Dynamic Moral Hazard with Manipulation of Output Reports

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Multi-period moral hazard problem
Agent produces successes – sales, breakthroughs, strategic deals, etc.
Agent can delay report of success
Why Does Hiding Matter

- In a one-period setting, MLRP is sufficient to prevent hiding.
- In multi-period settings delaying reports is a common concern:
  - “Earnings Management” – delaying reports to shareholder, has been extensively studied empirically in accounting research
  - “Sales Gaming” – timing sales according to incentives
This Paper

- Simple model to identify the main underlying economics:
  - Private saving of outcomes *does not equal* private saving of income
  - Optimal contract is more responsive to outcomes
  - Informativeness principle (sufficient statistic) may not apply
  - Hiding matters less if agent is risk neutral
  - Increases cost (to the principal) of agent’s risk averseness
  - Agent may be worse off

- General dynamic program framework
Related Literature

- Only paper in which agent can delay output reports we know of is Strulovici (2011). Very different focus.
- Zhu (2013) and Varas (2013) allow the agent to increase current productivity by sacrificing future productivity.
- Williams (2011) and Sannikov (2014) allow the agent’s actions to have long term effects.
- Fudenberg et al. (1990), Edmans et al. (2012), and others consider private savings.
Model - Production

- Agent’s effort $a \in A$, cost $c(a)$ increasing (convex)
- Discrete outcome $y \in \{0, 1, 2, \ldots, Y\}$, density $p(a, y) > 0$
- Principal’s value from outcome $y$ is $v(y)$, increasing in $y$ (concave)
- Higher effort yields higher expected outcome value
- Discrete time
Only positive payments to the agent

- Contract specifies for each history $h_t$ the required effort $a$ and output-dependent payments $w_y$
  - Dynamic problem also specifies continuation utility (or certainty equivalent) $U_y$

- Agent’s utility value for period effort cost $c$, payment $w$ and continuation utility (or certainty equivalent) $U$ is $u(c, w, U)$

- Discount factors can be different $(\delta_a, \delta_p)$
At the end of period $t$ with outcome $y_t$, the agent can report $y_t - 1$, the unreported unit is added to $y_{t+1}$

- Can be hidden again

- Only one output unit can be hidden at a time
Honest Reporting Incentive Compatibility (RIC)

For any proposed contract, let:

- $U(h_t)$ be the expected continuation utility for a complying agent.
- $\hat{U}(h_t)$ be the expected continuation utility for a “hiding” agent: an agent that started the current period with a stored success.

Note: hiding agent may choose different actions.

Let $h' = \langle h, (a, y) \rangle$ be the history $h$ followed by requested effort $a$ and outcome $y$.

**Proposition**

A contract is RIC iff for any history $h$ with requested action $a$, for all $y > 0$: $U(\langle h, (a, y) \rangle) \geq \hat{U}(\langle h, (a, y-1) \rangle)$.
Proposition

A contract is RIC iff for any history $h$ with requested action $a$, for all $y > 0$: $U(h, (a, y)) \geq \hat{U}(h, (a, y - 1))$

Proof

- Necessity: If the condition fails, the agent should hide
- Sufficiency: If the condition holds, lying reduces the agents’ expected utility
Dynamic Problem:

\[
V(U, \hat{U}) = \max_{a \in A, z \in A, U^y \geq 0, \hat{U}^y \geq 0} \sum_y \left[p(a, y) \left(v(y) - w_y + \delta_p V(U^y, \hat{U}^y)\right)\right] \quad \text{s.t.}
\]

\[
U = \sum_y p(a, y) \cdot u(c(a), w_y, \delta_a U^y)
\]

\[
a \in \arg \max \sum_y p(\tilde{a}, y) \cdot u(c(\tilde{a}), w_y, \delta_a U^y)
\]

\[
\hat{U} \geq \sum_{y < Y} \left[p(z, y) \cdot u(c(z), w_{y+1}, \delta_a U^{y+1})\right] + p(z, Y) \cdot u(c(z), w_Y, \delta_a \hat{U}^Y)
\]

\[
z \in \arg \max \sum_{y < Y} \left[p(\tilde{z}, y) \cdot u(c(\tilde{z}), w_{y+1}, \delta_a U^{y+1})\right]
\]

\[
+ p(\tilde{z}, Y) \cdot u(c(\tilde{z}), w_Y, \delta_a \hat{U}^Y)
\]

\[
\forall y > 0 \quad U^y = U^{\hat{U}^y-1}
\]
Dynamic Problem - Similar Parts

\[
V(U, \hat{U}) = \max_{a \in A, z \in A, U^y \geq 0, \hat{U}^y \geq 0} \sum_y \left[ p(a, y) \left( v(y) - w_y + \delta_p V(U^y, \hat{U}^y) \right) \right] \quad s.t.
\]

\[
U = \sum_y p(a, y) \cdot u(c(a), w_y, \delta_a U^y)
\]

\[
a \in \arg \max \sum_y p(\tilde{a}, y) \cdot u(c(\tilde{a}), w_y, \delta_a U^y)
\]
Dynamic Problem - Changed/New Parts

\[
V(U, \hat{U}) = \max_{a \in A, z \in A, U^y \geq 0, \hat{U}^y \geq 0} \sum_y \left[ p(a, y) \left( v(y) - w_y + \delta p \right) \right] \sum_y \left[ p(z, y) \cdot u(c(z), w_{y+1}, \delta_a U^{y+1}) \right] + p(z, Y) \cdot u(c(z), w_Y, \delta_a \hat{U}^Y)
\]

\[
\hat{U} \geq \sum_{y < Y} \left[ p(z, y) \cdot u(c(z), w_{y+1}, \delta_a U^{y+1}) \right] + p(z, Y) \cdot u(c(z), w_Y, \delta_a \hat{U}^Y)
\]

\[
z \in \arg \max \sum_{y < Y} \left[ p(\tilde{z}, y) \cdot u(c(\tilde{z}), w_{y+1}, \delta_a U^{y+1}) \right] + p(\tilde{z}, Y) \cdot u(c(\tilde{z}), w_Y, \delta_a \hat{U}^Y)
\]

\forall y > 0 \quad U^y = U^{y-1}
Dynamic Problem Simplifications / Observations

- Standard IC and regeneration constraint \((U = ...)\) unaffected
- Period return not directly affected
- No need to choose \(\hat{U}^y\) for \(y < Y\)
- Only two additional variables compared to the standard problem: hiding agent’s action \(z\) and \(\hat{U}^Y\)
- Two additional constraints: \(\hat{U} \geq ...\) and \(z \in ...\)
- If f.o.c. approach works without hiding, f.o.c. works with hiding
- Under some regularity conditions, the \(\hat{U}\) is an equality
- Managable computational burden – if you can solve the problem without hiding, you can solve with hiding
Three Reasons to Hide

- **Game the Rewards**
  - **Threshold contract:** Rewards only when \( y \geq y^* \), hide when \( y < y^* \) (and maybe when \( y > y^* \))
  - Decreasing rewards within period: e.g. \( y > y^* \) is so unlikely that it isn’t informative of action, rewards don’t increase: hide when \( y > y^* \)

- **Game the Contract Dynamics**
  - Contract expected to increase incentives
  - Value of marginal reward today lower than expected marginal value tomorrow

- **Insure**
  - Value of marginal reward today is lower than value of reducing variance tomorrow
Example 1 – Gaming the Rewards

- Two periods, no discounting
- Risk neutral agent: $u(c, w, U) = w - c + U$
- Two actions: $a \in \{L, H\}$
- $c(L) = 0, c(H) = c$
- Three outcomes $y \in \{0, 1, 2\}$, $v(y) = y$
- $p(a, 1) = p_a \cdot (1 - \lambda), \quad p(a, 2) = p_a \lambda, \quad p(a, 0) = 1 - p_a$
- $p_H > p_L$
- Note: outcomes 1 and 2 provide the same indication of the agent’s effort
Example 1 – Optimal Contract

- Assume contract without hiding sets $a_1 = a_2 = H, w(0) = 0$
- Optimal contract problem is to choose $w(1)$ and $w(2)$
- Without hiding, infinitely many stationary contracts

\[(1 - \lambda)w(1) + \lambda w(2) = \frac{c}{p_H - p_L}\]

- Easier to write $b(2) \equiv w(2) - w(1)$

\[w(1) + \lambda b(2) = \frac{c}{p_H - p_L}\]
Example 1 – Contract to Prevent Reward Gaming

1. Will a hiding agent work?

\[ w(1) + p_H b(2) - c \geq w(1) + p_L b(2) \iff b(2) \geq \frac{c}{p_H - p_L} \]

2. RIC if hiding agent works:

\[
\begin{align*}
  w(1) &\geq w(1) + p_H b(2) - c - (p_H(w(1) + \lambda b(2) - c)) \\
  &\iff w(1) \geq (1 - \lambda)b(2)
\end{align*}
\]

\[
\begin{align*}
  b(2) &\geq w(1) + p_H b(2) - c - (p_H(w(1) + \lambda b(2) - c)) \\
  &\iff b(2)(1 - p_H(1 - \lambda)) \geq w(1)(1 - p_H)
\end{align*}
\]

3. Solution: 
\[ b(2) = \frac{c}{p_H - p_L}, \quad w(1) = (1 - \lambda)\frac{c}{p_H - p_L} \]
Reward Gaming – Observations

1. Optimal contract is as profitable to the principal and agent.
2. Optimal contract is unique.
3. Optimal stationary rewards violate “sufficient statistic principle” ($w(2) \neq w(1)$).
4. Optimal stationary contract is convex:
   \[ w(2) - w(1) > w(1) - w(0) \]
5. Agent is rewarded even more for something completely out of his control.
Same problem, but with risk averse agent (CARA)

\[ u = -e^{c(a_1) + c(a_2) - w(y_1) - w(y_2)} \]

Unique optimal contract without hiding is stationary, sets \( w(1) = w(2) = w^* \)

Unique optimal stationary contract with hiding:

\[ b(2) \equiv w(2) - w(1) = w^* \]

\[ e^{-w(1)} = \frac{e^{-w^*}}{1 - \lambda + \lambda e^{-w^*}} \]
Optimal stationary contract is less profitable to the principal.
Same reward structure as for the risk neutral agent.
- Reward for second success in period higher than reward for first success.
Agent can only be worse off.
- IC still binds so certainty equivalent didn’t change.
- Increased cost may sway the principal to avoid work altogether.
Optimal rewards are not stationary (didn’t show here).
- In second period, can set $w(2) \approx w(1)$.
- Stationary contract more indicative of longer horizons with fixed actions.
Gaming Contract Dynamics

- Three actions \((a \in L, M, H)\), same outcome structure \((p_M \in (p_L, p_H))\)
- Optimal contract without hiding starts with \(a_1 = M\) and moves to \(a_2 = H\) if there is a success
  - Second period rewards are stronger
  - Hiding the second success is a bigger problem
- Optimal contract without hiding starts with \(a_1 = M\) and moves to \(a_2 = L\) if there is no success
  - Second period rewards are higher
  - Reward gaming only
Conclusion - Output Report Manipulation

- Interesting and important problem
- Many variations, similar structure
- Simplest models to flush out the economics:
  - Gaming the rewards
  - Insurance
  - Gaming the dynamics
- Storing output is NOT like storing payments