A Robust Redesign of High School Match

Sam Hwang\textsuperscript{1}

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\textsuperscript{1}University of British Columbia
Introduction

- Parents report rankings of schools to get their children assigned to schools
- Assignment policy often not incentive compatible
- Debate: should we switch to incentive compatible policy?
  1. Estimate the distribution of parents’ cardinal utilities for schools
  2. Simulate the benefit (efficiency) / cost (inequity) of the policy
- Contribution: relax the assumption about how strategic/unstrategic parents may be
Incentive-incompatible Boston Mechanism (BM) and Deferred Acceptance (DA)

- **Algorithm**
  - **Round 1.** Assign as many students as possible to their first choices
  - **Round k.** Assign as many remaining students as possible to their $k^{th}$ choices

- **Why not IC?**
  - By the end of Rd. 1, all the good schools will be already filled up.
  - You want to be assigned to some school in round 1
  - Avoid 1$^{st}$-ranking low-probability schools; 1$^{st}$-rank high-probability, good-enough schools

- **I-C alternative: DA**
Benefit and Cost of BM

- BM is more efficient than DA when everyone plays equilibrium / has same ordinal preferences
  - Experimental evidence suggests that 14 to 40% report truthfully
  - Students might have different ordinal preferences
- Truthfully reporting students may be penalized under BM under complete information about other’s preferences/lottery number
  - Students are not likely to have complete information
- Is the BM more efficient than DA with heterogeneity in strategic sophistication/ordinal preferences and by how much?
  - Yes, by 0.6 to 3.2 min. of per-capita daily commuting
- Is naivété in BM penalized without complete information?
  - Yes, more likely to be assigned to lesser favorite schools
Literature Review

   ▶ Strong behavioral assumptions: correctly predict assignment probabilities, fully optimize


5. Partially identified model: Romano et al. (2014)
Model

- Measure 1 of students, $i$
- $S$: a set of finite number of schools, $s$
  - $S = \{1, 2, 3\}$
- $q_s$: capacity of $s$
  - $q_1 = 0.3$, $q_2 = 0.4$, $q_3 = 0.5$
- $R = \{r_1, r_2, \ldots, r_m\}$: set of all rankings parents can report, $r$
  - $R = \{(1, 2, 3), (1, 3, 2), (2, 1, 3), \ldots\}$
- $B$: a set of beliefs about the distribution of rankings reported by parents
  - $b = (0.1, 0.15, 0.2, \ldots)$
The Boston Mechanism

- Equal priority, lottery is drawn from Unif[0,1], lower number is better

- A cut-off for a school in a round: highest (worst) lottery number that guarantees assignment to a school in the round

Example 1: $q_s = 0.3$, measure of applicants in round 1 $= 0.2$

- Cut-off: 1

Example 2: $q_s = 0.3$, measure of applicants in round 1 $= 0.6$

- Cut-off: $\frac{0.3}{0.6} = 0.5$
Cut-off Plot

- Critical rounds: the round at which cut-off $\in (0, 1)$
- Critical cutoffs: cut-offs at critical rounds
Previous literature assumes that strategic parents correctly predict the cut-off table.
### Calculation of Assignment Probabilities

<table>
<thead>
<tr>
<th>Rank</th>
<th>School</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>School 2</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>School 9</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>School 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>School</th>
<th>Round 1</th>
<th>Round 2</th>
<th>Round 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>School 2</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>School 9</td>
<td>1</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>School 10</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

#### Diagram

The diagram illustrates the cut-off values for different schools, with bars representing the assigned probabilities for each round. The cut-off values are 1, 0.5, and 0.3, with the unmatched cases represented by the tallest bar.
Model of Parents’ Decision Process

1. i’s vNM utility for school s: \( u_{is} = u(x_{is}, \epsilon_{is}; \theta) \)
   - Observable on school-student pair \( x_{is} \)
   - Unobservable \( \epsilon_{is} \)

2. Belief about the distribution of ranking reported by all parents \( b_i \in B \)

3. A reporting strategy \( \sigma_i \) maps utility and belief to a probability distribution over \( R \)
   - \( \sigma_i^r(u_i, b_i) > 0: \sigma_i \) "recommends" \( r \) to \( i \)

4. Draws \((x_{is}, \epsilon_{is}, b_i, \sigma_i)\) from \( \mu \)
Reporting Strategies

- Truth-telling strategy, fully-optimizing strategy
- Simple strategy: never recommends rankings that violate the simple rule
  - Simple rule: do not rank a school if you do not prefer it to higher-probability schools

<table>
<thead>
<tr>
<th></th>
<th>Sch 1</th>
<th>Sch 2</th>
<th>Sch 3</th>
<th>Sch 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{is}$</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Rd 1} & \text{Sch 1} & \text{Rd 2} & \text{Sch 2} & \text{Rd 3} & \text{Sch 3} & \text{Rd 3} & \text{Sch 4} \\
\hline
\text{Rd 1} & 0.3 & 0 & 0 & \text{Rd 2} & 1 & 0.5 & 0 \\
\text{Rd 3} & 1 & 1 & 1 & \text{Rd 3} & 1 & 0.7 & 0 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{School} & u_{is} & \text{Cut-offs} \\
\hline
\text{1st} & \text{School 1} & 20 & 0.3 \\
\text{2nd} & \text{School 2} & 15 & 0.5 \\
\text{3rd} & \text{School 3} & 10 & 1 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{School} & u_{is} & \text{Cut-offs} \\
\hline
\text{1st} & \text{School 1} & 20 & 0.3 \\
\text{2nd} & \text{School 4} & 18 & 0.7 \\
\text{3rd} & \text{School 3} & 10 & 1 \\
\hline
\end{array}
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\begin{array}{|c|c|c|}
\hline
\text{School} & u_{is} & \text{Cut-offs} \\
\hline
\text{1st} & \text{School 1} & 20 & 0.3 \\
\text{2nd} & \text{School 4} & 18 & 0.7 \\
\text{3rd} & \text{School 3} & 10 & 1 \\
\hline
\end{array}
\]

Proposition

*Truth-telling/fully-optimizing strategies are simple strategies*
Assumptions about Reporting Behavior

1. Everyone correctly predicts the ex-post critical round and the ranking of critical cut-offs

<table>
<thead>
<tr>
<th>Sch. 1</th>
<th>Rd. 1</th>
<th>Rd. 2</th>
<th>Rd. 3</th>
<th>Rd. 4</th>
<th>Rd. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sch. 2</td>
<td>1</td>
<td>1</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sch. 3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sch. 4</td>
<td>1</td>
<td>1.2</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sch. 5</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Cut-off table

<table>
<thead>
<tr>
<th>Critical Round</th>
<th>Cut-off ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sch. 1</td>
<td>3</td>
</tr>
<tr>
<td>Sch. 2</td>
<td>2</td>
</tr>
<tr>
<td>Sch. 3</td>
<td>5</td>
</tr>
<tr>
<td>Sch. 4</td>
<td>3</td>
</tr>
<tr>
<td>Sch. 5</td>
<td>1</td>
</tr>
</tbody>
</table>

Reduced Cut-off table

2. Everyone uses a simple strategy
### Identification

<table>
<thead>
<tr>
<th>School</th>
<th>Critical Round</th>
<th>Ranking of cut-offs</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>1</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>School 2</td>
<td>2</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>School 3</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>School 4</td>
<td>2</td>
<td>3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

**Table:**

#### Rankings

<table>
<thead>
<tr>
<th>Round</th>
<th>School</th>
<th>Ranking of cut-offs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>School 1</td>
<td>1</td>
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<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>School 2</td>
<td>2</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>School 3</td>
<td>4</td>
</tr>
</tbody>
</table>

#### Critical Values

- **Critical Values**
  - \( r_1 \)
  - \( r_2 \)

#### Conditions

- If \( u_{i2} < u_{i4} \), then \( \sigma^{r_1}(u_i, b_i) = 0 \)
- If \( u_{i2} \geq u_{i4} \), then \( \sigma^{r_1}(u_i, b_i) \in [0, 1] \)
- \( \sigma^{r_1}(u_i, b_i) \leq 1 \{ u_{i2} \geq u_{i4} \} \)
- \( \int \sigma^{r_1}(u_i, b_i) d\mu \leq \int 1 \{ u(x_{i2}, \epsilon_{i2}; \theta) \geq u(x_{i4}, \epsilon_{i4}; \theta) \} d\mu \)
- **Prob. parents submit \( r_1 \)**
- **Prob. \( u_{i2} \geq u_{i4} \)**
- Identified \( \theta = \{ \theta' : \text{All moment inequalities hold at } \theta' \} \)
Parameters To Be Estimated

- $u(x_{is}, \epsilon_{is}; \theta) = x_{is}\beta_i - d_{is} + \epsilon_{is}$

- Observables $x_{is}$
  1. Quality index, measured by the % of students scoring on or above average at a standardized test
  2. $\mathbb{1}\{\text{science magnet}\}$
  3. $\mathbb{1}\{\text{charter}\}$
  4. $\mathbb{1}\{\text{private}\}$
  5. $\mathbb{1}\{i \text{ boy } \cap s \text{ boys only}\}$
  6. $\mathbb{1}\{i \text{ girl } \cap s \text{ girls only}\}$

- Distance $d_{is}$: minutes spent in commuting from i to s

- Assume $\beta_i \sim \mathcal{N}(\mu, \Sigma)$, $\epsilon_{is} \sim \mathcal{N}(0, \sigma_\epsilon)$

- Estimate the confidence region of the true $\mu, \Sigma, \sigma_\epsilon$
Computation of Confidence Region

- Romano et al. (2014): Test each $\theta \in \Theta$ whether they should be in the confidence region

- Get two point-estimates under the assumptions that
  1. everyone is truth-telling
  2. everyone plays equilibrium

- $\Theta$: a 28-dimensional interval that contains the two point estimates in its interior

- Draw 4 million points; 9 points pass the test

- Confidence region: smallest 28-dimensional interval that contains the 9 points
### 95% Confidence Region of the True Parameter

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[\beta_i]$</th>
<th>Std[$\beta_i$]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quality Index</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Science magnet</td>
<td>$[12.3, 14.0]$</td>
<td>$[9.9, 14.2]$</td>
</tr>
<tr>
<td>Charter</td>
<td>$[-60.0, -54.1]$</td>
<td>$[39.3, 44.7]$</td>
</tr>
<tr>
<td>Private</td>
<td>$[3.2, 7.0]$</td>
<td>$[29.0, 35.5]$</td>
</tr>
<tr>
<td>Boys-only</td>
<td>$[7.0, 9.9]$</td>
<td>$[15.3, 21.4]$</td>
</tr>
<tr>
<td>Girls-only</td>
<td>$[-2.9, 1.0]$</td>
<td>$[8.2, 13.3]$</td>
</tr>
<tr>
<td>$\epsilon_{is}$</td>
<td>0</td>
<td>$[0.1, 0.14]$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Science</th>
<th>Charter</th>
<th>Private</th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>$[-0.99,-0.6]$</td>
<td>$[0.4,0.5]$</td>
<td>$[-0.8,-0.6]$</td>
<td>$[0.4,0.6]$</td>
</tr>
<tr>
<td>Science</td>
<td>$[-0.5,-0.3]$</td>
<td>$[0.4,0.7]$</td>
<td>$[0.7,-0.5]$</td>
<td>$[-0.6,-0.2]$</td>
</tr>
<tr>
<td>Charter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private</td>
<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>Girls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Simulation Procedure

- 9 parameter estimates in the confidence region
- For each of the 9 parameter estimates, fix the fraction of naive parents at \( X \% \)

1. Draw 80,000 \( u_i \)
2. Randomly choose naive parents from the population
3. Deferred Acceptance: let everyone report truthfully
4. The Boston Mechanism
   - Naive parents: report truthfully
   - Strategic parents: best respond to the rest of the parents
5. Run each mechanism 100 times with different lottery numbers

- Repeat 1 to 5 10 times
- Vary \( X \) from 0 to 100%, 10% increment
Simulation Results: Efficiency

- Measure of efficiency: \( \sum_{i \in I} \frac{1}{|I|} \) (Expected utility);
- Families under the Boston Mechanism are better off by 0.6 to 3.2 minutes on average than under Deferred Acceptance.
Simulation Results: Inequity

- Naive families are more likely to be assigned to lower ranked schools
- Strategic families are better off by 16 to 32 minutes in daily commuting time on average
Conclusion

- Can learn enough about preferences without strong assumptions on the behavior
  - Bounds are "tight", i.e. we got the answer we wanted
- Many decisions to make in designing the mechanism
  - Who gets what priority to which school
  - Tie-breaking scheme
  - The size of the matching market
- Bounds might not be tight enough to be informative for these decisions


