The system-wide effects of bank capital regulation on credit supply and risk-taking

Milton Harris† Christian C. Opp‡ Marcus M. Opp§

October 14, 2015

Abstract

We propose a tractable framework to examine the system-wide effects of bank capital requirements. In our model, banks can serve a socially beneficial role by financing firms that are credit rationed by public markets, but banks’ access to deposit insurance (or implicit guarantees) creates socially undesirable risk-shifting incentives. Equity ratio requirements reduce banks’ risk-taking incentives, but may also constrain banks’ balance sheets. When bank capital is not scarce, increases in equity-ratio requirements unambiguously improve welfare and the stability of the banking system. However, when bank capital is scarce, increased equity-ratio requirements induce credit rationing of both bank-dependent firms with positive NPV projects and risky firms with negative NPV. In this case, the net effects on welfare and the risk-taking of banks are ambiguous, as they depend on which type of credit rationing dominates. Our model provides conceptual guidance on the cyclicality of optimum capital requirements as well as their dependence on the development of public markets, the cross-sectional distribution of firms, and the quality of risk signals available to regulators.

PRELIMINARY AND INCOMPLETE

*We are grateful to Anat Admati, Michael Brennan, Elena Carletti, Sam Lee, George Pennachi, Giorgia Piacentino, Matt Spiegel and Jeremy Stein for discussions of earlier drafts of this paper as well as Peter DeMarzo, Nicolae Gârleanu, Valerie Shen, and Ivo Welch for helpful comments. In addition, we thank seminar participants at the Berkeley-Stanford joint seminar, Bocconi, INSEAD, London School of Economics, Stockholm School of Economics, UC San Diego, University of Chicago, University of Northern Carolina at Chapel Hill, the Bundesbank Frankfurt, the Federal Reserve Bank of New York, the Federal Reserve Board of Governors and feedback from the following conferences: NBER SI credit ratings 2013, FIRS 2014, NBER SI corporate finance 2014, WFA 2014, Barcelona GSE summer symposium 2015, Econometric Society World Meeting 2015.

†University of Chicago, Booth School of Business, e-mail: milt@chicagobooth.edu.
‡University of Pennsylvania, The Wharton School, e-mail: opp@wharton.upenn.edu.
§University of California, Berkeley (Haas), e-mail: mopp@haas.berkeley.edu.
1 Model setup

We consider a discrete-state, incomplete-markets economy with two dates, 0 and 1. At date 1, the aggregate state of the world $s \in S$ is realized. The ex-ante probability of state $s$ is denoted by $p_s > 0$. The economy consists of firms, public market investors, banks, and a regulator. All agents in the economy are risk-neutral, are subject to limited liability, and discount their respective payoffs at a rate of zero.

1.1 Firms

There is a continuum of firms of measure one, indexed by $f \in \Omega_f = [0, 1]$. Each firm $f$ is owned by a cashless entrepreneur who has access to a project that requires a fixed-scale investment $I$ at time 0, and produces state-contingent cash flows $C_s$ at time 1. Firm cash flows $C_s(q|e)$ are affected by the entrepreneur’s observable discrete quality type $q(f) \in \Omega_q$ and her unobservable binary effort choice $e(f) \in \{0, 1\}$. Firms have access to monitored financing from banks and unmonitored financing from public investors.

The firm’s moral hazard problem is as in Holmstrom and Tirole (1997). Shirking, $e = 0$, allows the entrepreneur to enjoy a private benefit of $B(q)$ when unmonitored, and 0 when monitored. As a result of this moral hazard problem, some firm types will be credit rationed by public markets, providing a role for bank monitoring.

Assumption 1 Parameters satisfy the following relations:
1) $B(q) + \mathbb{E}[C_s(q|e = 0)] - I < 0 \quad \forall q,$
2) $C_s(q|e = 0) < I \quad \forall s,$
3) $C_s(q|e = 1) > I$ for some $s$.

The first condition implies that a project never generates social value under shirking. We impose the second restriction for expositional reasons as it simplifies the firm’s incentive problem under public financing. If there existed firm types violating the third restriction, these firm types would never obtain financing in equilibrium. Thus, without loss of generality, we can eliminate these types from our setup. We note that condition 3 does not restrict the expected social surplus to be positive for all firm types, even under

1 None of our results depends on the fact that the private benefit under monitoring is zero. Banks are effective monitors as long as they reduce the private benefit of shirking below $B(q)$.

1
high effort. The expected social surplus conditional on high effort, \( e = 1 \), is given by:

\[
NPV(q) = \mathbb{E}[C_s(q|e = 1)] - I.
\]

### 1.2 Public market financing

Public capital markets are a potential source of funding for firms. There is a continuum of competitive public investors with sufficient wealth to finance all projects in the economy. At date 0, public market investors have access to the following investment opportunities: (1) securities issued by firms, (2) bank deposits and bank equity, and (3) a storage technology with zero interest. Sufficient wealth, competition, risk-neutrality, and access to the storage technology imply that public market investors’ expected rate of return is zero on all investments in equilibrium.

Given limited liability and risk-neutrality, firms optimally try to raise outside financing from public market investors in the form of debt (see Innes (1990), Tirole (2005)). Since public market investors cannot monitor, the firm’s incentive constraint for high effort, \( e = 1 \), under a debt contract with face value \( FV \) is:

\[
\mathbb{E}[\max\{C_s(q|e = 1) - FV, 0\}] \geq B(q) + \mathbb{E}[\max\{C_s(q|e = 0) - FV, 0\}].
\]

(1)

The individual rationality constraint of public investors requires that

\[
\mathbb{E}[\min\{C_s(q|e = 1), FV\}] \geq I.
\]

(2)

Competition among public investors ensures that the firm can extract all the surplus, that is, constraint (2) is binding. Moreover, Assumption 1 implies that, absent effort, the firm’s equity value is zero for any face value \( FV \) satisfying constraint (2), that is,

\[
\mathbb{E}[\max\{C_s(q|e = 0) - FV, 0\}] = 0.
\]

(3)

As a result, (1) and (2) are equivalent to the constraint \( NPV(q) \geq B(q) \). Thus, firms can raise public debt if and only if their project’s NPV exceeds the benefit of shirking. In contrast, firm types with low \( NPV \) or high agency rents \( B \), such as small and medium-sized firms, find it impossible to raise funds via public markets.
1.3 Banks

There is a continuum of competitive, ex-ante identical bankers of mass 1 indexed by \( b \in \Omega_b \). Each bank has initial wealth \( E_I > 0 \) in the form of cash at time 0, so that aggregate bank capital is \( E_I \). To avoid integer problems in the allocation of capital to firms, we assume that, for every bank \( b \), there are a continuum of firms. Banks can serve a social role as monitors: monitoring eliminates a firm’s private benefit of shirking, \( B(q) \). For simplicity, we presume that banks, when lending, have to retain the entire loan on their balance sheet.

**Banks’ outside financing frictions.** Banks are subject to outside financing frictions. To keep the baseline model as tractable as possible, we presume that outside equity issuance is associated with a deadweight cost of \( c(E_O) \) which is an increasing, convex function of the amount of outside equity financing \( E_O \). We show how this cost can arise endogenously in our model extension (see Appendix A), which features insights from two seminal papers in the banking literature. First, outside equity is costly relative to inside equity since it induces (socially) inefficient diversion of cash flows by bankers. The issuance of demand deposits is not subject these costs as demand deposits can stop diversion in progress (Calomiris and Kahn (1991)). Second, as in Diamond and Dybvig (1983), full insurance of deposits is useful as it preempts inefficient bank runs, that is, runs that are caused purely by coordination failures among depositors.

In sum, our setup features two deviations from Modigliani-Miller that will play distinct roles for our analysis. First, deposit insurance implies that total cash flows to all security-holders are weakly increasing in bank leverage. This friction generates risk-taking incentives. Second, the wedge between the cost of internal equity and outside equity is crucial to generate balance sheet effects when banks are constrained by bank capital regulation. We note that there are alternative modeling assumptions that generate balance sheet constraints, such as, equity issuance cost (see e.g., Hennessy and Whited (2007) for the empirical relevance), or, costly monitoring as in Holmstrom and Tirole (1997).

---

2 From a technical perspective, a finite number of banks would introduce cumbersome indivisibilities in the optimal asset allocation among banks.

3 Technically, imagine bankers are located on the interval \([0, 1]\) with total inside equity \( E_I \) and firms are located on a square \([0, 1] \times [0, 1]\) with a total financing demand \( I \).

4 Even absent deposit insurance, an asset substitution problem may arise after a bank has issued debt. However, incentives for risk shifting would be reduced since depositors would require higher yields from banks that take risks (in particular, in the presence of covenants that address banks’ asset choice), or not even invest.
framework also subsumes the polar case \( c(E_O) = 0 \).

**Balance sheet and asset returns.** Given the bank’s choices for the amount of outside equity, \( E_O \), and deposits, \( D \), and the bank’s investment in the storage technology \( Cash \), the total real (non-cash) investments by banks satisfy the standard accounting identity:

\[
A + Cash = E + D,
\]

where \( E = E_I + E_O \) is the book value of equity and \( A \) is the book value of non-cash assets.

Let \( x_i \geq 0 \) denote the portfolio weight of asset \( i \) in a bank’s non-cash asset portfolio and let \( r_s^i \) denote the associated state-contingent rate of return on asset \( i \). Then, the rate of return on a bank’s non-cash assets, \( A \), in state \( s \) is given by:

\[
r_s^A = \sum x_i r_s^i.
\]

**Objective.** At date 1 in state \( s \), the payoff for bank equity holders is given by

\[
(1 + r_s^A) A + Cash - D (1 + r_D)
\]
as long as the state-dependent payoffs from the asset portfolio and cash exceed the promised repayment to debt holders given the promised deposit rate \( r_D \), and 0 otherwise. The deposit rate \( r_D \) is set such that competitive depositors break even on average, taking into account potential guarantees by the regulator (see subsequent section). After having made the capital structure choices \( E_O \) and \( D \), the market value of equity is

\[
E_M = \mathbb{E} \left[ \max \left\{ (1 + r_A^s) A + Cash - D (1 + r_D), 0 \right\} \right].
\]

Pre issuance, the (inside) equity holders’ objective function is to maximize the value of their share, taking into account that the present value of outside equity has to equal outside equity holders’ initial investment, \( E_O \), and that outside equity issuances are associated with deadweight cost \( c(E_O) \):

\[
E_{I,M} = \max_{E_O,D,Cash,\{x_i\}} E_M - E_O - c(E_O).
\]
1.4 Regulator

Our regulatory tool set and interventions are motivated by the key regulatory features of bank regulation across the world. Hence, our modeling framework lends itself to analyze the comparative statics of existing tools, in particular capital requirements. One way to justify our focus on the existing toolset is to assume that the space of regulatory policies is incomplete, as in the literature on incomplete contracts. We discuss broader implications for optimal regulation design in the conclusion.

Deposit insurance. Deposit insurance and, more generally, implicit guarantees of short-term debt serve the purpose of avoiding bank runs (see Diamond and Dybvig (1983) and the model extension in Appendix A). Whenever the bank cannot repay depositors, i.e.,

\[(1 + r^A) A + \text{Cash} < D (1 + r_D)\]  \hspace{1cm} (9)

depositors receive the respective shortfall from the government (or the institution administering deposit insurance). As a result of this guarantee, competitive depositors are willing to provide funds to each bank at zero promised interest, independent of a bank’s asset choice,

\[r_D = 0\]  \hspace{1cm} (10)

Bank capital regulation The value of the subsidy from deposit insurance depends on the bank’s asset risk and leverage since the regulator effectively holds a short position in a put option on the bank’s asset value (as in Merton (1977)). Thus, to counter the resulting asset substitution incentives (see Kareken and Wallace (1978)), she prescribes that the book equity ratio of every bank, \(e \equiv \frac{E}{A}\), be above some minimum threshold. To simplify the exposition we shut down additional risk taking opportunities through security design by imposing that banks can invest only in minimum-risk securities issued by firms, that is, senior bank loans.

Following the guidelines under Basel I-III, capital requirements for loans to a particular firm \(f \in \rho_f\), may be a function of contractible signals about firm type \(\rho \in \Omega\) where \(\Omega\)

is a partition of $\Omega_f$. To fix ideas, one may think of $\rho_f$ as a credit rating of firm $f$ and the partition $\Omega_{\rho}$ being generated by the 24 possible credit rating bins. A bank’s overall equity-ratio constraint is then given by:

$$e \geq \sum_f x_f e(\rho_f).$$

(11)

For our subsequent comparative statics analysis, it is useful to express $e(\rho_f)$ as a product of a risk-weight, $rw(\rho_f)$, and an overall level of capital requirements, $e$, i.e., $e(\rho_f) = rw(\rho_f) \cdot e$.

Since, in reality, all signals of risk, such as credit ratings, asset classifications, and accounting variables, are noisy measures of firm quality $q$, the contractible signals do not allow the regulator to perfectly discriminate between different firm types $q$. The partition $\Omega_{\rho}$ captures this coarseness of the regulator’s contractible information set relative to the investors’ information set $\Omega_q$ in a general and flexible way. Coarseness gives rise to a residual moral hazard problem in the asset choice of banks. We do not impose any assumption on the signal generating process, except for the technical condition that each element of the cross-partition of $\Omega_{\rho}$ and $\Omega_q$ has positive mass.

**Welfare.** Let $\mu(f)$ be an indicator of whether firm $f$ is funded, i.e., $\mu(f) = 1$ if $f$ is funded and $\mu(f) = 0$ otherwise. We define total ex-ante surplus in the economy $W$, as the total expected surplus generated by funded firms net of banks’ deadweight cost of outside equity issuances:

$$W = \int_{f \in \Omega_f} NPV(q(f)) \mu(f) df - \int_{b \in \Omega_b} c(E_O(b)) db.$$  

(12)

We note that our model can easily accommodate ex-post taxation or distress costs resulting from bailouts of failing banks, since our model pins down the fraction of failing banks for any structure of capital requirements. However, while such additional distortions matter for welfare, they would not affect the ex-ante funding decisions of banks given capital requirements, which is the main focus of our analysis. For the sake of tractability, we will thus not introduce such costs until we discuss parameterized example economies in Section 6.

---

7 For the trivial partition, $\Omega_{\rho} = \{[0, 1]\}$, all firms look identical to the regulator. If, on the other hand, $\Omega_{\rho}$ is a refinement of $\Omega_q$ the regulator has no informational disadvantage relative to investors.
2 Analysis

2.1 Preliminaries

The overall game consists of 1) the regulator’s ex-ante choice of equity ratio requirements, \( e(\rho) \), and 2) the subgame of all private-party decisions in the economy given \( e(\rho) \). Our notion of equilibrium is subgame perfection:

Definition 1 Subgame perfect equilibrium

a) The regulator maximizes expected welfare \( W \) by choosing minimum equity capital requirements \( e(\rho) \).

b) Given its type \( q \), each firm \( f \) maximizes its expected value of profits by obtaining the cheapest source of financing that results in raising \( I \) units of capital.

c) Each bank \( b \) maximizes \( [8] \) by choosing its cash position, its outside equity \( E_O \), its equity ratio \( e \geq \sum x_f e(\rho_f) \), and its loan portfolio \( \{x_f\} \geq 0 \).

d) Public market investors invest in firm projects and deposits if and only if they expect to break-even.

In the following, we solve for equilibrium outcomes by backward induction. Most of our analysis focuses on the subgame after the regulator has chosen \( e(\rho) \).

2.2 Equilibrium given capital requirements

Our analysis of the equilibrium given capital requirements, \( e(\rho) \), proceeds as follows. We will first study the optimal behavior of an individual bank in partial equilibrium. Due to competition, an individual bank takes the returns on each loan \( f \), \( r^*_f \), as exogenously given. In a second step, we will characterize the system-wide implications of individually optimal behavior, thereby endogenizing the returns on all assets in the economy.

2.2.1 Individual bank problem

Using \( r_D = 0 \), the maximization problem of an individual bank \( [8] \) may be split into two parts: a problem of optimal outside equity issuance, \( E_O \), and the jointly optimal portfolio
and leverage choice \( e, x \), i.e.,

\[
E_{I,M} = E_I + \max_{E_O} \left[ (E_I + E_O) \max_{e,x} r_E(e,x) - c(E_O) \right],
\]

where

\[
r_E(e,x) = \mathbb{E} \left[ \max \left\{ \frac{\sum f x_f r_f^*}{e}, -1 \right\} \right].
\]

We interpret \( r_E(e,x) \) as the expected rate of return on bank book equity (ROE) before the cost of equity issuance. Consider now the inner (ROE) maximization problem given exogenous returns:

\[
\max_{e,x} r_E(e,x) \quad \text{s.t.} \quad e \geq e_{\min}(x),
\]

where we define:

\[
e_{\min}(x) = \sum f x_f (\rho_f),
\]

and, given a solution \( e^*, x^* \), define the bank’s default states and its complementary survival states as \( \Sigma_B(x^*, e^*) \) and \( \Sigma_B^c(x^*, e^*) \), respectively. In bank default states, \( \Sigma_B(x, e) \), the loss on assets is greater than the bank’s capital buffer, that is, \(-r_A^s > e\).

**Lemma 1** Jointly optimal bank leverage \( e^* \) and portfolio choices \( x^* \)

i) **Leverage:** The regulatory leverage constraint binds, \( e = e_{\min}(x^*) \), if either

1) there exists a portfolio \( x \) with strictly positive bank rents, \( r_E(e_{\min}(x), x) > 0 \), or

2) the bank’s deposit insurance pays off in some state \( s \) if the bank takes maximum leverage, \( r_A^s < -e_{\min}(x^*) \).

ii) **Portfolio choice:** Suppose there are \( n \) firm loans in the portfolio, then

1) All loans generate the same bank ROE conditional on bank survival, i.e.,

\[
\mathbb{E} \left[ r_f^* \sum_B^c(x^*, e^*) \right] / \xi(\rho_f) = k \quad \text{for some} \quad k \geq 0.
\]

2) \( \Gamma = (-r_f^s, \xi(\rho)) \in \mathbb{R}^n \times \mathbb{R}^n \) is comonotonic for each state \( s \)^8

---

^8 The set \( \Gamma \) is comonotonic, if for all \((r_1^*, r_2^*, ..., r_n^*)\) and \((\xi(\rho_1), \xi(\rho_2), ..., \xi(\rho_n))\) in \( \Gamma \) with \( r_f^* < \xi(\rho_f) \) for some \( f \in \{1, 2, ..., n\} \), it follows that \( r_i^* \leq \xi(\rho_i) \) for all \( i \in \{1, 2, ..., n\} \). Equivalently, \( r_f^* > \xi(\rho_f) \) implies that \( r_i^* \geq \xi(\rho_i) \). See Dhaene, Denuit, Goovaerts, Kaas, and Vyncke (2002a, 2002b).
Leverage: Part i.1 states that if the given asset returns allow banks to earn rents, \( r_E(e_{\min}(x), x) \), banks have a strict incentive to lever up their scarce capital up to the regulatory constraint. To understand condition 2, observe that upon a bank default in some state \( s \), government transfers to bank depositors, \( A \cdot (-r^s_A - e) \), are strictly decreasing in \( e \). Total payments to security holders are thus increasing in leverage, a key departure from the Modigliani-Miller framework. While these transfers accrue \textit{ex post} to depositors, competition among public market investors on the deposit rate ensures that the present value of these transfers, \( \mathbb{E}(A \max\{-r^s_A - e, 0\}) \), is passed on to bank equityholders \textit{ex ante}. Thus, shareholder value maximization requires the value of the deposit insurance guarantee be maximized for any given optimal portfolio \( x^* \).

Portfolio choice: Since bankers are risk neutral, portfolio diversification confers no benefit in our setup. In fact, naïve diversification of the loan portfolio is generally suboptimal as it lowers the subsidy derived from deposit insurance and hence reduces bankers’ ROE. Thus, abstracting from the supply of securities, a single-asset portfolio which maximizes

\[
\mathbb{E} \left[ \max_j \left\{ \frac{r^*_j}{\xi(\rho_j)}, -1 \right\} \right]
\]

is an optimal portfolio. Lemma 1 highlights that optimally designed loan portfolios may consist of multiple, imperfectly correlated firms as long as the following properties are satisfied. First, each loan in the portfolio must generate the same expected asset return per unit of required equity capital \( \xi(\rho_j) \), conditional on bank survival. Intuitively, a bank only cares about asset returns in survival states \( s \in \Sigma^c \). The comonotonicity property implies that for each state \( s \) losses on all individual loans in the portfolio, \(-r^s_j\), are either all above or all below their respective loan-specific capital requirement, \( \xi(\rho_j) \). Forming portfolios with such correlated downside risk allows banks to maximally exploit deposit guarantees.

Given a solution \( e^* \) and \( x^* \) yielding \( r^*_E = r_E(e^*, x^*) \), we can characterize the incentives of an individual bank to issue additional equity in a straightforward way.

**Lemma 2** \textit{Raising equity in partial equilibrium}

1) If \( r^*_E \leq c'(0) \), a bank does not raise equity.
2) A bank has a strict incentive to increase outside equity as long as \( r^*_E > c'(E_O) \).

2.2.2 Capital allocation in a competitive financial system

A competitive financial system is characterized by 1) competition across different classes of investors (banks vs. public market investors) and 2) competition within a given investor class. Both modes of competition matter for the allocation of capital to firms and equilibrium loan yields.

Only firm types with $NPV(q) \geq B(q)$ have access to funding from public markets. Due to perfect competition between public market investors, the offered public bond yield, $y_P(q)$, is set such that firms can extract all surplus, i.e.,

$$\mathbb{E} \left[ \min \left\{ y_P(q), \frac{C_s(q) - I}{I} \right\} \right] = 0.$$  

(17)

Public market investors thus realize the minimum of the promised yield, $y_P(q)$, and the underlying firm asset return, $(C_s(q) - I)/I$.

Banks have two competitive advantages over public markets. First, for credit-rationed firms with positive NPV projects, $0 < NPV(q) < B(q)$, banks’ monitoring ability may enable financing of socially valuable projects. Second, banks have access to insured deposit financing, which may allow banks to undercut the public loan yield $y_P(q)$ even for firm types that can be funded by competitive public markets.

Consistent with Lemma[1] ex-ante identical banks fund heterogeneous firms by choosing heterogenous portfolio strategies that yield the same expected return. This segmentation allows the banking sector as a whole to reap the maximum deposit insurance subsidy for each funded borrower, denoted as $\sigma(q_f, \rho_f)$, where formally:

$$\sigma(q_f, \rho_f) = I \cdot \mathbb{E} \left( \max \left\{ -r_f^s - \xi(\rho_f), 0 \right\} \right) = \mathbb{E} \left( \max \left\{ I (1 - \xi(\rho_f)) - C_s(q_f), 0 \right\} \right).$$  

(18)

To see how competition between banks induces segmentation consider the following example. A bank $b$ with overall asset portfolio payoffs sufficient to repay debt holders in all states would not be able to extract a financing subsidy by adding a “small” investment in risky firm types ($\sigma > 0$) to the portfolio. As a result, bank $b$ could not compete on loan terms offered by another bank with an efficient portfolio satisfying comonotonicity (see Lemma[1]).

---

10 The second equality follows from the fact that loan returns satisfy $r_f^s = \frac{C_s(q_f) - I}{I}$ in states when the firm cannot fully repay.
Combining both sources of competitive advantage, bank financing of a particular firm generates total private surplus $\Pi$.

$$
\Pi (f) = \text{NPV} (q_f) \mathbb{1}_{B(q_f) > \text{NPV}(q_f)} + \sigma (q_f, \rho_f).
$$

(19)

Figure 1 illustrates how the state-contingent cash-flows of borrowers in a two-state economy relate to $\Pi$, the $\text{NPV}$, and access to public market finance. Private surplus $\Pi$ and $\text{NPV}$ both increase toward the “north” and “east”. Financing of firms in the area between the $\Pi = 0$ line and the $\text{NPV} = 0$ line creates negative social $\text{NPV}$, but positive private surplus if funded by maximally levered banks. A maximally levered bank with an efficient loan portfolio does not care about payoffs below $1 - \varepsilon$ in the state $s = L$ (left of the vertical dashed line) or $s = H$ (below the horizontal dashed line). Firms in the corridor between $\text{NPV} = 0$ and $\text{NPV} = B$, such as the borrower type marked with a black cross, are firms with socially valuable investments that are capital rationed by public markets and hence require bank financing. Finally, borrowers above the $\text{NPV} = B$ line have access to public markets and bank finance.

Figure 1. Private surplus, social surplus, and access to public markets. The graph delineates firm cash flow combinations in two equiprobable macro-states $s \in \{L, H\}$ that give rise to zero private surplus, zero social surplus $\text{NPV} = 0$, and $\text{NPV} = B$. We use the following parametrization: $I = 1$, $B = 0.2$ and $\varepsilon = 0.3$. The firm marked with a black cross features cash flows $(C_L (f), C_H (f)) = (1.1, 1.1)$, so that $\bar{\text{NPV}} = \Pi = 0.1 < B$. 

\[11\]
In our following analysis, we characterize all financing decisions in the economy, as well as the division of surplus. We posit

**Assumption 2** Banks behave in the socially optimal way when indifferent.

Assumption 2 implies that banks do not fund firms with zero total private surplus when such funding generates a negative social NPV. Our analysis discusses the equilibrium funding decisions depending on the scarcity of bank capital.

**Definition 2** Given \( e(\rho) \), inside bank equity capital is scarce if the banking sector cannot fund all firms with positive total surplus using only inside equity, \( E_I \), and deposits, that is, if:

\[
E_I < I \int_{f: \Pi(f) > 0} e(\rho_f) \, df. \tag{20}
\]

**Case:** \( E_I \geq I \int_{f: \Pi(f) > 0} e(\rho_f) \, df \). We first discuss the case when banks’ inside equity capital is not scarce.

**Proposition 1** When inside bank equity capital is not scarce

1) banks fund all borrowers with \( \Pi(f) > 0 \),
2) of the remaining firms public markets fund all firms with \( \text{NPV}(q_f) \geq B(q_f) \),
3) funded firms extract private and social rents in the economy, that is,

\[
\Pi(f) + \text{NPV}(q_f) \mathbb{1}_{\text{NPV}(q_f) \geq B(q_f)}. \]

Since banks have excess inside equity capital they will finance all borrowers that offer positive total private surplus, that is, all borrowers above the \( \Pi = 0 \) line in Figure 1. This includes the set of all borrowers with positive-NPV projects, but also firms with negative-NPV projects provided that their effective financing subsidy, \( \sigma(q_f, \rho_f) \), is sufficiently high.

When bank capital is not scarce, competition between banks for borrowers on loan terms ensures that banks pass on all social and private advantages to borrowers. Given that banks’ equilibrium return is zero, we can solve for the bank loan yields \( y(f) \) for all borrowers funded by banks

\[
\mathbb{E} \left[ \max \left\{ \min \left\{ y(f), \frac{C_s(f) - I}{I} \right\}, -e(\rho_f) \right\} \right] = 0. \tag{21}
\]
The loan pricing condition by banks (21) is similar to the one by public market investors (17), except for the fact that, from a bank equity holder’s view, loan losses are capped by their required co-investment $\mathcal{E}(\rho_f)$. It is immediate, that when inside equity capital is not scarce, yields on loans to firms are increasing in capital requirements. Higher capital requirements imply that bankers can pass on a lower financing subsidy to firms (see (18)).

Our setup thus formalizes how risk signals, such as credit ratings, used for regulation will be reflected in prices, holding cash flow characteristics constant. The value implications of a ratings change from $\rho = AAA$ to $\rho = B$ thus depend on the corresponding change in capital requirements.

**Case:** $E_I < I \int_{f : \Pi(f) > 0} \mathcal{E}(\rho_f) df$. In the case when banks’ inside equity capital is scarce, banks cannot fund all borrowers in the economy, unless outside equity is raised. If the banking sector does not fund all borrowers, it will rationally shed the borrowers with the lowest profitability per unit of required equity capital. Banks’ private ranking of borrowers is determined by the private profitability index

$$PI(f) = \frac{\Pi(q_f, \rho_f)}{I\mathcal{E}(\rho_f)}.$$  

(22)

Since the private profitability index is a function of social surplus and deposit insurance subsidies (see (19)), it is generally not aligned with the social ranking of projects. In particular, the banking sector’s private ranking is misaligned when a sufficient fraction of projects feature the combination of high downside risk, low capital requirements, and negative NPV. These characteristics may well have been satisfied highly-rated structured securities in the years leading up to the Great Recession.

Let $PI_{E_I} = PI(f_M)$ denote the profitability index of the marginal funded firm by the banking sector, $f_M$, absent outside equity issuances. $PI_{E_I}$ solves:

$$E_I = I \int_{f : PI(f) \geq PI_{E_I}} \mathcal{E}(\rho_f) df.$$  

(23)

The value of $PI_{E_I}$ plays a crucial role in determining whether the banking sector has incentives to raise outside equity. If $c'(0) \geq PI_{E_I}$, the banking sector does not raise outside equity. In this case, the marginal firm $f_M$ pledges its entire surplus $\Pi(f_M)$ to banks to attract bank funding, so that the firm just receives its outside option, $NPV(q) \mathbb{1}_{NPV(q) \geq B(q)}$.

11 Opp, Opp, and Harris (2013) analyze how price effects of this kind create a feedback effect on the policies of credit rating agencies.
The banking sector’s return on equity from this loan is \( r_E^* = PI_E \). Loan yields for all other firms funded by banks adjust such that all banks earn an expected ROE of \( PI_{E_i} \), thus defining the equilibrium rate of return for the banking sector.

When \( c'(0) < PI_{E_1} \), the return on the marginal funded firm absent equity issuances exceeds the marginal cost of raising outside equity. Thus, an equilibrium without equity issuances cannot obtain. The banking sector raises outside equity up to the point where all firms with \( PI(f) > c'(E_O^*) \) are funded, where \( E_O^* \) refers to the equilibrium quantity that each bank raises.\(^{12}\) Competition among banks ensures that the equilibrium rate of return of the banking sector is given by \( r_E^* = c'(E_O^*) \). We summarize these insights in

**Proposition 2** When inside bank equity capital is scarce

1) no bank raises equity if \( c'(0) \geq PI_{E_0} \). Otherwise, \( E_O^* \) is the unique solution to

\[
E_I + E_O = \int_{I:PI(f)>c'(E_O)} e(\rho_f) \, df,
\]

2) banks fund all firms with \( PI(f) > r_E^* = \min\{PI_{E_i}, c'(E_O^*)\} \),
3) of the remaining firms public markets fund all firms with \( NPV(q_f) \geq B(q_f) \),
4) funded firms extract

\[
\Pi(f) \max\left\{1 - \frac{r_E^*}{PI(f)}, 0\right\} + NPV(q_f)\mathbb{1}_{NPV(q_f) \geq B(q_f)}.
\]

With scarce inside bank equity capital, it is possible that some firms with positive-NPV projects remain unfunded. Such capital rationing occurs for firms with both high agency rents, i.e., \( B(q_f) > NPV(q_f) \), and a profitability index below the hurdle rate, i.e., \( PI(f) < r_E^* \).

Scarcity not only affects the funding decisions of the economy, but also allows banks to earn equilibrium rents. Each firm that is funded by the banking sector now needs to pledge a fraction \( \frac{r_E^*}{PI(f)} \) of the private surplus \( \Pi(f) \) to bankers as bankers’ next best investment allows them to earn an ROE of \( r_E^* \). Since the equilibrium rate of return of the banking sector may be non-monotonic in capital requirements, it is no longer necessarily true that an increase in capital requirements leads to lower payoffs (higher loan yields) for firms with socially valuable projects. We will illustrate this possibility in our example section.

\(^{12}\) If \( c(E_O) \) is strictly convex, then symmetric outside equity issuances are the unique equilibrium outcome. If \( c(E_O) \) is linear, individual banks may raise different amounts of equity, but the aggregate amount of issuance is still uniquely determined.
Given the above characterization of economy-wide funding decisions and surplus allocations for any specification of capital requirements, we next conduct comparative statics of capital requirements.

### 2.3 Comparative statics of capital requirements

**Economy without capital requirements.** We first characterize an economy without capital ratio requirements as a benchmark, i.e., \( \varsigma(\rho_f) = 0 \) \( \forall f \). Definition 2 implies that bank capital is not scarce in this case. Proposition 1 applies.

**Corollary 1** In an economy without equity ratio requirements:
1) all firms are funded at a yield of \( y(i) = 0 \), and,
2) ex-ante welfare is given by \( W = \int_{f \in \Omega} NPV(f) \, df \).

Absent capital requirements banks finance themselves with infinite leverage \( (e = 0) \) to capture the upside in at least one state of the world while tax payers subsidize debtholders on the downside. Loan yields for funded firms are completely uninformative about default risk, \( y = 0 \), since the marginal buyer, the banking sector, does not price default risk due to deposit insurance (21). Public market investors cannot compete with banks for risky borrowers at these distorted yields.

To analyze welfare distortions of the unregulated equilibrium (see Corollary 1), it is useful to define the first-best outcome \( W^* \) which is

\[
W^* = \int_{f:NPV(q_f)>0} NPV(q_f) \, df. \tag{24}
\]

First-best welfare is achieved if banks exclusively fund all surplus-generating projects in the economy. The degree of welfare distortions in the unregulated economy relative to first-best depends on the degree of ex-ante overinvestment, i.e., how many surplus destroying projects are funded and, more generally, the expected taxation costs of bailouts to rescue failing banks. Since funding of negative NPV borrowers is a result of deposit insurance subsidies (and hence firms’ downside risk), the economy-wide welfare distortions vary with the cross-sectional distribution of borrower risk profiles.

**Channels of capital requirements.** In our framework, capital requirements operate via two channels, an incentive channel and a balance-sheet channel. The incentive chan-
nel has a strictly positive impact on welfare $W$. A system-wide symmetric increase in $\xi(\rho_f)$ lowers the implicit deposit insurance subsidy for each project (18) and thus aligns the incentives of banks, reflected in $PI(f)$, better with the social ranking of projects, $NPV(q) \mathbb{1}_{B(q) > NPV(q)}$.

These benefits have to be weighed against the potential cost of constraining the aggregate funding capacity of the banking sector, the balance-sheet channel. The welfare implications of such balance sheet reductions depend on the marginally funded project type.

**Proposition 3** Let $\xi(\rho_f) = rw(\rho_f) \cdot \xi$ and consider the comparative statics in $\xi$
1) If bank capital is not scarce, a marginal increase in $\xi$ weakly increases welfare.
2) If bank capital is scarce, a marginal increase in $\xi$ weakly increases welfare unless the marginal funded firm, $f_M$, satisfies, $B(q_f) > NPV(q_f) > 0$.

Intuitively, if bank capital is not scarce, only one of the channels, the incentive channel, is at work. Then, an increase in capital requirements may reduce the private profitability of some negative NPV projects below zero, so banks will stop financing these value-destroying projects without affecting the financing of positive NPV projects. Welfare weakly increases.

In contrast, when bank capital is scarce, balance sheet constraints may cause capital rationing of value-creating borrowers first. To see this, note that providing a loan to a high-risk, negative-NPV borrower may have a higher profitability index than a loan to a low-risk, positive-NPV borrower at a given level of capital requirements (see definitions in (19) and (22)). As a result, if this positive-NPV borrower is the marginal funded firm by the banking sector, an increase in capital requirements will induce the banking sector to stop funding the positive-NPV borrower, rather than the bad borrower. Moreover, for sufficiently high agency rents, $B > NPV > 0$, this borrower will find it impossible to substitute to public markets.

We have demonstrated that a local increase in capital requirements can induce capital rationing for good borrowers. In this case, somewhat counterintuitively, the asset portfolio of the average bank becomes riskier in response to higher equity-ratio requirements as the banking sector drops only projects of good low-risk borrowers while maintaining investments in bad high-risk borrowers.

By the same token, the model also highlights that the balance sheet channel need not be associated with a negative welfare effects. First, if the marginal borrower can
obtain funding from public markets, then capital rationing is of no concern. Second, if the marginal firm of the banking sector has a negative NPV project, the balance sheet channel reduces investments in welfare-destroying projects, leading to an increase in total surplus. In sum, without detailed knowledge about the marginal borrowers in an economy, increases in capital requirements have a non-trivial effect on welfare when bank capital is scarce. In contrast, with sufficient bank capital, the balance sheet channel is absent, and, locally, higher capital requirements unambiguously increase welfare.

Figure 2 illustrates the horse race between the incentives and the balance sheet channel in an example economy with 2,500 firm types and two macro-states (as in Figure 1). Each cross represents a funded firm type with cyan crosses indicating public funding and orange crosses indicating bank funding. We (initially) assume that outside equity issuance costs are prohibitively high. Moreover, for ease of illustration, all firm types are subject to the same capital requirement \( e \), which varies across panels (increasing from Panel A to Panel D, thereby “aligning” private incentives \( \Pi \) and \( NPV \) more and more).

**Upper panels.** In Panel A, capital requirements are absent (see Corollary\[\ref{lower:corollary:1}\]). There is overinvestment: All firms offering a positive return in at least one state, \( C_s(i) > I \), will obtain financing. Bank equity capital is not scarce. Thus, a (small) increase in capital requirements to \( e = 0.1 \) (see Panel B) only works through the incentives channel. Overinvestment, i.e., funding of firm types below the \( NPV = 0 \) line, is reduced.

**Lower panels.** A further increase in capital requirements to \( e = 0.3 \) (see Panel C) causes inside bank equity capital to be scarce. Now, some firm types offering positive private surplus cannot get (bank) financing, as indicated by red circles. The positive effect is that overinvestment is further reduced. The negative effect is that some bank-dependent firm types in the corridor between the \( NPV = 0 \) line and the \( NPV = B \) line are credit rationed. Interestingly, this occurs despite the fact that the banking sector has enough funding capacity to fund all bank-dependent firm types. Instead, it prefers to fund risky firm types with high payoffs in the good state (some of these types could have even been funded by public markets). As capital requirements are further increased to \( e = 0.7 \) (see Panel D), the associated decrease in the financing subsidy (see (18)) reduces the set of privately profitable loans by so much that it outweighs the increase in required capital per unit of funded project. Bank capital is no longer scarce. In particular, banks fulfill their social role by funding bank-dependent firms with positive NPV projects; without engaging in excessive risk-taking. In this example, sufficiently high capital requirements can thus achieve first-
best welfare. Moreover, somewhat surprisingly, higher capital requirements can lead to a less constrained financial sector, despite the absence of outside equity issuances.

**Outside equity issuances** So far, we have ruled out outside equity issuances. However, since equity issuances are only relevant when the banking sector is constrained, only Panel
C is affected by alternate assumptions on outside equity issuance costs. When \( c'(0) < PI_{E_i} = 33\% \), banks have an incentive to raise outside equity to fund additional borrower types, indicated by the green diamonds in Figure 3. The welfare effects of such equity issuances are generally ambiguous. On the one hand, banks lever up on the newly raised equity and provide loans to negative-NPV firms, thereby exacerbating overinvestment. On the other hand, it will lead to increased funding of bank-dependent positive NPV firms, i.e., a reduction in underinvestment, and, to crowding-out of public markets. The net welfare effect ultimately depends on the cross-sectional distribution of borrower types.

Figure 3. Effects of equity issuances. The graph plots funding decisions for the case of \( \epsilon = 0.3 \) (corresponding to Panel C of Figure 2) under the assumption that outside equity issuance cost are given by \( c(E_O) = 0.2E_O \). All other parameters are as in Figure 2.

References


