“Risk Factors and Asset Pricing: Evidence from China’s A-Share Market”¹

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ABSTRACT

Classic CAPM has long been criticized for wild errors in explaining the equity premia. This paper shows that part of the inaccuracy comes from the quality of the risk measurement. Evidence shows that an alternative single-factor model using the Aumann-Serrano economic index (the AS index) of riskiness dominates the conventional CAPM and the Fama-French three-factor model in China’s A-share stock market. We show that the AS index is able to capture information of higher-order moments in the systematic risk that is neglected in the CAPM and the Fama-French three-factor model. This paper also suggests that the momentum factor is particularly important in explaining the equity premia of China’s A-share market. The momentum factor is significantly related to the AS index. The two factors are also proved to be complementary rather than perfect substitutes for each other under different market regime.

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1. Introduction

The Capital Asset Pricing Model (CAPM) introduced by Treynor (1961,1962), Sharpe (1964) and Lintner (1965) is one of the most important asset pricing model in finance. Welch (2008) finds that 75% of professors in finance recommend the CAPM for asset pricing. Graham and Harvey (2001) find that 73.5% of CFOs use it to calculate the financial cost. The CAPM focuses on the sensitivity of risky assets’ premia against market excess returns and uses beta to measure the systematic risk of a given asset. By design, the CAPM intends to be mathematically straightforward and economically meaningful. However, the empirical performance of the CAPM is far from satisfactory. Major challenges against the CAPM come from market anomalies. Evidence of such anomalies that undermines the original CAPM rationality can be frequently found in practice. Academic literature on relevant topics is also extensive. Examples of anomalies include: the size effect identified by Banz (1981), who finds evidence that small companies tend to have higher risk-adjusted return than big companies; the excessive volatility phenomenon by Shiller (1980), who indicates that the stock price is simply too volatile if the market is efficient and the CAPM holds; the overreaction effect by De Bondt (1985); the momentum effect by Jegadeesh and Titman (1993).

To address these problems, scholars tried to modify the original CAPM by adding more risk factors. The three-factor model by Fama and French (1993) and four-factor model by Carhart (1997) are the most famous ones. Specifically, Fama and French (1993) add two more factors, the SMB (Small company minus big company) and HML (high book-to-market ratio minus low book-to-market ratio), to the original CAPM and Carhart (1997) adds another factor, the prior one-year return (PR1YR), which represents the momentum effect, into the Fama-French three-factor model. However, although the three-factor and four-factor models perform well in the American market in the 1990s, their performance in the global market is not as satisfactory.
Griffin (2002) finds that the Fama-French three-factor model performs better in the American market than the international market, as the error of the model is relatively larger in the international market. Fama and French (2012) also investigate the effect of market size, value, and momentum in the international market, and find that their regional models perform poorly on the size-momentum portfolios of Europe and Asia Pacific. Da et al. (2012) argue that the CAPM failed to consider the value of options of a firm to modify current projects and undertake new ones and contend that beta in the CAPM could be adjusted by the option proxies. More specifically, they propose that the return and beta are functions of current asset risk and future growth potential. Da et al. (2012) show that using option-adjusted equity premium and betas for estimation substantially improves the performance of popular stock pricing models for the US market.

A common focus of the various extended forms of CAPM is trying to find satisfactory factors which precisely capture tractable information on the risk premium, and there have been many techniques in building these ingredients linearly in equity return which helps improve the model. Therefore, documented extensions of the CAPM are to add relevant factors. Hence, the thread of literature which put efforts in augmenting the original CAPM has been based on firstly acknowledging the soundness of the CAPM. Thus, the classic beta coefficient, which represents the systematic risk of a given asset, should be intuitively correct. As we shall see in this paper, beta as a risk factor has its limitations. Particularly, it does not satisfy the duality axiom of Aumann and Serrano (2008). Therefore, a better risk measurement than beta could potentially improve the empirical performance.

The reason that the classic CAPM is empirically flawed could also be attributed to its limited capability of measuring higher moments of the excess returns, therefore the systematic risk itself indicated by beta\(^3\) can only proxy lower-moment risk for an individual asset, and ignore the risk of higher moments that is also hard to be diversified. This paper shows that it is possible to improve the performance of asset pricing models by simply revising the risk factors to include higher moments of returns.

\(^3\) The beta coefficient for each asset is calculated as the excess return of the asset as per unit of market excess return in the classic CAPM.
Any measurement of risk should be clearly defined and meet certain criteria. Aumann and Serrano (2008) provide two critical axioms, the duality axiom, and the homogeneity positive axiom, that riskiness index should satisfy and give a sample economic index of riskiness that satisfies those criteria. Homm and Pisgorsch (2012) recommend an estimation method to calculate the Aumann-Serrano riskiness index (the AS index, hereafter) under normal inverse Gaussian distribution. They show that one advantage of their estimated riskiness index over the traditional risk measurement is that it contains information on the third and fourth moments of asset returns. Therefore, the estimated AS index carries information not only on volatility but also on tail risk. This paper tries to identify and quantify additional information delivered by the AS index. Our study also considers the possibility that the AS index performs as an alternative risk factor to replace the beta coefficient in the conventional CAPM. We focus our research on China’s equity market, which has long been considered as irrational, extremely volatile and strongly influenced by policies. Properties as such can be interpreted as excessive kurtosis and heavy skew in econometric terms. The paper, therefore, extends factor models in an international context, especially explains markets with a risk profile of higher-orders being a norm. Tests are based on classic asset pricing models, option-adjusted equity premium and the AS factor. Corresponding evaluation and comparison on the performance of the AS factor in augmented asset pricing model and other model settings are subsequently conducted. Empirical evidence suggests that even a single-factor pricing model using the AS index to proxy the risk exhibits superior performance in China’s A-Share stock market. It fits the equity premium better than the CAPM and Fama-French three-factor models. By comparing explanatory powers of the AS factor and the momentum factor on equity premia, our study also shows the two are complementary to each other and partly correlated as well, which implies that part of the momentum effect can be rationalized as risk (of higher-order) compensation rather than pure irrational overreaction. But there is still momentum effect that cannot be explained by rational expectations.

The rest of the paper is structured as follows: Section 2 provides the methodology to estimate a risk measurement using the AS index and discusses risk factors; Section 3 describes the dataset and definitions of different variables; Section 4 is an empirical study on China’s A-Share stock market; Section 5 concludes the paper.
2. Riskiness index and risk factors

2.1 Characteristics of the Aumann-Serrano riskiness index

Aumann and Serrano (2008) give an axiomatic characterization of the riskiness index, which enables investors to compare the riskiness of two assets without a specific utility function or preference. Like the definition of Value at Risk (VaR), assets in terms of different levels of risk are mapped to a field of the real number so that the comparison between several risky assets becomes easy. Aumann and Serrano (2008) start with the definition of the risk aversion. An agent with a utility function \( u \) invests in a risky asset \( g \) at wealth \( w \) if\[ E\{u(w + g)\} > u\{w\}, \]
where \( E\{\cdot\} \) stands for expectation. We call agent \( i \) uniformly no less risk-averse than the agent \( j \) if whenever \( i \) accepts the asset at some wealth, \( j \) accepts that asset at any wealth. Aumann and Serrano (2008) also prove that \( i \) is no less risk averse than \( j \) if and only if \( \rho_i(w_i) \geq \rho_j(w_j) \) for all \( w \), (i.e., \( \min(\rho_i(w_i)) \geq \max(\rho_j(w_j)) \)), where \( \rho_i(w_i) \) is the absolute risk aversion of \( i \) with function \( u \).

Given the definition above, two axiomatic characterizations of the economic index of riskiness (\( Q(g) \) is a risk measure of asset \( g \)) are proposed:

**Duality:** If \( i \) is uniformly more risk averse than \( j \), \( i \) accepts \( g \) at \( w \), and \( Q(g) > Q(h) \), then \( j \) accepts \( h \) at \( w \). In short, the duality axiom tries to illuminate that, between two agents, if the one being more risk-averse accepts the riskier of two assets, then the other agent must accept the less risky one.

**Positive Homogeneity:** \( Q(tg) = tQ(g) \) for all positive numbers \( t \). Positive homogeneity reflects the cardinal nature of riskiness. If \( g \) is a gamble, then the riskiness of \( 2g \) should be “twice as” risky as \( g \), not just riskier. Similarly, \( tg \) should be “\( t \) times as” risky as \( g \).

According to the above two axioms, Aumann and Serrano (2008) give an implicit expression of the Aumann-Serrano riskiness index (the AS index) as follows:

\[
E\left( e^{-\frac{\kappa}{E^{AS}}} \right) = 1. \quad (2.1)
\]
Aumann and Serrano (2008) proved that the riskiness index as in (2.1) satisfies both the duality and positive homogeneity and it can be interpreted as the risk tolerance of a person with constant absolute risk aversion (CARA) who is indifferent between accepting and not accepting a given asset. This definition is consistent with the idea that less risk averse agents accept riskier assets. In addition, the higher the AS index is, the riskier the asset is.

Equation (2.1) defines the AS index but is silent on a feasible estimation method for empirical studies. Homm and Pigorsch (2012) propose a parametric estimator for the AS index. They show that if stock returns follow a normal distribution, the AS index is given by equation (2.2):

\[
AS_{\text{normal}} = \frac{\sigma_r}{2\mu_r}
\]

where \(\mu_r\) and \(\sigma_r\) are empirical estimates for the mean and variance of the stock return. The relationship between the AS index and Sharpe ratio under Gaussian is given by equation (2.3):

\[
AS_{\text{normal}} = \frac{\mu_r}{2 \times \text{Sharpe Ratio}^2}
\]

In this case, higher moments of the underlying stock returns do not play a role at all. However, as argued by Homm and Pigorsch (2012), stock returns do not follow a normal distribution and higher moments are particularly informative on tail risks. Therefore, a normal inverse Gaussian (NIG) distribution can be used to match the empirical distribution of stock returns. Parametric approach as such is well-established in the field of financial econometrics and statistics. The NIG distribution is implemented widely to model unconditional as well as conditional return distributions\(^4\). In this case, the AS index is given by equation (2.4):

\[
AS_{\text{NIG}} = \frac{3\kappa\mu - 4\mu \chi^2 - 6\chi \sigma + 9\sigma^2 / \mu}{18}
\]

where \(\chi\) represents the skewness and \(\kappa\) represents the kurtosis of the return distribution. These two higher-order parameters help capture information about tail risks. Following methods as

shown above by Homm and Pigorsch (2012), we construct the empirical AS index. However, this parameter estimation method requires the positive average return. Thus, we implemented the cumulative Aumann-Serrano Index rather than simple AS on a rolling basis. We will further discuss this issue in Section 2.3.

2.2 Further discussion on risk factors

Apart from the AS index calculation as shown in subsection 2.1. This subsection focuses on the other two different types of risk factors: beta of the CAPM model; the adjusted betas.

**Beta coefficient**

In the traditional CAPM, a measure of riskiness, termed as beta, is an asset’s correlation with the market portfolio. It is defined as equation (2.5):

$$
\beta_i = \frac{\text{cov}(r_i - r_f, r_m - r_f)}{\text{var}(r_m - r_f)}
$$

(2.5)

where $r_i$, $r_m$ and $r_f$ represents the return of riskiness asset, the stock market, and risk-free asset, respectively.

According to Aumann-Serrano axiomatic definition, we firstly verify if the beta meets the requirements of duality and positive homogeneity simultaneously.

**Duality**: Consider two rational individuals $i$ and $j$ whose CARA utility functions are defined as:

$$
U_k(r) = 1 - e^{-\alpha_k \omega (1+r)}
$$

(2.6)

where $\omega$ stands for the initial wealth of the investor, $\alpha_k$ $(k = i, j)$ represents the degree of risk aversion. A higher $\alpha_k$ represents a higher degree of risk aversion. Without loss of generality, we set the initial wealth $\omega$ to 1, $\alpha_i = 2$ for investor $i$, and $\alpha_j = 1$ for investor $j$ (Henceforth, $i$ is more risk averse.). By considering two risky assets $g$ and $h$, and market return $r_m$ in three
equally probable situations in Table 1, we obtain the expected utility for investor $i$ and $j$ as presented in Table 2.

[Table 1 and Table 2 about here]

According to the numerical levels of the expected utility, as reported in Table 2, investor $i$ will choose asset $h$ and investor $j$ will choose asset $g$. However, from the definition of beta, the beta of asset $g$ is 27.74 while the beta of asset $h$ is 86.60. This means that a more risk-averse investor $i$ will choose a riskier asset measured by beta. Clearly, beta violates the duality axiom of the riskiness index.

**Positive Homogeneity**: Because the definition of beta is defined by the return of an asset, the positive homogeneity is satisfied.

In summary, the beta in CAPM does not satisfy the duality axiom of riskiness. Therefore, it may introduce mistakes when measuring risks in asset pricing models.

**Adjusted betas**

This paper also adopts the approach by Da et al. (2012) to adjust equity returns and betas. According to Da et al. (2012), both equity returns and betas are influenced by project beta/return and other factors which are related to the option values of equities. Taylor expansions of equity returns and betas around their market counterparts can be done so as to separate the effects of other factors and project beta/return. More specifically, the return and the CAPM beta are defined in the following equation (2.7) and Equation (2.8):

\[
\mu_i = f(\mu_i^p, OF_i) = f(\mu_i^p, OF_M) + f_1(\mu_i^p - \mu_M^p) + f_2(OF_i - OF_M) + \varepsilon_i \tag{2.7}
\]

\[
\beta_i = g(\beta_i^p, OF_i) = g(\beta_i^p, OF_M) + g_1(\beta_i^p - \beta_M^p) + g_2(OF_i - OF_M) + \varepsilon_i \tag{2.8}
\]

where $\mu_i$ and $\beta_i$ are the unadjusted equity premium and beta on stock $i$, $\mu_i^p$ and $\beta_i^p$ are equity premium and beta on the project of firm $i$, respectively. $OF_i$ represents a vector of variables which are related to the option value of the stock $i$, $\mu_M^p$, $\beta_M^p$ and $OF_M$ are the market risk premium, market beta and market option value, respectively. $f_1$, $f_2$, $g_1$ and $g_2$ are the corresponding partial derivatives. Based on equation (2.7) and (2.8), Da et al. (2012) show that the option-adjusted equity returns and beta can be obtained by the following OLS regression.
\[ \mu_i = a(OF_i - OF_M) + \mu_{i \text{Adjusted}} \] (2.9)

\[ \beta_i = b(OF_i - OF_M) + \beta_{i \text{Adjusted}} \] (2.10)

where \(a\) and \(b\) are regression coefficients, \(\mu_{i \text{Adjusted}}\) and \(\beta_{i \text{Adjusted}}\) are the option-adjusted return and beta. To implement the option adjustments, we need to find proxies for \((OF_i - OF_M)\). Da et al. (2012) use the idiosyncratic volatility, return on asset (ROA) and the book-to-market ratio (BM) of firm \(i\) as proxies for the option value. Recent literature (Cao et al., 2008 and Bekaert et al., 2010) link growth options to firms’ idiosyncratic risks. The idiosyncratic volatility is then a common measure for the idiosyncratic risk. However, it ignores the higher moments which would imply the tail risk. As shown by Homm and Pisgorsch (2012), compared with traditional volatility measures, the AS index can be a more informative risk indicator because it captures higher moments of the return. Therefore, in this paper, two types of adjusted betas are considered: beta that is defined as Da et al. (2012) with adjusted option value (hereafter, the “growth-adjusted beta”). And the “risk-adjusted beta”, which is obtained by using the AS index of firm \(i\) as the regressor instead of \((OF_i - OF_M)\) in equation (2.10).\(^5\)

2.3 Cumulative Statistics

In fact, equation (2.4) holds based on the assumptions below:

\[ \mu > 0, \sigma^2 > 0, \kappa > 0, |\chi| < \sqrt{3\kappa - 5} \]

If we choose to construct the Aumann-Serrano index using a rolling window, it is highly likely that the average mean return is a negative value, which violates the assumption of this parameter estimation and makes this model flawed theoretically. Instead of calculating Aumann-Serrano Index on a rolling window, our paper chooses to use cumulative data, which Aumann-Serrano index is based on the information from time-0 to time-\(t\).

\[ AS_i = AS_i(r_0, r_1, ..., r_t) \]

\(^5\) Note that although the adjusted betas preclude the effect of real option values, they are still constructed based on the CAPM beta of firm \(i\). As we have discussed, it does not satisfy the duality axiom and hence may cause potential measurement errors as a risk indicator.
In this case, AS becomes an adapt process similar to the price series and turned into an integrated process (I(1) process). We plotted the HS300 index and its corresponding cumulative Aumann-Serrano index.

[Figure 1 about here]

Intuitively, Figure 1 shows the cumulative Aumann-Serrano index follows an integrated process and negatively correlates to the price. We therefore assume the cumulative AS index follows an asymptotic martingale process which can be expressed as:

$$\lim_{t \to \infty} E[AS_{t+1} | F_t] = AS_t$$

where $F_t$ is the filtration at time $t$ which includes all the currently available information. In other words, the current value of the cumulated AS index can be regarded as an estimate of the next period, and the change of it, $\Delta AS_t$, can be interpreted as the “unexpected” change of risk. $\Delta AS_t$ may result in increases/decrease in the expected return, which often leads to a decrease/increase in the current asset price.

3. Data description and variables

We collect time series dataset of daily frequency for each stock that is publicly traded in China’s A-share stock market from the year 2002 to the end of 2015. The A-share stocks are officially termed as the “RMB-denominated common stocks”, and traded in Shanghai and Shenzhen Stock Exchange. A-share stocks may only be initiated by China-based companies and traded in domestic currency\(^6\). Among all the stock markets in mainland China, the A-share stock market is the dominant one in terms of the total number of listed companies and gross market value. By the end of 2015, there were 2,592 public-listed companies with a market value of USD 4.9 trillion in the A-share market. Our cross-sectional test period starts from May, 2005 (just after the non-tradable stock reform) until the end of 2015.

\(^6\) As opposed to the A-share stocks, there is also a B-share stock market in mainland China (excluding Hong Kong), which is denominated in RMB but shares can only be traded using US dollar or Hong Kong dollar. Besides the A- and B-share stocks, there is also a H-share stocks, which refers to the shares of companies incorporated in mainland China but are traded in the Hong Kong Stock Exchange.
A brief description of our variables is as follows: the beta estimates are calculated as the slope coefficient of CAPM regressions; the Beta_MKT, Beta_SMB, Beta_HML are calculated as the slope coefficients in the Fama-French three-factor model; all estimates are based on the cumulative method. To calculate the growth-adjusted beta, we apply the approach of Da et al. (2012) to remove the effects of option value on the stock return and beta. The idiosyncratic volatility is calculated using the approach by Ang et al. (2006). The ROA and BM data are retrieved from the CSMAR database. The Fama-French three factors for the Chinese A-share market are retrieved from CSMAR as well. The risk-adjusted beta is constructed by removing the effect of the AS Index on the original beta. The first-order difference of the cumulative AS index is used in all regressions. Stock price momentum (hereafter, Momt) is calculated using the prior 20 days returns (including day t). All variables are defined in Table 3.

[Table 3 and Table 4 about here]

Table 4 summarizes pairwise correlations, and it shows: correlations between different risk measurements (CumAS_nig, AS_nig, Beta, Beta_MKT, Beta_SMB, Beta_HML) are statistically significant, but the AS_normal index only has a significant correlation with the AS_nig index and CumAS_nig. This may imply either the AS_normal is less informative compared with the AS_nig or it is wrong. Literature confirms the latter, which is consistent with a well-established recognition that daily return series in the stock market deviates away from Gaussian but converge to a normal inverse Gaussian (NIG) distribution\(^7\).

It is also clear from Table 4 that correlations between the CumAS_nig and beta, MKT, HML are significant (7\(^{th}\) row in Table 4, and at 5% significance level) and positive. But if we replace the CumAS_nig with AS_nig, the above correlations become significantly negative. After adjusting the “growth” and “risk”, correlations between the AS index (also CumAS_nig) and the Beta_PA, Beta_RA, Beta_PA&RA respectively become insignificant (6\(^{th}\) and 7\(^{th}\) column in Table 4), which suggests that the AS index contains information on the potential growth of a company. Recall now that a firm’s growth potential is related to the idiosyncratic risk and the AS index contains comprehensive information on risk associated with the firm compared with traditional measures.

\(^7\) For the NIG distribution and its application, please see Andersson (2001), Bollerslev et al. (2009), Eriksson et al. (2009), Zakamouline and Koekebakker (2009), Homm and Pigorsch (2012).
4. Empirical evidence from China’s A-share stock market

In this section, we estimate how the AS index correlates with stock returns and, most importantly, evaluate whether it could be a better factor to determine the risk-return relationship. **We build the riskiness index by calculating the change in the cumulative AS index. This measurement reflects the innovation of market perception of riskiness towards a specific asset.** After obtaining values of the riskiness for each cross-sectional panel, we implement the Fama-Macbeth (1973) approach to run cross-sectional regressions and thereby calculate the corresponding t-values. We also test the correlation between the AS index and stock price momentum and compare their explanatory power on stock returns.

4.1 Panel analysis

We estimate seven models based on our data panels and results of each model are exhibited as rows in Table 5. Columns in Table 5 contain estimated coefficients (betas) that correspond to different factors of the regressions. Beta, as shown in column 3 is the premium for the individual stock as per market excess return. Therefore, Model 1 with a zero intercept stands for a classic CAPM model. The estimated coefficients of Model 1 include a significantly positive intercept (alpha) with a value of 0.2505 (significant at a level of 1%), which suggests after adjusting for the systematic risk, there still exist abnormal returns for individual stocks. Hence, a significant alpha indicates either the CAPM model is mis-specified or beta itself is an incompetent measure of risk. Augmented and generalized models and explanations as such are well-documented. Therefore, from the results in Model 1, it is consistent to the literature that stock market violates the CAPM model because it does not fully explain individual stock returns. In China’s case, one proven explanation is irrationality happens more often since the number of individual investors dominates the number of institutional investors. This phenomenon is corroborated by many studies focusing on China’s A-share market. For instance, Wu and Xu (2004) point out that the return of Chinese market is determined by risk factors and some significant irrational events which often cause simultaneity in price movements. Another result
from Model 1 is that the estimated beta is negative and only marginally significant at 10% significance level, which implies that the beta of each stock may even hardly explain its return.

As mentioned before, violation of the CAPM revealed by Model 1 may imply beta is an imprecise measure of risk. We then use the risk-adjusted beta to replace the CAPM-beta and get results as shown by Model 2 in Table 5. Estimates of Model 2 show this model hardly captures any systematic risk and has a highly significant abnormal return. Model 3 uses the growth-adjusted beta to replace beta in the CAPM. In contrast to the results by Da et al. (2012) for the US market, Model 3 does not perform well in the Chinese market either. Model 4 combines both the risk and growth adjustment together in the Beta_RA&PA, and the performance does not improve. Model 5 shows the performance of a standard Fama-French three-factor model. All three factors are insignificant at 5% significant level but the constant term is highly significant. Therefore, the performance of the Fama-French three-factor model explains little. To sum up, none of the above model does a good job to explain China’s A-share market.

As we discussed in Section 2, betas and the adjusted betas fail to satisfy the duality axiom proposed by Aumann and Serrano (2008) and may omit valuable information regarding risk premium of higher orders. Hence, we re-construct Model 6 in Table 5 by simply using the AS index as a substitute for the beta in the classic CAPM (hereafter, Model 6 is defined as the “AS-CAPM”). Surprisingly, the estimated coefficient (before the AS-index) in the AS-CAPM is highly significant and the intercept becomes significant. It suggests that, in general, individual stock’s return volatility, after adjusted for higher-order risks, tends to negatively correlate to the systematic risk. And China’s A-share market does yield to abnormal returns by noticing a highly significant intercept of 0.1688. Now, by employing our definition of the riskiness as shown above (the change in the cumulative AS index, or the innovation of perceived risk of a given asset), the negative beta is interpreted as a current decrease in the innovation of the riskiness implying a higher expected return in the future.

To reconcile with empirical evidence that shows a significant abnormal return (or alpha), stories regarding certain types of irrationality are often told. Instead, our study argues that abnormal returns may always stand from time to time to compensate higher-order risk that equities may carry. Therefore, the alpha can be interpreted as a premium for the combined risk of low and high orders. Results in Model 6 therefore immediately fit this intuition that if a stock
carries more and more perceived risk (in terms of positive innovations of riskiness), it leads to big losses due to the beta risk but are instantly offset by a positive alpha. To the opposite, decreasing riskiness (negative innovations) may help accumulate abnormal returns. Theoretically, it is possible to hedge against the beta risk to harvest the abnormal return forever. But, it is very difficult to forecast and to hedge against high-order risk of individual stocks in real-life trading activities. Products such as CBOE SKEW index which tracks and forecast black swan events on stock level are pre-requisites.

Model 7 applied a similar cumulative-difference technique for beta, however, the result is similar to model 2, and the relationship between the change of beta and current period return is not significant.

[Table 5 about here]

In summary, results from Table 5 shows that using the AS index as a proxy for the risk factor outperforms the CAPM model, Fama-French three-factor model and the option-adjusted CAPM models. We now examine possible reasons and try to present explanations.

Our model, as Model 6 in Table 5, can be expressed as follows,

\[ R_{i,t} - R_{f,t} = \alpha_{i,t} + \sum_{k=1}^{n} Risk_{k,t} \cdot Premium_{k,t} + \epsilon_{i,t} \]  \hspace{1cm} (4.1)

It is natural to say that a pricing model as the form of Equation (4.1) is correct only if the risk is correctly specified. The estimated coefficient before the AS index in Model 6 of Table 5 can be seen as the risk premium associated to the risk measured by the AS index (hereafter, the “AS premium”).

We firstly examine the relationship between the market return and the AS premium. Table 6 presents two estimations of for the AS premium. There is no significant relationship between the market return and the AS premium, and the fittings are trivial. This indicates the explanatory power of the market excess return on the compensation for higher-order risk captured by the AS index is very limited. This may explain why the performance of the risk-adjusted beta model is dominated by the AS-CAPM. A possible extension using the Fama-French three factors (Model 2 in Table 6) shows similar features. Therefore, results in Table 6 suggest that the AS premium is somewhat beyond the scope of the traditional factor models.
4.2 The AS index and momentum

The last row in Table 4 summarizes the correlations between betas, the AS index, and the momentum of stock prices. Traditionally, the momentum is usually seen as a phenomenon of collective irrationality which may be related to investor over-reactions. However, if the momentum factor can be partly explained by some risk measurement, then the excess risk compensation under rationality still holds even for an evident behavior of momentum. That says even if momentum strategy generates an excess profit margin that could not be explained by classic factor modeling, it is possible that this abnormal return may not be a bonus to superior trading strategy. Instead, one part of the momentum profit merely means traders take on skew and/or tail risks which are impossible to diversify. Therefore, momentum is some yield on the higher-order risks, but not gift money to show off. Hence, it is important to investigate whether the AS index correlates with the momentum of stock prices.

Three models are considered: the CAPM augmented by the momentum factor; the AS-CAPM augmented by the momentum factor; a model with the momentum as the single factor. Results are shown in Table 7. Results of all three models show that momentum plays a very important (and robust) role to explain daily stock returns. Estimates of the coefficients before the momentum factor are both highly significant and consistent in all three models.

Model 1 of Table 7 shows the momentum factor improves the performance of a classic CAPM. Model 2 includes both the AS index and the momentum factor as independent variables. The estimated coefficient of the AS index, in this case, is both consistent in value (with Model 6 in Table 5) and significant at 1% level.

We use the Shapley variance decomposition to compare the explanatory power of the AS index and the momentum factor after orthogonalizing these two variables to the daily returns. Figure 2 and 3 present the results of the Shapley variance decomposition. In Figure 2, the x-axis represents the time line, and the y-axis represent the Shapley value, which stands for the
explanatory power of the AS index. Each spot in the figure represents the Shapley value of the AS index at each period. Figure 3 gives the corresponding quantile percentage of the AS index. Most of the observed values in Figure 2 are concentrated at the bottom, which suggests that momentum explains more variations of daily stock returns in most of the days. Figure 3 further confirms this intuition. However, there is still a substantial number of dates on which the Shapley percentage of the AS index is high. This means that the explanatory power of the AS index is also non-negligible. Interestingly, the dates on which the AS index has a high explanatory power tends to cluster around 2009 and 2015, which suggests that the AS index and momentum factor could have comparative advantages in different market regimes.

To further investigate this phenomenon, we classify the days on which the AS index outperforms the stock price momentum in explaining the equity premium and vice versa into two groups, and using the mean, variance, skewness and kurtosis of the prior 20 days HS index return on a rolling basis to quantify different market regimes in different days. Group 1 consists of the days on which the AS index significantly outperforms stock price momentum and Group 2 consists of the days on which stock price momentum significantly outperforms the AS index. We use two quantitative criteria for the classification. Criterion 1 is based on the absolute value of the Shapley percentage. Those days with higher than 90 Shapley percentage of the AS index go to Group 1 while those days with less than 10 Shapley percentage go to Group 2. Criterion 2 is based on percentiles of the sample. The top 10% Shapley percentage of AS index goes to Group 1 while the bottom 10% go to Group 2. In Table 8, we present the ANOVA analysis of the inter-group difference.

[Figure 2, Figure 3 and Table 8 about here]

From Table 8, we notice that there are significant differences in return distributions between the days on which the AS index performs better and the days on which stock price momentum performs better. From the ANOVA analysis, we discovered that there are significant differences in the variance, skewness and kurtosis of the return distribution between the two groups no matter which criterion is applied to group the dates. These results further confirm that the AS index and stock price momentum has their comparative advantages in explaining the equity premium in different market regimes. The AS index outperforms momentum in volatile market conditions, where big market swings, or even, market crash, tend to happen. But the
momentum factor dominates AS index when the market is relatively calm and shows some deterministic trending over time. Therefore, those two factors are complementary in a good stock pricing model for China’s A-share stock market.

5. Conclusion

Our paper focuses on the performance of alternative riskiness measurements in asset pricing models for the China’s stock market. Our paper shows that:

1. The CAPM and Fama-French Three-factor model with their corresponding risk-measurement, beta, cannot explain the equity premium in the China’s A-share stock market, due to the fact the beta violates the duality axiom in Aumann’s definition for risk-measurement. The empirical evidence further convinced our statement as the intercept term, i.e. pricing error, is highly significant while the regressor is not significant.

2. Da et al. (2012)’s adjusted-CAPM does not work for Chinese market as well, as the potential proxy variable overly absorbed the information in daily return and beta, and it does not solve its violation of duality axiom.

3. Our alternative AS-CAPM model explains the daily return better as the AS index not only satisfies Aumann’s two axiom but absorbed the higher-order risk of individual assets under extreme market regimes. The change of cumulative AS index, in another word, the innovative of perceived risk, is negatively related to current market return, which meets our intuition that, the higher level of risk, the higher level of expected return and the lower level of current stock price. Moreover, the market return and the three factors in the Fama-French model has limited explanatory power of the risk premium for the AS index.

4. AS index and momentum factor is complimentary in the asset pricing model. The AS index explained more in the volatile market while the momentum factor performs better when the market is relatively calm and shows some deterministic trending over time.
Our study shows that the change of market view on the AS index is one important and robust factor for asset pricing. Compared with the momentum factor, it is a most-be-concerned riskiness measurement under extreme market circumstances, when the market crash or black swan take place.
References


APPENDICES: ALL TABLES AND FIGURES

Table 1: Asset Returns and Utilities

<table>
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<th>1</th>
<th>2</th>
<th>3</th>
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<th>10%</th>
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<tbody>
<tr>
<td>0</td>
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<td>0.6321</td>
<td>0.8892</td>
<td>0.6671</td>
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<tr>
<td>10%</td>
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<td>0.8647</td>
<td>0.6321</td>
<td>10%</td>
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1, 2, 3 in the first column correspond to the three different situations, each has a probability 1/3.

Table 2: Expected Utility of Investors

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<th>Asset g</th>
<th>Asset h</th>
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<td>E(U_i)</td>
<td>0.8724</td>
<td>0.8728</td>
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<tr>
<td>E(U_j)</td>
<td>0.6463</td>
<td>0.6437</td>
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### Table 3: Variable Definition

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<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>Beta</td>
<td>Capital asset pricing model beta estimated using 20 days data prior to day t (including day t).</td>
</tr>
<tr>
<td>Beta_RA</td>
<td>Risk-adjusted beta</td>
</tr>
<tr>
<td>Beta_PA</td>
<td>Potential growth-adjusted beta</td>
</tr>
<tr>
<td>AS_nig</td>
<td>Aumann-Serrano index estimated by assuming the return distribution of normal inverse Gaussian distribution</td>
</tr>
<tr>
<td>AS_normal</td>
<td>Aumann-Serrano index estimated by assuming the return distribution of normal distribution</td>
</tr>
<tr>
<td>CumAS_nig</td>
<td>Aumann-Serrano index estimated using cumulative method by assuming the return distribution of normal inverse Gaussian distribution</td>
</tr>
<tr>
<td>Ivol</td>
<td>Idiosyncratic volatility estimates using the approach of Ang et al. (2006)</td>
</tr>
<tr>
<td>ROA</td>
<td>Return on total asset</td>
</tr>
<tr>
<td>BM</td>
<td>Book-to-market ratio</td>
</tr>
<tr>
<td>MKT</td>
<td>Market excess return of the Fama-French(1993) three-factor model</td>
</tr>
<tr>
<td>SMB</td>
<td>SMB factor of the Fama-French(1993) three-factor model</td>
</tr>
<tr>
<td>HML</td>
<td>HML factor of the Fama-French(1993) three-factor model</td>
</tr>
<tr>
<td>Momt</td>
<td>Stock price momentum using prior 20 days returns (including day t).</td>
</tr>
</tbody>
</table>
Table 4: Pairwise correlation of riskiness measurements

<table>
<thead>
<tr>
<th></th>
<th>Beta</th>
<th>Beta_MKT</th>
<th>Beta_SMB</th>
<th>Beta_HML</th>
<th>AS_nig</th>
<th>CumAS_nig</th>
<th>AS_normal</th>
<th>Beta_PA</th>
<th>Beta_RA</th>
<th>Beta_PA&amp;RA</th>
<th>Momt</th>
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<tr>
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<tr>
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<td>Beta_HML</td>
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<td>AS_nig</td>
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<td>-0.0023**</td>
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<td>CumAS_nig</td>
<td>0.0043**</td>
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<td>AS_normal</td>
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<td>Beta_RA</td>
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<td>0.0043**</td>
<td>0.0002</td>
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<td>0.7766***</td>
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<td>Beta_PA&amp;RA</td>
<td>0.6712**</td>
<td>0.0031*</td>
<td>0.0013</td>
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<td>0.5219***</td>
<td>0.0657**</td>
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<td>Momt</td>
<td>0.0659**</td>
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Source: CSMAR; daily data from 1st January 2002 to 31st December 2014; Pearson correlations of the CAPM beta, betas of the Fama-French three-factor model, Cumulative Aumann-Serrano Index(nig) and Aumann-Serrano Index (normal and nig), beta after potential growth adjustment, risk adjustment, both potential growth adjustment and risk adjustment and stock price momentum. See Table 3 for variable definitions. ***, **, * denotes significance at 1%, 5% and 10% respectively.
### Table 5: Panel Regression Results of the Asset Pricing Models

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<tr>
<th>Model</th>
<th>Constant</th>
<th>Beta</th>
<th>Beta_RA</th>
<th>Beta_PA</th>
<th>Beta_RA&amp;PA</th>
<th>Beta_MKT</th>
<th>Beta_SMB</th>
<th>Beta_HML</th>
<th>AS</th>
<th>Beta_Diff</th>
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</tr>
<tr>
<td>2</td>
<td>0.1435***</td>
<td>0.0044</td>
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<td>4</td>
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<tr>
<td>5</td>
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<td>-0.1139**</td>
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<td>6</td>
<td>0.1688***</td>
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<td>7</td>
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</table>

Source: CSMAR; daily data from 1st January 2002 to 31st December 2015; Dependent variable is the daily stock return. The stock return is adjusted using equation (3.5) in Model 2-4. Independent variables consist of CAPM beta, betas of the Fama-French three-factor model, risk-adjusted betas, potential growth-adjusted beta, and the Aumann-Serrano risk index. The data ranges from 2002.1.1 to 2014.12.31. The coefficient is the mean value of all the cross-sectional regression; and the t-value is calculated using Fama-Macbeth (1973) approach. Average number of observations of all cross-sections is 1623. ***, ** and * denotes the significance level at 1%, 5% and 10% respectively.
Table 6: Regressions of the Premium of the Aumann-Serrano Index

<table>
<thead>
<tr>
<th>Model</th>
<th>Constant</th>
<th>Market Return (MKT)</th>
<th>SMB</th>
<th>HML</th>
<th>R²</th>
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</thead>
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<tr>
<td>1</td>
<td>-0.0028***</td>
<td>-0.0002</td>
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<td>0.0003</td>
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<td></td>
<td>(-6.24)</td>
<td>(-1.01)</td>
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<tr>
<td>2</td>
<td>-0.0029***</td>
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<td>0.0005</td>
<td>-0.0001</td>
<td>0.0003</td>
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<td>(-6.28)</td>
<td>(0.59)</td>
<td>(0.65)</td>
<td>(-0.28)</td>
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</tr>
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</table>

Source: CSMAR; daily data from 1st January 2002 to 31st December 2014; The dependent variable is the risk premium for the AS index, i.e. the coefficient of the AS index for each cross-section; the independent variables are the market return and Fama-French three factors (MKT, SMB, HML). We apply the time-series regression and all series are stationary. The value in the parentheses is the t-value for each regressor. *** and ** denote the significance level at 1% and 5% respectively.

Table 7: Regressions with the Momentum Factor

<table>
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<tr>
<th>Model</th>
<th>Constant</th>
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<th>AS Index</th>
<th>Momt</th>
<th>R²</th>
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<td>(0.60)</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.0315</td>
<td>-0.0032***</td>
<td>0.0505***</td>
<td>(33.72)</td>
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<td></td>
<td>(-0.70)</td>
<td>(-6.07)</td>
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<tr>
<td>3</td>
<td>-0.0313</td>
<td></td>
<td>0.0506***</td>
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<td>0.0594</td>
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<tr>
<td></td>
<td>(-0.63)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: CSMAR; daily data from 1st January 2002 to 31st December 2014; Dependent variable is daily return of stocks, independent variables consists of CAPM beta, Aumann-Serrano risk index and the stock price momentum term. The coefficient is the mean value of all the cross-sectional regressions; and the t-value is calculated using Fama-Macbeth (1973) approach. Average number of observations of all cross-sections is 1623. *** and * denote the significance level at 1%, 5% and 10% respectively.
Table 8: ANOVA analysis

<table>
<thead>
<tr>
<th>Mean Value</th>
<th>Group 1</th>
<th>Group 2</th>
<th>ANOVA F-Value</th>
<th>P-Value</th>
</tr>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Criterion 1</td>
<td></td>
</tr>
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<td>Skewness</td>
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<td>Kurtosis</td>
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<td>15.0794</td>
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<td></td>
<td></td>
<td></td>
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<td>Mean</td>
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<td>15.2470</td>
<td>4.48</td>
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</table>

Source: CSMAR; daily data from 1st January 2002 to 31st December 2014; Criterion 1: The days with higher than 90 Shapley percentage of AS index go to group 1, while those days with less than 10 Shapley percentage go to group 2. Criterion 2: The top 10% Shapley percentage of AS index go to group 1, while the bottom 10% go to group 2. F-value and P-value are calculated by ANOVA. The mean, variance, skewness and kurtosis data is the correspondent statistics of prior 20 days market return.
**Figure 1:** Relationship between cumulative Aumann-Serrano Index and Price Series

The x-axis is date and the left y-axis is HS300 index and the right y-axis is corresponding cumulative AS index of HS300 index. The estimation of cumulative AS index starts from May. 2005 after the non-tradeable shares reform.

**Figure 2:** Shapley Percentage of Aumann-Serrano Index Term

The y-axis is Shapley percentage of AS index. The x-axis is time.
Figure 3: Quantile Percentage of Aumann-Serrano Index Term

The y-axis is the quantiles of Shapley percentage of AS index. The x-axis is the fraction of the data. The number of observations is 3003.