Social Network, Herding and Competition

By H. Henry Cao and Dongyan Ye[*]

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We analyze conditions under which fund managers herd to acquire information and trade on the same stock. This happens when fund managers have highly complementary signals, that is each manager has very imprecise information but taken together they have perfect information. However, the number of managers herding on the same stock cannot exceed three due to competition. When information sharing in a social network is introduced among managers, herding can occur for arbitrary number of managers and the set of parameters under which herding occurs is strictly larger. The benefit of social network increases with the social network size for highly complementary information. The optimal social network size decreases with the precision of managers' signals. With social network, fund managers can act in unison and maximize their combined profits. We then allow information sharing to be noisy and show that noisy communication of signals can be optimal and further expanding the set of parameters for herding to be optimal. We extend our model to multi-period and continuous time trading and show that our main results still go through in dynamic trading although the opportunity set that favors herding will be smaller due to more intensive competition. In addition, in continuous time trading, investors will not herd unless there is a social network to share information. However, they will never share their information to all in the network due to the resulting rat race among the managers.

Human beings are influenced by what their friends do, especially in their financial activities. For example, fund managers prefer to be close to financial centers, partly because they would like to know what other fund managers are investing. There are many venues with which fund managers can share ideas, including industry conferences, Internet clubs and private communications. In the hedge fund industry, top investors share ideas and learn from each other in conferences such as the Value Investing Congress, the Hedge Fund Activism and Shareholder Value Summit. With the spread of mobile communications, information exchanges through on line communities become very convenient. For example, Sumzero.com is an invitation only internet community open to hedge fund managers. Valueinvestorsclub.com provides another platform for top investors to share their best ideas. On xueqiu.com, investors dislcose their trading positions to each other.

Observation of each other's activities and private communication in a social network can result in herding, which is prevalent in financial activities. Some stocks are hot while others attract no attention even when the stocks have similar distributions in terms of fundamental value. When Warren Buffet buys stocks in China, other managers take notice. In the fund industry, a large literature has shown that herding among fund managers are prevalent. Lakonishock, Shleifer, and Vishny (1992) find evidence that pension funds engage in herding with a stronger effect in smaller stocks. Grinblatt, Titman and Wermers (1995) find a tendency for funds to buy and sell stocks at the same time in which a large number of funds are active. Kodres and Pritsker (1997) report herding in daily trading by large futures market institutional investors.

People not only watch what other people do, they also communicate with each other, although conversation can be noisy. With the current explosive development of social networks on the Internet, a huge amount of information gets passed

from one person to another through multimedia. Shiller and Pound (1989) argue that conversations among investors are very important in investment decisions. Shiller (2000) argues that: "This flow of conversation serves to exchange a wide variety of information, and also to reinforce memories of pieces of information to be held uncommon by the group." Hong, Kubik and Stein (2005) show that mutual fund managers in a given city tend to have trading behavior that covaries more strongly with other managers in the same city, as opposed to with managers in different cities. Feng and Seasholes (2004) find similar behavior in the Chinese stock market for geographically closed investors. Cohen, Frazznini and Malloy (2007) find that mutual fund managers who went to college together have similar trades. Ivkovic and Weisbenner (2007) find individual investors who live close by trade similar stocks. Pool, Stoffman and Yonker (2014) find that socially connected fund managers have more similar holdings and trades. Many fund managers place an identical trade at the same time. For example, in October 2008, multiple hedge funds trading in the Porsche/Volkswagen lost money when the deal went through. In August 2007, many hedge funds experience losses caused by one or more sizable hedge funds liquidating (Khadani and Lo (2007)). Using an account level dataset of all trades on the Istanbul Stock Exchange in 2005, Ozsoylev et. al. (2014) identify traders with similar trading behavior as linked in an empirical investor network (EIN). Consistent with the theory of information networks, they find that central investors earn higher returns and trade earlier than peripheral investors with respect to information events.

In this paper, we propose a theory of herding on information acquisition and trading on the same stock due to information complementarity. There is a public announcement that provides a noisy signal for the asset value. We assume that managers have different information about the noises in the public signal. Hence, when the managers combine their information, they have perfect information about the asset value.[1] We show that when the noise in managers' signals is

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[1] Perfect information is not important but it makes the analysis more tractable.

sufficiently large, they would prefer to acquire information and trade on the same stock, provided the total number of fund managers in the stock is less than or equal to three. In this case, herding on the same stock occurs because of implicit collusion when they trade together. Fund managers make more profits as their combined information is much more informative than their individual signals. However, the incentive to herd is hindered by competition, and herding occurs only when the number of managers is less than or equal three.

The implicit collusion through trading raises the possibility that managers may have incentives to interact with each other through social networks and share information among themselves directly. We derive conditions under which managers are better off sharing their information with each other. We show that with very noisy signals, managers' gains from information sharing increases with the size of social network. Intuitively, direct information sharing helps managers to have stronger information advantage over the market. As a result, information sharing increases the opportunity set with which herding on information acquisition and informed trading occurs. We show that in the presence of social network and information sharing, herding can occur when the number of fund managers is arbitrarily large.

Interestingly, for any social network size, there exist parameters such that managers trade in aggregate like a monopolist. In this case collusion is perfect in terms of expected profits. First of all, information sharing makes managers more informative. Secondly, competition is also minimized when the sufficient statistic of each fund manager are independent from each other in a social network. Fund manager's expected profits are non-monotonic with respect to the informativeness of the fund manager's signal under optimal social network size. Start with the case in which fund managers have very noisy signals such that the optimal social network is to include all managers. As the informativeness of manager's signal increases, it reaches a point in which all managers collude which maximize their expected profits. At this point, when managers' signals becomes more in-

formative, they start to trade more competitively and their expected profits will decrease. As the manger's signal's precision further increases, there will be a point such that the optimal network size drops by one. At this point, manager's expected profits will again increases with the informativeness of their signal and this zigzag will repeat itself until information sharing is no longer optimal.

Information sharing is a double edged sword. On one hand, it helps fund managers to have more precise information. On the other hand it also intensifies competition. Thus there exist an optimal size of social network among the fund managers. We show that the optimal size of social network decreases as managers have more precise information. Only when each fund manager's signal is sufficiently noisy, would their expected profits increase with the size of social network. In this case it is optimal for the social network to cover all managers.

Communication in a social network may not be complete and precise. Conversational learning can be noisy and information can be distorted during communication. With large social networks, fund managers may not be willing to share all of their signals in the social network. We analyze how noisy information sharing affects trading. We show that when social network size is sufficiently large, managers would like to add noises to the information shared in the network. Adding noises could reduce competition caused by information sharing and expand the set of parameters with which herding and information sharing are beneficial to managers.

In earlier literature, it has been shown that when informed investors trade dynamically, they compete more aggressively which erodes their profits. Consequently, it is important to examine how dynamic trading affects the incentive to share information. We extend our model to multi-period setting and continuous time trading. The incentive to share information still persists in dynamic trading. However, the opportunity set in which managers herd reduces and are willing to share information increases with trading frequency. In particular, herding will not occur without social network in the continuous time trading setting. Moreover,

managers will never share all of their information in continuous time trading due to a rat race in continuous time trading with homogeneous information.

Our paper is related to two strands of literature in finance. The first is herding. An extensive literature have shown that people herd because of reputation, social learning or relative compensation. Herding in our paper is different as it occurs due to complementarity in their information as without other managers in the market, each manager will end up with very noisy information and limited gains from it. Moreover, herding occurs due to collusion to reap higher profits and the potential to share information in a social network.

The second line of related research is on social network in financial markets. Social networks theory has been used to analyze contagion. Stein (2008) shows that when players have to bounce ideas off each other in order to come up with a new product, they are willing to exchange ideas when the discount rate is close to one and competition is not too high. In his model, players receive signals sequentially and alternate with each other. On the contrary, in our model fund managers act simultaneously and price formation is endogenous. Moreover, we study herding in the fund industry while Stein's focus is on venture capital. Colla and Mele (2009) also study the role of social network in a strategic setting. They focus on trade correlations among informed investors and show that informed investors have more correlated trades when they are close to each other in the network. Ozsoylev and Walden (2011) study general forms of network structure and provide conditions for existence of linear rational expectations equilibria.

The remainder of the paper is organized as follows. Section I introduces the model in the absence of social network. Section II analyzes the effects of social network on herding and profits to fund managers. Section III allows for noisy signals in personal communication. Section IV extends the model to multi-period trading and continuous time trading. Section V Concludes. Proofs are presented in the appendix.

I.   The Model

We consider an economy with N risk neutral fund managers who invest in a stock based on the classic model of Kyle (1985). In our model, there are two dates, time 0 and time 1. And in the financial market there exist one risk-free asset and one risky stock that the fund managers can invest in. The risk-free rate is taken to be zero.

Each fund manager receives a mean-zero signal s_i at time 0. We assume the signals and the liquidation value of the stock have a nondegenerate joint normal distribution that is symmetric in the signals.[2] Let v denote the expectation of the liquidation value conditional on the combined information of the fund managers. By normality, v is an affine function of s_i's. By rescaling the s_i if necessary, we can assume without loss of generality that

$$(1) \qquad v = \bar{v} + \sum_{i=1}^{N} s_i,$$

for a constant $\bar{v}$. This is a normalization adopted by Foster and Viswanathan (1996) and Back, Cao and Willard (2000). For simplicity, we assume $\bar{v} = 0$.

REMARK 1:   *Notice that our information structure allows for the signals to have negative correlation. To understand how this could happen in the economy, consider the following setting. At time zero, there is a public signal y about the stock value:*

$$(2) \qquad y = v - \sum_{i=1}^{N} s_i.$$

And fund manager i observes the noise in the public signal, s_i. We assume that the stock value v, the public signal y and the fund managers' private signals $\{s_i\}_{i=1}^{N}$ have zero means and a nondegenerate joint normal distribution that is symmetric

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[2]Symmetry means that the joint distribution of stock value v and private signals $\{s_i\}_{i=1}^{N}$ is invariant to a permutation of the indices $1, \ldots, N$.

in the private signals. The variance of v is σ_v^2 and the variance of s_i is the same across i and is denoted σ_s^2. Prior to observing the signal y, correlation coefficient of s_i with s_j for $i \neq j$ is ρ_0 and we assume that s_i are positively correlated, $0 \leq \rho_0 \leq 1$. The private signal s_i is assumed to be uncorrelated with the stock value v, $Cov[v, s_i] = 0$, $i = 1, \ldots, N$. In such a setting, although knowing private signal s_i alone doesn't help predicting the stock value, each fund manager has information advantage over the market as she knows how to interpret the public signal better.

Given the public signal, we can rewrite the stock value as

$$(3) \qquad\qquad v = y + \sum_{i=1}^{N} s_i.$$

Therefore given the public signal, the stock value is a sum of the signals of the fund managers. This provides an explanation for equation (1).

The conditional correlation between signals s_i and s_j given the public signal y is

$$(4) \qquad \rho_{s|y} = \frac{\rho_0 \sigma_v^2 - (1 - \rho_0)(1 + (N-1)\rho_0)\sigma_s^2}{\sigma_v^2 + (N-1)(1-\rho_0)(1+(N-1)\rho_0)\sigma_s^2}$$

The conditional correlation coefficient $\rho_{s|y}$ decreases in the variance of the private signal, σ_s^2. When σ_s^2 goes to infinity, $\rho_{s|y}$ goes to $-\frac{1}{N-1}$. When σ_s^2 goes to 0, $\rho_{s|y}$ goes to ρ_0. Although the unconditional correlation between private signals are always positive, the conditional correlation between them could be negative given the variance of the private signal is big enough.[3]

Let ϕ denote the "R-squared" of running a regression of v on Ns_i,

$$(5) \qquad\qquad \phi = \frac{\text{Var}[v]}{\text{Var}[Ns_i]}.$$

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[3]In Colla and Mele (2010), there are scenarios that the initial correlation between private signals is assumed to be negative. In our setting, the private signals are assumed to be positively correlated but the conditional correlation between them turns negative after observing the public signal.

It is the percentage of variance in v that is explained by the fund manager's information. This is a measure of the quality of each fund manager's information. If $\phi = 1$, then either $N = 1$ or the s_i's are perfectly correlated. In either case, each fund manager has perfect information about the stock value v. Letting ρ denote the correlation coefficient of s_i with s_j for $i \neq j$, one can compute ϕ for $N > 1$ as

$$(6) \qquad \phi = \frac{1}{N} + \frac{N-1}{N}\rho$$

Notice that due to symmetry, there is no i subscript on ϕ. It is easy to show that ϕ's range is (0,1] and that ϕ increases with ρ.

Definition We say that fund managers have complementary signals when $\phi < \frac{1}{N}$, independent signals when $\phi = \frac{1}{N}$ and substitutive signals when $\phi > \frac{1}{N}$.

As ϕ represents fraction of variance explained by each of the fund manager's signal, when $\phi < 1/N$, it means each manager standing alone, knows about less than $1/N$ of the variation of v. However, when managers can combine their information, they have perfect information. Therefore, managers have incentive to exchange signals and the stock payoff variance explained by the combined signals is more than a simple addition of explained variance with individual signals. On the contrary, when $\phi > 1/N$, each fund manager alone knows more than $1/N$ of the variation of v. Although managers know the stock value v perfectly if they combine their information, they might not benefit from exchanging signals because competition could become so fierce and trading profits are reduced by information sharing.

As in the usual Kyle (1985) setup, there is also a group of liquidity traders with order u which is normally distributed with mean zero and variance σ_u^2. u is independent with the liquidation value v and private signals $\{s_i\}_{i=1}^N$. There is a competitive market maker who sets the price at the conditional expected payoff given the aggregate order flow. All market participants are assumed to be risk

neutral. We first analyze the incentive to herd on information acquisition in a static setting and thereafter consider the role of social networks.

A.   Fund Managers' Profits in the Absence of Social Network

In this subsection, we derive the manager's expected profits of trading as a monopolist and trading together with other fund managers on the same stock. The case in which a fund manager trading alone in the absence of other fund managers is similar to that in Kyle (1985). Without loss of generality, we assume fund manager i is the only manager who trades on the stock. The fund manager behaves as a monopolist, at time 0, he submits an order of $x_i = \beta s_i$, keeping in mind of the price impact of his order. The market maker observes the total order flow $z = u + x_i$ and set the stock price to be the expectation of v given total order z:

(7)
$$p_0 = \lambda z$$

As all random variables are normally distributed, one can compute λ as

(8)
$$\lambda = \frac{\beta \mathrm{Cov}[v, s_i]}{\sigma_u^2 + \beta^2 \mathrm{Var}[s_i]}$$

The fund manager's expected profit at time 0 when submitting an order x_i is

$$\mathrm{E}[x_i(v - (\lambda(u + x_i)))|s_i] = x_i(\mathrm{E}[v|s_i] - \lambda x_i),$$

So the optimal order is

$$x_i = \frac{1}{2\lambda}\mathrm{E}[v|s_i] = \frac{\mathrm{Cov}[v, s_i]}{2\lambda \mathrm{Var}[s_i]} s_i.$$

Hence,

$$\beta = \frac{\text{Cov}[v, s_i]}{2\lambda \text{Var}[s_i]}. \tag{9}$$

Combining equation (8) and equation (9), one expresses β explicitly as

$$\beta = \sqrt{\frac{\sigma_u^2}{\text{Var}[s_i]}}. \tag{10}$$

Specifically, let σ_v^2 denote the variance of v, then we have following proposition,

PROPOSITION 1: *Let π_M denote the expected profits of a manager trading alone on a stock. Then*

$$\pi_M = \frac{\sqrt{\phi}\sigma_v\sigma_u}{2} \tag{11}$$

Similarly as in Kyle (1985), the fund manager's profit is proportional to the standard deviation of the explained part of the stock value by his signal, and the standard deviation of the liquidity order. In Kyle (1985), the fund manager knows the stock value v perfectly and hence $\phi = 1$.

When there are more than one fund manager trade together with other informed fund managers, the case is similar to the static model in Cao (1995) and Foster and Viswananthan (1996). Fund manager i submits an order of $x_i = \beta s_i$, the market maker observes a total order flow:

$$z = u + \sum_{i=1}^{N} x_i = u + \beta v.$$

and sets the stock price at the conditional expectation of v given the aggregate order flow,

$$p_0 = \text{E}[v|z] = \lambda z,$$

with $\lambda = \frac{\beta \text{Var}[v]}{\sigma_u^2 + \beta^2 \text{Var}[v]}$.

Fund manager i's expected profits while he submits an order of x_i are

$$\mathrm{E}\left[x_i\left(v-\left(\lambda(x_i+\sum_{j\neq i}\beta s_j)\right)\right)|s_i\right]=x_i\left[\left((1-\lambda\beta)\frac{\mathrm{Cov}[s_i,v]}{\mathrm{Var}[s_i]}+\lambda\beta\right)s_i-\lambda x_i\right]$$

So the optimal order x_i is

$$x_i=\frac{1}{2\lambda}\left((1-\lambda\beta)\frac{\mathrm{Cov}[s_i,v]}{\mathrm{Var}[s_i]}+\lambda\beta\right)s_i=\beta s_i.$$

One can solve β explicitly as

$$\beta=\sqrt{\frac{\sigma_u^2\mathrm{Cov}[s_i,v]}{\mathrm{Var}[v]\mathrm{Var}[s_i]}}=\sqrt{\frac{\sigma_u^2}{N\mathrm{Var}[s_i]}}$$

Each fund manager's expected profits are given below:

PROPOSITION 2: *Let π_C denote the expected profits of each fund manager trading together on a stock. Then*

(12)
$$\pi_C=\frac{\sqrt{\phi}\sigma_v\sigma_u}{\sqrt{N}(1+N\phi)}.$$

Interestingly, fund managers trade in unison like a monopolist when $\phi=1/N$. In the case that a monopolist knows the stock value v perfectly, she places an order $\frac{\sigma_u}{\sigma_v}v$ and earns expected profits of $\frac{1}{2}\sigma_u\sigma_v$. When there are multiple fund managers and $\phi=1/N$, they collectively place orders $\beta\sum_{i=1}^{N}s_i=\beta v$, with $\beta=\sqrt{\frac{\sigma_u^2\mathrm{Cov}[s_i,v]}{\mathrm{Var}[v]\mathrm{Var}[s_i]}}=\frac{\sigma_u}{\sigma_v}$, and earn expected profits of $N\times\frac{\sqrt{\phi}\sigma_u\sigma_v}{\sqrt{N}(1+N\phi)}=\frac{1}{2}\sigma_u\sigma_v$.

When the expected profits of a fund manager is higher when he trades together with other fund managers, this implies that the fund manager would prefer to herd together and trade on stocks in which other fund managers are already active. Comparing the expected profits with and without competition, we can derive conditions in which herding is optimal. For that purpose we assume that there is another stock with independent payoff v', private signals s_i', $i=1,...,N$.

In addition, the variance covariance matrix of v', s_i', $i = 1, ..., N$ is identical to that of v, s_i, $i = 1, ..., N$. Before trading starts, fund managers have to decide on which stock to acquire information. We analyze conditions under which all fund managers would decide to acquire information and trade on the same stock.

PROPOSITION 3: *When $N = 2, 3$, and*

$$\phi < \phi^* \equiv \frac{1}{N} \left(\sqrt{\frac{4}{N}} - 1 \right),$$

there exists a herding equilibrium in which all fund managers will herd to acquire information on the same stock. When $N \geq 4$, there does not exist such a herding equilibrium.

Intuitively, one would thought that fund managers prefer to analyze stocks that no other fund manager has been trading actively. Indeed, notice that $\phi^* \leq 1/N$ and thus when managers have substitutive signals, that is $\phi > 1/N$, herding will not occur. However, when $\phi < 1/N$, each fund manager has information that tells him less than $1/N$ of the variation in the stock payoff. This implies that combining fund managers signals together, each fund manager knows more about the variation in stock payoff than the simple addition of the explained variances using individual signals. However, the gains of cooperation and herding is also hindered by competition. When the number of fund managers is large, i.e., $N \geq 4$, fund managers will never herd on the same one stock. However, it's possible that there exists partial herding, in which a group of less than four managers herd on a stock and other groups herd on other stocks.[4]

When investors have complementary signals, investors herd as they would like

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[4]When a group of three fund managers herd on a stock, then each manager makes more profits than what he gains from forming a two member group with another manager and trading on anther stock. When forming a two-manager group, the variance of the liquidation value becomes $\hat{\sigma}_v^2 = \frac{4\phi}{3\phi+1}\sigma_v^2$ and ϕ becomes $\hat{\phi} = \frac{\phi^2}{3\phi+1}$. The ratio of expected profits that the manager earns when staying in a three-manager group and forming a new a two-manager group is $\frac{18\phi^2+3\phi+1}{\sqrt{54}\phi(1+3\phi)}$, which is larger than 1 when $\phi < \frac{1}{3}\left(\sqrt{\frac{4}{3}} - 1\right)$.

to analyze the same stock and trade together to reap more profits. An interesting question is whether investors would share their information directly. We next analyze the effect of social network and information sharing on the incentive to herd.

II. Social Network and Herding

In this section, we assume that fund managers are seated on a circle and each fund manager can observe the signals of up to $G - 1$ managers clockwise to his seat, $1 \leq G \leq N$.[5] We call G the size of social network. When $G = 1$, there is no social network and each fund manager can see only his own private signal. When $G = N$, each fund manager observes all private signals $\{s_i\}_{i=1}^N$.

Since fund manager i can observe the signals of fund manager $i+1, \ldots, i+G-1 - N\lfloor (i+G-1)/N \rfloor$, his information can be summarized by a sufficient statistic $\hat{s}_i(G)$

$$(13) \qquad \hat{s}_i(G) \equiv \frac{1}{G} \sum_{j=i}^{i_G} s_j,$$

here, $i_G \equiv i + G - 1 - N\lfloor (i+G-1)/N \rfloor$, where $\lfloor \cdot \rfloor$ is the largest smaller integer function. The stock value can still be expressed as the sum of signals $\hat{s}_i(G)$.

$$(14) \qquad v = \sum_{i=1}^N \hat{s}_i(G).$$

The symmetric information structure is maintained although the correlation structure is different. The traders that are seated close by will have more highly correlated signals. For example, the correlation between $\hat{s}_i(G)$ and $\hat{s}_{i+1}(G)$ is

$$\text{Corr}[\hat{s}_i(G), \hat{s}_{i+1}(G)] = \rho_0 + \frac{(G-1)(1-\rho_0)(1+G\rho_0)}{G(1+(G-1)\rho_0)}$$

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[5]In Colla and Mele (2010), fund manager i can observe the signals of manager $i \pm 1, \ldots, i \pm G_0$. In their setting, a manager observes $2G_0 + 1$ (which is always an odd number) signals. However, in our setting a manager can observe G signals, here G could be an arbitrary positive integer.

which is larger than ρ_0, the initial correlation coefficient between private signals, when $G > 1$ and $\rho_0 < 1$.[6] Similarly, the proportion of variance explained by $\hat{s}_i(G)$ is

$$\phi(G) \equiv \frac{\text{Var}[v]}{\text{Var}[N\hat{s}_i(G)]} = \frac{(N-1)G\phi}{N(G-1)\phi + N - G}.^7$$

Clearly, as fund managers share more information with each other, each manager knows more about the stock value. Therefore $\phi(G)$ is an increasing function of G.[8] When $G = 1$, i.e., there is no social networking and $\phi(G) = \phi$. When $G = N$, each fund manager observes all private signals and hence knows the stock value v perfectly, $\phi(G) = 1$.

LEMMA II.1: *When $\phi(G) < \frac{1}{N^2\phi}$, fund managers are better off in a social network sharing information.*

In Section I, we have shown that when fund managers trade in unison like a monopolist and their expected profits are maximized when $\phi = 1/N$. With social network, we have following result.

PROPOSITION 4: *For every $N > 2$ and $G > 1$, there exist*

$$(16) \qquad \hat{\phi} = \frac{N-G}{N[G(N-2)+1]} < \frac{1}{N}$$

such that $\phi(G) = 1/N$ and fund managers trade in unison like a monopolist.

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[6]One can show that $\text{Corr}[\hat{s}_i(G), \hat{s}_{i+j+1}(G)] = \text{Corr}[\hat{s}_i(G), \hat{s}_{i+j}(G)] - \frac{1-\rho_0}{G(1+(G-1)\rho_0)}$, which is smaller than $\text{Corr}[\hat{s}_i(G), \hat{s}_{i+j}(G)]$ given $\rho_0 < 1$ and manager $(i+j+1)_G$ doesn't see s_i.

[7]
$$\phi(G) = \frac{G\sigma_v^2}{N^2(1+(G-1)\rho_0)\text{Var}[s_i]}$$
$$= \frac{G\phi}{1+(G-1)\rho_0} \quad \left(\text{By } \phi = \frac{\sigma_v^2}{N^2\text{Var}[s_i]}\right)$$
$$= \frac{(N-1)G\phi}{N(G-1)\phi + (N-G)} \quad \left(\text{By } \rho_0 = \frac{N\phi-1}{N-1}\right)$$

[8]Rewriting $\phi(G) = \left(1 + \frac{1-\phi}{(N-1)\phi}\left(\frac{N}{G}-1\right)\right)^{-1}$, which is a increasing function in G given $\phi > 0$ and $1 \le G \le N$.

Fund managers know more about the stock value as they share more information with each other. With social network, the fund managers can have maximum profits with even noisier information. The precision of their private signals could reduce further as the size of social network increases, i.e., $\hat{\phi}$ decreases in G.

Above we have analyzed the precision of private signals (given G) for fund managers to have maximum profits. We next determine the size of social network for the fund managers to have maximum profits given the precision of private signal ϕ. [9]

PROPOSITION 5: *Let*

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\text{(17)} \qquad \hat{G} \equiv \left\lfloor \frac{N(1-\phi)}{1+N(N-2)\phi} \right\rfloor
$$

The optimal network size is

$$
\text{(18)} \qquad G^*(\phi) = \begin{cases} \hat{G} & \text{if} \quad \phi(\hat{G})\phi(\hat{G}+1) > \frac{1}{N^2} \\ \hat{G}, \hat{G}+1 & \text{if} \quad \phi(\hat{G})\phi(\hat{G}+1) = \frac{1}{N^2} \\ \hat{G}+1 & \text{if} \quad \phi(\hat{G})\phi(\hat{G}+1) < \frac{1}{N^2} \end{cases}.
$$

In Figure 1**A**, we plot the optimal size of social network as a function of the precision of each manager's signal ϕ. Interestingly, when $\phi \geq 1/N$,[10] it is optimal for fund managers not to share any information. Notice that the smaller ϕ, the larger is the optimal network size. We have $G^* = N$ if $\phi < \frac{1}{N(N(N-1)^2-(N-2))}$. So when fund managers have very noisy signals, they would like to share their information to all other managers. Only in this case, managers' gains from information sharing increases with the size of social network. As $\phi(G)$ remains smaller than $1/N$ but gets closer to $1/N$ as G increases, the fund managers expected profits

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[9]We can also have managers endogenously chose how many neighbors to share information on his right hand side to endogenies the size of the network. In this case the network size can arise as an equilibrium outcome. We decided not to pursue this further as there could be mixed strategies with asymmetric network sizes which can make the problem complicated without generating much new insight.

[10]Actually, $G^* = 1$ when $\phi > \frac{1+\sqrt{1+8(N-1)(N-2)}}{4N(N-1)}$ which is smaller than $1/N$.

increases in the size of social network.

In Figure 1**B**, we plot each fund manager's expected profits as a function of the precision of each manager's signal ϕ with optimal size of social network. Fund manager's expected profits are non-monotonic with respect to the informativeness of the fund manager's signal under optimal social network size. Start with the case in which fund managers have very noisy signals such that the optimal social network is to include all managers. As the informativeness of manager's signal increases, it reaches a point in which all managers collude which maximize their expected profits. At this point, when managers' signals become more informative, they start to trade more competitively and their expected profits will decrease. As the mangers' signals' precision further increases, there will be a point such that the optimal network size drops by one. At this point, manager's expected profits will again increases with the informativeness of their signal and this zigzag will repeat itself until social network is no longer optimal.
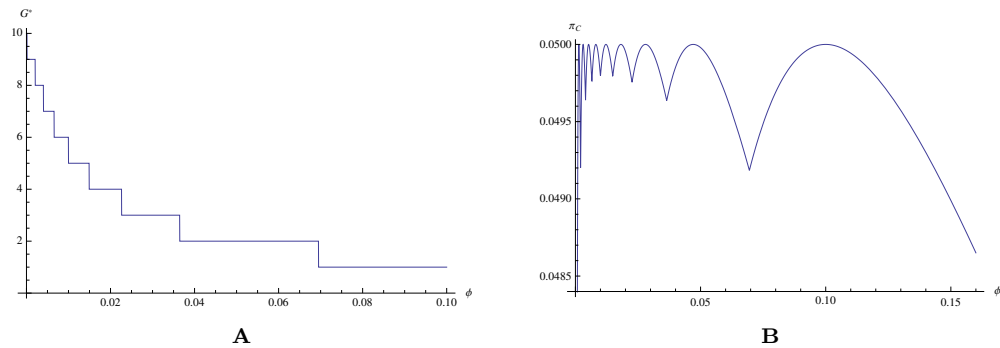

A

B

Figure 1.   Figure 1**A**: The optimal size of social network as a function of the precision of each manager's signal ϕ. Figure 1**B**: Each fund manager's expected profits as a function of the precision of each manager's signal ϕ. The number of fund managers $N = 10$.

In Section I, we show that fund managers would like to herd to acquire information and trade on the same stock, given the number of fund managers is less than four. Fund managers know more about the stock value as they share more

information with each other. So, with social network, fund managers with very imprecise private signals share information with each other and come up with better information about the stock value and it's possible for them to make more profits than what they gain while trading alone in which case there is no a social network to share information. We can show that under certain conditions herding to acquire information and trading on the same stock is optimal for arbitrary number of fund managers. Specifically, we have following proposition.

PROPOSITION 6: *For any $N > 1$, there exist G and ϕ such that all fund managers will herd to acquire information and trade on the same stock.*

III.   Noisy Communication in a Social Network

In last section we have shown that there could be gains in sharing information. However, information sharing can also hurt managers if they share too much information. Therefore it is interesting to see information sharing and herding are affected when a fund manager can choose to share how much of his information to other managers sitting close to him.

Now, fund manager i can send a noisy version of his private signal to managers up to $G - 1$ seats away from him clockwise, and at the same time he observes a noisy version of private signals of his $G - 1$ neighbors. Each fund manager adds a noise to his own private signal when sharing information with his neighbors. An alternative one to interpret this is that each informed investor have multiple pieces of information and they choose to share only a partial set of his information to people in his network. Specifically, fund manager i adds a noise η_i to his private signal s_i to form a noisy version of signal $s'_i = s_i + \eta_i$ that he shares with his $G - 1$ neighbors. At the same time, he observes

$$s'_j = s_j + \eta_j, j = i + 1, \ldots, i_G.$$

Here, $\{\eta_j\}_{j=1}^N$ are normally distributed with mean zero and variance σ_η^2, they are

mutually independent and are independent with other random variables in the economy. The precision of the noise η_i measures how much of noise a fund manager adds into his private signal. The larger the σ_η^2, the noisier is the information that the fund managers share with each other. When $\sigma_\eta^2 = 0$, the fund managers don't add any noise into their signals and it goes back the perfect information sharing case we analyze in Section II. At the other extreme case, $\sigma_\eta^2 = \infty$ and the information that the fund managers share with each other is useless and this case corresponds to that in Section II without information sharing or $G = 1$.

Now, fund manager i's information set contains: private signal s_i, the noise that he adds to his shared signal, η_i, and information that up to $G-1$ managers sitting away from him clockwise share to him, $\{s'_{i+1}, \ldots, s'_{i_G}\}$. Due to the symmetry of information structure, the sufficient statistics for $\{s_i, \eta_i, s'_{i+1}, \ldots, s'_{i_G}\}$ is $\{s_i, \eta_i, \sum_{j=i+1}^{i_G} s'_j\}$. Define

$$(19) \qquad \epsilon_i \equiv \frac{1}{G-1} \sum_{j=i+1}^{i_G} s'_j,$$

and

$$(20) \qquad \omega \equiv \sum_{i=1}^{N} \eta_i.$$

Then fund manager i's information set at time 0 is $S_i = \{s_i, \eta_i, \epsilon_i\}$.

LEMMA III.1: *The conditional expectations of v and ω under fund manager i's information set are*

$$(21) \qquad E[v|S_i] = \alpha_s^v s_i + \alpha_\eta^v \eta_i + \alpha_\epsilon^v \epsilon_i,$$

$$(22) \qquad E[\omega|S_i] = \alpha_s^\omega s_i + \alpha_\eta^\omega \eta_i + \alpha_\epsilon^\omega \epsilon_i,$$

with

$$(23) \quad \begin{bmatrix} \alpha_s^v \\ \alpha_\eta^v \\ \alpha_\epsilon^v \end{bmatrix} = \begin{bmatrix} \frac{(1+(N-1)\rho_0)(\sigma_\eta^2+(1-\rho_0)\sigma_s^2)}{\sigma_\eta^2+(1-\rho_0)(1+(G-1)\rho_0)\sigma_s^2} \\ 0 \\ \frac{(1+(N-1)\rho_0)(G-1)(1-\rho_0)\sigma_s^2}{\sigma_\eta^2+(1-\rho_0)(1+(G-1)\rho_0)\sigma_s^2} \end{bmatrix}, \quad \begin{bmatrix} \alpha_s^\omega \\ \alpha_\eta^\omega \\ \alpha_\epsilon^\omega \end{bmatrix} = \begin{bmatrix} \frac{-(G-1)\rho_0\sigma_\eta^2}{\sigma_\eta^2+(1-\rho_0)(1+(G-1)\rho_0)\sigma_s^2} \\ 1 \\ \frac{(G-1)\sigma_\eta^2}{\sigma_\eta^2+(1-\rho_0)(1+(G-1)\rho_0)\sigma_s^2} \end{bmatrix}.$$

Because the noise η_i that fund manager i adds to his private signal doesn't help predict the stock value v and η_i is independent of s_i and ϵ_i, we have $\alpha_\eta^v = 0$. Moreover, $\omega = \sum_{i=1}^N \eta_i$ and η_i doesn't have any power in predicting η_j $(j \neq i)$, so we have $\alpha_\eta^\omega = 1$. Each fund manager use the information ϵ_i that other managers share to him to help predicting the stock value, the improvement of predicting v from observing ϵ_i deceases as the fund managers add more noise to their signals. This can be seen clearly from that $\text{Var}[v|s_i] - \text{Var}[v|S_i] = \frac{\phi\sigma_v^2(G-1)(1-\rho_0)^2}{\sigma_\eta^2+(1-\rho_0)(1+(G-1)\rho_0)\sigma_s^2}$ decreases with σ_η^2. In the extreme case $\sigma_\eta^2 = \infty$, ϵ_i doesn't help to predict v and hence we have $\alpha_\eta^v = \infty$. On the other hand, it goes back to the perfectly information sharing case in Section II when $\alpha_\eta^v = 0$ and we have $E[v|S_i] = \frac{1+(N-1)\rho_0}{1+(G-1)\rho_0} \sum_{j=i}^{j_G} s_j$. Similarly, fund manager i uses ϵ_i to predict $v - \eta_i$ and at the same uses s_i to hedge the information about s_j contained in ϵ_i. One can see this clearly when $\sigma_\eta^2 = \infty$ and $E[\omega|S_i] = \eta_i + \sum_{j=i+1}^{i_G}(s_j + \eta_j) - (G-1)\rho_0 s_i = \sum_{j=i}^{i_G}\eta_j + \sum_{j=i+1}^{i_G}(s_j - \rho_0 s_i)$.

Fund manager i submits an order of

$$(24) \qquad\qquad x_i = \beta_s s_i + \beta_\eta \eta_i + \beta_\epsilon \epsilon_i$$

and the market maker observes a total order of

$$z = u + \sum_{i=1}^N x_i$$

$$= u + \sum_{i=1}^N (\beta_s s_i + \beta_\eta \eta_i + \beta_\epsilon \epsilon_i)$$

$$(25) \qquad\qquad = u + (\beta_s + \beta_\epsilon)v + (\beta_\eta + \beta_\epsilon)\omega$$

and sets the time 0 stock price to be

$$p_0 = \mathrm{E}[v|z] = \lambda z$$

with

(26)
$$\lambda = \frac{(\beta_s + \beta_\epsilon)\sigma_v^2}{\sigma_u^2 + (\beta_s + \beta_\epsilon)^2\sigma_v^2 + (\beta_\eta + \beta_\epsilon)^2\sigma_\omega^2}.$$

In this case, each fund manager's expected profits are given below:

PROPOSITION 7: *Let π_C^n denote the expected profits of each fund manager trading together on a stock. Then*

(27)
$$\pi_C^n = \lambda[\beta_s, \beta_\eta, \beta_\epsilon] \, Var[S_i][\beta_s, \beta_\eta, \beta_\epsilon]',$$

When the size of social network is sufficiently large, fund managers would be sharing too much information with each other when $\phi > \hat{\phi}$ at which $\phi(G) = 1/N$. In this case, managers would like to add noises to the information shared in the network. With perfect information sharing, the fund managers will not herd on information acquisition and trading anymore if $\phi > \hat{\phi}$. However, if they can add noise to their shared information, it would be still optimal to herd when ϕ is slightly larger than $\hat{\phi}$. We can see this clearly from figure 2**A**, in which we plot the boundary value of ϕ_h as a function of N with $G = N$. When $\phi < \phi_h$, herding on information acquisition and stock trading occurs. Notice that noisy communication can expand the set of parameters with which herding occurs. Similarly, in figure 2**B**, we plot the boundary value ϕ_s under which information sharing in a network is optimal and we notice that noisy communication increases $\phi(s)$ for all N.
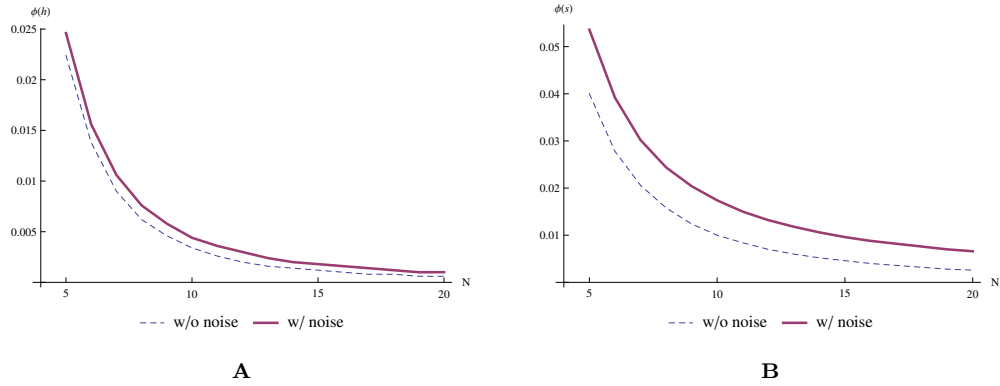
$\phi(h)$

$\phi(s)$

--- w/o noise — w/ noise

--- w/o noise — w/ noise

A

B

Figure 2.   Figure 2**A**: The boundary value of ϕ_h as a function of number of fund managers N, herding on information acquisition and stock trading occurs when $\phi < \phi(h)$. Figure 2**B**: The boundary value of $\phi(s)$ as a function of number of fund managers N, information sharing in a network is optimal when $\phi < \phi(s)$. The size of social network $G = N$. The dashed line is for the case with perfect communication and the solid line is for the case with noisy communication.

IV.   Dynamic Trading

Cao (1995), Foster and Viswananthan (1996), and Back, Cao and Willard (2000) show that dynamic trading can affect informed investors' trading strategy dramatically. In particular, when investors have the same information, their profits will be driven down to zero as trading approaches continuous time. It is interesting to examine how multi-period trading and continuous time trading affects the incentive to herd on information acquisition and to share information. For the ease of exposition, we leave the detailed presentation of pricing equations and informed investors' profits in the appendix and discuss the results belwo.

Intuitively, dynamic trading will intensify competition among informed investors and thus reduce the incentive to share information or herd on the same stock. In figure 3**A**, we plot the boundary value of ϕ_h as a function of number of trading periods with the size of social network $G = 2$ and the number of fund managers $N = 6$. When $\phi < \phi_h$, herding on information acquisition and stock trading occurs. Notice that dynamic trading can narrow the set of parameters
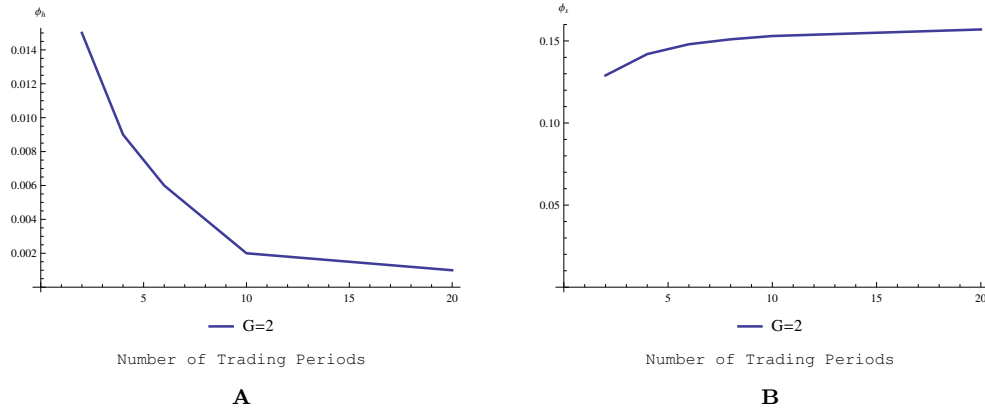
Figure 3.   Figure 3**A**: The boundary value of ϕ_h as a function of number of trading periods, herding on information acquisition and stock trading occurs when $\phi < \phi_h$. Figure 3**B**: The boundary value of ϕ_s as a function of number of trading periods, information sharing in a network is optimal when $\phi < \phi_s$. The number of fund managers $N = 6$ and the size of social network $G = 2$.

in which herding occurs. ϕ_h decreases with the number of trading periods and reaches 0 when the number of trading periods goes to infinity. Dynamic trading intensifies competition and in particular, an informed trader will always avoid herding in continuous time trading. On the contrary, in figure 3**B**, we plot the boundary value ϕ_s under which information sharing in a network is optimal and we notice that dynamic trading increases ϕ_s. This is because with limited information sharing, learning from each other is more beneficial with dynamic trading as the gains from learning outweighs competition. However, when the network size is large such that $G = N$, both ϕ_h and ϕ_s will decrease with the number of trading periods. When the number of trading periods goes to infinity, the model approaches to continuous-time trading. When $G = N$, all managers possess identical information about the stock value, and it will be revealled right away at the beginning of the trading period and the expected profits of each fund manager are driven down to 0. Consequently, it's never optimal to share information to all managers.

PROPOSITION 8: *In continuous time trading, investors will never herd in the absence of social network. In addition, investors will never share their signals with all managers. However, it is still optimal to share information and herd on information acquisition for $G < N$ when ϕ is sufficiently small.*

While dynamic trading makes competition more intensive, herding in the presence of information sharing in a partial social network can still be optimal.

V. Conclusion

Social networks are becoming increasingly important in daily life. It affects all areas of social life including investment activities. We analyze the role of social network on the incentives of fund managers to herd on information acquisition and trading. We show that when fund managers have noisy and complementary signals, they would like to herd. The formation of social network expands the set of economies in which herding will occur. The optimal size of the social network will be larger when investors have noisier signals.

Dynamic and around the clock trading can intensify competition among fund managers. As a result, the set of economies for herding to be optimal is smaller in the setting of continuous time trading. Nevertheless, information sharing and herding remains optimal for very noisy signals.

For simplicity, we have considered only trading on a single stock. It would be interesting to see when investors have multiple signals on many stocks, what information they would like to share. For example, they may want to share information on one stock but not the other, depending on the correlation structure of signals.

We have also limited our attention to settings in which a manager releases the same signal to others in his social network. As there are more channels for private communications in the age of Internet, it would be interesting to see how private communications with different versions of garbled signals would affect our results.

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Appendix

Proof of Proposition 1.

Due to the symmetry of the information structure, we have

$$(28) \qquad \phi = \frac{\mathrm{Var}[v]}{\mathrm{Var}[s_i]} \left(\frac{\mathrm{Cov}[s_i, v]}{\mathrm{Var}[v]} \right)^2 = \frac{\mathrm{Cov}[s_i, v]^2}{\mathrm{Var}[v]\mathrm{Var}[s_i]}$$

The fund manager's expected profits before observing his private signal are:

$$
\begin{aligned}
\pi_M &= \mathrm{E}[\mathrm{E}[\beta s_i(v - \lambda(u + \beta s_i))|s_i]] \\
&= \mathrm{E}\left[\frac{\beta \mathrm{Cov}[v, s_i]}{2\mathrm{Var}[s_i]} s_i^2 \right] \\
&= \frac{1}{2} \beta \mathrm{Cov}[v, s_i] \\
&= \frac{1}{2} \sqrt{\frac{\sigma_u^2}{\mathrm{Var}[s_i]}} \mathrm{Cov}[v, s_i] \\
&= \frac{1}{2} \sqrt{\phi} \sigma_u \sigma_v
\end{aligned}
$$

The second equation comes from equation (8), the fourth equation from equation (9), and the last equation from equation (28). Q.E.D.

Proof of Proposition 2.

Fund manager i's unconditional expected profits are

$$
\begin{aligned}
\pi_C &= \mathrm{E}[\mathrm{E}[\beta s_i(v - \lambda(u + \beta v))|s_i]] \\
&= \mathrm{E}\left[\beta(1 - \lambda\beta) \frac{\mathrm{Cov}[s_i, v]}{\mathrm{Var}[s_i]} s_i^2 \right] \\
&= \beta(1 - \lambda\beta)\mathrm{Cov}[s_i, v] \\
&= \frac{1}{1 + \sigma_v^2/(N\mathrm{Var}[s_i])} \sqrt{\frac{\sigma_u^2}{N\mathrm{Var}[s_i]}} \frac{\sigma_v^2}{N} \\
&= \frac{\sqrt{\phi}\sigma_u\sigma_v}{\sqrt{N}(1 + N\phi)}
\end{aligned}
$$

The second equation holds because $E[v|s_i] = \frac{\text{Cov}[s_i, v]}{\text{Var}[s_i]} s_i$, plugging the expressions of β and λ into the third equation gives the fourth equation, and the last equation comes from the definition of ϕ. Q.E.D.

Proof of Proposition 3. The fund managers will herd to acquire information and trade on the same stock if and only if $\pi_M < \pi_C$, which means $\sqrt{N}(1 + N\phi) < 2$, i.e., $\phi < \frac{1}{N}\left(\sqrt{\frac{4}{N}} - 1\right)$. Because the range of ϕ is $(0, 1]$, we must have $N < 4$. If $N \geq 4$, the fund managers will not herd to acquire information and trade on the same stock. Q.E.D.

Proof of Proposition 4. Given $N > 2$ and $G > 1$, it's straightforward to verify that we have $\phi(G) = 1/N$ when ϕ equals to

$$\hat{\phi} = \frac{N - G}{N[G(N-2) + 1]} = \frac{N - G}{N[(N - G) + (N - 1)(G - 1)]} < 1/N$$

So if the fund managers possess complementary private information and there exists a social network with size G, it's possible that they behave in unison like a monopolist. Q.E.D.

Proof of Equation (17). Differentiating π_C with respect to $\phi(G)$ gives

$$\frac{\partial \pi_C}{\partial \phi(G)} \propto \frac{N(1/N - \phi(G))}{2(1 + N\phi(G))\sqrt{\phi(G)}} \begin{cases} > 0, & \text{if} \quad \phi(G) < 1/N \\ = 0, & \text{if} \quad \phi(G) = 1/N \\ < 0, & \text{if} \quad \phi(G) > 1/N \end{cases}$$

The fund manager's expected profit is a concave function in $\phi(G)$. It increases in $\phi(G)$ when $\phi(G) < 1/N$, reaches its maximum at $\phi(G) = 1/N$, and decreases as $\phi(G) > 1/N$.

When $\phi(G) = 1/N$, we have $G = \frac{N(1-\phi)}{1+N(N-2)\phi}$. However, G has to be a natural

number. We have to compare which is larger, $\pi_C(\phi(\hat{G}))$ and $\pi_C(\phi(\hat{G}+1))$.

$$\pi_C(\phi(\hat{G}+1)) - \pi_C(\phi(\hat{G}))$$

$$\propto \frac{\phi(\hat{G}+1)}{(1+N\phi(\hat{G}+1))} - \frac{\phi(\hat{G})}{(1+N\phi(\hat{G}))}$$

$$\propto (\phi(\hat{G}+1) - \phi(\hat{G}))(1 - N^2\phi(\hat{G})\phi(\hat{G}+1)) \begin{cases} < 0, & \text{if} \quad \phi(\hat{G})\phi(\hat{G}+1) > \frac{1}{N^2}, \\ = 0, & \text{if} \quad \phi(\hat{G})\phi(\hat{G}+1) = \frac{1}{N^2}, \\ > 0, & \text{if} \quad \phi(\hat{G})\phi(\hat{G}+1) < \frac{1}{N^2}. \end{cases}$$

Q.E.D.

Proof of Proposition 6. Here, we need to find conditions under which $\pi_M < \pi_C$, which is equivalent to

$$\frac{\sqrt{\phi}\sigma_u\sigma_v}{2} < \frac{\sqrt{\phi(G)}\sigma_u\sigma_v}{\sqrt{N}(1+N\phi(G))}$$

(29)

$$\Leftrightarrow \quad \frac{1 - \sqrt{1-N^2\phi}}{\sqrt{N^3\phi}} < \sqrt{\phi(G)} = \sqrt{\frac{(N-1)G\phi}{N(G-1)\phi + N - G}} < \frac{1 + \sqrt{1-N^2\phi}}{\sqrt{N^3\phi}}.$$

Solutions of the above set of inequalities are the intersection of $0 < \phi < 1/N^2$ and solutions of the inequality $f(\phi) < 0$,

$$f(\phi) \equiv N^2(NG-1)^2\phi^2 - 2[(N^2+2N-2)G^2 - (N^3+3N-2)G+N^2]\phi + \left(5 - \frac{4}{N}\right)(N-G)\left(\frac{N^2}{5N-4} - G\right).$$

When $\phi = 1/N^2$,

$$f\left(\frac{1}{N^2}\right) = \left(\frac{(N-1)G}{N} - \frac{G-1}{N} - (N-G)\right)^2 > 0,$$

and

$$f(0) = \left(5 - \frac{4}{N}\right)(N - G)\left(\frac{N^2}{5N - 4} - G\right)\begin{cases} \leq 0, & \text{if} \quad N \geq G > \frac{N^2}{5N-4}, \\ \geq 0, & \text{if} \quad 1 \leq G \leq \frac{N^2}{5N-4}. \end{cases}$$

Considering the polynomial in G, $h(G) = (N^2 + 2N - 2)G^2 - (N^3 + 3N - 2)G + N^2$, which is a convex function of G and has

$$h(1) = -N(N - 1)^2 <= 0, \text{ and}$$

$$h\left(\frac{N^2}{5N - 4}\right) \propto -N^2(4N + 5)(N - 3) - (7N^2 + 18N + 24) < 0,$$

so we must have $h(G) \leq 0$ over $\left[\frac{N^2}{5N-4}, 1\right]$ when $N < 4$ or $\left[1, \frac{N^2}{5N-4}\right]$ when $N \geq 4$, which means $f(\phi)$ is increasing in ϕ when ϕ goes from 0 to $1/N^2$. That is there is no solutions to the set of inequalities (29) when $G \leq \frac{N^2}{5N-4}$.

By the Intermediate Value Theorem, there must be a $\phi \in (0, 1/N^2)$ such that $f(\phi) = 0$ if $G > \left\lfloor \frac{N^2}{5N-4} \right\rfloor$. So, the conditions under which inequalities (29) hold is,

$$N \geq G > \left\lfloor \frac{N^2}{5N - 4} \right\rfloor$$

$$0 < \phi < \bar{\phi}.$$

with $\bar{\phi} = \frac{[(N^2+2N-2)G^2 - (N^3+3N-2)G + N^2] + \sqrt{[(N^2+2N-2)G^2 - (N^3+3N-2)G + N^2]^2 - N(NG-1)^2(N-G)(N^2 - (5N-4)G)}}{N^2(NG-1)^2}$.

Proof of Lemma III.1.

All random variables are jointly normally distributed, so we have

$$E[v|S_i] = \text{Cov}[v, S_i]\text{Var}[S_i]^{-1}S_i$$

$$= \alpha_s^v s_i + \alpha_\eta^v \eta_i + \alpha_\epsilon^v \epsilon_i$$

with

$$
\begin{bmatrix} \alpha_s^v \\ \alpha_\eta^v \\ \alpha_\epsilon^v \end{bmatrix} = \begin{bmatrix} \sigma_v^2/N \\ 0 \\ \sigma_v^2/N \end{bmatrix} \begin{bmatrix} \sigma_s^2 & 0 & \rho_0\sigma_s^2 \\ 0 & \sigma_\eta^2 & 0 \\ \rho_0\sigma_s^2 & 0 & \frac{(1+(G-2)\rho_0)\sigma_s^2+\sigma_\eta^2}{G-1} \end{bmatrix}^{-1}
$$

$$
= \begin{bmatrix} \frac{(1+(N-1)\rho_0)(\sigma_\eta^2+(1-\rho_0)\sigma_s^2)}{\sigma_\eta^2+(1-\rho_0)(1+(G-1)\rho_0)\sigma_s^2} \\ 0 \\ \frac{(1+(N-1)\rho_0)(G-1)(1-\rho_0)\sigma_s^2}{\sigma_\eta^2+(1-\rho_0)(1+(G-1)\rho_0)\sigma_s^2} \end{bmatrix}
$$

The proof of equation (22) can proceed in the same way. Q.E.D.

Proof of Proposition 7. Fund manager i's expected profits when he submits an order of x_i given other fund managers apply the trading strategy as in equation (24),

$$
E[x_i(v - \lambda(u + (\beta_s + \beta_\epsilon)v + (\beta_\eta + \beta_\epsilon)\omega + x_i - (\beta_s s_i + \beta_\eta \eta_i + \beta_\epsilon \epsilon_i)))|S_i]
$$

$$
= x_i[(1 - \lambda(\beta_s + \beta_\epsilon))E[v|S_i] - \lambda(\beta_\eta + \beta_\epsilon)E[\omega|S_i] + \lambda(\beta_s s_i + \beta_\eta \eta_i + \beta_\epsilon \epsilon_i) - \lambda x_i]
$$

So the optimal x_i is

$$
x_i = \frac{1}{2\lambda}[((1 - \lambda(\beta_s + \beta_\epsilon))\alpha_s^v + \lambda\beta_s - \lambda(\beta_\eta + \beta_\epsilon)\alpha_s^\omega)s_i + ((1 - \lambda(\beta_s + \beta_\epsilon))\alpha_\eta^v + \lambda\beta_\eta - \lambda(\beta_\eta + \beta_\epsilon)\alpha_\eta^\omega)\eta_i
$$

$$
+ ((1 - \lambda(\beta_s + \beta_\epsilon))\alpha_\epsilon^v + \lambda\beta_\epsilon - \lambda(\beta_\eta + \beta_\epsilon)\alpha_\epsilon^\omega)\epsilon_i]
$$

which means

$$
(30\text{a}) \qquad \beta_s = \frac{(1 - \lambda(\beta_s + \beta_\epsilon))\alpha_s^v + \lambda\beta_s - \lambda(\beta_\eta + \beta_\epsilon)\alpha_s^\omega}{2\lambda}
$$

$$
(30\text{b}) \qquad \beta_\eta = \frac{(1 - \lambda(\beta_s + \beta_\epsilon))\alpha_\eta^v + \lambda\beta_\eta - \lambda(\beta_\eta + \beta_\epsilon)\alpha_\eta^\omega}{2\lambda}
$$

$$
(30\text{c}) \qquad \beta_\epsilon = \frac{(1 - \lambda(\beta_s + \beta_\epsilon))\alpha_\epsilon^v + \lambda\beta_\epsilon - \lambda(\beta_\eta + \beta_\epsilon)\alpha_\epsilon^\omega}{2\lambda}
$$

From equation (30), one can compute $\lambda(\beta_s + \beta_\epsilon)$ and $\lambda(\beta_\eta + \beta_\epsilon)$ as

$$(31a) \qquad \lambda(\beta_s + \beta_\epsilon) = \frac{(\alpha_s^v + \alpha_\epsilon^v)(1 + \alpha_\eta^\omega + \alpha_\epsilon^\omega) - (\alpha_\eta^v + \alpha_\epsilon^v)(\alpha_s^\omega + \alpha_\epsilon^\omega)}{(1 + \alpha_s^v + \alpha_\epsilon^v)(1 + \alpha_\eta^\omega + \alpha_\epsilon^\omega) - (\alpha_\eta^v + \alpha_\epsilon^v)(\alpha_s^\omega + \alpha_\epsilon^\omega)}$$

$$(31b) \qquad \lambda(\beta_\eta + \beta_\epsilon) = \frac{\alpha_\eta^v + \alpha_\epsilon^v}{(1 + \alpha_s^v + \alpha_\epsilon^v)(1 + \alpha_\eta^\omega + \alpha_\epsilon^\omega) - (\alpha_\eta^v + \alpha_\epsilon^v)(\alpha_s^\omega + \alpha_\epsilon^\omega)}$$

Plugging equation (31) into equation (26) gives

$$\lambda = \frac{1}{\sigma_u}\sqrt{\lambda(\beta_s + \beta_\epsilon)\sigma_v^2 - (\lambda^2(\beta_s + \beta_\epsilon)^2\sigma_v^2 + \lambda^2(\beta_\eta + \beta_\epsilon)^2\sigma_\eta^2)}$$

and β's can be computed from equation (30).

The expected profits that fund manager i earns at the equilibrium is

$$
\begin{aligned}
\pi_C^n &= \mathrm{E}[(\beta_s s_i + \beta_\eta \eta_i + \beta_\epsilon \epsilon_i)\mathrm{E}[v - \lambda(u + (\beta_s + \beta_\epsilon)v + (\beta_\eta + \beta_\epsilon)\omega)|S_i]] \\
&= \mathrm{E}[(\beta_s s_i + \beta_\eta \eta_i + \beta_\epsilon \epsilon_i)(((1 - \lambda(\beta_s + \beta_\epsilon))\alpha_s^v - \lambda(\beta_\eta + \beta_\epsilon)\alpha_s^\omega)s_i \\
&\quad + ((1 - \lambda(\beta_s + \beta_\epsilon))\alpha_\eta^v - \lambda(\beta_\eta + \beta_\epsilon)\alpha_\eta^\omega)\eta_i + ((1 - \lambda(\beta_s + \beta_\epsilon))\alpha_\epsilon^v - \lambda(\beta_\eta + \beta_\epsilon)\alpha_\epsilon^\omega)\epsilon_i)] \\
&= \lambda\mathrm{E}[(\beta_s s_i + \beta_\eta \eta_i + \beta_\epsilon \epsilon_i)^2] \\
&= \lambda[\beta_s, \beta_\eta, \beta_\epsilon]\mathrm{Var}[S_i][\beta_s, \beta_\eta, \beta_\epsilon]'.
\end{aligned}
$$

The second equation comes from Lemma III.1 and the third equation from equation (30). Q.E.D.

Proof of Proposition 8. In a dynamic trading setting, all is similar to our static setting except that N informed traders could buy and sell the stock over M periods. At time 0 before any trading takes place, the informed trader i receives a signal s_{i0}, $i = 1, \ldots, N$. The variance of s_{i0} is Λ_0, the covariance between any two signals s_{i0} and s_{j0} is Ω_0 and hence we have $\sigma_v^2 \equiv \Sigma_0 = N(\Lambda_0 + (N-1)\Omega_0)$. In addition to the informed traders, there are liquidity traders, whose trade in period m is u_m, the realization of a normally distributed random variable with mean zero and variance σ_u^2. There is also a risk-neutral and competitive market maker who observes the total order flow and sets the price equal to the conditional

expected value of the stock, based on the public information he received up to and including that period. Denote by x_{im} the informed trader i's order at time m, the market maker observes the total order flow, $y_m = \sum_{i=1}^{N} x_{im} + u_m$ and sets the price at time m so that:

$$p_m = \mathrm{E}[v|y_1, \ldots, y_m].$$

After m periods of trading, the market maker has observed order flows $(y_1, \ldots, y_m)$ and updates his estimate of the stock, v, to

$$p_m = \mathrm{E}[\sum_{i=1}^{N} s_{i0}|y_1, \ldots, y_m] = \sum_{i=1}^{N} \mathrm{E}[s_{i0}|y_1, \ldots, y_m] = \sum_{i=1}^{N} t_{im}.$$

Given the model structure described above, we are interested in linear Markov equilibria, where the demands of the informed traders, market maker learning about the signal vector, and market maker learning about the true value of the asset take the form:

(32) $$x_{im} = \beta_m s_{im},$$

(33) $$t_{im} = t_{im-1} + \zeta_m y_m,$$

(34) $$p_m = p_{m-1} + \lambda_m y_m.$$

For each informed trader, starting with $\hat{s}_{j0}^i = s_{j0}$, we recursively define:

$$\hat{y}_m^i = \sum_{j=1}^{N} \beta_m \hat{s}_{jm-1}^i + u_m, \hat{p}_n^i = \sum_{k=1}^{m} \lambda_k \hat{y}_k^i$$

$$\hat{t}_{jn}^i = \sum_{k=1}^{m} \xi_k \hat{y}_k^i, \hat{s}_{jn}^i = s_{j0} - \sum_{k=1}^{m} \xi_k \hat{y}_k^i$$

where $\hat{y}_m^i$ is the order flow that would have occurred in the mth round of trading if trader i had followed the equilibrium strategy in the first m periods of trading.

Similarly, after m rounds of trading, $\hat{p}_n^i$ is the price that prevails in the nth round of trading, $\hat{t}_{jn}^i$ is the market maker's conditional expected value of information trader i's information, and $\hat{s}_{jn}^i$ is the information that informed trader j has that is not known to the market maker, if trader i had followed the equilibrium strategy $(\beta \hat{s}_{i0}^i, \ldots, \beta \hat{s}_{in-1}^i)$ in the first n periods of trading.

We now conjecture the optimal strategy of a trader who has played an arbitrary, suboptimal strategy $x_{i1}, \ldots, x_{in-1}$. Given his past suboptimal play, the future strategy will not be the conjectured optimal strategy in equation (32).However, his conjectured optimal strategy from trading period m and beyond is of the form:

$$x_{ik} = \beta_k \hat{s}_{k-1}^i + \gamma_k (\hat{p}_{k-1}^i - p_{k-1}), \text{for } k = m \text{ to } M.$$

Thus the optimal strategy from this period and beyond is the same as the optimal strategy that would have occurred given past optimal play, plus a second term that depends on the difference between the price that would have occurred in this period had trader i followed the optimal strategy in the past and the actual price this period. The value function of trader i after stage $m - 1$ is conjectured to be:

(35) $\quad V_i(\hat{s}_{im-1}^i, \hat{p}_{m-1}^i - p_{m-1}) = \alpha_{m-1}(\hat{s}_{im-1}^i)^2 + \psi_{m-1}\hat{s}_{im-1}^i(\hat{p}_{m-1}^i - p_{m-1})$

(36) $\qquad\qquad\qquad\qquad\qquad\qquad + \mu_{m-1}(\hat{p}_{m-1}^i - p_{m-1})^2 + \delta_{m-1}.$

Given past optimal play by trader i, we have $\hat{p}_{m-1}^i = p_{m-1}$, so that only the first and fourth terms on the right hand of the value function remain. All parameters defined above could be solved from the difference equation system in the proposition 1 of Foster and Viswanathan (1996). Taking expectation of equation (35) at $m = 1$ gives the expected trading profits of trader i.

When the number of trading periods goes to infinity, the dynamic trading model of Foster and Viswanathan (1996) approaches the continuous-time trading model of Back, Cao, and Willard (2000). Similarly, we assume $N \geq 1$ risk-neutral informed traders continuously trade the stock over the time period $[0, 1)$. An

announcement is made at time 1 that reveals the liquidation value of the asset. At each time t prior to time 1, the asset price $P(t)$ is set through competition by risk-neutral market makers. Market makers observe the sum of the orders of the informed traders and liquidity traders. The cumulative order process of the liquidity traders is assumed to be a Wiener process U.

We are interested in only the linear equilibria, meaning that there are functions α, β, and λ such that the rate of trade of each informed trader i at each time t is

$$\alpha(t)P(t) + \beta(t)s_i$$

and the price changes according to

$$dP(t) = \lambda(t)\left\{dU(t) + \sum_{i=1}^{N}[\alpha(t)P(t) + \beta(t)s_i]dt\right\}.$$

Parameters such as α, β, and λ could be solved from Theorem 1 of Back, Cao, and Willard (2000). In equilibrium, the expected profit of each informed trader is

$$\frac{1}{N}\left(\frac{\sigma_v^2}{\kappa}\right)^{1/2}\int_1^{\infty} x^{-2/N}e^{-x(1-\phi)/N\phi}dx$$

with

$$\kappa = \int_1^{\infty} x^{2(N-2)/N}e^{-2x(1-\phi)/N\phi}dx.$$

Clearly, if the fund managers share all his information to all managers, they have identical information that will be revealed right away at the beginning of trading and their expected profits will be pulled to 0. So it will never be optimal to share all information with others. In Proposition 13 of Cao, Ma, and Ye (2015), they prove that the expected profits a fund manager earns while trading alone are always larger than what he's able to earn while trading with other fund managers, which means fund managers will never herd in the absence of social networking.

Figure 3**A** shows that there exists sufficiently small ϕ such that it's optimal for informed traders to share information and herd on information acquisition for

$G < N$. We next prove this result rigorously.