Breaking the Sovereign-Bank Diabolic Loop: A Case for ESBies

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The “diabolic loop” between sovereign and bank credit risk has been the hallmark of the 2009-12 sovereign debt crisis in the periphery of the euro area – Greece, Ireland, Italy, Portugal and Spain. In these vulnerable countries – the ECB’s terminology – the deterioration of sovereign creditworthiness reduced the market value of banks’ holdings of domestic sovereign debt. This reduced the perceived solvency of domestic banks and curtailed their lending activity. The resulting bank distress increased the chances that banks would have to be bailed out by the (domestic) government, which increased sovereign distress even further, engendering a “bailout loop”. Moreover, the recessionary impact of the credit crunch led to a reduction in tax revenue, which also contributed to weakening government solvency in these countries, triggering a “real-economy loop”. These two concomitant feedback loops are illustrated in Figure 1.

Notably, after 2012, the two loops started operating in reverse: following the “whatever it takes” speech by ECB President Mario Draghi on 26 July 2012, the market value of sovereign debt recovered, euro-area periphery banks started making capital gains on their domestic sovereign holdings, and their resulting increase in creditworthiness contributed to reduce sovereign stress. (However, the “real-economy loop” started operating in reverse much more slowly, as the resumption of lending by banks in the euro-area periphery have surfaced in 2014-15.)

There are three ingredients to the feedback loops. First, the home bias of banks’ sovereign debt portfolios, which makes their value and solvency dependent on the swings in the perceived solvency and market value of government debt (as documented by Altavilla, Simonelli and Pagano, 2015, and De Marco, 2014). Second, the inability of governments to commit not to bailout domestic banks, due to the fact that bailout is optimal once banks are distressed, which creates the feedback from bank to government distress. Third, free capital mobility, which ensures that international investors’ perceptions of future government solvency – whether warranted by fiscal fundamentals or not – are impounded in the market value of domestic government debt. To break these loops, policy must remove at least one of these three ingredients: either drastically reduce the domestic bias of banks’ sovereign exposures, or find ways in which government can credibly precommit to abstain from bailing out distressed domestic banks, or impose controls on international capital flows to prevent the flight to quality by domestic sovereign debt holders and bank depositors at times of sovereign stress. So far, only the last of these three policy remedies has been adopted in the cases of Cyprus and Greece, but only as the ex-post forced outcome of the extreme instances of the diabolic loop, not as an ex-ante policy to prevent its operation.

In this paper, we propose instead to attack the problem ex ante by eliminating altogether the home bias of banks’ sovereign debt portfolios in the euro area, which incidentally has greatly increased during the crisis (Battistini, Pagano and Simonelli, 2014; Altavilla, Simonelli and Pagano, 2015). We propose to do so by forcing banks to hold the senior tranche of a well-diversified portfolio of sovereign debt holdings, instead of the debt issued by their own domestic government, along the lines

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proposed by Brunnermeier et al. (2011). This seniority structure could be achieved via a simple securitization, whereby financial intermediaries use a well-diversified portfolio of euro-area sovereign bonds to back the issuance of a senior tranche, labeled “European Safe Bonds” (or ESBies), and a junior tranche, named “European Junior Bonds” (or EJBies). ESBies would have very little exposure to sovereign risk, owing to the “double protection” of diversification and of seniority: relative to a simple diversified portfolio of sovereign debt, ESBies would enjoy the additional protection provided by tranching, as the impact of a sovereign default would be absorbed mainly or exclusively by the junior tranche of the sovereign debt portfolio, which would not be held by banks. This is an important feature, as the existence of a safe asset is important for a well-functioning economy (Bolton and Jeanne, 2011) and for the conduct of monetary policy (Brunnermeier and Sannikov, 2015).

This paper shows both that restricting euro-area banks to hold ESBies would effectively isolate banks from domestic sovereign risk, and thereby defuse the “diabolic loop” between sovereign and bank credit risk described above. We also show that both of the two features of ESBies – diversification and tranching – are useful to this purpose. On one hand, the price of a diversified – but not tranched – sovereign debt portfolio would not be as insulated as ESBies from the swings in the perceived credit worthiness of euro-area governments, especially when such swings are correlated across countries due to a generalized (though possibly unwarranted) “flight to quality”. On the other hand, tranching sovereign debt at the level of each individual country has the disadvantage of producing an insufficient amount of safe domestic securities, especially in countries with weaker fiscal positions and limited sovereign debt issuance. In contrast, performing the tranching on a large pool of imperfectly correlated sovereign bonds would generate a very large issuance of an essentially risk-free euro-area sovereign asset, which could be not only enable banks to hold a liquid and safe sovereign bond, but would also be attractive for other classes of investors. And, last but not least, the issuance of such a security would not require any form of “fiscal solidarity” among euro-area governments: each government would remain entirely responsible for its own solvency, and the market price of its debt would remain a signal of its perceived solvency, barring “sunspots”, herding and other forms of irrational behavior by investors. This absence of joint liability stands in contrast to euro-bond proposals, such as the blue-red bond proposal by Von Weizsäcker and Delpla (2011).

It is natural to ask what is the “external validity” of the argument developed in this paper, i.e. to what extent it applies to other monetary unions. In particular, one may wonder why US banks are not exposed to the same kind of problems that euro-area banks have faced in 2009-12 – why for instance US banks in Puerto Rico were not destabilized by the default of the state in which they operate. There are two reasons for this: (i) since state government debt and municipal debt is given a favorable tax treatment if held by households, US banks do not invest in this type of debt; (ii) to the extent that US banks wish to invest in sovereign debt, they can invest in federal debt, which provides a common safe asset that effectively pools the tax revenue streams of all the states. This explains why Puerto Rico did not experience the same diabolic loop of, say, Greece or Cyprus, in spite of facing not dissimilar fiscal solvency problems.

The outline of the paper is as follows. Section 1 illustrates the diabolic loop in the context of a single country, and outlines the advantages from requiring banks to hold only senior domestic sovereign debt. In section 2, we introduce a two-country setting, and consider first the implications of requiring banks to hold a diversified portfolio of domestic and foreign sovereign debt, or alternatively the senior tranche of such portfolio, i.e. ESBies. The last section concludes.
I. One-Country Model

For expositional simplicity, we start by considering a single country with stochastic tax revenue, resulting in a high or low primary surplus. We show that a “sunspot-driven” repricing of the country’s sovereign risk can result in bank bailout and, when primary surplus is low, to sovereign default; while, absent such repricing, the government never defaults. Effectively, the sunspot acts an equilibrium selection device among two equilibria – one with bailout and possible default, and another with no bailout and no default. A key condition for the first equilibrium to exist – hence for the diabolic loop to arise – is that banks hold a sufficiently large fraction of the stock of domestic sovereign debt. In Section 2, we extend the analysis to a two-country setting, where the two countries are exposed to potential repricing of their sovereign risk, and explore how the parameter region in which the diabolic loop can arise changes if banks are constrained to diversify their sovereign exposures across the two countries or to hold the senior tranche of such a portfolio, i.e. ESBies.

Model

There are four domestic agents: the government, which prefers higher to lower output (as this is associated with greater tax revenue); dispersed depositors, who run on insolvent banks only if the government does not bail them out, and are also taxpayers; bank equity holders, who contribute all their equity financing initially, and hence cannot recapitalize banks subsequently; finally, international and domestic government bond-holders whose beliefs determine the price of sovereign debt, and therefore can lead to a repricing of sovereign risk upon observing a “sunspot”. For simplicity, all agents are risk neutral and there is no discounting, so that the risk-free interest rate is zero. Short-term deposits yield extra utility (“warm glow of liquidity”) compared to long-term government debt due to their convenience value in performing trans-

actions.

The model has four dates: 0, 1, 2, 3 and all consumption takes place at the final date 3. At $t = 0$, the government issues a unit of single long-term at price $B_0$ with face value $S$, which is repaid probabilistically in the last period: the primary surplus $S$ is a low $S$ with probability $\pi$ and a higher $S > S$ with probability $1 - \pi$. We denote by $B_t$ the price of the bond at each date $t$. Next, we denote by $\beta$ the share of debt owned by banks in the original period, the remaining fraction $1 - \beta$ being held by other risk-neutral investors. Hence, at time $t = 0$, banks hold $\beta B_0$ in sovereign debt on the asset side of their balance sheet, as well as an amount $L_0$ of loans to the real economy. Their liabilities are deposits $D_0$ and equity $E_0$.

At date $t = 1$ – the sun-spot stage – with probability $p$ a sunspot occurs and is observed by investors. When a sunspot is observed, government bond-holders become pessimistic: they expect a partial default in the last period, which in equilibrium will be a true belief. As a consequence, the price of the bond $B_1$ drops – we hence denote by $B_1$ the price of the bond conditional on the sunspot occurring. Hence, the banks suffer marked-to-market capital losses of size $-\alpha (B_1 - B_0)$. If this leads banks’ equity to drop below zero, banks are insolvent. We assume that they then cannot roll-over maturing loans of size $\psi L_0$, which implies that tax revenues correspondingly drops by $\tau \psi L_0$ in $t = 3$. For simplicity we assume that $\tau \psi L_0 \leq S$, so that the recovery rate is not negative.

In date $t = 2$ – the bail-out stage – the government must decide whether to bail out banks or not, before knowing its actual tax revenue. A bailout involves the issuance of additional government bonds, which are given to the banks as extra assets. If the government chooses not to bail-out, a fur-

\footnote{This is necessary to justify why there is a demand for bank deposits backed by sovereign debt. Otherwise, banks would not need to hold sovereign debt among their assets.}

\footnote{The sunspot carries no fundamental information about the likelihood of the primary surplus, revealed in $t = 2$.}
the primary surplus at \( t = 3 \) is only \( S \), after a bailout the government can only pay \( S - C \). Therefore, the price of debt at \( t = 1 \) is \( B_1 = S - \pi C \), so \( \pi C \equiv \Delta_1 \) is the price discount relative to its face value \( S \). The price of the debt in period 0 is the probability-weighted average of the sunspot and no-sunspot prices: \( B_0 = S - \pi p C \), with a price discount \( \pi p C \equiv \Delta_0 = p \Delta_1 \). Recalling the definition of bailout costs \( C \) and of the prices \( B_0 \) and \( B_1 \), the discount at \( t = 1 \) is

\[
\Delta_1 \equiv \pi C = \pi [\tau \psi L_0 - \alpha (B_1 - B_0) - E_0] = \frac{\pi (\tau \psi L_0 - E_0)}{1 - \alpha \pi (1 - p)}.
\]

Hence the bailout is avoided if banks are left with positive capital, i.e.,

\[
B_1 - B_0 = -(1 - p) \Delta_1 = \frac{(1 - p) \pi}{1 - \alpha \pi (1 - p)} (\tau \psi L_0 - E_0)
\]

and from (1). If instead banks’ equity is below the threshold \( E_0 \) in (2), then the sunspot leads to the diabolic-loop equilibrium. In this equilibrium, the price drop \( \Delta \) is higher in absolute value (i) the smaller bank equity \( E_0 \), (ii) the larger the fraction \( \alpha \) of sovereign debt held by banks (as this requires larger dilution of other debt holders), (iii) the higher the probability \( \pi \) of low fiscal surplus, and (iv) the smaller the sunspot probability \( p \) (as an unexpected sunspot is not priced in \( B_0 \)).

Hence, the diabolic loop can be avoided by requiring banks to meet the minimum equity threshold \( E_0 \), for a given desired sovereign debt portfolio \( \alpha \). Equivalently, one can impose on them an aggregate position limit on government bonds \( \alpha^* \), given their initial equity level \( E_0 \). Hence, the total supply of safe (diabolic-loop-free) assets to the banks is \( \alpha^* S \), since bonds are risk-free. This effectively sets a limit on the maximum amount of safe deposits that the banking system can generate.

The following Proposition summarizes our results.

**PROPOSITION 1:** (i) To avoid the diabolic loop, banks’ aggregate equity to

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3This assumption is only used to simplify calculations, but can easily be relaxed.
sovereign exposure ratio $\frac{E_0}{\alpha S}$ must be at least $\frac{E_0}{\alpha S} \equiv (1 - p) \pi \tau \psi L_0 - \frac{S}{S}$.

(a) The maximum amount of safe assets available to banks is $\alpha^* B_0 = \frac{E_0}{(1 - p) \pi \tau \psi L_0 - \frac{S}{S}} S$. Equivalently, $\frac{E_0}{\alpha S}$ is the minimum aggregate equity to sovereign debt ratio.

SOVEREIGN DEBT TRANCHEING

Instead of imposing an upper bound, $\alpha$, on bank holdings of debt given their equity $E_0$, sovereign debt can be split into a senior and a junior tranche, and banks can be required to hold only the senior tranche: then the diabolic loop will be ruled out if the face value, $F^s$, of the senior tranche (i.e. the tranching point) is sufficiently low; alternatively if banks are required to meet a sufficiently tight position limit on their holdings of the senior tranche, $\alpha^*$. Therefore, the diabolic loop equilibrium can be ruled out by picking appropriate pairs $(\alpha^*, F^s)$.

If the space $(\alpha^*, F^s)$ is split into a subset in which the diabolic loop occurs and one, $\mathcal{N}$, in which it does not, identifying the boundary of $\mathcal{N}$ will enable us to characterize the diabolic loop region. To do so, we compute senior bond prices under the diabolic-loop equilibrium and require that the losses associated with the sunspot reduce bank equity exactly to zero.

If the sunspot is not observed, debt is traded at its no default-value $S$, and the same holds for the senior tranche, which trades at $F^s$. If the sunspot is observed and banks require a recapitalization, the cost to the government is $C^* \equiv \tau \psi L_0 - \alpha (B_1^s - B_0^s) - E_0$, where $B_0^s$ denotes the price of the senior tranche. If the surplus at $t = 3$ is $S$, the government can repay its debt in full after incurring the cost $C^*$ because of Assumption 1, so that the senior tranche pays its face value $F^s$; if instead the surplus is $S$, the government can only pay $S - C^*$ and the senior tranche yields $F^s - [C^* - (S - F^s)]$, where $S - F^s$ is the loss absorbed by the junior tranche. Hence, the price of the senior tranche at $t = 1$ is $B_1^s = F^s - \pi [C^* - (S - F^s)]$. So the analysis is the same as in the case of no tranching except that $C$ is replaced by $C^* - (S - F^s)$.

This amounts to replacing $\tau \psi L_0$ in Equation (2) by $\tau \psi L_0 - (S - F^s)$. In other words, the bailout is avoided if

$$E_0 \geq \alpha^* (1 - p) [\tau \psi L_0 - (S - F^s)].$$

The pairs $(\alpha^*, F^s)$ on the boundary of the no-diabolic-loop subset $\mathcal{N}$ satisfy condition [4] with equality. The right-hand side of [4] is increasing in both $\alpha^*$ and $F^s$, which means that at the boundary if banks hold a larger fraction of the senior tranche $\alpha^*$, this tranche must have a lower face value $F^s$, and viceversa.

We now want to find the pair $(\alpha^{**}, F^{**}) \in \mathcal{N}$ that maximizes the total value of safe assets available to the banking system:

$$\max_{(\alpha^*, F^s) \in \mathcal{N}} \alpha^* F^s = \max_{(\alpha^*, F^s) \in \mathcal{N}} \frac{E_0 F^s}{\pi (1 - p) [\tau \psi L_0 - (S - F^s)].}$$

The maximand is decreasing in $F^s$, because $S > \tau \psi L_0$. Therefore, the maximization requires setting the optimal face value $F^{**}$ at the lowest possible value that meets [4] with equality. In turn, this requires setting $\alpha^{**}$ at its upper bound $\alpha^{**} = 1$, so that

$$F^{**} = S + \frac{E_0}{\pi (1 - p)} - \tau \psi L_0 < S,$$

where the inequality follows from Assumption 2. Since the solution for $\max_{(\alpha^*, F^s) \in \mathcal{N}} \alpha^* F^s$ differs from the no-tranching solution, tranching allows the economy to generate a larger amount of safe assets for the banking system. Interestingly, if banks’ equity is $E_0 > E_0^*$, then the junior bond is also risk-free, as a diabolic loop cannot occur.

PROPOSITION 2: (i) For a given security structure $F^s$, to avoid the diabolic loop, banks’ aggregate equity to sovereign exposure ratio $\frac{E_0}{\alpha^* F^s}$ must be at least $\frac{E_0}{\alpha^* F^s} \equiv (1 - p) \pi \tau \psi L_0 - (S - F^s)$, where the term $-(S - F^s)$ reflects the protection due to the junior tranche.

(ii) If $E_0 > E_0^*$, then the junior bond is also risk-free.
(iii) If $F^*$ is chosen, so as to maximize the amount of safe assets for the banking sector, tranching generates larger amounts of safe assets than no tranching. Equivalently, tranching lowers the equity that banks must hold per unit of sovereign exposure: 

$$\frac{E_{\pi_s}^p}{\alpha_s + p \sigma} = \frac{E_{\pi_s}^p}{S - \tau \psi L_0 + E_0 /[\pi (1 - p)]} < \frac{E_{\pi_s}^p}{\alpha_s}.$$ 

II. Two-Country Model

Let us now consider two symmetric countries. Their realization of the primary surplus absent bailout interventions is independently distributed: it equals $S$ occurs with probability $\pi$, and $\bar{S}$ with probability $1 - \pi$. Both countries issue government bonds with face value equal to $S$. Initially, we consider the case in which banks can hold a pooled asset consisting of sovereign bonds issued by both countries. Subsequently, we consider the case in which they are restricted to hold only the senior tranche of such a pooled portfolio, the ES-Bies.

Pooling

Suppose that, besides the domestic sovereign bond $\alpha$, banks also hold a security backed by equal proportions of the sovereign bonds of the two countries. The intermediary issuing this security backs it with a fraction $\beta$ of the domestic debt of both countries. In practice, it is as if banks own a fraction $\alpha + \frac{1}{2} \beta =: \alpha^p$ of the domestic sovereign debt and $\frac{1}{2} \beta$ of foreign sovereign debt. In our analysis we hold banks’ total sovereign portfolio $\alpha + \beta = \zeta$ fixed, but vary the two portfolio weights. Hence, $\alpha^p = \zeta + \alpha/2$.

For brevity, here we focus on the special case in which banks hold only the pooled asset. In this case, when banks require a recapitalization, the cost to the government is $\tau \psi L_0 - \alpha (B^p_1 - B^p_0) - E_0$, where $B^p_i$ now denotes the price of the pooled asset. When both countries have primary surplus $S$ at $t = 3$, which occurs with probability $(1 - \pi)^3$, they pay their debts in full. When one country has primary surplus $S$ and the other $\bar{S}$, which occurs with probability $2\pi (1 - \pi)$, the latter country can only pay $S - C$, and the pooled asset pays $S - \frac{1}{2} C$. When both countries have primary surplus $S$, which occurs with probability $\pi^2$, each country can only pay $S - C$, and the pooled asset pays the same. Therefore, the price of the pooled asset in period 1 is

$$B^p_1 = S - \left[\frac{1}{2} 2 \pi (1 - \pi) + \pi^2\right] C = S - \pi C.$$ 

This equation is identical to the case of a single country with no tranching. Therefore, the price of the pooled asset is also the same as of the single-country asset, $B^p_1 = B_1$, and Condition (2) that rules out the diabolic-loop equilibrium is also the same. Hence, in this two-country setting the sunspot affects banks in both countries identically, so that either a recapitalization is required in both countries or in none. In summary:

PROPOSITION 3: If banks only hold the pooled asset, then
(i) the diabolic loop either occurs in both countries or none;
(ii) the pooling outcome is identical to the single country outcome, with the same sunspot probability.

This proposition illustrates an important insight: simply requiring banks to hold a pooled asset – or an equivalently diversified portfolio of sovereign bonds – might actually lead to contagion across countries. In the next subsection, we show that this no longer holds if banks hold only the senior tranche of such a pooled asset, i.e. ES-Bies.

Pooling and Tranching

Also when the pool of sovereign bonds is tranched, symmetry implies that repricing of sovereign debt occurs either in both countries or none. But now the issue is whether such repricing affects only the junior tranche or the senior one as well: as banks are required to hold only the senior tranche, repricing may trigger a bailout only if the senior tranche incurs losses. As in the case where tranching occurs in a single country, we wish to characterize the set $N$ of pairs $(\alpha^E, F^E)$ that rule out the
diabolic-loop equilibrium. To do so, we initially compute prices of ESBies for a given \((\alpha^E, F^E)\) under a diabolic-loop equilibrium and require that bank equity remains non-negative. Consider the parameter region in which the senior tranche incurs losses when the (union-wide) sunspot is observed. There are two scenarios to be considered:

First, suppose equity \(E_0\) is large enough that ESBies incur losses only in the worse-case outcome at \(t = 3\), in which both countries have primary surplus realization. In this event which occurs with probability \(\pi^2\), junior bond holders are wiped out. Clearly, ESBies are better protected than a single country senior bond, where the low surplus realization occurs with probability \(\pi\).

Second, for lower equity levels \(E_0\) the diabolic loop might be so large that ESBies might incur losses also if only one of the two countries has a low primary surplus realization, an event that occurs with probability \(2\pi (1 - \pi)\). In this case the junior bond holder will be wiped out in three of the four possible surplus realizations.

In both scenarios after a sunspot ESBies will drop less in value than a senior bond of a single country would. Banks would need less equity to absorb the loss than in the case of single country tranching. Intuitively, tranching the pooled asset increases the number of states relative to a single country tranching and allows senior bond holders to push losses onto the junior bond holders in a greater number of states than in a single country tranching. Since banks do not hold junior bonds, they losses these occur do not contribute to the diabolic loop. But notice that insofar as the diabolic loop is avoided, these losses are an off equilibrium phenomenon so that even junior bonds are risk-free.

The arguments so far has taken the tranching point and the ESBies exposure of banks as given, but these two variables can be chosen (ex-ante) as so to maximize the value of the safe asset available to the banks. Formally, we seek the pair \((\alpha^{E^*}, F^{E^*})\) that maximizes \(\alpha^{E^*} F^{E^*}\) without triggering the diabolic. As before \(\alpha^{E^*} = 1\), but now the face value of ESBies is

\[
F^{E^*} = S - \left[ \tau \psi L_0 - \frac{E_0}{\pi (1 - \pi)} \right]
\]

in the first scenario and

\[
F^{E^*} = S - \frac{1}{(2 - \pi)} \tau \psi L_0 - \frac{E_0}{\pi (1 - \pi)}
\]

in the second scenario. Both exceed the face value of the senior of the single country given by \(E^s\).

**PROPOSITION 4:** (i) For a given tranching point \(F^E\), ESBies lower the required equity to sovereign exposure ratio compared to a single country tranching (assuming \(\alpha^E = \alpha^s\)).

(ii) If these lower equity to sovereign exposure ratios are upheld, junior bond remains risk-free.

(iii) If \(F^E\) and \(\alpha^E\) are chosen, so as to maximize the amount of safe assets for the banking sector, ESBies generate a larger amount of safe assets than tranching in a single country.

### III. Conclusion

This paper adds to a recent literature on the feedback loop between sovereign and bank solvency risk (Brunnermeier et al. 2011, Acharya, Drechsler and Schnabl, 2014; Cooper and Nikolov, 2013, Farhi and Tirole, 2015; Leonello, 2014), by providing a simple, workable model and using it to explore whether and how the loop can be defused by restricting banks’ portfolio of sovereign holdings. First, we find that what matters is the ratio of banks’ equity to their domestic sovereign exposures: the diabolic loop can equivalently be defused by raising banks’ equity requirements or by restricting their holdings of domestic sovereign debt. Second, requiring banks to hold only a senior tranche of domestic sovereign debt is more effective than requiring them to diversify their sovereign portfolios across countries. But a diversified sovereign debt portfolio is most effective jointly with tranching: requiring banks to hold only the senior tranche of such a portfolio – i.e. ESBies – reduces their equity requirement per dollar of sovereign holdings, relative to a regime where they are required to hold senior domestic sovereign debt only. Intuitively, the reason is that using both pooling and tranching allows to shift more of
the sovereign risk generated by each government onto the junior bond-holders, hence away from the banks of the corresponding country – thus eliminating the need for the government to intervene with a bailout, and the resulting diabolic loop. Accordingly, we show that ESBies generate a larger amount of safe assets than domestic debt tranching alone, if the split between senior and junior bonds is optimally designed. Finally, insofar as ESBies succeed in defusing the diabolic loop and associate endogenous risk, in equilibrium the junior bond is itself risk-free!

Bibliography


Figure 1. "Bailout and Real-economy Diabolic Loop. Source: Brunnermeier et al. (2011).
Mathematical Appendix

A1. Proof of Proposition 4

In the first scenario, which occurs with probability \( \pi^2 \), the pooled asset pays \( S - C^E \), and the senior tranche pays \( F^E - [C^E - (S - F^E)] \). In the second scenario, which has probability \( 2\pi(1 - \pi) \), the pooled asset pays \( \frac{1}{2}C^E \).

In the first scenario, in which ESBies only default in the state where surplus realization is \( S \) for both governments, the following inequality must hold

\[
S - \frac{1}{2}C^E \geq F^E. \tag{A1}
\]

If (A1) holds, then the price of the senior tranche in period 1 is \( B^E_1 = F^E - \pi^2[C^E - (S - F^E)] \). The analysis is the same as in the one-country case with tranching except that \( \pi \) is replaced by \( \pi^2 \). A recapitalization is not needed if

\[
E_0 \geq \alpha^E \pi^2(1 - p) \left[ \tau\psi L_0 - (S - F^E) \right]. \tag{A2}
\]

In the second scenario, where (A1) is violated, if one country has surplus \( \bar{S} \) and the other \( S \), the senior tranche receives \( F^E - \left( \frac{1}{2}C^E - (\bar{S} - F^E) \right) \) and its price \( t = 1 \) is

\[
B^E_1 = F^E - \left[ \frac{1}{2}2\pi(1 - \pi) + \pi^2 \right] C^E + \left[ 2\pi(1 - \pi) + \pi^2 \right] (\bar{S} - F^E)
= F^E - \pi \left[ C^E - (2 - \pi)(\bar{S} - F^E) \right].
\]

The analysis is the same is in the one-country case with tranching except that we must replace \( S - F^E \) by \( (2 - \pi)(\bar{S} - F^E) \). A recapitalization is not needed if

\[
E_0 \geq \alpha^E \pi (1 - p) \left[ \tau\psi L_0 - (2 - \pi)(\bar{S} - F^E) \right]. \tag{A3}
\]

Setting \( \alpha^E = \alpha^s \) in (A2) and (A3) and comparing them with (4), it immediate that the lower bound on equity to sovereign exposure ratio is less stringent with ESBies than with single country tranching. This completes part (i) of the proof.

The claim in part (ii) follows directly from the Equations (A2) and (A3) which rule out the diabolic loop equilibrium.

To prove the claim in part (iii) note that in the first scenario the pair \( (\alpha^{E^*}, F^{E^*}) \) that maximizes the value of the safe asset available to the banks satisfies (A2) with equality, and \( \alpha^{E^*} = 1 \) by the same argument as in the one-country case. The resulting value of the senior tranche is analogous to (5) in the one-country case with tranching:

\[
F^{E^*} = S + \frac{E_0}{\pi^2(1 - p)} - \tau\psi L_0. \tag{A4}
\]

Since \( \pi \) is no replaced by \( \pi^2 \), we have \( F^{E^*} > F^s^* \): pooling and tranching generates a larger supply of the safe asset than tranching in each country separately.

We must finally check that ESBies suffer no losses even in this worst-case scenario, i.e. (A1) is satisfied. Noting that

\[
C^{E^*} = \tau\psi L_0 - \alpha(B_1^{E^*} - B_0^{E^*}) - E_0 = \tau\psi L_0 + \alpha^{E^*}(1 - p)\Delta^{E^*} - E_0,
\]

and that in the two-country case with tranching \( \Delta^E \) is given by an equation analogous to (1) where \( \tau\psi L_0 \) is replaced by \( \tau\psi L_0 - (S - F^E) \) and \( \pi \) by \( \pi^2 \), the no-loss condition (A1)
can be rewritten as

\[(A5) \quad S - F^{E^*} - \frac{1}{2} \left[ \tau \psi L_0 + \alpha^{E^*} (1 - p) \frac{\pi^2 (\tau \psi L_0 - E_0 - (S - F^{E^*}))}{1 - \alpha^{E^*} \pi^2 (1 - p)} - E_0 \right] \geq 0.\]

We next set \(\alpha^{E^*} = 1\) and \(F^{E^*}\) equal to its value in \((A4)\). Because these values satisfy \((A2)\) with equality, the sum of the second and third term in the square bracket of \((A5)\) is zero, so using \((A4)\), \((A5)\) becomes

\[(A6) \quad E_0 \leq \frac{1}{2} \pi^2 (1 - p) \tau \psi L_0,\]

which is part of the parameter space for \(E_0\) we consider under Assumption \(\text{Assumption 2}\).

For the second scenario, in which \((A1)\) holds, going through the same steps as for the first scenario, we find that the pair \((\alpha^{E^*}, F^{E^*})\) that maximizes the value of safe investment available to the banks satisfies \(\alpha^{E^*} = 1\) and

\[(A7) \quad F^{E^*} = S - \frac{1}{2 - \pi} \left[ \tau \psi L_0 - \frac{E_0}{\pi (1 - p)} \right].\]

The face value \(F^{E^*}\) is larger than in the one-country case because by Assumption \(\text{Assumption 2}\) the term in square brackets is positive. We are in the second scenario, i.e. \((A1)\) is violated, if equity is in the region

\[\frac{1}{2} \pi^2 (1 - p) \tau \psi L_0 < E_0 < \pi (1 - p) \tau \psi L_0.\]