Good Disclosure, Bad Disclosure

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August 2015

Abstract
We study the real efficiency implications of releasing public information in a model with multiple dimensions of uncertainty where market prices convey information to real decision makers. Disclosing more precise public information has a positive direct effect of providing new information and an indirect effect of changing price informativeness. The direction of the indirect effect depends on the type of information being disclosed. Paradoxically, when disclosure is about a variable that real decision makers care to learn, the indirect effect is negative and attenuates the direct effect. Moreover, in markets which are very effective in aggregating private information, the negative indirect effect can be stronger than the direct effect, implying that better disclosure can reduce the overall quality of information available to decision makers and harm real efficiency.

Keywords: Disclosure, price informativeness, learning, real efficiency.

JEL Classifications: D61, G14, G30, M41

1 Introduction

One of the main conclusions often coming up following financial crises and failures is the need to provide more precise public information. Regulatory efforts of this kind go back at least to the Securities Exchange Act of 1934 and have been refined and reinforced many times over the years since then, such as in the Sarbanes-Oxley Act and more recently in the Dodd-Frank Act.\footnote{For instance, Sarbanes-Oxley Act was passed as an “act to protect investors by improving the accuracy and reliability of corporate disclosures made pursuant to the securities laws, and for other purposes.” Greenstone et al. (2006, p. 399) state: “Since the passage of the Securities Act of 1933 and the Securities Exchange Act of 1934, the federal government has actively regulated U. S. equity markets. The centerpiece of these efforts is the mandated disclosure of financial information.”}

The idea behind these regulations is to create an environment with more abundant public information that will allow investors to make more informed capital allocation decisions. These attempts come in multiple channels: requiring firms to provide better disclosure to their investors, addressing problems in credit rating agencies to make credit ratings a more precise and reliable source of public information, disclosing publicly banks’ stress-tests results, and others.

As is often the case with regulation, its intended consequences are very appealing and well understood: Providing information seems quite desirable as we want decision makers to make more informed and efficient decisions. However, unintended consequences often appear, reducing the effectiveness of the act, sometimes even creating overall undesirable outcomes. In the case of public disclosure of financial information, a very natural unintended consequence to consider is that the disclosure of such information might crowd out other types of information, possibly even making inferior the overall set of information available to decision makers.

In particular, going back at least to Hayek (1945), economists believe that market prices are an important source of information for real decision makers. They aggregate disperse pieces of information from many traders who trade in financial markets for their own profit.\footnote{For example, the FASB states: “The benefits of financial reporting information include better investment, credit, and similar resource allocation decisions, which in turn result in more efficient functioning of the capital markets and lower costs of capital for the economy as a whole.” (FASB Financial Accounting Series, NO.1260-001 July 6 2006, “Conceptual Framework for Financial Reporting: Objective of Financial Reporting and Qualitative Characteristics of Decision-Useful Financial Reporting Information,” Section QC53, p. 35.)}
motives. This aggregation process at the end generates a signal – the price – which can be very valuable in capital allocation decisions and would not be available without the trading process in the financial market. The trading process and the information aggregation are expected to be affected by disclosure of public information. The question then is whether the provision of more public information – via mandatory disclosure, credit ratings, or stress tests – encourages or discourages the processing of information via market prices. If the latter happens then another question is whether the provision of public information increases or decreases the overall quality of information available to real decision makers.

In this paper, we propose a model to examine these questions. Our model shows that improving the precision of publicly disclosed information sometimes improves and sometimes reduces the quality of information generated by the financial market. In the former case, we say that the financial market amplifies the direct positive effect that disclosure has on overall informativeness and efficiency. In the latter case, we say that it attenuates the direct effect, and in fact this can be so strong so as to generate an overall negative effect. Which results we get depends on the type of information being disclosed and the efficiency of the financial markets in aggregating disperse information. These subtle conclusions can be quite useful for policy purposes, as they can guide policymakers in deciding which information would be more valuable to disclose publicly and when.

In our model, speculators trade a risky asset in the financial market based on their disperse private signals and the available public information. As a result of the trading process, the price of the risky asset reveals some of the private information of speculators. Decision makers on the real side of the economy – who make investment/production decisions affecting the cash flows of the risky asset – base their decisions on the public information, their own private information, and the information in the price. Their actions establish the effect that public information has on the real economy. Our mechanism works through the interactions between the exogenous public information and the endogenous price information, both of which affect the forecast quality of real decision makers.

A key ingredient in our model is that the profitability of the production technology determining the asset cash flow is affected by two independent factors – factor $\tilde{a}$ and factor $\tilde{f}$ – such as a macro factor and a firm-specific factor as in Veldkamp and Wolfers (2007), or a
permanent factor and a transitory factor as in Liu, Wang, and Zha (2013). More generally, one can think of one factor representing the quality of the technology and the other one the demand for the products being produced by it (see Goldstein and Yang (2015)). The key difference between factors $\tilde{a}$ and $\tilde{f}$ in our model is based on the direction of information asymmetry between real decision makers and speculators: Relative to speculators, real decision makers know more about one factor ($\tilde{a}$) than the other ($\tilde{f}$), and hence they are more keen to learn about factor $\tilde{f}$ of which they are relatively uninformed.\footnote{So, the notation “$\tilde{a}$” refers to information already known by real decision makers, while the notation “$\tilde{f}$” means information that they need to forecast.}

There are different empirical settings that naturally fit our model. One example is a financially-constrained firm that needs funds from capital providers (such as banks) to make investments. Real decision makers in this example are the capital providers; the speculators are hedge funds or mutual funds who trade the firm’s shares. The asset is the firm’s stock, whose cash flow depends on the factors $\tilde{a}$ and $\tilde{f}$ and the capital provided by the capital providers. Public information can be disclosure mandated by the government from the firm or a credit rating published by a credit rating agency. If the firm is a financial institution, then the public disclosure can represent the results of a stress test. While we mostly refer to this example in our paper, another example to keep in mind is an index on a particular industry. Real decision makers are the managers of the companies included in the index, making investment decisions, and speculators are still mutual funds or hedge funds. The public information can be an aggregate statistic disclosed by the government about the state of the economy/industry. In both these settings, it is quite natural to think of multiple dimensions of uncertainty, for reasons described above, and having the real decision makers being relatively less informed about some of them.

Public disclosure has two effects on real decision makers’ forecast quality and hence on real efficiency in our model. The direct effect is to provide new information, and it is always positive: Real decision makers become better informed and make more efficient decisions after observing more information, no matter how noisy it is. The second effect is an indirect effect: Public disclosure affects speculators’ trading, which in turn affects the price informativeness about factor $\tilde{f}$ that real decision makers are trying to learn. This indirect
effect can be positive or negative. When it is positive, the direct effect is amplified, leading to a strong overall effect on real efficiency. When the indirect effect is negative, the direct effect is attenuated or even overturned, making the overall effect modest or even negative.

Whether the indirect effect is positive or negative depends on whether the public disclosure is primarily a signal about factor $\tilde{a}$ or about factor $\tilde{f}$. In the first case, the indirect effect is positive. To see this, note that, in this case, speculators’ trading is determined by the public signal (primarily) about $\tilde{a}$ and their private signals about $\tilde{a}$ and $\tilde{f}$. When the public signal mainly provides information about $\tilde{a}$ and when it becomes more accurate, speculators will put a higher weight on this public information and less on their private information in forecasting factor $\tilde{a}$. As a result, their private-information based trading will reflect more of their private information about factor $\tilde{f}$ than their private information about factor $\tilde{a}$. Then, the price better aggregates information about $\tilde{f}$ which is what real decision makers try to learn.

On the other hand, when public disclosure is mainly a signal about factor $\tilde{f}$ that real decision makers wish to learn, the indirect effect becomes negative. This is because a more accurate public signal about $\tilde{f}$ will lead speculators to put a lower weight on their own private information about $\tilde{f}$, which reduces the informational content about $\tilde{f}$ in the price. This negative indirect effect of disclosure will attenuate the positive direct effect, thereby making the overall effect modest. In addition, the indirect effect can be so strong that it even dominates the direct effect, leading to a negative overall effect of disclosure on real efficiency. This is true when the market aggregates speculators’ private information very effectively and so the loss of efficiency due to the reduction in price information is pronounced.

Summarizing the insights, we can see that disclosing more precise public information about the factor that real decision makers already know enables the market to better aggregate and reveal information about what real decision makers try to learn, thus making prices more informative and amplifying the direct positive effect of better disclosure on real efficiency. Paradoxically, disclosing more precise public information about the factor that real decision makers want to learn can backfire as it interferes with the ability of the market to reveal this type of information attenuating the positive direct effect of better disclosure. In cases where the market is very effective in processing information, the indirect effect can
be stronger than the direct effect, implying that better disclosure can reduce the overall quality of information available to decision makers and harm real efficiency. So, although it appears attractive to disclose information about some variable that relevant decision makers care to learn the most, the overall impact of such disclosure can be counter-productive.

The remainder of this section provides a review of related literature. Section 2 provides the description of the model. In Section 3, we characterize the equilibrium outcomes. Section 4 contains the main results on the effect of different types of disclosure on real efficiency. In Section 5, we provide some discussion of the key ingredients of the model and robustness analysis. Section 6 concludes. All proofs and additional technical material are in the appendix.

1.1 Related Literature

Our paper is related to the literature on the real effect of financial markets, where trading and prices in financial markets affect real investment/production decisions, which in turn affect firms’ cash flows. This is known as the “feedback effect.” Bond, Edmans, and Goldstein (2012) provide a review of this literature. Several papers provide related empirical evidence; see, e.g., Luo (2005), Chen, Goldstein, and Jiang (2007), Edmans, Goldstein, and Jiang (2012), Foucault and Frésard (2014). Solving a model with a feedback loop between the market price and the firm’s cash flows is known to be challenging and require non-standard approaches to modeling the financial market. We adopt in this paper the basic framework of Goldstein, Ozdenoren, and Yuan (2013). While they focus on coordination among speculators, we focus on patterns of trading on different types of information and how they are affected by public information, and so the model presented here is different along these dimensions. Our focus on the interaction between public information and the feedback effect is new to this literature (one exception is the paper by Bond and Goldstein (2015) which is discussed below).

The main question of our paper relates it to the accounting literature on the real effects of disclosure. Kanodia (2007) provides a review of this literature. More recently, Goldstein and Sapra (2013) provide a survey on the costs and benefits of disclosure in the context of banks’ stress tests. The tradeoff presented in this paper and the implications for disclosure
are new to this literature.

There is a large and diverse literature on the welfare implications of public information. Some early papers, such as Hirshleifer (1971) and Hakannson, Kunkel, and Ohlson (1982), have pointed out that public information destroys risk sharing opportunities and thereby impairs social welfare. A more recent line of research relies on payoff externalities and coordination motives across agents to show that public information release may harm welfare, for example, Morris and Shin (2002), Angeletos and Pavan (2004, 2007), Goldstein, Ozdenoren, and Yuan (2011), Vives (2013), and Colombo, Femminis, and Pavan (2014). The mechanism in our paper does not rely on risk sharing or externalities. Instead, our results are generated by the effect that disclosure has on the aggregation of information in financial markets and what this does to the overall amount of information available to real decision makers.

The mechanism in our paper is related to that in Amador and Weill (2010). They construct a monetary model and show that releasing public information about monetary and/or productivity shocks can reduce welfare through reducing the informational efficiency of the good price system, which relates to the indirect effect of disclosure in our financial-market model. One difference in our paper is that we build a model of a financial market, which is quite different, and so we analyze how these forces interact in the context of firms and security prices and the feedback loop between them. This creates differences in both the content and the techniques (see mention of the complications of the feedback-effect literature above). Perhaps more importantly, our paper highlights the different implications of different types of disclosure and so highlights that disclosure can be good or bad, depending on the type of information being disclosed and how effective the market is. This provides a rich set of implications for policy and empirical work. In contrast, in Amador and Weill (2010), there is only one dimension of information and the indirect effect of disclosing information is always negative. Importantly, the two dimensional uncertainty ($\tilde{a}$ and $\tilde{f}$) is crucial in driving the results in our model, because as we show in Section 5.1, the indirect effect of disclosure vanishes in an economy with one-dimensional uncertainty. This is not the case in Amador and Weill (2010).

Another related paper is by Bond and Goldstein (2015) who analyze a feedback model and discuss different implications of disclosure. In particular, their analysis also suggests
that disclosure can either reduce or raise price informativeness, depending on the type of information being disclosed. However, both their mechanism and results are different from ours. First, their mechanism works through a risk-return tradeoff faced by risk-averse traders, for whom disclosure affects the risk and return from the trade. In our model, all agents are risk neutral and so our results are not driven by any kind of risk-return tradeoff. Second, in Bond and Goldstein (2015), there is no direct positive effect of disclosing information, since this information is already known to the decision maker (the government in their model). Hence, they do not study when the indirect effect is strong enough to overcome the direct effect, as we do here.

A few other recent papers also present models in which disclosure harms price efficiency or investment efficiency, albeit through different channels. In Edmans, Heinle, and Huang (2013), only hard information (such as earnings) can be disclosed, and disclosing hard information distorts the manager’s investment incentives by changing the relative weight between hard and soft information. In Han, Tang, and Yang (2014), disclosure attracts noise trading which reduces price informativeness and harms managers’ learning quality. In Gao and Liang (2013), disclosure crowds out private information production, reduces price informativeness, and so harms managers’ learning and investments. In Banerjee, Davis, and Gondhi (2014), public information can lower price efficiency because facing more fundamental information traders choose to acquire non-fundamental information exclusively. Our results highlight the importance of disclosing different types of fundamental information, and they are not driven by anything related to manager incentives, attracting noise trading, or private-information production.

2 The Model

2.1 Environment

There are three dates, $t = 0, 1$, and 2. At date 0, a continuum $[0, 1]$ of “speculators” trade one risky asset based on their diverse private signals and a common public signal about factors related to the asset’s future cash flows. The equilibrium asset price aggregates speculators’
private information through their trading. Assuming a continuum of speculators endowed with diverse signals captures the idea that the financial market aggregates value-relevant information that is inherently dispersed among market participants. At date 1, a continuum [0, 1] of “real decision makers,” who see the public signal and the equilibrium asset price, make inference from the price to guide their actions, which in turn determine the cash flow of the risky asset that was traded in the previous period. As will become clear later, in our baseline model all real decision makers are identical, and so we can simply replace the continuum with a single real decision maker without affecting our analysis. We keep a continuum of real decision makers to admit a macro interpretation of our model, as we explain shortly. At date 2, the cash flow is realized, and all agents get paid and consume.

Our model admits two interpretations – a micro interpretation and a macro interpretation. At a micro level, the risky asset can be interpreted as a stock of a financially-constrained firm which needs capital from outside capital providers to make investments. Real decision makers in this case are capital providers, such as banks, equity investors, and venture capital firms. In this case, the public signal can be thought of as a credit rating released by a credit rating agency or any kind of information about the firm released or required by regulators (e.g., stress tests results in case of a financial firm). At a macro level, the risky asset can be interpreted as an index on a particular industry or on the aggregate stock market, and real decision makers are the managers of those companies included in the index. In this case, the public signal can be thought of as some aggregate statistics released by the government, a public institution, or a rating agency. Under both interpretations, speculators can be thought of as mutual and/or hedge funds who have private information about the future value of the asset. We are agnostic as to which one of the two interpretations is more suitable, but for simplicity, we have adopted the first one and refer to real decision makers as capital providers in presenting the model.

2.2 Investment

The firm in our economy has access to the following production technology:

$$ q(k_j) = \tilde{A}\tilde{F}k_j, $$
where $k_j$ is the amount of capital that the firm raises from capital provider $j$ at date 1, $q(k_j)$ is the date-2 output that is generated by the investment $k_j$, and $\bar{A} \geq 0$ and $\bar{F} \geq 0$ are two productivity factors. Let $\bar{a}$ and $\bar{f}$ denote the natural logs of $\bar{A}$ and $\bar{F}$, i.e., $\bar{a} \equiv \log \bar{A}$ and $\bar{f} \equiv \log \bar{F}$. We assume that $\bar{a}$ and $\bar{f}$ are normally distributed as follows:

$$\bar{a} \sim N(0, \tau_{a}^{-1}) \quad \text{and} \quad \bar{f} \sim N(0, \tau_{f}^{-1})$$

where $\bar{a}$ and $\bar{f}$ are mutually independent, and $\tau_{a} > 0$ and $\tau_{f} > 0$, respectively, are their precision (inverse of variance).

Factors $\bar{A}$ and $\bar{F}$ represent two dimensions of uncertainty that affect the cash flow of the traded firm. For example, one dimension can be a factor related to the aggregate economy, and the other one can be firm-specific (e.g., Greenwood, MacDonald, and Zhang, 1996, p. 97; Veldkamp and Wolfers, 2007). Also, $\bar{a}$ can be thought of as the permanent component in the total productivity and the factor $\bar{f}$ is the transitory component, as in Liu, Wang, and Zha (2013). More generally, cash flows depend on the demand for firms’ products and the technology they develop, and on the success of firms’ operations in traditional lines of business and in new speculative lines of business, and thus the feature of multiple dimensions of uncertainty follows directly. Several papers in the finance literature have also specified that the value of the traded security is affected by more than one fundamental; e.g., Froot, Scharfstein, and Stein (1992), Goldman (2005), Kondor (2012), and Goldstein and Yang (2015), among others. In Section 5.1, we show that this feature of two-dimensional uncertainty is essential in generating our results.

The two-dimensional uncertainty serves to capture the idea that real decision makers might be more informed about some particular aspect of the firm, which is a natural feature of the modern economy to the extent that decision makers often have comparative advantage in processing some types of information. So, in our setting, it is the nature of information asymmetry that characterizes the dichotomy between factors $\bar{a}$ and $\bar{f}$. Specifically, we assume that, relative to financial-market speculators, capital providers have better information about factor $\bar{a}$ than factor $\bar{f}$. In the baseline model analyzed in this section, we consider an extreme version of this asymmetric knowledge by assuming that capital providers know perfectly factor $\bar{a}$ but nothing about factor $\bar{f}$ beyond the prior distribution. Capital providers are
essentially identical in the baseline model, because they have access to the same investment technology and information set. In Section 5.3, we extend our model to equip each capital provider with differential noisy signals about the two factors, and show that our results go through as long as the signal quality about one factor is sufficiently different from the signal quality about the other factor.

At date $t=1$, each capital provider $j$ chooses the level of capital $k_j$. As in Goldstein, Ozdenoren, and Yuan (2013), providing capital incurs a private cost according to the following functional form:

$$c(k_j) = \frac{1}{2}ck_j^2,$$

where the constant $c > 0$ controls the size of the cost relative to the output $q(k_j)$. The cost can be the monetary cost of raising the capital or the private effort incurred in monitoring the investment.

We also follow Goldstein, Ozdenoren, and Yuan (2013) and assume that each capital provider $j$ captures proportion $\beta \in (0, 1)$ of the full output $q(k_j)$ by providing $k_j$, and thus his payoff from the investment is $\beta q(k_j)$. Capital provider $j$ chooses $k_j$ to maximize the payoff $\beta q(k_j)$ he captures from the firm minus his cost $c(k_j)$ of raising capital, conditional on his information set. All capital providers have the same information set, denoted by $\mathcal{I}_R$ (with the subscript “$R$” indicating “real decision makers”), which consists of factor $\tilde{a}$, a public signal $\tilde{w}$, and the asset price $\tilde{P}$ (we will elaborate on $\tilde{w}$ and $\tilde{P}$ in the subsequent subsections). Therefore, capital provider $j$ chooses $k_j$ to solve

$$\max_{k_j} E \left[ \beta \tilde{A} \tilde{F} k_j - \frac{1}{2}ck_j^2 \right] | \mathcal{I}_R.$$

The solution to this maximization problem is:

$$k_j^* = \frac{\beta \tilde{A} \tilde{E} (\tilde{F} \mid \mathcal{I}_R)}{c}.$$  \hspace{1cm} (1)

It is worth noting that the role of the parameter $\beta$ is to provide exposure to both capital providers and speculators to the cash flows from the investment. That is, for every $\beta$ strictly between 0 and 1, both have some exposure to the cash flow (capital providers get fraction $\beta$ and speculators get fraction $1 - \beta$), which is required for our model. As will become clear later, the exact value of $\beta$ does not have any effect on our results. At the cost of additional
complexity, one can endogenize $\beta$ (which we have done in additional analysis), but without much added insight into our main results, which remain identical.

### 2.3 Private and Public Information

Each speculator $i$ observes two private noisy signals about $\bar{a}$ and $\bar{f}$, respectively:

$$\bar{x}_i = \bar{a} + \bar{\xi}_{x,i} \text{ and } \bar{y}_i = \bar{f} + \bar{\xi}_{y,i},$$

where $\bar{\xi}_{x,i} \sim N(0, \tau_x^{-1})$ (with $\tau_x > 0$), $\bar{\xi}_{y,i} \sim N(0, \tau_y^{-1})$ (with $\tau_y > 0$), and they are mutually independent and independent of $\{\bar{a}, \bar{f}\}$. The market price $\bar{P}$ will aggregate speculators’ private signals $\{\bar{x}_i, \bar{y}_i\}$ through their trading in the financial market, and hence $\bar{P}$ will contain information about $\bar{a}$ and $\bar{f}$, which is useful for capital providers to make real investment decisions.

All agents, including speculators and capital providers, observe a public signal $\bar{\omega}$, which communicates a linear combination of the two productivity factors with some error as follows:

$$\bar{\omega} = \mu_a \bar{a} + \mu_f \bar{f} + \bar{\xi}_\omega,$$

where $\mu_a$ and $\mu_f$ are two constants, and $\bar{\xi}_\omega \sim N(0, \tau_\omega^{-1})$ (with $\tau_\omega \geq 0$) is independent of $\{\bar{a}, \bar{f}\}$. The constants $\mu_a$ and $\mu_f$ determine what information the signal $\bar{\omega}$ conveys. If $\frac{\text{Var}(\mu_a \bar{a})}{\text{Var}(\mu_f \bar{f})}$ is large, then the variations in $\mu_a \bar{a} + \mu_f \bar{f}$ are largely driven by factor $\bar{a}$, and so $\bar{\omega}$ is mainly a signal about $\bar{a}$. If $\frac{\text{Var}(\mu_a \bar{a})}{\text{Var}(\mu_f \bar{f})}$ is small, then by the same reason, $\bar{\omega}$ is primarily a signal about factor $\bar{f}$.

Parameter $\tau_\omega$ controls the precision of the public signal $\bar{\omega}$. The public signal can represent various types of public announcements. For example, it can be a rating provided by a credit rating agency for the debt issued by a firm. Another example is an announcement made by a firm, based on disclosure regulation imposed by the government. Finally, the public signal can also be some economic statistics published by government agencies or central banks (e.g., forward guidance provided by central banks about the future path of interest rates). For all these signals, a key question that is often debated is what is the optimal degree of precision of the information that is being provided. For example, a current policy debate is on how much information governments should release about the outcomes of bank stress tests. Or, people often think about what is the optimal level of precision of credit ratings.
This precision is captured in our model by the parameter $\tau_\omega$, and in our analysis, we will follow the literature (e.g., Morris and Shin, 2002; Amador and Weill, 2010) and conduct comparative statics exercises with respect to parameter $\tau_\omega$ to examine the optimal level of precision of the public information for maximizing real efficiency.

Note that in equation (2) we have specified that public information generally conveys information about both factors, as long as both $\mu_a$ and $\mu_f$ are not zero. This specification captures the idea that due to technical reasons the providers of public information may have difficulty in separating the two factors when collecting information or communicating it. For example, a credit rating is a signal about the overall credit worthiness of the firm, which combines the various dimensions that lead to the firm’s success (e.g., the productivity of the firm and the demand for its products). Alternatively, equation (2) can be viewed as a parsimonious modeling device to capture two types of disclosure. That is, if we set $\mu_a = 0$ and $\mu_f \neq 0$, then $\tilde{\omega}$ is a public signal about factor $\tilde{f}$, and similarly, if we set $\mu_f = 0$ and $\mu_a \neq 0$, then $\tilde{\omega}$ is a public signal about factor $\tilde{a}$. So by deriving equilibrium outcomes under the more general specification of (2), our analysis can naturally cover both degenerate cases. Finally, in Section 5.2, we also analyze a variation of our model by specifying two separate public signals – each of which conveys information about one factor, respectively – and show that our results hold.

2.4 Trading and Price Formation

At $t = 0$, speculators submit market orders as in Kyle (1985) to trade the risky asset in the financial market. They can buy or sell up to one unit of the risky asset, and thus speculator $i$’s demand for the asset is $d(i) \in [-1, 1]$. This position limit can be justified by borrowing/short-sales constraints faced by speculators. As argued by Goldstein, Ozdenoren, and Yuan (2013), the specific size of this position limit is not crucial, and what is crucial is that speculators cannot take unlimited positions. Speculators are risk neutral, and therefore they choose their positions to maximize the expected trading profits conditional on their information sets $\mathcal{I}_i = \{\tilde{x}_i, \tilde{y}_i, \tilde{\omega}\}$.

The traded asset is a claim on the portion of the aggregate output that remains after
removing capital providers’ share.\footnote{As explained in Goldstein, Ozdenoren, and Yuan (2013), for technical reasons, we do not assume that the asset is a claim on the net return from the investment.} Specifically, the aggregate output is

\[ \hat{Q} \equiv \int_0^1 q(k_j^*) \, dj = \hat{A} \hat{F} \int_0^1 k_j^* \, dj = \hat{A} \hat{F} K^*, \]

where \( K^* \equiv \int_0^1 k_j^* \, dj \) is the aggregate investment in equilibrium. So, after removing the \( \beta \) fraction of \( \hat{Q} \), the remaining \((1 - \beta)\) fraction constitutes the cash flow on the risky asset:

\[ \hat{V} \equiv (1 - \beta) \hat{Q} = (1 - \beta) \hat{A} \hat{F} K^*. \]

A speculator’s profit from buying one unit of the asset is given by \( \hat{V} - \hat{P} \), and similarly, his profit from shorting one unit is \( \hat{P} - \hat{V} \). So, speculator \( i \) chooses demand \( d(i) \) to solve:

\[ \max_{d(i) \in [-1,1]} d(i) \, E \left( \hat{V} - \hat{P} \, \bigg| \, I_i \right). \tag{3} \]

Since each speculator is atomistic and is risk neutral, he will optimally choose to either buy up to the one-unit position limit, or short up to the one-unit position limit. We denote the aggregate demand from speculators as \( D \equiv \int_0^1 d(i) \, di \), which is the fraction of speculators who buy the asset minus the fraction of those who short the asset.

As in Goldstein, Ozdenoren, and Yuan (2013), to prevent a price that fully reveals the factor \( \hat{f} \) to capital providers, we assume the following noisy supply curve provided by (unmodeled) liquidity traders:

\[ L \left( \hat{\xi}, \hat{P} \right) \equiv 1 - 2 \Phi \left( \hat{\xi} - \lambda \log \hat{P} \right), \tag{4} \]

where \( \hat{\xi} \sim N \left( 0, \tau_{\xi}^{-1} \right) \) (with \( \tau_{\xi} > 0 \)) is an exogenous demand shock independent of other shocks in the economy. Function \( \Phi(\cdot) \) denotes the cumulative standard normal distribution function, which is increasing. Thus, the supply curve \( L \left( \hat{\xi}, \hat{P} \right) \) is strictly increasing in the price \( \hat{P} \) and decreasing in the demand shock \( \hat{\xi} \). The parameter \( \lambda > 0 \) captures the elasticity of the supply curve with respect to the price,\footnote{See Banerjee, Davis, and Gondhi (2014) for a discussion on the economic relevance of price-dependent noise trading.} and it can be interpreted as the liquidity of the market in the sense of price impact: When \( \lambda \) is high, the supply is very elastic with respect to the price and thus, the demand from informed speculators can be easily absorbed by noise trading without moving the price very much. In our baseline model, we need to assume \( \lambda > 0 \) to determine the price, and in Section 5.4 we will relax this assumption by allowing speculators to observe prices and show that our main results are robust.
The basic features assumed in (4) are that the supply is increasing in price $\tilde{P}$ and also has a noisy component $\tilde{\xi}$, both of which are standard in the literature. It is also common in the literature to assume particular functional forms to obtain tractability. The specific functional form assumed here is close to that in Angeletos and Werning (2006), Hellwig, Mukherji, and Tsyvinski (2006), Dasgupta (2007), and Albagli, Hellwig, and Tsyvinski (2012, 2014). As usual, the noisy supply component $\tilde{\xi}$ represents trading coming from (unmodeled) agents who trade for liquidity or hedging needs (e.g., Dow and Rahi, 2003). We do not endogenize the actions of these traders in our setting, because doing so breaks the loglinear structure of the model, which makes an analytical characterization impossible.

The market clears by equating the aggregate demand $D$ from speculators with the noisy supply $L(\tilde{\xi}, \tilde{P})$:

$$D = L(\tilde{\xi}, \tilde{P}).$$

This market clearing condition will determine the equilibrium price $\tilde{P}$.

### 2.5 Equilibrium Definition

The exogenous parameters in our model are: $\tau_a$, the prior precision of factor $\tilde{a}$; $\tau_f$, the prior precision of factor $\tilde{f}$; $\tau_\omega$, the precision of public information; $\tau_x$, the precision of speculators’ private signals about factor $\tilde{a}$; $\tau_y$, the precision of speculators’ private signals about factor $\tilde{f}$; $\tau_\xi$, the precision of noise trading; $\lambda$, the elasticity of noisy supply; $\mu_a$, the loading on factor $\tilde{a}$ in the public signal; $\mu_f$, the loading on factor $\tilde{f}$ in the public signal; $\beta$, the fraction of the output captured by capital providers; and $c$, the parameter controlling the relative size of investment costs. So, the tuple

$$\mathcal{E} = \{\tau_a, \tau_f, \tau_\omega, \tau_x, \tau_\xi, \lambda, \mu_a, \mu_f, \beta, c\}$$

defines an economy. For a given economy, we consider an equilibrium which involves the optimal decisions of each player (capital providers and speculators) and the statistical behavior of aggregate variables ($K, D$, and $\tilde{P}$).

Each player’s optimal decisions will be a function of their information sets. For capital providers, their optimal investments $k_j^*$ given by (1) will be a function of their information set $\mathcal{I}_R = \{\tilde{\omega}, \tilde{P}, \tilde{\omega}\}$. That is, $k_j^* = k(\tilde{a}, \tilde{P}, \tilde{\omega})$. Since they are identical, the aggregate
investment function $K\left(\bar{a}, \bar{P}, \bar{\omega}\right)$ will be the same as the individual investment function: $K^* = K\left(\bar{a}, \bar{P}, \bar{\omega}\right) = k\left(\bar{a}, \bar{P}, \bar{\omega}\right)$. Speculators’ optimal trading strategies $d_i^*$ will be a function of their information set $\mathcal{I}_i = \{\bar{x}_i, \bar{y}_i, \bar{\omega}\}$. That is, $d_i^* = d(\bar{x}_i, \bar{y}_i, \bar{\omega})$. In aggregate, the noise terms $\bar{\varepsilon}_{x,i}$ and $\bar{\varepsilon}_{y,i}$ in their signals $\bar{x}_i$ and $\bar{y}_i$ will wash out, and so the aggregate demand $D$ for the risky asset is a function of $\bar{a}$, $\bar{f}$ and $\bar{\omega}$:

$$D = D\left(\bar{a}, \bar{f}, \bar{\omega}\right) = \int_0^1 d(\bar{x}_i, \bar{y}_i, \bar{\omega}) \, di = E\left[d(\bar{x}_i, \bar{y}_i, \bar{\omega}) \, \big| \, \bar{a}, \bar{f}, \bar{\omega}\right],$$

where the expectation is taken over $\left(\bar{\varepsilon}_x, \bar{\varepsilon}_y\right)$ in (6).

The market clearing condition (5) will therefore determine the price $\bar{P}$ as a function of productivity factors $\{\bar{a}, \bar{f}\}$, the public signal $\bar{\omega}$, and the noise trading shock $\bar{\xi}$: $\bar{P} = P\left(\bar{a}, \bar{f}, \bar{\omega}, \bar{\xi}\right)$. An equilibrium is defined formally as follows.

**Definition 1** An equilibrium consists of a price function, $P\left(\bar{a}, \bar{f}, \bar{\omega}, \bar{\xi}\right) : \mathbb{R}^4 \rightarrow \mathbb{R}$, an investment policy for capital providers, $k\left(\bar{a}, \bar{P}, \bar{\omega}\right) : \mathbb{R}^3 \rightarrow \mathbb{R}$, a trading strategy of speculators, $d(\bar{x}_i, \bar{y}_i, \bar{\omega}) : \mathbb{R}^3 \rightarrow [-1, 1]$, and the corresponding aggregate demand function for the asset $D\left(\bar{a}, \bar{f}, \bar{\omega}\right)$, such that:

(a) for capital provider $j$, $k\left(\bar{a}, \bar{P}, \bar{\omega}\right) = \frac{\beta AE(F|\bar{a}, \bar{P}, \bar{\omega})}{c}$;

(b) for speculator $i$, $d(\bar{x}_i, \bar{y}_i, \bar{\omega})$ solves (3);

(c) the market clearing condition (5) is satisfied; and

(d) the aggregate asset demand is given by (6).

### 3 Equilibrium Characterization

In this section, we illustrate the steps for constructing an equilibrium. It turns out that the equilibrium characterization boils down to a fixed point problem of solving the weight that speculators put on the signal $\bar{y}_i$ about factor $\bar{f}$ when they trade the risky asset. Specifically, we first conjecture a trading strategy of speculators and use the market clearing condition to determine the asset price and hence the information that capital providers can learn from the price. We then update capital providers’ beliefs and characterize their investment rule, which in turn determines the cash flow of the traded asset. Finally, given the implied price and cash flow in the first two steps, we solve for speculators’ optimal trading strategy, which
compares with the initial conjectured trading strategy to complete the fixed point loop.

3.1 Price Informativeness

We conjecture that speculators buy the asset if and only if a linear combination of their (private and public) signals is above a cutoff $g$, where $\phi_y$, $\phi_\omega$, and $g$ are endogenous parameters that will be determined in equilibrium. Note that $\tilde{x}_i + \phi_y \tilde{y}_i + \phi_\omega \tilde{\omega} > g$ is equivalent to

$$\frac{\tilde{z}_{x,i} + \phi_y \tilde{z}_{y,i}}{\sqrt{\tau_x + \phi_y^2 \tau_y}} > \frac{g - (\tilde{a} + \phi_y \tilde{f}) - \phi_\omega \tilde{\omega}}{\sqrt{\tau_x + \phi_y^2 \tau_y}},$$

and hence speculators’ aggregate purchase can be characterized by

$$1 - \Phi \left( \frac{g - (\tilde{a} + \phi_y \tilde{f}) - \phi_\omega \tilde{\omega}}{\sqrt{\tau_x + \phi_y^2 \tau_y}} \right).$$

Similarly, their aggregate selling is

$$\Phi \left( \frac{g - (\tilde{a} + \phi_y \tilde{f}) - \phi_\omega \tilde{\omega}}{\sqrt{\tau_x + \phi_y^2 \tau_y}} \right).$$

Thus, the net holding from speculators is:

$$D = 1 - 2\Phi \left( \frac{g - (\tilde{a} + \phi_y \tilde{f}) - \phi_\omega \tilde{\omega}}{\sqrt{\tau_x + \phi_y^2 \tau_y}} \right).$$ (7)

The market clearing condition (5) together with equations (4) and (7) indicates that

$$1 - 2\Phi \left( \frac{g - (\tilde{a} + \phi_y \tilde{f}) - \phi_\omega \tilde{\omega}}{\sqrt{\tau_x + \phi_y^2 \tau_y}} \right) = 1 - 2\Phi \left( \tilde{\xi} - \lambda \log \tilde{P} \right),$$

which implies that the equilibrium price is given by:

$$\tilde{P} = \exp \left( \frac{\tilde{a} + \phi_y \tilde{f} + \phi_\omega \tilde{\omega} - g}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} + \frac{\tilde{\xi}}{\lambda} \right).$$ (8)

Recall that capital providers have the information set $\{\tilde{a}, \tilde{f}, \tilde{\omega}\}$, and so they know the realizations of $\tilde{a}$ and $\tilde{\omega}$. As a result, the price $\tilde{P}$ is equivalent to the following signal in predicting factor $\tilde{f}$:

$$\tilde{s}_p = \frac{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}} \log \tilde{P} - \tilde{a} - \phi_\omega \tilde{\omega} + g}{\phi_y} = \tilde{f} + \tilde{\varepsilon}_p,$$ (9)

where

$$\tilde{\varepsilon}_p = \frac{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}} \tilde{\xi}}{\phi_y},$$ (10)

which has a precision of

$$\tau_p = \frac{1}{\text{Var}(\tilde{\varepsilon}_p)} = \frac{\phi_y^2 \tau_x \tau_y \lambda}{\tau_y + \phi_y^2 \tau_x \lambda}.$$ (11)

The endogenous precision $\tau_p$ captures how much information capital providers can learn.
from the price about factor $\tilde{f}$, which they do not know. As we will see, $\tau_p$ will affect real efficiency through guiding capital providers’ investment decisions. We will be interested in studying how the precision $\tau_\omega$ of the public information affects $\tau_p$ and then real efficiency. We will show that $\tau_\omega$ affects $\tau_p$ only through its effect on $\phi_y$, the weight that speculators put on their signal about factor $\tilde{f}$ when they trade. Specifically, if speculators trade more aggressively on their information about $\tilde{f}$ (i.e., when $\phi_y$ increases), the price will be more informative about factor $\tilde{f}$, all other things being equal. As a result, capital providers can glean more information from the price, which increases real efficiency.

### 3.2 Optimal Investment Policy

Capital providers have information set $\mathcal{I}_R = \{\tilde{a}, \tilde{P}, \tilde{\omega}\}$. We have already characterized how they use the price $\tilde{P}$ to form a signal $\tilde{s}_p$ in predicting the factor $\tilde{f}$. Regarding the public signal $\tilde{\omega}$ in (2), they can use their knowledge of $\tilde{a}$ to transform $\tilde{\omega}$ into the following signal in predicting $\tilde{f}$:

$$\tilde{s}_\omega \equiv \frac{\tilde{\omega} - \mu_a \tilde{a}}{\mu_f} = \tilde{f} + \mu_f^{-1} \tilde{\omega},$$

which has a precision of $\mu_f^2 \tau_\omega$. That is, capital providers’ information set is equivalent to:

$$\mathcal{I}_R = \{\tilde{a}, \tilde{s}_p, \tilde{s}_\omega\},$$

where the two signals $\tilde{s}_p$ and $\tilde{s}_\omega$ are useful for predicting $\tilde{f}$.

By Bayes’ rule and equation (1), we compute capital providers’ optimal investment as follows:

$$k_j^* = \exp \left[ \left( \log \frac{\beta}{c} + \frac{1}{2 \tau_f + \mu_f^2 \tau_\omega + \tau_p} \right) + \tilde{a} + \frac{\mu_f^2 \tau_\omega}{\tau_f + \mu_f^2 \tau_\omega + \tau_p} \tilde{s}_\omega + \frac{\tau_p}{\tau_f + \mu_f^2 \tau_\omega + \tau_p} \tilde{s}_p \right].$$

(12)

### 3.3 Optimal Trading Strategy

Using the expression of $\tilde{P}$ in (8), the cash flow expression $\tilde{V} = (1 - \beta) \tilde{A} \tilde{F} K^*$, and the investment rule in (12), we can compute the expected price and cash flow conditional on
speculator \(i\)’s information set \(\{\bar{x}_i, \bar{y}_i, \bar{\omega}\}\) as follows:

\[
E\left( \bar{P} \mid \bar{x}_i, \bar{y}_i, \bar{\omega} \right) = \exp \left( b^v_x \bar{x}_i + b^v_y \bar{y}_i + b^v_\omega \bar{\omega} \right),
\]

\[
E\left( \bar{V} \mid \bar{x}_i, \bar{y}_i, \bar{\omega} \right) = \exp \left( b^v_x \bar{x}_i + b^v_y \bar{y}_i + b^v_\omega \bar{\omega} \right),
\]

where the coefficients \(b\)’s are given in the appendix.

Speculator \(i\) will choose to buy the asset if and only if

\[
E\left( \bar{V} \mid \bar{x}_i, \bar{y}_i, \bar{\omega} \right) > E\left( \bar{P} \mid \bar{x}_i, \bar{y}_i, \bar{\omega} \right).
\]

Thus, we have

\[
(b^v_x - b^p_x) \bar{x}_i + (b^v_y - b^p_y) \bar{y}_i + (b^v_\omega - b^p_\omega) \bar{\omega} > (b^p_y - b^p_\omega).
\]

Recall that we conjecture speculators’ trading strategy as buying the asset whenever \(\bar{x}_i + \phi_y \bar{y}_i + \phi_\omega \bar{\omega} > g\). So, to be consistent with our initial conjecture, we require that in equilibrium,

\[
\phi_y = \frac{b^v_y - b^p_y}{b^v_x - b^p_x},
\]

\[
\phi_\omega = \frac{b^v_\omega - b^p_\omega}{b^v_x - b^p_x},
\]

provided that \((b^v_x - b^p_x) > 0\). The right-hand-side \(\frac{b^v_y - b^p_y}{b^v_x - b^p_x}\) of (15) depends only on \(\phi_y\) (through the term of \(\phi_y\) in \(b^v_y\) and \(b^p_y\) and the term of \(\tau_p\) in \(b^v_x\) and \(b^p_x\)). Therefore, we use (15) to compute \(\phi_y\), and then plug this solved \(\phi_y\) into equation (16) to compute \(\phi_\omega\).

To summarize, we have the following characterization proposition.

**Proposition 1** The equilibrium is characterized by the weight \(\phi_y\) that speculators put on the private signal \(\bar{y}_i\) about factor \(\bar{f}\), and \(\phi_y\) is determined by condition (15), as long as \(b^v_x > b^p_x\).

4 The Effect of Disclosure

4.1 Measurement of Efficiency

In this section we study the implications of disclosure in the model, focusing on real efficiency— the surplus generated by real investment decisions. Ideally, we should conduct a full welfare analysis by examining how public disclosure affects the expected utility levels of all agents in the economy. However, as we mentioned before, in order to solve the model in closed form, we have assumed that noise traders trade the risky asset according to equation (4) to meet
their unmodeled liquidity/hedging needs, and it is challenging to endogenize noise trading fully. This precludes a welfare analysis on these (unmodeled) noise traders. So, we focus our analysis on real efficiency implications.\footnote{We have conducted separate analysis on the effect of different kinds of disclosure on speculators’ profits and found similar results concerning the different effects of different types of disclosure.}

It is important to note that, to the extent that noise traders’ unmodeled liquidity/hedging needs are not affected by public disclosure, real efficiency is a reasonable measure for the aggregate welfare.\footnote{We thank Alessandro Pavan for suggesting this interpretation.} Specifically, there are three categories of agents in the economy – speculators, noise traders, and capital providers. The welfare of speculators can be measured by their ex-ante expected trading profit evaluated in equilibrium. As for noise traders, we can follow the microstructure literature to say that their welfare is the trading profit plus additional unmodelled gain from satisfying any hedging/liquidity needs \cite[e.g.,][]{chowdhry1991, subrahmanyam1991, leland1992}. Under this interpretation, what speculators gain in trading is exactly equal to what noise traders lose, and so the total welfare of these two groups of traders is the exogenous hedging/liquidity benefit, which we take not to be affected by disclosure. Then, the only term remaining in the overall welfare calculation is the welfare of capital providers, $E \left[ \beta q \left( k^*_j \right) - c \left( k^*_j \right) \right]$, and we can show that in equilibrium, it is equal to real efficiency scaled by a constant $\frac{\beta}{2-\beta}$. As a result, the total welfare across all agents in the economy is proportional to real efficiency.

Our specific measure of real efficiency follows \cite{goldstein2013} reflecting the expected net benefit of investment evaluated in equilibrium:

$$ RE = E \left[ \tilde{A} \tilde{F} K^* - \int c \left( k^*_j \right) dj \right] $$
$$ = \frac{\beta}{c} \left( 1 - \frac{\beta}{2} \right) \exp \left[ \frac{2}{\tau a} + \frac{2}{\tau f} - Var \left( \frac{\tilde{f}^*}{\tilde{s}_\omega, \tilde{s}_p} \right) \right], \quad (17) $$

where

$$ Var \left( \frac{\tilde{f}^*}{\tilde{s}_\omega, \tilde{s}_p} \right) = \frac{1}{\tau_f + \mu^2 \tau + \tau_p}. \quad (18) $$

In our model, disclosure affects real efficiency through changing capital providers’ information set. The more precise information that capital providers have, the more efficient are their investment decisions. This fact is clearly captured by expression (17): Recall that real
decision makers know factor $\tilde{a}$ and so they only need to forecast the other factor $\tilde{f}$, and so the term $Var\left(\tilde{f} \mid \tilde{s}_\omega, \tilde{s}_p\right)$ captures the efficiency loss due to remaining uncertainty relative to a full information economy.

Equation (18) demonstrates that the quality of public disclosure, measured by $\tau_\omega$, has two effects on the overall quality of capital providers’ information (and hence real efficiency). The first is a direct effect of providing new information, which is related to the term $\mu_\omega^2 \tau_\omega$ in (18). The second effect is an endogenous indirect effect: Public information affects the trading of speculators (more specifically, the loading $\phi_y$ on private information about $\tilde{f}$), and hence the price informativeness about factor $\tilde{f}$, which in turn affects the amount of information that capital providers can learn from the price (i.e., the term $\tau_p$ in (18)). Formally, by (17) and (18), we have

$$\frac{\partial RE}{\partial \tau_\omega} \propto \frac{\partial (\tau_f + \mu_\omega^2 \tau_\omega + \tau_p)}{\partial \tau_\omega} = \frac{\partial \tau_p}{\partial \tau_\omega} + \frac{\partial \mu_\omega^2}{\partial \tau_\omega},$$

(19)

where,

$$\frac{\partial \tau_p}{\partial \tau_\omega} = \frac{2\tau_p \tau_y}{\phi_y (\tau_y + \phi_y^2 \tau_x)} \frac{\partial \phi_y}{\partial \tau_\omega},$$

(20)

which follows from applying the chain rule to equation (11).

Clearly, in equation (19), the direct effect of better disclosure is always positive, as long as $\mu_\omega > 0$ (i.e., as long as public disclosure has some information about factor $\tilde{f}$): When public disclosure becomes more precise, capital providers can learn more valuable information directly from it and use it to improve their investment decisions. However, the indirect effect $\frac{\partial \tau_p}{\partial \tau_\omega}$ can be positive or negative. This is because more precise public disclosure can lead speculators to trade either more or less aggressively on their private information about $\tilde{f}$, making the price, which serves as an additional source of information for capital providers, either more or less precise as a signal about $\tilde{f}$. If the indirect effect is positive, then the direct effect of disclosure is amplified, leading to a very strong positive effect of public disclosure on real efficiency. By contrast, if the indirect effect is negative, then it attenuates or even overturns the direct effect, making the overall effect of disclosure modest or even negative.

In the following two subsections, we show that the sign of the indirect effect depends on the type of information being disclosed. As we will show, the key difference is coming
from whether the capital providers are trying to learn from the market about the factor on which information is being disclosed or not. Specifically, releasing public information about factor $\tilde{a}$ generates a positive indirect effect, while releasing public information about factor $\tilde{f}$ generates a negative indirect effect, which can dominate the positive direct effect so that disclosing information about $\tilde{f}$ can harm real efficiency, provided that the market aggregates speculators’ private information effectively.

4.2 The Effect of Disclosure about Factor $\tilde{a}$

As we discussed before, when $\frac{\text{Var}(\mu_a \tilde{a})}{\text{Var}(\mu_f \tilde{f})}$ is very large, the public signal $\tilde{\omega}$ in (2) is primarily a signal about factor $\tilde{a}$. For simplicity, we assume $\mu_f = 0$ and normalize $\mu_a$ as 1, so that $\tilde{\omega}$ degenerates to

$$\tilde{\omega} = \tilde{a} + \tilde{\varepsilon}_\omega.$$  

In this case, since capital providers know factor $\tilde{a}$ perfectly and the public signal $\tilde{\omega}$ does not provide information about the other factor $\tilde{f}$, the direct effect of public disclosure vanishes (i.e., $\frac{\partial \mu_f^2 \tau}{\partial \tau_x} = \mu_f^2 = 0$ in (19)). Therefore, the only channel for public disclosure to affect real efficiency is through its indirect effect on the endogenous precision of the information that capital providers can learn from the asset price (i.e., $\frac{\partial RE}{\partial \tau_x} \propto \frac{\partial \tau_x}{\partial \tau_x}$ in (19)).

Computing the terms $b$’s in equation (15) for the case where the public signal $\tilde{\omega}$ provides information only about factor $\tilde{a}$ and assuming that the supply elasticity $\lambda$ is very large, we get that $b_x^p$ and $b_y^p$ approach 0, and thus the expression in (15) determining $\phi_y$ degenerates to

$$\phi_y \approx \frac{b_y^v}{b_x^v} = \left(1 + \frac{\tau_p}{\tau_f + \tau_p}\right) \frac{\tau_y}{\tau_f + \tau_y}.$$  

(21)

Intuitively, when the supply elasticity $\lambda \to \infty$, the market is very liquid and so prices do not move that much (see equation (8)). Hence, traders mainly use their information to update about cash flows and not so much about prices.\(^8\) Then, the relative weight $\phi_y$ they put on their signal $\tilde{y}_i$ (about factor $\tilde{f}$) in their trading rule is determined by the extent to which they use signal $\tilde{y}_i$ to update about cash flow relative to the extent they use signal $\tilde{x}_i$ (about

\(^8\)Note that in this case, we always have $b_x^v > b_y^v$, so that the condition in Proposition 1 is always satisfied.
factor \( \tilde{a} \) to update about cash flow. This is the ratio \( \frac{b_v^y}{b_v^x} \).

Using the expression of \( \tau_p \) in (11) and applying the implicit function theorem to (21), we can show
\[
\frac{\partial \phi_y}{\partial \tau_{\omega}} = 1 - \frac{\phi_y}{(\tau_f + \tau_p)(\tau_f + 2\tau_p)(\tau_f + \phi_y^2 \tau_{\omega})} > 0.
\] (22)

That is, more precise public disclosure about factor \( \tilde{a} \) causes speculators to trade more aggressively on their private information about the other factor \( \tilde{f} \).

To see the intuition note that in the expression of \( E \left( \tilde{V} \mid \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \right) \) in (14) the private signal \( \tilde{x}_i \) and the public signal \( \tilde{\omega} \) are useful for predicting \( \tilde{a} \), while the private signal \( \tilde{y}_i \) is useful for predicting \( \tilde{f} \). When \( \tau_{\omega} \) increases, so that the public signal becomes a more informative signal about \( \tilde{a} \), speculators put a higher weight \( b_v^\omega \) on the signal \( \tilde{\omega} \) and a lower weight \( b_v^x \) on the signal \( \tilde{x}_i \) in predicting \( \tilde{a} \). Other things equal, this increases \( \phi_y \) given that \( \phi_y = \frac{b_v^\omega}{b_v^x} \) as we see in (21). This effect is captured by the numerator of (22). Moreover, there is a further “multiplier effect” captured by the denominator in (22): The increase in \( \phi_y \) improves \( \tau_p \) in (11), and so capital providers glean more information on \( \tilde{f} \) from the price, making the asset cash flow \( \tilde{V} \) more responsive to \( \tilde{f} \) through capital providers’ investments. This, in turn, causes speculators to rely more on their private signal \( \tilde{y}_i \) – which is a signal about \( \tilde{f} \) – in making their forecasts, which increases \( b_v^y \) in (14). So, \( \phi_y \) increases further given that \( \phi_y = \frac{b_v^\omega}{b_v^x} \). This amplification chain continues on and on till it converges to a much higher level of \( \phi_y \). Note that this second multiplier effect depends on the fact that the cash flows from the traded security are endogenous and affected by market prices, whereas the first basic effect would exist even in a model where the cash flows from the traded security do not depend on market prices (as in Subrahmanyam and Titman (1999) and Foucault and Gehrig (2008)).

Overall, since \( \frac{\partial \phi_y}{\partial \tau_{\omega}} > 0 \) by (22), we have \( \frac{\partial \tau_{\omega}}{\partial \tau_p} > 0 \) as well in (20). That is, capital providers learn more information about factor \( \tilde{f} \) from the price. Thus, real efficiency improves with better disclosure. Greater disclosure about factor \( \tilde{a} \) allows the market to do a better job aggregating information about factor \( \tilde{f} \), and this increases real efficiency. To summarize, we have the following proposition.

**Proposition 2** Suppose that the supply elasticity \( \lambda \) is high and that public information \( \tilde{\omega} \) is
a signal about factor $\tilde{a}$ (i.e., $\mu_a = 1$ and $\mu_f = 0$).

(a) There exists a unique equilibrium that is characterized by the relative weight $\phi_y > 0$ on private signals $\tilde{y}_i$ about the other factor $\tilde{f}$ in speculators’ trading strategy, which is determined by equation (21).

(b) Increasing the precision $\tau_\omega$ of public disclosure

(i) increases the relative weight $\phi_y > 0$ on private signals $\tilde{y}_i$ (i.e., $\frac{\partial \phi_y}{\partial \tau_\omega} > 0$);

(ii) increases the precision $\tau_p$ that capital providers learn from the price regarding factor $\tilde{f}$ (i.e., $\frac{\partial \tau_p}{\partial \tau_\omega} > 0$), and so the indirect effect is positive; and

(iii) increases real efficiency $RE$ (i.e., $\frac{\partial RE}{\partial \tau_\omega} > 0$).

Panels (a1) and (a2) of Figure 1 graphically illustrate Proposition 2. Here, we simply set the precision of all random variables to be 1; that is, $\tau_a = \tau_f = \tau_x = \tau_y = \tau_\xi = 1$. The patterns are quite robust with respect to changes in these precision parameter values. We set the supply elasticity $\lambda$ at 1. We choose $\mu_a = 0.8$ and $\mu_f = 0.2$, so that public disclosure $\tilde{\omega}$ is mainly a signal about factor $\tilde{a}$. Note that under this parameter configuration, $\tilde{\omega}$ also provides information about factor $\tilde{f}$, and the direct effect of disclosure is positive (i.e., $\mu_f > 0$ in equation (19)). \footnote{In Section 5.3, we extend the model to allow capital providers to see noisy signals about both factors, and in that extension, even a public signal of the form $\tilde{a} + \tilde{\varepsilon}_\omega$, which corresponds to $\mu_f = 0$ and $\mu_a = 1$ in (2), has a positive direct effect on real efficiency.}

In Panel (a1), we plot the weight $\phi_y$ that speculators put on the private signal $\tilde{y}_i$ against the precision $\tau_\omega$ of the public signal. In Panel (a2), we plot three variables against $\tau_\omega$: (i) $\mu_f^2 \tau_\omega$, the direct effect of public disclosure on capital providers’ forecast precision by providing new information about $\tilde{f}$; (ii) $\tau_p$, the indirect effect of public disclosure on capital providers’ forecast precision by affecting the informational content in the price; and (iii) $\mu_f^2 \tau_\omega + \tau_p$, which is a proxy for real efficiency, since by (17) and (18), real efficiency $RE$ is a monotonic transformation of $\mu_f^2 \tau_\omega + \tau_p$.

We see that in Panel (a1), as Proposition 2 predicts, increasing the precision $\tau_\omega$ of the public signal increases the weight $\phi_y$ that speculators put on the private signal $\tilde{y}_i$ about factor $\tilde{f}$. This in turn means that in Panel (a2), the precision $\tau_p$ of information that capital providers learn from the price increases, because $\tau_p$ increases with $\phi_y$ by (11). Clearly, the
direct effect $\mu_f^2 \tau_\omega$ of disclosure increases with $\tau_\omega$ as well in Panel (a2). As a result, the indirect effect of disclosure amplifies the direct effect, and the overall effect of disclosure is to increase real efficiency $\mu_f^2 \tau_\omega + \tau_p$.

### 4.3 The Effect of Disclosure about Factor $\tilde{f}$

When $\frac{Var(\mu_\tilde{f})}{Var(\mu_f)}$ is very small, public disclosure $\tilde{\omega}$ in (2) is primarily a signal about $\tilde{f}$. For simplicity, we assume $\mu_\tilde{a} = 0$ and normalize $\mu_f$ as 1, so that $\tilde{\omega}$ degenerates to

$$\tilde{\omega} = \tilde{f} + \tilde{\varepsilon}_\omega.$$ 

In this case, both effects of public disclosure are active in equation (19). First, the public signal $\tilde{\omega}$ directly benefits capital providers by providing information that they wish to learn. Second, it affects the trading behavior of speculators and hence the informational content in the price, thereby indirectly affecting capital providers’ forecast.

Similarly to the steps taken in the previous subsection, we derive the terms $b$’s and take the supply elasticity $\lambda$ to be very large, so that $\phi_y$ degenerates to

$$\phi_y \approx \frac{b^w_y}{b^w_x} = \frac{\tau_y (\tau_f + 2\tau_p + \tau_\omega)}{(\tau_f + \tau_p + \tau_\omega)(\tau_f + \tau_y + \tau_\omega)}.$$

Applying the implicit function theorem to the above equation, we can show:

$$\frac{\partial \phi_y}{\partial \tau_\omega} = -\phi_y \left( \frac{\tau_p}{(\tau_f + 2\tau_p + \tau_\omega)(\tau_f + \tau_y + \tau_\omega)} + \frac{1}{\tau_f + \tau_y + \tau_\omega} \right) < 0.$$

That is, more precise public disclosure about factor $\tilde{f}$ causes speculators to trade less aggressively on their own private information about $\tilde{f}$.

The intuition for this result goes as follows: When the public information $\tilde{\omega}$ is mainly a signal about $\tilde{f}$, increasing its precision $\tau_\omega$ will decrease the weight $b^w_y$ on speculators’ own private signal $\tilde{y}_i$ in predicting $\tilde{f}$. This directly decreases $\phi_y$, given that $\phi_y = \frac{b^w_y}{b^w_x}$. In addition, there is a multiplier effect, as captured by the denominator in (24): The decrease in $\phi_y$ reduces $\tau_p$, which causes capital providers to glean less information about $\tilde{f}$, making the asset value $\tilde{V}$ less sensitive to $\tilde{f}$. So, in anticipation of this outcome, speculators trade more aggressively on their private information $\tilde{x}_i$ about the other factor $\tilde{a}$ and less aggressively
on information $\bar{y}_i$ about $\tilde{f}$ (that is, $b^i_x$ becomes higher and $b^i_y$ becomes lower), which further reduces $\phi_y$ through (23), until the equilibrium value $\phi_y$ reaches a much lower level. As before, only the multiplier effect depends on the fact that the cash flows from the traded security are endogenous and affected by market prices.

Since $\frac{\partial b^i_x}{\partial \tau_\omega} < 0$, we have $\frac{\partial \tau_\omega}{\partial \tau_\omega} < 0$ as well. That is, capital providers learn less information from the price as a result of more disclosure about factor $\tilde{f}$, so that the indirect effect of disclosing information about factor $\tilde{f}$ is negative in (19). This negative indirect effect attenuates the positive direct effect, causing the overall effect of disclosure on real efficiency to be modest or even negative. This result presents a paradox: Recall that factor $\tilde{f}$ is the variable that capital providers care to learn, still disclosing more information about it publicly gives rise to a counter-productive indirect effect through affecting the price informativeness, and this indirect effect may overturn the positive direct effect reducing real efficiency overall.

We show that the negative indirect effect is stronger than the positive direct effect when public information is relatively imprecise ($\tau_\omega$ is small) and the precision $\tau_\xi$ of noise trading is large. The intuition is as follows. First, when the disclosure level is sufficiently high, the positive direct effect always dominates. For instance, if $\tau_\omega \to \infty$, capital providers would know factor $\tilde{f}$, and the allocation would be the first best, which achieves the maximum real efficiency. So, only when the disclosure level $\tau_\omega$ is low, is it possible for the negative indirect effect to dominate. Second, suppose $\tau_\omega$ is low. When there is little noise trading ($\tau_\xi$ is large), the market aggregates speculators’ private information effectively. Then, since the indirect effect operates through price informativeness, it is particularly strong in this case. By contrast, when $\tau_\xi$ is small, the market has a lot of noise trading, and its information aggregation role is limited, thereby weakening disclosure’s indirect effect via price informativeness.

Overall, greater disclosure about factor $\tilde{f}$ interferes with the ability of the market to aggregate information about this factor. This tends to reduce real efficiency. The effect might be so strong as to outweigh the positive direct effect that precise disclosure about $\tilde{f}$ has on real efficiency. To summarize, we have the following proposition.

**Proposition 3** Suppose that the supply elasticity $\lambda$ is high and that public information $\bar{\omega}$ is a signal about factor $\tilde{f}$ (i.e., $\mu_a = 0$ and $\mu_f = 1$).

(a) There exists a unique equilibrium that is characterized by the relative weight $\phi_y > 0$
on private signals $\tilde{y}_i$ about factor $\tilde{f}$ in speculators’ trading strategy, which is determined by equation (23).

(b) Increasing the precision $\tau_\omega$ of public disclosure

(i) decreases the relative weight $\phi_y$ on private signals $\tilde{y}_i$ (i.e., $\frac{\partial \phi_y}{\partial \tau_\omega} < 0$);

(ii) decreases the precision $\tau_p$ that capital providers learn from the price regarding the factor $\tilde{f}$ (i.e., $\frac{\partial \tau_p}{\partial \tau_\omega} < 0$), and so the indirect effect is negative;

(iii) increases real efficiency $RE$ at high levels of disclosure (i.e., $\frac{\partial RE}{\partial \tau_\omega} > 0$ for large $\tau_\omega$); and

(iv) decreases (increases) real efficiency $RE$ at low levels of disclosure if the precision $\tau_\xi$ of noise trading is large (small) (i.e., for small $\tau_\omega$, $\frac{\partial RE}{\partial \tau_\omega} < 0$ if $\tau_\xi$ is large, and $\frac{\partial RE}{\partial \tau_\omega} > 0$ if $\tau_\xi$ is small).

Panels (b1)–(c2) of Figure 1 graphically illustrate Proposition 3. As in Panels (a1)–(a2), we set $\tau_a = \tau_f = \tau_x = \tau_y = \lambda = 1$ in all these four panels. But here we set $\mu_a = 0.2$ and $\mu_f = 0.8$, so that public information $\tilde{\omega}$ is primarily a signal about factor $\tilde{f}$. In Panels (b1)–(b2), we choose $\tau_\xi = 0.5$, so that the level $\frac{1}{\tau_\xi}$ of noise trading is relatively high and the market does not aggregate private information that much. In Panels (c1)–(c2), we choose $\tau_\xi = 10$, and thus the level of noise trading is low and the market aggregates private information effectively.

In Panels (b1) and (c1), we see that, consistent with Proposition 3, the relative weight $\phi_y$ that speculators put on private information $\tilde{y}_i$ decreases with the precision $\tau_\omega$ of public disclosure. This translates to a decreasing $\tau_p$ as a function of $\tau_\omega$ in Panels (b2) and (c2), which corresponds to the negative indirect effect of disclosure. As a result, the direct effect of increasing $\tau_\omega$, as manifested by the increasing $\mu_f^2 \tau_\omega$, is attenuated by the negative indirect effect in both panels.

In addition, in Panel (b2) where $\tau_\xi$ is relatively small, the direct effect dominates and real efficiency $(\mu_f^2 \tau_\omega + \tau_p)$ increases with $\tau_\omega$. By contrast, in Panel (c2) where $\tau_\xi$ is relatively large, the indirect effect dominates for low levels of disclosure while the direct effect dominates for high levels of disclosure, so that there exists a U-shape between real efficiency and disclosure. This U-shape pattern shares some similarity to the main result of Morris and Shin (2002, p. 1529), and so it has similar implications for optimal disclosure. That is, there may be technical constraints in achieving precision beyond some upper bound, so that a social
planner may be restricted to choosing a disclosure level $\tau_\omega$ from some given interval $[0, \bar{\tau}_\omega]$. Then, we will see a “bang-bang” solution to the choice of optimal $\tau_\omega^*$ in which the socially optimal real efficiency entails either providing no public information at all (i.e., setting $\tau_\omega^* = 0$), or providing the maximum feasible amount of public information (i.e., setting $\tau_\omega^* = \bar{\tau}_\omega$).

5 Discussion and Robustness

In this section, we provide additional discussion and demonstrate the robustness of the model. First, in Subsection 5.1, we show that having two dimensions of uncertainty is crucial for our main results. Then, in the following three subsections, we show that our results are robust to various changes in the structure of the model.

5.1 The Role of Two Dimensional Uncertainty

Suppose we shut down the uncertainty related to factor $\tilde{a}$ by letting $\tau_a$ approach infinity, so that $\tilde{a}$ becomes common knowledge. Then, speculators will no longer rely on their signals $\tilde{x}_i$ in forming their trading strategies. We thus conjecture that speculators buy the asset whenever $\tilde{y}_i + \phi_{\omega^1}\tilde{\omega} > g_{\omega^1}$, where $\phi_{\omega^1}$ and $g_{\omega^1}$ are endogenous parameters. We can follow similar steps as in Section 3 and show that speculators’ aggregate demand for the risky asset is $D_{\omega^1} = 1 - 2\Phi \left( \frac{g_{\omega^1} - \phi_{\omega^1}}{\sqrt{\tau_y}} \right)$. So, using market clearing condition (5), we can find that the equilibrium price would change to:

$$\tilde{P}_{\omega^1} = \exp \left( \frac{\tilde{f} + \tilde{\xi}}{\lambda \sqrt{\tau_y}} - \frac{g_{\omega^1}}{\lambda \sqrt{\tau_y}} + \frac{\phi_{\omega^1}}{\lambda \sqrt{\tau_y}} \right).$$

Given that capital providers know public information $\tilde{\omega}$, the price $\tilde{P}_{\omega^1}$ is equivalent to the following signal in predicting $\tilde{f}$:

$$\tilde{s}_{p^1} = \tilde{f} + \sqrt{\tau_y} \tilde{\xi},$$

which has a precision of

$$\tau_{p^1} = \frac{1}{\text{Var} \left( \sqrt{\tau_y} \tilde{\xi} \right)} = \tau_y \tau_\xi.$$

Clearly, the amount $\tau_{p^1}$ of information that capital providers learn from the price is not
affected by the public information precision $\tau_{\omega}$, which therefore shuts down the mechanism emphasized in our analysis. So, all our main results, such as the amplification or attenuation of the direct effect of disclosure and the negative overall effect on real efficiency, vanish in this alternative economy with unidimensional uncertainty.

**Proposition 4** In the economy with unidimensional uncertainty, disclosure does not affect the amount of information that capital providers learn from prices and so the indirect effect of disclosure is inactive.

### 5.2 Two Public Signals

In our baseline model, in (2) we specify public disclosure as a signal about a linear combination of both factors. Although we think this specification is reasonable – for instance, the providers of public disclosure have technical constraints in separating the two factors – it is also important to ensure that our results hold when there are two separate public signals, each of which conveys information about one factor only. Specifically, in this subsection, we assume two pieces of public information as follows:

$$\tilde{\omega}_a = \tilde{a} + \tilde{\epsilon}_{\omega a} \quad \text{and} \quad \tilde{\omega}_f = \tilde{f} + \tilde{\epsilon}_{\omega f},$$

where $\tilde{\epsilon}_{\omega a} \sim N(0, \tau_{\omega a}^{-1})$ (with $\tau_{\omega a} \geq 0$) and $\tilde{\epsilon}_{\omega f} \sim N(0, \tau_{\omega f}^{-1})$ (with $\tau_{\omega f} \geq 0$) are mutually independent and independent of other random variables. Parameters $\tau_{\omega a}$ and $\tau_{\omega f}$ control the precision of the two public signals, respectively. All the other features of the model are the same as before.

We conjecture that speculators buy the asset whenever $\bar{x}_i + \phi_y \bar{y}_i + \phi_{\omega a} \tilde{\omega}_a + \phi_{\omega f} \tilde{\omega}_f > g$, where $\phi$’s and $g$ are endogenous parameters. Following similar steps as in the baseline model, we can show that the price information to capital providers is still given by a signal $\tilde{s}_p$, as specified by equations (9)–(11). Now capital providers have the information set $\{\tilde{a}, \tilde{\omega}_a, \tilde{\omega}_f, \tilde{s}_p\} = \{\bar{a}, \tilde{\omega}_a, \tilde{\omega}_f, \tilde{s}_p\}$. By noting that $\tilde{\omega}_a$ is redundant given $\bar{a}$, capital provider $j$’s decision problem at $t = 1$ is:

$$k^*_j = \arg\max_{k_j} E \left( \frac{\beta \tilde{A} \tilde{F} k_j}{c} \mid \tilde{\omega}_f, \tilde{s}_p \right) = \frac{\beta \tilde{A} E \left( \tilde{F} \mid \tilde{\omega}_f, \tilde{s}_p \right)}{c}$$

$$= \exp \left[ \log \frac{\beta}{c} + \frac{1}{2} \frac{1}{\tau_f + \tau_{\omega f} + \tau_p} \right] + \bar{a} + \frac{\tau_{\omega f}}{\tau_f + \tau_{\omega f} + \tau_p} \tilde{\omega}_f + \frac{\tau_p}{\tau_f + \tau_{\omega f} + \tau_p} \tilde{s}_p.$$
We can also compute real efficiency as
\[ RE = \frac{\beta}{c} \left( 1 - \frac{\beta}{2} \right) \exp \left( \frac{2}{\tau_a} + \frac{2}{\tau_f} - \frac{1}{\tau_f + \tau_\omega + \tau_p} \right). \]

Back to date 0, speculators forecast the cash flow and the asset price as follows:
\[
E \left( \tilde{V} \mid \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \right) = \exp \left( b^v_0 + b^v_x \tilde{x}_i + b^v_y \tilde{y}_i + b^v_{\omega a} \tilde{\omega}_a + b^v_{\omega f} \tilde{\omega}_f \right),
\]
\[
E \left( \tilde{P} \mid \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \right) = \exp \left( b^p_0 + b^p_x \tilde{x}_i + b^p_y \tilde{y}_i + b^p_{\omega a} \tilde{\omega}_a + b^p_{\omega f} \tilde{\omega}_f \right),
\]
where the coefficients \( b \)'s are given in the appendix. They will purchase one unit of the asset if and only if \( E \left( \tilde{V} \mid \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \right) > E \left( \tilde{P} \mid \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \right) \). So, comparing with the initially conjectured trading strategy, we have the following system determining the equilibrium:
\[
\phi_y = \frac{b^v_y}{b^v_x}, \phi_{\omega a} = \frac{b^v_{\omega a}}{b^v_x - b^v_y}, \phi_{\omega f} = \frac{b^v_{\omega f}}{b^v_x - b^v_y}, \]
where the first equation has only one unknown \( \phi_y \).

We can show that our results in Propositions 2 and 3 continue to hold in this economy with two independent public signals. Formally, we have the following proposition.

**Proposition 5** Suppose that the supply elasticity \( \lambda \) is high in the economy with two independent public signals about the two factors.

(a) There exists a unique equilibrium characterized by the relative weight \( \phi_y > 0 \) on private signals about factor \( \tilde{f} \) in speculators’ trading strategy.

(b) Increasing the precision \( \tau_{\omega a} \) of public disclosure about factor \( \tilde{a} \) increases the relative weight \( \phi_y \) in speculators’ trading strategy, the precision \( \tau_p \) that capital providers learn from the price, and real efficiency \( RE \).

(c) Increasing the precision \( \tau_{\omega f} \) of public disclosure about factor \( \tilde{f} \) decreases the relative weight \( \phi_y \) and the precision \( \tau_p \) that capital providers learn from the price, and it increases real efficiency \( RE \) if and only if \( \tau_\omega \) is large or the precision \( \tau_\xi \) of noise trading is small.

### 5.3 Capital Providers Receive Noisy Signals about Factors

In our baseline model, we have assumed that capital providers know factor \( \tilde{a} \) perfectly and know nothing about factor \( \tilde{f} \), so that they care only about the price’s informational content about \( \tilde{f} \). In this subsection, we extend our model by assuming that capital providers receive noisy signals about both factors, and show that all our results go through as long as capital
providers wish to learn one productivity factor more than the other.

Specifically, we now endow each capital provider \( j \) with two private signals

\[
\tilde{z}_j = \tilde{a} + \tilde{z}_{z,j} \quad \text{and} \quad \tilde{s}_j = \tilde{f} + \tilde{s}_{s,j},
\]

where \( \tilde{z}_{z,j} \sim N(0, \tau_{z}^{-1}) \) (with \( \tau_{z} > 0 \)) and \( \tilde{s}_{s,j} \sim N(0, \tau_{s}^{-1}) \) (with \( \tau_{s} > 0 \)) are mutually independent and they are independent of all other random variables. We keep intact all the other features of the model. Our baseline model corresponds to the case of \( \tau_{z} = \infty \) and \( \tau_{s} = 0 \). If \( \frac{\tau_{a}}{\tau_{a}} \) and \( \frac{\tau_{s}}{\tau_{s}} \) are sufficiently different, then capital providers are more keen to learn one factor than the other.

We still consider trading strategies such that speculators buy the asset if and only if

\[
\tilde{x}_i + \tilde{y}_i + \tilde{y}_i > g,
\]

where \( \tilde{x}_i \) and \( \tilde{y}_i \) are endogenous parameters determined in equilibrium. So, their aggregate demand \( D(\tilde{a}, \tilde{f}, \tilde{\omega}) \) is still given by equation (7), and the market-clearing condition still implies a price function \( P(\tilde{a}, \tilde{f}, \tilde{\omega}, \tilde{\xi}) \) in equation (8). However, because now capital providers do not observe \( \tilde{a} \) perfectly, the price is no longer a signal about \( \tilde{f} \) given by (9); but instead, it is a signal about both \( \tilde{a} \) and \( \tilde{f} \) as follows:

\[
\tilde{s}_{p}^{\text{ext}} = \frac{\tilde{a}}{\phi_y} + \tilde{f} + \tilde{\varepsilon}_p,
\]

where \( \tilde{\varepsilon}_p \) is still defined by \( \tilde{\varepsilon}_p = \frac{\sqrt{\tau_{x}^{-1} + \phi_y^2 \tau_{y}^{-1}}}{\phi_y} \).

Each capital provider \( j \)'s optimal investment decision is:

\[
k_j^* = \arg \max_{k_j} E \left( \beta \tilde{A} F k_j - \frac{c}{2} k_j^2 \right) = \frac{\beta E \left( e^{\tilde{a} + \tilde{f}} \tilde{z}_j, \tilde{s}_j, \tilde{\omega}, \tilde{s}_{p}^{\text{ext}} \right)}{c}.
\]

We can still show that capital providers follow a loglinear investment rule:

\[
k_j^* = \exp \left( h_0 + h_z \tilde{z}_j + h_s \tilde{s}_j + h_{\omega} \tilde{\omega} + h_p \tilde{s}_{p}^{\text{ext}} \right),
\]

where \( h \)'s are endogenous constants that depend on \((\tau_{a}, \tau_{f}, \tau_{z}, \tau_{s}, \mu_{a}, \mu_{f}, \phi_y, \tau_{p})\). We then follow steps similar to the baseline model and show that the characterization of the equilibrium boils down to one equation in terms of the loading \( \phi_y \) that speculators put on their private signals \( \tilde{y}_i \). The complexity of the inference problem induced by the price signal \( \tilde{s}_{p}^{\text{ext}} \) in (25) precludes a full analytical characterization of the equilibrium, and therefore we rely on numerical analysis.

[INSERT FIGURE 2 HERE]
In the right panels of Figure 2, we plot real efficiency, \( \text{RE} = E \left[ \tilde{A} \tilde{F} K^* - j c (k_j^*) dj \right] \), against the precision \( \tau_\omega \) of public information. In the left panels, we also plot the direct and indirect effects of public information on the inference problem of capital providers. Specifically, in the first-order condition (26) of capital providers’ decision problem, they wish to forecast the total productivity \( \tilde{a} + \tilde{f} \) using the information set \( \{ \tilde{z}_j, \tilde{s}_j, \tilde{\omega}, \tilde{a}_{p}^{\text{ext}} \} \). Note that capital providers have private information \( \{ \tilde{z}_j, \tilde{s}_j \} \), and the forecast precision given their own information is \( \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j)} \). After adding the public disclosure \( \tilde{\omega} \), their forecast precision increases to \( \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j, \tilde{\omega})} \), and so the difference of \( \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j)} - \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j, \tilde{\omega})} \) captures how much extra precision capital providers obtain by directly observing the public signal \( \tilde{\omega} \) (This is consistent with using \( \mu_2^2 \tau_\omega = \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{a}, \tilde{s}, \tilde{\omega})} - \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{a}, \tilde{s})} \) to measure the direct learning from disclosure in the baseline model). So, we measure the direct effect of disclosure as follows:

\[
\text{Direct Effect} \equiv \frac{\partial}{\partial \tau_\omega} \left[ \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j, \tilde{\omega}))} - \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j)} \right],
\]

which is always positive. Similarly, we will use the difference \( \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j, \tilde{\omega}, \tilde{a}_{p}^{\text{ext}})} - \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j, \tilde{\omega})} \) to capture how much extra information capital providers can learn from the price, and therefore we define the indirect effect of disclosure as follows:

\[
\text{Indirect Effect} \equiv \frac{\partial}{\partial \tau_\omega} \left[ \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j, \tilde{\omega}, \tilde{a}_{p}^{\text{ext}}))} - \frac{1}{\text{Var}(\tilde{a} + \tilde{f} | \tilde{z}_j, \tilde{s}_j, \tilde{\omega})} \right].
\]

The indirect effect can be positive or negative.

In all panels of Figure 2, we set \( \tau_a = \tau_f = \tau_x = \tau_y = \lambda = 1 \), \( \beta = \frac{1}{2} \), and \( c = 1 \). We also choose \( \tau_z = 5 \) and \( \tau_s = 1 \), so that capital providers know more about factor \( \tilde{a} \) than factor \( \tilde{f} \). In Panels (a1) and (a2), we set \( \mu_a = 0.8 \) and \( \mu_f = 0.2 \), making public information \( \tilde{\omega} \) primarily a signal about factor \( \tilde{a} \). We also arbitrarily choose \( \tau_\xi = 1 \) in these two panels. In contrast, in the remaining four panels (Panels (b1)–(c2)), we set \( \mu_a = 0.2 \) and \( \mu_f = 0.8 \), making public information \( \tilde{\omega} \) primarily a signal about factor \( \tilde{f} \). Also, in Panels (b1) and (b2), we choose \( \tau_\xi = 0.5 \), so that the market does not aggregate private information that much, and in Panels (c1) and (c2), we choose \( \tau_\xi = 10 \) to make the market aggregate private information effectively.

We see that Figure 2 delivers the same message as Figure 1. In Panels (a1) and (a2), when
public information is mainly a signal about $\bar{a}$, increasing the precision of public disclosure both directly and indirectly benefits capital providers’ learning. That is, the indirect effect amplifies the direct effect of disclosure, leading to a positive total effect on real efficiency. In Panels (b1)–(c2), when public information is mainly a signal about $\bar{f}$, public disclosure directly improves but indirectly harms capital providers’ learning. That is, the indirect effect attenuates the direct effect, and the overall effect of disclosure is ambiguous. In addition, when the market does not aggregate speculators’ private information effectively, the positive direct effect of public disclosure always dominates and the overall effect of disclosure is to improve real efficiency (Panels (b1) and (b2)). In contrast, when the market aggregates speculators’ private information effectively, the indirect effect dominates for small levels of disclosure, so that the overall effect is that real efficiency can decrease with disclosure precision (Panels (c1) and (c2)).

5.4 Speculators Submit Demand Schedules

In the baseline model, we have assumed that speculators submit market orders and that noise trading depends on prices to clear the market. Now we consider a variation in which speculators submit price-contingent demand schedules, so that they can effectively condition their trades on prices. That is, as in Albagli, Hellwig, and Tsyvinski (2012, 2014), each speculator decides how many shares to trade at the prevailing price $\bar{P}$, in exchange for cash. This variation serves two purposes. First, we can check the robustness of our main results. Second, in this alternative setting, we can set $\lambda = 0$ to make noise trading independent of prices and still we can clear the market through speculators’ trading.

Now we conjecture that speculators buy the asset whenever $\bar{x}_i + \phi_y \bar{y}_i + \phi_w \bar{\omega} - \phi \bar{p} > g$, where $\bar{p} \equiv \log \bar{P}$ and $\phi$’s and $g$ are endogenous parameters. Then, we follow similar steps as in the baseline model and show that the price is

$$\bar{P} = \exp \left( \frac{-g + \bar{a} + \phi_y \bar{f} + \phi_w \bar{\omega} + \sqrt{\tau_y^{-1} + \phi_y^2 \tau_y^{-1} \xi}}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1} + \phi_p}} \right),$$

which, to capital providers, is still a signal $\bar{s}_p$ in predicting $\bar{f}$, as specified by equations (9)–(11). So, capital providers’ decision problem does not change and their investment policy is
still given by (12). Accordingly, the expression of real efficiency is still given by (17).

However, when we go back to date $0$, speculators’ forecast problem changes. They now can condition on the information in prices to make forecast about future cash flow as follows:

$$E \left( V_i | \bar{x}_i, \bar{y}_i, \bar{\omega}, \bar{p} \right) = b_0^v + b_x^v \bar{x}_i + b_y^v \bar{y}_i + b_{\omega}^v \bar{\omega} + b_p^v \bar{p},$$

where $b$’s are given in the appendix. Since they know the price $\bar{p}$, they do not need to forecast it. As a result, speculator $i$ will buy the asset if and only if

$$b_0^v + b_x^v \bar{x}_i + b_y^v \bar{y}_i + b_{\omega}^v \bar{\omega} + b_p^v \bar{p} \geq (1 - b_p^v) \bar{p} > 0,$$

which compares with the conjectured trading strategy, yielding the following equations that determine the equilibrium:

$$\phi_y = \frac{b_y^v}{b_x^v}, \phi_{\omega} = \frac{b_{\omega}^v}{b_x^v}, \text{ and } \phi_p = \frac{1 - b_p^v}{b_x^v}.$$

We can show that the supply elasticity $\lambda$ in noise trading does not affect the value of $\phi_y$. As a result, it does not affect the information precision $\tau_p$ that capital providers can learn from the price and hence real efficiency in equilibrium. Its only role is to change the value of $\phi_p$. We summarize this result in the following proposition.

**Proposition 6** In the economy where speculators submit price-contingent demand schedules, the supply elasticity $\lambda$ in noise trading has no effect on real efficiency.

In Figure 3, we numerically examine the implications of disclosure in this economy. Similar to Figure 1, we here have set $\tau_a = \tau_f = \tau_x = \tau_y = \tau_{\xi} = \lambda = 1$. In Panels (a1) and (a2), we choose $\mu_a = 0.8$ and $\mu_f = 0.2$ to make public disclosure mainly a signal about factor $\bar{a}$, while in Panels (b1)–(c2) we choose $\mu_a = 0.2$ and $\mu_f = 0.8$ to make public disclosure mainly a signal about factor $\bar{f}$. We find that our main results continue to hold qualitatively. First, consistent with Proposition 2, disclosing information about $\bar{a}$ makes speculators trade more aggressively on their private information about $\bar{f}$ in Panel (a1), which in turn improves capital providers’ learning from the price in Panel (a2). This means that the indirect effect of disclosing information about $\bar{a}$ is positive and it can amplify the direct effect, thereby making the overall effect positive. Second, consistent with Proposition 3, disclosing information about $\bar{f}$ causes speculators to trade less aggressively on private information about $\bar{f}$ in Panels (b1) and (c1), thereby harming capital providers’ learning from the price in Panels
(b2) and (c2). That is, the indirect effect of disclosing information about $\tilde{f}$ is negative and it attenuates the positive direct effect on real efficiency.

[INSERT FIGURE 3 HERE]

However, we notice that Panel (c2) of Figure 3 is different from our previous Panel (c2) of Figure 1. Here, we find that even when the variance of noise trading is relatively small, the indirect effect does not dominate the direct effect. What accounts for this difference is the following. In this alternative economy, speculators can also observe prices, and so part of the effect of disclosure on the weights $\phi_y$ that speculators put on their private signals about $\tilde{f}$ is absorbed by the price information when they make predictions about the asset’s cash flow. This weakens the indirect effect which operates through the responsiveness of $\phi_y$ to disclosure. Despite the difference between Panel (c2) of Figure 3 and Panel (c2) of Figure 1, we want to emphasize that our main message continues to be valid. That is, when disclosure is about factor $\tilde{f}$, the indirect effect is negative (i.e., $\tau_p$ is decreasing in both Panels (b2) and (c2) of Figure 3), and this negative indirect effect becomes stronger as the noise trading level in the market becomes smaller (i.e., $\mu_f^2 \tau_\omega + \tau_p$ is increasing in Panel (b2), while it is almost flat in Panel (c2)).

6 Conclusion

Public disclosure of information has been an important component of financial regulation for many years. One key question is whether the provision of more public information – via mandatory disclosure, credit ratings, stress tests, or macro statistics – improves real efficiency. In a world with other channels for learning, providing more public disclosure can crowd out other types of information. This is particularly relevant in the context of financial markets where prices are thought to provide useful information to decision makers. In this paper, we propose a framework to study these issues in a setting with multiple dimensions of uncertainty. We find that public information release generally has two effects on real efficiency through affecting real decision makers’ forecast quality. The direct effect is to provide new information, and it is always positive. The indirect effect works through influencing the information aggregation function of financial markets, and it can be positive.
or negative, depending on the type of information being disclosed. Paradoxically, the indirect effect tends to be negative when disclosure is about a variable that real decision makers want to learn. Moreover, when there is little noise trading in financial markets, the negative indirect effect can dominate the direct effect, implying that better disclosure can harm real efficiency. Thus, although it appears attractive to disclose information about some variable that relevant decision makers care to learn the most, the overall impact of such disclosure can be counter-productive.

Appendix

The Expressions of the Coefficients \( b \)'s in Equations (13) and (14)

Define \( \Delta^p \equiv Var \left( \tilde{a} + \phi_y \tilde{f} \right| \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \), and let \( \delta_x^p, \delta_y^p, \) and \( \delta_\omega^p \) be the loadings of \( \tilde{x}_i, \tilde{y}_i \) and \( \tilde{\omega} \) in the expression of \( E \left( \tilde{a} + \phi_y \tilde{f} \right| \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \), respectively. Then, we have:

\[
\begin{align*}
\delta_0^p &= \frac{-g}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} + \frac{1}{2 \lambda^2 \tau_{\xi}} + \frac{\Delta^p}{2 \lambda^2 (\tau_x^{-1} + \phi_y^2 \tau_y^{-1})}, \\
\delta_x^p &= \frac{\delta_y^p}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}, \quad b_y^p = \frac{\delta_y^p}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}, \quad \text{and} \quad b_\omega^p = \frac{\delta_\omega^p + \phi_\omega}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}.
\end{align*}
\]

Similarly, define \( \Delta^v \equiv Var \left( \tilde{a} + \frac{1}{2^{-\frac{\tau_p}{\tau_f + \mu^2 \tau_\omega + \tau_p}} \mu_a} \tilde{f} \right| \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \), and let \( \delta_x^v, \delta_y^v, \) and \( \delta_\omega^v \) be the loadings of \( \tilde{x}_i, \tilde{y}_i \) and \( \tilde{\omega} \) in the expression of \( E \left( \tilde{a} + \frac{1}{2^{-\frac{\tau_p}{\tau_f + \mu^2 \tau_\omega + \tau_p}} \mu_a} \tilde{f} \right| \tilde{x}_i, \tilde{y}_i, \tilde{\omega} \), respectively. Then, we have:

\[
\begin{align*}
\delta_0^v &= \left[ \log \frac{\beta (1 - \beta)}{c} \right] + \tau_p \left( \frac{\tau_f + \mu^2 \tau_\omega + \tau_p}{2 \tau_p} \right) + \frac{1}{2 \tau_p} \left( \frac{\tau_f + \mu^2 \tau_\omega + \tau_p}{2 \tau_p} \right)^2 + \frac{\Delta^v}{2} \left( 2 - \frac{\mu^2 \tau_\omega}{\tau_f + \mu^2 \tau_\omega + \tau_p} \right)^2, \\
\delta_x^v &= \left( 2 - \frac{\mu^2 \tau_\omega}{\tau_f + \mu^2 \tau_\omega + \tau_p} \right) \delta_x^v, \quad b_y^v = \left( 2 - \frac{\mu^2 \tau_\omega}{\tau_f + \mu^2 \tau_\omega + \tau_p} \right) \delta_y^v, \quad \text{and} \\
\delta_\omega^v &= \left( 2 - \frac{\mu^2 \tau_\omega}{\tau_f + \mu^2 \tau_\omega + \tau_p} \right) \delta_\omega^v.
\end{align*}
\]
Proof of Proposition 2

Given that Part (b) has been proved in the text, we here prove Part (a). By the expression of \( \tau_p \) in (11), we have \( \lim_{\phi_y \to 0} \frac{b^v_y}{\bar{b}^v} > 0 \) and \( \lim_{\phi_y \to \infty} \frac{b^v_y}{\bar{b}^v} < \infty \) in equation (21). So, by the intermediate value theorem, we know that there exists \( \phi_y > 0 \) satisfying equation (21).

We next prove the uniqueness. If we can prove that at the equilibrium level of \( \phi_y \), the right-hand-side (RHS) \( \frac{b^v_y}{\bar{b}^v} \) in equation (21) always crosses the 45 degree line from above, then the equilibrium is unique. That is, we need to show \( \frac{\partial}{\partial \phi_y} \frac{b^v_y}{\bar{b}^v} < 1 \) for those values of \( \phi_y \) satisfying equation (21). Direct computation shows

\[
\frac{\partial}{\partial \phi_y} \frac{b^v_y}{\bar{b}^v} = \frac{\tau_y}{\tau_f + \tau_y} \frac{\tau_f}{\tau_f + \tau_y} \frac{\partial \tau_p}{\partial \phi_y}.
\]

(A1)

By the expression of \( \tau_p \) in (11), we can compute

\[
\frac{\partial \tau_p}{\partial \phi_y} = \tau_y \tau_y \frac{2 \phi_y + \tau_p}{\tau_y + \phi_y^2 \tau_x}^2.
\]

(A2)

which is plugged in (A1), yielding

\[
\frac{\partial}{\partial \phi_y} \frac{b^v_y}{\bar{b}^v} = \frac{\tau_y}{\tau_f + \tau_y} \frac{\tau_f}{\tau_f + \tau_y} \frac{2 \phi_y + \tau_p}{\tau_y + \phi_y^2 \tau_x}^2.
\]

(A3)

By (21), we have

\[
\frac{\tau_y}{\tau_f + \tau_y} = \frac{\phi_y}{1 + \frac{\tau_p}{\tau_f + \tau_p}},
\]

which is plugged into (A3), yielding

\[
\frac{\partial}{\partial \phi_y} \frac{b^v_y}{\bar{b}^v} = \frac{1}{1 + \frac{\tau_p}{\tau_f + \tau_p}} \tau_f \tau_y \tau_y \frac{2 \phi_y + \tau_p}{\tau_y + \phi_y^2 \tau_x}^2.
\]

(A4)

By the expression of \( \tau_p \) in (11), we have

\[
\phi_y \tau_x = \frac{\tau_p \tau_y}{\tau_y \tau_y - \tau_p},
\]

(A5)

which is plugged into (A4),

\[
\frac{\partial}{\partial \phi_y} \frac{b^v_y}{\bar{b}^v} = \frac{2 \tau_p \tau_f}{(\tau_f + 2 \tau_p)(\tau_f + \tau_p)} \frac{\tau_y \tau_y - \tau_p}{\tau_y \tau_y} < 1,
\]

since \( 2 \tau_p \tau_f < (\tau_f + 2 \tau_p)(\tau_f + \tau_p) \) and \( 0 < \tau_y \tau_y - \tau_p < \tau_y \tau_y \). QED.
Proof of Proposition 3

Part (a). We follow the same methodology of proving Part (a) of Proposition 2. First, in (23), we have \( \lim_{y \to 0} \frac{b^v_y}{y_2} > 0 \) and \( \lim_{y \to \infty} \frac{b^v_y}{y_2} < \infty \), and so by the intermediate value theorem, we have the existence of the equilibrium.

Second, for the uniqueness, we will show that in (23), the RHS always crosses the 45 degree line from above at equilibrium. That is, we establish \( \frac{\partial}{\partial \phi_y} \frac{b^v_x}{x^2} < 1 \). Direct computation shows

\[
\frac{\partial}{\partial \phi_y} \frac{b^v_y}{b^v_x} = \frac{\frac{\tau_y}{\tau_f + \tau_y + \tau_w} \frac{\tau_f + \tau_w}{\tau_a + \tau_x} (\tau_f + \tau_p + \tau_w)^2 y^x \frac{2\phi_y^2 \tau_x \tau_y}{(\tau_y + \phi_y^2 \tau_x)^2}.
\]

(A6)

By (23), we have

\[
\frac{\tau_y}{\tau_f + \tau_y + \tau_w} \frac{\tau_f + \tau_w}{\tau_a + \tau_x} = \frac{\phi_y (\tau_f + \tau_p + \tau_w)}{\tau_f + 2\tau_p + \tau_w},
\]

which is plugged into (A6), yielding

\[
\frac{\partial}{\partial \phi_y} \frac{b^v_y}{b^v_x} = \frac{1}{\tau_f + 2\tau_p + \tau_w} \frac{\tau_f + \tau_w}{\tau_f + \tau_p + \tau_w} y^x \frac{2\phi_y^2 \tau_x \tau_y}{(\tau_y + \phi_y^2 \tau_x)^2}.
\]

(A7)

Then, inserting (A5) into (A7), we have

\[
\frac{\partial}{\partial \phi_y} \frac{b^v_y}{b^v_x} = \frac{2\tau_p (\tau_f + \tau_w)}{\tau_f + 2\tau_p + \tau_w (\tau_f + \tau_p + \tau_w)} \frac{y^x \tau_x - \tau_p}{\tau_y \tau_x} < 1.
\]

Part (b). Since the other results have been proved in the text, we only need to examine the real efficiency implications. By (24) and (A5), we can compute the indirect effect of disclosure as

\[
\frac{\partial r_p}{\partial \tau_w} = -2\tau_p \frac{\tau_y \tau_x - \tau_p}{\tau_y \tau_x} \frac{\tau_p}{\tau_f + \tau_p + \tau_w} \frac{\tau_f + \tau_p + \tau_w}{\tau_f + 2\tau_p + \tau_w} \frac{1}{\tau_f + \tau_y + \tau_w} + \frac{1}{\tau_f + \tau_y + \tau_w}.
\]

Clearly, as \( \tau_w \to \infty \), we have \( \frac{\partial r_p}{\partial \tau_w} \to 0 \), and so disclosure only has the positive direct effect. This establishes Part (b.iii).

To show Part (b.iii), we examine the behavior of the indirect effect \( \frac{\partial r_p}{\partial \tau_w} \) at \( \tau_w = 0 \). Consider the process of \( \tau_x \to \infty \) or \( \tau_x \to 0 \). If \( \lim_{\tau_x} \frac{f_1(\tau_x)}{f_2(\tau_x)} = 0 \), then we denote \( f_1 = o(f_2) \), meaning that \( f_1 \) converges at a faster rate than \( f_2 \). If \( \lim_{\tau_x} \frac{f_1(\tau_x)}{f_2(\tau_x)} \) is bounded (but different from 0), then we denote \( f_1 = O(f_2) \), meaning that \( f_1 \) and \( f_2 \) converge at the same rate. By (23) and (11), we have \( \phi_y = O(1) \) and \( \tau_p = O(\tau_x) \). By (24) and the orders of \( \phi_y \) and \( \tau_p \), we
have
\[ \frac{\partial \phi_y}{\partial \tau} \bigg|_{\tau=0} = -\frac{\phi_y}{\tau_f + \tau_y} + o(1). \]

So, by (20), we have
\[ \frac{\partial \tau_p}{\partial \tau} \bigg|_{\tau=0} = -\frac{2\phi_y^2 \tau_x^2}{(\tau_y + \phi_y^2 \tau_x)^2} \frac{\tau_y}{(\tau_f + \tau_y) \tau_y} + o(1). \]

Thus, by (19), we have
\[ \frac{\partial RE}{\partial \tau} \bigg|_{\tau=0} \propto 1 + \frac{\partial \tau_p}{\partial \tau} \bigg|_{\tau=0} = 1 - \frac{2\phi_y^2 \tau_x^2}{(\tau_y + \phi_y^2 \tau_x)^2} \frac{\tau_y}{(\tau_f + \tau_y) \tau_y} + o(1), \]

when \( \mu_f = 1 \) and \( \mu_a = 0 \). As a result, \( \frac{\partial RE}{\partial \tau} \bigg|_{\tau=0} < 0 \) for sufficiently large \( \tau_y \), and \( \frac{\partial RE}{\partial \tau} \bigg|_{\tau=0} > 0 \) for sufficiently small \( \tau_y \). QED.

**Proof of Proposition 5**

The coefficients \( b \)'s in speculators forecast about the cash flow and the price are
\[
\begin{align*}
b^0_x &= -\frac{g}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} + \frac{1}{2\lambda^2 \tau_y} + \frac{1}{2\lambda^2} \left( \frac{1}{\tau_a + \tau_x + \tau_y + \tau_f} + \frac{\phi_y^2}{\tau_f + \tau_y + \tau_y + \tau_f} \right), \\
b^0_p &= \frac{1}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} \left( \phi_y + \frac{\tau_y}{\tau_a + \tau_x + \tau_y} \right), \\
b^0_{\omega a} &= \frac{1}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} \left( \phi_y + \frac{\tau_{\omega a}}{\tau_a + \tau_x + \tau_y} \right), \\
b^0_{\omega f} &= \frac{1}{\lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} \left( \phi_y + \frac{\tau_{\omega f}}{\tau_f + \tau_y + \tau_y + \tau_f} \right), \\
b^v_x &= \frac{2\tau_x}{\tau_a + \tau_x + \tau_y + \tau_{\omega a}}, \\
b^v_p &= \left( 1 + \frac{\tau_p}{\tau_f + \tau_y + \tau_{\omega f}} \right) \frac{\tau_y}{\tau_f + \tau_y + \tau_{\omega f}}, \quad \text{and} \\
b^v_{\omega a} &= \frac{2\tau_{\omega a}}{\tau_a + \tau_x + \tau_y + \tau_{\omega a}}, \\
b^v_{\omega f} &= \left( 1 + \frac{\tau_p}{\tau_f + \tau_y + \tau_{\omega f}} \right) \frac{\tau_{\omega f}}{\tau_f + \tau_y + \tau_{\omega f}}.
\end{align*}
\]

**Part (a).** As \( \lambda \to \infty \), we have \( b^p_x \to 0 \) and \( b^p_y \to 0 \), and so \( \phi_y \) is determined by
\[
\phi_y = \frac{b^v_y}{b^v_x} = \left( 1 + \frac{\tau_p}{\tau_f + \tau_y + \tau_{\omega f}} \right) \frac{\tau_y}{\tau_f + \tau_y + \tau_{\omega f}}. \tag{A8}
\]

When \( \phi_y = 0 \), we have \( \frac{\partial \phi_y}{\partial \phi_y} \bigg|_{\phi_y=0} > 0 \) in (A8). When \( \phi_y \to \infty \), we have \( \frac{\partial \phi_y}{\partial \phi_y} \bigg|_{\phi_y=\infty} < \infty \). So, by the intermediate value theorem, there exists a \( \phi_y > 0 \) satisfying equation (A8), which
establishes the existence of the equilibrium.

We prove the uniqueness by showing that the RHS of (A8) crosses 45 degree line from above, that is, at equilibrium, we have \( \frac{\partial}{\partial y} \frac{b^v}{b^w} < 1 \). Specifically, direct computation shows

\[
\frac{\partial}{\partial \phi_y} \frac{b^v}{b^w} = \frac{\tau_y}{\tau + \tau_f + \tau_{\omega f}} \left[ \frac{\tau_f + \tau_{\omega f}}{(\tau_f + \tau_{\omega f} + \tau_p)^2} \right] \tau_y \tau \xi \frac{2 \phi_y \tau_x \tau_y}{(\tau_y + \phi_y \tau_x)^2}.
\]

Then, by (A8), we have

\[
\frac{\tau_f + \tau_{\omega f}}{\tau_f + \tau_{\omega f} + \tau_p} = \frac{\phi_y (1 + \phi_{\omega f})}{\phi_y (1 + \phi_{\omega f} + \tau_p)}
\]

which is plugged into the above equation, together with (A5), yielding

\[
\frac{\partial}{\partial \phi_y} \frac{b^v}{b^w} = \frac{2 \tau_p (\tau_f + \tau_{\omega f})}{(\tau_f + 2 \tau_p + \tau_{\omega f})(\tau_f + \tau_{\omega f} + \tau_p)} \frac{\tau_y \tau \xi - \tau_p}{\tau_y \tau \xi} < 1.
\]

**Part (b).** Applying the implicit function theorem to equations (11) and (A8), we can show

\[
\frac{\partial \phi_y}{\partial \tau_{\omega f}} = \frac{\phi_y}{\tau + \tau_f + \tau_{\omega f}} \frac{2 \tau_y (\tau_f + \tau_{\omega f})}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} \frac{\partial \tau_p}{\partial \tau_{\omega f}} \frac{\partial \tau_y}{\partial \tau_{\omega f}} > 0.
\]

The other results follow directly from \( \frac{\partial \tau_p}{\partial \tau_{\omega f}} = \frac{2 \tau_y (\tau_f + \tau_{\omega f})}{\phi_y (\tau_f + \tau_{\omega f} + \tau_p)} \frac{\partial \tau_y}{\partial \tau_{\omega f}} + \frac{\partial \tau_y}{\partial \tau_{\omega f}} \frac{\partial \tau_p}{\partial \tau_{\omega f}} \) and \( \frac{\partial \tau_p}{\partial \tau_{\omega f}} \frac{\partial \tau_y}{\partial \tau_{\omega f}} \frac{\partial \tau_y}{\partial \tau_{\omega f}} \frac{\partial \tau_p}{\partial \tau_{\omega f}} \).

**Part (c).** Applying the implicit function theorem to equations (11) and (A8), we can show

\[
\frac{\partial \phi_y}{\partial \tau_{\omega f}} = -\frac{\phi_y}{\tau + \tau_f + \tau_{\omega f}} \frac{1}{\left(\frac{\tau_f + \tau_{\omega f} + \tau_p}{\tau_f + \tau_{\omega f} + \tau_p}\right)} \frac{\tau_f + \tau_{\omega f} + \tau_p}{\tau_f + \tau_{\omega f} + \tau_p} \frac{2 \tau_p \tau_y}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} \frac{\partial \phi_y}{\partial \tau_{\omega f}} < 0.
\]

By (11), direct computation shows

\[
\frac{\partial \tau_p}{\partial \tau_{\omega f}} = -\left[\frac{\tau_p}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} + \frac{\tau_f + \tau_{\omega f} + \tau_p}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} \right] \frac{2 \tau_p \tau_y}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} \frac{\partial \tau_p}{\partial \tau_{\omega f}} < 0.
\]

Plugging (A5) into the above equation yields

\[
\frac{\partial \tau_p}{\partial \tau_{\omega f}} = -\left[\frac{\tau_p}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} + \frac{\tau_f + \tau_{\omega f} + \tau_p}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} \right] \frac{2 \tau_p \tau_y}{(\tau_f + \tau_{\omega f} + \tau_p)(\tau_f + \tau_{\omega f} + \tau_p)} \frac{\partial \tau_p}{\partial \tau_{\omega f}}
\]

and thus, as \( \tau_{\omega f} \to \infty \), we have \( \frac{\partial \tau_p}{\partial \tau_{\omega f}} \to 0 \) by noting that \( \tau_p \) is bounded. So, we will have \( \frac{\partial \tau_p}{\partial \tau_{\omega f}} \to 1 \) as \( \tau_{\omega f} \to \infty \).

Now suppose \( \tau_{\omega f} = 0 \) and we consider the process of \( \tau_{\xi} \to 0 \) or \( \tau_{\xi} \to \infty \). By equations (11) and (A8), we know that \( \phi_y = O(1) \) and \( \tau_p = O(\tau_{\xi}) \). So, combining this order
information with equation (A9), we have
\[ \frac{\partial \phi_y}{\partial \tau_{\omega f}} \bigg|_{\tau_{\omega f}=0} = -\frac{\phi_y}{\tau_f + \tau_y} + O(1). \]

Then, by (11), we have
\[ \frac{\partial \tau_p}{\partial \tau_{\omega f}} \bigg|_{\tau_{\omega f}=0} = -\frac{2\phi_y^2 \tau_x \tau_y^2}{(\tau_y + \phi_y^2 \tau_x)^2 (\tau_f + \tau_y)} \tau_{\xi} + O(\tau_{\xi}), \]
which implies that \( \frac{\partial \text{RE}}{\partial \tau_{\omega a}} \bigg|_{\tau_{\omega f}=0} \propto \frac{\partial (\tau_f + \tau_{\omega f} + \tau_p)}{\partial \tau_{\omega f}} = 1 + \frac{\partial \tau_p}{\partial \tau_{\omega f}} \bigg|_{\tau_{\omega f}=0} > 0 \) if and only if \( \tau_{\xi} \) is sufficiently small. QED.

**Proof of Proposition 6**

Speculator \( i \)'s information set is \( \{\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{p}\} \). To speculators, the price is equivalent to the following signal
\[ \tilde{t}_p \equiv \left( \lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1} + \phi_p} \right) \tilde{p} + g - \phi_{\omega} \tilde{\omega} = \tilde{a} + \phi_y \tilde{f} + \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1} \tilde{\xi}}. \]

Define \( \theta \equiv \left( 2 - \frac{\mu_f^2 \tau_{\omega}}{\tau_f + \mu_f^2 \tau_{\omega} + \tau_p \mu_f} - \frac{\tau_p}{\tau_f + \mu_f^2 \tau_{\omega} + \tau_p \phi_y} \right) \). Let \( \Delta^v \equiv \text{Var} \left( \theta \tilde{a} + \tilde{f} \bigg| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{t}_p \right) \) and let \( \delta_x, \delta_y, \delta_{\omega 0}, \) and \( \delta_{p0} \) be the loadings of \( \tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \) and \( \tilde{t}_p \) in the expectations \( E \left( \theta \tilde{a} + \tilde{f} \bigg| \tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{t}_p \right) \), respectively. Then, we can compute
\[
\begin{align*}
b_0^c &= \left( \log \frac{\beta (1 - \beta)}{c} \right) + \frac{1}{2} \frac{1}{\tau_f + \mu_f^2 \tau_{\omega} + \tau_p} + \frac{\Delta^v}{2} + \left( \delta_{p0} + \frac{\tau_p}{\tau_f + \mu_f^2 \tau_{\omega} + \tau_p \phi_y} \right) g, \\
b_x^c &= \delta_x, \quad b_y^c = \delta_y, \quad b_{\omega}^c = \left( \delta_{\omega 0} + \frac{\mu_f^2 \tau_{\omega}}{\tau_f + \mu_f^2 \tau_{\omega} + \tau_p \mu_f} - \frac{1}{\tau_f + \mu_f^2 \tau_{\omega} + \tau_p \phi_y} \right) \phi_{\omega}, \\
\text{and} \quad b_p^c &= \left( \delta_{p0} + \frac{\tau_p}{\tau_f + \mu_f^2 \tau_{\omega} + \tau_p \phi_y} \right) \left( \lambda \sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1} + \phi_p} \right). 
\end{align*}
\]

Note that \( \phi_y = \frac{b_y^c}{b_x^c} = \frac{\delta_y}{\delta_x} \), and that \( \delta_x \) and \( \delta_y \) are not affected by parameter \( \lambda \). Therefore, the equilibrium value of \( \phi_y \) is not affected by \( \lambda \). Real efficiency implications follow directly from equations (11) and (17). QED.

**References**


Figure 1: Implications of Disclosure for Trading and Real Efficiency

This figure plots the trading and real efficiency implications of public information release in the baseline model. Parameter $\tau_\omega$ controls the precision of public information. Parameter $\phi_y$ measures speculators’ trading aggressiveness on their private information about factor $\tilde{f}$ that capital providers care to learn. Parameter $\tau_p$ is the endogenous precision of the information that capital providers can learn from the price. In all panels, we have set $\tau_a = \tau_f = \tau_y = \tau_s = \tau_\xi = \lambda = 1$. In Panels (a1) and (a2), $\mu_a = 0.8$, $\mu_f = 0.2$, and $\tau_\xi = 1$. In Panels (b1) and (b2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 0.5$. In Panels (c1) and (c2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 10$. 
Figure 2: Real Efficiency Effect of Disclosure in Economies Where Capital Providers Receive Noisy Signals about Both Factors

This figure plots the real efficiency implications of public information in the extended economies in which capital provider receive noisy signals about factors $\bar{a}$ and $\bar{f}$. The left panels plot the direct and indirect effects of disclosure on capital providers’ forecast problem. The right panels plot real efficiency against the precision of public disclosure. In all panels, we have set $\tau_{a} = \tau_{f} = \tau_{y} = \tau_{s} = \tau_{\xi} = \lambda = 1$, $\tau_{z} = 5$, $\beta = 1/2$, and $c = 1$. In Panels (a1) and (a2), $\mu_{a} = 0.8$, $\mu_{f} = 0.2$, and $\tau_{\xi} = 1$. In Panels (b1) and (b2), $\mu_{a} = 0.2$, $\mu_{f} = 0.8$, and $\tau_{\xi} = 0.5$. In Panels (c1) and (c2), $\mu_{a} = 0.2$, $\mu_{f} = 0.8$, and $\tau_{\xi} = 10$. 
Figure 3: Implications of Disclosure for Trading and Real Efficiency in Economies Where Speculators Observe Prices

This figure plots the trading and real efficiency implications of public information release in economies where speculators submit demand schedules. Parameter $\tau_\omega$ controls the precision of public information. Parameter $\phi_y$ measures speculators’ trading aggressiveness on their private information about factor $\tilde{f}$. Parameter $\tau_p$ is the endogenous precision of the information that capital providers can learn from the price. In all panels, we have set $\tau_a = \tau_f = \tau_y = \tau_s = \tau_\xi = \lambda = 1$. In Panels (a1) and (a2), $\mu_a = 0.8$, $\mu_f = 0.2$, and $\tau_\xi = 1$. In Panels (b1) and (b2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 0.5$. In Panels (c1) and (c2), $\mu_a = 0.2$, $\mu_f = 0.8$, and $\tau_\xi = 10$. 