

Financial Intermediation Chains in an OTC Market*

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Abstract

This paper analyzes financial intermediation chains in a search model with an endogenous intermediary sector. We show that the chain length and price dispersion among inter-dealer trades are decreasing in search cost, search speed, and market size, but increasing in investors' trading needs. Using data from the U.S. corporate bond market, we find evidence broadly consistent with these predictions. Moreover, as search speed approaches infinity, the search equilibrium does *not* always converge to the centralized-market equilibrium: prices and allocation converge, but the trading volume may not. Finally, the efficiency of the intermediary sector size is analyzed.

JEL Classification Numbers: G10.

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1 Introduction

Financial intermediation chains appear to be getting longer over time, that is, more and more layers of intermediaries are involved in financial transactions. For instance, with the rise of securitization in the U.S., the process of channeling funds from savers to investors is getting increasingly complex (Adrian and Shin (2010)). This multi-layer nature of intermediation also appears in many other markets. For example, the average *daily* trading volume in the Federal Funds market is more than ten times the aggregate Federal Reserve balances (Taylor (2001)). The trading volume in the foreign exchange market appears disproportionately large relative to international trade.¹

These examples suggest the prevalence of intermediation chains. What determines the chain length? How does it respond to the changes in economic environment? What are the implications on asset prices, trading volume, and investor welfare? To analyze these questions, we need theories that endogenize the chain of intermediation. The literature so far has not directly addressed these issues. Our paper attempts to fill this gap.

The full answer to the above questions is likely to be complex and hinges on a variety of issues (e.g., transaction cost, trading technology, regulatory and legal environment, firm boundary). As an initial step, however, we abstract away from many of these aspects to analyze a simple model of an over-the-counter (OTC) market, and assess its predictions empirically.²

In the model, investors have heterogeneous valuations of an asset. Their valuations change over time, leading to trading needs. When an investor enters the market to trade, he faces a delay in locating his trading partner. In the mean time, he needs to pay a search cost each period until he finishes his transaction. Due to the delay and search cost, not all investors choose to stay in the market continuously, giving rise to a role of intermediation. Some investors choose to be intermediaries. They stay in the market continuously and act as

¹According to the Main Economic Indicators database, the *annual* international trade in goods and services is around \$4 trillion in 2013. In that same year, however, the Bank of International Settlement estimates that the *daily* trading volume in the foreign exchange market is around \$5 trillion.

²OTC markets are enormous. According to the estimate by the Bank for International Settlements, the total outstanding OTC derivatives is around 711 trillion dollars in December 2013.

dealers. Once they acquire the asset, they immediately start searching to sell it to someone who values it more. Similarly, once they sell the asset, they immediately start searching to buy it from someone who values it less. In contrast, other investors act as *customers*: once their trades are executed, they leave the market to avoid the search cost. We solve the model in closed-form, and the main implications are the following.

First, when the search cost is lower than a certain threshold, there is an equilibrium with an endogenous intermediary sector. Investors with intermediate valuations of the asset choose to become dealers and stay in the market continuously, while others (who have with high or low valuations) choose to be customers, and leave the market once their transactions are executed. Intuitively, if an investor has a high valuation of an asset, once he obtains the asset, there is little benefit for him to stay in the market since the chance of finding someone with an even higher valuation is low. Similarly, if an investor has a low valuation of the asset, once he sells the asset, there is little benefit for him to stay in the market. In contrast to the above equilibrium, when the search cost is higher than the threshold, there is an equilibrium with no intermediary. Only investors with very high or low valuations enter the market, and they leave the market once their trading needs are satisfied. In this case, the investors with intermediate valuations have weak trading needs, and choose to stay out of the market to avoid search cost.

Second, at each point in time, there is a continuum of prices for the asset. When a buyer meets a seller, their negotiated price depends on their specific valuations. The delay in execution in the market makes it possible to have multiple prices for the asset. Naturally, as the search speed improves, the price dispersion reduces, and converges to zero when the search speed goes to infinity.

Third, we characterize two equilibrium quantities on the intermediary sector, which can be easily measured empirically. The first is the *dispersion ratio*, the price dispersion among inter-dealer trades divided by the price dispersion among all trades in the economy.³ The second is the *length* of the intermediation chain, the average number of layers of intermediaries

³For convenience, we use “intermediary” and “dealer” interchangeably, and refer to the transactions among dealers as “inter-dealer trades.”

for all customers' transactions. Intuitively, both variables reflect the size of the intermediary sector. When more investors choose to become dealers, the price dispersion among inter-dealer trades is larger (i.e., the dispersion ratio is higher), and customers' transactions tend to go through more layers of dealers (i.e., the chain is longer).

Our model implies that both the dispersion ratio and the chain length are decreasing in the search cost, the speed of search, and the market size, but are increasing in investors' trading frequency. Intuitively, a higher search cost means that fewer investors find it profitable to be dealers, leading to a smaller intermediary sector and hence a smaller dispersion ratio and chain length. Similarly, with a higher search speed or a larger market size, intermediation is less profitable because customers can find alternative trading partners more quickly. This leads to a smaller intermediary sector (relative to the market size). Finally, when investors need to trade more frequently, the higher profitability attracts more dealers and so increases the size of the intermediary sector.

We test these predictions using data from the U.S. corporate-bond market. The Trade Reporting and Compliance Engine (TRACE) records transaction prices, and identifies traders with the Financial Industry Regulatory Authority (FINRA) membership as "dealers," and others as "customers." This allows us to construct the dispersion ratio and chain length.

We run Fama-MacBeth regressions of the dispersion ratio and chain length of a corporate bond on proxies for search cost, market size, the frequency of investors' trading needs. Our evidence is broadly consistent with the model predictions. It is worth noting the difference between the dependent variables in the two regressions: The dispersion ratio is constructed based on price data while the chain length is based on quantity data. Yet, for almost all our proxies, their coefficient estimates have the same sign across the two regressions, as implied by our model. For example, relative to other bonds, investment-grade bonds' price dispersion ratio is on average larger by 0.007 ($t = 2.62$), and their chain length is longer by 0.245 ($t = 32.17$). If one takes the interpretation that it is less costly to make market for investment-grade bonds (i.e., the search cost is lower), then this evidence is consistent with our model prediction that the dispersion ratio and chain length are decreasing in search cost. We also include in our regressions five other variables as proxies for search cost, the

frequency of investors' trading needs, and market size. Among all 12 coefficients, 11 are highly significant and consistent with our model predictions.⁴

Fourth, when the search speed goes to infinity, the search-market equilibrium does *not* always converge to a centralized-market equilibrium. Specifically, in the case without intermediary (i.e., the search cost is higher than a certain threshold), as the search speed goes to infinity, all equilibrium quantities (prices, volumes, and allocations) converge to their counterparts in the centralized-market equilibrium. However, in the case with intermediaries (i.e., the search cost is lower than the threshold), as the search speed goes to infinity, all the prices and asset allocations converge but the trading volume in the search-market equilibrium remains higher than that in the centralized-market equilibrium. Moreover, this volume difference is larger if the search cost is smaller, and converges to infinity when the search cost goes to 0.

Intuitively, in the search market, intermediaries act as “middlemen” and generate “excess” trading. As noted earlier, when the search speed increases, the intermediary sector shrinks. However, thanks to the faster search speed, each dealer executes more trades, and the total excess trading volume is higher. As the search speed goes to infinity, the trading volume in the search market remains significantly higher than that in a centralized market. Moreover, the volume difference increases when the search cost becomes smaller because a smaller search cost implies a larger intermediary sector, which leads to a higher excess trading volume in the search market.

This insight sheds light on why a centralized-market model has trouble explaining trading volume, especially in an environment with a small transaction cost. We argue that even for the U.S. stock market, it seems plausible that some aspects of the market are better captured by a search model. For example, the cheaper and faster trading technology in the last a few decades made it possible for investors to exploit many high frequency opportunities that used to be prohibitive. Numerous trading platforms were set up to compete with main exchanges; hedge funds and especially high-frequency traders directly compete with traditional market

⁴The only exception is the coefficient for issuance size in the price dispersion ratio regression. As explained later, we conjecture that this is due to dealers' inventory capacity constraint, which is not considered in our model.

makers. The increase in turnover in the stock market in the last a few decades was likely to be driven partly by these “intermediation” trades.

Fifth, the relation between dispersion ratio, chain length and investors’ welfare is ambiguous. As noted earlier, a higher dispersion ratio and longer chain may be due to a lower search cost. In this case, they imply higher investors welfare. On the other hand, they may be due to a slower search speed. In that case, they imply lower investors welfare. Hence, the dispersion ratio and chain length are not clear-cut welfare indicators.

Finally, we examine the efficiency of the intermediary sector in our model by comparing its size with the size of the intermediary sector that would be chosen by a social planner. Our results are reminiscent of the well-known Hosios (1990) condition that efficiency is achieved only for a specific distribution of bargaining powers. In particular, we find that the size of the intermediary sector is efficient if buyers and sellers have the same bargaining power, but is generally inefficient otherwise.

1.1 Related literature

Our paper belongs to the recent literature that analyzes OTC markets in the search framework developed by Duffie, Garleanu, and Pedersen (2005). This framework has been extended to include risk-averse agents (Duffie, Garleanu, and Pedersen (2007)), unrestricted asset holdings (Lagos and Rocheteau (2009)). It has also been adopted to analyze a number of issues, such as security lending (Duffie, Garleanu, and Pedersen (2002)), liquidity provision (Weill (2007)), on-the-run premium (Vayanos and Wang (2007), Vayanos and Weill (2008)), cross-sectional returns (Weill (2008)), portfolio choices (Garleanu (2009)), liquidity during a financial crisis (Lagos, Rocheteau, and Weill (2011)), price pressure (Feldhutter (2012)), order flows in an OTC market (Lester, Rocheteau, and Weill (2014)), commercial aircraft leasing (Gavazza 2011), high frequency trading (Pagnotta and Philippon (2013)), the roles of benchmarks in OTC markets (Duffie, Dworzak, and Zhu (2014)), adverse selection and repeated contacts in opaque OTC markets (Chang (2014), Zhu (2012)) the effect of the supply of liquid assets (Shen and Yan (2014)) as well as the interaction between corporate default decision and liquidity (He and Milbradt (2013)). Another literature follows Kiyotaki

and Wright (1993) to analyze the liquidity value of money. In particular, Lagos and Wright (2005) develop a tractable framework that has been adopted to analyze liquidity and asset pricing (e.g., Lagos (2010), Lester, Postlewaite, and Wright (2012), and Li, Rocheteau, and Weill (2012), Lagos and Zhang (2014)). Trejos and Wright (2014) synthesize this literature with the studies under the framework of Duffie, Garleanu, and Pedersen (2005).

Our paper is related to the literature on the trading network of financial markets, see, e.g., Viswanathan and Wang (2004) analyze inter-dealer trades. Gofman (2010), Babus and Kondor (2012), Malamud and Rostek (2012), Chang and Zhang (2015). Atkeson, Eisfeldt, and Weill (2014) analyze the risk-sharing and liquidity provision in an endogenous core-periphery network structure. Neklyudov (2014) analyzes a search model with investors with heterogeneous search speeds to study the implications on the network structure.

Intermediation has been analyzed in the search framework (e.g., Rubinstein and Wolinsky (1987), and more recently Wright and Wong (2014), Nosal Wong and Wright (2015)). However, the literature on financial intermediation chains has been recent. Adrian and Shin (2010) document that the financial intermediation chains are becoming longer in the U.S. during the past a few decades. Li and Schurhoff (2012) document the network structure of the inter-dealer market for municipal bonds. Di Maggio, Kermani, and Song (2015) analyze the trading relation during a financial crisis. Glode and Opp (2014) focuses on the role of intermediation chain in reducing adverse selection. Afonso and Lagos (2015) analyze an OTC market for Federal Funds. The equilibrium in their model features an intermediation chain, although they do not focus on its property.

Our paper is closest to Hugonnier, Lester, and Weill (2014). They analyze a model with investors with heterogeneous valuations, highlighting that heterogeneity magnifies the impact of search frictions. In order to analyze intermediation, we generalize their model to include search cost and derive the intermediary sector, price dispersion ratio, and the intermediation chain, and also conduct empirical analysis of the intermediary sector.

The rest of the paper is as follows. Section 2 describes the model and its equilibrium. Section 3 analyzes the price dispersion and intermediation chain. Section 4 contrasts the search market equilibrium with a centralized market equilibrium. Section 5 examines the

efficiency of the size of the intermediary sector. Section 6 tests the empirical predictions. Section 7 concludes. All proofs are in the appendix.

2 Model

Time is continuous and goes from 0 to ∞ . There is a continuum of investors, and the measure of the total population is N . They have access to a riskless bank account with an interest rate r . There is an asset, which has a total supply of X units with $X < N$. Each unit of the asset pays \$1 per unit of time until infinity. The asset is traded at an over-the-counter market.

Following Duffie, Garleanu, and Pedersen (2005), we assume the matching technology as the following. Let N_b and N_s be the measures of buyers and sellers in the market, both of which will be determined in equilibrium. A buyer meets a seller at the rate λN_s , where $\lambda > 0$ is a constant. That is, during $[t, t + dt)$ a buyer meets a seller with a probability $\lambda N_s dt$. Similarly, a seller meets a buyer at the rate λN_b . Hence, the probability for an investor to meet his partner is proportional to the population size of the investors on the other side of the market. The total number of matched pairs per unit of time is $\lambda N_s N_b$. The search friction reduces when λ increases, and disappears when λ goes to infinity.

Investors have different types, and their types may change over time. If an investor's current type is Δ , he derives a utility $1 + \Delta$ when receiving the \$1 coupon from the asset. One interpretation for a positive Δ is that some investors, such as insurance companies, have a preference for long-term bonds, as modeled in Vayanos and Vila (2009). Another interpretation is that some investors can benefit from using those assets as collateral and so value them more, as discussed in Bansal and Coleman (1996) and Gorton (2010). An interpretation of a negative Δ can be that the investor suffers a liquidity shock and so finds it costly to carry the asset on his balance sheet. We assume that Δ can take any value in a closed interval. Without loss of generality, we normalize the interval to $[0, \bar{\Delta}]$.

Each investor's type changes independently with intensity κ . That is, during $[t, t + dt)$, with a probability κdt , an investor's type changes and is independently drawn from a random

variable, which has a probability density function $f(\cdot)$ on the support $[0, \bar{\Delta}]$, with $f(\Delta) < \infty$ for any $\Delta \in [0, \bar{\Delta}]$. We use $F(\cdot)$ to denote the corresponding cumulative distribution function.

Following Duffie, Garleanu, and Pedersen (2005), we assume each investor can hold either 0 or 1 unit of the asset. That is, an investor can buy 1 unit of the asset only if he currently does not have the asset, and can sell the asset only if he currently has it.

2.1 Investors' choices

All investors are risk-neutral and share the same time discount rate r . They face a search cost of c per unit of time, with $c \geq 0$. That is, when an investor searches to buy or sell in the market, he incurs a cost of $c dt$ during $[t, t + dt)$. An investor's objective function is given by

$$\sup_{\theta_\tau} \mathbf{E}_t \left[\int_t^\infty e^{-r(\tau-t)} [\theta_\tau(1 + \Delta_\tau) - \mathbf{1}_\tau c] d\tau - \int_t^\infty e^{-r(\tau-t)} P_\tau d\theta_\tau \right],$$

where $\theta_\tau \in \{0, 1\}$ is the investor's holding in the asset at time τ ; Δ_τ is the investor's type at time τ ; $\mathbf{1}_\tau$ is an indicator variable, which is 1 if the investor is searching in the market to buy or sell the asset at time τ , and 0 otherwise; and P_τ is the asset's price that the investor faces at time τ and will be determined in equilibrium.

We will focus on the steady-state equilibrium. Hence, the value function of a type- Δ investor with an asset holding θ_t at time t can be denoted as $V(\theta_t, \Delta)$. That is, the distribution of all other investors' types is not a state variable, since it stays constant over time in the steady state equilibrium.

A non-owner (whose θ_t is 0) has two choices: search to buy the asset or stay inactive. We use $V_n(\Delta)$ to denote the investor's expected utility if he chooses to stay inactive, and follows the optimal strategy after his type changes. Similarly, we use $V_b(\Delta)$ to denote the investor's expected utility if he searches to buy the asset, and follows the optimal strategy after he obtains the asset or his type changes. Hence, by definition, we have

$$V(0, \Delta) = \max(V_n(\Delta), V_b(\Delta)). \quad (1)$$

An asset owner (whose θ_t is 1) has two choices: search to sell the asset or stay inactive.

We use $V_h(\Delta)$ to denote the investor's expected utility if he chooses to be an inactive holder, and follows the optimal strategy after his type changes. Similarly, we use $V_s(\Delta)$ to denote the investor's expected utility if he searches to sell, and follows the optimal strategy after he sells his asset or his type changes. Hence, we have

$$V(1, \Delta) = \max(V_h(\Delta), V_s(\Delta)). \quad (2)$$

We conjecture, and will verify later, that in equilibrium, equation (1) implies that a non-owner's optimal choice is given by

$$\begin{cases} \text{stay out of the market if } \Delta \in [0, \Delta_b), \\ \text{search to buy the asset if } \Delta \in (\Delta_b, \bar{\Delta}], \end{cases} \quad (3)$$

where the cutoff point Δ_b will be determined in equilibrium. A type- Δ_b non-owner is indifferent between staying out of the market and searching to buy the asset. Note that due to the search friction, a buyer faces delay in his transaction. In the meantime, his type may change, and he will adjust his action accordingly. Similarly, we conjecture that equation (2) implies that an owner's optimal choice is

$$\begin{cases} \text{search to sell his asset if } \Delta \in [0, \Delta_s), \\ \text{stay out of the market if } \Delta \in (\Delta_s, \bar{\Delta}], \end{cases} \quad (4)$$

where the Δ_s will be determined in equilibrium. A type- Δ_s owner of the asset is indifferent between the two actions. A seller faces potential delay in his transaction. In the meantime, if his type changes, he will adjust his action accordingly. If an investor succeeds in selling his asset, he becomes a non-owner and his choices are then described by equation (3).

Suppose a buyer of type $x \in [0, \bar{\Delta}]$ meets a seller of type $y \in [0, \bar{\Delta}]$. The surplus from the transaction is

$$S(x, y) = \underbrace{[V(1, x) + V(0, y)]}_{\text{total utility after trade}} - \underbrace{[V(0, x) + V(1, y)]}_{\text{total utility before trade}}. \quad (5)$$

The pair can agree on a transaction if and only if the surplus is positive. We assume that the buyer has a bargaining power $\eta \in (0, 1)$, i.e., the buyer gets η of the surplus from the transaction, and hence the price is given by

$$P(x, y) = V(1, x) - V(0, x) - \eta S(x, y), \text{ if and only if } S(x, y) > 0. \quad (6)$$

The first two terms on the right hand side reflect the value of the asset to the buyer: the increase in the buyer's expected utility from obtaining the asset. Hence, the above equation implies that the transaction improves the buyer's utility by $\eta S(x, y)$.

We conjecture, and verify later, that when a buyer and a seller meet in the market, the surplus is positive if and only if the buyer's type is higher than the seller's:

$$S(x, y) > 0 \text{ if and only if } x > y. \quad (7)$$

That is, when a pair meets, a transaction occurs if and only if the buyer's type is higher than the seller's type. With this conjecture, we obtain investors' optimality condition in the steady state as the following.

$$V_h(\Delta) = \frac{1 + \Delta + \kappa \mathbf{E} [\max \{V_h(\Delta'), V_s(\Delta')\}]}{\kappa + r}, \quad (8)$$

$$V_s(\Delta) = \frac{1 + y - c}{\kappa + r} + \frac{\lambda(1 - \eta)}{\kappa + r} \int_{\Delta}^{\bar{\Delta}} S(x, \Delta) \mu_b(x) dx + \frac{\kappa \mathbf{E} [\max \{V_h(\Delta'), V_s(\Delta')\}]}{\kappa + r}, \quad (9)$$

$$V_n(\Delta) = \frac{\kappa \mathbf{E} [\max \{V_n(\Delta'), V_b(\Delta')\}]}{\kappa + r}, \quad (10)$$

$$V_b(\Delta) = -\frac{c}{\kappa + r} + \frac{\lambda\eta}{\kappa + r} \int_0^{\Delta} S(\Delta, x) \mu_s(x) dx + \frac{\kappa \mathbf{E} [\max \{V_b(\Delta), V_n\}]}{\kappa + r}, \quad (11)$$

where Δ' is a random variable with a PDF of $f(\cdot)$.

2.2 Intermediation

Decision rules (3) and (4) determine whether intermediation arises in equilibrium. There are two cases. In the first case, $\Delta_b \geq \Delta_s$, there is no intermediation. When an investor has a trading need, he enters the market. Once his transaction is executed, he leaves the market and stays inactive. In the other case $\Delta_b < \Delta_s$, however, some investors choose to be intermediaries and stay in the market continuously. If they are non-owners, they search to buy the asset. Once they receive the asset, however, they immediately search to sell the asset. For convenience, we call them "dealers."

Details are illustrated in Figure 1. Panel A is for the case without intermediation, i.e., $\Delta_b \geq \Delta_s$. If an asset owner's type is below Δ_s , as in the upper-left box, he enters the market

to sell his asset. If successful, he becomes a non-owner and chooses to be inactive since his type is below Δ_b , as in the upper-right box. Similarly, if a non-owner's type is higher than Δ_b , as in the lower-right box, he enters the market to buy the asset. If successful, he becomes an owner and chooses to be inactive because his type is above Δ_s , as in the lower-left box.

The dashed arrows in the diagram illustrate investors' choices to enter or exit the market when their types change. Suppose, for example, an owner with a type below Δ_s is searching in the market to sell his asset, as in the upper-left box. Before he meets a buyer, however, if his type changes and becomes higher than Δ_s , he will exit the market and become an inactive owner in the lower-left box. Finally, note that all investors in the interval (Δ_s, Δ_b) are inactive regardless of their asset holdings.

Panel B illustrates the case with intermediation, i.e., $\Delta_b < \Delta_s$. As in Panel A, asset owners with types below Δ_s enter the market to sell their assets. However, they have two different motives. If a seller's type is in $[0, \Delta_b)$, as in the upper-left box, after selling the asset, he will leave the market and become an inactive non-owner in the upper-right box. For convenience, we call this investor a "true seller." This is to contrast with those sellers whose types are in (Δ_b, Δ_s) , as in the middle-left box. We call them "intermediation sellers," because once they sell their assets and become non-owners (i.e., move to the middle-right box), they immediately search to buy the asset in the market since their types are higher than Δ_b . Similarly, we call non-owners with types in $(\Delta_s, \bar{\Delta}]$ "true buyers" and those with types in (Δ_b, Δ_s) "intermediation buyers."

In the intermediation region (Δ_b, Δ_s) , investors always stay in the market. If they are asset owners, they search to sell their assets. Once they become non-owners, however, they immediately start searching to buy the asset. They buy the asset from those with low types and sell it to those with high types, and make profits from their intermediation services.

What determines whether intermediation arises in equilibrium? Intuitively, a key determinant is the search cost c . Investors are only willing to become intermediaries when the expected trading profit is enough to cover the search cost. We will see later that the intermediation equilibrium arises if $c < c^*$, and the no-intermediation equilibrium arises if $c \geq c^*$, where c^* is given by equation (74) in the appendix.

Our formulation captures two important features of the intermediation sector. First, while customers leave the market once they finish their trades, intermediaries stay in the market continuously. Second, relative to intermediaries, customers tend to have more extreme valuations of the asset. For tractability, however, we also adopt some simplifications. For instance, all investors are assumed to be ex ante identical. One consequence is that the intermediaries in our model have a chance to become customers after shocks to their types. This is not as unrealistic as it appears: Of course, in reality, the identities of “dealers” and “customers” are persistent. However, identities do switch when, for example, new dealers enter, or existing dealers exit the market. For instance, Lehman Brothers was a major dealer for corporate bonds before it filed for bankruptcy in 2008. After this shock, Lehman Brothers is more like a customer in this market, trying to sell its holdings. More generally, however, traders’ identities are perhaps more persistent than implied by our formulation. In practice, some institutions specialize and act as dealers for an extended period of time. This feature can be captured in our framework by introducing a switching cost. It is natural to expect that, with this cost, investors will not switch their identities between dealers and customers, unless they experience very large shocks to their types. However, this extension makes the model much less tractable and we leave it to future research.

2.3 Demographics

We will first focus on the intermediation equilibrium case, and leave the analysis of the no-intermediation case to Section 4.3. Due to the changes in Δ and his transactions in the market, an investor’s status (type Δ and asset holding θ) changes over time. We now describe the evolution of the population sizes of each group of investors. Since we will focus on the steady-state equilibrium, we will omit the time subscript for simplicity.

We use $\mu_b(\Delta)$ to denote the density of buyers, that is, buyers’ population size in the region $(\Delta, \Delta + d\Delta)$ is $\mu_b(\Delta)d\Delta$. Similarly, we use $\mu_n(\Delta)$, $\mu_s(\Delta)$, and $\mu_h(\Delta)$ to denote the density of inactive non-owners, sellers, and inactive asset holders, respectively.

In the steady state, the cross-sectional distribution of investors’ types is given by the probability density function $f(\Delta)$. Hence, the following accounting identity holds for any

$\Delta \in [0, \bar{\Delta}]$:

$$\mu_s(\Delta) + \mu_b(\Delta) + \mu_n(\Delta) + \mu_h(\Delta) = Nf(\Delta). \quad (12)$$

Decision rules (3) and (4) imply that for any $\Delta \in (\Delta_s, \bar{\Delta}]$,

$$\mu_n(\Delta) = \mu_s(\Delta) = 0. \quad (13)$$

In the steady state, the group size of inactive holders remains a constant over time, implying that for any $\Delta \in (\Delta_s, \bar{\Delta}]$,

$$\kappa\mu_h(\Delta) = \kappa Xf(\Delta) + \lambda N_s \mu_b(\Delta). \quad (14)$$

The left hand side of the above equation is the “outflow” from the group of inactive holders: The measure of inactive asset holders in interval $(\Delta, \Delta + d\Delta)$ is $\mu_h(\Delta) d\Delta$. During $[t, t + dt)$, a fraction κdt of them experience changes in their types and leave the group. Hence, the total outflow is $\kappa\mu_h(\Delta) d\Delta dt$. The right hand side of the above equation is the “inflow” to the group: A fraction κdt of asset owners, who have a measure of X , experience type shocks and $\kappa Xf(\Delta) d\Delta dt$ investors’ new types fall in the interval $(\Delta, \Delta + d\Delta)$. This is captured by the first term in the right hand side of (14). The second term reflects the inflow of investors due to transactions. When buyers with types in $(\Delta, \Delta + d\Delta)$ acquire the asset, they become inactive asset holders, and the size of this group is $\lambda N_s \mu_b(\Delta) d\Delta dt$. Similarly, for any $\Delta \in [0, \Delta_b)$, we have

$$\mu_b(\Delta) = \mu_h(\Delta) = 0, \quad (15)$$

$$\kappa\mu_n(\Delta) = \kappa(N - X)f(\Delta) + \lambda N_b \mu_s(\Delta). \quad (16)$$

For any $\Delta \in (\Delta_b, \Delta_s)$, we have

$$\mu_n(\Delta) = \mu_h(\Delta) = 0, \quad (17)$$

$$\kappa\mu_s(\Delta) = \kappa Xf(\Delta) - \lambda\mu_s(\Delta) \int_{\Delta}^{\bar{\Delta}} \mu_b(x) dx + \lambda\mu_b(\Delta) \int_0^{\Delta} \mu_s(x) dx. \quad (18)$$

2.4 Equilibrium

Definition 1 *The steady-state equilibrium with intermediation consists of two cutoff points Δ_b and Δ_s , with $0 < \Delta_b < \Delta_s < \bar{\Delta}$, the distributions of investor groups $(\mu_b(\Delta), \mu_s(\Delta))$,*

$\mu_n(\Delta)$, $\mu_h(\Delta)$, and asset prices $P(x, y)$, such that

- the asset prices $P(x, y)$ are determined by (6),
- choices (3) and (4) are optimal for all investors,
- $(\mu_b(\Delta), \mu_s(\Delta), \mu_n(\Delta), \mu_h(\Delta))$ are time invariant, i.e., satisfy (12)–(18),
- market clears:

$$\int_0^{\bar{\Delta}} [\mu_s(\Delta) + \mu_h(\Delta)] d\Delta = X. \quad (19)$$

Theorem 1 *If $c < c^*$, where c^* is given by equation (74), there exists a unique steady-state equilibrium with $\Delta_b < \Delta_s$. The value of Δ_b is given by the unique solution to*

$$c = \frac{\lambda\kappa\eta X}{[\kappa + r + \lambda N_b(1 - \eta)](\kappa + \lambda N_b)} \int_0^{\Delta_b} F(x) dx, \quad (20)$$

the value of Δ_s is given by the unique solution to

$$c = \frac{\lambda\kappa(1 - \eta)(N - X)}{(\kappa + r + \lambda\eta N_s)(\kappa + \lambda N_s)} \int_{\Delta_s}^{\bar{\Delta}} [1 - F(x)] dx, \quad (21)$$

where N_s and N_b are given by (54) and (56).

The distributions of investor groups $(\mu_b(\Delta), \mu_s(\Delta), \mu_n(\Delta), \mu_h(\Delta))$ are given by equations (46)–(53).

When a type- x buyer ($x \in (\Delta_b, \bar{\Delta}]$) and a type- y seller ($y \in [0, \Delta_s)$) meet in the market, they will agree to trade if and only if $x > y$, and their negotiated price is given by (6), with the value function $V(\cdot, \cdot)$ given by (69)–(72).

This theorem shows that when the cost of search is smaller than c^* , there is a unique intermediation equilibrium. Investors whose types are in the interval (Δ_b, Δ_s) choose to be dealers. They search to buy the asset if they do not own it. Once they obtain the asset, however, they immediately start searching to sell it. They make profits from the differences in purchase and sale prices to compensate the search cost they incur. In contrast to these intermediaries, sellers with a type $\Delta \in [0, \Delta_s)$ and buyers with a type $\Delta \in (\Delta_b, \bar{\Delta}]$ are true buyers and true sellers, and they leave the market once they finish their transactions.

Note that investors' type distributions $(\mu_b(\Delta), \mu_s(\Delta), \mu_n(\Delta), \mu_h(\Delta))$ determine the speed with which investors meet their trading partners, which in turn determines investors' type distributions. The equilibrium is the solution to this fixed-point problem. The above theorem shows that the distributions can be computed in closed-form, making the analysis of the equilibrium tractable.

To illustrate the equilibrium, we define $R(\Delta)$, for $\Delta \in [0, \bar{\Delta}]$, as

$$R(\Delta) \equiv \frac{\mu_s(\Delta) + \mu_h(\Delta)}{\mu_b(\Delta) + \mu_n(\Delta)}.$$

That is, $R(\Delta)$ is the density ratio of asset owners (i.e., sellers and inactive holders) to nonowners (i.e., buyers and inactive nonowners). It has the following property.

Proposition 1 *In the equilibrium in Theorem 1, $R(\Delta)$ is weakly increasing in Δ : $R'(\Delta) > 0$ for $\Delta \in (\Delta_b, \Delta_s)$, and $R'(\Delta) = 0$ for $\Delta \in [0, \Delta_b) \cup (\Delta_s, \bar{\Delta}]$.*

The above proposition shows that high- Δ investors are more likely to be owners of the asset in equilibrium. The intuition is the following. As noted in (7), when a buyer meets a seller, transaction happens if and only if the buyer's type is higher than the seller's. Hence, if a nonowner has a higher Δ he is more likely to find a willing seller. On the other hand, if an owner has a higher Δ he is less likely to find a willing buyer. Consequently, in equilibrium, the higher the investor's type, the more likely he is an owner.

Proposition 2 *In the equilibrium in Theorem 1, we have $\frac{\partial P(x,y)}{\partial x} > 0$ and $\frac{\partial P(x,y)}{\partial y} > 0$.*

The price of each transaction is negotiated between the buyer and the seller, and depends on the types of both. Since there is a continuum of buyers and a continuum of sellers, there is a continuum of equilibrium prices at each point in time. The above proposition shows that the negotiated price is increasing in both the buyer's type and the seller's type. Intuitively, the higher the buyer's type x , the more he values the asset. Hence, he is willing to pay a higher price. On the other hand, the higher the seller's type y , the less eager he is in selling the asset. Hence, only a higher price can induce him to sell.

3 Intermediation Chain and Price Dispersion

If a true buyer and a true seller meet in the market, the asset is transferred without going through an intermediary. On other occasions, however, transactions may go through multiple dealers. For example, a type- Δ dealer may buy from a true seller, whose type is in $[0, \Delta_b)$, or from another dealer whose type is lower than Δ . Then, he may sell the asset to a true buyer, whose type is in $(\Delta_s, \bar{\Delta}]$, or to another dealer whose type is higher than Δ . That is, for an asset to be transferred from a true seller to a true buyer, it may go through multiple dealers.

What is the average *length* of the intermediation chain in the economy? To analyze this, we first compute the aggregate trading volumes for each group of investors. We use TV_{cc} to denote the total number of shares of the asset that are sold from a true seller to a true buyer (i.e., “customer to customer”) per unit of time. Similarly, we use TV_{cd} , TV_{dd} , and TV_{dc} to denote the numbers of shares of the asset that are sold, per unit of time, from a true seller to a dealer (i.e., “customer to dealer”), from a dealer to another (i.e., “dealer to dealer”), and from a dealer to a true buyer (i.e., “dealer to customer”), respectively. To characterize these trading volumes, we denote $F_b(\Delta)$ and $F_s(\Delta)$, for $\Delta \in [0, \bar{\Delta}]$, as

$$F_b(\Delta) \equiv \int_0^\Delta \mu_b(x) dx,$$

$$F_s(\Delta) \equiv \int_0^\Delta \mu_s(x) dx.$$

That is, $F_b(\Delta)$ is the population size of buyers whose types are below Δ , and $F_s(\Delta)$ is population size of sellers whose types are below Δ .

Proposition 3 *In the equilibrium in Theorem 1, we have*

$$\text{TV}_{cc} = \lambda F_s(\Delta_b) [N_b - F_b(\Delta_s)], \quad (22)$$

$$\text{TV}_{cd} = \lambda F_s(\Delta_b) F_b(\Delta_s), \quad (23)$$

$$\text{TV}_{dc} = \lambda [N_s - F_s(\Delta_b)] [N_b - F_b(\Delta_s)], \quad (24)$$

$$\text{TV}_{dd} = \lambda \int_{\Delta_b}^{\Delta_s} [F_s(\Delta) - F_s(\Delta_b)] dF_b(\Delta). \quad (25)$$

The above proposition characterizes the four types of trading volumes. For example, true sellers are those whose types are below Δ_b . The total measure of those investors is $F_s(\Delta_b)$. True buyers are those whose types are above Δ_s , and so the total measure of those investors is $N_b - F_b(\Delta_s)$. This leads to the trading volume in (22). The results on \mathbb{TV}_{cd} and \mathbb{TV}_{dc} are similar. Note that in these 3 types of trades, every meeting results in a transaction, since the buyer's type is always higher than the seller's. For the meetings among dealers, however, this is not the case. When a dealer buyer meets a dealer seller with a higher Δ , they will not be able to reach an agreement to trade. The expression of \mathbb{TV}_{dd} in (25) takes into account the fact that transaction occurs only when the buyer's type is higher than the seller's.

With these notations, we can define the length of the intermediation chain as

$$L \equiv \frac{\mathbb{TV}_{cd} + \mathbb{TV}_{dc} + 2\mathbb{TV}_{dd}}{\mathbb{TV}_{cd} + \mathbb{TV}_{dc} + 2\mathbb{TV}_{cc}}. \quad (26)$$

This definition implies that L is the average number of layers of dealers in the economy. To see this, let us go through the following three simple examples.⁵ First, suppose there is no intermediation in the economy and true buyers and true sellers trade directly. In this case, we have $\mathbb{TV}_{cd} = \mathbb{TV}_{dc} = \mathbb{TV}_{dd} = 0$. Hence $L = 0$, that is, the length of the intermediation chain is 0. Second, suppose a dealer buys one unit of the asset from a customer and sells it to another customer. We then have $\mathbb{TV}_{cd} = \mathbb{TV}_{dc} = 1$ and $\mathbb{TV}_{dd} = \mathbb{TV}_{cc} = 0$. Hence, the length of the intermediation chain is 1. Third, suppose a dealer buys one unit of the asset from a customer and sells it to another dealer, who then sells it to a customer. We then have $\mathbb{TV}_{cd} = \mathbb{TV}_{dc} = 1$, $\mathbb{TV}_{dd} = 1$, and $\mathbb{TV}_{cc} = 0$. Hence, the chain length is 2. In the following, we will analyze the effects of search speed λ , search cost c , market size X , and trading need κ on the intermediation chain.

3.1 Search cost c

Proposition 4 *In the equilibrium in Theorem 1, $\frac{\partial \Delta_b}{\partial c} > 0$ and $\frac{\partial \Delta_s}{\partial c} < 0$, that is, the total population size of the intermediary sector is decreasing in c .*

⁵The validity of the measure in (26) does not depend on the assumption that investors can only hold 0 or 1 unit of the asset.

Intuitively, investors balance the gain from trade against the search cost. The search cost has a disproportionately large effect on dealers since they stay active in the market constantly. Hence, when the search cost c increases, fewer investors choose to be dealers and so the size of the intermediary sector becomes smaller, i.e., the interval (Δ_b, Δ_s) shrinks. Consequently, the smaller intermediary sector leads to a shorter intermediation chain, as summarized in the following proposition.

Proposition 5 *In the equilibrium in Theorem 1, $\frac{\partial L}{\partial c} < 0$, that is, the length of the financial intermediation chain is decreasing in c .*

When c increases to c^* , the interval (Δ_b, Δ_s) shrinks to a single point and the intermediary sector disappears. Hence, we have $\lim_{c \rightarrow c^*} L = 0$. On the other hand, as c decreases, more investors choose to be dealers, leading to more layers of intermediation and a longer chain in the economy. What happens when c goes to zero?

Proposition 6 *In the equilibrium in Theorem 1, when c goes to 0, we obtain:*

$$\begin{aligned} \Delta_b &= 0, & \Delta_s &= \bar{\Delta}, \\ N_s &= X, & N_b &= N - X, \\ L &= \infty. \end{aligned}$$

As the search cost c diminishes, the intermediary sector (Δ_b, Δ_s) expands. When c goes to 0, (Δ_b, Δ_s) becomes the whole interval $(0, \bar{\Delta})$. That is, almost all investors (except zero measure of them at 0 and $\bar{\Delta}$) are intermediaries, constantly searching in the market. Hence, $N_s = X$ and $N_b = N - X$, that is, virtually every asset holder is trying to sell his asset and every non-owner is trying to buy. Since virtually all transactions are intermediation trading, the length of the intermediation chain is infinity.

This proposition demonstrates that our model is a generalization of Hugonnier, Lester, and Weill (2014), where the search cost c is 0. Their analysis highlights that heterogeneity magnifies the impact of search frictions, while our focus is the endogenous intermediation sector size and the resulting intermediation chains.

3.2 Search speed λ

Proposition 7 *In the equilibrium in Theorem 1, when λ is sufficiently large, $\frac{\partial \Delta_s - \Delta_b}{\partial \lambda} < 0$, that is, the intermediary sector shrinks when λ increases; $\frac{\partial L}{\partial \lambda} < 0$, that is, the length of the financial intermediation chain is decreasing in λ .*

The intuition for the above result is the following. As the search technology improves, a customer has a better outside option when he trades with a dealer. This is because the customer can find an alternative trading partner more quickly, if the dealer were to turn down the trade. As a result, intermediation is less profitable and the dealer sector shrinks, leading to a shorter intermediation chain.

3.3 Market size X

To analyze the effect of the market size X , we keep the ratio of investor population N and asset supply X constant. That is, we let

$$N = \phi X, \tag{27}$$

where ϕ is a constant. Hence, when the issuance size X changes, the population size N also changes proportionally. We impose this condition to shut down the effect from the change in the ratio of asset owners and non-owners in equilibrium.

Proposition 8 *In the equilibrium in Theorem 1, under condition (27), when λ is sufficiently large, $\frac{\partial \Delta_s - \Delta_b}{\partial X} < 0$, that is, the intermediary sector shrinks when the market size increases; $\frac{\partial L}{\partial X} < 0$, that is, the length of the financial intermediation chain is decreasing in the size of the market X .*

Intuitively, when the market size gets larger, it becomes easier for an investor to meet his trading partner. Hence, the effect is similar to that from an increase in the search speed λ . From the intuition in Proposition 7, we obtain that the length of the financial intermediation chain is decreasing in the size of the market.

3.4 Trading need κ

Proposition 9 *In the equilibrium in Theorem 1, when λ is sufficiently large, $\frac{\partial(\Delta_s - \Delta_b)}{\partial\kappa} > 0$, and $\frac{\partial L}{\partial\kappa} > 0$, that is, the intermediary sector expands and the length of the intermediation chain increases when the frequency of investors' trading need increases.*

The intuition for the above result is as follows. Suppose κ increases, i.e., investors need to trade more frequently. This makes it more profitable for dealers. Hence, the intermediary sector expands as more investors choose to become dealers, leading to a longer intermediation chain.

3.5 Price dispersion

Theorem 1 shows that there is a continuum of prices for the asset in equilibrium. How is the price dispersion related to search frictions? It seems reasonable to expect the price dispersion to decrease as the market frictions diminishes. However, this intuition is not complete, and the relationship between price dispersion and search frictions is more subtle.

To see this, we use D to denote the price dispersion

$$D \equiv P_{\max} - P_{\min}, \quad (28)$$

where P_{\max} and P_{\min} are the maximum and minimum prices, respectively, among all prices. Proposition 2 implies that

$$P_{\max} = P(\bar{\Delta}, \Delta_s), \quad (29)$$

$$P_{\min} = P(\Delta_b, 0). \quad (30)$$

That is, P_{\max} is the price for the transaction between a buyer of type $\bar{\Delta}$ and a seller of type Δ_s . Similarly, P_{\min} is the price of the transaction between a buyer of type Δ_b and a seller of type 0. The following proposition shows that effect of the search speed on the price dispersion.

Proposition 10 *In the equilibrium in Theorem 1, when λ is sufficiently large, $\frac{\partial D}{\partial\lambda} < 0$.*

The intuition is the following. When the search speed is faster, investors do not have to compromise as much on prices to speed up their transactions, because they can easily find alternative trading partners if their current trading partners decided to walk away from their transactions. Hence, the dispersion across prices becomes smaller when λ increases.

However, the relation between the price dispersion and the search cost c is more subtle. As the search cost increases, fewer investors participate in the market. On the one hand, this makes it harder to find a trading partner and so increases the price dispersion as the previous proposition suggests. There is, however, an opposite driving force: Less diversity across investors leads to a smaller price dispersion. In particular, as noted in Proposition 4, Δ_s is decreasing in c , that is, when the search cost increases, only investors with lower types are willing to pay the cost to try to sell their assets. As noted in (29), this reduces the maximum price P_{\max} . On the other hand, when the search cost increases, only investors with higher types are willing to buy. This increases the minimum price P_{\min} . Therefore, as the search cost increases, the second force decreases the price dispersion. The following proposition shows that the second force can dominate.

Proposition 11 *In the equilibrium in Theorem 1, the sign of $\frac{\partial D}{\partial c}$ can be either positive or negative. Moreover, when c is sufficiently small, we have $\frac{\partial D}{\partial c} < 0$.*

Price dispersion in OTC markets has been documented in the literature, e.g., Green, Hollifield, and Schurhoff (2007). Jankowitsch, Nashikkar, and Subrahmanyam (2011) proposes that price dispersion can be used as a measure of liquidity. Our analysis in Proposition 10 confirms this intuition that the price dispersion is larger when the search speed is lower, which can be interpreted as the market being less liquid. However, Proposition 11 also illustrates the potential limitation, especially in an environment with a low search cost. It shows that the price dispersion may decrease when the search cost is higher.

3.6 Price dispersion ratio

To further analyze the price dispersion in the economy, we define *dispersion ratio* as

$$DR \equiv \frac{P_{max}^d - P_{min}^d}{P_{max} - P_{min}}, \quad (31)$$

where P_{\max}^d and P_{\min}^d are the maximum and minimum prices, respectively, among inter-dealer transactions. That is, DR is the ratio of the price dispersion among inter-dealer transactions to the price dispersion among all transactions.

This dispersion ratio measure has two appealing features. First, somewhat surprisingly, it turns out to be easier to measure DR than D . Conceptually, price dispersion D is the price dispersion at a point in time. When measuring it empirically, however, we have to compromise and measure the price dispersion during *a period of time* (e.g., a month or a quarter), rather than at an instant. As a result, the asset price volatility directly affects the measure D . In contrast, the dispersion ratio DR alleviates part of this problem since asset price volatility affects both the numerator and the denominator. Second, as noted in Proposition 11, the effect of search cost on the price dispersion is ambiguous. In contrast, our model predictions on the price dispersion ratio are sharper, as illustrated in the following proposition.

Proposition 12 *In the equilibrium in Theorem 1, we have $\frac{\partial DR}{\partial c} < 0$; when λ is sufficiently large, we have $\frac{\partial DR}{\partial \lambda} < 0$, $\frac{\partial DR}{\partial \kappa} > 0$, and under condition (27) we have $\frac{\partial DR}{\partial X} < 0$.*

Intuitively, DR is closely related to the size of the intermediary sector. All these parameters (c, λ, X , and κ) affect DR through their effects on the interval (Δ_b, Δ_s) . For example, as noted in Proposition 4, when the search cost c increases, the intermediary sector (Δ_b, Δ_s) shrinks, and so the price dispersion ratio DR decreases. The intuition for the effects of all other parameters (λ, X , and κ) is similar.

In summary, both DR and L are closely related to the size of the intermediary sector. All the parameters of (c, λ, X , and κ) affect both DR and L through their effects on the size of the intermediary sector, i.e., the size of the interval (Δ_b, Δ_s) . Indeed, by comparing the above results with Propositions 5, 7, 8, and 9, we can see that, for all four parameters (c, λ, X , and κ), the effects on DR and L have the same sign.

3.7 Welfare

What are the welfare implications from the intermediation chain? For example, is a longer intermediation chain an indication of higher or lower investors' welfare? Propositions 5–12 have shed some light on this question. In particular, a longer intermediation chain is a sign of a lower c , a lower λ , a higher κ , or a lower X , which have different welfare implications. Hence, the chain length and dispersion ratio are not clear-cut indicators of investors' welfare.

For example, a lower c means that more investors search in equilibrium. Hence, high- Δ investors can obtain the asset more quickly, leading to higher welfare for all investors. On the other hand, a lower λ means that investors obtain their desired asset positions more slowly, leading to lower welfare for investors. Therefore, if the intermediation chain L becomes longer because of a lower c , it is a sign of higher investor welfare. However, if it is due to a slower search speed λ , it is a sign of lower investor welfare. A higher κ means that investors have more frequent trading needs. If L becomes longer because of a higher κ , holding the market condition constant, this implies that investors have lower welfare. Finally, if L becomes longer because of a smaller X , it means that investors execute their trades more slowly, leading to lower welfare for investors. To formalize the above intuition, we use W to denote the average expected utility across all investors in the economy. The relation between investors' welfare and those parameters is summarized in the following proposition.

Proposition 13 *In the equilibrium in Theorem 1, we have $\frac{\partial W}{\partial c} < 0$; when λ is sufficiently large, we have $\frac{\partial W}{\partial \lambda} > 0$, $\frac{\partial W}{\partial \kappa} < 0$, and under condition (27) $\frac{\partial W}{\partial X} > 0$.*

4 On Convergence

When the search friction disappears, does the search market equilibrium converge to the equilibrium in a centralized market? Since Rubinstein and Wolinsky (1985) and Gale (1987), it is generally believed that the answer is yes. This convergence result is also demonstrated in Duffie, Garleanu, and Pedersen (2005), the framework we adopted.

However, we show in this section that as the search technology approaches perfection (i.e., λ goes to infinity) the search equilibrium does *not* always converge to a centralized

market equilibrium. In particular, consistent with the existing literature, the prices and allocation in the search equilibrium converge to their counterparts in a centralized-market equilibrium, but the trading volume may not.

4.1 Centralized market benchmark

Suppose we replace the search market in Section 2 by a centralized market and keep the rest of the economy the same. That is, investors can execute their transactions without any delay. The centralized market equilibrium consists of an asset price P_w and a cutoff point Δ_w . All asset owners above Δ_w and nonowners below Δ_w stay inactive. Moreover, each nonowner with a type higher than Δ_w buys one unit of the asset instantly and each owner with a type lower than Δ_w sells his asset instantly, such that all investors find their strategies optimal, the distribution of all groups of investors remain constant over time, and the market clears. This equilibrium is given by the following proposition.

Proposition 14 *In this centralized market economy, the equilibrium is given by*

$$\Delta_w = F^{-1}\left(1 - \frac{X}{N}\right), \quad (32)$$

$$P_w = \frac{1 + \Delta_w}{r}. \quad (33)$$

The total trading volume per unit of time is

$$\mathbb{T}\mathbb{V}_w = \kappa X \left(1 - \frac{X}{N}\right). \quad (34)$$

As shown in (33), the asset price is determined by the marginal investor's valuation Δ_w . Asset allocation is efficient since (almost) all investors whose types are higher than Δ_w are asset owners, and (almost) all investors whose types are lower than Δ_w are nonowners. Trading needs arise when investors' types change. In particular, an asset owner becomes a seller if his new type is below Δ_w and a nonowner becomes a buyer if his new type is above Δ_w . In this idealized market, they can execute their transactions instantly. Hence, at each point in time, the total measure of buyers and sellers are infinitesimal, and the total trading volume during $[t, t + dt)$ is $\mathbb{T}\mathbb{V}_w dt$.

4.2 The limit case of the search market

Denote the total trading volume in the search market economy in Section 2 as

$$\text{TV} \equiv \text{TV}_{cc} + \text{TV}_{cd} + \text{TV}_{dc} + \text{TV}_{dd}. \quad (35)$$

The following proposition reports asymptotic properties of the search equilibrium.

Proposition 15 *When λ goes to infinity, the equilibrium in Theorem 1 is given by*

$$\lim_{\lambda \rightarrow \infty} \Delta_b = \lim_{\lambda \rightarrow \infty} \Delta_s = \Delta_w, \quad (36)$$

$$\lim_{\lambda \rightarrow \infty} P(x, y) = P_w \text{ for any } x < y, \quad (37)$$

$$\lim_{\lambda \rightarrow \infty} \mu_h(\Delta) = \begin{cases} Nf(\Delta) & \text{if } \Delta > \Delta_w, \\ 0 & \text{if } \Delta < \Delta_w, \end{cases} \quad (38)$$

$$\lim_{\lambda \rightarrow \infty} \mu_n(\Delta) = \begin{cases} 0 & \text{if } \Delta > \Delta_w, \\ Nf(\Delta) & \text{if } \Delta < \Delta_w, \end{cases} \quad (39)$$

$$\lim_{\lambda \rightarrow \infty} \mu_b(\Delta) = \lim_{\lambda \rightarrow \infty} \mu_s(\Delta) = 0, \quad (40)$$

$$\lim_{\lambda \rightarrow \infty} \frac{\text{TV} - \text{TV}_w}{\text{TV}_w} = \log \frac{\hat{c}}{c}, \quad (41)$$

where \hat{c} is a constant, with $\hat{c} > c$, and is given by

$$\hat{c} = \sqrt{\int_0^{\Delta_w} \frac{F(x)}{F(\Delta_w)} dx} \sqrt{\int_{\Delta_w}^{\bar{\Delta}} \frac{1 - F(x)}{1 - F(\Delta_w)} dx}. \quad (42)$$

As λ goes to infinity, many aspects of the search equilibrium converge to their counterparts in a centralized market equilibrium. First, the interval (Δ_b, Δ_s) shrinks to a single point at Δ_w (equation (36)), and the size of the intermediary sector goes to zero. Second, all transaction prices converge to the price in the centralized market, as shown in equation (37). Third, the asset allocation in the search equilibrium converges to that in the centralized market. As shown in equations (38)–(40), almost all investors whose types are higher than Δ_w are inactive asset holders, and almost all investors whose types are lower than Δ_w are inactive nonowners. The population sizes for buyers and sellers are infinitesimal.

However, there is one important difference. The equation (41) shows that as λ goes to infinity, the total trading volume in the search market equilibrium is higher than the volume

in the centralized market equilibrium. This is surprising, especially given the result in (36) that the size of the intermediary sector shrinks to 0.

It is worth emphasizing that this result is not a mathematical quirk from taking limit. Rather, it highlights an important difference between a search market and an idealized centralized market. Intuitively, the excess trading in the search market is due to intermediaries, who act as middlemen, buying the asset from one investor and selling to another. As λ increases, the intermediary sector shrinks. However, thanks to the faster search technology, each intermediary can execute more trades such that the total excess trading induced by intermediaries *increases* with λ despite the reduction of the intermediary sector size. As λ goes to infinity, the trading volume in the search market remains significantly higher than that in a centralized market.

As illustrated in (41), the difference between $\mathbb{T}\mathbb{V}$ and $\mathbb{T}\mathbb{V}_w$ is larger when the search cost c is smaller, and approaches infinity when c goes to 0. As noted in Proposition 4, the smaller the search cost c , the larger the intermediary sector. Hence, the smaller the search cost c , the larger the excess trading generated by middlemen.

These results shed some light on why centralized market models have trouble explaining trading volume, especially in markets with small search frictions. Even in the well-developed stock market in the U.S., some trading features are perhaps better captured by a search model. Over the past a few decades, the cheaper and faster technology makes it possible for investors to exploit opportunities that were prohibitive with a less developed technology. Numerous trading platforms were set up to compete with main exchanges; hedge funds and especially high-frequency traders directly compete with traditional market makers. It seems likely that the increase in turnover in the stock market in the past a few decades was driven partly by the decrease in the search frictions in the market. Intermediaries, such as high frequency traders, execute a large volume of trades to exploit opportunities that used to be prohibitive.

In summary, our analysis suggests that a centralized market model captures the behavior of asset prices and allocations when market frictions are small. However, it is not well-suited for analyzing trading volume, even in a market with a fast search speed, especially in the

case when the search cost is small.

4.3 Equilibrium without intermediation

Our discussion so far has focused on the case $c < c^*$. We now briefly summarize the analysis for the other case. As noted in Section 3.1, when c increases to c^* , the interval (Δ_b, Δ_s) shrinks to a point and the intermediary sector disappears. As one might have expected, intermediaries disappear in the equilibrium for the case of $c \geq c^*$.

Similar to the analysis in Section 2, we can construct an equilibrium for the case $c \geq c^*$. The only difference is that as described in Panel A of Figure 1, two cutoff points Δ_b and Δ_s are such that $\Delta_b \geq \Delta_s$. In the equilibrium in Theorem 1, investors with intermediate valuations become intermediaries and stay in the market continuously. In contrast, in this case with a higher search cost, investors with intermediate valuations choose not to participate in the market. Only those with strong trading motives (buyers with types higher than Δ_b and sellers with types lower than Δ_s) are willing to pay the high search cost to participate in the market. In the limit case where λ goes to infinity, as in Proposition 15, equations (36)–(40) still hold. However, we now have

$$\lim_{\lambda \rightarrow \infty} \text{TV} = \text{TV}_w.$$

That is, as λ goes to infinity, both Δ_b and Δ_s converge to Δ_w . The inactive sector shrinks to a point. Moreover, the prices, allocation, and the trading volume *all* converge to their counterparts in a centralized market equilibrium. This result further confirms our earlier intuition that, in the intermediation equilibrium in Section 2, the difference between TV and TV_w is due to the extra trading generated by intermediaries acting as middlemen.

4.4 Alternative matching functions

Section 4.2 shows that the non-convergence result on volume is due to the fact that while λ increases, the intermediary sector shrinks but each one can trade more quickly. The higher trading speed dominates the reduction in the size of the intermediary sector. One natural question is whether this result depends on the special matching function in our model. As explained in Section 2, for tractability, we adopt the matching function $\lambda N_b N_s$. Does our

non-convergence conclusion depend on this assumption?

To examine this, we modify our model to have a more general matching function: We now assume that the matching function is $\lambda Q(N_b, N_s)$, where $Q(\cdot, \cdot)$ is homogeneous of degree k ($k > 0$) in N_b and N_s . The matching function in our previous analysis, $\lambda N_b N_s$, is a special case with homogeneity of degree 2. The rest of the model is kept the same as in Section 2. We construct an intermediation equilibrium that is similar to the one in Theorem 1, and let λ go to infinity to compare the limit equilibrium with the centralized market equilibrium.

The conclusions based on this general matching function remain the same as those in Section 4.2. When λ goes to infinity, both the prices and allocation converge to their counterparts in a centralized market equilibrium, but the trading volume does not. Interestingly, the trading volume in this generalized model converges to exactly the same value as in our previous model, and is given by (41).

5 Efficiency

This section analyzes the efficiency of the intermediary sector size. Specifically, let's imagine a social planner, who can determine the choices of all investors. That is, the social planner chooses the two cutoff points in (3) and (4) to maximize the average of all investors' expected utility over their life time. Investors follow this decision rule set by the social planner, and face the same market frictions as described in Section 2. Compared to this social planner equilibrium, does the decentralized equilibrium in Section 3 have efficient amount of intermediaries? In other words, if we use Δ_b^e and Δ_s^e to denote the two cut-off points in the social planner case, the question becomes: does the interval (Δ_b^e, Δ_s^e) coincide with (Δ_b, Δ_s) in Theorem 1? The asymptotic analysis in the following proposition shows that this is generally not the case.

Proposition 16 *Suppose λ is sufficiently large. If $\eta = 1/2$, the intermediary sector in the decentralized equilibrium is close to that in the social planner case:*

$$\Delta_b = \Delta_b^e + o(\lambda^{-1/2}), \quad (43)$$

$$\Delta_s = \Delta_s^e + o(\lambda^{-1/2}). \quad (44)$$

If $\eta \neq 1/2$, however, the decentralized equilibrium may have too much or too little intermediation.

The above results are reminiscent of the Hosios (1990) condition that efficiency is achieved only for a specific distribution of bargaining powers between buyers and sellers. The matching function we adopted is symmetric for buyers and sellers, and our proposition shows that the efficiency is achieved when the buyers and sellers have the same bargaining power. In the case of $\eta \neq 1/2$, however, the decentralized equilibrium is generally inefficient. We illustrate in the proof of this proposition that the decentralized equilibrium may have too much or too little intermediation, depending on the distribution of investors' types $F(\cdot)$.

6 Empirical Analysis

In this section, we conduct empirical tests of the model predictions on the intermediation chain length L and the price dispersion ratio DR . We choose to analyze the U.S. corporate bond market, which is organized as an OTC market. Moreover, a large panel dataset is available that makes it possible to conduct the tests reliably. Finally, some of the propositions in Section 3 were proved under the condition that λ is sufficiently large. It might be natural to expect that the search speed in the corporate bond market in the U.S. is sufficiently fast.

6.1 Hypotheses

Our analysis in Section 3 provides predictions on the effects of search cost c , market size X , trading need κ , and search technology λ . There is perhaps little variation in the search technology λ across corporate bonds in our sample during 2002–2012. Hence, our empirical analysis will focus on the cross-sectional analysis on the effects of c , X , and κ .

Specifically, we obtain a number of observable variables that can be used as proxies for these three parameters. Table 1 summarizes the interpretations of our proxies and model predictions. We use issuance size as a proxy for the market size X . Another variable that captures the effect of market size is bond age, i.e., the number of years since issuance. The idea is that after a corporate bond is issued, as time goes by, a larger and larger fraction of

the issuance reaches long-term buy-and-hold investors such as pension funds and insurance companies. Hence, the active size of the market becomes smaller as the bond age increases. With these interpretations, Propositions 8 and 12 imply that the intermediation chain length L and price dispersion ratio DR should be decreasing in the issuance size, but increasing in bond age.

We use turnover as a proxy for the frequency of investors' trading need κ . The higher the turnover, the more frequent the trading needs are. Propositions 9 and 12 imply that the chain length L and dispersion ratio DR should be increasing in turnover.

As proxies for the search cost c , we use credit rating, time to maturity, and effective bid-ask spread. The idea is that these variables are related to the cost that dealers face. For example, all else being equal, it is cheaper for dealers to make market for investment-grade bonds than for high-yield or non-rated bonds, perhaps because dealers face less inventory risk and less capital charge for holding investment-grade bonds. Hence, our interpretation is that the search cost c is smaller for investment-grade bonds. Moreover, bonds with longer maturities are more risky, and so more costly for dealers to make market (i.e., c is higher). Finally, everything else being equal, a larger effective bid-ask spread implies a higher profit for dealers (i.e., c is lower). With these interpretations, Propositions 4 and 12 imply that the chain length L and price dispersion ratio DR should be larger for investment-grade bonds, and for bonds with shorter time to maturity or larger bid-ask spreads.

Our goal here is to assess if our model can describe the behavior of intermediation chains and price dispersion in the corporate bond market. We are certainly not drawing causality inferences. Rather, we attempt to examine if the correlations appear consistent with the model implications in equilibrium. We keep in mind the possible endogeneity of the independent variables, especially the effective bid-ask spread, and re-run our analysis after dropping this variable.

6.2 Data

Our sample consists of corporate bonds that were traded in the U.S. between July 2002 and December 2012. We combine two databases: the Trade Reporting and Compliance Engine

(TRACE) and the Fixed Income Securities Database (FISD). TRACE contains information about corporate bond transactions, such as date, time, price, and volume of a transaction. The dataset also classifies all transactions into “dealer-to-customer” or “dealer-to-dealer” transactions.⁶ We rely on this classification to construct our measure of chain length L and price dispersion ratio DR .

The FISD database contains information about a bond’s characteristics, such as bond type, date and amount of issuance, maturity, and credit rating. We merge the two databases using 9-digit CUSIPs. The initial sample from TRACE contains a set of 64,961 unique CUSIPs; among them, 54,587 can be identified in FISD. We include in our final sample corporate debentures (\$8.5 trillion total issuance amount, or 62% of the sample), medium-term notes (\$2.2 trillion total issuance amount, or 16% of the sample), and convertibles (\$0.6 trillion issuance amount, or 4% of the sample). In total, we end up with a sample of 25,836 bonds with a total issuance amount of \$11.3 trillion.

We follow the definition in (26) to construct the chain length L for each corporate bond during each period, where $TV_{cd} + TV_{dc}$ is the total dealer-to-customer trading volume and TV_{dd} is the total dealer-to-dealer trading volume during that period. In our data, $TV_{cc} = 0$, that is, there is no direct transaction between two customers. Hence, the chain length is always larger than or equal to 1.

We obtain the history of credit ratings on the bond level from FISD. For each bond, we construct its credit rating history at the daily frequency: for each day, we use credit rating by S&P if it is available, otherwise, we use Moody’s rating if it is available, and use Fitch’s rating if both S&P and Moody’s ratings are unavailable. In the case that a bond is not rated by any of the three credit rating agencies, we classify it as “not rated.” We use the rating on the last day of the period to create a dummy variable IG , which equals one if a bond has an investment-grade rating, and zero otherwise.

We use *Maturity* denote the time to maturity of a bond, measured in years, use *Age* to denote the time since issuance of a bond, denominated in years, use *Size* to denote issuance

⁶According to *TRACE User Guide*, FINRA members are classified as “dealers” and non-FINRA member institutions and retail accounts are classified as “customers.”

size of a bond, denominated in million dollars, and use *Turnover* to denote the total trading volume of a bond during the period, normalized by its *Size*.

To measure the effective bid-ask spread of a bond, denoted as *Spread*, we follow Bao, Pan, and Wang (2011) to compute the square root of the negative of the first-order autocovariance of changes in consecutive transaction prices during the period, which is based on Roll (1984)'s measure of effective bid-ask spread.

We follow the definition in equation (31) to construct the price dispersion ratio, *DR*, for each bond and time period, where P_{\max}^d and P_{\min}^d are the maximum and minimum transaction prices among dealer-to-dealer transactions according to the classification by TRACE, and P_{\max} and P_{\min} are the maximum and minimum transaction prices among all transactions.

6.3 Summary statistics

Table 2 reports the summary statistics for variables measured at the monthly frequency. To rule out extreme outliers, which are likely due to data error, we winsorize our sample by dropping observations below the 1st percentile and above 99th percentile. For the overall sample, the average chain length is 1.73. There is significant variation. The chain length is 7.00 and 1.00 at the 99th and 1st percentiles, respectively. For investment-grade bonds, the average chain length is 1.81 and the 99th percentile is 7.53, both higher than their counterparts for the overall sample.

The average price dispersion ratio is 0.50 for the overall sample, and 0.51 for investment-grade bonds. For the overall sample, the average turnover is 0.08 per month and the average issuance size is \$462 million. Investment-grade bonds have a larger average issuance size of \$537 million, and a turnover of 0.07. The effective bid-ask spread is 1.43% for the overall sample, and 1.32% for the investment-grade subsample. The average bond age is around 5 years and the time to maturity is around 8 years.

6.4 Cross-sectional analysis

We run Fama-MacBeth regressions of chain length on the variables in Table 1, and the results are reported in Table 3. As shown in column 1, the signs of all coefficients are consistent

with the model predictions, and all coefficients are highly significantly different from 0. The coefficient for *IG* is 0.245 ($t = 32.17$) implying that, holding everything else constant, the chain length for investment-grade bonds is longer than that for other bonds by 0.245 on average, which is significant given that the mean chain length is 1.73.

The coefficient for *Turnover* is 0.199 ($t = 11.48$), suggesting that the chain length increases with the frequency of investors' trading needs. The coefficients for *Size* and *Age* are -0.012 ($t = 3.73$) and 0.025 ($t = 23.92$), implying that the chain length is decreasing in the size of the market. Also consistent with the model prediction, the coefficient for *Maturity* is significantly negative. The coefficient for *Spread* is 0.073 ($t = 17.17$). Under the interpretation that a higher spread implies a lower search cost for dealers, this is consistent our model that the chain length is decreasing in the search cost.

We then run another Fama-MacBeth regression, using the price dispersion *DR* as the dependent variable. Our model predicts that the signs of coefficients for all the variables should be the same as those in the regression for *L*. As shown in the third column of Table 3, five out of the six coefficients have the same sign as those in the regression for *L* in column 1. For example, as shown in the third column of Table 3, the coefficient for *IG* is 0.007 ($t = 2.62$) implying that, holding everything else constant, the price dispersion for investment grade bonds is larger than that for other bonds by 0.007 on average. Similarly, as implied by our model, the coefficients for other variables such as *Turnover*, *Age*, *Maturity*, and *Spread* are all significant and have the same sign as in the regression for *L*.

The only exception is the coefficient for *Size*. Contrary to our model prediction, the coefficient is significantly positive. Intuitively, our model implies that, for a larger bond, it is easier to find trading partners. Hence, it is less profitable for dealers, leading to a smaller intermediary sector, and consequently a shorter intermediation chain and a smaller price dispersion ratio. However, our evidence is only consistent with the implication on the chain length. One conjecture is that our model abstracts away from the variation in transaction size and dealers' inventory capacity constraints. For example, in our sample, the monthly maximum transaction size for the largest 10% of the bonds is more than 50 times larger than that for the smallest 10% of the bonds. When facing extremely large transactions from

customers, with inventory capacity constraints, a dealer may have to offer price concessions when trading with other dealers, leading to a larger price dispersion ratio. However, this channel has a much weaker effect on the chain length, which reflects the *average* number of layers of intermediation and so is less sensitive to the transactions of extreme sizes. As a result, our model prediction on the chain length holds but the prediction on the price dispersion does not.

As a robustness check, we reconstruct all variables at the quarterly frequency and repeat our analysis. As shown in the second and fourth columns, the results at the quarterly frequency are similar to those at the monthly frequency. The only difference is that the coefficient for *Maturity* becomes insignificant. Finally, we acknowledge the potential endogeneity concern for the independent variables, especially *Spread*. Hence, we rerun our regressions after dropping *Spread*, and our results remain very similar for all other variables.

In summary, despite its simple structure, our model appears to describe reasonably well the intermediary sector in the U.S. corporate bond market. Especially, the dispersion ratio DR is constructed based on price data while the chain length L is based on quantity data. Yet, for almost all our proxies, their coefficients have the same sign across the two regressions for DR and L , as implied by our model.

7 Conclusion

We analyze a search model with an endogenous intermediary sector and an intermediation chain. The equilibrium is characterized in closed-form. Our model shows that the length of the intermediation chain and price dispersion ratio are decreasing in search cost, search speed, market size, but are increasing in investors' trading need. Based on the data from the U.S. corporate bond market, our evidence is broadly consistent with the model predictions.

As search frictions diminish, the search market equilibrium does *not* always converge to a centralized market equilibrium. In particular, the prices and allocations in the search market equilibrium converge to their counterparts in a centralized market equilibrium, but the trading volume does not converge in the case with intermediaries. The difference between

the two trading volumes across the two equilibria increases when the search cost becomes smaller, and approaches infinity when the search cost goes to zero. These results suggest that a centralized market model captures the behavior of asset prices and allocations when market frictions are small. However, it is not well-suited for analyzing trading volume, even in a market with a fast search speed, especially in the case when the search cost is small.

8 Appendix

Proof of Theorem 1

The proof is organized as follows. Step I, we take Δ_b , Δ_s and decision rules (3) and (4) as given to derive densities $\mu_s(\Delta)$, $\mu_b(\Delta)$, $\mu_n(\Delta)$, $\mu_h(\Delta)$. Step II, from the two indifference conditions at Δ_b and Δ_s , we obtain equations (20) and (21) that pin down Δ_b and Δ_s . Step III, we verify that decision rules (3) and (4) are indeed optimal for all investors.

Step I. We now show that $\mu_i(\Delta)$ for $i = b, s, h, n$ are given by following. For $\Delta \in [0, \Delta_b)$,

$$\mu_b(\Delta) = \mu_h(\Delta) = 0, \quad (45)$$

$$\mu_n(\Delta) = \frac{\kappa(N - X) + \lambda N_b N}{\kappa + \lambda N_b} f(\Delta), \quad (46)$$

$$\mu_s(\Delta) = \frac{\kappa X}{\kappa + \lambda N_b} f(\Delta). \quad (47)$$

For $\Delta \in (\Delta_b, \Delta_s)$,

$$\mu_n(\Delta) = \mu_h(\Delta) = 0, \quad (48)$$

$$\mu_s(\Delta) = \frac{Nf(\Delta)}{2} \left[1 - \frac{N - NF(\Delta) - X - \frac{\kappa}{\lambda}}{\sqrt{[N - NF(\Delta) - X - \frac{\kappa}{\lambda}]^2 + 4\frac{\kappa}{\lambda}(N - X)[1 - F(\Delta)]}} \right], \quad (49)$$

$$\mu_b(\Delta) = \frac{Nf(\Delta)}{2} \left[1 + \frac{N - NF(\Delta) - X - \frac{\kappa}{\lambda}}{\sqrt{[N - NF(\Delta) - X - \frac{\kappa}{\lambda}]^2 + 4\frac{\kappa}{\lambda}(N - X)[1 - F(\Delta)]}} \right]. \quad (50)$$

For $\Delta \in (\Delta_s, \bar{\Delta}]$,

$$\mu_n(\Delta) = \mu_s(\Delta) = 0, \quad (51)$$

$$\mu_b(\Delta) = \frac{\kappa(N - X)}{\kappa + \lambda N_s} f(\Delta), \quad (52)$$

$$\mu_h(\Delta) = \frac{\kappa X + \lambda N_s N}{\kappa + \lambda N_s} f(\Delta). \quad (53)$$

From (3) and (4), we have (45), (48), and (51). Substituting (51) into (12), we obtain

$$\mu_b(\Delta) + \mu_h(\Delta) = Nf(\Delta).$$

From the above equation and (14), we obtain (52) and (53). The market clearing condition

(19), together with (45) and (48), implies that

$$\int_{\Delta_s}^{\bar{\Delta}} \mu_h(\Delta) d\Delta + N_s = X.$$

Substituting (53) into the above equation, we get an equation of N_s ,

$$N_s^2 + \left(\frac{\kappa}{\lambda} + N - X - NF(\Delta_s) \right) N_s - \frac{\kappa X}{\lambda} F(\Delta_s) = 0$$

from which we get

$$N_s = \frac{1}{2} \sqrt{\left[\frac{\kappa}{\lambda} + N - X - NF(\Delta_s) \right]^2 + 4 \frac{\kappa X}{\lambda} F(\Delta_s)} - \frac{1}{2} \left[\frac{\kappa}{\lambda} + N - X - NF(\Delta_s) \right]. \quad (54)$$

The derivation for the region $\Delta \in [0, \Delta_b]$ is similar. We obtain (46) and (47), with

$$N_b^2 + \left(\frac{\kappa}{\lambda} - N + X + NF(\Delta_b) \right) N_b - \frac{\kappa}{\lambda} (N - X) [1 - F(\Delta_b)] = 0. \quad (55)$$

Solving the above equation for N_b , we obtain

$$N_b = \frac{N - NF(\Delta_b) - X - \frac{\kappa}{\lambda}}{2} + \frac{1}{2} \sqrt{\left[N - NF(\Delta_b) - X - \frac{\kappa}{\lambda} \right]^2 + 4 \frac{\kappa}{\lambda} (N - X) [1 - F(\Delta_b)]}. \quad (56)$$

The derivation for the region $\Delta \in (\Delta_b, \Delta_s)$ is the following. We rewrite (18) as

$$\kappa \frac{dF_s(\Delta)}{d\Delta} = \kappa X f(\Delta) - \lambda [N_b - F_b(\Delta)] \frac{dF_s(\Delta)}{d\Delta} + \lambda F_s(\Delta) \frac{dF_b(\Delta)}{d\Delta}. \quad (57)$$

After some algebra, we get

$$\kappa \frac{dF_s(\Delta)}{d\Delta} = \kappa X f(\Delta) - \frac{d}{d\Delta} [\lambda (N_b - F_b(\Delta)) F_s(\Delta)].$$

Integrating both sides from Δ_b to $\Delta \in (\Delta_b, \Delta_s)$, we have

$$\kappa [F_s(\Delta) - F_s(\Delta_b)] = \kappa X [F(\Delta) - F(\Delta_b)] - \lambda [(N_b - F_b(\Delta)) F_s(\Delta) - N_b F_s(\Delta_b)], \quad (58)$$

where we have used the fact that $F_b(\Delta_b) = 0$.

Substituting (47) into the definition of $F_s(\cdot)$, we have

$$F_s(\Delta_b) = \frac{\kappa X}{\kappa + \lambda N_b} F(\Delta_b). \quad (59)$$

Substituting (48) into (12), we get

$$\mu_s(\Delta) + \mu_b(\Delta) = Nf(\Delta). \quad (60)$$

We can rewrite the above equation as

$$\frac{dF_b(\Delta)}{d\Delta} + \frac{dF_s(\Delta)}{d\Delta} = Nf(\Delta).$$

Integrating both sides from Δ_b to $\Delta \in (\Delta_b, \Delta_s]$, after some algebra, we obtain

$$F_s(\Delta) = F_s(\Delta_b) - F_b(\Delta) + N[F(\Delta) - F(\Delta_b)]. \quad (61)$$

Substituting (59) and (61) into (58), we get a quadratic equation of $F_b(\Delta)$, from which we obtain the solution for $F_b(\Delta)$. Differentiating it with respect to Δ , we obtain $\mu_b(\Delta)$ in (50). From (60) we obtain $\mu_s(\Delta)$ in (49).

Step II. Let's first determine $V_n(\Delta)$ and $V_h(\Delta)$ for $\Delta \in [0, \bar{\Delta}]$. Equation (10) implies that $V_n(\Delta)$ is a constant for all Δ . We denote it by $V_n \equiv V_n(\Delta)$. Equation (8) implies that $V_h(\Delta)$ is linear in Δ with a positive slope

$$\frac{dV_h(\Delta)}{d\Delta} = \frac{1}{\kappa + r}. \quad (62)$$

We now compute the slope for $V_s(\Delta)$ for the region $\Delta \in [0, \Delta_b)$. From (9), we have

$$\begin{aligned} V_s(\Delta) &= \frac{1 + \Delta - c}{\kappa + r} + \frac{\kappa \mathbf{E}[\max\{V_h(\Delta'), V_s(\Delta')\}]}{\kappa + r} \\ &\quad + \frac{\lambda(1 - \eta)}{\kappa + r} \int_{\Delta_b}^{\Delta_s} [V_s(x) + V_n - V_b(x) - V_s(\Delta)] \mu_b(x) dx \\ &\quad + \frac{\lambda(1 - \eta)}{\kappa + r} \int_{\Delta_s}^{\bar{\Delta}} [V_h(x) + V_n - V_b(x) - V_s(\Delta)] \mu_b(x) dx. \end{aligned}$$

Differentiating both sides of the equation with respect to Δ , we obtain

$$\frac{dV_s(\Delta)}{d\Delta} = \frac{1}{\kappa + r + \lambda(1 - \eta) N_b}. \quad (63)$$

Similarly, for $\Delta \in (\Delta_b, \Delta_s)$, we get

$$\frac{dV_s(\Delta)}{d\Delta} = \frac{1}{\kappa + r} - \frac{\lambda(1 - \eta)}{\kappa + r} \left[\frac{dV_s(\Delta)}{d\Delta} - \frac{dV_b(\Delta)}{d\Delta} \right] \int_{\Delta}^{\bar{\Delta}} \mu_b(x) dx. \quad (64)$$

Let's now determine the slope for $V_b(\Delta)$ for $\Delta \in (\Delta_s, \bar{\Delta}]$. From (11), we have

$$\begin{aligned} V_b(\Delta) = & - \frac{c}{\kappa + r} + \frac{\kappa E [\max \{V_b(\Delta'), V_n\}]}{\kappa + r} \\ & + \frac{\lambda\eta}{\kappa + r} \int_0^{\Delta_b} [V_h(\Delta) + V_n(x) - V_b(\Delta) - V_s(x)] \mu_s(x) dx \\ & + \frac{\lambda\eta}{\kappa + r} \int_{\Delta_b}^{\Delta_s} [V_h(\Delta) + V_b(x) - V_b(\Delta) - V_s(x)] \mu_s(x) dx. \end{aligned}$$

Differentiating both sides with respect to Δ , after some algebra, we obtain

$$\frac{dV_b(\Delta)}{d\Delta} = \frac{1}{\kappa + r} \frac{\lambda\eta N_s}{\kappa + r + \lambda\eta N_s} \text{ for } \Delta \in (\Delta_s, \bar{\Delta}]. \quad (65)$$

Similarly, for $\Delta \in (\Delta_b, \Delta_s)$, we have

$$\frac{dV_b(\Delta)}{d\Delta} = \frac{\lambda\eta}{\kappa + r} \left[\frac{dV_s(\Delta)}{d\Delta} - \frac{dV_b(\Delta)}{d\Delta} \right] \int_0^{\Delta} \mu_s(x) dx. \quad (66)$$

From (64) and (66), we can solve for the $\frac{dV_s(\Delta)}{d\Delta}$ and $\frac{dV_b(\Delta)}{d\Delta}$ for $\Delta \in (\Delta_b, \Delta_s)$. Then, we have the following

$$\frac{dV_s(\Delta)}{d\Delta} = \begin{cases} \frac{1}{\kappa+r+\lambda(1-\eta)N_b} & \text{for } \Delta \in [0, \Delta_b) \\ \frac{1}{\kappa+r} \frac{\kappa+r+\lambda\eta F_s(\Delta)}{\kappa+r+\lambda(1-\eta)[N_b-F_b(\Delta)]+\lambda\eta F_s(\Delta)} & \text{for } \Delta \in (\Delta_b, \Delta_s) \end{cases}. \quad (67)$$

$$\frac{dV_b(\Delta)}{d\Delta} = \begin{cases} \frac{1}{\kappa+r} \frac{\lambda\eta F_s(\Delta)}{\kappa+r+\lambda(1-\eta)[N_b-F_b(\Delta)]+\lambda\eta F_s(\Delta)} & \text{for } \Delta \in (\Delta_b, \Delta_s) \\ \frac{1}{\kappa+r} \frac{\lambda\eta N_s}{\kappa+r+\lambda\eta N_s} & \text{for } \Delta \in (\Delta_s, \bar{\Delta}] \end{cases}. \quad (68)$$

From the above expressions for the slopes, we obtain the following

$$V_n = \frac{\kappa}{r} \int_{\Delta_b}^{\Delta_s} \frac{dV_b(z)}{dz} [1 - F(z)] dz + \frac{\kappa}{r} \frac{1}{\kappa + r} \frac{\lambda\eta N_s}{\kappa + r + \lambda\eta N_s} \int_{\Delta_s}^{\bar{\Delta}} [1 - F(z)] dz. \quad (69)$$

$$V_b(\Delta) = V_n + \begin{cases} \int_{\Delta_b}^{\Delta} \frac{dV_b(z)}{dz} dz \text{ for } z \in [\Delta_b, \Delta_s] \\ \int_{\Delta_b}^{\Delta_s} \frac{dV_b(z)}{dz} dz + \frac{1}{\kappa+r} \frac{\lambda\eta N_s}{\kappa+r+\lambda\eta N_s} (\Delta - \Delta_s) \text{ for } z \in (\Delta_s, \bar{\Delta}] \end{cases}, \quad (70)$$

$$V_h(\Delta) = V_h(\Delta_s) + \frac{\Delta - \Delta_s}{\kappa + r}, \quad (71)$$

where

$$V_h(\Delta_s) = \frac{1+\Delta_s}{r} - \frac{\kappa}{r} \frac{\int_0^{\Delta_b} F(z) dz}{\kappa + r + \lambda N_b (1 - \eta)} - \frac{\kappa}{r} \int_{\Delta_b}^{\Delta_s} \frac{dV_s(z)}{dz} F(z) dz + \frac{\kappa}{r} \frac{\int_{\Delta_s}^{\bar{\Delta}} [1 - F(z)] dz}{\kappa + r},$$

$$V_s(\Delta) = V_s(\Delta_b) + \begin{cases} \frac{\Delta - \Delta_b}{\kappa + r + \lambda N_b(1 - \eta)} & \text{for } z \in [0, \Delta_b) \\ \int_{\Delta_b}^{\Delta} \frac{dV_s(z)}{dz} dz & \text{for } z \in (\Delta_b, \bar{\Delta}] \end{cases}, \quad (72)$$

where

$$V_s(\Delta_b) = V_h(\Delta_s) - \int_{\Delta_b}^{\Delta_s} \frac{dV_s(z)}{dz} dz.$$

We now verify the conjecture in (7). Suppose $x \in (\Delta_b, \Delta_s)$ and $y \in (\Delta_b, \Delta_s)$. Define $\xi(\Delta)$ for $\Delta \in (\Delta_b, \Delta_s)$ as

$$\xi(z) \equiv \frac{dV_s(z)}{dz} - \frac{dV_b(z)}{dz}. \quad (73)$$

Then we have $S(x, y) = \int_y^x \xi(z) dz$. Hence, $S(x, y) > 0$ if and only if $x > y$. The verification for other values for x and y is straightforward.

We now derive the value for Δ_b and Δ_s . Substituting $\Delta = \Delta_b$ into (11), we then obtain

$$V_b(\Delta_b) = -\frac{c}{\kappa + r} + V_n + \frac{\lambda \eta}{\kappa + r} \frac{\kappa X}{\kappa + \lambda N_b} \frac{\int_0^{\Delta_b} F(x) dx}{\kappa + r + \lambda(1 - \eta) N_b}.$$

Substituting the indifference condition $V_b(\Delta_b) = V_n$ into the above equation, we obtain (20). From the monotonicity of the right hand side of (20) and its boundary conditions at $\Delta_b = 0$ and $\Delta_b = \bar{\Delta}$, we know that equation (20) has a unique solution $\Delta_b \in [0, \bar{\Delta}]$. Similarly, substituting $\Delta = \Delta_s$ in (9), after some algebra, we obtain

$$V_s(\Delta_s) = V_h(\Delta_s) - \frac{c}{\kappa + r} + \frac{\lambda(1 - \eta)}{\kappa + r} \frac{\kappa(N - X)}{\kappa + \lambda N_s} \frac{\int_{\Delta_s}^{\bar{\Delta}} [1 - F(x)] dx}{\kappa + r + \lambda \eta N_s}.$$

Substituting the indifference condition $V_s(\Delta_s) = V_h(\Delta_s)$ into the above equation, we obtain (21). From the monotonicity of the right hand side of (21) and its boundary conditions at $\Delta_s = 0$ and $\Delta_s = \bar{\Delta}$, we know that equation (21) has a unique solution $\Delta_s \in [0, \bar{\Delta}]$.

Equation (20) implies that Δ_b is increasing in c . Let's denote the function as $\Delta_b(c)$. Similarly, equation (21) implies that Δ_s is decreasing in c . Let's denote the function as $\Delta_s(c)$. Define c^* as the solution to

$$\Delta_b(c^*) = \Delta_s(c^*). \quad (74)$$

From the monotonicity and boundary conditions, the above equation has a unique solution. Moreover, for any $c < c^*$, we have $\Delta_b < \Delta_s$.

Step III. We now verify that a non-owner's optimal choice is given by (3) and that an owner's optimal choice is given by (4). We can prove both by contradiction.

Let's first consider the case for an owner with $\Delta \in (\Delta_s, \bar{\Delta}]$. Suppose this owner deviates from the equilibrium choice (4), i.e, rather than staying inactive, he searches in the market during a period $[t, t + dt)$ and then returns to the equilibrium strategy (3) and (4). Let's use $\widehat{V}_o(\Delta)$ to denote the investor's expected utility if he follows this alternative strategy:

$$\begin{aligned} \widehat{V}_o(\Delta) &= (1 + \Delta - c) dt + \kappa \mathbf{E} [\max \{V_h(\Delta'), V_s(\Delta')\}] dt \\ &\quad + \lambda dt (1 - \eta) \int_{\Delta}^{\bar{\Delta}} \widehat{S}(x, \Delta) \mu_b(x) dx + e^{-r dt} (1 - \kappa dt) V_h(\Delta), \end{aligned}$$

where $\widehat{S}(x, \Delta)$ denotes the trading surplus if this owner meets a buyer of type $x > \Delta$:

$$\widehat{S}(x, \Delta) = V_h(x) + V_b(\Delta) - V_b(x) - \widehat{V}_o(\Delta),$$

where we have used the result that the trading surplus is negative if the buyer's type is lower than the owner. For the owner to deviate, it has to be the case that $\widehat{V}_o(\Delta) > V_h(\Delta)$. Hence, the trade surplus is bounded by

$$\widehat{S}(x, \Delta) < V_h(x) + V_b(\Delta) - V_b(x) - V_h(\Delta).$$

Substituting (70) into the right hand side of the above inequality, we obtain

$$\widehat{S}(x, \Delta) < \frac{x - \Delta}{\kappa + r + \lambda \eta N_s}. \quad (75)$$

By comparing $\widehat{V}_o(\Delta)$ and $V_h(\Delta)$, we obtain

$$\widehat{V}_o(\Delta) - V_h(\Delta) = -c dt + \lambda dt (1 - \eta) \int_{\Delta}^{\bar{\Delta}} \widehat{S}(x, \Delta) \mu_b(x) dx. \quad (76)$$

Substituting (75) and (52) into the above equation, we obtain

$$\widehat{V}_o(\Delta) - V_h(\Delta) < -\frac{\lambda (1 - \eta) \kappa (N - X) \int_{\Delta_s}^{\Delta} [1 - F(x)] dx}{(\kappa + \lambda N_s) (\kappa + r + \lambda \eta N_s)} dt < 0. \quad (77)$$

This contradicts $\widehat{V}_o(\Delta) > V_h(\Delta)$. The proofs for other values for Δ and the decision rule (3) are similar.

Proof of Propositions 1–3

Propositions 1 and 2 can be obtained by differentiation. To prove Proposition 3, note that $\mathbb{T}\mathbb{V}_{cc}$ is the total volume of trades between sellers with types $[0, \Delta_b)$, whose population size is $F_s(\Delta_b)$, and buyers with types $(\Delta_s, \bar{\Delta}]$, whose population size is $N_b - F_b(\Delta_s)$. Note that any meeting between the two groups will lead to a trade. Hence, the total volume is given by (22). By the same logic, we obtain $\mathbb{T}\mathbb{V}_{cc}$ and $\mathbb{T}\mathbb{V}_{dc}$ in (23) and (24).

$\mathbb{T}\mathbb{V}_{dd}$ is the total volume of trades between sellers with types $y \in (\Delta_b, \Delta_s)$ and buyers with types $x \in (\Delta_b, \Delta_s)$. However, trade occurs if and only if $x > y$. For any $\Delta \in (\Delta_b, \Delta_s)$, the density of buyers is $dF_b(\Delta)$. They only trade with sellers whose types are below Δ , and whose population size is $F_s(\Delta) - F_s(\Delta_b)$. Hence, type- Δ investors' trading volume is $\lambda[F_s(\Delta) - F_s(\Delta_b)]dF_b(\Delta)$. Integrating this volume for $\Delta \in (\Delta_b, \Delta_s)$, we obtain (25).

Proof of Proposition 4–6

Based on equations (20) and (56), after some tedious algebra, we obtain $\frac{d\Delta_b}{dc} > 0$ and $\frac{dN_b}{dc} < 0$. Similarly, Equations (21) and (54) imply $\frac{d\Delta_s}{dc} > 0$ and $\frac{dN_s}{dc} < 0$. Differentiating L with respect to c , we can obtain $\frac{dL}{dc} < 0$. From (20) and (21), we can see that $c = 0$ implies that $\Delta_b = 0$, and $\Delta_s = \bar{\Delta}$. This implies that $N_b = N - X$, $N_s = X$, and $L = \infty$.

Proof of Proposition 7

Let's first conduct asymptotic analysis when λ is sufficiently large. Denote the limit of Δ_b and Δ_s under $\lambda \rightarrow \infty$ by

$$\begin{aligned}\Delta_b^\infty &\equiv \lim_{\lambda \rightarrow \infty} \Delta_b, \\ \Delta_s^\infty &\equiv \lim_{\lambda \rightarrow \infty} \Delta_s.\end{aligned}$$

We can rewrite (20) as

$$\lambda N_b^2 + \left(\kappa + \frac{\kappa + r}{1 - \eta} \right) N_b + \frac{\kappa(\kappa + r)}{(1 - \eta)\lambda} = \frac{\kappa\eta X}{(1 - \eta)c} \int_0^{\Delta_b} F(x) dx. \quad (78)$$

When λ goes to infinity, the right hand side of (78) converges to a positive constant (it is

easy to see that $\Delta_b^\infty \neq 0$):

$$\lim_{\lambda \rightarrow \infty} \frac{\kappa\eta X}{(1-\eta)c} \int_0^{\Delta_b} F(x) dx = \frac{\kappa\eta X}{(1-\eta)c} \int_0^{\Delta_b^\infty} F(x) dx.$$

Hence, the left hand side of (78) also converges to this positive constant, which implies

$$N_b = \frac{M_b}{\sqrt{\lambda}} + o(\lambda^{-1/2}), \text{ where } M_b = \sqrt{\frac{\kappa\eta X}{(1-\eta)c} \int_0^{\Delta_b^\infty} F(x) dx}. \quad (79)$$

Substituting the above equation into (55),

$$[NF(\Delta_b) - N + X] \left(\frac{M_b}{\sqrt{\lambda}} + o(\lambda^{-1/2}) \right) + \frac{1}{\lambda} (M_b^2 - \kappa(N - X)[1 - F(\Delta_b^\infty)]) + o\left(\frac{1}{\lambda}\right) = 0. \quad (80)$$

The above equation implies that

$$NF(\Delta_b) - N + X = O\left(\frac{1}{\sqrt{\lambda}}\right).$$

From the above equation, we have

$$\begin{aligned} \Delta_b^\infty &= \Delta_w, \\ \Delta_b - \Delta_b^\infty &= O\left(\frac{1}{\sqrt{\lambda}}\right). \end{aligned} \quad (81)$$

where $\Delta_w \equiv F^{-1}\left(\frac{N-X}{N}\right)$. Hence, we can write (81) as

$$\Delta_b = \Delta_w + \frac{m_b}{\sqrt{\lambda}} + o(\lambda^{-1/2}), \quad (82)$$

where m_b is a constant. Substituting this expression of Δ_b into (80), and setting the coefficient of $1/\lambda$ to zero, we obtain

$$m_b = \frac{1}{Nf(\Delta_w)} \left[\frac{\kappa X \left(1 - \frac{X}{N}\right)}{M_b} - M_b \right].$$

Following a similar logic, we obtain

$$\begin{aligned} \Delta_s^\infty &= \Delta_w \\ N_s &= \frac{M_s}{\sqrt{\lambda}} + o(\lambda^{-1/2}) \text{ with } M_s = \sqrt{\frac{\kappa(1-\eta)(N-X)}{\eta c} \int_{\Delta_w}^{\bar{\Delta}} [1 - F(x)] dx}, \end{aligned} \quad (83)$$

$$\Delta_s = \Delta_w + \frac{m_s}{\sqrt{\lambda}} + o(\lambda^{-1/2}) \text{ with } m_s = \frac{1}{Nf(\Delta_w)} \left[M_s - \frac{\kappa X \left(1 - \frac{X}{N}\right)}{M_s} \right]. \quad (84)$$

Finally, we can verify that $\Delta_s > \Delta_b$ in the asymptotic case if $m_s > m_b$, which can be shown as equivalent to $c < \hat{c}$. With the expressions of Δ_b , Δ_s , N_b , N_s we can obtain all other equilibrium quantities in the asymptotic case.

With the above results, we can now prove Proposition 7 by expanding L as the following

$$L = \ln \frac{\hat{c}}{c} + \frac{Z}{\sqrt{\lambda}} g\left(\frac{c}{\hat{c}}\right) + o\left(\lambda^{-1/2}\right), \quad (85)$$

where Z is a positive constant and is given by

$$Z \equiv \frac{\kappa}{2Nc} \left(\sqrt{\frac{\eta X}{(N-X)(1-\eta)}} \int_{\underline{\Delta}}^{\Delta_w} \frac{F(y)}{F(\Delta_w)} dy + \sqrt{\frac{(N-X)(1-\eta)}{\eta X}} \int_{\Delta_w}^{\bar{\Delta}} \frac{1-F(x)}{1-F(\Delta_w)} dx \right),$$

and $g(\cdot)$ is the following function

$$g(x) \equiv 3x - \left(1 + \frac{1}{x}\right) \ln x - 1, \text{ for } x \in [0, 1].$$

It is easy to show that $g(x) > 0$. Hence, (85) implies $\frac{dL}{d\lambda} < 0$ when λ is sufficiently large.

Proof of Propositions 8–9

Equation (85) implies that when λ is sufficiently large, we have $\frac{\partial L}{\partial \kappa} > 0$, and that under condition (27), we have $\frac{\partial L}{\partial X} < 0$. To prove the rest of the two propositions, we expand $\Delta_s - \Delta_b$ as the following

$$\Delta_s - \Delta_b = \frac{Y}{\sqrt{\lambda}} + o\left(\lambda^{-1/2}\right),$$

where Y is given by

$$Y = \frac{1 - \frac{c}{\hat{c}}}{\phi \sqrt{X} f(\Delta_w)} \left[\sqrt{\frac{\kappa \eta}{(1-\eta)c}} \int_{\underline{\Delta}}^{\Delta_w} F(y) dy + \sqrt{\frac{\kappa(1-\eta)}{\eta c}} (\phi - 1) \int_{\Delta_w}^{\bar{\Delta}} [1 - F(x)] dx \right].$$

The above equation implies that $\frac{\partial(\Delta_s - \Delta_b)}{\partial \kappa} > 0$, and under condition (27), $\frac{\partial(\Delta_s - \Delta_b)}{\partial X} < 0$.

Proof of Proposition 10 and 11

When λ is sufficiently large, we can expand D as

$$D = \frac{\sqrt{c}}{\sqrt{\lambda}} \left[\frac{\Delta_w}{\sqrt{\frac{\kappa(1-\eta)X}{\eta} \int_0^{\Delta_w} F(x) dx}} + \frac{(\bar{\Delta} - \Delta_w)}{\sqrt{\frac{\kappa\eta(N-X)}{(1-\eta)} \int_{\Delta_w}^{\bar{\Delta}} [1 - F(x)] dx}} \right] + o\left(\frac{1}{\sqrt{\lambda}}\right).$$

It is direct to see that $\frac{\partial D}{\partial \lambda} < 0$ and $\frac{\partial D}{\partial c} > 0$ in this case.

If c is close to zero, D can be expanded as

$$D = \int_{\Delta_b}^{\Delta_s} \xi(z) dz + O(\sqrt{c}),$$

where $\Delta_s = \bar{\Delta} - O(\sqrt{c})$ and $\Delta_b = O(\sqrt{c})$. It can be shown that $\frac{\partial}{\partial c} \left(\int_{\Delta_b}^{\Delta_s} \xi(z) dz \right) < 0$, so we obtain $\frac{\partial D}{\partial c} < 0$ when c is sufficiently small.

Proof of Proposition 12

From the definition, we have

$$\begin{aligned} P_{\max}^d &= P(\Delta_s, \Delta_s), \\ P_{\min}^d &= P(\Delta_b, \Delta_b). \end{aligned}$$

Substituting them and (29) and (30) into (31), and differentiating it, we obtain $\frac{\partial DR}{\partial c} < 0$.

It is easy to show that

$$\begin{aligned} P_{\max}^d - P_{\min}^d &= O(\lambda^{-1}), \\ P_{\max} - P_{\min} &= O(\lambda^{-1/2}). \end{aligned}$$

It follows that $DR = O(\lambda^{-1/2})$. Therefore, $\frac{\partial DR}{\partial \lambda} < 0$ when λ is sufficiently large. Similarly, we can show that, when λ is sufficiently large, we have

$$\begin{aligned} \frac{\partial}{\partial \kappa} (P_{\max}^d - P_{\min}^d) &> 0, \\ \frac{\partial}{\partial \kappa} (P_{\max} - P_{\min}) &< 0, \end{aligned}$$

which implies $\frac{\partial DR}{\partial \kappa} > 0$. Furthermore, under the condition in (27), we can show

$$\begin{aligned} P_{\max}^d - P_{\min}^d &= O\left(\frac{1}{\lambda X}\right), \\ P_{\max} - P_{\min} &= O\left(\frac{1}{\sqrt{\lambda X}}\right), \end{aligned}$$

which implies that $DR = O\left(\frac{1}{\sqrt{\lambda X}}\right)$. Therefore, when λ is sufficiently large, we have $\frac{\partial DR}{\partial X} < 0$.

Proof of Proposition 13

The average expected utility across all investors in the economy is defined by

$$W \equiv \frac{1}{N} \sum_{i \in \{b,s,h,n\}} \left[\int_0^{\bar{\Delta}} V_i(\Delta) \mu_i(\Delta) d\Delta \right].$$

When λ is sufficiently large, we have the following

$$W = W_w - \frac{m_w}{\sqrt{\lambda}} + o\left(\lambda^{-1/2}\right),$$

where W_w is average expected utility in a centralized market and is given by

$$W_w = \frac{1}{r} \int_{\Delta_w}^{\bar{\Delta}} (1 + \Delta) d\Delta,$$

and m_w is given by

$$m_w = \frac{1}{r} \sqrt{\frac{\kappa c}{\eta(1-\eta)X}} \left[\sqrt{\frac{1}{\phi} \left(1 - \frac{1}{\phi}\right) \int_{\Delta_w}^{\bar{\Delta}} [1 - F(x)] dx} + \frac{1}{\phi} \sqrt{\int_{\underline{\Delta}}^{\Delta_w} F(x) dx} \right].$$

By examining m_w , we can obtain all the conclusions in this proposition.

Proof Proposition 14

In a centralized market, transactions can be executed instantly, hence, all investors whose types are higher than Δ_w are holding the total X units of the asset. This implies

$$F(\Delta_w) = 1 - \frac{X}{N}, \tag{86}$$

which leads to (32). Similar to the proof for Theorem 1, we can obtain the expected utility of an asset owner $V_o^c(\Delta)$ and of a non-owner $V_n^c(\Delta)$

$$\begin{aligned} V_o^c(\Delta) &= \frac{1 + \Delta + \kappa \mathbf{E}[\max\{V_n^c(\Delta), V_n^c(\Delta) + P_w\}]}{\kappa + r}, \\ V_n^c(\Delta) &= \frac{\kappa \mathbf{E}[\max\{V_o^c(\Delta) - P_w, V_n^c(\Delta)\}]}{\kappa + r}. \end{aligned}$$

The indifference condition of a type- Δ_w investor is

$$V_o^c(\Delta) = V_n^c(\Delta_w) + P_w.$$

The above three equations lead to (33). During $[t, t + dt)$, $\kappa X dt$ investors' types change. $F(\Delta_w)$ of them have new types below Δ_w , and sell their assets. Hence, the trading volume is given by (34).

Proof Proposition 15

From the asymptotic analysis in the proof of Proposition 7, we obtain (36)–(40). Substituting them into (22)–(25), we obtain (41).

Proof Proposition 16

The social welfare, denoted by W^e , is the discounted sum of all realized cash flows from holding the asset net of total search cost, i.e.,

$$W^e = \frac{1}{r} \int_0^{\bar{\Delta}} (1 + \Delta) [\mu_h(\Delta) + \mu_s(\Delta)] d\Delta - \frac{1}{r} c(N_b + N_s).$$

where $\mu_h(\cdot)$ and $\mu_s(\cdot)$ are given in Theorem 1. After some algebra, the social planner's the first-order condition with respect to Δ_s can be simplified to

$$c = \int_{\Delta_s}^{\bar{\Delta}} \frac{1 - F(\Delta)}{1 - F(\Delta_s)} d\Delta \frac{\frac{X - N_s}{1 - F(\Delta_s)} (2N_s - X + \frac{\kappa}{\lambda}) + (NX - 2NN_s - \frac{\kappa X}{\lambda})}{NN_s + \frac{\kappa X}{\lambda}}.$$

When λ is sufficiently large, we have the following asymptotics

$$\begin{aligned} \Delta_s^e &= \Delta_w + \frac{m_s^e}{\sqrt{\lambda}} + o(\lambda^{-1/2}), \\ N_s^e &= \frac{M_s^e}{\sqrt{\lambda}} + o(\lambda^{-1/2}), \end{aligned}$$

where

$$\begin{aligned} m_s^e &= \frac{1}{Nf(\Delta_w)} \left[M_s^e - \frac{\kappa X (1 - \frac{X}{N})}{M_s^e} \right], \\ M_s^e &= \sqrt{\frac{\eta}{1 - \eta}} M_s. \end{aligned}$$

Similarly, the first order condition with respect to Δ_b is given by

$$c = \frac{\kappa X \int_0^{\Delta_b} F(\Delta) d\Delta}{(\kappa + \lambda N_b)^2}.$$

When λ is sufficiently large, we have the following asymptotics

$$\begin{aligned} \Delta_b^e &= \Delta_w + \frac{m_b^e}{\sqrt{\lambda}} + o(\lambda^{-1/2}), \\ N_b^e &= \frac{M_b^e}{\sqrt{\lambda}} + o(\lambda^{-1/2}), \end{aligned}$$

where

$$m_b^e = \frac{1}{Nf(\Delta_w)} \left[\frac{\kappa X \left(1 - \frac{X}{N}\right)}{M_b^e} - M_b^e \right],$$

$$M_b^e = \sqrt{\frac{1-\eta}{\eta}} M_b.$$

When $\eta = \frac{1}{2}$, we have $M_b^e = M_b$ and $M_s^e = M_s$. From (82) and (84), we obtain (43) and (44). Otherwise, the decentralized equilibrium is generally inefficient. For example, in the case of $\eta > \frac{1}{2}$, the intermediary sector in the decentralized equilibrium is too big (i.e., $\Delta_s - \Delta_b > \Delta_s^e - \Delta_b^e$) if $\frac{\eta \hat{c}_b}{(1-\eta)\hat{c}_s} > 1$, and is too small if $\frac{\eta \hat{c}_b}{(1-\eta)\hat{c}_s} < 1$, where

$$\hat{c}_b \equiv \int_0^{\Delta_w} \frac{F(\Delta)}{F(\Delta_w)} d\Delta,$$

$$\hat{c}_s \equiv \int_{\Delta_w}^{\bar{\Delta}} \frac{1-F(\Delta)}{1-F(\Delta_w)} d\Delta.$$

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Table 1: **Model Predictions**

This table summarizes the model predictions. The first column are the variables that we will measure empirically. The second column reports the variables in our model, for which the variable in the first column is a proxy. The third column reports the predicted relation with the length of the intermediation chain L and the price dispersion ratio DR . L is the ratio of the volume of transactions generated by dealers to that generated by customers, and is defined in (26). DR is the price dispersion among inter-dealer trades divided by the price dispersion among all trades, and is defined in (31). $Size$ is the initial face value of the issuance size of the corporate bond, denominated in million dollars. Age is the time since the issuance, denominated in years. $Turnover$ is the total trading volume of a bond in face value during the period, normalized by $Size$. IG is a dummy variable, which is 1 if the bond is rated as investment grade, and 0 otherwise. $Maturity$ is the the time until maturity of a bond, measured in years. $Spread$ of a bond is the square root of the negative of the first-order autocovariance of changes in consecutive transaction prices of the bond.

Variable	Proxy for	Relation with L and DR
$Size$	X	—
Age	X	+
$Turnover$	κ	+
IG	c	+
$Maturity$	c	—
$Spread$	c	+

Table 2: **Summary Statistics**

This table reports the summary statistics of the variables defined in Table 1, all of which are measured at the monthly frequency. For each variable, the table reports its mean, standard deviation, the 99th, 75th, 50th, 25th, and 1st percentiles, as well as the number of observations.

		Mean	S.D.	99%	75%	50%	25%	1%	Obs.
<i>L</i>	All	1.73	0.96	7.00	2.10	1.36	1.02	1.00	862109
	IG	1.81	0.97	7.53	2.25	1.48	1.05	1.00	526272
<i>DR</i>	All	0.50	0.31	1.00	0.76	0.54	0.25	0.00	683379
	IG	0.51	0.31	1.00	0.75	0.54	0.27	0.00	436993
<i>Turnover</i> (per month)	All	0.08	0.12	1.02	0.10	0.04	0.01	0.00	866831
	IG	0.07	0.11	0.76	0.08	0.03	0.01	0.00	528698
<i>Spread</i> (%)	All	1.43	1.46	14.88	1.81	1.02	0.56	0.05	590883
	IG	1.32	1.24	6.77	1.69	0.97	0.54	0.04	372473
<i>Size</i> (\$million)	All	462	1645	3000	500	275	150	2.00	866832
	IG	537	2029	3000	600	300	175	3.11	528698
<i>Age</i> (year)	All	4.86	4.50	18.91	6.91	3.73	1.64	0.02	866832
	IG	5.06	4.56	18.89	7.32	3.91	1.71	0.04	528698
<i>Maturity</i> (year)	All	8.19	9.35	33.37	9.57	5.08	2.37	0.08	866523
	IG	8.67	9.91	35.17	10.08	5.00	2.25	0.08	528434

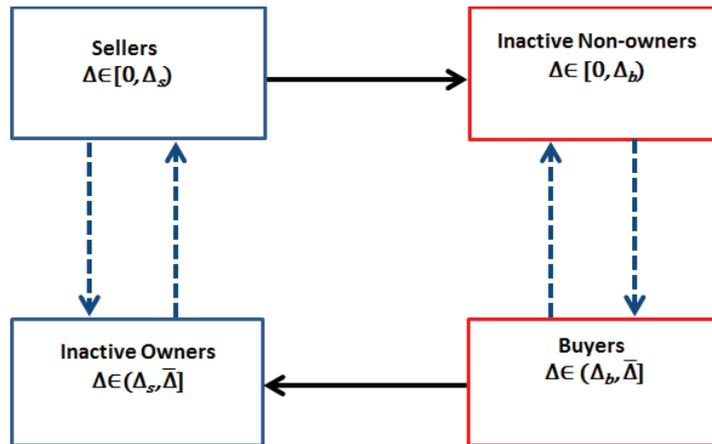
Table 3: **Regression Results**

This table reports the estimated coefficients from Fama-MacBeth regressions of intermediation chain length L and price dispersion ratio DR on a number of independent variables, at monthly and quarterly frequencies. All variables are defined in Table 1. T -statistics are reported in parentheses. The superscripts *, **, *** indicate significance levels of 10%, 5%, and 1%, respectively.

	L		DR	
	Monthly	Quarterly	Monthly	Quarterly
IG	0.245*** (32.17)	0.239*** (20.43)	0.007*** (2.62)	0.004 (1.14)
$Turnover$	0.199*** (11.48)	0.118*** (10.47)	0.217*** (26.58)	0.107*** (15.59)
$Size(\times 10^{-3})$	-0.012*** (3.73)	-0.008* (1.66)	0.021*** (15.17)	0.016*** (8.88)
Age	0.025*** (23.92)	0.019*** (13.92)	0.001*** (5.39)	0.002*** (5.47)
$Maturity$	-0.001*** (3.72)	0.000 (0.08)	-0.001*** (6.00)	0.000 (0.40)
$Spread$	0.073*** (17.17)	0.049*** (8.22)	0.004*** (4.47)	0.003** (2.54)

Figure 1: The evolution of demographics.

Panel A: The case without intermediation: $\Delta_b \geq \Delta_s$



Panel B: The case with intermediation: $\Delta_b < \Delta_s$

