1 Introduction

Crude oil is one of the most important commodities affecting economic activities. In particular, a better understanding of crude oil markets would be crucial for portfolio allocation and risk management and asset pricing in practice. Crude oil markets closely move together. Thus, it is important to consider that financial market agents consider such co-movement, in particular, upper and lower tail dependencies of oil prices across the crude oil different markets. The analysis of the dependence structure could allow them to have portfolio selection and hedging strategies. It is also crucial to understand the volatility of the oil prices because persistent changes in volatility can expose the crude oil market participants to risk.

The motivation of this article is that we propose a new approach using the conditional leverage effect that can imply the asymmetric tail dependence. We investigate two types of asymmetries: the asymmetry in tail dependence between West Texas Intermediate (WTI) and Brent crude oil returns and the asymmetry in volatility of WTI returns conditional on Brent returns. Our contribution to the literature is threefold. First, our study provides better understanding of asymmetry of tail dependence and of asymmetry of conditional leverage effects by using relatively new techniques: copulas and asymmetric generalized autoregressive conditional heteroskedasticity (GARCH) regression models, respectively. Second, we try to find a clear relationship between the two apparently different asymmetries. The findings from two different approaches are congruent, in that there is no asymmetry of tail dependence and no conditional leverage effect in crude oil prices. Lastly, we develop the Threshold-Bilinear GARCH (TBL-GARCH) to the asymmetric TBL-GARCH regression. The model is flexible to be used in different data series.

There is a limited amount of recent literature on using copulas to study the dependence across financial markets. Chang (2012a) investigates both the interdependence between spot and futures returns and the individual dynamic process of the return series. Li and Yang (2013) attempt to find the relationships between the volatility of rubber futures and the oil index via the copula-based GARCH model. Other studies focus on the dependence structure and co-movements in stock markets using copulas (see Mensah and Alagidede (2017) for the dependence structure across African stock markets, and for Nguyen *et al.* (2016) the dependence structure between degree of dependency between gold and stock markets.

While the above literature focused on the dependence structure and co-movements, we utilize the copula approach in order to assess asymmetry in the dependence structure between crude oil markets. The asymmetry of tail dependence in Uhm *et al.* (2012) is a case in which the level of dependence at the upper tail is not equal to the level of dependence at the lower tail. Tong *et al.* (2013) find asymmetry in tail dependence between crude and heating oil returns and between crude oil and jet fuel returns by using the asymmetric copulas. Reboredo (2011) employs various copula models with time-invariant and time-varying dependence structures and finds no evidence of asymmetric tail dependence between different crude oil spot market prices for WTI, Brent, Dubai, and Maya.

To study the asymmetry of tail dependence between WTI and Brent crude oil returns, we specify a joint model for dependence with various dependence structures. For example, the Gaussian copula has no-tail dependence, the Clayton copula has lower tail dependence, the Plackett copula has symmetric tail dependence, the Frank copula has symmetric tail dependence, the Gumbel copula has upper tail dependence, the Student-t copula has symmetric tail dependence, and the symmetrized Joe-Clayton (SJC) copula by Patton (2006) allows for asymmetric tail dependence and nest symmetry as a special case.

One of the well-established features in the financial time series is the leverage effect (Black 1976; Engle and Ng 1993). It is well-known that the impact of a negative shock on volatility is stronger than that of positive one. This characteristic is captured by the asymmetric volatility. A number of recent empirical studies have been dedicated to examining asymmetry in crude oil volatility. Fan *et al.* (2008) find a spillover effect between WTI and Brent crude oil prices and an asymmetric leverage effect in the WTI returns by using various specifications of the GARCH models. Nomikos and Andriosopoulos (2012) also find a leverage effect for WTI spot markets and an inverse leverage effect for the natural gas spot prices. However, another empirical studies (e.g., Agnolucci 2009; Cheong 2009; Chang 2012b) find no leverage effect for the WTI market.

We examine the conditional leverage effect of the WTI return series given Brent oil prices by employing three different asymmetric GARCH regression models, such as Threshold GARCH (T-GARCH), Bilinear GARCH (BL-GARCH), and TBL-GARCH regression models. The T-GARCH model proposed by Zakoian (1994) is typically used to capture the leverage or asymmetric effect in the volatility by introducing thresholds in the volatility equation. The BL-GARCH model proposed by Storti and Vitale (2003) captures the asymmetry by including the interactions between past observations and volatilities in the volatility equation. Recently Choi *et al.* (2012) proposed the asymmetric GARCH model featuring both threshold effect and bilinear structure. In this paper, we develop the TBL-GARCH to a regression type model in order to examine the conditional leverage effect of oil prices. A comparative study between the three asymmetric GARCH regression models is carried out in this article. All alternative specifications consistently show the evidence that revels no conditional leverage effect, while the best fitting model is BL-GARCH in our study.

The remainder of this paper is structured as follows. The next section introduces the econometric methodologies, such as copula methods and asymmetric GARCH models. In Section 3, we discuss our empirical results regarding the asymmetry of tail dependence and the asymmetry of conditional leverage effect in oil prices. Concluding remarks are presented in Section 4.

2 Econometric Methodology

2.1 Copula Methods

The dependence structure of a set of random variables is contained within F. The idea of separating F into one part which describes the dependence structure and other parts which describe only the marginal behavior has led to the concept of a copula. A copula is a multivariate uniform distribution representing a way of trying to extract the dependence structure of the random variables from the joint distribution function. It is a useful approach to understanding and modeling dependent random variables. Every joint distribution can be written as $F_{XY}(x,y) = C(F_X(x), F_Y(y))$ where F_X and F_Y are marginal distributions. A bivariate copula is a function $C : [0,1]^2 \rightarrow [0,1]$, whose domain is the entire unit square with the following three properties:

- $C(u,0) = C(0,v) = 0, \quad \forall u, v \in [0,1]$
- $C(u,1) = C(1,u) = u, \quad \forall u \in [0,1]$
- $C(u_1,v_1) - C(u_1,v_2) - C(u_2,v_1) + C(u_2,v_2) \geq 0, \ \forall u_1,u_2,v_1,v_2 \in [0,1]$ such that $u_1 \leq u_2$ and $v_1 \leq v_2$.

The coefficients of upper and lower tail dependence of (X, Y) are defined by Nelsen (2006) as

$$\tau^L = \lim \epsilon \rightarrow \infty P[U \leq \epsilon | V \leq \epsilon] = \lim \epsilon \rightarrow \infty P[V \leq \epsilon | U \leq \epsilon] \text{ and}$$
$$\tau^U = \lim \epsilon \rightarrow 1 P[U > \epsilon | V > \epsilon] = \lim \epsilon \rightarrow 1 P[V > \epsilon | U > \epsilon] \tag{1}$$

where $\tau^L \in [0,1]$ and $\tau^U \in [0,1]$. The Gaussian copula has $\tau^U = \tau^L = 0$, meaning that in the extreme tails of the distribution, the variables are independent. The normal distribution is the most common assumption in finance but it does not have tail-dependence while the Gumbel copula has right tail dependence $\tau^U = 2 - 2^{\frac{1}{2}}$. Survival copula has left tail dependence. Clayton's copula has contours that are quite a bit more peaked for negative events than they are for joint positive events.

Patton (2006) points out one major drawback of the Joe-Clayton copula: even when the two tail dependence measures are equal, there is still some (slight) asymmetry in the

Joe-Clayton copula. The Joe-Clayton copula (Joe, 1997) is:

$$C_{JC}(u, \upsilon|\tau^U, \tau^L) = 1 - (1 - \{[1 - (1 - u)^\kappa]^{-\gamma} + [1 - (1 - \upsilon)^\kappa]^{-\gamma} - 1\}^{-1/\gamma})^{1/\kappa},$$

where $\kappa = 1/\log_2(2 - \tau^U)$, $\gamma = -1/\log_2(\tau^L)$, $\tau^U \in (0, 1)$, and $\tau^L \in (0, 1)$. To investigate asymmetry of dependence by using both tail dependencies, Patton (2006) proposes the symmetrized Joe-Clayton (SJC) copula:

$$C_{SJC}(u, \upsilon|\tau^U, \tau^L) = 0.5 \times [C_{JC}(u, \upsilon|\tau^U, \tau^L) + C_{JC}(1 - u, 1 - \upsilon|\tau^U, \tau^L) + u + \upsilon - 1].$$

A copula model used in econometrics is the time-varying normal copula :

$$C^G(u, \upsilon|\rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(\upsilon)} \frac{1}{2\pi\sqrt{(1 - \rho^2)}} \exp\left\{\frac{-(r^2 - 2\rho rs + s^2)}{2(1 - \rho^2)}\right\} drds \ ,$$

where Φ^{-1} is the inverse of the standard normal distribution function and

$$\rho_t = \tilde{\Lambda}\left\{\omega_\rho + \beta_\rho \rho_{t-1} + \alpha_\rho \cdot \frac{1}{10} \sum_{j=1}^{10} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(\upsilon_{t-j})\right\},$$

where $\tilde{\Lambda} = (1 - e^{-x})(1 + e^{-x})^{-1} = \tanh(x/2)$ is the modified logistic transformation as Patton (2006).

2.2 Asymmetric GARCH Models

The main source of asymmetry in the volatility of the financial time series is related to the so called leverage effect first noted by Black (1976). As shown in many empirical studies, positive and negative innovations have different impacts on future volatility. There is a long list of variations of GARCH models that consider the asymmetry. In this article, we study the existence of conditional leverage effects by using asymmetric GARCH regression models for WTI oil given Brent oil prices. For the asymmetric GARCH regression model, we have three different asymmetric GARCH specifications: the T-GARCH model (Glosten *et al.*, 1993; Zakoian, 1994), which considers a threshold effect in modeling volatility, the BL-GARCH model (Storti and Vitale, 2003) with the bilinear structure, and the TBL-GARCH model (Choi *et al.*, 2012), which combines the threshold effect with the bilinear structure in order to generate a broader class of asymmetric GARCH models. For more details about the asymmetric GARCH models, see Engle and Ng (1993).

Let e_t be independent identically distributed random variables with zero mean and unit variance, for $t = 1, 2, \cdots, T$. The AR(1)-T-GARCH(1, 1) regression model is expressed as

$$\begin{aligned}
\text{Mean equation :} \quad Y_t &= a + b \times X_t + \epsilon_t, \;\; \epsilon_t = \phi_1 \times \epsilon_{t-1}, \;\; \epsilon_{t-1} = \sqrt{h_{t-1}} e_{t-1}, \;\; \text{where } \; \epsilon_{t-1} \sim N(0, h_{t-1}) \\
\text{Variance equation :} \quad h_t &= \alpha_0 + \alpha_{11} \times (\epsilon^+_{t-1})^2 + \alpha_{12} \times (\epsilon^-_{t-1})^2 + \beta_1 \times h_{t-1} \qquad (2)
\end{aligned}$$

where $\alpha_0 > 0$, α_{11}, $\alpha_{12} \geq 0$, $\epsilon^+ = \max(\epsilon, 0)$, $\epsilon^- = \max(-\epsilon, 0)$, $\epsilon = \epsilon^+ - \epsilon^-$ and $|\epsilon| = \epsilon^+ + \epsilon^-$, and $\beta_1 \geq 0$. The case of $\alpha_{11} < \alpha_{12}$ corresponds to a leverage effect frequently observed in T-GARCH(1, 1) (Choi *et al.*, 2012). Similarly, the AR(1)-BL-GARCH(1, 1) regression model is expressed as

$$
\begin{aligned}
\text{Mean equation}: \quad Y_t &= a + b \times X_t + \epsilon_t, \;\; \epsilon_t = \phi_1 \times \epsilon_{t-1}, \;\; \epsilon_{t-1} = \sqrt{h_{t-1}} e_{t-1}, \;\; \text{where } \; \epsilon_{t-1} \sim N(0, h_{t-1}) \\
\text{Variance equation}: \quad h_t &= \alpha_0 + \alpha_1 \times \epsilon_{t-1}^2 + \beta_1 \times h_{t-1} + \gamma_1 \times \epsilon_{t-1} \times \sqrt{h_{t-1}} \qquad (3)
\end{aligned}
$$

where $\alpha_0 > 0, \;\; \alpha_1 \geq 0, \;\; -\infty < \gamma_1 < \infty$, and $\beta_1 \geq 0$. A leverage effect corresponds to $\gamma_1 < 0$ and the BL-GARCH model is known to be capable of accommodating shift-features in h_t (Choi *et al.*, 2012), and the leverage effects are explained by the interactions between past observations and volatilities. Thus, the BL-GARCH model is a useful tool for modeling time varying conditional variances and leverage effects in financial time series (Storti and Vitale, 2003). In this paper, we develop the TBL-GARCH to a TBL-GARCH regression type model in order to examine the conditional leverage effect volatility of WTI oil prices given Brent oil prices. The AR(1)-TBL-GARCH(1, 1) regression model is expressed as

$$
\begin{aligned}
\text{Mean equation}: \quad Y_t &= a + b \times X_t + \epsilon_t, \;\; \epsilon_t = \phi_1 \times \epsilon_{t-1}, \;\; \epsilon_{t-1} = \sqrt{h_{t-1}} e_{t-1} \;\; \text{where } \; \epsilon_{t-1} \sim N(0, h_{t-1}) \\
\text{Variance equation}: \quad h_t &= \alpha_0 + \alpha_{11} \times (\epsilon_{t-1}^{+})^2 + \alpha_{12} \times (\epsilon_{t-1}^{-})^2 + \gamma_{11} \times \epsilon_{t-1}^{+} \times \sqrt{h_{t-1}} \\
&\quad + \gamma_{12} \times \epsilon_{t-1}^{-} \times \sqrt{h_{t-1}} + \beta_1 \times h_{t-1}, \qquad (4)
\end{aligned}
$$

where $\alpha_0 > 0, \;\; \alpha_{11}, \alpha_{12} \geq 0, \;\; \epsilon^{+} = \max(\epsilon, 0), \epsilon^{-} = \max(-\epsilon, 0), \epsilon = \epsilon^{+} - \epsilon^{-}$ and $|\epsilon| = \epsilon^{+} + \epsilon^{-}, -\infty < \gamma_{11}, \gamma_{12} < \infty$ and $\beta_1 \geq 0$.

Table 1: Summary Statistics for Brent and WTI.

	Mean	Median	Minimum	Maximum	St.D	Skewness	Kurtosis
Brent	0.040	0.034	-3.827	3.953	1.079	0.111	3.687
WTI	-0.004	0.054	-4.172	6.072	1.244	0.282	4.624

3 Empirical Results

3.1 Descriptive Statistics

Our dataset contains daily crude oil prices such as Brent and WTI from January 3, 2013 to October 6, 2014.[1] Let S_t be an observed daily crude oil price process in discrete time, $t = 1, 2, \cdots, n$ and $r_t = log\left(\frac{S_t}{S_{t-1}}\right)$ is the rates of return of the oil prices at time t.

We want to know the relationship between the most heavily traded crude oils, Brent and WTI. Table 1 presents the descriptive statics of our sample dataset. All return series of Brent and WTI are close to zero mean. Brent and WTI are slightly skewed to the right relative to the normal distribution and the kurtoses of all return series of Brent and WTI are higher than three. Their excess kurtoses are significantly positive, indicating that they have heavy tails relative to the normal distribution.

We inspect the Autocorrelation Function (ACF) and Partial ACF (PACF) of the residual series. The ACF and PACF plots in Figure 1 show that there is no significant autocorrelation left in the residuals. We also employ a formal method such as the McLeod-Li test in order to test serial correlations and the volatility clustering effects of Brent oil price returns. The figure displays that the p-values in most lags, except for at 10, 11, 12, and 14 lags, are

[1] The dataset is obtained from the currency database retrieval system provided by Professor Werner Antweiler's website at UBC's Sauder School of Business, (http://fx.sauder.ubc.ca/data.html).

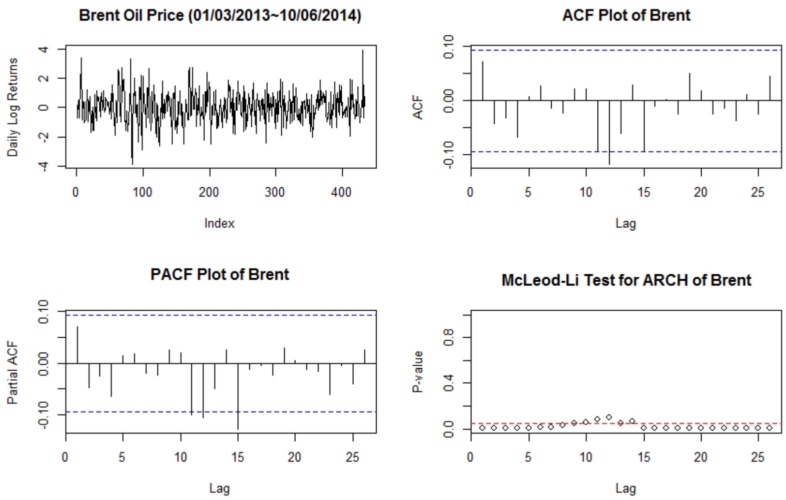


Figure 1: Daily log-returns of Brent.

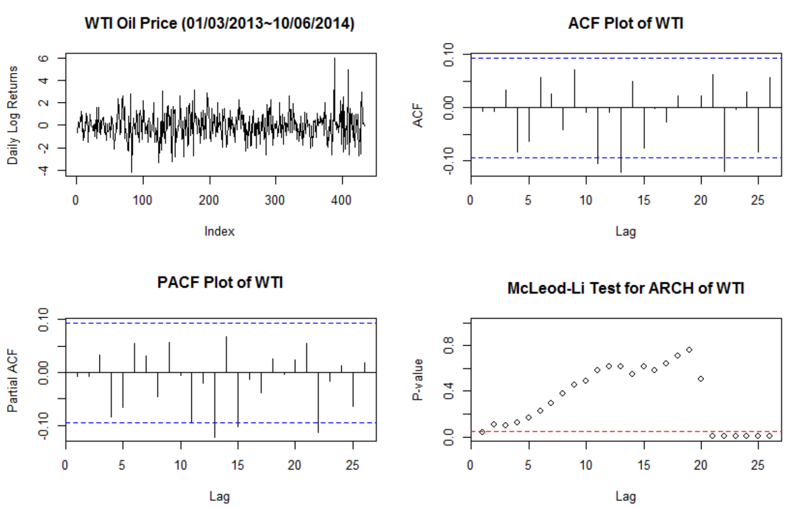


Figure 2: Daily log returns of WTI

Table 2: Granger-Causality Wald test by VAR(1) model

Causality	Brent $\rightarrow$ WTI	WTI $\rightarrow$ Brent
DF	1	1
Chi-Square	13.67	0.89
p-value	0.000	0.346

smaller than the 5% significance level. Therefore, we are able to reject the null hypothesis of homoscedasticity and that the Brent return series have the Autoregressive Conditional Heteroskedastic (ARCH) effect, which is in fact, typically observed in the financial time series. In Figure 2, it is obvious that there exist weak ARCH effects at the 5% significance level at 1 and beyond 21 lags (long-term lags) in the return series. From visual inspection, we find the existence of conditional heteroskedasticity in the return series of the crude oil prices. Therefore, we consider the Generalized ARCH (GARCH) model (Bollerslev, 1986) in order to eliminate the serial dependence of Brent and WTI. In Section 3.3, we will use the asymmetric GARCH regression models to investigate the conditional leverage effect.

To select the dependent and independent variables for our empirical analysis in the next section, we examine the relationship between Brent and WTI. An obvious implication is that shocks on a specific market quickly affect the other crude oil prices. The Granger-Causality Wald test in Table 2 shows that the unidirectional relationship from Brent to WTI is statistically significant at the 1% significance level. This indicates that Brent oil prices do Granger cause WTI oil prices. However, WTI oil prices do not Granger cause Brent oil prices. The empirical test provides evidence that information on previous Brent returns plays a dominant role in explaining future returns of WTI over our sample period.

Table 3: Testing for Stock-Watson's common trends using differencing filter

H_0: Rank=m	H_1: Rank=s	Eigenvalue	Filter	5% Critical Value	Lag
1	0	0.588	-178.38	-14.1	2
2	0	0.594	-175.89	-8.8	
	1	0.495	-218.54	-23.0	

Based on the Granger-Causality test results, we will consider WTI as a response variable and Brent as an explanatory variable in the Gaussian copula marginal regression in the next subsections.

We also test whether the oil markets are cointegrated. Stock and Watson (1988) observe that every time series in a conitegrated set can be expressed in terms of common stochastic trends. We use Stock and Watson's (1988) proposed statistics for common trends testing. Table 3 shows the output considering the AR(2) case. The first column contains the null hypothesis that the k-dimensional time series has m common stochastic trends; the alternative used for the test in the second column, indicating that the time series has s common trends, where $s < m$. The test statistic for testing for 2 versus 1 common trends (-218.54) is less than the critical value (-23.0). Therefore, the test rejects the null hypothesis, which means that the series has a single common trend. Thus Brent and WTI return series are conintegrated with a single common trend. For further investigation, we will use the tail dependence by employing diverse copula functions in the next subsection. The Granger-Causality Wald test and Stock-Watson's common trend test yield evidence that Brent and WTI oil prices have a single common trend along with the fact that Brent has more influence on the prices of WTI.

3.2 Asymmetry of Tail Dependence by Copula

We employ the most popular copula functions to investigate the dependence between WTI and Brent oil market returns during our sample period. Our goal in this analysis is to examine the asymmetry of tail dependence by using copula models. As we discussed earlier, the return series of Brent and WTI are not normally distributed. The finding is a good reason for us to use a copula approach in order to examine the structural dependence between Brent and WTI oil prices. For a marginal distribution model, we consider the nonlinear-asymmetric GARCH (NAGARCH) of Engle and Ng (1993), which specifies the volatility h_t^2 as follows:

$$h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i (a_{t-i} - \gamma_i h_{t-i})^2 + \sum_{j=1}^{p} \beta_j h_{t-j}^2$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, for $i = 1, \dots, q$, and $\beta_j \geq 0$ for $j = 1, \dots, p$. In this model, the news impact curve, which measures the possible asymmetric impact of good and bad news at $t-1$ on the conditional variance of t, shifts to the right by $\gamma_i h_{t-i}$. In particular, by applying a family of NAGARCH models to Brent and WTI oil prices, we are able to avoid the serial dependence in the component time series (Kojadinovic and Yan, 2010).

We generate the vector of standardized residuals from the NAGARCH(1, 1) model, where an error follows Student-t distribution, and then we use the obtained residuals to various copula models in order to investigate tail dependence between the crude oil prices. With these standardized residuals of Brent and WTI, we carry out this empirical exercise by using equation (1). The analysis of tail dependence has been used in the financial time series in order to investigate the behavior of the random variables during extreme events. We examine

Table 4: Statistics for copulas with Brent and WTI

Copula type	Log-Likelihood	AIC	BIC	τ^L	τ^U
Gaussian	-92.753	-185.502	-185.493	0	0
Clayton	**-73.903**	**-147.801**	**-147.792**	**0.474**	**0**
Rotated Clayton	**-65.075**	**-130.146**	**-130.136**	**0**	**0.445**
Plackett	-82.803	-165.601	-165.591	0	0
Frank	-83.931	-167.857	-167.848	0	0
Gumbel	**-78.910**	**-157.814**	**-157.805**	**0**	**0.448**
Rotated Gumbel	**-84.541**	**-169.077**	**-169.068**	**0.455**	**0**
Student's t	-92.707	-185.405	-185.386	0	0
SJC	**-86.831**	**-173.653**	**-173.635**	**0.403**	**0.344**
Time-Varying Gaussian	**-93.048**	**-186.083**	**-186.055**		
Time-Varying rotated Gumbel	-86.121	-172.227	-172.199		
Time-Varying SJC	-90.126	-180.225	-180.169		

the probability of the extreme events of an extremely large increase or decrease of both Brent and WTI prices by using tail dependence.

Table 4 presents the lower and the upper tail dependencies of various copula models for oil price returns. There is a symmetric tail dependence between two variables when the lower tail dependence coefficient τ^L equals the upper one τ^U. The symmetrized Joe-Clayton copula introduced by Patton (2006) exhibits that τ^L of 0.403 is slightly larger than τ^U of 0.344, but there is no significant difference because the estimated values of τ^L andτ^U are roughly similar for the Clayton and the rotated Clayton copulas and for the Gumbel and the rotated Gumbel copulas. Therefore, Table 4 provides evidence against asymmetric tail dependence between WTI and Brent crude oil prices. The results of the goodness of fit test are also summarized in the table. They show that the Time-Varying Gaussian copula is the best fitting one among the twelve different copula models based on the model selection criteria.

Figure 3 displays the Time-Varying Gaussian copula correlations, obtained from the best

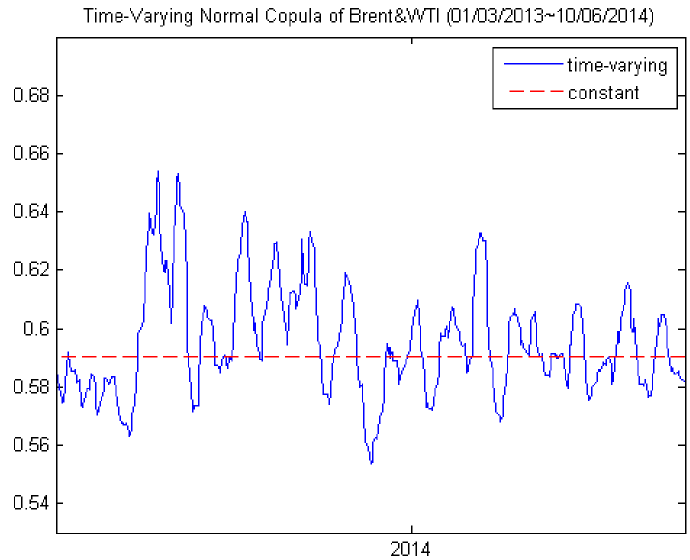


Figure 3: Correlations by the Time-Varying Gaussian copula model for crude oil returns

fitting copula model. The average time-varying correlation is 0.59, indicating that a strong comovement between the Brent and WTI crude oil markets. An increase in prices in an oil market leads to increase in the extreme dependence level of the oil prices in the other market.

3.3 Asymmetry of Conditional Leverage Effect

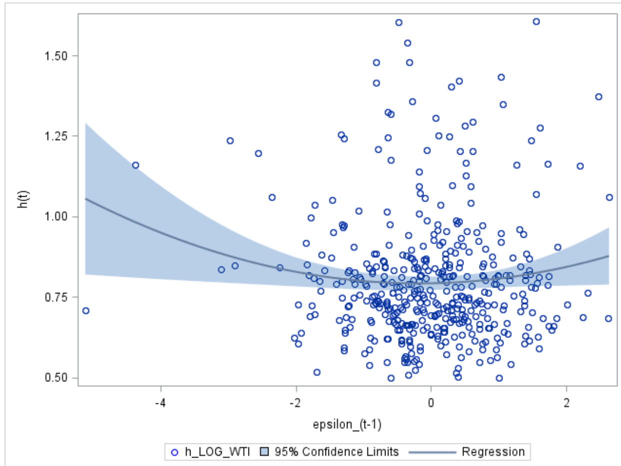
We estimate the three different asymmetric GARCH models discussed in Section 2.2. Table 5 presents the estimated coefficients of the three different asymmetric GARCH regression models for the entire sample period. The table also reports that BL-GARCH is the best fitting model. Overall, the alternative asymmetric GARCH specifications consistently show that there is no conditional leverage effect. This result is congruent with our previous findings of no significant difference in the tail dependence at the lower and upper tails.

Table 5: Empirical results of three asymmetric GARCH specifications

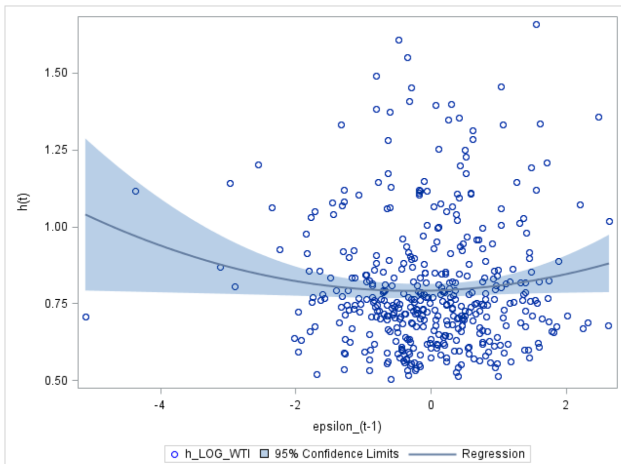
Model	T-GARCH		BL-GARCH		TBL-GARCH	
Parameter	Estimate	p-value	Estimate	p-value	Estimate	p-value
a	-0.050	0.217	-0.046	0.256	-0.050	0.212
b	**0.682**	**0.000**	**0.684**	**0.000**	**0.677**	**0.000**
AR(1)(ϕ_1)	**-0.144**	**0.004**	**-0.145**	**0.004**	**-0.143**	**0.004**
DF	**10.065**	**0.010**	**9.813**	**0.010**	**9.396**	**0.007**
α_0	0.018	0.3559	0.023	0.272	0.028	0.250
α_1			0.027	0.085		
α_{11}	0.026	0.134			0.020	0.587
α_{12}	0.027	0.259			0.131	0.078
γ_1			0.018	0.587		
γ_{11}					0.009	0.944
γ_{12}					-0.234	0.135
β_1	**0.946**	**0.000**	**0.939**	**0.000**	**0.973**	**0.000**
AIC	1,220.974		**1,220.661**		1,221.876	
BIC	1,253.540		**1,253.227**		1,262.583	
Log Likelihood	-602.487		-602.330		-600.938	

We have explained the relationship between asymmetry by tail dependence and the conditional leverage effect of the target variable given the auxiliary variable by asymmetric GARCH regression models in this section. Our empirical study shows that these two different approaches consistently provide the convincing and congruent evidence of both no asymmetry and no conditional leverage effect. The brief theoretical justification of our empirical result in this paper is that since both copula tail dependence and the conditional leverage effect are calculated by using the same conditional probability distribution function of WTI given on Brent, see equations (1)-(4). We may infer that both the copula tail dependence and the conditional leverage effect share the property of asymmetry.

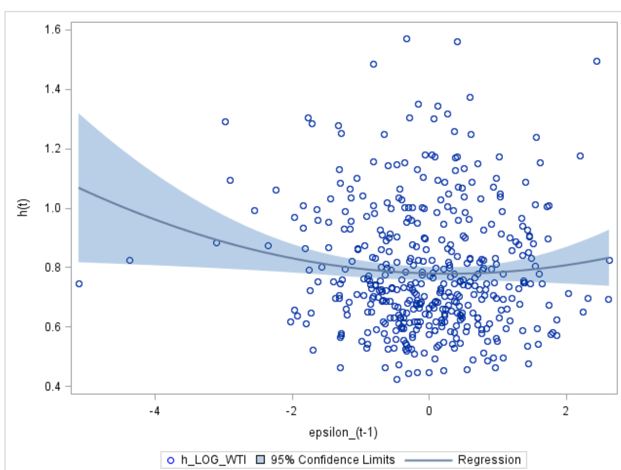
The news impact curve (NIC) is the functional relationship between conditional variance at time t and the shock term (error term) at time $t-1$. Figure 4 displays the NICs from three different asymmetric GARCH regression specifications of WTI on Brent. We observe



(a) New Impact Curve of AR(1)-TGARCH(1, 1) Model



(b) New Impact Curve of AR(1)-BL-GARCH(1, 1) Model



(c) New Impact Curve of AR(1)-TBL-GARCH(1, 1) Model

Figure 4: New impact curves for WTI given on Brent

that all three panels show slightly asymmetric NICs, which are slightly steeper for negative shocks than they are for positive shocks. However, the p-values of the test of the asymmetry are not statistically significant as shown in Table 5. We find no evidence of leverage effects of WTI return series conditional on Brent oil prices.

4 Conclusion

From a short-term predictability perspective, we have examined whether the asymmetry in the tail dependence and volatility is pronounced between crude oil markets. We first provided evidence, by using the Granger-Causality test, that previous Brent returns play a dominant role in explaining future returns of WTI over our sample period. In addition, we found that Brent and WTI returns are cointegrated with a single common trend.

After generating standardized residuals of Brent and WTI oil prices, we have employed various copula model specifications in order to investigate the tail dependence. We found evidence of symmetric tail dependence between WTI and Brent return series. This result implies that returns across crude oil markets exhibit equal correlation and tend to move together during extreme downturns and upturns in our sample. We also analyzed the leverage effect of WTI oil prices conditional on Brent returns by using three asymmetric GARCH regression models. Particularly, in this article, we developed the TBL-GARCH regression model to investigate the existence of a conditional leverage effect of WTI oil prices. For the best fitting model - BL-GARCH - the volatility asymmetry was not found during our sample period. Two different approaches have provided the identical empirical findings that there is neither asymmetry nor conditional leverage effect in our sample data.

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