

Disagreement in Optimal Security Design

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Abstract

Which security does a firm optimally issue when it is more optimistic than its financiers about the characteristics of the asset? In our basic model, either debt or debt plus barrier options are optimal. When multiple assets with identical characteristics are available, pooling can be strictly preferred to selling optimal securities backed by the individual assets. When investors disagree amongst themselves, selling multiple tranches can be optimal. In a stylized dynamic extension, convertible securities commonly used in VC financing naturally emerge.

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1 Introduction

Which security does a firm optimally issue when firm and market agree to disagree about the firm’s cash-flow distribution? We consider an issuer who has more optimistic views about the firm’s future profits than the market does, consistent with empirical evidence (e.g., [Malmendier et al., 2007, 2011](#)).¹ We show that such disagreement can generate various commonly observed financial contracts.

We study a model in which an issuer owns an asset that will pay uncertain cash-flows at a future date. To raise capital, the issuer designs a security backed by this asset, with the objective to sell the security to the market. Following [DeMarzo and Duffie \(1999\)](#), we assume the issuer discounts future cash-flows more than the market does. Such differences in discounting arise naturally when the issuer has profitable investment opportunities, or when the issuer faces credit constraints. Our key assumption is that the issuer is more optimistic about the asset’s cash-flow distribution than her financiers, who find bad outcomes relatively more likely. The issuer’s problem is to design the monotonic security that maximizes her expected payoffs, which are given by the market price of the security (based on the market’s less optimistic beliefs) plus the expected discounted retained cash-flows (evaluated with her own more optimistic beliefs).

Our analysis delivers four main results. First, we characterize conditions under which debt or debt plus barrier options, respectively, are optimal. Second, we show that selling a security backed by a pool of several underlying assets can be strictly preferred to selling individual asset-backed securities. Third, when market participants disagree among themselves, selling separate tranches instead of a single security can be optimal. Fourth, in a model with multiple

¹[Kreps \(1990\)](#) and [Morris \(1995\)](#) argue that economic theory lacks a fundamental rationale for the ubiquitous common prior assumption. Disagreement generally arises when agents have heterogeneous priors, even when they have access to the same information ([Aumann, 1976](#); [Acemoglu et al., 2006](#)); it can also arise when agents are ambiguity averse and use a max-min decision rule ([Gilboa and Schmeidler, 1989](#)). Of course, managerial optimism can be time-varying ([Shleifer et al., 2015](#)), as can be the market’s expectations ([Greenwood and Shleifer, 2014](#)), implying time-variation in the optimal security design.

financing rounds, the optimal security is convertible preferred stock, a security commonly used in venture capital (VC) financing.

The intuition for the optimality of debt is straightforward. Because the issuer is more optimistic than the market, she finds it optimal to sell cash-flows in the left tail of the cash-flow distribution. By contrast, it is optimal for the issuer to retain the cash-flows in the right tail, since the market assigns a relatively low value to them. Under some conditions, the security that implements the desired trade is debt; under others, it is optimal to add a barrier option to the debt contract.² This result may help explain why a large fraction of corporate bonds are callable (Duffee, 1998),³ and also rationalizes “write-off bonds” (Vallée, 2013). When pre-existing debt encumbers the balance sheet, debt overhang obtains – that is, the firm optimally stops selling securities. This prediction contrasts with that of some asymmetric information theories, in which over-levered firms issue equity (e.g., Fulghieri et al., 2013). Moreover, in the traditional “pecking order” model (Myers and Majluf, 1984), only the worst firms issue equity. The fact that firms issue equity when stock prices and sentiment are high (e.g., Marsh, 1982; Baker and Wurgler, 2002; Erel et al., 2011; McLean and Zhao, 2014; Farre-Mensa, 2015) is inconsistent with that prediction. By contrast, firms in our model only sell the firm when investors are more confident – consistent with the above evidence – and when agreement between issuer and market is high – documented empirically by Dittmar and Thakor (2007).

The pooling result is more intricate. With heterogenous beliefs, pooling several assets allows the issuer to design securities that are better tailored to the relatively pessimistic beliefs of investors. Intuitively, while outside investors might be very pessimistic (relative to the issuer) about the probability of an individual asset delivering high profits, they may not

²Excessive betting between contracting parties such as discussed in Börgers (2014) does not arise in our context as we study the design of an asset-backed security under limited liability. In contrast to Caballero and Farhi (2014), we allow the market to be complete.

³Indeed, the combination of debt plus a barrier option can be interpreted as callable debt.

be as pessimistic about the event that at least one of several assets pays off a high return. As a result, an issuer who owns multiple assets may find it strictly optimal to combine them and sell a “senior” security backed by the pool of assets. The following example illustrates this result.

Example 1. Consider first an issuer who owns a single asset, which can either pay a return of 1 or a return of 0. The market believes that the probability of the asset paying off is $\frac{1}{3}$; the issuer believes in an upside probability of $\frac{2}{3}$. The issuer discounts future cash-flows with a factor of 0.6, whereas the market does not discount. The market is therefore willing to pay $\frac{1}{3}$ for the asset, whereas the asset is worth $\frac{2}{3} \cdot 0.6 = 0.4$ to the issuer. Because $0.4 > \frac{1}{3}$, the issuer retains the asset.

Consider now an issuer who owns two of these assets with *iid* returns. The issuer’s payoff from retaining the two assets is 0.8, which is strictly larger than her payoff from selling two individual securities, each backed by an asset. Suppose instead that the issuer sells a “senior” security backed by the pool of assets that pays 1 if at least one asset pays off and zero otherwise. Because the market believes the probability that both assets don’t pay off is $(\frac{2}{3})^2$, investors are willing to pay $1 - (\frac{2}{3})^2 = \frac{5}{9}$ for the security. At the same time, the security is worth $(1 - (\frac{1}{3})^2) \cdot 0.6 = \frac{8}{15} < \frac{5}{9}$ to the issuer. Because the issuer retains a cash-flow of 1 in the event that both assets pay off, her expected payoff from selling this security is $\frac{5}{9} + (\frac{2}{3})^2 \cdot 0.6 \approx 0.822 > 0.8$.

In our model, differences in beliefs between the issuer and the market – and not changes in the level of risk as reflected in the literature (e.g., [Coval et al., 2009](#)) – are crucial for pooling to be strictly optimal. Indeed, because the issuer discounts future cash-flows more than the market, with homogenous beliefs it is always optimal for the issuer to sell the entire cash-flows of the assets that she owns. As a result, when issuer and market share the same beliefs,

the issuer is indifferent between pooling her assets or selling them as separate concerns.⁴

We then extend our analysis to allow for multiple types of investors, who have different beliefs about the asset’s cash-flow distribution. We show that, with divergence of beliefs among investors, it can be optimal to offer to the market separate tranches instead of a single security. This can include retention of the most junior tranche by the issuer.

Finally, in a stylized extension with multiple financing rounds, securities with a conversion feature become optimal. Such securities are used in most VC financing contracts (see, e.g., [Gompers and Lerner, 2001](#); [Kaplan and Strömberg, 2003, 2004](#)). As before, we assume that the issuer (here: the entrepreneur) is more confident about the project’s prospects than the lender (here: the VC).⁵ Because the entrepreneur assigns a relatively low probability to states in which performance is bad, she finds it relatively cheap to rescind cash-flows to the VC in such states, who values these states more highly. At the time of initial contracting, the entrepreneur also secures an option for a future financing round that enables her to expand the project conditional on good interim performance.⁶ Because the VC finds good performance relatively unlikely, she finds it cheap to write that option to the entrepreneur, who is more optimistic. Conditional on refinancing, the VC obtains an equity stake, allowing her to break even.⁷

The paper proceeds as follows. Section 2 discusses the related literature. Section 3 in-

⁴We also show that the pooling result breaks down when the correlation between the underlying assets increases, as was the case in the recent financial crisis. This feature is consistent with the dynamics of securitization reported, e.g., in [Coval et al. \(2009\)](#); [Chernenko et al. \(2013\)](#).

⁵See [Bernardo and Welch \(2001\)](#); [Cooper et al. \(1988\)](#); [Koellinger et al. \(2007\)](#); [Puri and Robinson \(2007\)](#) for supportive evidence.

⁶Our predictions do not rely on assuming that the strike of the refinancing option is determined with certainty at the time of initial contracting. Indeed, the fact that disagreement is reduced by learning about project quality over time is one of the key reasons for using convertible securities in early-stage financing, rather than securities that require a precise agreed-upon valuation of the project at the time of contracting.

⁷Importantly, the key assumption that makes the financier secure part of the upside is that the project’s required investment and the upside potential are high – in other words, when the payoff profile is highly skewed. Indeed, in practice many less risky entrepreneurial ventures are financed with straight (bank) debt, whereas VC financing with convertibles obtains only for projects with relatively high investment needs and high potential payoffs ([Cochrane, 2005](#)).

troduces the basic model and derives the optimality of debt. Sections 4 and 5 present the results on pooling and tranching. In section 6, convertibles naturally arise in a setting with multiple financing rounds.

2 Related Literature

While this paper is the first to formally investigate the role of disagreement in optimal security design, informal mentions of the idea go back at least to [Modigliani and Miller \(1958\)](#). These authors write (excerpts from p. 292): “Grounds for preferring one type of financial structure to another still exist within the framework of our model. If the owners of a firm discovered a major investment opportunity which they felt would yield much more than [the market’s discount rate], they might well prefer not to finance it via common stock. A better course would be to finance the project initially with debt. Still another possibility might be to [issue] a convertible debenture.”

The model we offer is “non-informational” because we do not assume asymmetric information between firm and investors; instead, the two parties knowingly disagree. In contrast to [Brunnermeier et al. \(forthcoming\)](#); [Simsek \(2013b,a\)](#), the disagreement is not between different sets of traders, but between the issuer and the market. A similarity of our basic setting to Simsek’s is that optimists borrow from pessimists. However, the contract used is endogenous in our setting.

Several previous papers invoke optimistic issuers or other non-standard beliefs to explain financing and capital structure choices ([Heaton, 2002](#); [Coval and Thakor, 2005](#); [Dittmar and Thakor, 2007](#); [Hackbarth, 2008](#); [Landier and Thesmar, 2009](#); [Boot and Thakor, 2011](#); [Gervais et al., 2011](#); [Bayar et al., 2011](#); [Adam et al., 2014](#); [Bayar et al., forthcoming](#)) as well as asset prices ([Geanakoplos, 2010](#)). However, these papers do not study optimal security design in the sense of [Allen and Gale \(1988\)](#) because the state space and/or contracting space is more

constrained.⁸ An exception is [Garmaise \(2001\)](#), who shows that tranching can be optimal in a model in which there is disagreement among investors and in which the prices of securities are determined through a first price auction. By contrast to [Shleifer et al. \(2015\)](#), who study the influence of managerial beliefs on investment, we take the asset – i.e., the investment project – as given, and study how insiders’ versus outsiders’ beliefs affect how the asset is optimally financed.

Because there is open disagreement in our model rather than asymmetric information between issuer and market, our contribution is sharply distinguished from contributions that rationalize particular security designs, including the predominance of debt, with adverse selection or an informational advantage of insiders over the market ([Myers and Majluf, 1984](#); [Innes, 1990](#); [Nachman and Noe, 1994](#)), from papers that point out the fragility of the same “pecking order” result to the choice of specific off-equilibrium beliefs ([Noe, 1988](#)), to who has the private information ([Inderst and Mueller, 2006](#); [Axelson, 2007](#)), and to particular distributional assumptions (e.g., [Nachman and Noe, 1990, 1994](#); [Fulghieri et al., 2013](#)). Similarly, the pooling results in [DeMarzo \(2005\)](#) rely on asymmetric information whereas ours don’t; hence, our result is robust to the critique that informational asymmetries oftentimes are not overcome by pooling ([Arora et al., 2013](#)).⁹ Our theory also makes no use of moral hazard as a driver of the optimal security as in [Admati and Pfleiderer \(1994\)](#); [Bergemann and Hege \(1998\)](#); [Schmidt \(2003\)](#); [Antic \(2014\)](#); [Hébert \(2014\)](#). Lastly, investors in our model do not suffer from limited channel capacity, which can also render debt optimal ([Yang, 2013](#)).

In sum, previous papers have investigated the effect of disagreement on several financial decisions including the choice of leverage; others have investigated the effect of informational and other frictions on optimal security design. Our paper contributes to this literature by

⁸Disagreement is related to outcome variables other than capital structure by [Boot et al. \(2006, 2008\)](#), [Adrian and Westerfield \(2009\)](#), and [Dicks and Fulghieri \(2015\)](#). The observation that the optimal securities can be sold also to employees relates our paper to [Hellman and Puri \(2000\)](#), [Oyer and Schaefer \(2005\)](#), and [Bergman and Jenter \(2007\)](#).

⁹The pooling results are loosely related to mergers and divestitures, e.g., [Fluck and Lynch \(1999\)](#).

providing a full and formal analysis of the effect of disagreement on optimal security design. The insight that disagreement can generate rich predictions that previous models generated with other frictions is important, because different empirical proxies capture these different frictions. The paper hence contributes a starting point for a more nuanced empirical investigation of precisely which securities firms issue, and why. Moreover, our comparatively simple model of disagreement can generate a variety of securities for which the literature thus far as employed separate models.

3 Basic Model

3.1 Payoffs, Beliefs, and Objectives

The model that we study is as follows. At date $t = 0$, there is an issuer who owns an asset which will yield state-contingent payoffs $X \in \mathbb{R}_+^K$ at date $t = 1$. That is, there is a finite set of possible states of nature $S = \{1, \dots, K\}$, and the issuer's asset pays an amount $X_s \in \mathbb{R}_+$ at state $s \in S$. We assume that $X_s > 0$ for all $s \in S$ (i.e., even in the worst state the asset generates strictly positive cash-flows) and that there exists at least one pair of states $s, s' \in S$ such that $X_s > X_{s'}$ (i.e., the asset is risky). Without loss of generality, we order the states in a way such that $X_1 \leq X_2 \leq \dots \leq X_K$.

The issuer's problem is to design a security $F \in \mathbb{R}_+^K$ backed by the cash-flows X in order to sell it in the market. Thus, security F must be such that $0 \leq F_s \leq X_s$ for all $s \in S$. Let π^I be the probability distribution over S that represents the issuer's beliefs. We assume that $\pi_s^I > 0$ for all $s \in S$. After selling the security, the issuer retains $X - F$ of the cash-flows generated by the asset. Following [DeMarzo and Duffie \(1999\)](#) we assume that the issuer discounts retained cash-flows at a rate that is higher than the market rate (which is normalized to 1).¹⁰ Thus, the issuer attaches a value of $\delta \sum_{s \in S} \pi_s^I (X_s - F_s)$ to retained

¹⁰The assumption that the issuer discounts future cash-flows at a higher rate than the market is a

cash-flows, where $\delta < 1$ is the issuer's discount rate. The payoff of an issuer who sells to the market a security F at a price p is then given by

$$p + \delta \sum_{s \in S} \pi_s^I (X_s - F_s).$$

The new feature of our analysis is the way in which the market evaluates the security that the issuer designs. We assume that the market has different beliefs about the cash-flow distribution of the underlying asset than the issuer. Let π^M be the probability distribution over S that describes the market's beliefs, with $\pi_s^M > 0$ for all $s \in S$. For most of our results, we assume that π^I first-order stochastically dominates π^M (i.e., for all $s \in S$, $\sum_{s' \leq s} \pi_{s'}^I \leq \sum_{s' \leq s} \pi_{s'}^M$), so that the issuer is more optimistic about the asset's return than the market.¹¹ The price that the market is willing to pay for security F is

$$p(F) := \sum_{s \in S} \pi_s^M F_s.$$

Overall, the issuer's payoff from selling security F is

$$U(F) := p(F) + \delta \sum_{s \in S} \pi_s^I (X_s - F_s).$$

Definition 1. We say that security F is *monotonic* if F_s and $X_s - F_s$ are both increasing in s .

metaphor for example for a situation in which the issuer has some profitable investment opportunity. Also, the assumption will hold if the issuer faces credit constraints or, as in the case of financial entities, minimum-capital requirements.

¹¹There is an alternative interpretation for the differences in beliefs in our model: if we interpret π^I and π^M as risk-neutral probabilities, then π^I and π^M will be different whenever issuer and market have different preferences, even if they share the same beliefs over S . If issuer and market are expected utility maximizers and share the same beliefs over S , then the issuer's risk-neutral probabilities π^I will first-order stochastically dominate the market's risk-neutral probabilities π^M only under the assumption that the issuer is less risk-averse than the market. Because the standard assumption is that the risk-bearing capacity of the market is larger than that of the issuer, we find the heterogeneous-beliefs interpretation more adequate.

We will assume that the issuer is restricted to sell monotonic securities. The restriction to monotone securities is standard in the literature of optimal security design; see, for instance, [Innes \(1990\)](#); [DeMarzo and Duffie \(1999\)](#). In [Appendix B](#) we show how our results change when we relax this restriction.

Let \mathcal{F} be the set of feasible securities

$$\mathcal{F} := \{F \in \mathbb{R}^K : 0 \leq F_s \leq X_s \forall s \in S \text{ and } F \text{ monotonic}\}.$$

The issuer’s problem is to find the security F within the set of feasible securities \mathcal{F} that maximizes her payoff, taking as given the market’s beliefs (that diverge from the issuer’s), and thus recognizing how much the market will pay for the security. Formally, the issuer’s problem is

$$\sup_{F \in \mathcal{F}} U(F). \tag{1}$$

3.2 Optimal Security Design with Divergent Beliefs

In this section we present the solution to problem (1). We start with two preliminary results. All proofs are in [Appendix A](#).

Lemma 1. *If F solves the issuer’s problem (1), then there exists $s \in S$ such that $F_s = X_s$.*

[Lemma 1](#) says that an optimal security must pay all available cash-flows at least at one state. The intuition behind this result is as follows. Regardless of how pessimistic the market’s beliefs π^M are, the market will always value additional risk-free cash-flows at their face value. Since the issuer discounts retained cash-flows at a higher rate than the market does (i.e. since $\delta < 1$), it is strictly optimal to add these risk-free cash-flows to a security. As a result, a security F such that $F_s < X_s$ can never be optimal.¹²

¹²Note that our model has a “variable capital raised” property – it is endogenous how much capital will

Our next result shows that the optimal security pays all of the asset's cash-flows at states at which cash-flows are below some given cutoff. For any $s \in S$, let $A_s := \{s, s + 1, \dots, K\}$ be the event that the asset yields cash-flows weakly larger than X_s . For all $s \in S$, let $\pi^I(A_s) := \sum_{s' \geq s} \pi_{s'}^I$ and $\pi^M(A_s) := \sum_{s' \geq s} \pi_{s'}^M$ be, respectively, the probability that the issuer and market assign to event A_s . The assumption that π^I first-order stochastically dominates π^M implies that $\pi^I(A_s) \geq \pi^M(A_s)$ for all $s \in S$.

Lemma 2. *Suppose there exists a state $k \in S$ such that $\pi^M(A_s) > \delta\pi^I(A_s)$ for all $s \leq k$. Then, if F solves (1), $F_s = X_s$ for all $s \leq k$.*

To see the intuition for Lemma 2, consider a state $s \leq k$ (so that $\pi^M(A_s) > \delta\pi^I(A_s)$), and let F^ϵ be a security with $F_{s'}^\epsilon = \epsilon \mathbf{1}_{\{s': s' \geq s\}}$ for all $s' \in S$; i.e., F^ϵ pays $\epsilon > 0$ if the state is weakly larger than s and pays zero otherwise. Since $\pi^M(A_s) > \delta\pi^I(A_s)$, the market values security F^ϵ strictly more than the issuer does. As a result, any security $F \in \mathcal{F}$ with $F_s < X_s$ is strictly dominated by security $\tilde{F} \in \mathcal{F}$ with $\tilde{F}_{s'} = F_{s'} + F_{s'}^\epsilon$ for all $s' \in S$.

Since $\pi^M(A_1) = \pi^I(A_1) = 1$, there is always a state $k \in S$ such that $\pi^M(A_s) > \delta\pi^I(A_s)$ for all $s \leq k$. This, together with Lemma 2, implies the following remark.

Remark 1. There exists $k \in \{1, 2, \dots, K\}$ such that the optimal security has $F_s = X_s$ for all $s \leq k$.

Remark 1 states that the optimal security must always be a combination of debt plus (possibly) another security which only pays at states $s > k$. The face value of the debt included in the optimal security depends on the degree of belief heterogeneity between the issuer and the market.

The following result builds on Lemmas 1 and 2 to characterize the optimal security.

Proposition 1. *An optimal security is given by:*

be raised. This feature is in contrast to other models in the literature that operate under the assumption that a fixed amount of capital needs to be raised for investment.

(i) $F_1 = X_1$;

(ii) For all $s \in S \setminus 1$:

$$F_s = \begin{cases} X_s - X_{s-1} + F_{s-1} & \text{if } \pi^M(A_s) \geq \delta\pi^I(A_s), \\ F_{s-1} & \text{if } \pi^M(A_s) < \delta\pi^I(A_s). \end{cases}$$

Proposition 1 characterizes the optimal security when issuer and market have different opinions about the underlying asset's performance. The key value that determines the shape of the optimal security at each state s is the ratio between $\pi^M(A_s)$ and $\pi^I(A_s)$; i.e., the ratio of the probability that the market and issuer assign to profits being larger than X_s . If $\frac{\pi^M(A_s)}{\pi^I(A_s)} \geq \delta$, the optimal security F pays the largest possible amount in state s ; if $\frac{\pi^M(A_s)}{\pi^I(A_s)} < \delta$, the optimal security pays the least possible amount (in both cases, subject to the constraint that F is monotonic).

The following corollaries immediately follow.

Corollary 1. *If $\pi^M(A_s) \geq \delta\pi^I(A_s)$ for all $s \in S$, then it is optimal to sell the entire firm; i.e., $F_s = X_s$ for all $s \in S$.*

Corollary 2. *If $\pi^M(A_s) < \delta\pi^I(A_s)$ for all $s > 1$, then risk-less debt is an optimal security; i.e., $F_s = X_1$ for all $s \in S$.*

Corollary 3. *Suppose there exists $k \in S \setminus \{1, K\}$ such that $\pi^M(A_s) \geq \delta\pi^I(A_s)$ if and only if $s \leq k$. Then, risky debt with face value X_k is an optimal security; i.e., $F_s = \min\{X_s, X_k\}$ for all $s \in S$.*

Corollaries 1, 2 and 3 show that the optimal security can take the form of a debt contract. The face value of the debt contract ranges from the lowest state, which makes risk-less debt

the optimal security, to risky debt with face value $X_k \in (X_1, X_K)$, to risky debt that always defaults (because the face value is equal to the highest possible cash-flow), which can be interpreted as an equity issuance.

In sum, our model predicts that firms issue debt when there is much disagreement between optimistic issuers and less optimistic markets, whereas the entrepreneur sells the whole firm when the market is less pessimistic. This prediction is in stark contrast to several theories of security design based on asymmetric information. Most prominently, the traditional “pecking order” hypothesis holds that firms issue equity only as a “last resort” (e.g., [Myers, 1984](#)) – hence, only the worst firms that have run out of other options issue equity. The empirical evidence is arguably more consistent with the disagreement prediction that the relative optimism of investors versus firms drives issuance decisions: [Farre-Mensa \(2015\)](#) analyses firms that are hit with negative cash-flow shocks and thus face a need to issue securities (a decrease in δ in our model), and shows that firms whose stock is overvalued issue equity, whereas undervalued firms issue debt. Similar in spirit, [Erel et al. \(2011\)](#) and [McLean and Zhao \(2014\)](#) find that equity issuance is cyclical and higher amid positive investor sentiment, whereas firms turn to issuing safer securities during market downturns.

Optimality of debt

Corollary 3 establishes conditions under which the optimal security is debt. These conditions are satisfied under natural assumptions on the beliefs of the issuer and market. Indeed, suppose that π^I and π^M are such that $\frac{\pi_s^I}{\pi_s^M}$ is increasing in s ; i.e., $\frac{\pi_s^I}{\pi_s^M}$ satisfy the Monotone Likelihood Ratio Property (MLRP). Note that $\frac{\pi_s^I}{\pi_s^M}$ increasing in s implies that $\frac{\pi^M(A_s)}{\pi^I(A_s)}$ is decreasing in s . In this case, if $\delta \in \left(\frac{\pi^M(A_K)}{\pi^I(A_K)}, \frac{\pi^M(A_2)}{\pi^I(A_2)} \right]$, the optimal security F is such that $F_s = \min \{X_s, X_k\}$ for some $k \in S \setminus \{1, K\}$. The following corollary summarizes this discussion.

Corollary 4. *If $\frac{\pi_s^I}{\pi_s^M}$ satisfy MLRP and $\delta \in \left(\frac{\pi^M(A_K)}{\pi^I(A_K)}, \frac{\pi^M(A_2)}{\pi^I(A_2)} \right]$, there exists $k \in S \setminus \{1, K\}$ such*

that the optimal security is debt with face value X_k .

Debt plus barrier options

The following corollary to Proposition 1 shows that securities other than straight debt can also be optimal.

Corollary 5. *Suppose that there exists $k, k' \in S$, with $k+1 < k'$, such that $\pi^M(A_s) \geq \delta\pi^I(A_s)$ if and only if either $s \leq k$ or $s = k'$. Then, the optimal security is*

$$F_s = \begin{cases} \min \{X_s, X_k\} & \text{if } s < k', \\ X_{k'} - X_{k'-1} + X_k & \text{if } s \geq k'. \end{cases}$$

The security in Corollary 5 can be thought of as a combination of debt with face value X_k plus a barrier option that pays $X_{k'} - X_{k'-1}$ in the event that the asset yields a payoff weakly larger than $X_{k'}$.¹³ Such a security can be interpreted as a write-off bond: the face value, and thus the payoff, of such contracts is discontinuously higher when cash-flows exceed a certain threshold (Vallée, 2013).¹⁴

In appendix B, for completeness we relax the standard assumption (which can be micro-founded with hidden managerial actions) that the security to be issued must be monotonic. The analysis shows that the function of the assumption is similar to the one it serves in the existing literature (Innes, 1990).

¹³In their model with asymmetric information, Nachman and Noe (1994) also show that securities similar to the one in Corollary 5 can be optimal.

¹⁴The security in Corollary 5 can also be interpreted as callable debt.

3.3 Debt Overhang

We now extend our baseline setting to consider the problem of an issuer who has debt outstanding that is backed by the cash-flows that her asset will generate, and who is considering to issue a new security backed by the remaining cash-flows. Formally, suppose the issuer has debt outstanding with face value $D < X_K$. The issuer wants to design a security $F \in \mathbb{R}_+^K$ to sell to the market, with F backed by the remaining cash-flows she owns; i.e., F such that $0 \leq F_s \leq X_s - \min\{X_s, D\}$ for all $s \in S$. As before, we restrict the issuer to design securities such that both the cash-flows that she pays and the cash-flows that she retains are monotone in the underlying asset's cash-flows; that is, securities F such that F_s and $X_s - F_s - \min\{X_s, D\}$ are increasing in s . Let \mathcal{F}_D denote the set of feasible securities; i.e., $\mathcal{F}_D := \{F \in \mathbb{R}^K : 0 \leq F_s \leq X_s - \min\{X_s, D\} \forall s \in S \text{ and } F_s \text{ and } X_s - F_s - \min\{X_s, D\} \text{ are increasing in } s\}$.

The issuer's problem is

$$\sup_{F \in \mathcal{F}_D} U_D(F), \quad (2)$$

where for any $F \in \mathcal{F}_D$,

$$U_D(F) := p(F) + \delta \sum_{s \in S} \pi_s^I (X_s - \min\{X_s, D\} - F_s) = \sum_{s \in S} \pi_s^M F_s + \delta \sum_{s \in S} \pi_s^I (X_s - \min\{X_s, D\} - F_s).$$

Let $s_D = \max\{s \in S : X_s \leq D\}$, and note that any security $F \in \mathcal{F}_D$ must be such that $F_s = 0$ for all $s \leq s_D$. The next proposition shows that the issuer may cease to issue any security when she has pre-existing debt outstanding.

Proposition 2. *Suppose the issuer already has debt outstanding with face value D . Then, if $\pi^M(A_s) < \delta \pi^I(A_s)$ for all $s > s_D$, the solution to (2) is $F_s = 0$ for all $s \in S$.*

This result shows the consistency of our approach with the well-known debt overhang

problem. Notably, our firm stops the issuance of all securities when it becomes over-levered. While the result in This prediction contrasts with that of informational theories of security design, in which the firm may start to issue equity instead of debt when it has preexisting debt (see [Fulghieri et al., 2013](#) for an excellent discussion). These opposing predictions can in principle be used to test apart these two theories (i.e., disagreement vs. asymmetric information). The empirical evidence in [Erel et al. \(2011\)](#) indicates that low market sentiment can indeed lead firms to not access credit markets.

4 Pooling

This section shows how an issuer who has more optimistic beliefs than the market can strictly benefit from pooling different assets and designing a security backed by the cash-flows generated by the pool of assets. By exploring a new mechanism that can lead to pooling, this result speaks to a key question in security design that has become a central item of the policy discussion in the aftermath of the financial crisis. We begin by illustrating this result by means of a simple example.

4.1 A Simple Example

Suppose that there are two possible states, $S = \{1, 2\}$. Let $0 < X_1 < X_2$ be the cash-flows that the issuer's asset generates under each of the states, and let $\pi^I \in (0, 1)$ and $\pi^M \in (0, 1)$ be, respectively, the probability the issuer and market assigns to state 1. As above, we assume that the issuer is more optimistic than the market, so $\pi^I < \pi^M$.

Consider first the problem of designing a security backed by the asset described above. By [Lemma 1](#) and the restriction to monotonic securities, an optimal security F has $F_1 = X_1$ and $F_2 \geq F_1 = X_1$. The market price of security F is $p(F) = \pi^M X_1 + (1 - \pi^M)F_2$, and the

issuer's payoff from selling this security is

$$p(F) + \delta(1 - \pi^I)(X_2 - F_2) = X_1 + (1 - \pi^M - \delta(1 - \pi^I))(F_2 - X_1) + \delta(1 - \pi^I)(X_2 - X_1).$$

The optimal security has $F_2 = X_1$ if $\pi^M > 1 - \delta(1 - \pi^I)$ and $F_2 = X_2$ if $\pi^M < 1 - \delta(1 - \pi^I)$.

Consider next the case in which the issuer has two identical assets, X^1 and X^2 , with cash-flows that are independently and identically distributed. Each of the assets can produce cash-flows in $\{X_1, X_2\}$. As before, the issuer (market) believes that asset X^i yields cash-flow X_1 with probability π^I (π^M). Assume that $\pi^M > 1 - \delta(1 - \pi^I)$, so that the optimal security backed by asset X^i has $F_s = X_1$ for $s = 1, 2$. The issuer's profits from selling the securities separately are

$$2p(F) + 2\delta(1 - \pi^I)(X_2 - F_2) = 2X_1 + 2\delta(1 - \pi^I)(X_2 - X_1). \quad (3)$$

Suppose instead that the issuer pools the two assets and sells a single security backed by the cash-flows generated by the pool. Let $Y = X^1 + X^2$, and consider a security $F_Y = \min\{Y, X_1 + X_2\}$; i.e. F_Y is debt with face value equal to $X_1 + X_2$. The price that the market is willing to pay for security F_Y is $p(F_Y) = (\pi^M)^2 2X_1 + (1 - (\pi^M)^2)(X_1 + X_2)$, and the issuer's payoff from selling this security is

$$p(F_Y) + \delta(1 - \pi^I)^2(2X_2 - X_2 - X_1) = 2X_1 + (1 - (\pi^M)^2 + \delta(1 - \pi^I)^2)(X_2 - X_1) \quad (4)$$

Comparing equations (3) and (4), the issuer strictly prefers to pool the assets and sell security F_Y if $\pi^M < \sqrt{1 - \delta(1 - (\pi^I)^2)}$. Therefore, for $\pi^M \in \left(1 - \delta(1 - \pi^I), \sqrt{1 - \delta(1 - (\pi^I)^2)}\right)$, pooling is strictly optimal for the issuer.¹⁵ This simple example suggests that changes in belief divergence between issuers and the market may relate to the time-series variation in

¹⁵Note that $\sqrt{1 - \delta(1 - \pi^2)} > 1 - \delta(1 - \pi)$ for all $\pi \in (0, 1)$.

the issuance of asset-backed securities (Chernenko et al., 2013).

4.2 General Framework

We now present a general result. Consider an issuer who owns two assets, X^1 and X^2 , with *iid* returns. Let $S = \{1, \dots, K\}$ and let $\{X_s\}_{s \in S}$ be the possible cash-flow realizations of asset X^i . Without loss of generality we assume that $X_1 \leq X_2 \leq \dots \leq X_K$.

Let π^I and π^M be two probability distributions over S , with π^I and π^M representing, respectively, the beliefs of issuer and market. The issuer is more optimistic than the market, and we model this by assuming that π^I first-order stochastically dominates π^M . As before, we assume that the issuer discounts future profits at rate $\delta < 1$, whereas the market discounts future profits at rate 1. The following definition generalizes Definition 1 to the current environment:

Definition 2. We say that security F backed by asset $Y = X^1 + X^2$ is $X^1 X^2$ -monotonic if:

- (i) for all $s' \in S$, $F_{s,s'}$ and $X_s + X_{s'} - F_{s,s'}$ are increasing in s , and
- (ii) for all $s \in S$, $F_{s,s'}$ and $X_s + X_{s'} - F_{s,s'}$ are increasing in s' .

Let \mathcal{F}_Y be the set of feasible securities:

$$\mathcal{F}_Y := \left\{ F \in \mathbb{R}^{|\hat{S}|} : 0 \leq F_{s,s'} \leq X_s + X_{s'} \forall (s, s') \in \hat{S} \text{ and } F \text{ is } X^1 X^2\text{-monotonic} \right\}.$$

The price that the market is willing to pay for security $F \in \mathcal{F}_Y$ is

$$p_Y(F) := \sum_{s \in S} \sum_{s' \in S} \pi_s^M \pi_{s'}^M F_{s,s'}. \tag{5}$$

The issuer's payoff from selling security $F \in \mathcal{F}_Y$ is

$$U_Y(F) := p_Y(F) + \delta \sum_{s \in S} \sum_{s' \in S} \pi_s^I \pi_{s'}^I (X_s + X_{s'} - F_{s,s'}).$$

The optimal pooled security solves

$$\sup_{F \in \mathcal{F}_Y} U_Y(F). \tag{6}$$

Let $F^* \in \mathcal{F}$ be the optimal security backed by a single asset X^i . The issuer's payoff from selling two individual securities, each backed by one asset, is $2U(F^*)$. The following result provides sufficient conditions under which $\sup_{F \in \mathcal{F}_Y} U_Y(F) > 2U(F^*)$; i.e., under which pooling the securities is strictly optimal.

Proposition 3. *Assume that:*

(i) *there exists $k \in S \setminus \{K\}$ such that $\pi^M(A_s) \geq \delta \pi^I(A_s)$ if and only if $s \leq k$, and*

$$(ii) \frac{\pi^M(A_{k+1})}{\delta \pi^I(A_{k+1})} > \frac{2 - \pi^I(A_{k+1})}{2 - \pi^M(A_{k+1})}.$$

Then, $\sup_{F \in \mathcal{F}_Y} U_Y(F) > 2U(F^)$.*

Proposition 3 generalizes the example of section 4.1 to the current setting. As in the example, pooling the assets allows the issuer to design securities that are better tailored to the relatively pessimistic beliefs of investors. In turn, this makes the issuer strictly better off than selling the two securities as separate concerns.

We stress that the restrictions in Definition 2 do not necessarily imply that security $F \in \mathcal{F}_Y$ will be monotonic in $Y = X^1 + X^2$; that is, $F(Y)$ and $Y - F(Y)$ need not be increasing in Y . To restrict attention to securities that are monotonic in Y , let $\tilde{S} = \{1, \dots, k^2\}$ be a relabeling of the set of states in $\hat{S} = S \times S$ such that $Y_{\tilde{s}+1} \geq Y_{\tilde{s}}$ for all $\tilde{s} \in \tilde{S}$. Let $\tilde{\pi}^I$ and $\tilde{\pi}^M$ be, respectively, the issuer's and market's beliefs over \tilde{S} (which are derived from

π^I and π^M). Since π^I first-order stochastically dominates π^M , it follows that $\tilde{\pi}^I$ first-order stochastically dominates $\tilde{\pi}^M$. With this notation, security F backed by asset Y is monotonic in Y if $F_{\tilde{s}}$ and $Y_{\tilde{s}} - F_{\tilde{s}}$ are both increasing in \tilde{s} . Let

$$\mathcal{F}_Y^* := \left\{ F \in \mathbb{R}^{|\tilde{S}|} : 0 \leq F_s \leq Y_s \forall s \in \tilde{S} \text{ and } F \text{ is monotonic} \right\}.$$

When restricted to issue securities in \mathcal{F}_Y^* , the issuer's problem is

$$\sup_{F \in \mathcal{F}_Y^*} U_Y(F). \tag{7}$$

Note that the solution to (7) is characterized by Proposition 1 (using beliefs $\tilde{\pi}^I$ and $\tilde{\pi}^M$ instead of π^I and π^M). With the solution to (7) in hand, one can easily check if $\sup_{F \in \mathcal{F}_Y^*} U_Y(F) > 2U(F^*)$.¹⁶

We conclude the discussion of pooling by pointing out the limitations of our results. Specifically, the underlying assets' returns need not be *iid*, but the results depend on the correlations not being too high. In Appendix C, we generalize the simple example presented above for non-zero correlations and characterize the set of parameters for which pooling is optimal. The fact that pooling ceases to be optimal when correlations between the underlying assets increase too much is consistent with the time-series variation in the issuance of asset-backed securities (Chernenko et al., 2013).

5 Tranching

This section extends our basic framework in section 3 to allow for heterogeneity of beliefs among investors. We show that, in this setting, it may be optimal for the issuer to sell

¹⁶Note that the example in Section 4.1 already shows that $\sup_{F \in \mathcal{F}_Y^*} U_Y(F) > 2U(F^*)$, since the pooled security in that example is monotonic in Y .

multiple tranches to the market.

As in section 3, we consider an issuer who owns an asset which will yield state-contingent payoffs at date $t = 1$. Let $\{X_s\}_{s \in S}$ be the possible cash-flow realizations of the asset, where $S = \{1, \dots, K\}$ is the set of possible states of nature. Let π^I be a probability distribution over S representing the issuer's beliefs about the possible state realizations. The issuer discounts future cash-flows at rate $\delta < 1$, while market participants discount them at rate 1.

There are two types of investors in the market, $\tau = t_1, t_2$. The two types of investors differ in their beliefs about the cash-flow distribution of the asset that the issuer owns. Let π^τ be a probability distribution over S representing the beliefs of investors of type τ . We assume that the issuer is more optimistic than both types of investors: for $\tau = t_1, t_2$, π^I first-order stochastically dominates π^τ . To keep the analysis simple, we further assume that for $\tau = t_1, t_2$, there exists $s_\tau \in S \setminus \{1, K\}$ with $s_{t_1} \neq s_{t_2}$ such that $\pi^\tau(A_s) \geq \delta \pi^I(A_s)$ if and only if $s \leq s_\tau$. Without further loss of generality, assume that $s_{t_1} < s_{t_2}$.

For any security $F \in \mathcal{F}$, the price that investors of type τ are willing to pay is $p^\tau(F) := \sum_s \pi_s^\tau F_s$. The profits that the issuer gets from selling security F to investors of group τ are

$$U^\tau(F) := p^\tau(F) + \delta \sum_s \pi_s^I (X_s - F_s).$$

For $\tau = t_1, t_2$, let F^τ be the security that solves $\sup_{F \in \mathcal{F}} U^\tau(F)$. By Proposition 1 and our assumptions on beliefs, F^τ has $F_s^\tau = \min\{X_s, X_{s_\tau}\}$ for all $s \in S$. Note that if the issuer designs a single security $F \in \mathcal{F}$ to sell to the market, the largest payoff she can obtain is $\max\{U^{t_1}(F^{t_1}), U^{t_2}(F^{t_2})\}$.

Consider next an issuer who designs two different securities, F^1 and F^2 , both of them backed by the cash-flows generated by asset X_s ; i.e., with $0 \leq F_s^1 + F_s^2 \leq X_s$ for all $s \in S$.

Definition 3. We say that securities F^1 and F^2 are *jointly monotonic* if F_s^1 and F_s^2 are increasing in s and if $X_s - F_s^1 - F_s^2$ is increasing in s .

Let \mathcal{F}^T be the set of feasible securities

$$\mathcal{F}^T := \{F^1, F^2 \in \mathbb{R}_+^K : 0 \leq F_s^1 + F_s^2 \leq X_s \forall s \in S \text{ and } F^1 \text{ and } F^2 \text{ are jointly monotonic}\}.$$

For any $F^1, F^2 \in \mathcal{F}^T$, let

$$U^T(F^1, F^2) := \max\{p^{t_1}(F^1), p^{t_2}(F^1)\} + \max\{p^{t_1}(F^2), p^{t_2}(F^2)\} + \delta \sum_s \pi_s^I (X_s - F_s^1 - F_s^2),$$

be the payoff that the issuer obtains from selling this pair of securities to the market. The issuer's problem is

$$\sup_{(F^1, F^2) \in \mathcal{F}^T} U^T(F^1, F^2). \quad (8)$$

Our goal is to identify sufficient conditions on the investors' beliefs under which offering two tranches is strictly better than selling one security; i.e., under which $\sup_{(F^1, F^2) \in \mathcal{F}^T} U^T(F^1, F^2) > \max\{U^{t_1}(F^{t_1}), U^{t_2}(F^{t_2})\}$.

Assumption 1. *There exists $\hat{s} \in S$ with $\hat{s} + 1 \leq s_{t_1}$ such that $\sum_{s \leq s'} \pi_s^{t_1} < \sum_{s \leq s'} \pi_s^{t_2}$ for all $s' \leq \hat{s}$ and $\sum_{s \leq s'} \pi_s^{t_1} \geq \sum_{s \leq s'} \pi_s^{t_2}$ for $s' \geq \hat{s} + 1$, with strict inequality for $s' \neq K$.*

Assumption 1 states that the c.d.f's $\Pi_s^\tau := \sum_{s \leq s'} \pi_s^\tau$ of the two types of investors cross at exactly one point. When the two types of investors assign the same value to the underlying asset (i.e., when $\sum_s \pi_s^{t_1} X_s = \sum_s \pi_s^{t_2} X_s$), Assumption 1 implies that π^{t_1} second-order stochastically dominates π^{t_2} ; i.e., type t_2 investors perceive the asset to be more risky than type t_1 investors. Note that Assumption 1 implies that $\pi^{t_1}(A_s) > \pi^{t_2}(A_s)$ for all $s \leq \hat{s} + 1, s \neq 1$ and $\pi^{t_1}(A_s) < \pi^{t_2}(A_s)$ for all $s > \hat{s} + 1$.

Proposition 4. *Under Assumption 1, $\sup_{(F^1, F^2) \in \mathcal{F}^T} U^T(F^1, F^2) > \max\{U^{t_1}(F^{t_1}), U^{t_2}(F^{t_2})\}$.*

Proposition 4 establishes that tranching can be optimal when an issuer faces different types of investors, with different beliefs about the cash-flow distribution of the issuer's asset.

We prove Proposition 4 by showing that, when Assumption 1 holds, selling an individual security is strictly dominated by selling securities $(F^1, F^2) \in \mathcal{F}^T$, with $F_s^1 = \min\{X_s, X_{\hat{s}+1}\}$ (i.e., F^1 is debt with face value $X_{\hat{s}+1}$) and

$$F_s^2 = \begin{cases} 0 & \text{if } s \leq \hat{s} + 1, \\ X_s - X_{\hat{s}+1} & \text{if } s \in (\hat{s} + 1, s_{t_2}], \\ X_{s_{t_2}} - X_{\hat{s}+1} & \text{if } s > s_{t_2}. \end{cases}$$

Security F^1 , which can be thought of as a senior tranche, is bought by investors of type t_1 . Security F^2 , which can be thought of as a junior tranche which only pays off when the asset's returns are larger than $X_{\hat{s}+1}$, is bought by investors of type t_2 . Finally, the issuer only retains cash-flows $X_s - X_{s_{t_2}}$ at states $s > s_{t_2}$.

6 Convertibles

This section provides a simple dynamic extension of the disagreement framework. The stylized version of the model presented here introduces the possibility of financing a project in multiple stages and contracts between the issuer (here called an entrepreneur) and investor (the bank or venture capitalist) that can depend on interim performance. We show that convertible securities that are used in venture capital financing naturally arise under the assumption that the issuer and investor's beliefs differ; a key assumption is that the investment project requires a relatively large investment and has high upside potential – i.e., a highly skewed payoff profile. To be able to most clearly illustrate the role of belief differences, we abstract away from some more intricate features of VC contracts and the frictions that generate them, including the role of moral hazard, adverse selection, taxes, control and monitoring rights, etc..

The setup is as follows. The entrepreneur is endowed with an investment opportunity,

which requires an initial investment I_0 at time $t = 0$, and offers in period $t = 2$ a risky payoff. There are two states of nature, $\{H, L\}$ (high or low). In the interim period, $t = 1$, a public and contractible signal is observed, which we specify below. In response to the signal, there are two options for the project:

- the project can be left as is, in which case the returns of the investment at state $s \in \{H, L\}$ are X_s , with $X_H > X_L > 0$;
- the project can be expanded by way of an interim investment $I_1 > 0$, in which case the returns of the investment at state $s \in \{H, L\}$ are $K \times X_s$, where $K > 1$.

Let $\pi^E \in (0, 1)$ and $\pi^{VC} \in (0, 1)$ be, respectively, the entrepreneur's and the venture capitalist's initial beliefs that the realized state at $t = 2$ will be H . We assume that $\pi^E > \pi^{VC}$, so the entrepreneur is more optimistic about the project's outcome than the VC.

The interim signal at time $t = 1$, σ , can take either of two values: $\sigma \in \{h, l\}$. We assume that signals $\sigma = h, l$ are informative about the state of nature: the entrepreneur and the VC believe that

$$P(\sigma = h|s = H) = P(\sigma = l|s = L) = \alpha > \frac{1}{2}.$$

For $\sigma \in \{h, l\}$ and for $i = E, VC$, let $\pi^i(\sigma)$ denote the probability that i assigns to the state being H after observing signal σ :

$$\pi^i(l) = \frac{(1 - \alpha)\pi^i}{(1 - \alpha)\pi^i + \alpha(1 - \pi^i)} < \pi^i \quad \pi^i(h) = \frac{\alpha\pi^i}{\alpha\pi^i + (1 - \alpha)(1 - \pi^i)} > \pi^i.$$

The contract, in exchange for which the entrepreneur receives funding from a competitive VC sector, specifies two things:

- (i) an expansion decision $\mathbf{1}(\sigma) \in \{0, 1\}$ to be made at $t = 1$ as a function of the signal σ ; $\mathbf{1}(\sigma) = 1$ denotes expanding the firm and $\mathbf{1}(\sigma) = 0$ denotes not expanding the firm; and
- (ii) repayments $z = (z_L(\sigma), z_H(\sigma))$ from the entrepreneur to the VC to be made at $t = 2$: for $s \in \{H, L\}$ and $\sigma \in \{h, l\}$, $z_s(\sigma) \times (1 + (K - 1)\mathbf{1}(\sigma))$ is the repayment at state s if signal σ was observed at the interim stage (with $z_s(\sigma) \in [0, X_s]$).

To show how convertible securities can be optimal under belief heterogeneity, we make the following parametric assumptions:

Assumption 2. *(i) the VC believes that the project is profitable enough to invest I_1 at $t = 1$ only after observing signal $\sigma = h$:*

$$(K - 1) (\pi^{VC}(h)X_H + (1 - \pi^{VC}(h))X_L) > I_1 > K (\pi^{VC}(l)X_H + (1 - \pi^{VC}(l))X_L).$$

(ii) the VC believes that the project is profitable but risky:

$$\pi^{VC}X_H + (1 - \pi^{VC})X_L > I_0 > X_L.$$

The entrepreneur's expected payoff from contract $(z, \mathbf{1})$ is

$$\begin{aligned} U^E(z, \mathbf{1}) &:= [(1 - \alpha)\pi^E(X_H - z_H(l)) + \alpha(1 - \pi^E)(X_L - z_L(l))] (1 + \mathbf{1}(l)(K - 1)) \\ &\quad + [\alpha\pi^E(X_H - z_H(h)) + (1 - \alpha)(1 - \pi^E)(X_L - z_L(h))] (1 + \mathbf{1}(h)(K - 1)) \end{aligned} \quad (9)$$

The VC's payoff from this contract is

$$\begin{aligned}
U^{VC}(z, \mathbf{1}) &:= [(1 - \alpha)\pi^{VC}z_H(l) + \alpha(1 - \pi^{VC})z_L(l)](1 + \mathbf{1}(l)(K - 1)) \\
&\quad + [\alpha\pi^{VC}z_H(h) + (1 - \alpha)(1 - \pi^{VC})z_L(h)](1 + \mathbf{1}(h)(K - 1)) \quad (10) \\
&\quad - \rho_l \mathbf{1}(l)I_1 - \rho_h \mathbf{1}(h)I_1 - I_0,
\end{aligned}$$

where ρ_l and ρ_h denote, respectively, the probability that the VC assigns to the signal taking values l and h , respectively, i.e., $\rho_l = (1 - \alpha)\pi^{VC} + \alpha(1 - \pi^{VC})$ and $\rho_h = \alpha\pi^{VC} + (1 - \alpha)(1 - \pi^{VC})$.

The problem of the entrepreneur is

$$\max_{(z, \mathbf{1})} U^E(z, \mathbf{1}) \text{ s.t.} \quad (11)$$

$$U^{VC}(z, \mathbf{1}) \geq 0, \quad (\text{BE})$$

$$K(\pi^{VC}(\sigma)z_H(\sigma) + (1 - \pi^{VC}(\sigma))z_L(\sigma)) \geq I_1 \text{ if } \mathbf{1}(\sigma) = 1. \quad (\text{EC})$$

Constraint (BE) is the VC's break-even condition. Constraint (EC) requires that, if the VC expands the project at $t = 1$, her expected return should cover the investment cost.

Proposition 5. *If Assumption 2 holds, the solution to (11) is such that:*

- (i) *the project is expanded if and only if $\sigma = h$; i.e., $\mathbf{1}(l) = 0$ and $\mathbf{1}(h) = 1$;*
- (ii) *for $\sigma \in \{l, h\}$, $z_L(\sigma) = X_L$;*
- (iii) *$z_H(l)$ and $z_H(h)$ are such that (BE) holds with equality.*

Proposition 5 characterizes the main properties of the solution to (11). Part (i) follows immediately from Assumption 2. Part (ii) follows from the fact that the entrepreneur is

relatively more optimistic than the VC, and so the cheapest way to satisfy the VC's break-even condition is to repay the entire cash-flows at the low state. (This feature is reminiscent of the results in the optimality of debt in section 3.2.)

Proposition 5 does not pin down what the exact payments at state H are.¹⁷ However, under further parametric conditions, convertible preferred stock is an optimal contract:

$$\rho_H K X_L + \rho_l X_L < I_0 + \rho_h I_1 < K [(1 - \alpha)\pi^{VC} X_H + \alpha(1 - \pi^{VC})X_L] + \rho_l X_L. \quad (12)$$

The first inequality in equation (12) states that the VC does not break even under a contract that specifies repayments $z_s(\sigma) = X_L$ for $s \in \{L, H\}$ and $\sigma \in \{l, h\}$. The second inequality in equation (12), on the other hand, states that the VC makes a strict profit under a contract that specifies repayments $z_H(l) = z_L(l) = z_L(h) = X_L$ and $z_H(h) = X_H$.

Corollary 6. *Suppose that Assumption 2 and (12) hold. Then, the following contract solves (11):*

- (i) *the project is expanded if and only if $\sigma = h$; i.e., $\mathbf{1}(l) = 0$ and $\mathbf{1}(h) = 1$;*
- (ii) *$z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$.*
- (iii) *$z_H(l) = X_L$ and $z_H(h) \in (X_L, X_H)$ such that (BE) holds with equality.*

The optimal contract in Corollary 6 can be implemented by convertible security that promises $\min\{R, K \times X_L\}$ (where R is the return of the project) to the VC, and gives the VC the following options: (i) after observing interim performance, choose whether or not to invest in expanding the project; and (ii) if the project is expanded, choose whether or not

¹⁷Indeed, given the linearity of the entrepreneur and the VC's payoffs, there is a continuum of optimal contracts. Increasing $z_H(l)$ by Δ allows the entrepreneur to reduce $z_H(h)$ by $\frac{1-\alpha}{\alpha} \frac{1}{K} \Delta$ (so that the break even constraint is still satisfied with equality). This change in the contract leaves the entrepreneur indifferent since $-\Delta\pi^E(1 - \alpha) + \frac{1-\alpha}{\alpha} \frac{1}{K} \Delta\alpha\pi^E K = 0$.

to convert the original security into a fraction $z_H(h)$ of equity after observing the realized profits.

This example illustrates that our disagreement-based theory of security design can explain the emergence of convertible contracts between entrepreneurs and financiers in a natural way: the entrepreneur's relative optimism that the project will go well is the driving force not only behind the entrepreneur's venture itself, but also behind the financing vehicle that helps her realize the project. Aside from its simplicity, an attractive feature of the model presented here is that highly skewed projects (those with high investment needs and high potential payoffs when everything goes well) receive financing with convertible securities as typically used in VC; optimistic entrepreneurs with less ambitious projects can also finance their ventures with straight debt.

Lastly, while we emphasize the role of disagreement, of course, other frictions including moral hazard, asymmetric information, and taxes are also important in the context of financing young firms and can explain other more intricate features of VC contracts from which our model abstracts away. Indeed, previous work on VC financing has highlighted how these frictions shape the types of contracts that a VC will optimally offer to an entrepreneur. For instance, [Schmidt \(2003\)](#) shows how convertibles can be an optimal way of inducing both the entrepreneur and the VC to put costly effort into the project. [Bergemann and Hege \(1998\)](#) show that convertible securities can be optimal in a dynamic environment with moral hazard in which both the entrepreneur and the VC need to learn about the feasibility of the project. Finally, convertible securities can be an optimal way of allocating control rights when these rights cannot be separated from cash-flow rights (i.e., [Marx \(1998\)](#)). Our theory complements these studies by highlighting a new force that makes convertibles optimal. Moreover, unlike theories based on control-rights, convertible securities are optimal in our theory even when control rights can be separated from cash-flow rights, which is typically the case in real world VC financing (i.e., [Kaplan and Strömberg \(2003\)](#)).

7 Conclusion

This paper offers a theory of optimal security design based on the premise that the issuer is more optimistic about the asset's return than the market. In particular, disagreement about the right tail of the cash-flow distribution determines which security will be issued. As information about the tails, by definition, is scarce, and therefore agreement about the precise tail characteristics is generally unlikely, we think of the theory as widely applicable. The most frequently issued security – debt – indeed arises as an optimal security in our model. Risk-free debt and equity arise as special cases – the former when there is more disagreement about the right tail, and the latter when the market is more confident about the likelihood of right-tail outcomes and hence there is less disagreement. A mild variation in distributional assumptions generates call provisions that are common features of debt contracts in the real world. Moreover, we find that when the balance sheet is encumbered with pre-existing debt, debt overhang occurs and the firm ceases to issue any security. We also show that issuing securities backed by a pool of assets (instead of issuing one security per asset) can be optimal. When there is disagreement among investors, the issuer optimally sells different tranches to the market. Finally, in a stylized setting with multiple financing rounds, convertible securities similar those observed in typical venture capital contracts are optimal.

In sum, we find that disagreement between issuer and market helps explain a variety of real-world security designs that have thus far required multiple distinct models and frictions as explanations. For tractability, our model abstracts away from frictions that are known to be important for security design, such as moral hazard, adverse selection, taxes, etc. Combining disagreement with these frictions may help researchers explain other intricate features of real-world financial contracts.

A Proofs

Proof of Lemma 1

Proof. Suppose F solves (1), but $F_s < X_s \forall s \in S$. Thus, there exists $\lambda > 0$ such that $F_s + \lambda \leq X_s \forall s \in S$. Let F' denote the security with $F'_s = F_s + \lambda \forall s \in S$. Note that, if F is monotonic, then so is F' . If the issuer sells security F' instead of F her total payoff is given by

$$\begin{aligned}
 U(F') &= \sum_{s \in S} \pi_s^M F'_s + \delta \sum_{s \in S} \pi_s^I (X_s - F'_s) \\
 &= \sum_{s \in S} \pi_s^M F_s + \lambda + \delta \sum_{s \in S} \pi_s^I (X_s - F_s) - \delta \lambda \\
 &= U(F) + \lambda(1 - \delta) > U(F).
 \end{aligned}$$

Hence, F cannot be optimal. □

Proof of Lemma 2

Proof. Suppose that $\pi^M(A_s) > \delta \pi^I(A_s)$ for all $s \leq k$ and let F be an optimal security. Towards a contradiction, suppose that $F_s < X_s$ for some $s \leq k$. Let $j = \min\{s \leq k : F_s < X_s\}$ and let $\epsilon > 0$ be such that $F_j + \epsilon = X_j$. Let F' be a security with $F'_s = F_s = X_s$ for all $s < j$ and $F'_s = F_s + \epsilon$ for all $s \geq j$. Since F is a monotonic security, it follows that $F'_s \leq X_s$ for all s and that F' is also monotonic; i.e., $F' \in \mathcal{F}$. Note that

$$\begin{aligned}
 U(F') &= \sum_{s \in S} \pi_s^M F'_s + \delta \sum_{s \in S} \pi_s^I (X_s - F'_s) \\
 &= \sum_{s \in S} \pi_s^M F_s + \delta \sum_{s \in S} \pi_s^I (X_s - F_s) + (\pi^M(A_j) - \delta \pi^I(A_j)) \epsilon \\
 &= U(F) + (\pi^M(A_j) - \delta \pi^I(A_j)) \epsilon > U(F),
 \end{aligned}$$

where the last inequality follows since $\pi^M(A_s) > \delta\pi^*(A_s)$ for all $s \leq k$ and since $\epsilon > 0$. This contradicts the assumption that F is an optimal security. Hence, it must be that $F_s = X_s$ for all $s \leq k$. \square

Proof of Proposition 1

Proof. The proof is by induction on s . Let F be an optimal security. Note first that, from Lemma 2 and Remark 1, there exists $k \in S$ such that $\pi^M(A_s) > \delta\pi^I(A_s)$ and $F_s = X_s$ for all $s \leq k$. Let $k^* \in S$ be the largest such k , and note that the statement in the proposition is true for all $s \leq k^*$.

Suppose next that the statement in the proposition is true for all $s \leq s'$ for some $s' \geq k^*$. We now show that the statement is also true for $s = s' + 1$. There are two cases to consider: (i) $\pi^M(A_{s'+1}) \geq \delta\pi^I(A_{s'+1})$, and (ii) $\pi^M(A_{s'+1}) < \delta\pi^I(A_{s'+1})$. Consider first case (i). Since F is monotonic, it must be that $F_{s'+1} \in [F_{s'}, F_{s'} + X_{s'+1} - X_{s'}]$. Suppose by contradiction that $F_{s'+1} < F_{s'} + X_{s'+1} - X_{s'}$ and define $\epsilon := F_{s'} + X_{s'+1} - X_{s'} - F_{s'+1} > 0$. Let \tilde{F} be a security such that $\tilde{F}_s = F_s$ for all $s \leq s'$ and $\tilde{F}_s = F_s + \epsilon$ for all $s \geq s' + 1$. Since F is monotonic, it follows that $\tilde{F} \in \mathcal{F}$. Note that

$$\begin{aligned} U(\tilde{F}) &= \sum_{s \in S} \pi_s^M \tilde{F}_s + \delta \sum_{s \in S} \pi_s^I (X_s - \tilde{F}_s), \\ &= \sum_{s \in S} \pi_s^M F_s + \delta \sum_{s \in S} \pi_s^I (X_s - F_s) + (\pi^M(A_{s'+1}) - \delta\pi^I(A_{s'+1})) \epsilon \\ &\geq U(F), \end{aligned}$$

where we used $\pi^M(A_{s'+1}) \geq \delta\pi^I(A_{s'+1})$. Since F was assumed to be optimal, security \tilde{F} with $\tilde{F}_{s'+1} = F_{s'} + X_{s'+1} - X_{s'}$ is also optimal.¹⁸

¹⁸Note that $U(\tilde{F}) > U(F)$ whenever $\pi^M(A_{s'+1}) > \delta\pi^I(A_{s'+1})$; i.e., in this case security F is not optimal. When $\pi^M(A_{s'+1}) = \delta\pi^I(A_{s'+1})$, securities F and \tilde{F} give the issuer the same payoff.

Consider next case (ii), and suppose that $F_{s'+1} > F_{s'}$. Let \hat{F} be a security such that $\hat{F}_s = F_s$ for all $s \leq s'$ and $\hat{F}_s = F_s - (F_{s'+1} - F_{s'})$ for all $s \geq s' + 1$. Since F is monotonic, $\hat{F} \in \mathcal{F}$. Moreover,

$$\begin{aligned} U(\hat{F}) &= \sum_{s \in S} \pi_s^M \hat{F}_s + \delta \sum_{s \in S} \pi_s^I (X_s - \hat{F}_s), \\ &= \sum_{s \in S} \pi_s^M F_s + \delta \sum_{s \in S} \pi_s^I (X_s - F_s) - (\pi^M(A_{s'+1}) - \delta \pi^I(A_{s'+1})) (F_{s'+1} - F_{s'}) \\ &> U(F), \end{aligned}$$

where we used $\pi^M(A_{s'+1}) < \delta \pi^I(A_{s'+1})$ and $F_{s'+1} > F_{s'}$. This contradicts F being an optimal security. Therefore, if F is optimal it must be that $F_{s'+1} = F_{s'}$. \square

Proof of Proposition 2

Proof. Towards a contradiction, suppose that the security $F \in \mathcal{F}_D$ that solves (2) has $F_s > 0$ for some $s > s_D$. Let $s' = \min\{s \in S : F_s > 0\}$. Let $\epsilon \in (0, F_{s'})$, and let F' be the security with $F'_s = F_s = 0$ for all $s < s'$ and $F'_s = F_s - \epsilon$ for all $s \geq s'$. Note that, since $F \in \mathcal{F}_D$, it must be that $F' \in \mathcal{F}_D$. Note further that

$$\begin{aligned} U_D(F') &= \sum_{s \in S} \pi_s^M F'_s + \delta \sum_{s \in S} \pi_s^I (X_s - \min\{X_s, D\} - F'_s) \\ &= \sum_{s \in S} \pi_s^M F_s - \pi^M(A_{s'})\epsilon + \delta \sum_{s \in S} \pi_s^I (X_s - \min\{X_s, D\} - F_s) + \delta \pi^I(A_{s'})\epsilon \\ &= U_D(F) + \epsilon(\delta \pi^I(A_{s'}) - \pi^M(A_{s'})) > U_D(F), \end{aligned}$$

where the strict inequality follows since $\delta \pi^I(A_{s'}) > \pi^M(A_{s'})$ for all $s > s_D$ and since $\epsilon > 0$. Hence, F cannot be an optimal security. \square

Proof of Proposition 3

Proof. By Corollary 3, under assumption (i) the optimal security backed by a single asset X^i is $F^* = \min\{X_s, X_k\}$. Note that selling two individual securities F^* , each backed by one of the assets, is the same as selling security $\tilde{F} \in \mathcal{F}_Y$ such that

$$\tilde{F}_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_k + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_k & \text{if } s \leq k, s' > k, \\ 2X_k & \text{if } s > k, s' > k. \end{cases}$$

Consider security the following alternative security $F \in \mathcal{F}_Y$

$$F_{s,s'} = \begin{cases} X_s + X_{s'} & \text{if } s, s' \leq k, \\ X_{k+1} + X_{s'} & \text{if } s > k, s' \leq k, \\ X_s + X_{k+1} & \text{if } s \leq k, s' > k, \\ X_k + X_{k+1} & \text{if } s > k, s' > k. \end{cases}$$

Note that, for any beliefs π over S ,

$$\begin{aligned} \sum_s \sum_{s'} \pi_s \pi_{s'} (F_{s,s'} - \tilde{F}_{s,s'}) &= \sum_{s=1}^k \pi_s \left(\sum_{s'=k+1}^K \pi_{s'} (X_{k+1} - X_k) \right) + \sum_{s=k+1}^K \pi_s \sum_{s'=1}^K \pi_{s'} (X_{k+1} - X_k) \\ &= (2 - \pi(A_{k+1})) \pi(A_{k+1}) (X_{k+1} - X_k). \end{aligned} \tag{13}$$

Note then that

$$\begin{aligned}
U_Y(F) - 2U(F^*) &= U_Y(F) - U_Y(\tilde{F}) \\
&= p_Y(F) - p_Y(\tilde{F}) + \delta \sum_s \sum_{s'} \pi_s^I \pi_{s'}^I (\tilde{F}_{s,s'} - F_{s,s'}) \\
&= \sum_s \sum_{s'} \pi_s^M \pi_{s'}^M (F_{s,s} - \tilde{F}_{s,s'}) + \delta \sum_s \sum_{s'} \pi_s^I \pi_{s'}^I (\tilde{F}_{s,s'} - F_{s,s'}) \\
&= ((2 - \pi^M(A_{k+1}))\pi^M(A_{k+1}) - \delta(2 - \pi^I(A_{k+1}))\pi^I(A_{k+1})) (X_{k+1} - X_k) > 0,
\end{aligned}$$

where we used equation (13) and condition (ii) in the statement of the Proposition. \square

Proof of Proposition 4

Proof. Suppose the issuer sells two assets, F^1 and F^2 , with $F_s^1 = \min\{X_s, X_{\hat{s}+1}\}$ and

$$F_s^2 = \begin{cases} 0 & \text{if } s \leq \hat{s} + 1, \\ X_s - X_{\hat{s}+1} & \text{if } s \in (\hat{s} + 1, s_{t_2}], \\ X_{s_{t_2}} - X_{\hat{s}+1} & \text{if } s > s_{t_2}. \end{cases}$$

Note that $(F_1, F_2) \in \mathcal{F}^T$.

Note that for any security F with F_s increasing in s and for any beliefs π over S ,

$$\begin{aligned}
\sum_s \pi_s F_s &= \pi_1 F_1 + \sum_{s \geq 2} \pi_s (F_s - F_1 + F_1) \\
&= F_1 + \sum_{s \geq 2} \pi_s (F_s - F_1 - F_2 + F_2) \\
&= F_1 + \pi(A_2) (F_2 - F_1) + \sum_{s \geq 3} \pi_s (F_s - F_2 + F_3 - F_3) \\
&\vdots \\
&= F_1 + \sum_{s \geq 2} \pi(A_s) (F_s - F_{s-1})
\end{aligned} \tag{14}$$

By equation (14), for $\tau = t_1, t_2$,

$$p^\tau(F^1) = \sum_s \pi_s^\tau F_s^1 = X_1 + \sum_{s \in [2, \hat{s}+1]} \pi^\tau(A_s) (X_s - X_{s-1}),$$

Assumption 1 implies that $p^{t_1}(F^1) > p^{t_2}(F^1)$; i.e., t_1 -investors are willing to pay more for security F^1 than t_2 -investors. Similarly, using again equation (14), for $\tau = t_1, t_2$,

$$p^\tau(F^2) = \sum_s \pi_s^\tau F_s^2 = \sum_{s \in [\hat{s}+2, s_{t_2}]} \pi^\tau(A_s) (X_s - X_{s-1}).$$

Assumption 1 implies that $p^{t_2}(F^2) > p^{t_1}(F^2)$; i.e., t_2 -investors are willing to pay more for security F^2 than t_1 -investors. Therefore,

$$\begin{aligned}
U^T(F_1, F_2) &= p^{t_1}(F^1) + p^{t_2}(F^2) + \delta \sum_s \pi_s^I (X_s - F_s^1 - F_s^2) \\
&= X_1 + \sum_{s \in [2, \hat{s}+1]} \pi^{t_1}(A_s) (X_s - X_{s-1}) + \sum_{s \in [\hat{s}+2, s_{t_2}]} \pi^{t_2}(A_s) (X_s - X_{s-1}) \\
&\quad + \delta \sum_{s \in [s_{t_2}+1, K]} \pi^I(A_s) (X_s - X_{s-1}),
\end{aligned} \tag{15}$$

where the second equality in (15) follows since, by equation (14), $\sum_s \pi_s^I(X_s - F_s^1 - F_s^2) = \sum_{s \in [s_{t_2}+1, K]} \pi^I(A_s)(X_s - X_{s-1})$.

On the other hand, the issuer's highest payoff from selling a single security is $\max\{U^{t_1}(F^{t_1}), U^{t_2}(F^{t_2})\}$. Since $F_s^\tau = \min\{X_s, X_{s_\tau}\}$,

$$\begin{aligned} U^\tau(F^\tau) &= p^\tau(F^\tau) + \delta \sum_{s > s_\tau} \pi_s^I(X_s - X_{s_\tau}) \\ &= X_1 + \sum_{s \in [2, s_\tau]} \pi^\tau(A_s)(X_s - X_{s-1}) + \delta \sum_{s \in [s_\tau+1, K]} \pi^I(A_s)(X_s - X_{s-1}). \end{aligned} \quad (16)$$

Using (15) and (16):

$$\begin{aligned} U^T(F_1, F_2) - U^{t_1}(F^{t_1}) &= \sum_{s \in [\hat{s}+2, s_{t_2}]} \pi^{t_2}(A_s)(X_s - X_{s-1}) + \delta \sum_{s \in [s_{t_2}+1, K]} \pi^I(A_s)(X_s - X_{s-1}) \\ &\quad - \sum_{s \in [\hat{s}+2, s_{t_1}]} \pi^{t_1}(A_s)(X_s - X_{s-1}) - \delta \sum_{s \in [s_{t_1}+1, K]} \pi^I(A_s)(X_s - X_{s-1}) \\ &= \sum_{s \in [\hat{s}+2, s_{t_1}]} (\pi^{t_2}(A_s) - \pi^{t_1}(A_s))(X_s - X_{s-1}) \\ &\quad + \sum_{s \in [s_{t_1}+1, s_{t_2}]} (\pi^{t_2}(A_s) - \delta \pi^I(A_s))(X_s - X_{s-1}) > 0, \end{aligned}$$

where the inequality follows since $\pi^{t_2}(A_s) > \pi^{t_1}(A_s)$ for all $s \in [\hat{s} + 2, s_{t_1}]$ (Assumption (1)) and since $\pi^{t_2}(A_s) \geq \delta \pi^I(A_s)$ for all $s \leq s_{t_2}$. Similarly,

$$U^T(F_1, F_2) - U^{t_2}(F^{t_2}) = \sum_{s \in [2, \hat{s}+1]} (\pi^{t_1}(A_s) - \pi^{t_2}(A_s))(X_s - X_{s-1}) > 0,$$

where the strict inequality follows since $\pi^{t_2}(A_s) < \pi^{t_1}(A_s)$ for all $s \in [2, \hat{s}]$ (Assumption (1)).

Therefore, $U^T(F_1, F_2) > \max\{U^{t_1}(F^{t_1}), U^{t_2}(F^{t_2})\}$. \square

Proofs of Proposition 5 and Corollary 6

Lemma 3. *Let $(z, \mathbf{1})$ be a solution to (11). If $z_H(\sigma) > 0$ for $\sigma \in \{l, h\}$, then it must be that $z_L(\sigma) = X_L$.*

Proof. Suppose by contradiction that $z_H(\sigma) > 0$ and $z_L(\sigma) < X_L$ for $\sigma \in \{l, h\}$. Suppose first that $\sigma = h$, and consider a contract $(\tilde{z}, \mathbf{1})$ such that $\tilde{z}_s(l) = z_s(l)$ for $s = H, L$, $\tilde{z}_L(h) = z_L(h) + \epsilon$ and $\tilde{z}_H(h) = z_L(h) - \frac{1-\alpha}{\alpha} \frac{1-\pi^{VC}}{\pi^{VC}} \epsilon$, with $\epsilon > 0$. Note that contract $(\tilde{z}, \mathbf{1})$ gives the VC the same expected payoff as contract $(z, \mathbf{1})$. Note further that

$$\begin{aligned} U^E(\tilde{z}, \mathbf{1}) - U^E(z, \mathbf{1}) &= (1 + \mathbf{1}(h)(K - 1)) \left(\alpha \pi^E \frac{1-\alpha}{\alpha} \frac{1-\pi^{VC}}{\pi^{VC}} \epsilon - (1-\alpha)(1-\pi^E)\epsilon \right) \\ &= (1 + \mathbf{1}(h)(K - 1)) \frac{(1-\alpha)\epsilon}{\pi^{VC}} (\pi^E(1-\pi^{VC}) - \pi^{VC}(1-\pi^E)) > 0, \end{aligned}$$

where we used $\pi^E > \pi^{VC}$. This contradicts the assumption that $(z, \mathbf{1})$ is optimal.

Suppose next that $\sigma = l$. Consider a contract $(\tilde{z}, \mathbf{1})$ such that $\tilde{z}_s(h) = z_s(h)$ for $s = H, L$, $\tilde{z}_L(l) = z_L(l) + \epsilon$ and $\tilde{z}_H(l) = z_L(l) - \frac{\alpha}{1-\alpha} \frac{1-\pi^{VC}}{\pi^{VC}} \epsilon$, with $\epsilon > 0$. Note that contract $(\tilde{z}, \mathbf{1})$ gives the VC the same expected payoff as contract $(z, \mathbf{1})$, and

$$\begin{aligned} U^E(\tilde{z}, \mathbf{1}) - U^E(z, \mathbf{1}) &= (1 + \mathbf{1}(l)(K - 1)) \left((1-\alpha)\pi^E \frac{\alpha}{1-\alpha} \frac{1-\pi^{VC}}{\pi^{VC}} \epsilon - \alpha(1-\pi^E)\epsilon \right) \\ &= (1 + \mathbf{1}(l)(K - 1)) \frac{\alpha\epsilon}{\pi^{VC}} (\pi^E(1-\pi^{VC}) - \pi^{VC}(1-\pi^E)) > 0. \end{aligned}$$

Again this contradicts the assumption that $(z, \mathbf{1})$ is optimal. □

Lemma 4. *Let $(z, \mathbf{1})$ be a solution to (11). Under Assumption 2, $z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$.*

Proof. Let $(z, \mathbf{1})$ be a solution to (11). The conditions in Assumption 2 imply that, in order for the VC to break even, it must be that $z_H(h) > 0$ and/or $z_H(l) > 0$. If both of these quantities are strictly positive, then the result follows from Lemma 3.

Suppose next that $z_H(h) = 0$ and $z_H(l) > 0$. By Lemma 3, $z_L(l) = X_L$. Towards a contradiction, suppose $z_L(h) < X_L$. Let $(\hat{z}, \mathbf{1})$ be an alternative contract with $\hat{z}_L(\sigma) = z_L(\sigma)$ for $\sigma = h, l$, $\hat{z}_H(l) = z_H(l) - \epsilon$ and $\hat{z}_H(h) = z_H(h) + \frac{1-\alpha}{\alpha} \frac{1+\mathbf{1}(l)(K-1)}{1+\mathbf{1}(h)(K-1)} \epsilon$, with $\epsilon > 0$. Contract $(\hat{z}, \mathbf{1})$ gives entrepreneur and VC the same expected payoff as contract $(z, \mathbf{1})$, so its also an optimal contract. But this contradicts Lemma 3, since $\hat{z}_L(h) = z_L(h) < X_L$ and $\hat{z}_H(h) > 0$. Hence, if $(z, \mathbf{1})$ is an optimal contract with $z_H(h) = 0$ and $z_H(l) > 0$, it must that $z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$.

Finally, consider the case with $z_H(h) > 0$ and $z_H(l) = 0$. By Lemma 3, $z_L(h) = X_L$. Towards a contradiction, suppose $z_L(l) < X_L$. Let $(\hat{z}, \mathbf{1})$ be an alternative contract with $\hat{z}_L(\sigma) = z_L(\sigma)$ for $\sigma = h, l$, $\hat{z}_H(l) = z_H(l) + \epsilon$ and $\hat{z}_H(h) = z_H(h) - \frac{1-\alpha}{\alpha} \frac{1+\mathbf{1}(l)(K-1)}{1+\mathbf{1}(h)(K-1)} \epsilon$. Again, contract $(\hat{z}, \mathbf{1})$ gives entrepreneur and VC the same expected payoff as contract $(z, \mathbf{1})$, so its also an optimal contract. But this contradicts Lemma 3, since $\hat{z}_L(l) = z_L(l) < X_L$ and $\hat{z}_H(l) > 0$. Hence, if $(z, \mathbf{1})$ is an optimal contract with $z_H(h) > 0$ and $z_H(l) = 0$, it must that $z_L(\sigma) = X_L$ for $\sigma \in \{l, h\}$. \square

Proof of Proposition 5. Part (ii) follows from Lemma 4.

We now prove part (i). Note first that, under Assumption 2, any optimal contract $(z, \mathbf{1})$ must be such that $\mathbf{1}(l) = 0$: indeed, under the condition (i) in Assumption 2, there are no feasible repayments z that satisfy constraint (EC) for $\sigma = l$ when $\mathbf{1}(l) = 1$.

We now show that, under an optimal contract, $\mathbf{1}(h) = 1$. Suppose that there exists an optimal contract $(z, \mathbf{1})$ with $\mathbf{1}(h) = 0$. Let $(\tilde{z}, \tilde{\mathbf{1}})$ be an alternative contract with $\tilde{\mathbf{1}}(h) = 1$, $\tilde{\mathbf{1}}(l) = \mathbf{1}(l) = 0$, $\tilde{z}_L(\sigma) = z_L(\sigma) = X_L$, $\tilde{z}_H(l) = z_H(l)$ and

$$\begin{aligned} \tilde{z}_H(h) &= \frac{\rho_h I_1 + \alpha \pi^{VC} z_H(h) + (1-\alpha)(1-\pi^{VC})z_L(h)}{K\alpha\pi^{VC}} - \frac{1-\alpha}{\alpha} \frac{1-\pi^{VC}}{\pi^{VC}} z_L(h) \\ &= \frac{\rho_h I_1 + \alpha \pi^{VC} z_H(h) + (1-\alpha)(1-\pi^{VC})X_L}{K\alpha\pi^{VC}} - \frac{1-\alpha}{\alpha} \frac{1-\pi^{VC}}{\pi^{VC}} X_L, \end{aligned}$$

where the second equality follows since, under any optimal contract, $z_L(h) = X_L$. Note that the VC's expected payoff under contract $(\tilde{z}, \tilde{\mathbf{1}})$ is the same as her expected payoff under contract $(z, \mathbf{1})$. Note further that

$$\begin{aligned} U^E(\tilde{z}, \tilde{\mathbf{1}}) - U^E(z, \mathbf{1}) &= K\alpha\pi^E(X_H - \tilde{z}_H(h)) - \alpha\pi^E(X_H - z_H(h)) \\ &= \alpha\pi^E \left[(K-1)X_H + (K-1)\frac{1-\alpha}{\alpha}\frac{1-\pi^{VC}}{\pi^{VC}}X_L - \frac{\rho_h I_1}{\alpha\pi^{VC}} \right] \\ &= \frac{\pi^E}{\pi^{VC}} \left[(K-1)\alpha\pi^{VC}X_H + (1-\alpha)(1-\pi^{VC})X_L - \rho_h I_1 \right] > 0. \end{aligned}$$

Hence, if $(z, \mathbf{1})$ is an optimal contract it must have $\mathbf{1}(h) = 1$. This establishes part (i).

Finally, part (iii) follows since any optimal contract $(z, \mathbf{1})$ must be such that $U^{VC}(z, \mathbf{1}) = 0$. Further, we note that if there exists an optimal contract $(z, \mathbf{1})$ such that $U^{VC}(z, \mathbf{1}) = 0$ and such that (EC) is satisfied with slack, then there exists a continuum of optimal contracts. Indeed, increasing $z_H(l)$ by ϵ allows the entrepreneur to reduce $z_H(h)$ by $\frac{1-\alpha}{\alpha}\frac{1}{K}\epsilon$ while still satisfying the VC's break even condition. This change in the contract leaves the entrepreneur indifferent since $-\epsilon\pi^E(1-\alpha) + \frac{1-\alpha}{\alpha}\frac{1}{K}\epsilon\alpha\pi^E K = 0$. \square

Proof of Corollary 6. Parts (i) and (ii) follow from Proposition (5). Finally, when the first inequality in (12) holds, the VC must get strictly more than X_L at state $s = H$ when $\sigma = l$ and/or $\sigma = h$ (otherwise the VC does not break even). When the second inequality in (12) holds, there exists $z \in (X_L, X_H)$ such that

$$I_0 + \rho_h I_1 = K \left[(1-\alpha)\pi^{VC}z + \alpha(1-\pi^{VC})X_L \right] + \rho_l X_L. \quad (17)$$

By equation (17), the VC breaks even under a contract $(z, \mathbf{1})$ with $\mathbf{1}(h) = 1$, $\mathbf{1}(l) = 0$, $z_L(l) = z_L(h) = z_H(l) = X_L$ and $z_H(h) = z$. Finally, since $I_0 > \rho_L X_L$ (by Assumption (2)),

equation (17) implies that

$$K [(1 - \alpha)\pi^{VC}z + \alpha(1 - \pi^{VC})X_L] > \rho_H I_1,$$

so that (EC) holds. Hence, by Proposition (5), contract $(z, \mathbf{1})$ is optimal. □

B Generalization to Non-monotonic Securities

Throughout section 3.2, we restricted the issuer to sell securities that are monotonic. For completeness, we now briefly describe how our results are modified if we drop this restriction.

Let

$$\mathcal{F}^u := \{F \in \mathbb{R}^K : 0 \leq F_s \leq X_s\}$$

be the unrestricted set of securities backed by asset X . Without the restriction to monotonic securities, the issuer's payoff is

$$\sup_{F \in \mathcal{F}^u} U(F). \tag{18}$$

The following result characterizes the optimal security when we relax the restriction to monotonic securities.

Proposition 6. *The solution to (18) is*

$$F_s = \begin{cases} X_s & \text{if } \pi_s^M > \delta\pi_s^I, \\ a \in [0, X_s] & \text{if } \pi_s^M = \delta\pi_s^I, \\ 0 & \text{if } \pi_s^M < \delta\pi_s^I. \end{cases}$$

Proof. For any $s \in S$, the payoff that the issuer gets from selling cash-flows $F_s \in [0, X_s]$

at state s is $\pi_s^M F_s$, while the payoff she gets from retaining those cash-flows is $\delta \pi_s^I F_s$. It is optimal for the issuer to set $F_s = X_s$ if $\pi_s^M > \delta \pi_s^I$, and to set $F_s = 0$ if $\pi_s^M < \delta \pi_s^I$. Finally, the issuer is indifferent between setting $F_s = a \in [0, X_s]$ if $\pi_s^M = \delta \pi_s^I$. \square

To gain intuition about the shape of the security that solves (18), suppose $\frac{\pi_s^I}{\pi_s^M}$ is increasing in s and let $k = \max \left\{ s : \frac{\pi_s^M}{\pi_s^I} \geq \delta \right\}$. Then, by Proposition 6, the security that solves (18) is such that

$$F_s = \begin{cases} X_s & \text{if } s \leq k \\ 0 & \text{if } s > k. \end{cases}$$

Hence, the restriction to monotonic securities imposed previously serves the same purpose as in the existing literature: indeed, without this restriction, the same security obtains as in a standard asymmetric information or moral hazard framework, see, e.g., Innes (1990). Also, the security is the same as the one Simsek (2013a) obtains in a setting with disagreement among investors.

C Generalization of the Simple Pooling Example

This appendix extends the example of section 4.1 to allow for non-zero correlation between the assets to be securitized.

As in section 4.1, suppose the issuer owns two assets, X^1 and X^2 , each of which can generate a return in $\{X_1, X_2\}$ (with $X_1 < X_2$). In contrast to section 4.1, suppose that the returns of assets X^1 and X^2 are correlated. Let $sk \in \hat{S} = \{11, 12, 21, 22\}$ denote the event that asset 1's return is X_s and asset 2's return is X_k . The beliefs of the issuer and market over the set of possible return realizations are, respectively, $\hat{\pi}^I$ and $\hat{\pi}^M$. For $j = I, M$, $\hat{\pi}_{sk}^j$ denotes the probability that j assigns to the event sk . We assume that the assets are symmetric, so that $\hat{\pi}_{12}^j = \hat{\pi}_{21}^j$ for $j = I, M$. The *iid* case of section 4.1 is the special case with $\hat{\pi}_{sk}^j = \pi_s^j \pi_k^j$

for $j = I, M$ and for all $sk \in \hat{S}$. The case in which the issuer and market believe the two assets to be positively and perfectly correlated has $\hat{\pi}_{12}^j = \hat{\pi}_{21}^j = 0$ for $j = I, M$.

Suppose first that the issuer sells two individual securities, each backed by an asset. By Lemma 1 and the restriction to monotonic securities, an optimal security F has $F_1 = X_1$ and $F_2 \geq F_1$. The price that the market is willing to pay for security F is $p(F) = X_1(\hat{\pi}_{11}^M + \hat{\pi}_{12}^M) + F_2(\hat{\pi}_{21}^M + \hat{\pi}_{22}^M)$; the issuer's payoff from selling this security is

$$p(F) + \delta(X_2 - F_2)(\hat{\pi}_{21}^M + \hat{\pi}_{22}^M) = X_1(\hat{\pi}_{11}^M + \hat{\pi}_{12}^M) + F_2(\hat{\pi}_{21}^M + \hat{\pi}_{22}^M) + \delta(X_2 - F_2)(\hat{\pi}_{21}^I + \hat{\pi}_{22}^I). \quad (19)$$

The issuer finds it optimal to set $F_2 = X_1$ if $\delta(\hat{\pi}_{21}^I + \hat{\pi}_{22}^I) > \hat{\pi}_{21}^M + \hat{\pi}_{22}^M$ and $F_2 = X_2$ if $\delta(\hat{\pi}_{21}^I + \hat{\pi}_{22}^I) \leq \hat{\pi}_{21}^M + \hat{\pi}_{22}^M$; that is, the issuer finds it optimal to set $F_2 = X_1$ whenever the market assigns a sufficiently low probability to the event that an individual asset has high returns. In what follows we maintain the assumption that $\delta(\hat{\pi}_{21}^I + \hat{\pi}_{22}^I) > \hat{\pi}_{21}^M + \hat{\pi}_{22}^M$, so that an issuer who sells individual securities F^1 and F^2 , each backed respectively by asset X^1 and X^2 , finds it optimal to set $F_s^1 = F_s^2 = X_1$ for $s = 1, 2$.

Suppose next that the issuer pools the two assets and sells a single security backed by cash-flows $Y = X^1 + X^2$. Consider a security $F_Y = \min\{Y, X_1 + X_2\}$. The price that the market is willing to pay for security F_Y is $p(F_Y) = \hat{\pi}_{11}^M 2X_1 + (1 - \hat{\pi}_{11}^M)(X_1 + X_2)$, and the issuer's payoff from selling this security is

$$p(F_Y) + \delta\hat{\pi}_{22}^I(X_2 - X_1) = \hat{\pi}_{11}^M 2X_1 + (1 - \hat{\pi}_{11}^M)(X_1 + X_2) + \delta\hat{\pi}_{22}^I(X_2 - X_1). \quad (20)$$

Comparing (19) and (20), the issuer strictly prefers selling security F_Y backed by the pool of assets than selling the two individual securities $F_s^1 = F_s^2 = X_1$ for $s = 1, 2$ if and only if $2\hat{\pi}_{21}^M + \hat{\pi}_{22}^M = 1 - \hat{\pi}_{11}^M > \delta(1 - \pi_{11}^I) = \delta(2\hat{\pi}_{21}^I + \hat{\pi}_{22}^I)$. Combining this with $\delta(\hat{\pi}_{21}^I + \hat{\pi}_{22}^I) > \hat{\pi}_{21}^M + \hat{\pi}_{22}^M$,

the issuer strictly prefers to pool the assets and sell security F_Y if

$$\hat{\pi}_{11}^M \in (1 - \delta(\hat{\pi}_{21}^I + \hat{\pi}_{22}^I) - \hat{\pi}_{21}^M, 1 - \delta(2\hat{\pi}_{21}^I + \hat{\pi}_{22}^I)). \quad (21)$$

The condition in (21) is identical to the condition in section 4.1 when the two assets are *iid*. If the issuer and the market both perceive the asset to be perfectly correlated (so that $\hat{\pi}_{21}^j = 0$ for $j = 1, 2$), the condition in (21) can never be satisfied, and hence pooling does not obtain.¹⁹

¹⁹We can also consider the case in which the issuer believes that the two assets are perfectly correlated, but the market believes that the correlation is less than perfect (i.e., $\hat{\pi}_{21}^I = 0 < \hat{\pi}_{21}^M$). In this case, condition (21) becomes $\hat{\pi}_{11}^M \in (1 - \delta\hat{\pi}_{22}^I - \hat{\pi}_{21}^M, 1 - \delta\hat{\pi}_{22}^I)$.

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