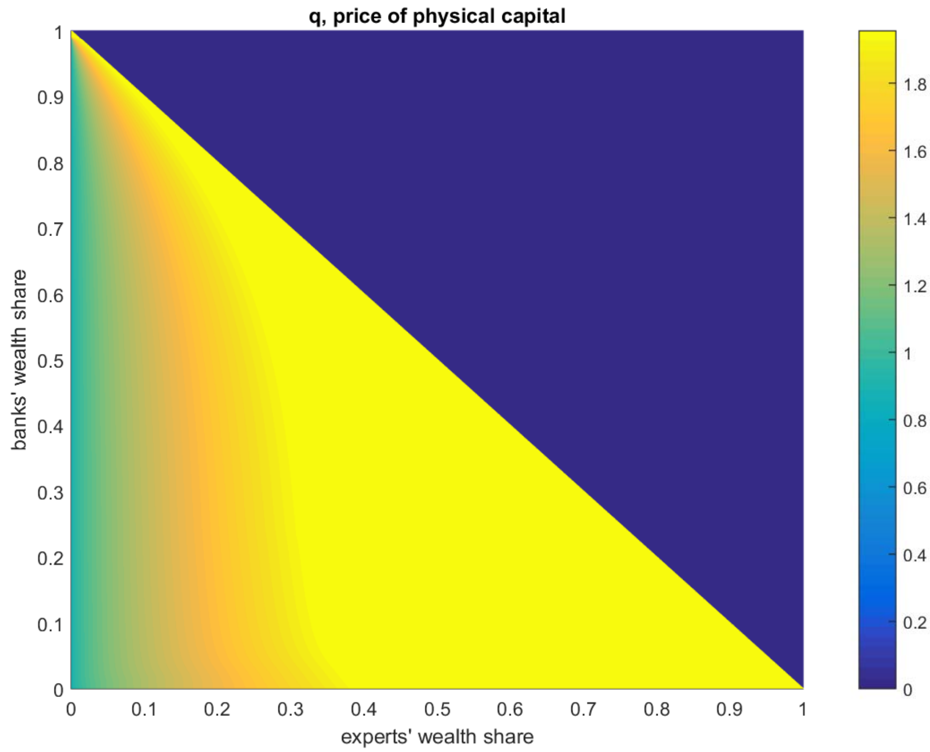

Figure 1: The price of physical capital. For parameter values, see the beginning of Section 4.

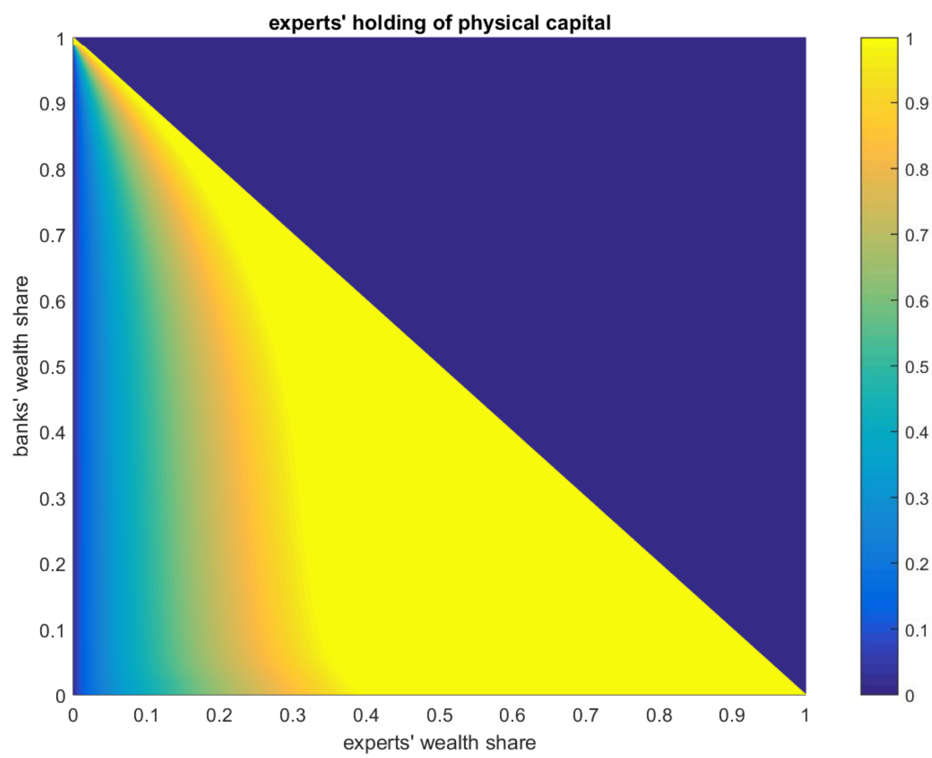

Figure 2: The fraction of physical capital that experts hold. For parameter values, see the beginning of Section 4.

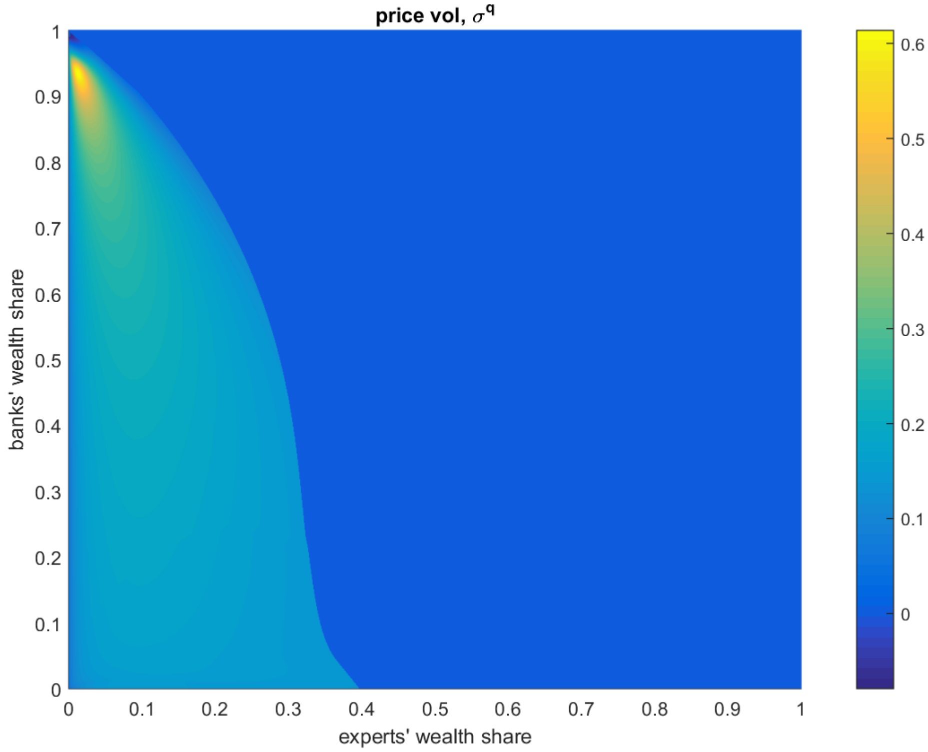

Figure 3: The volatility of the price of physical capital. For parameter values, see the beginning of Section 4.

4.2 Endogenous Risk and Amplification Mechanism

The exogenous Brownian shocks hit both experts' net worth and bank capital in the economy. The impact of the exogenous shock is amplified through the following two inter-connected vicious spirals. The decline in experts' net worth lowers their holdings of physical capital, which depresses its price and hurts experts' net worth. In addition, the decline in bank capital raises the cost of obtaining bank loans and deter risky firms from raising external funds. And, this also lowers the aggregate productivity and pushes down the price of physical capital, which in turn impairs net worth of both expert and banker sectors further.

To clearly illustration the amplification mechanism, we rewrite equation (24)

$$q\sigma^q = \frac{q_\omega \omega(\alpha b^0 + (1-\alpha)b^\lambda(1-\lambda) + (1-\alpha)l(1-\lambda)) + q_\eta \eta(\lambda x - 1)}{1 - \frac{q_\omega}{q}\omega(\alpha b^0 + (1-\alpha)b^\lambda(1-\lambda) - (1-\alpha)l(1-\lambda)) - \frac{q_\eta}{q}\eta(\lambda x - 1)}\sigma.$$

We can see that the magnitude of endogenous risk depends on i) the sensitivity of the price of physical capital to the change of wealth shares of experts and bankers, q_ω and q_η; ii) the exposure of their wealth shares to the aggregate risk.

Figure 3 indicates that endogenous risk is low when experts hold all physical capital in

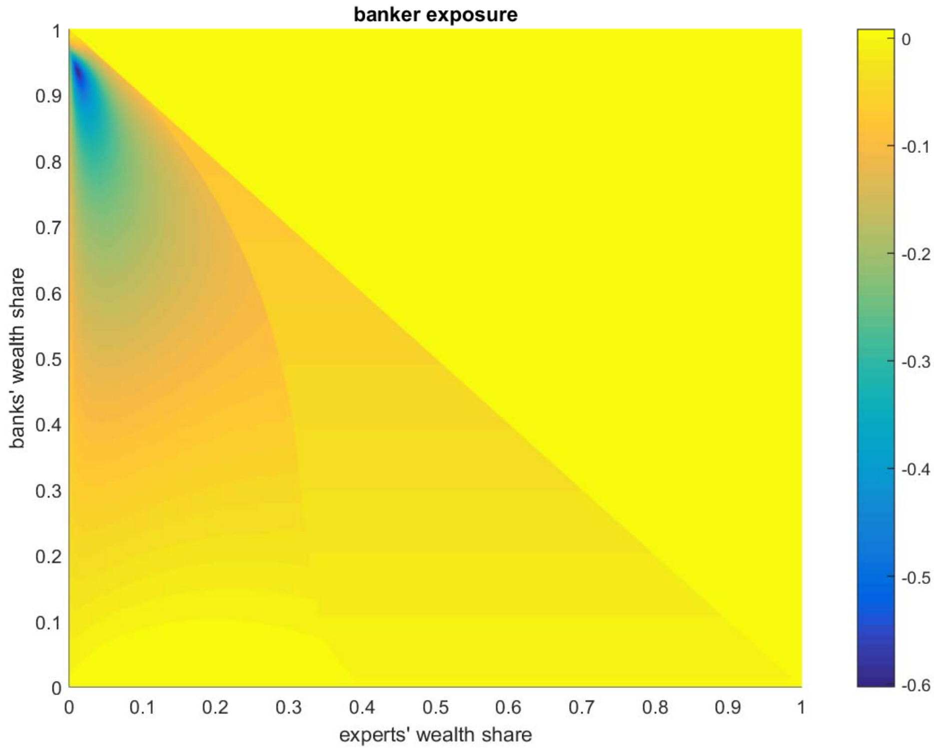

Figure 4: Bankers' exposure to the aggregate risk. For parameter values, see the beginning of Section 4.

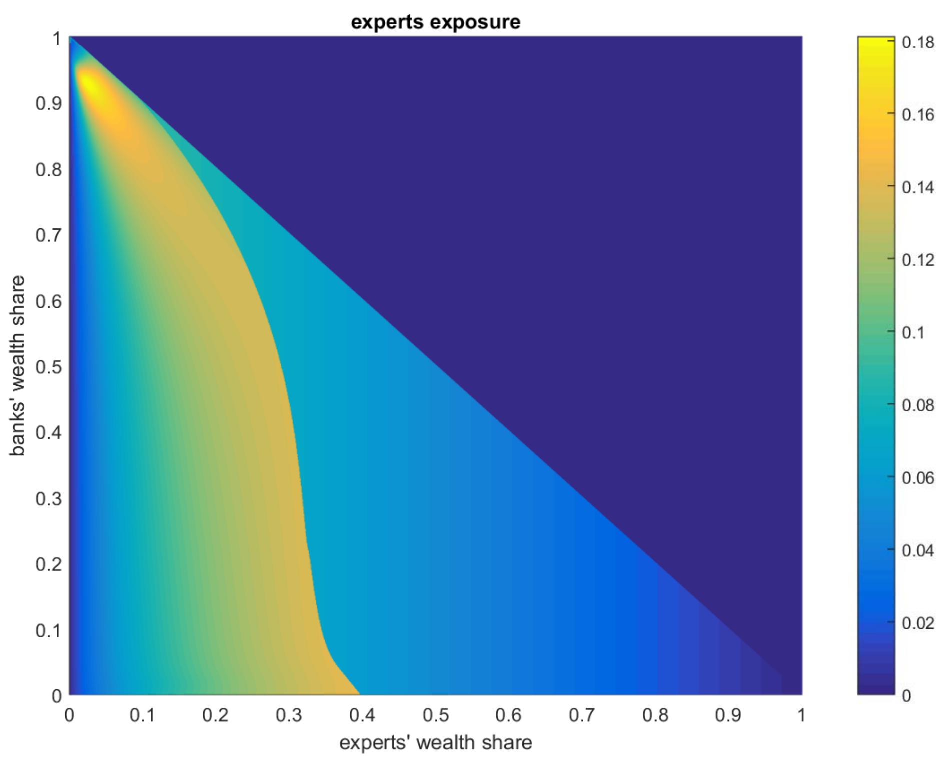

Figure 5: Experts' exposure to the aggregate risk. For parameter values, see the beginning of Section 4.

the economy (Figure 2). When asset fire sales occur, endogenous risk increases. Figure 3 also shows that the economy is extremely unstable when the total wealth of the economy is concentrated in the banking sector and the productive sector only possesses a slim fraction of total wealth in the economy (the upper left region of Figure 3). The underlying reason of this result is straightforward. Endogenous risk originates from the risk of asset fire-sales, which ultimately depends on the net worth of the productive sector (Figure 5) instead of the intermediary sector (Figure 4). Our result highlights that an overly-capitalized financial intermediary sector could be harmful for financial stability and also problematic for the entire economy.

4.3 Endogenous Fluctuation of Intermediation Costs

Costs of both bond-financing and bank-financing consist of two components: the cost of liquidation and the interest rate charged by creditors. Bank-financing dominates bond-financing in terms of the cost of liquidation, $\lambda\kappa < \lambda\kappa^d$. With respect to the interest payment, firms only pay the risk-free rate for issuing corporate debt regardless of their risks. In contrast, raising external funds from banks involves compensating banks for their exposures to both exogenous risk and endogenous risk, $x_t\lambda^2\left(\sigma+\sigma_t^q\right)^2$ as well as the unit intermediation cost τ. Recall

$$r_t^\lambda = r_t + \tau 1_{x_t>1} - x_t\lambda^2\left(\sigma+\sigma_t^q\right)^2.$$

One particular feature of bank-financing in our model is that its cost fluctuates endogenously in the dynamics of the economy. Bankers who are financial intermediaries in the economy channel funds provided by normal households to more productive experts. However, financial intermediaries cannot issue outside equity to normal households due to asymmetric information problem modelled in papers such as He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2014). As a result, bankers can only issue risk-free debt to normal households. The interest rate spread $r_t^\lambda - r_t$ that financial intermediaries earn from loans made to risky firms depends on three components: banks' leverage, x_t, the exposure to a risky firm's credit event λ, and the magnitude of endogenous risk σ_t^q. When the banking sector is well capitalized, it is relatively resilient to adverse exogenous shocks. Hence, both x_t and σ_t^q are small in economic booms, and thus risky firms find it more profitable to raise credit from banks in economic upturns. In downturns, however, when the banking sector is not financially healthy, banks become less tolerant of taking risks and endogenous risk also goes up. Overall, the rise in the cost of bank-financing in downturns squeezes risky firms to more costly bond-financing or self-financing, which of course hurts the aggregate productivity.

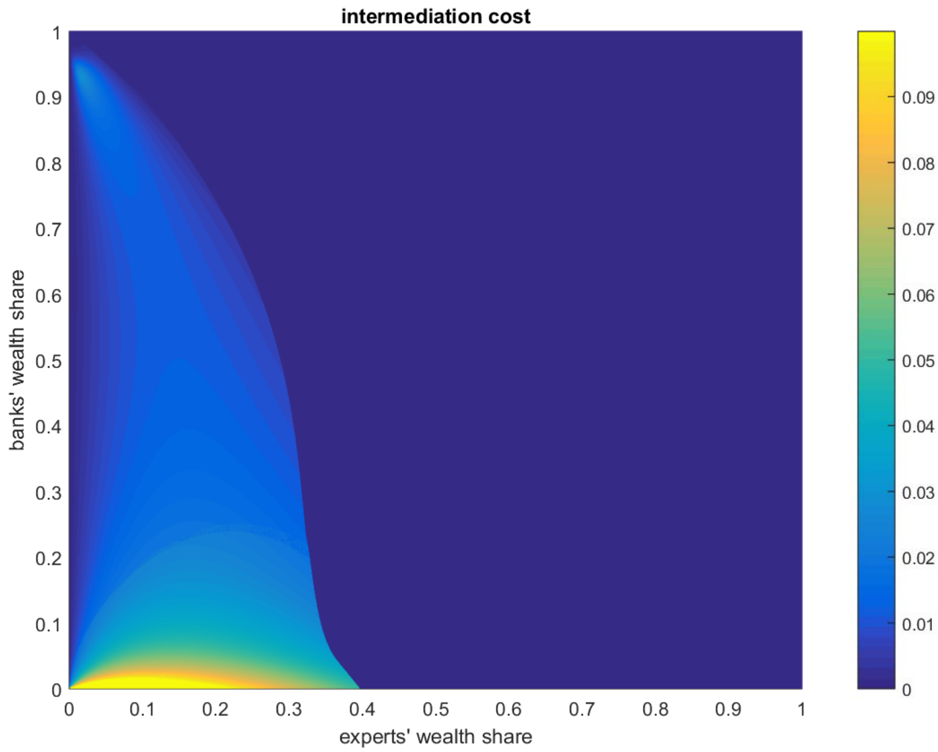

Figure 6: Intermediation cost $r^{\lambda} - r$. For parameter values, see the beginning of Section 4.

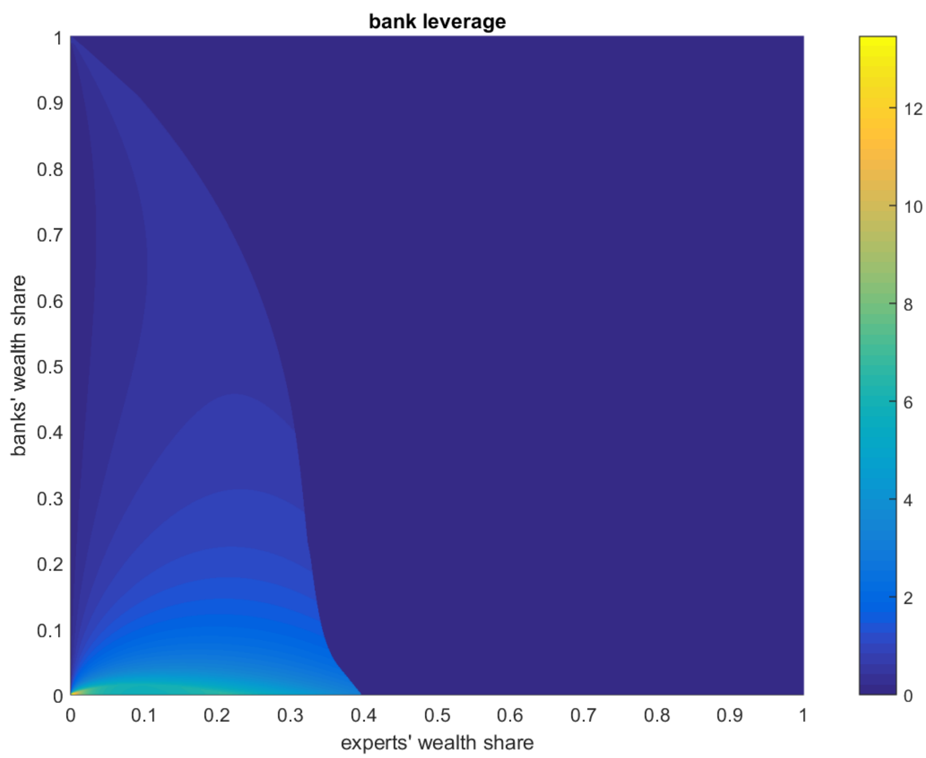

Figure 7: Bank leverage x. For parameter values, see the beginning of Section 4.

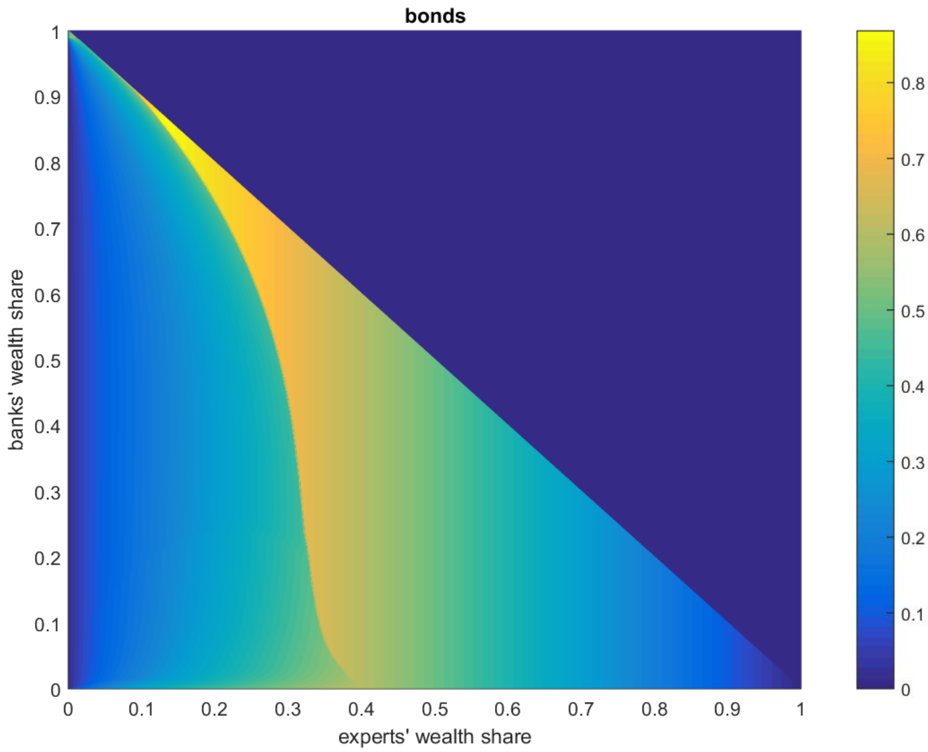

Figure 8: Outstanding corporate bonds as a fraction of total wealth in the economy. For parameter values, see the beginning of Section 4.

Figure 6 indicates that i) the intermediation cost is high when the banking sector is poorly capitalized (lower left areas of both Figure 6 and 7) and ii) when high endogenous risk also leads to elevated intermediation cost even when the banking sector is overly-capitalized (upper left area of Figure 6). In addition, Figure 7 shows that bank leverage is counter-cyclical, which is standard in the literature.

4.4 Heterogeneity of Bond-Financing and Bank-Financing

Bond-financing is acyclical in our model Figure 8. As the economy evolves into economic booms, the share of outstanding corporate debt in total wealth goes up. This is primarily the consequence of safe firms' high debt-to-equity ratio due to low endogenous risks. Our paper highlights that the credit market of direct finance can also benefit from the development of the financial intermediary sector. In economic downturns, the share of corporate debt is also high because 1) risky firms switch to bond-financing due to the rising cost of bank-financing, and 2) firms take high leverage due to high returns from holding physical capital.

In contrast, Panel b in Figure 9 shows that bank-financing is clearly pro-cyclical. This is true since bank-financing is pro-cyclical at both intensive margin and extensive margin:

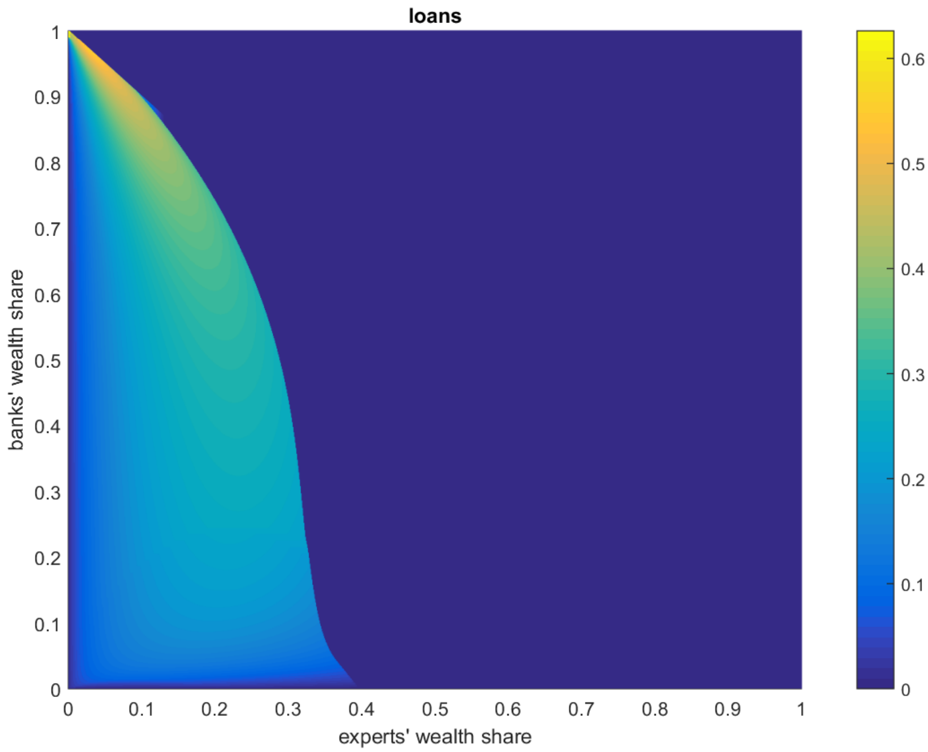

Figure 9: Outstanding bank loans as a fraction of total wealth in the economy. For parameter values, see the beginning of Section 4.

all risky firms choose bank-financing and they take high leverage in economic booms when endogenous risk is low.

The substitution of bond credit for bank credit in economic downturns has significant price effects in equilibrium. When bank loans are very expensive, risky firms have to replace bank credit with bond-financing. Noticing that bond-financing involves more costly liquidation than bank-financing does, the rising borrowing cost for firms exerts downward pressure on the price of physical capital. This explains why the magnitude of endogenous risks goes up when a large proportion of firms replacing bank credit with bond credit.

Overall, our model accounts for two facts of bond-financing and bank-financing in business cycles. The first fact is that bank-financing is more volatile and cyclical than bond-financing in the long-run as Becker and Ivashina (2014) document. The second fact, which Adrian et al. (2012) and many other papers have highlighted, is that the drastic decline in intermediated finance during big recessions such as 2007-09 financial crisis is partially made up by the increase in direct finance.

The reason why our model can capture the two facts has to do with two features of our framework: a feature on the technical side and a feature on the economics side. The technical feature is that our continuous-time frame allows for the full characterization of the dynamics of the economy. Thus, we do not only know the property of the equilibrium around the steady state but also we can precisely observe the equilibrium outcome in extreme states. Sometimes, properties of the equilibrium could be quite different in different states of the economy as we have noticed in our framework.

The other feature is that our framework highlights the dynamics of endogenous risks and these dynamics have substantial effects on the dynamics of bond-financing. In particular, as the banking sector becomes more and more financially healthy, endogenous risks becomes lower and lower, which in turn actually help firms issuing more corporate debt. This result implies the outstanding corporate debt in the economy is not monotonic in the state of the economy.

5 Bank Regulation: Price Control

5.1 Reserve Requirement

Following Drechsler et al. (2014), we associate the reserve requirement that banks face to the price control over banks' liabilities. Suppose a bank lends out deposits worth $n_t(x_t-1)$, the reserve requirement implies that the bank must hold reserve worth $\nu n_t(x_t-1)$. For simplicity, we assume that banks earn return $r_t-\tilde{r}$ from holding reserve and that the government redistributes its revenue back to banks such that the price control has no wealth effect. In particular, the bank's dynamic budget constraint becomes

$$\begin{aligned}\frac{\mathrm{d}n_t}{n_t} &= x_t\big(r_t^\lambda \mathrm{d}t+\lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t\big)+\nu(x_t-1)(r_t-\tilde{r}-r_t)\mathbf{1}_{x_t>1}\mathrm{d}t\\ &\quad+(1-x_t)(r_t+\tau)\mathbf{1}_{x_t>1}\mathrm{d}t-\frac{c_t}{n_t}\mathrm{d}t+s_t\mathrm{d}t\\ &= x_t\big(r_t^\lambda \mathrm{d}t+\lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t\big)+(1-x_t)(r_t+(\tau+\tau_0)\mathbf{1}_{x_t>1})\mathrm{d}t-\frac{c_t}{n_t}\mathrm{d}t+s_t\mathrm{d}t, \qquad (25)\\ &= x_t\big(r_t^\lambda \mathrm{d}t+\lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t\big)+(1-x_t)(r_t+\tau\mathbf{1}_{x_t>1})\mathrm{d}t-\frac{c_t}{n_t}\mathrm{d}t \qquad (26)\end{aligned}$$

where $\tau_0\equiv\nu\tilde{r}$ and s_t is the government-subsidy-to-wealth ratio for bankers. Budget constraint (26) is the same as the budget constraint in the baseline model (5), which comes from our assumption that the combined effect of the reserve requirement and the government redistribution has zero wealth effect, i.e., $\nu\tilde{r}(x_t-1)=s_t$.

The price control still has a real impact due to the increase in the marginal cost of raising external funds for banks. Given the reserve requirement, bankers' first-order condition becomes

$$x_t = \frac{r_t^\lambda - (\tau + \tau_0)\mathbf{1}_{x_t>1} - r_t}{\lambda^2(\sigma + \sigma_t^q)^2}.$$

The above equation implies that the price control could potentially affect the cost and supply of bank loans as well as endogenous risk. We next simulate our model and illustrate how price control on banks' liabilities influence the entire economy.

In our simulation, we randomize initial states of 10,000 economies and simulate for 500 years.[5] In the end, we calculate the average values of endogenous variables that we are interested in.

5.2 Price and Quantity

On the financial side, tightening the price control discourages bank-financing, and bond-financing increases to offset the loss of the credit supply. Row 1 in Table 1 indicates that the increase in banks' marginal funding cost is almost fully translated into the increase in the funding cost of loan borrowers, i.e., risky firms. The natural consequence is that risky firms switch to bond-financing (row 4 in Table 1). Nevertheless, risky firms' leverage $b_t^\lambda + l_t$ still declines as the price control tightens (row 5 in Table 1). As risky firms hold less physical capital, its price drops and the rate of return for holding physical capital increases, which explains why safe firms' leverage increases as the price control tightens (row 3 in Table 1). Overall, the share of outstanding bonds rises (row 7 in Table 1) and the share of outstanding loans declines (row 8 in Table 1) as banks' marginal funding cost increases due to the price control. As a result, the loan-to-bond rate declines (row 9 in Table 1).

Although safe firms' leverage is higher as the price control becomes tighter, it does not overturn the effect that tightening price control lowers endogenous risk and the average productivity (row 2 and 12 in Table 1). It is the declining leverage of risky firms that drives key macroeconomic variables. The decreased endogenous risk due to the price control also explains a seemly surprising result that banks' leverage increases when the regulation they faces becomes tighter (row 6 in Table 1). The reason underlying this result is that the banking sector transfers most of the increased funding cost to its borrowers, i.e., risky firms and at the same time the decreased leverage of risky firms lowers endogenous risk, which in turn helps the banking sector obtain higher leverage.

As the price control tightens, the supply of bank loans still declines even though banks' leverage increases. The key reason is that the wealth share of the banking sector declines

[5] We randomize $\{\omega_i, i = 1, 2, ..., 10000\}$ according to the uniform distribution over $(0, 1)$ and generate η_i according to the uniform distribution over $(0, 1 - \omega_i)$ for each i.

due to bank regulation. Recall that an agent's consumption is a proportion of her wealth, the decline in bankers' consumption indicates the decrease of their wealth share (row in Table 2). Bankers' wealth share declines mainly because *i*) bank regulation directly raises their funding cost and *2*) regulation indirectly lowers the risk premium compensating their exposure to aggregate risk due to the decreased endogenous risk.

Table 1: Impact of Price Control: Price and Quantity

τ_0	0	0.002	0.006	0.01	0.014	0.018	0.022	0.026	
Price									
$r_t^\lambda - r_t$	0.0580	0.0599	0.0633	0.0672	0.0710	0.0750	0.0787	0.0824	(1)
$q_t\sigma_t^q$	0.2112	0.2103	0.2082	0.2066	0.2056	0.2032	0.2015	0.2007	(2)
Quantity on the financial side									
b_t^0	3.332	3.3535	3.3958	3.4405	3.4752	3.5385	3.5758	3.5941	(3)
b_t^λ	0.0066	0.0080	0.0125	0.0198	0.0272	0.0472	0.0811	0.1443	(4)
$b_t^\lambda + l_t$	5.4813	5.408	5.2502	5.0984	4.915	4.8144	4.6253	4.3555	(5)
x_t	3.2494	3.2589	3.2626	3.2922	3.3096	3.3409	3.3558	3.3719	(6)
$\alpha\omega_t b^0 + (1-\alpha)\omega_t b^\lambda$	0.2296	0.2310	0.2356	0.2376	0.2414	0.2416	0.2457	0.2537	(7)
$(1-\alpha)\omega l_t$	0.1218	0.1194	0.1144	0.1095	0.1044	0.0989	0.0930	0.0864	(8)
loan-to-bond ratio	0.6409	0.6265	0.5950	0.5672	0.5369	0.5112	0.4782	0.4352	(9)
Quantity on the real side									
μ_t^K	0.0728	0.0728	0.0733	0.0729	0.0732	0.0721	0.0722	0.0735	(10)
TFP	0.1179	0.1176	0.1179	0.1171	0.1169	0.1151	0.1149	0.1160	(11)

[1] We randomize initial states of 10,000 economies, stimulate them for 500 years, and report mean values of above endogenous variables.
[2] $b_t^\lambda + l_t$ is risky firms' leverage.
[3] $\alpha\omega_t b^0 + (1-\alpha)\omega_t b^\lambda$ is outstanding corporate bonds as a fraction of total wealth
[4] $(1-\alpha)\omega l_t$ is outstanding bank loans as a fraction of total wealth
[5] TFP is the average productivity of the entire economy.

Our simulation result illustrates the transmission mechanism of the price-control type bank regulation. Tightening reserve requirement raises the marginal funding cost for banks, which in turn transfer this marginal cost to its loan borrowers, i.e., risky firms. Facing increased funding cost, although risky firms can switch to bond-financing their overall leverage still declines. While the positive consequence is the decreased endogenous risk, the average productivity of the entire economy declines as the reserve requirement tightens.

With respect to the growth of the economy's size, i.e. K_t, the message from our simulation is unclear. The underlying reason is the following. When the marginal cost of bank loans increases, both safe and risky firms issue more corporate bonds. Although the raised holding of physical capital by safe firms contributes to the growth of physical capital, the high liquidation cost related to bond-financing by risky firms depresses the accumulation of physical capital. Therefore, the net effect is indeterminate.

Table 2: **Impact of Price Control: Experts, Bankers, and Households**

τ_0	0	0.002	0.006	0.01	0.014	0.018	0.022	0.026	
wealth growth rate									
expert	0.6430	0.6452	0.6403	0.6445	0.6388	0.6522	0.6464	0.6244	(1)
banker	0.0226	0.0244	0.0213	0.0244	0.0225	0.0258	0.0259	0.0248	(2)
household	0.0909	0.0923	0.0900	0.0911	0.0892	0.0891	0.0889	0.0888	(3)
wealth volatility									
expert	1.0378	1.0364	1.0307	1.029	1.0247	1.0269	1.022	1.0108	(4)
banker	0.4168	0.4174	0.4167	0.4199	0.4228	0.4301	0.4395	0.4598	(5)
household	0.1534	0.1532	0.1520	0.1520	0.1514	0.1523	0.1517	0.1501	(6)
decomposition of GDP									
experts' consumption	0.0390	0.0391	0.0394	0.0394	0.0396	0.0394	0.0396	0.0404	(7)
bankers' consumption	0.0154	0.0151	0.0145	0.0137	0.0130	0.0123	0.0116	0.0106	(8)
households' consumption	0.3844	0.3853	0.3883	0.3897	0.3897	0.3967	0.3969	0.3958	(9)
investment	0.5571	0.5550	0.5493	0.5459	0.5437	0.5352	0.5335	0.5334	(10)
intermediation	0.0053	0.0052	0.0051	0.0049	0.0047	0.0046	0.0043	0.0040	(11)

[1] We randomize initial states of 10,000 economies, stimulate them for 500 years, and report mean values of above endogenous variables.

[2] Variables underneath "decomposition of GDP" are fractions of GDP.

5.3 Experts, Bankers, and Households

The influence of reserve requirement on individual agents is mixed. This is especially true for growth rates of their wealth (row 1-3 in Table 2). In terms of volatilities of their wealth, the picture is more clear. Tightening the price control makes wealth paths of both experts and households more stable (row 4 and 6 in Table). These are consequence of risky firms' decreased leverage and low endogenous risk. In contrast, as the price control tightens the volatility of bankers' wealth increases due to the increase in their leverage (row 5 in Table 2). Overall, tightening bank regulation benefits both experts and households and hurts bankers in terms of riskiness of their wealth. Similarly, as the government tightens reserve requirement consumption shares of both experts and households increase and bankers' consumption share declines. In addition, we need to take into account that the average FTP declines at the same time. With respect to the other two uses of final output, investment and intermediation, we observe that both of their shares in GDP decline as the price control tightens. The decline in investment is due to the decreased average productivity and the consequential decrease in the return for holding physical capital. Since the total supply of bank loans declines, the relevant intermediation cost falls accordingly.

6 Conclusion

In this paper, we present a dynamic general framework, in which firms choose either bond-financing and/or bank-financing and banks channel credit from savers to borrowers. The intermediation cost of bank-financing fluctuates endogenously because the risk-premium that banks ask for depends on the financial health of the banking sector. We investigate macroeconomic impacts of bank regulation in our framework. For bank regulation, we particularly focus on reserve requirement as the price control. Our model highlights the indirect channel through which bank regulation stabilizes financial markets and the entire economy. In particular, the channel is that the increase in banks' funding cost is almost completely passed to banks' borrowers. Our model shows that unlike financial intermediaries both experts and households in the real sector benefit from bank regulation. A natural extension of our current model is to investigate impacts of quantity control such as capital requirement on the aggregate economy and financial markets.

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Appendix

A Proofs

Proof of Lemma 1.

To apply Ito's Lemma, we first have

$$\mathrm{d}\big(q_t K_t\big) = q_t K_t(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t + q_t K_t(\sigma + \sigma_t^q)\mathrm{d}Z_t.$$

Given the above equation, equation 19, and Ito's Lemma, we have

$$\begin{aligned}
\mathrm{d}\omega_t = {} & \frac{W_t}{q_t K_t}\Big(R_t + \alpha b_t^0(R_t - r_t) + (1-\alpha)b_t^\lambda(R_t - \lambda\kappa^d - r_t) + (1-\alpha)l_t(R_t - \lambda\kappa - r_t^\lambda) - \frac{c_t}{W_t}\Big)\mathrm{d}t \\
& - \frac{W_t}{q_t K_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t - \frac{W_t}{q_t K_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda)\Big)(\sigma + \sigma_t^q)^2\mathrm{d}t \\
& + \frac{W_t}{q_t K_t}(\sigma + \sigma_t^q)^2\mathrm{d}t + \frac{W_t}{q_t K_t}\Big(1 + \alpha b_t^0 + (1-\alpha)(b_t^\lambda + l_t)(1-\lambda)\Big)(\sigma + \sigma_t^q)\mathrm{d}Z_t - \frac{W_t}{q_t K_t}(\sigma + \sigma_t^q)\mathrm{d}Z_t \\
\frac{\mathrm{d}\omega_t}{\omega_t} = {} & \mu_t^\omega \mathrm{d}t + \sigma_t^\omega \mathrm{d}Z_t.
\end{aligned}$$

Given bankers' Euler equation (15), the law of motion for W_t can be rewritten as

$$\frac{\mathrm{d}N_t}{N_t} = \Big(x_t^2\lambda^2(\sigma + \sigma_t^q)^2 + r_t + \tau\mathbf{1}_{x_t>1} - \frac{c_t}{N_t}\Big)\mathrm{d}t + x_t\lambda(\sigma + \sigma_t^q)\mathrm{d}Z_t.$$

Hence,

$$
\begin{aligned}
\mathrm{d}\eta_t &= \frac{N_t}{q_t K_t}\Big(x_t^2\lambda^2(\sigma+\sigma_t^q)^2 + r_t + \tau\mathbf{1}_{x_t>1} - \frac{c_t}{N_t}\Big)\mathrm{d}t - \frac{N_t}{q_t K_t}(\mu_t^q + \mu_t^K + \sigma\sigma_t^q)\mathrm{d}t \\
&\quad - \frac{N_t}{q_t K_t}x_t\lambda(\sigma+\sigma_t^q)^2\mathrm{d}t + \frac{N_t}{q_t K_t}(\sigma+\sigma_t^q)^2\mathrm{d}t + \frac{N_t}{q_t K_t}x_t\lambda(\sigma+\sigma_t^q)\mathrm{d}Z_t - \frac{N_t}{q_t K_t}(\sigma+\sigma_t^q)\mathrm{d}Z_t \\
\frac{\mathrm{d}\eta_t}{\eta_t} &= \mu_t^\eta\mathrm{d}t + \sigma_t^\eta\mathrm{d}Z_t.
\end{aligned}
$$

■