# Consumer Search and Retail Market Structure<sup>\*</sup>

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#### Abstract

This paper proposes a framework for studying how consumer search and multiproduct purchase affect retail market structure. To provide one-stop shopping convenience and attract more consumers, single-product shops supplying different products can merge to form a multiproduct retailer. The merger, however, also changes the market structure and affects price competition. When the search friction is not too high, the equilibrium market structure is asymmetric and multiproduct and single-product retailers coexist. This often leads to the weakest price competition and is the worst for consumers among all possible market structures. Due to the endogeneity of the market structure, reducing the search friction does not necessarily induce lower market prices and higher consumer welfare.

**Keywords:** consumer search, conglomerate merger, multiproduct pricing, retail market structure

JEL classification: D11, D43, D83, L13

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## 1 Introduction

Retailing is an important sector of the economy. Although retailers compete along many dimensions, product assortment plays a key role in determining where consumers shop. Since consumers are (increasingly) time-constrained, they value the chance to buy a basket of products in one place. Over time retailers have responded to this by significantly increasing the size of their product selections.<sup>1</sup> The Food Marketing Institute estimates that between 1975 and 2013, the number of products in an average US supermarket increased from around 9,000 to almost 44,000. At least some of this increase can be attributed to retailers stocking new product lines, as they attempt to provide consumers with a one-stop shopping experience. For example, Wal-Mart has expanded into pharmacies and clinical services, whilst drugstores like Walgreens have started selling fresh food and grocery items. Nevertheless, retail product assortments are extremely heterogenous - for every Wal-Mart or Amazon, there are many specialist retailers with much narrower product selections.

This paper provides a framework for studying equilibrium retail market structure, which allows for the existence of multiproduct firms, and in which consumers buy multiple products and face search frictions. The model is used to answer several interesting and topical questions. For example, what is the nature of competition between large and small retailers? Why might some firms choose to remain small? How is retail market structure affected by consumer search frictions (and therefore how might it differ across offline and online retailing)?<sup>2</sup> Does the market function better, both for consumers and welfare more generally, when retail assortments are large?

We begin by solving a benchmark model with two products (or product categories), each of which is sold by two single-product retailers. Consumers wish to buy one unit of each product, but differ with respect to their shopping costs. Some consumers ("shoppers") are able to visit all retailers without incurring any cost, while other consumers ("non-shoppers") are time-constrained and are only able to visit one retailer. We consider a two-stage game. At the first stage, single-product retailers choose whether or not to engage in conglomerate merger i.e. sell two different products in the same store.

<sup>&</sup>lt;sup>1</sup>Messinger and Narasimhan (1997) provide empirical evidence that time-saving convenience is the most important driver of the growth in supermarket store size.

 $<sup>^{2}</sup>$ The online market structure appears to be more asymmetric than the offline one. For instance in 2012 Amazon sold more than its top 12 online competitors combined.

At the second stage, all participants observe the resulting market structure, and retailers then engage in price competition. Shoppers learn all prices and so buy each product at the lowest price available. Non-shoppers do not observe prices, but visit a store which they believe will offer them the highest (expected) surplus. The fraction of non-shoppers is interpreted as a measure of the search friction in the market.

The paper first solves for equilibrium pricing in each of the three possible market structures: i.e., four single-product firms, two multiproduct firms, and an asymmetric market with one multiproduct firm and two single-product firms. Prices are drawn using mixed strategies, because firms face the usual trade-off of pricing low to attract shoppers, or pricing high to exploit non-shoppers. As we explain below, the existing related literature focuses on settings where firms have identical product ranges. As such, our analysis of the pricing equilibrium for the case of an asymmetric market structure is of independent interest. Here we show that the multiproduct firm charges higher prices than the single-product firms (in the sense of first order stochastic dominance), and yet still attracts a disproportionate share of non-shoppers. Intuitively this is because the multiproduct firm offers one-stop shopping convenience, which more than compensates for the higher prices that it charges. Moreover, we also demonstrate that this asymmetric market structure often leads to the weakest price competition. This is because the multiproduct retailer focuses more on exploiting its one-stop shopping convenience through higher prices, which softens competition with the two single-product retailers.

Using the previous insight, the paper then proceeds to characterize equilibrium market structure. A conglomerate merger is shown to have two distinct effects. Firstly, when two single-product firms merge they provide one-stop shopping convenience, and so are searched by more non-shoppers (a "search order effect"). Secondly though, the merger changes market structure - and either softens or strengthens price competition, depending upon whether the new market structure is asymmetric or not (a "price competition effect"). Consequently there is no equilibrium with four single-product firms: if two of them engage in a conglomerate merger, they enjoy higher demand and also soften price competition. More interestingly, the size of the search friction then determines whether or not a second conglomerate merger occurs. In particular the price competition effect dominates – and so the equilibrium market structure is asymmetric – if and only if the search friction is relatively low. Thus the model is able to generate both symmetric and asymmetric market structures, depending upon parameters.<sup>3</sup> We

<sup>&</sup>lt;sup>3</sup>Some suggestive evidence of a smaller search cost leading to more asymmetric market structure is

also analyze the welfare consequences of conglomerate mergers, and show amongst other results that a conglomerate merger which leads to an asymmetric market structure is detrimental for consumers, due to the way in which it relaxes price competition.

Finally we show how these insights are robust to various extensions. In one extension we allow for an arbitrary number of firms, and allow consumers to differ over how many products they wish to purchase. The equilibrium market structure is shown to respond to both the search friction and consumer needs in a very natural way. For example, the market is more likely to consist only of multiproduct firms when (i) many consumers are interested in buying both products, and (ii) the search friction is relatively large. In another extension we relax the assumption that non-shoppers may only visit one firm. We do this by introducing a new parameter, which captures the benefits of one-stop shopping generated by having a multiproduct firm. Implications for equilibrium pricing and retail market structure are more complicated than in the benchmark model, but qualitatively the same. Finally in another extension we introduce product differentiation and allow consumers to engage in explicit sequential search. As in the benchmark model, the asymmetric market structure leads to the weakest price competition, and arises in equilibrium whenever the search cost is small enough. Moreover, in contrast to the basic model the multiproduct retailer in this setting charges lower prices than its single-product rivals.

**Related literature**: Our paper is related to a recent body of literature on multiproduct search when firms supply and consumers need multiple products. For example, Zhou (2014) shows how multiproduct consumer search generates a joint search effect, which creates complementarity between even physically independent products. This induces lower market prices compared to the case with single-product search, and can also cause equilibrium price to decrease with the search cost. Rhodes (2015) demonstrates that consumer multiproduct purchase from a multiproduct firm can help avoid the standard hold-up problem which arises in the single-product search case. He also shows how a multiproduct retailer's low advertised prices can signal a low price image on its unadvertised products. Lal and Matutes (1994), McAfee (1995), and Shelegia (2012) examine the related issue of when and how multiproduct firms might correlate their

provided by Goldmanis, Hortacsu, Syverson, and Emre (2010). In a single-product search environment, they show that lower search costs push business towards a small number of very large retailers (in terms of market shares, instead of the product range).

product prices. However all these papers assume an exogenously given market structure where firms sell the same range of products. By contrast in this paper the market structure is endogenous, and we show that the size of search frictions can play an important in shaping the market structure, and in particular an asymmetric market structure where some firms sell a larger range of products than others, can emerge as an equilibrium outcome.<sup>4</sup>

Also related is the literature on shopping costs and agglomeration. Baumol and Ide (1956) were perhaps the first to propose that multiproduct retailers could be attractive to consumers, because they help to reduce time and transportation costs. Stahl (1982) shows that single-product firms have an incentive to co-locate (e.g., in a shopping mall), provided they are not too closely substitutable. Even though co-location intensifies price competition, it also expands demand since consumers become more willing to incur the transportation cost necessary to buy the products. Meanwhile in models where consumers can only learn about prices or match values through search, firms may have incentives to locate near each other either to offer consumers a higher chance of a good product match (Wolinsky, 1983), or as a way of guaranteeing consumers that they will face low prices (Dudey, 1990 and Non, 2010). Moraga-González and Petrikaitė (2013) show that when a subset of firms with differentiated versions of a product merge and sell all their products in a single shop, they can become prominent and induce consumers to search them before the other firms in the market. However in all these papers consumers buy only one product, and so any one-stop shopping convenience does not arise from consumers' need to buy multiple products. Nevertheless, in practice this is an important feature of many retail markets.

In our model the advantage of forming a multiproduct firm is being able to provide consumers with one-stop shopping convenience. Another potential advantage is that a multiproduct firm can use sophisticated pricing strategy such as bundling to compete better with its competitors. Nalebuff (2000) studies this issue in an environment with

<sup>&</sup>lt;sup>4</sup>One exception is Johnson (2014) who considers a model where consumers have a behavioral bias (such that they make unplanned purchases), and one firm is exogenously able to carry more products than another. In his model firms choose which products to select, and what prices to charge for them, but do not choose whether to be large or small. Another exception is Shaked and Sutton (1990) who consider a duopoly model where each firm can choose to supply one or two varieties of a product. They assume that each variety cannot be stocked by more than one firm, and so their model does not allow for the coexistence of large and small firms.

product differentiation and shows that a multiproduct firm (e.g., Microsoft) can use bundling strategy to obtain a significant advantage in competing with single-product rivals, especially when the number of products is relatively large. However if the rival single-product firms merge and also use bundling strategy, then the resulting bundleagainst-bundle competition will lead to low market prices and harm all the firms. As a result, an asymmetric market structure can also be sustained as an equilibrium outcome. (Without shopping frictions, bundle-against-bundle competition is never an equilibrium outcome in Nalebuff's model but a fragmented market with all small firms can be.) We focus on retail markets and argue that even if multiproduct firms do not adopt bundling strategy (e.g., in many retail markets such as the grocery industry we do not observe store-wide bundling), the existence of shopping frictions can still favor a multiproduct firm from the demand side and lead to interesting economics on market structure.

In another closely related paper Thanassoulis (2011) introduces both bundling and one-stop shopping convenience. His model is designed to investigate the convergence trend in the multimedia market where more and more companies are offering bundled services (e.g., the Phone-Internet-TV bundle). He uses a merger framework like ours, and he introduces one-stop shopping convenience in a Hotelling setting by assuming that if consumers buy both products from a single firm they pay a smaller unit transportation cost. In Thanassoulis (2011), the first pair of single-product firms always want to merge because of the resulting one-stop shopping convenience, but due to some modelling specifics the second pair of firms always remain separate to avoid the otherwise fierce price competition. Then the only possible equilibrium market structure is asymmetric.

Our framework endogenizes retail market structure by considering mergers among firms which sell products in different categories. As such, our paper is related to the literature on conglomerate mergers (i.e., mergers between suppliers of different products which do not compete with each other). Since conglomerate mergers do not eliminate competitors, economists and policymakers often hold a benevolent view of it (see Church, 2008, for a survey). However our model shows that conglomerate mergers can soften price competition and thus harm consumers when there is search friciton in the market. In an independent and concurrent work, Chen and Rey (2015) also consider incentives for conglomerate merger. They use a different framework in which consumers are perfectly informed about prices but have heterogenous shopping costs. They find that conglomerate merger can also soften price competition, but that it does not harm consumers (at least when bundling is infeasible). In addition, due to their modelling assumptions a second conglomerate merger is not profitable in their twoproduct framework, since it leads to Bertrand competition. Thus unlike our model, they always predict existence of an asymmetric market structure. Our framework (with search frictions and imperfect information) is perhaps more suited to studying retail markets, whilst their model (with shopping costs and perfect information) might fit better business-to-business markets.

The rest of the paper proceeds as follows. Section 2 outlines the basic model, characterizes price distributions in various market structures, and solves for the equilibrium market structure. Section 3 considers various extensions and shows the robustness of the main results from the basic model. Section 4 presents an alternative model with differentiated products and explicit consumer search. Section 5 then concludes with a discussion of future avenues for research. All omitted proofs are available in the appendix.

# 2 A Basic Model

A unit mass of consumers is interested in buying two products 1 and 2. Each consumer has unit demand, and is willing to pay up to v for each product.<sup>5</sup> Initially there are four single-product firms in the market: two of them, denoted by  $1_A$  and  $1_B$ , sell a homogenous product 1, and the other two, denoted by  $2_A$  and  $2_B$ , sell a homogenous product 2. Supplying each product involves a constant marginal cost which we normalize to zero. Consumers are divided into two groups. A fraction  $\alpha \in (0, 1)$  of consumers are shoppers, who can visit firms costlessly and multi-stop shop. The remaining fraction  $1 - \alpha$  of consumers are non-shoppers, who can visit only one firm (but can do so costlessly).<sup>6</sup>

We assume that conglomerate merger between two firms supplying different products is possible and costless. We also assume that horizontal merger between two firms selling the same product is *not* permitted (or is too costly), for instance due to antitrust policy. We consider a two-stage game. In the first stage, each pair of firms  $(1_k, 2_k)$ , k = A, Bhas the opportunity to merge. Their merger decisions can be simultaneous (in which

<sup>&</sup>lt;sup>5</sup>The analysis can be extended to allow for elastic demand without changing the main result.

<sup>&</sup>lt;sup>6</sup>For example, non-shoppers do not visit another firm because the cost of doing so is prohibitively high.

case we focus on pure strategy equilibria) or sequential. In the second stage, after observing the market structure firms choose their prices simultaneously. To highlight the effect of one-stop shopping convenience, we assume that merger does not affect the marginal cost of supplying each product.

Since shoppers visit all firms and observe all prices, they buy each product at the lowest price, provided the price is below their willingness-to-pay v. If two firms offer the same product at the same price, shoppers randomly choose where to purchase the product. Non-shoppers form (rational) expectations about each firm's price distribution, and choose to visit the firm which they believe will give them highest surplus. Once they arrive at a firm, they will buy all the products there provided their prices do not exceed v. The merger outcome is assumed to be observable to all consumers, so even non-shoppers know, before they search, whether a firm is supplying one or two products. The assumption that product range is observable but price is not is plausible in many cases, given price often varies over time frequently but product range is relatively stable. In turn, each firm sets its prices to maximize expected profits, given consumer search strategies and the other firms' pricing strategies. We assume that multiproduct firms charge separate prices for each product, and therefore do not use bundling.

Notice that  $1-\alpha$  can be regarded as a measure of search frictions in the market. Our primary aim in this paper is to understand how these search frictions affect equilibrium market structure. In order to do this, we need to solve for equilibrium pricing in all three potential market structures: (i) if no merger occurs, a market with four independent single-product firms, (ii) if only one pair of firms merges, an asymmetric market structure with one multiproduct firm and two single-product firms, and (iii) if both pairs of firm choose to merge, a symmetric market with two multiproduct firms. As discussed in the introduction, we believe that the pricing outcome for the asymmetric case is of independent interest. This is because the search literature rarely investigates the case where multiproduct firms compete with single-product firms, despite its obvious relevance. Notice that in our model non-shoppers can consume both products only if there is at least one multiproduct firm in the market. Hence, given inelastic demand in our model, having a multiproduct firm is the only way to overcome search frictions and improve market efficiency.

Finally, a few comments on our modelling assumptions. When firms sell a homogenous product in a search market, the standard way to avoid the Diamond (1971) paradox is to make consumers differentially informed. One approach - which we use here - is to assume that some consumers observe one firm's price, whilst other consumers observe at least two firms' prices. This induces firms to adopt a mixed pricing strategy, which creates price dispersion. (See, for example, Varian, 1980; Burdett and Judd, 1983.) The other approach would be to assume that non-shoppers search sequentially, and incur a cost each time they visit a firm. (See, for example, Stahl, 1989.) However, this is complicated to analyze in a multiproduct context, because typically there are multiple mixed-strategy pricing equilibria which are not outcome equivalent (see McAfee, 1995).<sup>7</sup> An alternative way to avoid the Diamond paradox is to introduce product differentiation. (See Wolinsky, 1986, and Anderson and Renault, 1999.) In section 4, we provide a model with product differentiation and show that the main results are qualitatively similar to those of our baseline model.

We adopt a conglomerate merger framework to study endogenous retail market structure. A possible alternative framework is to allow each retailer to choose its product range directly. In that case, to generate an asymmetric market with at least one multiproduct and two single-product retailers (one for each product), we need at least three firms in the market. Since each firm can choose to supply product 1, product 2, or both, the total number of pricing subgames we need to analyze will be much larger than in the current framework. The current conglomerate merger framework is a more parsimonious way to capture the main insights we intend to deliver.

## 2.1 Pricing under different market structures

As a preliminary step, it is useful to first solve a simpler model in which two singleproduct firms compete over price, and in which some consumers are 'captive' (able to buy from only one exogenously given firm) and others are 'non-captive' (able to buy from either firm). The following lemma reports equilibrium pricing in this game. (None of the results are new in the literature. Results (i) and (ii) can be found in the inelastic demand version of Varian, 1980. Result (iii) can be found in Narasimhan, 1988, or Baye, Kovenock, and de Vries, 1992.)

**Lemma 1** Consider a simultaneous pricing game between two firms A and B which supply a homogenous product at zero cost. Let  $N_k$ , k = A, B, be the number of captive

<sup>&</sup>lt;sup>7</sup>In the most interesting class of equilibria, a firm's prices are negatively correlated across its products. However the characterization of this equilibrium is complex.

consumers who visit firm k only. Suppose  $N_A \ge N_B \ge 0$  with at least one strict inequality. Let S > 0 be the number of non-captive consumers.

(i) There is no pure-strategy Nash equilibrium.

(ii) If  $N_A = N_B = N > 0$ , the unique equilibrium is that each firm charges a random price drawn from the atomless price distribution

$$F(p) = 1 - \frac{N}{S} \left(\frac{v}{p} - 1\right) \tag{1}$$

which has support [p, v] with

$$\underline{p} = \frac{N}{N+S}v \ . \tag{2}$$

(iii) If  $N_A > N_B \ge 0$ , the unique equilibrium is that firm A charges a random price drawn from the price distribution

$$F_A(p) = 1 + \frac{N_B}{S} - \left(\frac{N_B}{S} + \lambda\right)\frac{v}{p}$$
(3)

which has support [p, v] with

$$\underline{p} = \frac{N_A}{N_A + S} v \tag{4}$$

and has a mass point on v with size

$$\lambda = \frac{N_A - N_B}{N_A + S} , \qquad (5)$$

and firm B charges a random price drawn from the atomless price distribution

$$F_B(p) = 1 - \frac{N_A}{S} \left(\frac{v}{p} - 1\right) \tag{6}$$

which has support [p, v).

This lemma provides two useful observations for the case where one firm has more captive consumers than the other. In particular, suppose that  $N_A > N_B$ . Firstly, equations (3) and (6) imply that the two density functions  $f_A$  and  $f_B$  have the form of a constant times  $v/p^2$ . Secondly, using  $\lambda$  in (5) one can verify that

$$f_A(p) = (1 - \lambda)f_B(p) \tag{7}$$

for  $p \in [\underline{p}, v)$ . This indicates that firm A charges higher prices than firm B in the sense of first-order stochastic dominance (FOSD). Intuitively this is because firm A has

relatively more incentive to extract surplus from its captive consumers by pricing high, than compete for non-captive consumers by pricing low.

We now return to our set-up, and use Lemma 1 to study equilibrium pricing in each of the three possible market structures outlined above, starting with the simplest case of four independent single-product firms. (In the welfare analysis, total welfare is defined as the sum of industry profit and aggregate consumer surplus.)

**Lemma 2** With four independent single-product firms, non-shoppers randomly visit one firm and each firm uses a mixed pricing strategy as characterized in Lemma 1(ii) with  $N = \frac{1}{4}(1-\alpha)$  and  $S = \alpha$ . Each firm earns profit  $\frac{1}{4}(1-\alpha)v$  and industry profit is  $(1-\alpha)v$ . Total welfare is  $(1+\alpha)v$  and aggregate consumer surplus is  $2\alpha v$ .

**Proof.** We first show that in equilibrium non-shoppers must randomly visit one firm, such that each firm has  $\frac{1}{4}(1-\alpha)$  non-shoppers. Suppose, in contrast, that some firm *i* has strictly more non-shoppers than another firm  $j \neq i$ . Lemma 1 implies that firm *i* charges a higher price than firm *j* in the sense of FOSD, which is inconsistent with non-shoppers' search behavior. Therefore in equilibrium each firm must have the same number of non-shoppers. The equilibrium characterization then just follows from substituting  $N_A = N_B = \frac{1}{4}(1-\alpha)$  and  $S = \alpha$  into Lemma 1. When a firm charges the monopoly price *v*, only  $\frac{1}{4}(1-\alpha)$  non-shoppers buy from it. Thus each firm's equilibrium profit is  $\frac{1}{4}(1-\alpha)v$  and industry profit is  $(1-\alpha)v$ . Since non-shoppers are only able to buy one product but shoppers buy both products, total welfare is  $(1-\alpha)v + \alpha 2v = (1+\alpha)v$ . Aggregate consumer surplus is therefore  $2\alpha v$ .

Another simple case is when both pairs of firms choose to merge, such that the market has two multiproduct firms. It is without loss of generality to focus on an equilibrium where each firm charges independent prices across the two products.<sup>8</sup>

**Lemma 3** With two multiproduct firms, non-shoppers randomly visit one firm, and each firm uses a mixed pricing strategy for each product according to a price distribution

<sup>&</sup>lt;sup>8</sup>Given the rival firm's pricing strategy F (which can be a joint price distribution) and non-shoppers' shopping behavior, only the two marginal distributions of F matter for a firm's profit calculation. Therefore, for any equilibrium where each firm uses a joint price distribution, we can construct an alternative equilibrium with the same outcome by having each firm choose prices independently across their products according to the marginal distributions of F.

as characterized in Lemma 1(ii) with  $N = \frac{1}{2}(1-\alpha)$  and  $S = \alpha$ . Each firm earns profit  $\frac{1}{2}(1-\alpha)v$  from each product and industry profit is  $2(1-\alpha)v$ . Total welfare is 2v and aggregate consumer surplus is  $2\alpha v$ .

**Proof.** The argument that non-shoppers must randomly visit one firm in equilibrium is similar to Lemma 2. Then we must have  $N_A = N_B = \frac{1}{2}(1 - \alpha)$ , and the equilibrium characterization simply follows from Lemma 1. The welfare calculation is straightforward since now all consumers buy both products.

Compared to the case with four single-product firms, products are now more expensive. This is because in the former case each single-product firm has only one quarter of the non-shoppers, whilst now each multiproduct firm has half the non-shoppers. This is due to the assumption that a non-shopper can visit only one firm, even if all firms supply a single product.

Lastly consider the case where  $1_A$  and  $2_A$  merge to form a multiproduct firm A, whilst  $1_B$  and  $2_B$  remain as single-product firms. Intuitively, non-shoppers tend to visit the multiproduct firm A since they can find both products there. However firm A can exploit this advantage and charge higher prices than its single-product rivals. This may in turn discourage non-shoppers from visiting firm A. Therefore *ex ante* it is not clear whether in equilibrium all non-shoppers visit the multiproduct firm A. We now demonstrate that it depends on how many non-shoppers there are in the population. In particular, Lemma 4 below shows that when the fraction of non-shoppers is high, not all the non-shoppers visit firm A in equilibrium. This prevents the multiproduct firm from charging too high prices, and thus rationalizes non-shoppers' search behavior. Nevertheless, in all cases the multiproduct firm attracts a disproportionate share of non-shoppers, because it offers them one-stop shopping convenience.

In more detail, suppose that  $N_A$  non-shoppers visit firm A and  $N_B$  non-shoppers visit each single-product firm B in equilibrium. Then the pricing equilibrium in each product market is described as in result (iii) of Lemma 1 with  $S = \alpha$ . To check the optimality of non-shoppers' search decision, we need to compare their expected surplus from visiting firm A

$$2\int_{\underline{p}}^{v} (v-p)f_A(p)dp$$

with that from visiting a single-product firm B

$$\int_{\underline{p}}^{v} (v-p) f_B(p) dp$$

By using (5) and (7), the former is weakly higher if and only if

$$2(1-\lambda) \ge 1 \Leftrightarrow \frac{N_B + S}{N_A + S} \ge \frac{1}{2} .$$
(8)

This then allows us to state the following result:

**Lemma 4** Consider the asymmetric market structure with one multiproduct firm A and two single-product firms  $1_B$  and  $2_B$ .

(i) If there are more shoppers than non-shoppers in the market (i.e., if  $\alpha \geq \frac{1}{2}$ ), then all non-shoppers visit the multiproduct firm A. Firm A uses a pricing strategy  $F_A$  for each of its products, and firms  $1_B$  and  $2_B$  use a pricing strategy  $F_B$ , both characterized as in Lemma 1(iii) with  $N_A = 1 - \alpha$ ,  $N_B = 0$  and  $S = \alpha$ . Firm A's per product profit is  $(1-\alpha)v$ , each single-product firm's profit is  $\alpha(1-\alpha)v$ , and industry profit is  $2(1-\alpha^2)v$ . Total welfare is 2v, and aggregate consumer surplus is  $2\alpha^2 v$ .

(ii) If there are fewer shoppers than non-shoppers in the market (i.e., if  $\alpha < \frac{1}{2}$ ), then a fraction  $\frac{1}{2(1-\alpha)}$  of non-shoppers visit the multiproduct firm A and the remaining nonshoppers randomly visit a single-product firm. Firm A uses a pricing strategy  $F_A$  for each of its products, and firms  $1_B$  and  $2_B$  use a pricing strategy  $F_B$ , both as characterized in Lemma 1(iii) with  $N_A = \frac{1}{2}$ ,  $N_B = \frac{1}{4}(1-2\alpha)$  and  $S = \alpha$ . Firm A's per product profit is  $\frac{1}{2}v$ , each single-product firm's profit is  $\frac{1}{4}v$ , and industry profit is  $\frac{3}{2}v$ . Total welfare is  $(\frac{3}{2} + \alpha)v$  and aggregate consumer surplus is  $\alpha v$ .

**Proof.** Notice that in equilibrium each single-product firm must have the same number of non-shoppers, such that  $N_B = \frac{1}{2}(1 - \alpha - N_A)$ . Notice also that in equilibrium the multiproduct firm must receive some non-shoppers, otherwise it would charge lower prices in the sense of FOSD than the single-product firms, invalidating non-shoppers' search behavior.

(i) An equilibrium with all non-shoppers visiting firm A exists if (8) holds with  $N_A = 1 - \alpha$ ,  $N_B = 0$  and  $S = \alpha$ . This yields the condition  $\alpha \ge \frac{1}{2}$ . The equilibrium price distributions follow from Lemma 1(iii). From (5) we have  $\lambda = 1 - \alpha$ . In equilibrium, firm A's profit from each product is  $N_A v = (1 - \alpha)v$ , and each single-product firm's profit is  $(N_B + S\lambda)v = \alpha(1 - \alpha)v$ . Hence, industry profit is  $2(1 - \alpha^2)v$ . Total welfare

is 2v since even non-shoppers buy both products by visiting the multiproduct firm. Aggregate consumer surplus is then  $2\alpha^2 v$ .

(ii) Now consider the case with  $\alpha < \frac{1}{2}$ . The only possible equilibrium is that a fraction  $X \in (0,1)$  of the non-shoppers visit the multiproduct firm A such that  $N_A = (1 - \alpha)X$  and  $N_B = \frac{1}{2}(1 - \alpha)(1 - X)$ . According to (8), non-shoppers will be indifferent between visiting a multiproduct firm and a single-product firm only if  $\lambda = \frac{1}{2}$ , or equivalently

$$\frac{N_B + S}{N_A + S} = \frac{1}{2} \Leftrightarrow X = \frac{1}{2(1 - \alpha)}$$

(Notice that  $X \in (\frac{1}{2}, 1)$  given  $\alpha \in (0, \frac{1}{2})$ .) Then  $N_A = \frac{1}{2}$  and  $N_B = \frac{1}{4}(1 - 2\alpha)$ . In equilibrium, firm A's profit from each product is  $N_A v = \frac{1}{2}v$  and each single-product firm's profit is  $(N_B + S\lambda) v = \frac{1}{4}v$ . Industry profit is then  $\frac{3}{2}v$ . Since only a fraction X of the non-shoppers now buy both products, the total number of consumers who buy both products is  $\alpha + (1 - \alpha)X = \alpha + \frac{1}{2}$ . So total welfare is  $(\frac{1}{2} + \alpha)2v + (\frac{1}{2} - \alpha)v = (\frac{3}{2} + \alpha)v$ . Then aggregate consumer surplus is  $\alpha v$ .

In either case,  $N_A > N_B$  and so the multiproduct firm charges higher prices (in the sense of FOSD) than its single-product rivals. This is because the multiproduct firm offers one-stop shopping convenience. This prediction may not fit the casual observation that large retailers are often cheaper than small ones. Remember, however, that to highlight the effect of one-stop shopping convenience our model has assumed away any possible cost synergy from the conglomerate merger. In reality larger retailers may be more able to take advantage of economy of scale in operations and may also have better deals from upstream suppliers. This can induce them to charge lower prices on average. In addition, it can be shown that the multiproduct firm's price has a greater variance than a single-product firm's price.<sup>9</sup> Given the multiproduct firm's price has a mass point at v, this tends to suggest that the multiproduct firm holds sales more often than its single-product rivals.

For convenience, we summarize all relevant welfare variables in the following table:

<sup>&</sup>lt;sup>9</sup>The proof again uses  $f_A = (1 - \lambda)f_B$ . The details are available upon request.

	4 sp firms	2 mp firms	asymmetric	asymmetric
			$(\alpha \ge 1/2)$	$(\alpha < 1/2)$
Per product profit	$\frac{1}{4}(1-\alpha)v$	$\frac{1}{2}(1-\alpha)v$	$A: \ (1-\alpha)v$	$A: \frac{1}{2}v$
			$B: \ \alpha(1-\alpha)v$	$B: \frac{1}{4}v$
Industry profit	$(1-\alpha)v$	$2(1-\alpha)v$	$2(1-\alpha^2)v$	$\frac{3}{2}v$
Consumer surplus	$2\alpha v$	$2\alpha v$	$2\alpha^2 v$	$\alpha v$
Total welfare	$(1+\alpha)v$	2v	2v	$(\frac{3}{2} + \alpha)v$

Table 1: Profit and welfare comparison across market structures

## 2.2 Equilibrium market structure

We can now examine the equilibrium market structure when both pairs of firms  $(1_A, 2_A)$ and  $(1_B, 2_B)$  have the opportunity to merge before engaging in price competition. The basic trade-off in the merger decision is as follows. If a pair of firms choose to merge, they become more attractive to non-shoppers because of the resulting one-stop shopping convenience. This induces more non-shoppers to visit the merged multiproduct firm ("the search order effect"). Meanwhile, the merger also affects price competition in the market. When the other pair of firms do not merge, the merger creates an asymmetric market structure. This softens price competition ("the price competition effect"): multiproduct firm A can charge higher prices because it has more non-shoppers to exploit, and this further relaxes price competition among firms. As we can see from Table 1, all firms benefit in the asymmetric market structure compared to the initial situation with four single-product firms. As a result, the initial situation with four single-product firms cannot be an equilibrium outcome, because both effects move in the same direction.<sup>10</sup>

If the other pair of firms also choose to merge, they can win back some non-shoppers since they now also provide one-stop shopping convenience. However the merger also restores symmetry and thus may intensify price competition. Therefore whether or not the second pair of firms chooses to merge will depend on the relative strengths of the search order and price competition effects. As we can see from Table 1, when  $\alpha \geq \frac{1}{2}$  (i.e., when there are more shoppers than non-shoppers in the market), the price competition effect is stronger and so the second pair of firms have no incentive to merge. But when

<sup>&</sup>lt;sup>10</sup>Of course, if conglomerate merger involves a sufficiently high fixed cost, then the initial situation can remain as an equilibrium outcome.

 $\alpha < \frac{1}{2}$ , the search order effect dominates and the second pair of firms will merge.<sup>11</sup>

The following result reports the equilibrium market structure and welfare outcomes (the welfare outcomes are straightforward from Table 1) :

**Proposition 1** (i) When there are more shoppers than non-shoppers in the market (i.e., when  $\alpha \geq \frac{1}{2}$ ), the unique (pure-strategy) equilibrium outcome is that the market has a multiproduct firm and two single-product firms. Compared to the initial situation, this leads to higher industry profit and total welfare but lower aggregate consumer surplus. (ii) When there are more non-shoppers than shoppers in the market (i.e., when  $\alpha < \frac{1}{2}$ ), the unique equilibrium outcome is that the market has two multiproduct firms. Compared to the initial situation, this leads to higher industry profit and total welfare are more non-shoppers than shoppers in the market (i.e., when  $\alpha < \frac{1}{2}$ ), the unique equilibrium outcome is that the market has two multiproduct firms. Compared to the initial situation, this leads to higher industry profit and total welfare, and aggregate consumer surplus remains unchanged.

In either case, the market outcome with conglomerate merger improves total welfare relative to the initial situation with four single-product firms. This is simply because of a positive market coverage effect from conglomerate merger—now non-shoppers are able to buy both products by visiting a multiproduct firm. (Notice that in the current setting with inelastic demand, the price competition effect of conglomerate merger does not affect total welfare.) In either case, the market outcome also improves industry profit relative to the initial situation. This is because price competition is relaxed either because of the resulting asymmetric market structure (when  $\alpha \geq \frac{1}{2}$ ) or because more non-shoppers visit each multiproduct firm (when  $\alpha < \frac{1}{2}$ ).

In either case, the market outcome does not improve consumer welfare relative to the initial situation. In the case with an asymmetric market outcome, consumers suffer from the conglomerate merger because the negative price competition effect dominates the positive market coverage effect. In the case with two multiproduct firms, these two effects happen to cancel out each other.

The fraction of non-shoppers (i.e.,  $1 - \alpha$ ) is a measure of search frictions in this baseline model. Proposition 1 implies that when the search friction increases, the equilibrium market structure moves from an asymmetric one to a symmetric one with two multiproduct firms. Intuitively, this is because the search order effect becomes more important as the search friction becomes larger. Since the asymmetric market structure is often associated with weaker price competition, this also suggests that a

<sup>&</sup>lt;sup>11</sup>Notice that in the case with  $\alpha \geq \frac{1}{2}$ , if the two pairs of firms make their merger decisions simultaneously, there are two asymmetric pure-strategy equilibria and one mixed-strategy equilibrium.

higher search friction does not always harm consumers due to the endogeneity of market structure. Figure 1 below describes how total welfare (the top horizontal line), industry profit (the thick solid lines), and aggregate consumer surplus (the dashed lines) vary with  $\alpha$  when v = 1. Total welfare is independent of  $\alpha$  because demand is inelastic. Consumer surplus jumps down at  $\alpha = \frac{1}{2}$  and therefore does not always decrease in the search friction. Similarly industry profit jumps up at  $\alpha = \frac{1}{2}$  and therefore does not always increase in the search friction.<sup>12</sup>

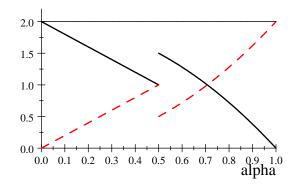


Figure 1: Welfare and the number of shoppers

## 3 Robustness Discussion

This section aims to show that the main insights from the base model are robust to various extensions.

## 3.1 Allowing non-shoppers to multi-stop shop

We now relax our earlier assumption that non-shoppers can only visit one firm (and so can only buy one good when all firms are single-product). To do this as simply as possible, we assume that at the beginning of the game non-shoppers can choose to either i) visit one (single-product or multiproduct) firm at zero cost, or ii) visit two single-product firms at a cost s > 0.<sup>13</sup> Notice that we may then loosely interpret s as

<sup>&</sup>lt;sup>12</sup>With an elastic demand, it can be shown that the threshold for an asymmetric market structure is still  $\alpha = \frac{1}{2}$ . Total welfare varies with  $\alpha$  in a similar way as consumer surplus does in Figure 1, and it jumps down at  $\alpha = \frac{1}{2}$ .

<sup>&</sup>lt;sup>13</sup>We assume away the possibility that non-shoppers can visit two multiproduct firms, or one multiproduct firm and one single-product firm. This greatly simplifies the analysis and captures the idea that consumers tend to be more keen on finding both products they want than searching for lower

a measure of one-stop shopping convenience generated by having a multiproduct firm.

First, consider equilibrium pricing in the two symmetric market structures. When there are four single-product firms, non-shoppers will search two firms with different products provided that s is not too large. Henceforth we assume that s is small enough such that this is the case.<sup>14</sup> For reasons analogous to those in the benchmark model, each firm must then receive the same mass of non-shoppers i.e.  $N = (1 - \alpha)/2$ . When instead there are two multiproduct firms, the model is identical to the one that we solved earlier, and hence each firm again receives a mass  $N = (1 - \alpha)/2$  of non-shoppers. Consequently using Lemma 1, equilibrium price distributions are now *identical* in the two symmetric market structures, and given by

$$F(p) = 1 - \frac{1 - \alpha}{2\alpha} \left(\frac{v}{p} - 1\right)$$

Industry profit is therefore the same in both symmetric market structures. However consumer surplus is higher when there are two multiproduct firms, because non-shoppers no longer have to incur s.

Second, consider the asymmetric market structure. Let  $X(s, \alpha)$  and  $Y(s, \alpha)$  denote the fractions of non-shoppers who visit a multiproduct firm and both single-product firms respectively. We can then show the following:

**Lemma 5** When the market is asymmetric, there exists a unique equilibrium. The nature of this equilibrium depends on the thresholds  $\tilde{s}(\alpha) < \dot{s}(\alpha) < \bar{s}(\alpha)$  which are defined in the appendix.

i) When  $\alpha \geq 1/2$  non-shoppers search either the multiproduct firm, or both singleproduct firms. Moreover  $X(0, \alpha) = 1/2$  and  $X(s, \alpha) = 1$  for all  $s \geq \bar{s}(\alpha)$ . In addition for  $s \in (0, \bar{s}(\alpha))$ ,  $X(s, \alpha)$  is strictly increasing in s and uniquely solves

$$s = 2\left(1-\alpha\right)v\frac{2X-1}{\left(1-\alpha\right)X+\alpha}\left[1+\frac{\left(1-\alpha\right)X}{\alpha}\ln\left(\frac{\left(1-\alpha\right)X}{\left(1-\alpha\right)X+\alpha}\right)\right]$$
(9)

ii) When  $\alpha < 1/2$  and  $s \in (0, \tilde{s}(\alpha)]$  non-shoppers search either the multiproduct firm, or both single-product firms. Moreover  $X(s, \alpha)$  is uniquely defined by (9), and satisfies  $X(\tilde{s}(\alpha), \alpha) = (2 - \alpha)/3(1 - \alpha).$ 

prices when the two products are already available for purchase.

<sup>&</sup>lt;sup>14</sup>In particular we assume that  $s \leq v \left[ 1 + \left(\frac{1-\alpha}{2\alpha}\right) \ln \left(\frac{1-\alpha}{1+\alpha}\right) \right].$ 

iii) When  $\alpha < 1/2$  and  $s \in (\tilde{s}(\alpha), \dot{s}(\alpha))$  non-shoppers randomize between searching the multiproduct firm, both single-product firms, and one of the single-product firms. Moreover

$$Y(s,\alpha) = \frac{2X(s,\alpha)(1-\alpha) - 1}{1-\alpha}$$
(10)

whilst  $X(s, \alpha)$  is strictly decreasing in s and uniquely solves

$$s = v \left[ 1 + \frac{1 - \alpha}{\alpha} X \ln \left( \frac{(1 - \alpha) X}{(1 - \alpha) X + \alpha} \right) \right]$$
(11)

In addition  $X(\tilde{s}(\alpha), \alpha) = (2 - \alpha)/3(1 - \alpha)$  and  $X(\dot{s}(\alpha), \alpha) = 1/2(1 - \alpha)$ .

iv) When  $\alpha < 1/2$  and  $s > \dot{s}(\alpha)$  non-shoppers search the multiproduct firm with probability  $X(s,\alpha) = 1/2(1-\alpha)$ , and with complementary probability search one singleproduct firm.

The equilibrium is more complicated than in the benchmark model, but the interpretation behind Lemma 5 is straightforward. For instance consider the case where  $\alpha < 1/2$ . When s is relatively low (below  $\tilde{s}(\alpha)$ ), non-shoppers randomize over where to shop, with some searching the multiproduct firm, and others searching both of the single-product firms. As s increases it becomes more attractive to search the multiproduct firm and avoid paying s. Therefore to ensure that non-shoppers are willing to randomize, the multiproduct firm's relative prices should increase, which is achieved by having more non-shoppers search it. However when s is sufficiently large (above  $\tilde{s}(\alpha)$ ), the multiproduct firm becomes so expensive that non-shoppers also find attractive the option of searching only one single-product firm. Therefore at this point some non-shoppers also search just one single-product firm. As s increases, fewer and fewer non-shoppers opt to search both single-product firms. Eventually s becomes so large (above  $\dot{s}(\alpha)$ ) that no non-shopper searches both single-product firms. At this point the equilibrium is exactly the same as in our earlier benchmark model.

Now consider the implications for equilibrium retail market structure:

**Proposition 2** The equilibrium market structure is asymmetric if and only if  $\alpha \geq \alpha^*(s)$ , where

$$\alpha^*(s) = \begin{cases} \frac{1+\ln(\frac{1}{2})}{3[1+\ln(\frac{1}{2})]-\frac{s}{v}} & \text{if } s \in (0, v \, [1-\ln 2]) \\ \frac{1}{2} & \text{if } s \ge v \, [1-\ln 2] \end{cases}$$

Otherwise the market has two multiproduct firms.

Qualitatively the market structure is the same as in the benchmark model. A first conglomerate merger is always profitable, because the merging firms soften competition and attract higher demand. Intuitively, the merged entity attracts a disproportionate share of non-shoppers, because it enables them to buy both products without needing to incur the additional cost s > 0. A second conglomerate merger is then profitable if and only if  $\alpha$  is sufficiently low. As in the benchmark model, a second merger has both a price and search order effect, with the latter dominating when there are relatively few shoppers in the market. Finally, we are also able to show that the asymmetric market structure is worst for consumers, but best for industry profit provided that  $\alpha$  is sufficiently large.

## 3.2 Allowing firms to choose product ranges

This section discusses an alternative approach to endogenize market structure. Instead of allowing single-product firms to merge before they engage in price competition, suppose that each firm in the market can choose whether to supply one product only or both products. Suppose there are three firms (A, B and C) in the market. This is the minimum number of firms needed to generate the asymmetric market structure with at least one multiproduct firm and one single-product firm for each product.<sup>15</sup> To make it profitable for each firm to stock at least one product, let us assume that the fixed cost of stocking one product is sufficiently low. For simplicity we normalize it to zero. Let  $\Delta > 0$  denote the incremental fixed cost of stocking the second product.<sup>16</sup> The model is otherwise the same as before. In particular, a fraction  $\alpha$  of consumers are shoppers who can visit any firm freely, and the remaining fraction  $1 - \alpha$  of consumers are non-shoppers who can visit only one firm.

We aim to derive conditions for an asymmetric market structure with one multiproduct firm and two single-product firms supplying different products. Without loss of generality, consider the hypothetical equilibrium where firm A supplies both products, firms B and C supply product 1 and product 2, respectively. Using Lemma 4, we can derive the profit outcome in this scenario: If  $\alpha \geq \frac{1}{2}$ , firm A earns profit  $2(1-\alpha)v - \Delta$ ,

<sup>&</sup>lt;sup>15</sup>The basic logic in the analysis of the three-firm case can be applied to the case with more firms, but considering more firms does not generate new insights.

<sup>&</sup>lt;sup>16</sup>Without the fixed cost, all firms will choose to supply both products. Notice that in the merger framework, when two firms choose to merge, each firm faces an opportunity cost which is the profit it could make if it remained as a single-product firm.

and firms B and C each earn  $\alpha(1-\alpha)v$ . If  $\alpha < \frac{1}{2}$ , firm A earns profit  $v - \Delta$ , and firms B and C each earn  $\frac{1}{4}v$ .

There are two possible deviations to consider: (i) Suppose a single-product firm, say, firm *B* unilaterally deviates and stocks both products. Then the market has two multiproduct firms *A* and *B* and a single-product firm *C* supplying product 2 only. As we show in the proof of the proposition below, in this scenario non-shoppers randomly visit one of the two multiproduct firms, and firm *B*'s profit becomes  $(1 - \alpha)v - \Delta$ . (ii) Suppose the multiproduct firm *A* unilaterally deviates and drops one product, say, product 2. Then the market has two single-product firms *A* and *B* supplying product 1 and one single-product firm *C* supplying product 2. This is a simple situation: Since firm *C* is the only supplier of product 2, it charges a monopoly price. Then non-shoppers will visit either firm *A* or *B*, and the price competition among *A* and *B* is a standard Varian model with two firms. Therefore, firm *A*'s deviation profit is  $\frac{1}{2}(1 - \alpha)v$  since it has half of the non-shoppers.

The following result reports the condition for the asymmetric market structure to be an equilibrium outcome.

**Proposition 3** It is an equilibrium that one firm supplies both products and the other two firms each supply a different product if

$$\alpha \ge \frac{1}{2}$$
 and  $(1-\alpha)^2 v \le \Delta \le \frac{3}{2}(1-\alpha)v$ ,

or if

$$\frac{1}{6} < \alpha < \frac{1}{2} \quad and \quad (\frac{3}{4} - \alpha)v \leq \Delta \leq \frac{1}{2}(1 + \alpha)v \ .$$

The asymmetric market structure we are interested in can arise only if  $\alpha > \frac{1}{6}$ . This is consistent with the observation from the merger model where the asymmetric market structure arises only if the fraction of non-shoppers is not too large. For a given  $\alpha > \frac{1}{6}$ , it further requires  $\Delta$  to be neither too small nor too big. When  $\Delta$  is too small, firms tends to stock both products, and when  $\Delta$  is too large, firms tends to stock one product only.

We are able to derive the conditions for every possible market structure. The details are lengthy, but the key observation is that when  $\alpha$  is given, the number of multiproduct firms tends to decrease as  $\Delta$  increases; when  $\Delta$  is given (and not too large), the number of multiproduct firms tends to increase as  $\alpha$  decreases (i.e., as the search friction increases). This is consistent with what we learned from the merger model.

#### **3.3** More firms and heterogeneous consumers

This section extends the basic model in two aspects: we consider n pairs of firms  $(1_j, 2_j)$ ,  $j = 1, \dots, n$ , and we also allow for the coexistence of both single-product and multiproduct consumers. Suppose each product is needed by a consumer with probability  $\gamma > 0$ . The purchase interest is independent across products, and also independent of whether the consumer is a shopper or non-shopper. Then a fraction  $\gamma^2$  of consumers want to buy both products, a fraction  $\gamma(1 - \gamma)$  need only product 1, a fraction  $\gamma(1 - \gamma)$  need only product 2, and the remainder have no demand for either product. The baseline model in section 2 corresponds to the case with n = 2 and  $\gamma = 1$ .

Compared to the baseline model, this extended model is less straightforward to analyze, mainly because the price competition in an asymmetric market structure with  $n \geq 3$  can become intricate. We report the equilibrium market structure in the following result and relegate all the details to the appendix.

**Proposition 4** Suppose a pair of single-product firms choose not to merge when they are indifferent.

(i) If  $\gamma \leq 1 - 1/n$ , the market has  $\lceil n\gamma \rceil$  multiproduct firms.

(ii) If  $\gamma > 1 - 1/n$ , the market has either n - 1 or n multiproduct firms. If  $\alpha$  is sufficiently large, there are n - 1 multiproduct firms, and if  $\alpha$  is sufficiently small, there are n multiproduct firms. When n = 2, the market has one multiproduct firm only if and only if  $\alpha \geq \frac{\gamma}{1+\gamma}$ .

When  $\gamma$  is relatively small (such that relatively many consumers are interested in only one product), an asymmetric market structure always arises. When  $\gamma$  is relatively large, at most one pair of single-product firms will remain in the market. Whether all firms merge or not depends on the search friction, in a way that is qualitatively the same as in the baseline model.<sup>17</sup>

<sup>&</sup>lt;sup>17</sup>With  $n \ge 3$ , a non-shopper's search problem becomes less tractable, and this makes it difficult to derive a cut-off result on  $\alpha$  as we do in the case with n = 2.

# 4 An Alternative Model with Differentiated Products

To make the baseline model tractable we assumed that some consumers can costlessly visit all firms, whilst others can visit only one firm. In this section we provide an alternative model where consumers can choose how many firms to visit, but visiting each firm is costly. As we pointed out earlier, however, this creates some complications when firms sell homogeneous products and there is more than one multiproduct firm in the market. Therefore in order to have a relatively tractable model, we introduce horizontal product differentiation. We argue that the basic insight about how the size of the search friction affects market structure carries over to this alternative model.

Consider a two-stage game as before, starting with four independent single-product firms. Suppose that firms  $i_A$  and  $i_B$ , i = 1, 2, supply two horizontally differentiated versions of product i at zero cost. Following Wolinsky (1986) and Anderson and Renault (1999), we use the random utility framework to model product differentiation. In particular the match utility of each product i is a random draw from a common distribution  $F_i(u)$  with support  $[\underline{u}, \overline{u}]$  and density  $f_i(u)$ . The realization of the match utility is i.i.d. across consumers and firms, as consistent for example with consumers having idiosyncratic tastes. As in the baseline model, we assume that the two products are symmetric (i.e.,  $F_1 = F_2 = F$ ). If a consumer buys a product with match utility uand pays price p, she obtains surplus u - p.

Initially consumers have imperfect information about each product's match utility and price, although they know the match utility distribution F(u) and also hold rational expectations about each firm's pricing strategy. To reach a firm and learn its product match utility and price, a consumer has to incur a search cost s > 0. The search process is sequential. To capture the idea of one-stop shopping convenience, we assume that the search cost is the same whether a consumer visits a single-product or a multiproduct firm. To have active search in each possible market structure, we assume that the search cost is not too high:

$$s < \int_{\underline{u}}^{\overline{u}} (u - \underline{u}) dF(u) .$$
(12)

This condition guarantees that at least some consumers will search actively even in the single-product case. For simplicity, we make two more assumptions which are often adopted in the literature: (i) Consumers have free recall. That is, they can return to a

previously visited firm without paying any extra cost. (ii) The market is fully covered. That is, all consumers buy both products in equilibrium. (This is the case, for example, when consumers have an additional high basic valuation for each product.)

In the following, we first derive the pricing equilibrium in each possible market structure, and then examine the equilibrium market structure.

A market with four single-product firms. With four single-product firms, a consumer's search process is separable across the two product markets. In each market, we have a duopoly version of the sequential search model in Anderson and Renault (1999). Consider the market for product *i*. We look for a symmetric equilibrium where both firms charge the same price  $p_0$  and consumers search in a random order (i.e., half of the consumers visit firm  $i_A$  first and the other half visit firm  $i_B$  first). In the symmetric equilibrium, the optimal stopping rule is characterized by a reservation utility level *a* which solves

$$\int_{a}^{\overline{u}} (u-a)dF(u) = s .$$
(13)

(The left-hand side is the expected benefit from sampling the second firm when the first firm offers match utility a.) This equation has a unique solution  $a \in (0, \overline{u})$  given the search cost condition (12). In equilibrium a consumer will buy immediately at the first visited firm if and only if its match utility is no less than a.

As explained in the appendix, the first-order condition for  $p_0$  to be the equilibrium price is<sup>18</sup>

$$\frac{1}{p_0} = f(a)[1 - F(a)] + 2\int_{\underline{u}}^a f(u)^2 du \; .$$

In equilibrium, firms share the market equally, so each firm earns profit  $\Pi_0 = \frac{1}{2}p_0$ . In the uniform distribution example with F(u) = u, we have  $a = 1 - \sqrt{2s}$  and condition (12) requires  $s < \frac{1}{2}$ . The first-order condition then implies that

$$p_0 = \frac{1}{2 - \sqrt{2s}} \ . \tag{14}$$

It is depicted as the dashed curve in Figure 2a below.

A market with two multiproduct firms. With two multiproduct firms, we have a multiproduct search model as analyzed in Zhou (2014). Let  $p_m$  denote the equilibrium

<sup>&</sup>lt;sup>18</sup>We can show that if p[1 - F(p)] is concave, then the first-order condition is also sufficient for defining the equilibrium price. See Appendix B in Anderson and Renault (1999) for other conditions which ensure the existence of a symmetric pure-strategy pricing equilibrium.

price for each product. We first report the optimal stopping rule in equilibrium where both firms charge the same prices. Consider a consumer who visits firm A first. After visiting firm A she faces the following options: stop searching and buy both products, or buy one product and keep searching for the other, or keep searching for both products. Given that the search cost occurs at the firm level and consumers have free recall, the second option is always dominated by the third. If the consumer continues to visit firm B, she can thereafter freely mix and match among the two firms. Therefore the consumer will stop searching and buy both products at firm A if the match utilities  $(u_{1A}, u_{2A})$  satisfy

$$\int_{u_{1A}}^{\overline{u}} (u_{1B} - u_{1A}) dF(u_{1B}) + \int_{u_{2A}}^{\overline{u}} (u_{2B} - u_{2A}) dF(u_{2B}) \le s \; .$$

(The left-hand side is the expected benefit from sampling firm B.) The equality of this condition defines a reservation frontier  $u_{2A} = \phi(u_{1A})$ , where  $\phi(\cdot)$  is a decreasing and convex function. If the match utilities  $(u_{1A}, u_{2A})$  at firm A are such that  $u_{2A} \ge \phi(u_{1A})$ , the consumer will stop searching. Otherwise she will continue to visit firm B.

We refer the reader to Zhou (2014) for details of how to derive the equilibrium price. For a general distribution, the first-order condition for  $p_m$  is<sup>19</sup>

$$\frac{1}{p_m} = \int_a^{\overline{u}} [1 - F(\phi(u))] f(\phi(u)) dF(u) + \int_a^{\overline{u}} [1 - F(u)] f(\phi(u)) dF(u) + 2 \int_{u_2 \le \phi(u_1)} f(u_1)^2 f(u_2) d\mathbf{u} + \int_a^{\overline{u}} [1 - F(u)] f(\phi(u)) dF(u) dF(u) + 2 \int_{u_2 \le \phi(u_1)} f(u_1)^2 f(u_2) d\mathbf{u} + \int_a^{\overline{u}} [1 - F(u)] f(\phi(u)) dF(u) dF(u)$$

where a is defined in (13). In equilibrium firms share the market equally, so each firm's per product profit is  $\Pi_m = \frac{1}{2}p_m$ . In the uniform distribution example, the first-order condition implies that

$$p_m = \frac{1}{2 - (\frac{1}{2}\pi - 1)s} , \qquad (15)$$

where  $\pi \approx 3.14$  is the mathematical constant. It is depicted as the lowest solid curve in Figure 2a below.

As proved in Zhou (2014),  $p_m < p_0$ , i.e., products become cheaper when singleproduct firms merge into two multiproduct firms. This differs from the result in the basic model in section 2, and arises due to the following joint-search effect. Intuitively when a firm reduces one product's price, more consumers who visit it first will stop

<sup>&</sup>lt;sup>19</sup>As explained in Zhou (2014), in general it is hard to derive a simple sufficient condition for the existence of a symmetric pure-strategy equilibrium. But for many common distributions (including the uniform distribution), the first-order condition is sufficient for defining the equilibrium price.

searching and buy both products. That is, reducing one product's price can increase the demand for the other product as well. Hence the two products behave like complements, inducing each firm to price more aggressively. In the basic model in section 2, no consumers have a real search decision to make and so this joint-search effect does not arise.

An asymmetric market. Consider the asymmetric case with a multiproduct firm A and two single-product firms  $1_B$  and  $2_B$ . Let  $p_A$  be the multiproduct firm's price and  $p_B$  be each single-product firm's price. We look for an equilibrium where all consumers visit the multiproduct firm first. Notice that the search cost of visiting each single-product firm is separable. Then a consumer's search decision when she is at the multiproduct firm is also separable between the two products: she will continue to visit the single-product firm  $i_B$  if and only if the multiproduct firm's product i has a surplus less than  $a-p_B$ , where a is defined in (13). Therefore unlike the case with two multiproduct firms, there is no joint-search effect here. The multiproduct firm competes with its smaller rivals in two separate single-product markets where consumers search non-randomly. (As such, the pricing problem is similar to the one studied by Armstrong, Vickers, and Zhou, 2009, where one firm is prominent and always visited first by consumers.)

As explained in the appendix, the first-order conditions for the equilibrium prices  $(p_A, p_B)$  are<sup>20</sup>

$$p_A = \frac{Q(\Delta)}{Q'(\Delta)} \tag{16}$$

and

$$p_B = \frac{1 - Q(\Delta)}{Q'(\Delta) - [1 - F(a)]f(a - \Delta)} , \qquad (17)$$

where  $\Delta \equiv p_B - p_A$ , and  $Q(\Delta) \equiv 1 - \int_{\underline{u}}^{a-\Delta} [1 - F(u + \Delta)] dF(u)$  is the equilibrium demand for firm A's product *i*. (Then  $1 - Q(\Delta)$  is the equilibrium demand for firm  $i_B$ .) Firm A's per product profit is  $\Pi_A = p_A Q(\Delta)$  and each single-product firm's profit is  $\Pi_B = p_B(1 - Q(\Delta))$ . This analysis implicitly assumes that all consumers visit firm A first and that  $a - \Delta > \underline{u}$ . The following result provides a condition for the system of (16) and (17) to have a solution  $\Delta \in (0, a - \underline{u})$ . With  $\Delta > 0$  i.e.  $p_A < p_B$ , the consumer search order is indeed optimal, because the multiproduct firm both offers lower prices and provides one-stop shopping convenience.<sup>21</sup>

<sup>&</sup>lt;sup>20</sup>Similar as in the case with four single-product firms, the first-order conditions are also sufficient for defining the equilibrium prices if p[1 - F(p)] is concave.

<sup>&</sup>lt;sup>21</sup>Armstrong, Vickers, and Zhou (2009) show a similar result without assuming full market coverage,

**Lemma 6** Suppose 1 - F is strictly logconcave and condition (12) holds. Then the system of (16) and (17) has a solution  $\Delta \in (0, a - \underline{u})$ .

Therefore, under the regularity condition there is an equilibrium in this asymmetric market where the multiproduct firm is cheaper than its single-product rivals and all consumers visit the multiproduct firm first. The prediction that  $p_A < p_B$  is different to what we observed in the basic model with homogenous products. Here, a consumer will visit a single-product firm only if she is unsatisfied with the multiproduct firm's product. Therefore when a consumer searches a single-product firm, she reveals something about her preferences. This gives the single-product firm an extra monopoly power and induces it to charge a higher price.<sup>22</sup> Nevertheless both  $p_A$  and  $p_B$  tend to be higher than  $p_0$  and  $p_m$  as we will see below. This is similar to the basic model, where price competition was typically softest when the market structure was asymmetric.

In the uniform distribution example, the system of (16) and (17) simplifies to

$$p_A = \frac{1}{1 - \Delta} [1 - a + \Delta + \frac{1}{2} (a^2 - \Delta^2)], \quad p_B = 1 - \frac{1}{2} (a - \Delta) ,$$

where  $a = 1 - \sqrt{2s}$ . It has a unique solution:

$$p_A = \frac{1}{16}(3K - 5a - 5)$$
,  $p_B = \frac{1}{16}(K - 7a + 9)$ ,

where  $K \equiv \sqrt{17a^2 - 30a + 49}$ . They are respectively depicted as the second highest and the highest solid curve in Figure 2a below.

Equilibrium market structure. For a general match utility distribution, it is hard to compare profits and study the equilibrium market structure. To make progress, we focus on the uniform distribution case with F(u) = u. It can be verified that  $p_A > p_0$ for any  $s \in (0, \frac{1}{2})$  and so the four prices can be ranked as  $p_m < p_0 < p_A < p_B$ . (They are depicted in Figure 2a below.) As in the basic model with homogenous products, the asymmetric market structure generates the highest market prices.

but they focus on the uniform distribution case.

<sup>&</sup>lt;sup>22</sup>Following this argument, one may conjecture another possible equilibrium in which consumers visit the two single-product firms first and they charge lower prices than the multiproduct firm. In order for this to be an equilibrium, the price difference has to be large enough to compensate consumers for the extra search cost incurred by visiting single-product firms first. It is analytically difficult to exclude this possibility in general, but in the uniform distribution example it can be ruled out.

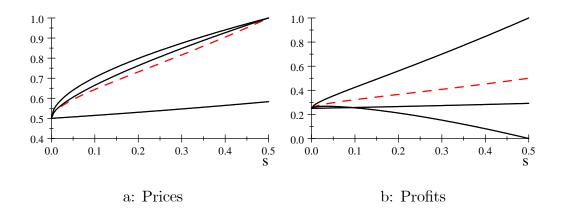


Figure 2: Price and profit comparison with differentiated products

Figure 2b compares per product profit across market structures. The dashed curve is  $\Pi_0$  (each firm's profit in the case with four single-product firms), the middle (almost horizontal) solid curve is  $\Pi_m$  (each firm's per product profit in the case with two multiproduct firms), and the other high and low curves are respectively  $\Pi_A$  and  $\Pi_B$ (the multiproduct firm's per product profit and each single-product firm's profit in the asymmetric case). A few observations follow: (i)  $\Pi_A > \Pi_0$ , so starting from the initial situation with four single-product firms, each pair of firms have a unilateral incentive to merge. (ii)  $\Pi_B > \Pi_m$  if and only if s is less than about 0.092. We can then deduce that if s is less than 0.092, the (pure-strategy) equilibrium outcome is an asymmetric market with a multiproduct firm and two single-product firms. Numerical simulations also show that industry profit is higher in this asymmetric case than in the initial situation with four single-product firms. On the other hand, if s is greater than 0.092, the market has two multiproduct firms. In this case, industry profit is lower than in the initial situation and so firms end up in a prisoner's dilemma.

Therefore, in this uniform distribution case, the search friction affects market structure in much the same way as it did in the basic model with homogeneous products. Again, there is a trade-off between the search order effect and the price competition effect. In particular when there is already a multiproduct retailer in the market, a merger between the remaining single-product firms (i) makes them more prominent in consumers' search order, boosting their demand, but (ii) also intensifies price competition. The latter effect dominates when s is small, whilst the former effect dominates otherwise.

In the uniform distribution case, one can also investigate how the magnitude of the

search cost affects consumer surplus and total welfare. The outcome is qualitatively similar as in Figure 1(b): due to the endogeneity of the market structure, reducing the search cost does not always improve consumers surplus and total welfare.

# 5 Conclusion

The paper has analyzed equilibrium market structure, within a model of consumer search where firms are able to engage in conglomerate mergers. The model permits a simple and tractable analysis of multiproduct competition, which we believe will be useful in other applications. Amongst other results, we have shown that the size of the search friction plays a key role in determining whether the equilibrium market structure is symmetric or not. In particular when search frictions are relatively low, some single-product firms choose to remain unmerged, in order to weaken the amount of price competition in the market. In equilibrium, larger retailers earn more profit per-product, and thus benefit from their greater size. This is due to their ability to offer one-stop shopping convenience, which expands their pool of potential customers. An asymmetric market structure delivers the weakest price competition, and as such minimizes consumer surplus and often maximizes industry profit. Consequently our model challenges the often benign view of conglomerate merger. Of course the model neglects an important practical consideration - namely that conglomerate mergers may be costly to propose, but could also generate synergies and therefore long-term cost savings. Nevertheless introducing this into the model would not change the main qualitative insights.

We believe that the current framework opens up several promising directions for future research. One such avenue is the question of store location choice. Consider a model in which single-product firms can choose to locate at any one of a number of retail outlets e.g. shopping malls. Some consumers have time to visit all locations, whilst others are time-constrained and may visit only one. This set-up is isomorphic to our baseline model – and hence our existing results imply that when the search friction is small, a mixture of shopping malls and isolated firms will co-exist in the market. Another interesting avenue for future research would be to use our current model to investigate when and how conglomerate mergers may deter entry into a market. Finally, and more broadly, it would also be interesting to think about market structure not as the result of conglomerate mergers, but as the result of firms optimally choosing the size and contents of their product line. We intend to think more about these possibilities in future work.

#### **Appendix: Omitted Proofs and Details**

**Proof of Lemma 1**: These results can be found in the existing literature. We are providing proofs here for completeness.

(i) The proof is standard and so omitted.

(ii) We first verify that this is an equilibrium. Since the other firm is using the atomless price distribution F, a firm's profit at p < v is p[N + S(1 - F(p))], whilst its profit at p = v is Nv. The expression for F in (1) equalizes these two profits, such that each firm is indifferent among all prices in  $[\underline{p}, v]$ , where the lower bound  $\underline{p}$  in (2) is derived from  $F(\underline{p}) = 0$ . It is also clear that neither firm has an incentive to charge a price below  $\underline{p}$ . Varian (1980) proves there are no other symmetric equilibria, whilst Baye, Kovenock, and de Vries (1992) prove there are no asymmetric equilibria either.

(iii) Again we begin by verifying that this is an equilibrium. Consider firm A first. Given that firm B is using the equilibrium strategy  $F_B$ , A's profit at p < v is  $p[N_A + S(1 - F_B(p))]$ , whilst its profit at p = v is  $N_A v$ . The expression for  $F_B$  in (6) equalizes these two profits. The lower bound of the support  $\underline{p}$  in (4) is derived from  $F_B(\underline{p}) = 0$ . Firm A is then indifferent among all prices between  $\underline{p}$  and v, and also has no incentive to charge a price below p.

Now consider firm B. Given that firm A is using the equilibrium strategy  $F_A$ , B's profit at p < v is  $p[N_B + S(1 - F_A(p))]$ . When p converges to v from below, B's profit converges to  $v[N_B + S\lambda]$  since  $F_A$  has a mass point of size  $\lambda$  at p = v. The expression for  $F_A$  in (3) equalizes these two profits. Given the mass point of  $F_A$ , firm B never wants to charge a price exactly at p = v because it is dominated by a price slightly below v. Hence the support of  $F_B$  is open at v.  $\lambda$  in (5) is derived from  $F_A(\underline{p}) = 0$ . Then firm B has no incentive to charge a price below  $\underline{p}$  either. Baye, Kovenock, and de Vries (1992) establish uniqueness of this equilibrium (ours is a special case of their section V).

**Proof of Proposition 4.** The number of non-shoppers who need both products is  $N_b = (1 - \alpha)\gamma^2$ , and the number of non-shoppers who need only product i, i = 1, 2, is  $N_i = (1 - \alpha)\gamma(1 - \gamma)$ . The number of shoppers for each product is  $S = \alpha\gamma$ . Henceforth we call these two types of non-shoppers  $N_b$  and  $N_i$  respectively. Denote by  $\pi_m(k)$ ,

 $k \ge 1$ , a multiproduct firm's per product profit when there are k multiproduct firms in the market, and by  $\pi_s(k)$ ,  $k \le n - 1$ , a single-product firm's profit when there are k multiproduct firms in the market.

We first consider the simple case where no firms merge or all firms merge.

**Claim 1** With all single-product firms (k = 0) or all multiproduct firms (k = n), non-shoppers search randomly and each firm's (per product) profit is respectively

$$\pi_s(0) = \frac{1-\alpha}{2n} \gamma \left(2-\gamma\right) v \quad and \quad \pi_m(n) = \frac{1-\alpha}{n} \gamma v \;.$$

**Proof.** With all single-product firms, multiproduct non-shoppers visit a firm randomly and single-product non-shoppers visit a relevant firm randomly,<sup>23</sup> so the number of non-shoppers each firm has is

$$\frac{N_b}{2n} + \frac{N_i}{n} = \frac{1-\alpha}{2n}\gamma\left(2-\gamma\right)$$

This determines each firm's profit  $\pi_s(0)$ .

With all multi-product firms, all non-shoppers visit a firm randomly, and so the number of non-shoppers each firm has at the product level is

$$\frac{N_b}{n} + \frac{N_i}{n} = \frac{1-\alpha}{n}\gamma \;.$$

This determines each firm's per product profit  $\pi_m(n)$ .

We now turn to an asymmetric market structure with  $1 \le k \le n-2$  such that at least two pairs of single-product firms remain. (The asymmetric case with k = n - 1will be treated separately.) We need the following two results from Baye, Kovenock, and de Vries (1992).

Claim 2 (Asymmetric Varian Model) Consider a Varian pricing game where n firms supply a homogenous product and consumers have identical valuations v. Suppose there are S > 0 shoppers in the market. Suppose each firm  $j \in \{1, \dots, l\}$  has  $N_A > 0$ non-shoppers and each firm  $j \in \{l + 1, \dots, n\}$  has  $N_B < N_A$  non-shoppers. Let  $F_A$  be the price distribution used by the first l firms, and let  $F_B$  be the price distribution used by the remaining n - l firms. Then  $F_A$  FOSD  $F_B$ , and if  $l \leq n - 2$  (i.e., if the second group has at least two firms),  $F_A$  degenerates at the monopoly price v.

<sup>&</sup>lt;sup>23</sup>It is impossible that non-shoppers search non-randomly. If they did, one firm would have more nonshoppers than another, such that it would charge higher prices and thereby contradict non-shoppers' search behavior.

See Appendix B in Baye, Kovenock, and de Vries (1992) for the proof.

Claim 3 (Asymmetric Equilibrium in Symmetric Varian Model) In the Varian pricing game described in Claim 2 with  $n \ge 3$ , if all firms equally share the nonshoppers (i.e., if  $N_A = N_B$ ), then as well as the standard symmetric equilibrium, there exist asymmetric equilibria where a group of firms  $j \in \{1, \dots, l\}$ ,  $l \le n - 2$ , adopt a price distribution  $F_A$  with support  $[\underline{p}, r] \cup \{v\}$  where r < v, and the rest of the firms  $j \in \{l+1, \dots, n\}$  adopt an atomless price distribution  $F_B$  with support  $[\underline{p}, v]$ . Moreover,  $F_A$  FOSD  $F_B$ , and  $F_A = F_B$  for  $p \in [p, r]$ .

This result is from Theorem 1 in Baye, Kovenock, and de Vries (1992).

The following result reports the equilibrium outcome when  $1 \le k \le n-2$ .

**Claim 4** When  $1 \le k \le n-2$ ,  $N_b$  visit multiproduct firms and  $N_i$  randomize if  $\gamma < \frac{k}{n}$ , and  $N_b$  randomize and  $N_i$  visit single-product firms if  $\gamma > \frac{k}{n}$ . All firms have the same per product profit:

$$\pi_m(k) = \pi_s(k) = \begin{cases} \frac{1-\alpha}{n}\gamma v & \text{if } \gamma < \frac{k}{n} \\ \frac{1-\alpha}{2n-k}\gamma(2-\gamma)v & \text{if } \gamma > \frac{k}{n} \end{cases}.$$

**Proof.** We first exclude the possibility that all non-shoppers (i.e., both  $N_b$  and  $N_i$ ) search in a deterministic way or in a random way. The proof consists of four steps. (i) It is impossible that all non-shoppers visit multiproduct firms. If that were the case, the single-product firms would sell only to shoppers and so set price equal to zero. The multiproduct firms would sell only to non-shoppers and so charge v. This would contradict the optimality of non-shoppers' search behavior.

(ii) It is also impossible that all non-shoppers visit single-product firms. If that were the case, single-product firms would charge higher prices than multiproduct firms in the sense of FOSD. (If  $k \ge 2$ , single-product firms would actually charge the monopoly price.) But this again would render non-shoppers' search behavior non-optimal.

(iii) It is also impossible that all  $N_b$  visit multiproduct firms and all  $N_i$  visit singleproduct firms supplying product *i* (except in the edge case  $\frac{N_b}{k} = \frac{N_i}{n-k}$  which we ignore). If that were the case, each multiproduct firm would have  $\frac{N_b}{k}$  non-shoppers per product, and each single-product firm would have  $\frac{N_i}{n-k}$  non-shoppers. If  $\frac{N_b}{k} < \frac{N_i}{n-k}$ , then according to Claim 2 single-product firms would charge higher prices than multiproduct firms. But then  $N_i$ 's search behavior could not be justified. If  $\frac{N_b}{k} > \frac{N_i}{n-k}$ , then according to Claim 2 multiproduct firms would charge the monopoly price v since  $k \leq n-2$ . But then  $N_b$ 's search behavior could not be justified.

(iv) It is also impossible that both  $N_b$  and  $N_i$  randomize their search behavior.  $N_i$  would randomize only if multiproduct firms and single-product firms supplying product i provide the same expected consumer surplus from purchasing their product i. But then  $N_b$  would favor visiting a multiproduct firm.

As a result, either  $N_b$  or  $N_i$  randomize in equilibrium. First, consider an equilibrium where  $N_i$  randomize and  $N_b$  visit multiproduct firms. Let X be the probability that  $N_i$ visits a multiproduct firm. Then a single-product firm has  $\frac{(1-X)N_i}{n-k}$  non-shoppers, and a multiproduct firm has  $\frac{XN_i+N_b}{k}$  non-shoppers per product. They must be equal to each other, otherwise using Claim 2 one type of firm charges higher prices than the other, and then  $N_i$ 's search behavior could not be justified. Therefore,

$$\frac{(1-X)N_i}{n-k} = \frac{XN_i + N_b}{k} \Leftrightarrow X = \frac{k/n - \gamma}{1 - \gamma} \; .$$

which is only positive if  $\gamma < \frac{k}{n}$ . In this case, one can verify that the number of non-shoppers each firm has at the product level is  $\frac{1-\alpha}{n}\gamma$ . This implies the profit outcome.

Second, consider an equilibrium where  $N_b$  randomize and  $N_i$  visit single-product firms. Let X be the probability that  $N_b$  visits a multiproduct firm. Then a multiproduct firm has  $\frac{XN_b}{k}$  non-shoppers per product, and a single-product firm has  $\frac{(1-X)N_b}{2(n-k)} + \frac{N_i}{n-k}$  nonshoppers. (Notice that those  $N_b$  who visit single-product firms will randomly choose one among 2(n - k) of them.) Again, they must be equal to each other, otherwise using Claim 2 either multiproduct firms would charge the monopoly price (which would contradict  $N_b$ 's search behavior), or single-product firms would charge higher prices (which would contradict  $N_i$ 's search behavior). Therefore,

$$\frac{XN_b}{k} = \frac{(1-X)N_b}{2(n-k)} + \frac{N_i}{n-k} \Leftrightarrow X = \frac{2-\gamma}{2n\gamma/k-\gamma} \; .$$

which is only less than 1 if  $\gamma > \frac{k}{n}$ . In this case, one can verify that the number of non-shoppers each firm has at the product level is  $\frac{1-\alpha}{2n-k}\gamma(2-\gamma)$ . This implies the profit outcome.<sup>24</sup>

<sup>&</sup>lt;sup>24</sup>However note that the pricing equilibrium is not symmetric. To justify  $N_b$ 's search behavior, we need an asymmetric pricing equilibrium where multiproduct firms charge higher prices than singleproduct firms. Given  $k \leq n-2$ , this is possible according to Claim 3. In this equilibrium each firm's per product profit is  $\frac{1-\alpha}{2n-k}\gamma(2-\gamma)v$ . Details of the equilibrium characterization are available upon request.

Finally, we study the case with k = n - 1 such that only one pair of single-product firms remain in the market. By a similar logic as in the proof of Claim 4, one can show that in this case it is impossible that all non-shoppers (i.e., both  $N_b$  and  $N_i$ ) visit multiproduct firms or single-product firms, and it is also impossible that all nonshoppers search in a random way. The only difference compared to the case with  $k \leq n-$ 2 is that now it is possible that  $N_b$  visit multiproduct firms and  $N_i$  visit single-product firms supplying product *i*. This is because given there is only one pair of single-product firms, with this configuration of non-shoppers' search behavior multiproduct firms will no longer charge the monopoly price, and so  $N_b$ 's search behavior can potentially be justified. We now have three possible types of equilibrium to consider.

Before we proceed, it is useful to first study an asymmetric Varian model where each of the first n - 1 firms has  $N_A$  non-shoppers and the last firm has  $N_B < N_A$ non-shoppers. (This is the case with l = n - 1 in Claim 2.) Let  $F_A$  and  $F_B$  be the price distributions used by the two types of firms respectively.  $F_A$  has a mass point at v, and let  $\lambda$  be its size. Then the two indifference conditions are:

$$p[N_A + S(1 - F_A)^{n-2}(1 - F_B)] = vN_A$$
,

and

$$p[N_B + S(1 - F_A)^{n-1}] = v(N_B + S\lambda^{n-1})$$

From the first condition, we can derive the common lower bound of the two price distributions:

$$\underline{p} = \frac{N_A}{S + N_A} v \; .$$

Substituting this into the second condition yields

$$\lambda^{n-1} = \frac{N_A - N_B}{S + N_A} \; .$$

Then the profit outcome is

$$\pi_A = vN_A \quad \text{and} \quad \pi_B = vN_A \frac{S+N_B}{S+N_A}$$
 (18)

The following result reports the market outcome when k = n - 1.

**Claim 5** (i) When k = n - 1 and  $\gamma \leq 1 - \frac{1}{n}$ ,  $N_b$  visit multiproduct firms and  $N_i$  randomize, and each firm has the same per product profit

$$\pi_m(n-1) = \pi_s(n-1) = \frac{1-\alpha}{n} \gamma v \; .$$

(ii) When k = n - 1 and  $\gamma > 1 - \frac{1}{n}$ , either  $N_b$  visit multiproduct firms and  $N_i$  visit single-product firms, in which case the profit outcome is

$$\pi_m(n-1) = \frac{1-\alpha}{n-1}\gamma^2 v \quad and \quad \pi_s(n-1) = \frac{\gamma^2 [1-(1-\alpha)\gamma]}{\gamma+(n-1)\frac{\alpha}{1-\alpha}} v ,$$
(19)

or  $N_b$  randomize and  $N_i$  visit single-product firms, in which case the profit outcome is

$$\pi_m(n-1) = X \frac{1-\alpha}{n-1} \gamma^2 v \quad and \quad \pi_s(n-1) = \frac{X \gamma^2 \left[1 - \frac{1+X}{2} (1-\alpha)\gamma\right]}{X \gamma + (n-1) \frac{\alpha}{1-\alpha}} v , \qquad (20)$$

where  $X \in (\frac{n-1}{n+1}\frac{2-\gamma}{\gamma}, 1)$  is the probability that a multiproduct non-shopper visits a multiproduct firm.

**Proof.** (i) For  $N_b$  to visit multiproduct firms and  $N_i$  to randomize, all firms must have the same number of non-shoppers at the product level, otherwise  $N_i$ 's search behavior could not be justified. This can happen only if  $\frac{N_b}{n-1} \leq N_i$ , or  $\gamma \leq 1 - \frac{1}{n}$ . Let X be the probability that  $N_i$  visits a multiproduct firm. Then we need

$$\frac{N_b + XN_i}{n-1} = (1-X)N_i \ .$$

From this one can solve  $X \in (0, 1)$  and verify that each firm has  $\frac{1}{n}(1-\alpha)\gamma$  non-shoppers per product. This implies the profit result.

(ii) First, consider an equilibrium where  $N_b$  visit multiproduct firms and  $N_i$  visit single-product firms. In this case,  $\gamma > 1 - \frac{1}{n}$  implies that a multiproduct firm has more non-shoppers at the product level than a single-product firm (i.e.,  $\frac{N_b}{n-1} > N_i$ ). Let  $F_s$  be the price distribution used by a single-product firm, and let  $F_m$  be the price distribution used by a multiproduct firm. Then the above general analysis of the asymmetric Varian model applies with  $N_A = \frac{N_b}{n-1}$ ,  $N_B = N_i$ ,  $F_A = F_m$ , and  $F_B = F_s$ , since  $N_A > N_B$ . Then the profit outcome (19) is from (18). For this equilibrium to be sustained, we need

$$2\int_{\underline{p}}^{v} (v-p) dF_m(p) \ge \int_{\underline{p}}^{v} (v-p) dF_s(p) .$$

$$(21)$$

Second, consider an equilibrium where  $N_b$  randomize and  $N_i$  visit single-product firms. This equilibrium can happen only if multiproduct firms charge higher prices. This requires that each multiproduct firm has more non-shoppers per product. Let Xbe the probability that a multiproduct non-shopper visits a multiproduct firm. Then the above general analysis applies with  $N_A = \frac{XN_b}{n-1}$ ,  $N_B = N_i + \frac{1}{2}(1-X)N_b$ ,  $F_A = F_m$ , and  $F_B = F_s$ , if

$$N_A > N_B \Leftrightarrow X > \frac{n-1}{n+1} \frac{2-\gamma}{\gamma}$$
.

Then the profit outcome (20) is from (18). For  $N_b$  to randomize, we need

$$2\int_{\underline{p}}^{v} (v-p) \, dF_m(p) = \int_{\underline{p}}^{v} (v-p) \, dF_s(p) \quad .$$
(22)

This determines X.

Due to the complication of the price distributions, in general it is difficult to find the precise primitive conditions for (21) or (22) to hold. (But one of them must hold when  $\gamma > 1 - \frac{1}{n}$ .) However in the case with n = 2, one can check that (21) holds if  $\alpha \ge \frac{3\gamma-2}{3\gamma-1}$  and (22) holds if  $\alpha < \frac{3\gamma-2}{3\gamma-1}$ . In the latter case, (22) implies  $X = \frac{1+(1-\alpha)(1-\gamma)}{2(1-\alpha)\gamma}$ .

When  $\gamma \leq 1 - \frac{1}{n}$ , the profit outcome is fully characterized in each possible market structure. By using Claims 1, 4, and 5, one can readily verify that the market has  $\lceil n\gamma \rceil$  multiproduct firms.

When  $\gamma > 1 - \frac{1}{n}$ , the situation is more complicated. But we can first claim that there will be at least n - 1 multiproduct firms in the market. With  $\gamma > 1 - \frac{1}{n}$ , we must have  $\gamma > \frac{k}{n}$  for  $1 \le k \le n - 2$ . Then according to Claim 4, when  $1 \le k \le n - 2$ , each firm's per product profit is  $\pi_m(k) = \pi_s(k) = \frac{1-\alpha}{2n-k}\gamma(2-\gamma)v$ . This is increasing in k. Hence, it suffices to show that  $\pi_s(n-2) < \pi_m(n-1)$ . If the profit outcome (19) applies when k = n - 1, this condition becomes

$$\frac{1-\alpha}{n+2}\gamma(2-\gamma) < \frac{1-\alpha}{n-1}\gamma^2 \Leftrightarrow \gamma > \frac{2(n-1)}{2n+1} \ .$$

This is guaranteed by  $\gamma > 1 - \frac{1}{n}$ . If the profit outcome (20) applies when k = n - 1, the condition becomes

$$\frac{1-\alpha}{n+2}\gamma(2-\gamma) < X\frac{1-\alpha}{n-1}\gamma^2 \Leftrightarrow \frac{2-\gamma}{n+2} < X\frac{\gamma}{n-1} \ .$$

This is guaranteed by  $X > \frac{n-1}{n+1} \frac{2-\gamma}{\gamma}$ .

To determine whether all single-product firms will merge, we need to compare  $\pi_s(n-1)$  with  $\pi_m(n)$ . If the profit outcome (19) applies, then one can check that  $\pi_s(n-1) < \pi_m(n)$  iff  $\alpha < \frac{\gamma}{1+\gamma}$ . If the profit outcome (20) applies, we do not have a clear comparison between  $\pi_s(n-1)$  and  $\pi_m(n)$ , because the X in  $\pi_s(n-1)$  cannot be explicitly solved from (22) (except for the case with n = 2 as we will discuss below). To make progress, we consider two extreme cases with  $\alpha \approx 1$  or  $\alpha \approx 0$ .

When  $\alpha \approx 1$  (i.e., when almost all consumers are shoppers), both price distributions  $F_m$  and  $F_s$  should almost degenerate around zero. Therefore (21) must hold, i.e., the

profit outcome (19) applies. Since now  $\alpha > \frac{\gamma}{1+\gamma}$ , we should have  $\pi_s(n-1) > \pi_m(n)$ . Therefore, the last pair of single-product firms will choose not to merge.

When  $\alpha \approx 0$  (i.e., when almost all consumers are non-shoppers), both price distributions  $F_m$  and  $F_s$  should almost degenerate at the monopoly price. Then it is not clear whether (21) holds or not. If the profit outcome (19) applies, we already know that  $\pi_s(n-1) < \pi_m(n)$  for  $\alpha < \frac{\gamma}{1+\gamma}$ . If the profit outcome (20) applies, without solving X we can also show  $\pi_s(n-1) < \pi_m(n)$ . This is because when  $\alpha \approx 0$ ,

$$\pi_s(n-1) < \pi_m(n) \Leftrightarrow 1 - \frac{1+X}{2}\gamma < \frac{1}{n} .$$

This condition is satisfied given  $\gamma > 1 - \frac{1}{n}$  and  $X > \frac{n-1}{n+1}\frac{2-\gamma}{\gamma}$ . Therefore, no matter which profit outcome applies, the last pair of single-product firms will choose to merge.

Finally, let us consider the case with n = 2. In this case as we have pointed out everything can be solved explicitly. It is then straightforward to check that  $\pi_s(n-1) \ge \pi_m(n)$  (i.e., the last pair of single-product firms will not choose to merge) iff  $\alpha \ge \frac{\gamma}{1+\gamma}$ . This completes the proof.

**Proof of Lemma 5:** As in the benchmark model, the multiproduct firm must attract a positive mass of non-shoppers, whilst each single-product firm must attract the same number of non-shoppers. Let  $f_A(p)$  denote the multiproduct firm's price distribution, and  $\lambda \ge 0$  denote its mass point. Let  $f_B(p) = f_A(p) / (1 - \lambda)$  denote a single-product firm's price distribution. A non-shopper can either search the multiproduct firm, two single-product firms, or one single-product firm, which yield respective payoffs of:

$$2(1-\lambda)\int_{\underline{p}}^{v} f_B(p)(v-p)\,dp \tag{23}$$

$$2\int_{\underline{p}}^{v} f_B(p)(v-p)\,dp-s\tag{24}$$

$$\int_{\underline{p}}^{v} f_B(p) \left(v - p\right) dp \tag{25}$$

Step 1. Look for an equilibrium where all non-shoppers search the multiproduct firm. Then  $N_A = (1 - \alpha)$  and  $N_B = 0$  such that using Lemma 1, (23) exceeds (25) if  $\alpha \ge 1/2$ , and (23) exceeds (24) if  $s \ge \bar{s}(\alpha)$  where

$$\bar{s}(\alpha) = 2(1-\alpha)v\left[1 + \left(\frac{1-\alpha}{\alpha}\right)\ln(1-\alpha)\right]$$

Step 2. Look for an equilibrium where  $X \in (0, 1]$  non-shoppers search the multiproduct firm, and the other 1 - X search both single-product firms. Then  $N_A = (1 - \alpha) X$  and  $N_B = (1 - \alpha) (1 - X)$ . Lemma 1 implies that (23) equals (24) if and only if X satisfies equation (9) from earlier. It is straightforward to show that the solution  $X(s, \alpha)$  to equation (9) satisfies i)  $X(0, \alpha) = 1/2$ , ii)  $X(\bar{s}(\alpha), \alpha) = 1$ , and iii)  $X_s(s, \alpha) > 0$  for all  $s \in (0, \bar{s}(\alpha))$ . The threshold  $\tilde{s}(\alpha)$  is defined such that  $X(\tilde{s}(\alpha), \alpha) = (2 - \alpha)/3(1 - \alpha)$ . In order for this equilibrium to exist, (23) must also exceed (25), which requires that

$$X \le \frac{2-\alpha}{3\left(1-\alpha\right)}.\tag{26}$$

Note that (26) is always satisfied if  $\alpha \ge 1/2$ , and for  $\alpha < 1/2$  is satisfied if and only if  $s \in (0, \tilde{s}(\alpha)]$ .

Step 3. Look for an equilibrium where X > 0 non-shoppers search the multiproduct firm, Y > 0 search both single-product firms, and 1 - X - Y > 0 search one single-product firm. Then  $N_A = (1 - \alpha) X$  and  $N_B = (1 - \alpha) (1 - X + Y)/2$ . Using Lemma 1, (23), (24) and (25) are all equal if and only if equations (10) and (11) hold. Let  $X(s, \alpha)$  be the solution to equation (11), note that again  $X(\tilde{s}(\alpha), \alpha) =$  $(2 - \alpha)/3(1 - \alpha)$ , and let the threshold  $\dot{s}(\alpha)$  be defined such that  $X(\dot{s}(\alpha), \alpha) =$  $1/2(1 - \alpha)$ . It is straightforward to show that  $X_s(s, \alpha) < 0$ . In order for this to be an equilibrium, we require that Y > 0; using (10) this requires  $X > 1/2(1 - \alpha)$ , which is only possible if  $\alpha < 1/2$  and  $s < \dot{s}(\alpha)$ . We also require Y < 1 - X which using (10) requires  $\alpha < 1/2$  and  $s > \tilde{s}(\alpha)$ .

Step 4. Look for an equilibrium where X > 0 non-shoppers search the multiproduct firm, and the other 1 - X > 0 search one single-product firm. From the benchmark model we know this is only possible if  $\alpha < 1/2$ , in which case  $X = 1/2(1 - \alpha)$ . Since  $N_A = 1/2$  and  $N_B = 1/4$  it is easy to show that (24) is dominated by (23) and (25) if and only if  $s \ge \dot{s}(\alpha)$ . Finally we note that it is straightforward (but lengthy) to show that  $\tilde{s}(\alpha) < \dot{s}(\alpha) < \bar{s}(\alpha)$  for all  $\alpha \in (0, 1/2)$ .

Step 5. In all cases Lemma 1 can then be used to calculate equilibrium price distributions.

**Proof of Proposition 2:** First note that four single-product firms is not an equilibrium. According to Lemma 5 in the asymmetric market structure, the multiproduct firm attracts  $X(s, \alpha) > 1/2$  non-shoppers. Therefore if two single-product firms deviate to form a conglomerate, their per-product profit strictly increases from  $(1 - \alpha) v/2$  to  $(1 - \alpha) X(s, \alpha)/2$ .

Second consider the incentives of the remaining pair of single-product firms to merge. If they merge, they earn  $(1 - \alpha) v/2$  per-product. If they remain unmerged, each earns (using Lemma 5, and again letting  $Y(s, \alpha)$  denote the fraction of non-shoppers that search both single-product firms)

$$v\left[\left(1-\alpha\right)\frac{1-X\left(s,\alpha\right)+Y\left(s,\alpha\right)}{2}+\alpha\left(1-\alpha\right)\frac{3X\left(s,\alpha\right)-Y\left(s,\alpha\right)-1}{X\left(s,\alpha\right)\left(1-\alpha\right)+\alpha}\right]$$
(27)

i) Suppose either  $\alpha \ge 1/2$ , or else  $\alpha < 1/2$  and  $s \le \tilde{s}(\alpha)$ . Using Lemma 5, (27) exceeds  $(1 - \alpha) v/2$  if and only if

$$X \le \frac{\alpha}{1 - \alpha} \tag{28}$$

Clearly (28) is satisfied for  $\alpha \ge 1/2$ , and not satisfied for  $\alpha \le 1/3$ . For  $\alpha \in (1/3, 1/2)$ , (28) is satisfied for  $s \approx 0$  since  $X(0, \alpha) = 1/2$ , but fails for  $s = \tilde{s}(\alpha)$  since  $X(\tilde{s}(\alpha), \alpha) > \alpha/(1-\alpha)$ . Therefore since  $X_s(s, \alpha) > 0$ , there exists a critical  $s^*(\alpha)$  defined by  $X(s^*(\alpha), \alpha) = \alpha/(1-\alpha)$  such that (28) is just satisfied. Inverting  $s^*(\alpha)$  gives the cutoff  $\alpha^*(s)$  in the proposition. ii) Suppose instead that  $\alpha < 1/2$  and  $s > \tilde{s}(\alpha)$ . Using Lemma 5 it is straightforward to show that (27) is strictly less than  $(1-\alpha)v/2$ .

**Proof of Proposition 3:** We first derive the pricing equilibrium in the first deviation, i.e., when both firms A and B are multiproduct firms and firm C sells only product 2. Let  $F_1$  be the price distribution of product 1 adopted by A and B. Let  $F_2$  be the price distribution of product 2 adopted by A and B, and let  $\hat{F}_2$  be the price distribution of product 2 adopted by C.

We conjecture an equilibrium where all non-shoppers randomly visit one of the two multiproduct firms. (This will be verified to be true later.) Since each multiproduct firm has half of the non-shoppers,  $F_1$  is determined in

$$p\left[\frac{1-\alpha}{2} + \alpha(1-F_1(p))\right] = \frac{1-\alpha}{2}v .$$

Then

$$F_1(p) = 1 - \frac{1-\alpha}{2\alpha} \left(\frac{v}{p} - 1\right) ,$$

and the lower bound of the distribution is

$$\underline{p}_1 = \frac{1-\alpha}{1+\alpha}v$$

For product 2, since the single-product firm C does not have any non-shoppers, its multiproduct competitors' price distribution has a mass point at the top. Following the standard logic,  $F_2$  and  $\hat{F}_2$  are determined in the following system of equations:

$$p\left[\frac{1-\alpha}{2} + \alpha(1-F_2(p))(1-\hat{F}_2(p))\right] = \frac{1-\alpha}{2}v$$

and

$$p\alpha(1-F_2(p))^2 = \alpha\lambda^2 v ,$$

where  $\lambda$  is the size of  $F_2$ 's mass point at v. Then one can solve that

$$F_2(p) = 1 - \sqrt{\frac{1 - \alpha v}{1 + \alpha p}} ,$$
$$\hat{F}_2(p) = 1 - \frac{\sqrt{1 - \alpha^2}}{2\alpha} \left(\sqrt{\frac{v}{p}} - \sqrt{\frac{p}{v}}\right) ,$$
$$\lambda = \sqrt{\frac{1 - \alpha}{1 + \alpha}} ,$$

and the common lower bound of the two distributions is

$$\underline{p}_2 = \frac{1-\alpha}{1+\alpha}v = \underline{p}_1 \ .$$

Notice that  $\hat{F}_2(p) < F_1(p)$  for  $p > \underline{p}$ . This is because

$$\frac{\sqrt{1-\alpha^2}}{2\alpha} \left( \sqrt{\frac{v}{p}} - \sqrt{\frac{p}{v}} \right) > \frac{1-\alpha}{2\alpha} \left( \frac{v}{p} - 1 \right) \Leftrightarrow p > \frac{1-\alpha}{1+\alpha} v$$

Then firm C's product 2 is more expensive than the multiproduct firms' product 1. Therefore, all non-shoppers prefer to visit a multiproduct firm. This verifies the conjectured equilibrium. (Following a similar logic, one can also check that there is no equilibrium where some non-shoppers visit the single-product firm C.)

Now we derive the conditions for neither deviation to be profitable:

(i) Suppose  $\alpha \geq \frac{1}{2}$ . The first deviation is not profitable if

$$\alpha(1-\alpha)v \ge (1-\alpha)v - \Delta$$

and the second deviation is not profitable if

$$2(1-\alpha)v - \Delta \ge \frac{1-\alpha}{2}v$$

The two conditions simplify to

$$(1-\alpha)^2 v \le \Delta \le \frac{3}{2}(1-\alpha)v$$
.

(The latter inequality also ensures that firm A earns a positive profit in the hypothetical equilibrium.)

(ii) Suppose  $\alpha < \frac{1}{2}$ . The first deviation is not profitable if

$$\frac{1}{4}v \ge (1-\alpha)v - \Delta \; ,$$

and the second deviation is not profitable if

$$v - \Delta \geq \frac{1 - \alpha}{2} v$$

The two conditions simplify to

$$(\frac{3}{4} - \alpha)v \le \Delta \le \frac{1+\alpha}{2}v$$
.

(The latter inequality also ensures that firm A earns a positive profit in the hypothetical equilibrium.)

#### The omitted details of the model with product differentiation.

The case with four single-product firms. To derive the equilibrium price for product i, suppose firm  $i_A$  unilaterally deviates and charges a price  $p'_0$ . The consumers who visit it first will stop searching and buy immediately if  $u_A - p'_0 \ge a - p_0$ . This generates demand  $\frac{1}{2}[1 - F(a - p_0 + p'_0)]$  since only half the consumers visit firm  $i_A$  first. For those who continue to visit firm  $i_B$ , they will return and buy from firm  $i_A$  if  $u_A - p'_0 \ge u_B - p_0$ . This generates demand

$$\frac{1}{2}\Pr[u_B - p_0 < u_A - p'_0 < a - p_0] = \frac{1}{2}\int_{\underline{u}}^{a - p_0 + p'_0} F(u_A - p'_0 + p_0)dF(u_A) .$$

Another demand source is consumers who visit firm  $i_B$  first. Since those consumers hold an equilibrium belief about firm  $i_A$ 's price, they visit firm  $i_B$  if  $u_B < a$ . They will then buy at firm  $i_A$  if  $u_A - p'_0 > u_B - p_0$ . This generates demand

$$\frac{1}{2} \int_{\underline{u}}^{a} [1 - F(u_B - p_0 + p'_0)] dF(u_B)$$

From these three demand components, one can check that the equilibrium demand slope is the negative of  $\frac{1}{2}f(a)[1-F(a)] + \int_{\underline{u}}^{a} f(u)^{2} du$ . Meanwhile, in equilibrium each firm sells to half the consumers due to symmetry and the assumption of full market coverage. Therefore, the first-order condition for  $p_0$  is (14).

The asymmetric case. Consider the market for product *i*. The demand for the multiproduct firm's product, if it charges  $p'_A$  while its single-product rival  $i_B$  sets the equilibrium price  $p_B$ , is

$$[1 - F(a - p_B + p'_A)] + \int_{\underline{u}}^{a - p_B + p'_A} F(u - p'_A + p_B) dF(u) .$$
<sup>(29)</sup>

This is explained as follows. All consumers visit firm A first. The first term is consumers who find  $u_A - p'_A \ge a - p_B$  and so buy immediately. The second term is consumers who find  $u_A - p'_A < a - p_B$  and so search firm  $i_B$ , but who subsequently return to buy from firm A because  $u_B - p_B < u_A - p'_A$ . The demand for firm  $i_B$ 's product, if it charges price  $p'_B$  while firm A sets its equilibrium price  $p_A$ , is

$$\int_{\underline{u}}^{a-p_B+p_A} [1 - F(u - p_A + p'_B)] dF(u) .$$
(30)

This is because all consumers visit firm A first, and hold an equilibrium belief about firm  $i_B$ 's price. Therefore they search firm  $i_B$  if  $u_A - p_A < a - p_B$ , and then buy from it if  $u_B - p'_B > u_A - p_A$ .

Define  $\Delta \equiv p_B - p_A$ , and

$$Q(\Delta) \equiv 1 - \int_{\underline{u}}^{a-\Delta} [1 - F(u+\Delta)] dF(u) \; .$$

Here  $Q(\Delta)$  is the equilibrium demand for firm A (i.e., (29) evaluated at  $p'_A = p_A$ ), and  $1 - Q(\Delta)$  is the equilibrium demand for firm  $i_B$  (i.e., (30) evaluated at  $p'_B = p_B$ ). Due to the assumption of full market coverage, they only depend on the price difference  $\Delta$ . Then it is straightforward to derive the first-order conditions stated in the main text. **Proof of Lemma 6:** From (16) and (17), we derive an equation of  $\Delta$ :

$$\Delta = \frac{1 - Q(\Delta)}{Q'(\Delta) - f(a - \Delta)[1 - F(a)]} - \frac{Q(\Delta)}{Q'(\Delta)} \equiv \Phi(\Delta) .$$

Using

$$Q'(\Delta) = [1 - F(a)]f(a - \Delta) + \int_{\underline{u}}^{a - \Delta} f(u + \Delta)dF(u) ,$$

we can check that  $\Phi(0) > 0$  if

$$\frac{1}{F(a) - \frac{1}{2}F(a)^2} - \frac{f(a)[1 - F(a)]}{\int_{\underline{u}}^{\underline{a}} f(u)dF(u)} < 2 .$$

This must hold because

$$\int_{\underline{u}}^{a} f(u) dF(u) = \int_{\underline{u}}^{a} \frac{f(u)}{1 - F(u)} [1 - F(u)] dF(u) < \frac{f(a)}{1 - F(a)} [F(a) - \frac{1}{2}F(a)^{2}] .$$

where the inequality is because logconcavity of 1 - F implies  $\frac{f(u)}{1 - F(u)}$  increases in u.

On the other hand, using L'Hôpital's rule one can check that

$$\Phi(a - \underline{u}) = \frac{1 - F(a)}{f(a)} - \frac{1}{f(\underline{u})[1 - F(a)]} < \frac{1}{f(\underline{u})}[1 - \frac{1}{1 - F(a)}] < 0 < a - \underline{u} .$$

where the first inequality again uses logconcavity of 1 - F. Therefore,  $\Phi(\Delta) = \Delta$  has a solution between 0 and  $a - \underline{u}$ .

## References

- ANDERSON, S., AND R. RENAULT (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30(4), 719–735.
- ARMSTRONG, M., J. VICKERS, AND J. ZHOU (2009): "Prominence and Consumer Search," RAND Journal of Economics, 40(2), 209–233.
- BAUMOL, W., AND E. IDE (1956): "Variety in Retailing," *Management Science*, 3(1), 93–101.
- BAYE, M., D. KOVENOCK, AND C. DE VRIES (1992): "It Takes Two to Tango: Equilibria in a Model of Sales," *Games and Economic Behavior*, 4(4), 493–510.
- BURDETT, K., AND K. JUDD (1983): "Equilibrium Price Dispersion," *Econometrica*, 51(4), 955–969.
- CHEN, Z., AND P. REY (2015): "A Theory of Conglomerate Mergers," manuscript from private communication.
- CHURCH, J. (2008): "Conglomerate Mergers," in *Issues in Competition Law and Policy*, ed. by W. Collins. Chicago: American Bar Association.
- DIAMOND, P. (1971): "A Model of Price Adjustment," Journal of Economic Theory, 3(2), 156–168.
- DUDEY, M. (1990): "Competition by Choice: The Effect of Consumer Search on Firm Location Decisions," *American Economic Review*, 80(5), 1092–1104.
- GOLDMANIS, M., A. HORTACSU, C. SYVERSON, AND O. EMRE (2010): "E-Commerce and the Market Structure of Retail Industries," *Economic Journal*, 120(545), 651– 682.
- JOHNSON, J. (2014): "Unplanned Purchases and Retail Competition," mimeo.
- LAL, R., AND C. MATUTES (1994): "Retail Pricing and Advertising Strategies," Journal of Business, 67(3), 345–370.

- MCAFEE, P. (1995): "Multiproduct Equilibrium Price Dispersion," Journal of Economic Theory, 67(1), 83–105.
- MESSINGER, P., AND C. NARASIMHAN (1997): "A Model of Retail Formats Based on Consumers' Economizing on Shopping Time," *Marketing Science*, 16(1), 1–23.
- MORAGA-GONZALEZ, J., AND V. PETRIKAITE (2013): "Search Costs, Demand-Side Economies, and the Incentives to Merge under Bertrand Competition," *RAND Journal of Economics*, 44(3), 391–424.
- NALEBUFF, B. (2000): "Competing Against Bundles," in *Incentives, Organization,* and Public Economics, ed. by G. Myles, and P. Hammond. Oxford University Press.
- NARASIMHAN, C. (1988): "Competitive Promotional Strategies," *Journal of Business*, 61(4), 427–449.
- NON, M. (2010): "Isolation or Joining a Mall? On the Location Choice of Competing Shops," mimeo.
- RHODES, A. (2015): "Multiproduct Retailing," *Review of Economic Studies*, 82(1), 360–390.
- SHAKED, A., AND J. SUTTON (1990): "Multiproduct Firms and Market Structure," RAND Journal of Economics, 21(1), 45–62.
- SHELEGIA, S. (2012): "Multiproduct Pricing in Oligopoly," International Journal of Industrial Organization, 30(2), 231–242.
- STAHL, D. (1989): "Oligopolistic Pricing with Sequential Consumer Search," American Economic Review, 79(4), 700–712.
- STAHL, K. (1982): "Location and Spatial Pricing Theory with Nonconvex Transportation Cost Schedules," Bell Journal of Economics, 13(2), 575–582.
- THANASSOULIS, J. (2011): "Is Multimedia Convergence To Be Welcome?," Journal of Industrial Economics, 59(2), 225–253.
- VARIAN, H. (1980): "A Model of Sales," American Economic Review, 70(4), 651–659.
- WOLINSKY, A. (1983): "Retail Trade Concentration due to Consumers' Imperfect Information," *Bell Journal of Economics*, 14(1), 275–282.

——— (1986): "True Monopolistic Competition as a Result of Imperfect Information," *Quarterly Journal of Economics*, 101(3), 493–511.

ZHOU, J. (2014): "Multiproduct Search and the Joint Search Effect," American Economic Review, 104(9), 2918–2939.