Distributive Conflict, Growth, and the 'Entrepreneurial State'

Daniele Tavani[*], Luca Zamparelli[†‡]

August 30, 2016

Abstract

In this paper, we introduce a twofold role for the public sector in the Goodwin (1967) growth cycle model. The government collects income taxes in order to: (a) invest in infrastructure capital, which directly affects the production possibilities of the economy; (b) finance publicly funded research, which augments the growth rate of labor productivity. We first focus on a special case in which labor productivity growth depends entirely on public research, and show that: (i) provided that the output-elasticity of infrastructure is greater than the elasticity of labor productivity growth to public R&D, there exists a tax rate τ^* that maximizes the long-run labor share, but not a growth-maximizing tax rate; (ii) the long-run labor share is always increasing in the share of public spending in infrastructure, and (iii) the presence of public R&D is not enough to stabilize the distributive conflict. We then study a more general model with induced technical change where, as is well known in the literature, the distributive conflict is resolved in the long run. With induced technical change: (iv) the labor share-maximizing tax rate is the same as in the special case; (v) the long-run share of labor is always increasing in the share of public spending in infrastructure, and (vi) maximizing growth requires to levy a tax rate in excess of τ^*.

Keywords: Public R&D, Goodwin growth cycle, optimal fiscal policy

JEL Classification: D33, E11, O38

[*]Department of Economics, Colorado State University. 1771 Campus Delivery, Fort Collins, CO 80523-1771. Email: daniele.tavani@colostate.edu.

[†]Department of Social Sciences and Economics, Sapienza University of Rome. P.le Aldo Moro 5, Rome Italy 00185. Email: luca.zamparelli@uniroma1.it.

[‡]We thank participants to the Analytical Political Economy Workshop 2016 for very helpful comments on a previous draft. The usual disclaimer applies.

1 Introduction

The seminal paper on the growth cycle by Goodwin (1967) provides a representation of the interaction between the accumulation of capital and the functional income distribution in a market economy. Savings out of profit incomes finance investment in physical capital. Capital accumulation raises the demand for labor, which in turn puts upward pressure on real wages relative to labor productivity, thus increasing the share of output accruing to workers. Once the labor share picks up, profitability suffers, and accumulation slows down. Employment will recede, and real wages will fall relative to labor productivity. At this point, profitability is restored, and accumulation can pick up again. As a result, the model produces endless cycles of employment and labor share around its steady state, which is fully determined by the capitalists' propensity to save, the exogenous growth rate of population, the exogenous growth rate of labor productivity, and the degree of conflict in the labor market. Since the steady state is never reached, we can say that the distributional conflict determining the growth cycle is never settled (van der Ploeg, 1987).

The original model has been refined and extended along several dimensions. On the one hand, there is a series of contributions, surveyed in Mohun and Veneziani (2006), addressing the dynamic problems related to the so-called 'structural instability' of the Goodwin growth cycle, namely that the endless cycles predicted by the model are not robust to small modifications of its main assumptions. Related, but with an eye on whether or not the distributive conflict is resolved in the long run, there is the literature on the role played by induced labor-augmenting technical change –that is, the dependence of labor productivity growth on the share of labor in production– in shaping cyclical growth and distribution patterns initiated by Shah and Desai (1981), further elaborated by van der Ploeg (1987), and recently revived by Foley (2003); Julius (2006). Desai (1973) introduces anticipated inflation as a stabilizing force, while financial considerations led Keen (1995) to find chaotic behavior, and a stabilizing role for the government. Finally, some contributions have been made in order to provide what could be seen as microeconomic foundations for the growth cycle. An early game-theoretic attempt can be found in Mehrling (1986), while a more recent model based on dynamic bargaining appears in Tavani (2013).

However, with the notable exception of Glombowski and Kruger (1984) who introduce taxation and unemployment benefits in the Goodwin model, little effort has been devoted to understand how the presence of a public sector affects the cyclical behavior of distribution and employment on the one hand, and the long run equilibrium around which they fluctuate on the other. This paper is an attempt in that direction. We consider a government sector collecting taxes in order to finance its expenditure on two alternative uses: investment in infrastructure capital, as well as investment in research and development (R&D). The accumulation of public capital increases the productivity of private capital stock, while public R&D augments labor productivity growth. Both uses of funds have strong public goods features.

Analyzing this twofold role of the public sector in the Goodwin model is relevant for several reasons. First, the two types of investment have opposite effects on the demand for labor and, in turn, on the labor share. A higher level of infrastructure capital enables private capital to employ more workers, while investment in R&D makes individual workers more productive thus lowering the economy's labor requirements. The growth rate of the labor force being exogenous, changes in labor demand are accommodated in equilibrium by adjustments in the labor share.[1] Hence, both the size (that is, the tax rate) and the composition (that is, the relative share of infrastructure vs. R&D spending) of government expenditure affect the distribution of income.

Second, our analysis provides a framework to study the role of the 'entrepreneurial state' in an economy's growth, employment, and distribution path. Recent influential work by Mariana Mazzucato (Mazzucato, 2013) has advocated for active industrial policies and a strong involvement of governments in the development of new technologies. In our model, assuming that productivity growth depends on public R&D investment makes long-run growth and employment dependent on fiscal policy. Moreover, embedding public R&D in the Goodwin model enables us to emphasize the distributive implications of promoting innovation, as the equilibrium labor share is affected by labor productivity growth. Distributive considerations, as they pertain to the active role of the state on innovation, are mostly absent in Mazzucato (2013).

To gain intuition on these linkages, we first study a special case of the model in which labor productivity growth depends entirely on public research and show that, provided that the output-elasticity of public infrastructure is greater than the elasticity of labor productivity growth to public R&D, there exists a tax rate τ^* that maximizes the labor share at the steady state, but not a growth-maximizing –or equivalently, an employment-maximizing–

[1] In our model, we actually assume a constant population size, which enables us to get closed form solutions. Including a constant population growth rate would not change the qualitative features of our results.

tax rate. Further, the steady state labor share is always increasing in the share of taxes spent in infrastructure investment; finally, we show that the presence of public R&D is not enough to stabilize the distributive conflict. We then study a more general model with induced technical change where, as is well known in the literature, the distributive conflict is resolved in the long run. With induced technical change, the long run value of the labor share is maximized at the same tax rate as in the special case, and again it is always increasing in the share of taxes spent in infrastructure investment; but maximizing growth and employment requires to levy a tax rate in excess of τ^*. Hence, our analysis shows that the wage share- and growth- maximizing tax rates do not coincide, and are related to the output and innovation elasticities of infrastructure and R&D investment.

Third, Goodwin-type employment-distribution dynamics seems to actually occur in industrialized countries –and the US in particular– at least qualitatively (Barrales and von Arnim, 2015; Fiorio, Mohun and Veneziani, 2013; Harvie, 2000). Yet, both the period of the cycles and the steady state around which the cycles happen appear quite volatile (as documented in Barrales and von Arnim 2016; Tavani and Zamparelli 2015), but the Goodwin (1967) model is of little help in identifying policy shocks that can contribute to these changes. In fact, the traditional parameters that shift the steady state of the Goodwin (1967) model are basically policy-invariant. Conversely, we clearly identify policies that can affect the long run equilibrium and the dynamics of the model. On the one hand, changes in both the tax rate and the composition of government spending between infrastructure and R&D –two explicit policy variables– will determine a shift in the long-run values for the growth rate, the labor share, and employment. On the other hand, the relative strength of public R&D *vs.* private incentives such as induced technical change on labor productivity growth will influence the stability property of the steady state and the period of the cycles: the stronger the weight of induced technical change on labor productivity growth, the faster the convergence to the steady state. Absent induced technical change, the model generates a limit cycle.

Fourth, the productive role of public infrastructure –or government spending in general– on GDP growth and income distribution is well understood in the mainstream economic literature (Aschauer, 1989, 2000; Barro, 1990; Devarajan et al., 1996; Glomm and Ravikumar, 1997; Holtz-Eakin, 1994; Irmen and Kuehnel, 2009; Turnovsky, 2015), and several empirical contributions have studied the effects of public R&D on growth and on private R&D (see for example Cohen et al., 2002; Levy, 1990); but the theoretical literature on public research –be it mainstream or not– is surprisingly thin (exceptions being Konishi (2016); Spinesi (2013)). Moreover, to the best of our knowledge no paper has attempted to investigate simultaneously the productive role of public investment in physical capital and public R&D.

The remainder of the paper is organized as follows. Section 2 outlines the main features of the model. A special case without induced technical change is analyzed in Section 3, while Section 4 studies the more general model. Section 5 concludes. Proofs of our main results are provided in the Appendix.

2 Basic Elements of the Model

2.1 Production, Income Shares, and Accumulation

We consider a one-good, closed economy with a government sector. The final good Y is produced by competitive firms using fixed proportions of aggregate capital stock $\tilde{K}$ and effective labor AL. We follow our previous work on this topic (Tavani and Zamparelli, 2016) in assuming aggregate capital to be a twice continuouosly differentiable, linearly homogeneous function $H : \mathbb{R}^2_+ \to \mathbb{R}$ of public capital X and private capital K, which amounts to impose imperfect substitutability between the two stocks. Denoting the public-to-private capital ratio by χ, we have

$$Y = H(X, K) = KH\left(\frac{X}{K}, 1\right) \equiv Kh(\chi) = AL. \tag{1}$$

At each moment in time, firms take the output/capital ratio $h(\chi)$ as a given. For concreteness, we assume that $h(\chi)$ has constant elasticity, denoted by $\eta \equiv \frac{h'(\chi)}{h(\chi)}\chi$. Each of the $L = h(\chi)K/A$ employed workers in the economy receives the same real wage w. Denoting the share of labor in output by $\omega = w/A$, firms' profits before taxes are $\Pi = Y - wL = Y(1 - \omega)$.

As it is customary in two-class models, we assume that savings occurs out of capital income only. In order to derive closed form solutions to our model, we assume a constant saving rate $s \in (0, 1)$ and rule out depreciation. With time flowing continuously, the growth rate of capital stock is

$$g_K \equiv \frac{\dot{K}}{K} = sh(\chi)(1 - \omega)(1 - \tau), \tag{2}$$

where τ is the the tax rate on profits. Finally, and again for the sake of obtaining closed form solutions, we assume a constant labor force N.

2.2 Government

The government sector taxes both profits and wages at the same rate $\tau \in [0,1]$. This is equivalent to levying an income tax on the overall economy. In fact, total tax receipts for the government are $\tau(wL+\Pi) = \tau[\omega+(1-\omega)]Y = \tau h(\chi)K$. Taxes collected by the government have two purposes: on the one hand, they finance the accumulation of public capital, $\dot{X}$. On the other hand, tax revenues finance publicly-funded R&D investment R_G. Denoting by $\theta \in (0,1)$ the fraction of government spending that goes to public investment, and imposing a balanced budget, we have the following relations:

$$g_X \equiv \frac{\dot{X}}{X} = \theta\tau\frac{h(\chi)}{\chi}, \tag{3}$$

$$\frac{R_G}{Y} = (1-\theta)\tau. \tag{4}$$

2.3 Innovation

We assume that labor productivity growth g_A is log-linear in the share of public R&D in output R_G/Y and on the labor share via induced technical change:

$$\begin{aligned} g_A \equiv \frac{\dot{A}}{A} &= \lambda\left(\frac{R_G}{Y}\right)^{\phi}\omega^{\beta} \\ &= \lambda\left[(1-\theta)\tau\right]^{\phi}\omega^{\beta}, \;\; \phi \in (0,1), \;\; \beta \in [0,1). \end{aligned} \tag{5}$$

Our innovation technology has two components. The first is based on the endogenous growth literature, which generally considers the flow of newly produced technologies $\dot{A}$ to depend positively on R&D inputs (R_G), and on the existing level of technology itself (A). This specification has two features: a linear spill-over from the stock of technology to the production of new ideas, which is useful to produce endogenous growth; and a normalization of R&D investment which excludes explosive growth. The latter is typically justified with the argument of increasing complexity of discovering new ideas, or the dilution argument of R&D investment over an increasing number of sectors. New ideas are made available freely and immediately to the private sector. The peculiarity of our assumption is that R&D investments are carried out of by the public sector only. While being an obvious simplification, this assumption highlights the importance of the entrepreneurial state emphasized by Mazzucato (2013).

The role of the private sector in promoting labor productivity growth is captured by the second component of the innovation technology. We follow the induced innovation hypothesis in assuming that the growth rate of labor productivity depends on the labor share. A higher labor share represents higher unit labor costs for individual firms, which then have an incentive to save on labor requirements and introduce labor-saving innovations.

2.4 Dynamics of the Public-to-Private Capital Ratio

One of the main implications of introducing infrastructure spending by the government sector in the model is that the public-to-private capital ratio becomes a state variable of the model. Its law of motion is:

$$g_\chi \equiv \frac{\dot{\chi}}{\chi} = g_X - g_K = h(\chi)\left[\frac{\theta\tau}{\chi} - s(1-\omega)(1-\tau)\right] \tag{6}$$

2.5 Dynamics of Employment

As in the basic Goodwin model, we consider the employment rate $e \equiv L/N$ as a state variable of our setup. Given equation (1) and the assumed constancy of population, the evolution of the employment rate over time is:

$$\begin{aligned} g_e \equiv \frac{\dot{e}}{e} &= \eta g_\chi + g_K - g_A \\ &= \eta h(\chi)\left[\frac{\theta\tau}{\chi} - s(1-\omega)(1-\tau)\right] \\ &= sh(\chi)(1-\omega)(1-\tau) - \lambda\left[(1-\theta)\tau\right]^\phi \omega^\beta. \end{aligned} \tag{7}$$

2.6 Dynamics of Income Shares

The third state variable of the model is the labor share. In typical Goodwin (1967) fashion, we assume that the real wage grows with employment, according to a version of the Phillips Curve: $\dot{w}/w = f(e)$, $f'(\cdot) > 0$. Therefore, using (5), we have:

$$\begin{aligned} g_\omega \equiv \frac{\dot{\omega}}{\omega} &= f(e) - g_A \\ &= f(e) - \lambda\left[(1-\theta)\tau\right]^\phi \omega^\beta \end{aligned} \tag{8}$$

Goodwin assumed a strictly convex function $f(e)$. In what follows, we impose $f(e) = e^{1/\delta}, \delta \in (0,1)$.

We thus have a three-dimensional dynamical system formed by equations (6), (7), and (8) to study. We first focus on a special case of the model where there is no role for induced technical change, that is with $\beta = 0, \phi \in (0,1)$. This is in line with the lack of distributive considerations in Mazzucato (2013). Such a special case is very tractable analytically, and quite close to the original Goodwin (1967) model. An important difference, however, is that fiscal policy matters in the long run for distributive purposes.

3 A Special Case

In order to characterize the steady state, let us begin with the public-to-private capital stock ratio. Setting $g_\chi = 0$, we first find the following *isocline*:

$$\chi(\omega) = \frac{\theta\tau}{s(1-\omega)(1-\tau)}, \tag{9}$$

which is increasing in the labor share. A higher labor share reduces private capital accumulation; being public capital accumulation decreasing in χ, a higher level of χ is required to ensure $g_X - g_K = 0$.

Next, setting $g_e = 0$ in equation (7) we can characterize the isocline relating the labor share to the public-to-private capital ratio:

$$1-\omega(\chi) = \frac{1}{(1-\eta)s(1-\tau)}\left[\frac{\lambda\left[(1-\theta)\tau\right]^{\phi}}{\chi^{\eta}} - \frac{\eta\theta\tau}{\chi}\right]. \tag{10}$$

Along the isocline, the labor share is U-shaped in the public-to-private capital ratio. In order to obtain the steady state value of the labor share in terms of parameters only, substitute the value $\chi(\omega)$ from equation (9):

$$1-\omega_{ss} = \frac{\lambda^{\frac{1}{1-\eta}}(1-\theta)^{\frac{\phi}{1-\eta}}}{s(1-\tau)\tau^{\frac{\eta-\phi}{1-\eta}}\theta^{\frac{\eta}{1-\eta}}}. \tag{ω}$$

The steady state share of labor is always increasing in the proportion of tax revenues spent on the accumulation of public capital (θ). Accumulating public capital raises labor demand, while investing in public R&D allows the economy to economize on labor requirements. Thus, a shift in the composition of government expenditure in favor of public investment puts pressure on the exogenous labor force, which is then able to capture a larger share of output.

With respect to the tax rate, if the output-elasticity of public capital is greater than the innovation-elasticity of public research, the steady state labor share is hump-shaped in the tax rate. In fact, we can state the following result.

Proposition 1. *Suppose that $1 > \eta > \phi$. Then, there exists an interior value $\tau^* = \frac{\eta-\phi}{1-\phi} \in (0,1)$ such that the steady state labor share is maximized.*

Proof. See Appendix A.

The intuition for our result is the following. Government spending has two effects on the labor share. On the one hand, public infrastructure investment reinforces capital accumulation: it increases employment everything else equal, thus putting pressure on real wages relative to labor productivity. The strength of this effect on the labor share depends on the output elasticity of public capital η. On the other hand, public R&D increases labor productivity, thus lowering unit labor costs in production everything else equal. The strength of this effect on the labor share is captured by the R&D elasticity ϕ. If $\eta < \phi$, the labor share is always decreasing in the tax rate. The negative effect of innovation on labor demand is stronger than the positive capital accumulation effect; labor demand falls relative to the labor force, and the labor share decreases. In this case, distributive considerations would push the government sector to levy a tax rate as small as possible; but this would reduce funds for both infrastructure and R&D spending. If instead $\eta > \phi$, the public sector can levy taxes so that the two effects balance each other, and the labor share is maximized. □

Next, we can find the steady state public-to-private capital stock ratio by plugging ω_{ss} into (9):

$$\chi_{ss} = \left(\frac{\theta}{\lambda}\right)^{\frac{1}{1-\eta}} \frac{\tau^{\frac{1-\phi}{1-\eta}}}{(1-\theta)^{\frac{\phi}{1-\eta}}}. \tag{χ}$$

Intuitively, the long-run public-to-private capital ratio rises with the tax rate and the share of government expenditure employed in public physical capital investment. Finally, the steady state employment rate is found, from (8), as:

$$e_{ss} = \left\{\lambda[(1-\theta)\tau]^{\phi}\right\}^{\delta}. \tag{e}$$

Real wages being a positive function of employment, a higher labor productivity growth requires a higher employment rate to stabilize the labor share. At a steady state, $g_{Y,ss} = g_{A,ss} = f(e_{ss})$, so that long run growth and employment move together, and the growth maximizing policy and the employment maximizing policy coincide. Higher taxes and a higher share of tax revenues invested in public R&D simultaneously raise both labor productivity growth and employment. Hence, there is no growth- or employment-maximizing strategy in this case. The government only faces the constraint that at $\tau = 1$ private capital accumulation drops to zero.

Regarding the stability properties of the steady state, Appendix D shows that this special case gives rise to a limit cycle. This result is similar to the Goodwin (1967) cycle, though it involves the additional state variable χ.

3.1 Simulation Results

We used *Mathematica* to numerically simulate the special case discussed in this section. The output elasticity of public capital for the United States can be found in the literature. Despite the initial estimates by Aschauer (1989) were in the magnitude of 40%, more recent research, surveyed in Isaksson (2009), suggest to calibrate η around .15. In order to restrict our attention to the interesting case where $\eta > \phi$, we set $\phi = .05$ for this simulation round.

In order to calibrate government spending, a figure for infrastructure (and water) spending can be obtained from the US Congressional Budget Office. The post-war federal average for the US is 2.4% of GDP which, since in our model $\dot{X} = \theta\tau Y$, anchors $\theta\tau = .024$. On the other hand, we found National Science Foundation figures for the share of public financing of innovation in GDP around $1.2\% = (1-\theta)\tau$. We can thus calibrate both the composition parameter θ and the tax rate τ using these two equations. Notice that, given the small size of the two average values for government spending to match, the solution will return a pretty low tax rate (which is the variable that scales government spending in our model). This, however, is harmless, because in our framework the only two uses of government spending are infrastructure spending and public R&D. Thus, the values obtained for τ and θ using our calibration strategy are those consistent with a (hypothetical) government sector only performing these two roles and running a balanced budget. Our values are $\theta = .667, \tau = .036$.[2] Using these values we can then calibrate λ, the scale parameter in the innovation function (5), and the wage-Phillips curve parameter δ in order to match a long-run growth rate of 2% and a long-run unemployment rate of 5%. The final parameter to calibrate is the saving rate. We calibrate it using equation (ω) to match a long-run value for the labor share of 2/3, in line with the average labor share in post-war United States. We thus use $s = .06$ for this round.[3]

Figure 1 displays the simulation results over 400 periods. From an initial condition with a labor share of .7 and an employment rate of .89, the dynamics quickly approach the limit cycle showing the familiar counter-clockwise cycles in the labor share - employment plane (left panel). The right panel

[2] Notice also that this calibration is not 'normative', but 'positive' in nature, because it does not require the tax rate to be at its labor share-maximizing value.

[3] Notice that the simulated employment rate can in principle leave the unit square, even though in our calibration it does not. This is a well-known limitation of the Goodwin model, pointed out by Desai et al. (2006). Avoiding the issue altogether would imply to drastically modify the wage-Phillips curve, and would come at the expenses of the tractability of the model. Notice finally that these simulations are only illustrative of the limit cycle that occurs under this special case of the model.

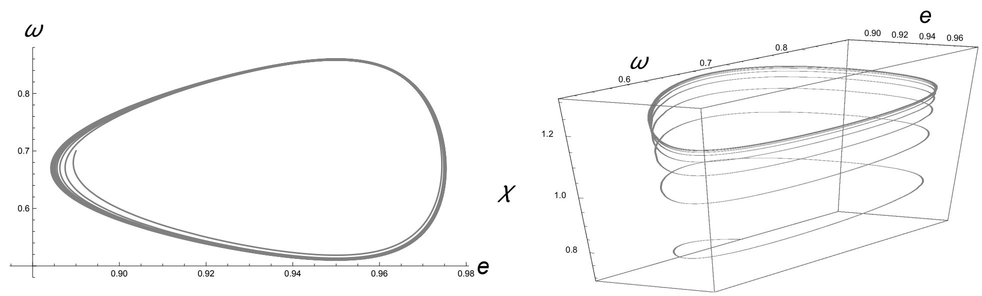

Figure 1: Simulation results with $\beta = 0$.

displays the full three dimensional plot.

4 A more General Model with Induced Technical Change

Let us now consider the more general case for the innovation technology, that is when we allow for a positive influence of the labor share on labor productivity growth via induced technical change. Public and private capital accumulation are independent of labor productivity growth, so that the evolution of the public-to-private capital ratio χ in the general model is not affected by the generalization in the innovation technology. Hence, equation (9) is still the solution to $g_\chi = 0$.

The general innovation technology does, however, change the dynamics of employment. Setting $g_e = 0$ in equation (7) when $\beta > 0$, and using equation (9) yields the steady state labor share as the solution to

$$\frac{\omega_{ss}^{\beta}}{(1-\omega_{ss})^{1-\eta}} = \frac{[s(1-\tau)]^{1-\eta}\,\tau^{\eta-\phi}\theta^{\eta}}{\lambda(1-\theta)^{\phi}}. \qquad (\omega')$$

Despite (ω') not having an explicit solution, we can show that the tax rate and the composition of public expenditure have the same effect on the labor share as in (ω). In fact, total differentiation of (ω') shows that the steady state labor share is still hump-shaped in the tax rate, and that the labor share maximizing tax rate is once again $\tau^* = \frac{\eta-\phi}{1-\phi} \in (0,1)$, provided that $1 > \eta > \phi$, as in Proposition (1). Appendix A provides a proof. Even in the general case, and for the same reason, if $\eta < \phi$ the labor share is always decreasing in the tax rate.

With respect to the composition of public expenditure, notice that the left hand side of (ω') is an increasing function of the labor share. The right hand side is a positive function of θ, so that raising the share of taxes spent on the accumulation of public capital has a positive effect on the labor share.

Productivity growth also influences the dynamics of income shares, as it is clear from equation (8). As before, this equation solves for the steady state employment rate, which under $\beta > 0$ becomes:

$$e_{ss} = \left\{ \lambda[(1-\theta)\tau]^{\phi} \omega_{ss}^{\beta} \right\}^{\delta}. \qquad (e')$$

The induced innovation hypothesis establishes a positive relation between the steady state employment rate and labor share. This feature of the model appealingly fits with the notion of a *wage curve,* as estimated by Blanchflower and Oswald (1995). We show in the Appendix A that the growth and employment maximizing tax rate satisfies $\tau > \frac{\eta-\phi}{1-\phi}$. The intuition is clear. Productivity growth depends on the tax rate both directly, as taxes finance public R&D investment, and indirectly via the influence of the tax rate on the labor share. Since the first effect is always positive, the growth-maximizing tax rate must be higher than the labor share-maximizing one.

On the other hand, the composition of public expenditure has two opposite effects on growth and employment. The share of tax revenues spent on capital accumulation increases the labor share and thus has a positive influence on productivity growth; at the same time, however, it may harm growth by reducing public R&D investment. We show in Appendix B that there may exist a growth-maximizing composition of public expenditure (θ^*) where the two effects, at the margin, offset each other. In general, θ^* will be a function of the elasticities of infrastructure spending and public R&D, of the private incentives to save on labor costs, and of the overall saving rate of the economy; but we cannot find a closed- form solution for these relations.

As far as stability is concerned, Appendix C shows that the steady state is a stable focus, in line with the literature which introduced a dependence of labor productivity growth on the labor share in the Goodwin model (Foley, 2003; Julius, 2006; Shah and Desai, 1981; van der Ploeg, 1987). Figure 2 shows the results of a 400 periods simulation round obtained under $\beta = .25$. while Figure 3 displays the results of a simulation run under $\beta = .5$. The initial conditions on the labor share and the employment rate are the same as above: in both figures, the left panel presents a two-dimensional slice of the plot, and it clearly displays both the counterclockwise movement and the converging path to the steady state. The right panel displays the full three dimensional plot as before. It is clear that convergence to the steady

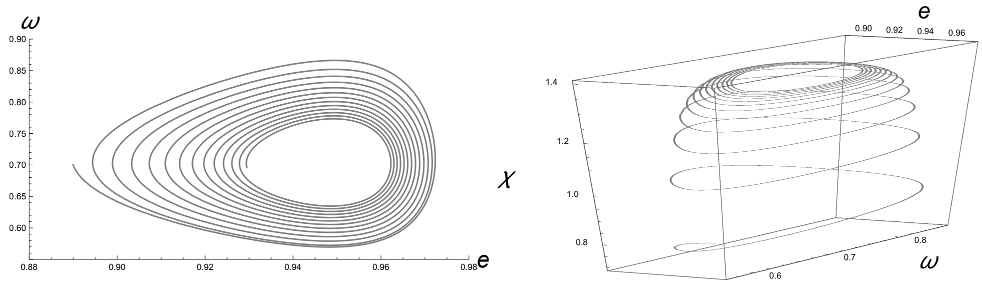

Figure 2: Simulation results of the model with induced technical change ($\beta = .25$, everything else as before).

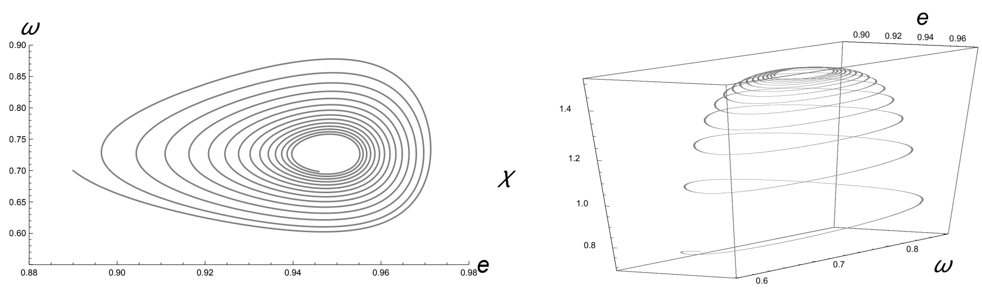

Figure 3: Simulation results of the model with induced technical change ($\beta = .5$, everything else as before).

state occurs faster the higher the value of the elasticity parameter β.

5 Conclusion

In this paper, we introduced a government sector as provider of public infrastructure as well as public R&D in an otherwise off-the-shelf growth cycle model based on Goodwin (1967), with and without induced technical change. We showed that such modification delivers important insights toward an understanding of the role of policy making in shaping the growth, employment, and distribution path of an economy. On the one hand, the accumulation of public capital fosters employment and wage growth, as well as having a positive level effect on GDP; on the other hand, public R&D increases labor productivity which keeps employment and the labor share in check.

A general conclusion of our model is that the growth-maximizing tax rate (when there is one) and the labor share-maximizing tax rate do not coincide, with the implication that the public sector is not neutral with respect to an economy's steady state growth and distribution path. The extent of the difference depends on the elasticities of infrastructure spending and public R&D, on the private incentives to save on labor costs, and on the overall saving rate of the economy. In this regard, our model provides additional channels to evaluate the impact of policy making on long-run growth and employment on the one hand, and income distribution on the other.

Interestingly enough, however, the mere presence of a government sector engaged in a time-invariant allocation policy over infrastructure and innovation is not enough to dampen the growth cycle: just like in the induced technical change literature, a negative feedback of the labor share onto its own rate of change is necessary for stability to occur, and this negative feedback is unrelated to policymaking in our framework. With respect to the endless (or not) dynamic unfolding of the distributive conflict between labor and capital, an entrepreneurial state setting its fiscal policy with a certain long-run goal appears like a neutral spectator.

A Proof of Proposition 1

A.1 Special Case

Maximizing the labor share is equivalent to minimizing the natural logarithm of its complement (that is, the share of profits) $1-\omega_{ss}$ as written in the RHS of equation (ω). We have that

$$\frac{\partial \ln(1-\omega_{ss})}{\partial \tau} = \frac{1}{1-\tau} - \left(\frac{\eta-\phi}{1-\eta}\right)\frac{1}{\tau},$$

and

$$\frac{\partial^2 \ln(1-\omega_{ss})}{\partial \tau^2} = \frac{1}{(1-\tau)^2} + \left(\frac{\eta-\phi}{1-\eta}\right)\frac{1}{\tau^2} > 0.$$

Because the steady state profit share is a convex function of τ, the first order condition $\partial \ln(1-\omega_{ss})/\partial\tau = 0$ is necessary and sufficient for a minimum. It has an interior solution in

$$\tau^* = \frac{\eta-\phi}{1-\phi} \in (0,1).$$

A.2 General Model

In order to prove the same result in the general model, total differentiate (ω') with respect to ω and τ to find

$$\left(\frac{(1-\omega_{ss})\beta+(1-\eta)\omega_{ss}}{\omega_{ss}^{1-\beta}(1-\omega_{ss})^{2-\eta}}\right)d\omega=\frac{s^{1-\eta}\theta^{\eta}}{\lambda(1-\theta)^{\phi}}\left(\frac{\eta-\phi-\tau(1-\phi)}{(1-\tau)^{\eta}\tau^{1-(\eta-\phi)}}\right)d\tau,$$

hence,

$$\frac{d\omega}{d\tau}=\frac{s^{1-\eta}\theta^{\eta}}{\lambda(1-\theta)^{\phi}}\frac{\omega_{ss}^{1-\beta}\left(1-\omega_{ss}\right)^{2-\eta}}{(1-\tau)^{\eta}\tau^{1-(\eta-\phi)}}\frac{[\eta-\phi-\tau(1-\phi)]}{[(1-\omega_{ss})\beta+(1-\eta)\omega_{ss}]}. \tag{11}$$

The denominator is always positive. It follows that $Sign\frac{d\omega}{d\tau}=Sign[\eta-\phi-\tau(1-\phi)]$, which proves that the labor share is maximized by $\tau^*=\frac{\eta-\phi}{1-\phi}$.

B Growth maximizing tax rate and composition of public expenditure

Taking logs in equation (5) evaluated at the steady state, we have $\ln g_A=\ln\left[\lambda(1-\theta)^{\phi}\tau^{\phi}\omega(\tau,\theta)_{ss}^{\beta}\right]=\ln\lambda+\phi\ln(1-\theta)+\phi\ln\tau+\beta\ln\omega(\tau,\theta)_{ss}$. Hence, $\frac{d\ln g_A}{d\tau}=\dfrac{\phi}{\tau}+\dfrac{\beta}{\omega}\frac{d\omega}{d\tau}$. Setting $\frac{d\ln g_A}{d\tau}=0$,while using 11 we have

$$\frac{1}{\tau}\left[\phi+\frac{\beta}{\lambda}\frac{\left(1-\omega_{ss}\right)^{2-\eta}}{\omega_{ss}^{\beta}}\frac{s^{1-\eta}\theta^{\eta}}{(1-\theta)^{\phi}}\frac{\tau^{\eta-\phi}}{(1-\tau)^{\eta}}\frac{[\eta-\phi-\tau(1-\phi)]}{[(1-\omega_{ss})\beta]+(1-\eta)\omega_{ss}]}\right]=0,$$

or

$$\phi=\frac{\beta}{\lambda}\frac{\left(1-\omega_{ss}\right)^{2-\eta}}{\omega_{ss}^{\beta}}\frac{s^{1-\eta}\theta^{\eta}}{(1-\theta)^{\phi}}\frac{\tau^{\eta-\phi}}{(1-\tau)^{\eta}}\frac{[\tau(1-\phi)-(\eta-\phi)]}{[(1-\omega_{ss})\beta+(1-\eta)\omega_{ss}]},$$

which requires $\tau(1-\phi)>\eta-\phi$, or $\tau>\frac{\eta-\phi}{1-\phi}$.

With respect to the growth maximizing composition of public expenditure (θ^*), totally differentiate (ω') with respect to ω and τ to find

$$\frac{d\omega}{d\theta}=\left(\frac{[s(1-\tau)]^{1-\eta}\tau^{\eta-\phi}}{\lambda}\right)\left(\frac{\eta-\theta(\eta-\phi)}{\theta^{1-\eta}(1-\theta)^{1+\phi}}\right)\left(\frac{\omega_{ss}^{\beta-1}\left(1-\omega_{ss}\right)^{2-\eta}}{(1-\omega_{ss})\beta+(1-\eta)\omega_{ss}]}\right).$$

Next, set $\frac{d\ln g_A}{d\theta} = \dfrac{-\phi}{1-\theta} + \dfrac{\beta}{\omega}\frac{d\omega}{d\theta} = 0$, to find

$$\frac{[s(1-\tau)]^{1-\eta}\tau^{\eta-\phi}}{\lambda}\frac{\eta-\theta^*(\eta-\phi)}{[(1-\omega_{ss})\beta+(1-\eta)\omega_{ss}]}\frac{(1-\omega_{ss})^{2-\eta}}{\omega_{ss}^{2-\beta}} = \frac{\phi}{\beta}\theta^{*(1-\eta)}(1-\theta^*)^{\phi}.$$

C Stability Analysis: General Model

Linearization of the system formed by equations (6), (7) and (8) around its steady state position, when $\beta \in (0,1)$, yields the following Jacobian matrix:

$$J(\chi_{ss}, e_{ss}, \omega_{ss}) \quad = \begin{bmatrix} J_{11} & 0 & J_{13} \\ 0 & 0 & J_{23} \\ 0 & J_{32} & J_{33} \end{bmatrix},$$

with

$$\begin{aligned} J_{11} &= -\tau\theta\chi_{ss}^{\eta-1} < 0; \\ J_{13} &= s(1-\tau)\chi_{ss}^{1+\eta} > 0; \\ J_{23} &= -e_{ss}\left\{(1-\eta)s(1-\tau)\chi_{ss}^{\eta} + \lambda\beta[1-\theta)\tau]^{\phi}\omega^{\beta-1}\right\} < 0; \\ J_{32} &= \delta^{-1}e_{ss}^{\frac{1-\delta}{\delta}}\omega_{ss} > 0; \\ J_{33} &= -\lambda\beta\left((1-\theta)\tau\right)^{\phi}\omega_{ss}^{\beta} < 0. \end{aligned}$$

The Routh-Hurwitz necessary and sufficient conditions for stability of the steady state require that:

1. $TrJ < 0$. We have that $TrJ = J_{11} + J_{33} < 0$ as required.

2. $DetJ < 0$. We have that $DetJ = J_{11} \times (-J_{23}J_{32}) < 0$ as required.

3. $PmJ > 0$, where PmJ denotes the sum of the principal minors of J. In fact, $PmJ = -J_{23}J_{32} + J_{11}J_{33} > 0$ as required.

4. Finally, we need to check that $-PmJ + DetJ/TrJ < 0$. Since $TrJ < 0$, the condition can be rewritten as $DetJ > TrJ(PmJ)$. We have $-J_{11}J_{23}J_{32} > (J_{11}+J_{33})\left[-J_{23}J_{32}+J_{11}J_{33}\right] = -J_{11}J_{23}J_{32} - J_{33}J_{23}J_{32} + J_{11}^2J_{33} + J_{11}J_{33}^2, \iff 0 > J_{33}\big(-J_{23}J_{32} + J_{11}^2 + J_{11}J_{33}\big)$, which is always true.

D Stability Analysis: Special Case

Linearization of the system formed by equations (6), (7) and (8) around its steady state position, evaluated at $\beta = 0$, yields the following Jacobian matrix:

$$J(\chi_{ss}, e_{ss}, \omega_{ss}) = \begin{bmatrix} J_{11} & 0 & J_{13} \\ 0 & 0 & J_{23} \\ 0 & J_{32} & 0 \end{bmatrix},$$

with

$$\begin{aligned} J_{11} &= -\tau\theta\chi_{ss}^{\eta-1} < 0; \\ J_{13} &= s(1-\tau)\chi_{ss}^{1+\eta} > 0; \\ J_{23} &= -(1-\eta)s(1-\tau)\chi_{ss}^{\eta}e_{ss} < 0; \\ J_{32} &= \delta^{-1}e_{ss}^{\frac{1-\delta}{\delta}} > 0. \end{aligned}$$

The Routh-Hurwitz necessary and sufficient conditions for stability of the steady state require that:

1. $TrJ < 0$. We have that $TrJ = J_{11} < 0$ as required.
2. $DetJ < 0$. We have that $DetJ = J_{11} \times (-J_{23}J_{32}) < 0$ as required.
3. $PmJ > 0$, where PmJ denotes the sum of the principal minors of J. It is easy to check that, in fact, $PmJ = -J_{23}J_{32} > 0$ as required.
4. Finally, we need to check that $-PmJ + DetJ/TrJ < 0$. This condition is violated. In fact, $DetJ/Tr_J = Pm_1J = PmJ$, so we have $-PmJ + PmJ = 0$. As argued by Julius (2006), when the fourth condition goes from negative (see the previous appendix) through zero the Hopf's bifurcation theorem implies that the system has a family of closed orbits in a neighborhood of the steady state. This is happening as β goes from positive to zero.

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