

# Explaining Exchange Rate Anomalies in a Model with Taylor-rule Fundamentals and Consistent Expectations\*

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## Abstract

We introduce boundedly-rational expectations into a standard asset-pricing model of the exchange rate, where cross-country interest rate differentials are governed by Taylor-type rules. Agents augment a lagged-information random walk forecast with a term that captures news about Taylor-rule fundamentals. The coefficient on fundamental news is pinned down using the moments of observable data such that the resulting forecast errors are close to white noise. The model generates volatility and persistence that is remarkably similar to that observed in monthly exchange rate data for Canada, Japan, and the U.K. Regressions performed on model-generated data can deliver the well-documented forward premium anomaly.

Keywords: *Exchange rates, Uncovered interest rate parity, Forward premium anomaly, Random-walk expectations, Excess volatility.*

JEL Classification: D83, D84, E44, F31, G17.

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# 1 Introduction

Efforts to explain movements in exchange rates as a rational response to economic fundamentals have, for the most part, met with little success. More than thirty years ago, Meese and Rogoff (1983) demonstrated that none of the usual economic variables (money supplies, real incomes, trade balances, inflation rates, interest rates, etc.) could help forecast future exchange rates better than a simple random walk forecast. With three decades of additional data in hand, researchers continue to confirm the Meese-Rogoff results.<sup>1</sup> While some tenuous links between fundamentals and exchange rates have been detected, the empirical relationships are generally unstable (Bacchetta and Van Wincoop 2004, 2013), hold only at 5 to 10 year horizons (Chinn 2006), or operate in the wrong direction, i.e., exchange rates may help predict fundamentals but not vice versa (Engel and West 2005).<sup>2</sup> The failure of fundamental variables to improve forecasts of future exchange rates has been called the “exchange rate disconnect puzzle.”

Another puzzle relates to the “excess volatility” of exchange rates. Like stock prices, exchange rates appear to move too much when compared to changes in observable fundamentals.<sup>3</sup> Engel and West (2006) show that a standard rational expectations model can match the observed persistence of real-world exchange rates, but it substantially underpredicts the observed volatility. West (1987) makes the point that exchange rate volatility can be reconciled with fundamental exchange rate models if one allows for “regression disturbances,” i.e., exogenous shocks that can be interpreted as capturing shifts in *unobserved* fundamentals. Similarly, Balke, Ma and Wohar (2013) find that unobserved factors (labeled “money demand shifters”) account for most of the volatility in the U.K./U.S. exchange rate using data that extends back more than a century. An innovative study by Bartolini and Gioginianni (2001) seeks to account for the influence of unobserved fundamentals using survey data on exchange rate expectations. The study finds “broad evidence...of excess volatility with respect to the predictions of the canonical asset-pricing model of the exchange rate with rational expectations” (p. 518).

A third exchange rate puzzle is the so-called “forward-premium anomaly.” In theory, a currency traded at a premium in the forward market predicts a subsequent appreciation of that currency in the spot market. In practice, there is a close empirical link between the observed forward premium and cross-country interest rate differentials, consistent with the *covered* interest parity condition. Hence, theory predicts that a low interest rate currency should, on average, appreciate relative to a high interest rate currency because the subsequent appreciation compensates investors for the opportunity cost of holding a low interest rate bond. The theoretical slope coefficient from a regression of the observed exchange rate change

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<sup>1</sup>For surveys of this vast literature, see Rossi (2013), Cheung, Chinn and Pascual (2005), and Sarno (2005).

<sup>2</sup>Specifically, Engel and West (2005) find Granger causality running from exchange rates to fundamentals. Recently, however, Ko and Ogaki (2015) demonstrate that this result is not robust after correcting for the small-sample size.

<sup>3</sup>Early studies applied to exchange rate volatility include Huang (1981) and Wadhvani (1987). Lansing and LeRoy (2014) provide a recent update on evidence of excess volatility in the U.S. stock market.

on the prior interest rate differential is exactly equal to one. This prediction of the theory, known as the *uncovered* interest parity (UIP) condition, is grossly violated in the data. As shown by Fama (1984), regressions of the observed exchange rate change on the prior interest rate differential yield estimated slope coefficients that are typically negative and significantly different from one. Like the other two puzzles, the forward-premium anomaly has stood the test of time.<sup>4</sup>

The wrong sign of the slope coefficient in UIP regressions can be reconciled with no-arbitrage and rational expectations if investors demand a particular type of risk premium to compensate for holding an uncovered currency position. By failing to account for a potentially time-varying risk premium that may co-move with the interest rate differential, the standard UIP regression may deliver a biased estimate of the slope coefficient.<sup>5</sup> Recently, Lustig and Verdelhan (2007) provide some evidence that carry-trade profits (excess returns from betting against UIP) may reflect a compensation for risk that stems from a negative correlation between the carry-trade profits and investors' consumption-based marginal utility.<sup>6</sup> However, Burnside (2011) points out that their empirical model is subject to weak identification such that there is no concrete evidence for the postulated connection between carry-trade profits and fundamental risk. Moreover, Burnside, Eichenbaum, and Rebelo (2011) show that there is no statistically significant covariance between carry-trade profits and conventional risk factors. Verdelhan (2010) develops a rational model with time-varying risk premiums along the lines of Campbell and Cochrane (1999). The model implies that rational domestic investors will expect low future returns on risky foreign bonds in good times (due to an expected appreciation of the domestic currency) when risk premia are low and domestic interest rates are high relative to foreign interest rates. Hence, the model delivers the prediction that the domestic currency will appreciate, on average, when domestic interest rates are high—thus violating the UIP condition, as in the data. Unfortunately, the idea that investors expect low future returns on risky assets in good times is strongly contradicted by a wide variety of survey evidence from both stock and real estate markets. The survey evidence shows that investors typically expect high future returns on risky assets in good times, not low future returns.<sup>7</sup> Overall, the evidence suggests that rationally time-varying risk premiums are not a convincing explanation for the empirical failure of UIP.

This paper develops a model that can account for numerous quantitative features of real-world exchange rates, including the three anomalies described above. The key aspect of our approach is the way in which agents' expectations are modeled. Starting from a standard asset-pricing model of the exchange rate, we postulate that agents augment a simple random

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<sup>4</sup>For recent evidence, see Baillie and Chang (2011) and Baillie and Cho (2014).

<sup>5</sup>See Engel (1996) for a survey of this large literature and Engel (2014) for a review of new developments in this field.

<sup>6</sup>See also Lustig et al. (2014).

<sup>7</sup>For additional details, see Amromin and Sharpe (2014), Greenwood and Shleifer (2014), Jurgilas and Lansing (2013), and Williams (2013).

walk forecast with news about fundamentals. Fundamentals in our model are determined by cross-country interest rate differentials which, in turn, are described by Taylor-type rules, along the lines of Engel and West (2005, 2006). We solve for a “consistent expectations equilibrium,” in which the coefficient on fundamental news in the agent’s subjective forecast rule is pinned down using the observed covariance between exchange rate changes and fundamental news. This learnable equilibrium delivers the result that the forecast errors observed by an agent are close to white noise, making it difficult to detect any misspecification of the subjective forecast rule.<sup>8</sup>

We demonstrate that our consistent expectations model can generate volatility and persistence that is remarkably similar to that observed in monthly bilateral exchange rate data (relative to the U.S.) for Canada, Japan, and the U.K. over the period 1974 to 2012. We show that regressions performed on model-generated data can deliver the forward-premium anomaly, whereby a high interest rate currency tends to appreciate, thus violating the UIP condition. Moreover, the estimated slope coefficient in the model regressions can vary over a wide range when estimated using a 15-year (180-month) rolling sample period. This result is consistent with the wide range of coefficient estimates observed across countries and time periods in the data.<sup>9</sup>

In our model, agents’ perceived law of motion (PLM) for the exchange rate is a driftless random walk that is modified to include an additional term involving fundamental news, i.e., the innovation to the AR(1) driving process that is implied by the Taylor-rule based interest rate differential. To maintain generality regarding agents’ information sets, we allow a fraction of market participants to construct the random walk component of the forecast using contemporaneous information about the exchange rate while the remainder employ lagged information about the exchange rate. Our setup captures an idea originally put forth in an informal way by Froot and Thaler (1990), who suggested that the empirical failure of the UIP condition might be linked to the fact that some investors “may need some time to think about trades before executing them, or that they simply cannot respond quickly to recent information.”<sup>10</sup>

The agents’ PLM has only one parameter that can be estimated within the model by running a regression of the exchange rate change on fundamental news, both of which are observable to the agent.<sup>11</sup> The agents’ subjective forecast rule can be viewed as boundedly-

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<sup>8</sup>The equilibrium concept that we employ was originally put forth by Hommes and Sorger (1998). A closely-related concept is the “restricted perceptions equilibrium” described by Evans and Honkapohja (2001, Chapter 13). For other applications of consistent expectations to asset pricing or inflation, see Sögner and Mitlöhner (2002), Branch and McGough (2005), Evans and Ramey (2006), Lansing (2009, 2010), Hommes (2013), and Hommes and Zhu (2014).

<sup>9</sup>For evidence of variability in estimated UIP slope coefficients, see Bansal (1997), Flood and Rose (2002), Baillie and Chang (2011), Baillie and Cho (2014), and Ding and Ma (2013).

<sup>10</sup>A somewhat similar information structure has been successfully employed in “sticky information” models of U.S. inflation dynamics. See Mankiw and Reis (2002) and Carroll (2003).

<sup>11</sup>Lansing (2010) employs a similar random walk plus fundamentals forecast rule in a standard Lucas-type asset pricing model to account for numerous quantitative features of long-run U.S. stock market data.

rational because the resulting actual law of motion (ALM) for the exchange rate exhibits a near-unit root with innovations that depend on Taylor-rule fundamentals.

We show that regardless of the starting value for the coefficient on fundamental news in the subjective forecast rule, a standard real-time learning algorithm will converge to the vicinity of the fixed point which defines the unique consistent expectations (CE) equilibrium. The backward-looking nature of the agent’s forecast rule when using lagged exchange rate information is the crucial element needed to generate the forward-premium anomaly. By introducing a modest forecast weight on the lagged exchange rate, the CE model shifts the temporal relationship between the expected exchange rate change and the interest rate differential, thus flipping the sign of the slope coefficient in the UIP regression.

Our setup is motivated by two important features of the data: (1) real-world exchange rates exhibit near-random walk behavior, and (2) exchange rates and fundamentals do exhibit some tenuous empirical links. For example, Andersen, Bollerslev, and Diebold (2003) employ high frequency data to show that fundamental macroeconomic news surprises induce shifts in the exchange rate. When we apply the CE model’s forecast rule to exchange rate data for Canada, Japan, and the U.K. (relative to the U.S. dollar), we find that the inclusion of fundamental news together with the lagged exchange rate can typically improve forecast accuracy relative to an otherwise similar random walk forecast that omits the fundamental news term.

Using consensus survey data of a large number of financial institutions that report 3-month ahead forecasts of exchange rates (relative to the U.S.) for Canada, Japan, and the U.K., we show that changes in the interest rate differential (a proxy for fundamental news) are helpful in explaining the forecasted changes in the exchange rates—a result that is consistent with the CE model’s forecast rule. In particular, the data shows that survey respondents tend to forecast a currency appreciation during periods when the change of interest rate differential is positive. The CE model’s forecast rule is also consistent with survey data which shows that the vast majority of professional forecasters use both technical analysis (chart patterns of past exchange rate movements) and fundamental economic data to construct their exchange rate forecasts (Dick and Menkhoff 2013 and Ter Ellen, Verschoor, and Zwinkels 2013).

## 1.1 Related Literature

This paper relates to some previous literature that has employed models with distorted beliefs to account for the behavior of exchange rates. Gourinchas and Tornell (2004) postulate that agents have distorted beliefs about the law of motion for fundamentals. Related mechanisms are proposed by Burnside et al. (2011), Ilut (2012), and Yu (2013). In contrast, we postulate that agents have distorted beliefs about the law of motion for exchange rates, not fundamentals. The distorted beliefs in our model affect not only the dynamics of the exchange rate, but also the dynamics of the equilibrium interest rate differential via the Taylor-type rule.

Bacchetta and Van Wincoop (2007) introduce “random walk expectations” into an exchange rate model with risk aversion and infrequent portfolio adjustments. Unlike our setup,

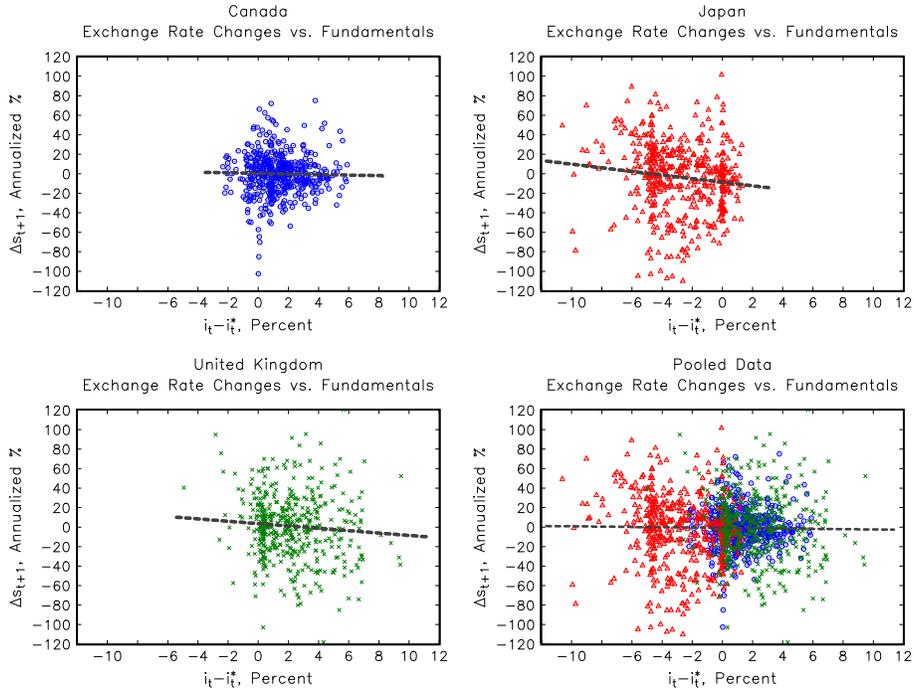


Figure 1: The slope of the fitted relationship between the observed exchange rate change and the prior month's interest rate differential is negative, confirming the forward-premium anomaly.

the agent's subjective forecast in their model completely ignores fundamentals. While their model can account for the forward-premium anomaly, it relies on exogenous shocks from ad hoc noise traders to account for the observed volatility of exchange rate changes.

Chakraborty and Evans (2008) introduce constant-gain learning about the reduced-form law of motion for the exchange rate. The agent in their model employs the correct (i.e., rational) form for the law of motion, but the estimated parameters are perpetually updated using recent data. They show that statistical variation in the estimated parameters may cause the UIP condition to be violated, particularly in small samples. However, their model does not account for excess volatility of the exchange rate. Mark (2009) develops a model with perpetual learning about the Taylor-rule coefficients that govern the cross-country interest rate differential. He shows that the model can account for major swings in the real deutschmark/euro-dollar exchange rate over the period 1976 to 2007.

The subjective forecast rules in our model can be interpreted as being comprised of both chartist and fundamentalist elements. In that sense, our setup is related to a large literature on forecast switching models with heterogeneous interacting agents, i.e., chartist versus fundamentalist-type traders.<sup>12</sup>

<sup>12</sup>For examples applied to exchange rates, see Frankel and Froot (1990), De Grauwe and Grimaldi (2006), and Markiewicz (2012), among others.

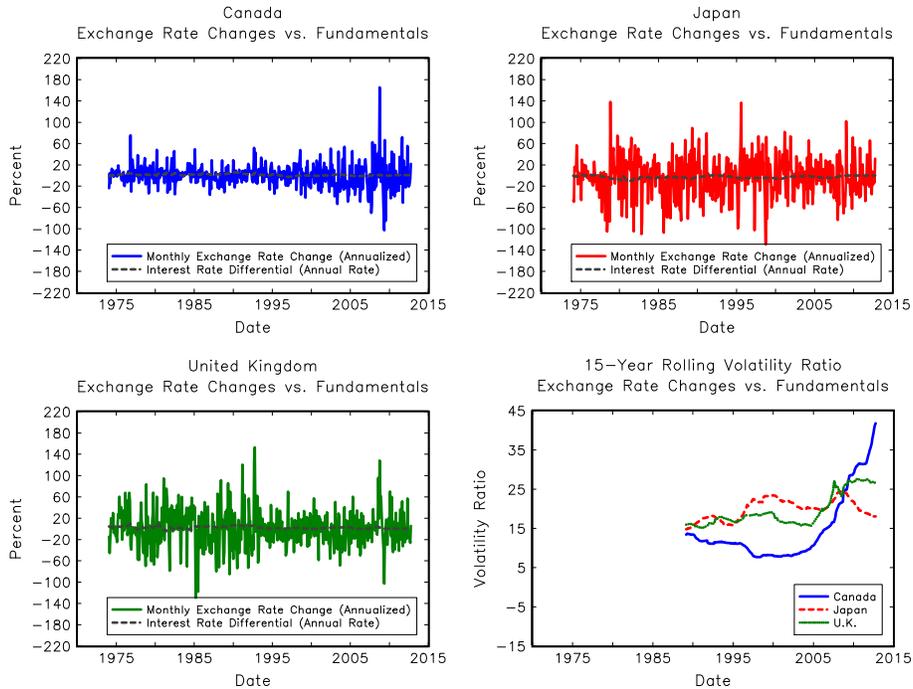


Figure 2: The volatility of exchange rate changes is 10 to 40 times higher than the volatility of the cross-country interest rate differential (relative to the U.S).

## 2 Data on Exchange Rates and Fundamentals

According to UIP theory, the cross-country interest rate differential should be a key explanatory variable for subsequent exchange rate changes. Figure 1 plots scatter diagrams of monthly bilateral exchange rate changes (in annualized percent) versus the prior month's average short-term nominal interest rate differential (in percent) for three pairs of countries, namely, Canada/U.S., Japan/U.S. and U.K./U.S. The data covers the period from January 1974 through October 2012.<sup>13</sup> The bottom right panel of Figure 1 shows a scatter diagram of the pooled data. Figure 2 shows time series plots of the same data, with the bottom right panel depicting the relative volatility of exchange rate changes to interest rate differentials, where volatilities are computed using a 15-year rolling sample period.

Three aspects of Figures 1 and 2 stand out. First, there is no tight systematic relationship between monthly exchange rate changes and the prior month's interest rate differential (exchange rate disconnect puzzle). Second, the volatility of exchange rate changes is 10 to 40 times higher than the volatility of the interest rate differential (excess volatility puzzle). Third, the dashed lines in Figure 1 show a negative slope in the fitted relationship between the ob-

<sup>13</sup>Exchange rate changes are computed as the log difference of sequential end-of-month values and then annualized. Interest rates are annualized 3-month government bond yields. All data are from the IMF's International Financial Statistics database.

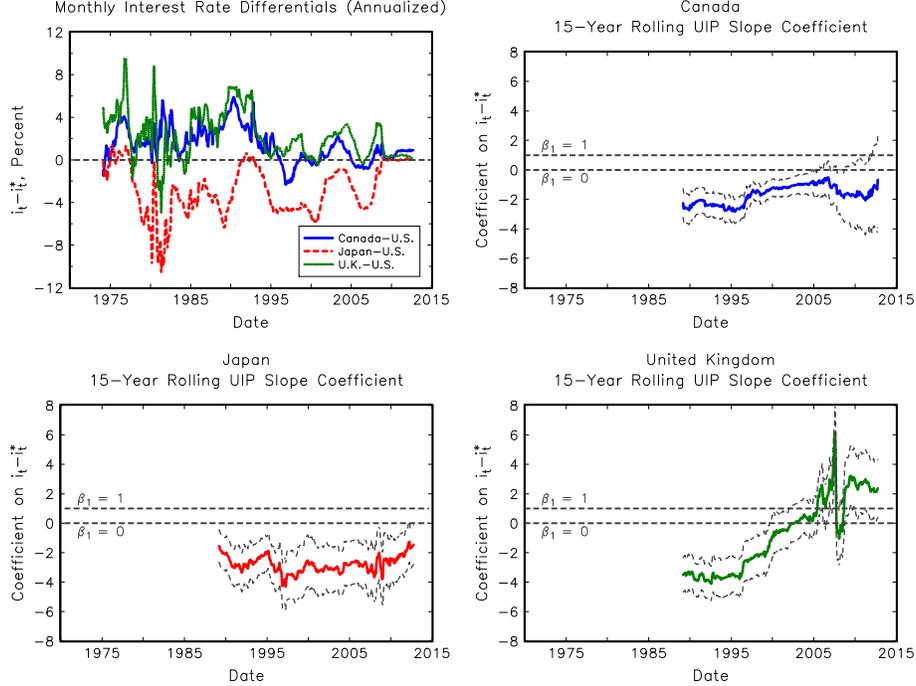


Figure 3: Estimated UIP slope coefficients lie mostly in mostly in negative territory and exhibit substantial time variation.

served exchange rate change and the prior month’s interest rate differential (forward-premium anomaly).

The top left panel of Figure 3 plots the interest rate differentials that are used to predict exchange rate changes via a standard UIP regression that takes the form:

$$\Delta s_{t+1} = \beta_0 + \beta_1(i_t - i_t^*) + \varepsilon_{t+1}, \quad (1)$$

where  $s_t \equiv \log(S_t)$  is the logarithm of the nominal exchange rate (home currency per US dollar) and  $\Delta s_{t+1} \equiv s_{t+1} - s_t$  is the monthly percent change (annualized) from period  $t$  to  $t + 1$ . The short-term nominal interest rate differential is  $i_t - i_t^*$ , where  $i_t$  is the rate for either Canada, Japan, or the U.K. and  $i_t^*$  is the U.S. interest rate. Under UIP (which assumes rational expectations and risk-neutral investors), we have  $\beta_0 = 0$  and  $\beta_1 = 1$ , with the error term  $\varepsilon_{t+1}$  reflecting white-noise rational forecast errors.

The remaining panels of Figure 3 plot the estimated slope coefficients  $\beta_1$  for each country together with the associated standard error bands using a 15-year rolling sample period. The horizontal dashed lines mark the values  $\beta_1 = 1$  and  $\beta_1 = 0$ . For all three countries, the rolling estimates of  $\beta_1$  lie mostly in negative territory, thus violating the UIP condition. Table 1 provides the full-sample estimates for  $\beta_1$ . The point estimates are all negative, consistent with fitted lines shown in Figure 1. Moreover, the 95 percent confidence intervals based on the

reported standard errors exclude the theoretical prediction of  $\beta_1 = 1$  in their coverages.<sup>14</sup>

Table 1. Full-Sample UIP Slope Coefficients

	Canada	Japan	U.K.	Pooled Data
$\hat{\beta}_1$	-0.29	-1.86	-1.21	-0.16
<i>Std. Error</i>	(0.65)	(0.75)	(0.76)	(0.29)

Note: Sample period is 1974.m1 to 2012.m10.

The consistently negative estimates for  $\beta_1$  reported here and elsewhere in the literature have interesting economic implications. In practice, the results imply that a carry-trade strategy (taking a long position in high-interest currency while shorting a low-interest currency) can deliver substantial excess returns, where excess returns are measured by  $(i_t - i_t^*) - \Delta s_{t+1}$ . When  $\beta_1 < 1$ , the future excess return can be predicted using the current interest rate differential  $i_t - i_t^*$ , which raises doubts about market efficiency.<sup>15</sup> Efforts to account for the predictability of excess returns in the data can be classified into two main approaches: (1) linking excess returns to some form of compensation for bearing risk, or (2) allowing for departures from fully-rational expectations. Empirical evidence using conventional risk factors argues against the first approach (Burnside, Eichenbaum, and Rebelo 2011). In this paper, we follow the second approach.

Another notable feature of the UIP regressions, evident in Figure 3, is the substantial time variation in the estimated slope coefficient for a given country. Baillie and Chang (2011) and Baillie and Cho (2014) employ time-varying parameter regressions to capture this feature of the data. Bansal (1997) shows that the sign of  $\beta_1$  appears to be correlated with the sign of the interest rate differential, but his results do not generalize to other sample periods or countries. Ding and Ma (2013) develop a model of cross-border portfolio reallocation that can help explain a time-varying  $\beta_1$  estimate. Our model can deliver a negative and statistically significant estimate of  $\beta_1$  in long-sample regressions as well as substantial time-variation in the estimated slope coefficient in 15-year rolling regressions. The time variation in the estimated slope coefficient arises for two reasons: (1) the actual law of motion that governs  $\Delta s_{t+1}$  in the consistent expectations equilibrium turns out to differ in significant ways from the UIP regression equation (1), and (2) the volatility of  $\Delta s_{t+1}$  in the consistent expectations equilibrium is much higher than the volatility of  $i_t - i_t^*$ .

<sup>14</sup>Using a dataset of 23 countries for the sample period of the 1990s, Flood and Rose (2002) obtain positive estimated values of  $\beta_1$  using pooled data. However, they acknowledge (p. 257) that “pooling is a dubious procedure” given the heterogeneity in the individual country estimates of  $\beta_1$ .

<sup>15</sup>Cochrane (2001, p. 394) points out that return predictability is directly related to the phenomenon of excess volatility.

### 3 Model

The framework for our analysis is a standard asset-pricing model of the exchange rate. Fundamentals are given by cross-country interest rate differentials which, in turn, are described by Taylor-type rules, in the spirit of Engel and West (2005, 2006). Given our data, the home country in the model represents either Canada, Japan, or the U.K. while the foreign country represents the United States (denoted by  $*$  variables).

We postulate that the home country central bank sets the short-term nominal interest rate according to the following Taylor-type rule

$$i_t = \theta i_{t-1} + (1 - \theta) \{ g_\pi \pi_t + g_y y_t + g_s [s_t - \kappa s_{t-1} - (1 - \kappa) \bar{s}_t] \} + \eta_t, \quad (2)$$

where  $i_t$  is the short term nominal interest rate,  $\pi_t$  is the inflation rate (log difference of the price level),  $y_t$  is the output gap (log deviation of actual output from potential output),  $s_t$  is the log of the nominal exchange rate (home currency per U.S. dollar),  $s_{t-1}$  is the lagged exchange rate, and  $\bar{s}_t \equiv p_t - p_t^*$  is a benchmark exchange rate implied by the purchasing power parity (PPP) condition, where  $p_t$  is the domestic price level and  $p_t^*$  is the foreign price level.<sup>16</sup> When  $\kappa = 0$ , the central bank reacts to  $s_t - \bar{s}_t$  which is the deviation of the exchange rate from the PPP benchmark, consistent with the models employed by Engel and West (2005, 2006). When  $\kappa = 1$ , the central bank reacts to the exchange rate change  $\Delta s_t = s_t - s_{t-1}$ , consistent with the empirical policy rule estimates of Lubik and Schorfheide (2007) and Justiniano and Preston (2010) for a variety of industrial countries. Motivated by the empirical evidence, we set  $\kappa \simeq 1$ .<sup>17</sup> The term  $\eta_t$  represents an exogenous monetary policy shock. In contrast to Engel and West (2005, 2006), we allow for interest-rate smoothing on the part of the central bank, as governed by the parameter  $\theta > 0$ . For the remaining reaction function parameters, we follow standard practice in assuming  $g_\pi > 1$ , and  $g_y, g_s > 0$ . In other words, the central bank responds more than one-for-one to movements in inflation and raises the nominal interest rate in response to a larger output gap or a depreciating home currency ( $\Delta s_t > 0$ ).

The foreign (i.e., U.S.) central bank sets the short-term nominal interest rate according to

$$i_t^* = \theta i_{t-1}^* + (1 - \theta) [g_\pi \pi_t^* + g_y y_t^*] + \eta_t^*, \quad (3)$$

where we assume that the reaction function parameters  $\theta$ ,  $g_\pi$ , and  $g_y$  are the same across countries. Subtracting equation (3) from equation (2) yields the following expression for the cross-country interest rate differential

$$i_t - i_t^* = \theta (i_{t-1} - i_{t-1}^*) + (1 - \theta) \{ g_\pi (\pi_t - \pi_t^*) + g_y (y_t - y_t^*) + g_s [s_t - \kappa s_{t-1} - (1 - \kappa) \bar{s}_t] \} + \eta_t - \eta_t^*. \quad (4)$$

<sup>16</sup>We omit constant terms from equation (2) because our empirical application of the central bank reaction function makes use of demeaned data.

<sup>17</sup>As noted below, we impose the parameter restriction  $0 \leq \kappa < 1$  to ensure the existence of a unique rational expectations solution of the model.

Assuming risk-neutral, rational investors, the uncovered interest rate parity condition implies

$$E_t s_{t+1} - s_t = i_t - i_t^*, \quad (5)$$

where  $E_t s_{t+1}$  is the rational forecast of next period's log exchange rate.<sup>18</sup> The UIP condition says that a negative interest rate differential  $i_t - i_t^* < 0$  will exist when rational investors expect a home currency appreciation, i.e., when  $E_t s_{t+1} < s_t$ . The expected appreciation compensates investors for the opportunity cost of holding a low interest rate domestic bond rather than a high interest rate foreign bond. Rational expectations implies  $E_t s_{t+1} = s_{t+1} - \varepsilon_{t+1}$ , where  $\varepsilon_{t+1}$  is a white-noise forecast error. Hence, theory predicts that, on average, a low interest rate currency should appreciate relative to a high interest rate currency such that  $E(s_{t+1} - s_t) < 0$ .

Substituting the cross-country interest rate differential (4) into the UIP condition (5) and solving for  $s_t$  yields the following no-arbitrage condition that determines the equilibrium exchange rate

$$s_t = bE_t s_{t+1} + \kappa(1-b)s_{t-1} + x_t, \quad b \equiv \frac{1}{1 + (1-\theta)g_s} < 1 \quad (6)$$

where  $b$  is the effective discount factor and  $x_t$  is the fundamental driving variable defined as

$$x_t \equiv -b\theta(i_{t-1} - i_{t-1}^*) - b(1-\theta)[g_\pi(\pi_t - \pi_t^*) + g_y(y_t - y_t^*) - g_s(1-\kappa)(p_t - p_t^*)] - b(\eta_t - \eta_t^*), \quad (7)$$

where we have made the substitution  $\bar{s}_t = p_t - p_t^*$ .<sup>19</sup>

The no-arbitrage condition (6) shows that the equilibrium exchange rate  $s_t$  depends on the agent's conditional forecast  $E_t s_{t+1}$ , the lagged exchange rate  $s_{t-1}$ , and the fundamental driving variable  $x_t$ . When  $0 \leq \kappa < 1$ , the sum of the coefficients on  $E_t s_{t+1}$  and  $s_{t-1}$  is less than unity which ensures the existence of a unique rational expectations solution. The general form of equation (6), whereby the current value of an endogenous variable depends in part on its own expected value and a lagged value appears in a wide variety of economic models, such as the hybrid New Keynesian Phillips Curve (Galí, et al. 2005).

The macroeconomic variables that enter the definition of  $x_t$  exhibit a high degree of persistence in the data. We therefore model the behavior of the fundamental driving variable using the following stationary AR(1) process

$$x_t = \rho x_{t-1} + u_t, \quad u_t \sim N(0, \sigma_u^2), \quad |\rho| < 1, \quad (8)$$

where the parameter  $\rho$  governs the degree of persistence. While some studies allow for a unit root in the law of motion for fundamentals, we maintain the assumption of stationarity for consistency with most of the literature. In practice, it is nearly impossible to distinguish

<sup>18</sup>More precisely, the UIP condition is  $E_t S_{t+1}/S_t = (1 + i_t) / (1 + i_t^*)$ . Following standard practice, we take logs of both sides and ignore the Jensen's inequality term such that  $\log(E_t S_{t+1}) \simeq E_t \log(S_{t+1})$ .

<sup>19</sup>The basic form of equations (6) and (7) will remain unchanged if we assume that the foreign central bank also reacts to the exchange rate, but with a smaller reaction coefficient  $g_s^* < g_s$ . In this case, the effective discount factor becomes  $b = 1 / [1 + (1-\theta)(g_s - g_s^*)]$ .

between a unit root process and one that is stationary but highly persistent given a finite sample size (Cochrane 1991).

Given values for  $x_t$  and  $s_t$ , we can recover the current-period interest rate differential as follows

$$i_t - i_t^* = -\frac{1}{b}x_t + \left(\frac{1-b}{b}\right)(s_t - \kappa s_{t-1}). \quad (9)$$

Empirical estimates of central bank reaction functions typically imply  $\theta$  values in the range of 0.8-0.9 together with small values for  $g_s$  such that  $b \simeq 1$ . In this case, the equilibrium dynamics for  $i_t - i_t^*$  will be very similar to the equilibrium dynamics for  $-x_t$ . We will make use of the inverse relationship between the interest rate differential and the fundamental driving variable  $x_t$  in our discussion of the results.

### 3.1 Rational Expectations

Proposition 1 shows that the no-arbitrage condition (6) delivers a unique rational expectation solution.

**Proposition 1.** *When fundamentals are governed by equation (8), there is unique solution to the no-arbitrage condition (6) under rational expectations (RE), as given by*

$$s_t = a_s s_{t-1} + a_x x_t,$$

$$a_s = \frac{1 - \sqrt{1 - 4\kappa b(1-b)}}{2b}, \quad a_x = \frac{1}{1 - b(a_s + \rho)}$$

*Proof:* See Appendix A.

Our parameter restriction  $0 \leq \kappa < 1$  implies the result  $0 \leq a_s < (1-b)/b$ . When  $b \simeq 1$ , the equilibrium coefficient on the lagged exchange rate must be a small positive number such that  $a_s \simeq 0$ . The result  $0 \leq a_s < (1-b)/b$  further implies  $(1-b\rho)^{-1} < a_x < [b(1-\rho)]^{-1}$ . When  $b$  and  $\rho$  are both close to unity, the equilibrium coefficient  $a_x$  will turn out to be a relatively large positive number. Since  $a_s \simeq 0$ , the equilibrium exchange rate approximately inherits the persistence properties of the fundamental driving variable  $x_t$ . Since  $x_t$  is very persistent in the data, the RE model predicts a persistent exchange rate level.<sup>20</sup>

The unconditional moments for  $s_t$  and  $\Delta s_t$  implied by the RE model are contained in Appendix B. The UIP condition (5) together with the assumption of rational expectations implies  $Var(i_t - i_t^*) = Cov(\Delta s_{t+1}, i_t - i_t^*)$ . Hence, the RE model predicts the following slope coefficient from a UIP regression

$$\beta_1 = \frac{Cov(\Delta s_{t+1}, i_t - i_t^*)}{Var(i_t - i_t^*)} = 1. \quad (10)$$

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<sup>20</sup>For the baseline model calibration, the equilibrium coefficients turn out to be  $a_s = 0.0196$  and  $a_x = 20.23$ .

### 3.2 Consistent Expectations

Real-world exchange rates exhibit near-random walk behavior. A naive forecast rule that uses only the most recently-observed exchange rate almost always outperforms a fundamentals-based forecast (Rossi 2013). In addition to its predictive accuracy, a random walk forecast has the advantage of economizing on computational and informational resources. As described many years ago by Nerlove (1983), “Purposeful economic agents have incentives to eliminate errors up to a point justified by the costs of obtaining the information necessary to do so...The most readily available and least costly information about the future value of a variable is its past value” (p. 1255).

Still, there is evidence that market participants pay attention to fundamentals. A recent study by Dick and Menkhoff (2013) uses survey data to analyze the methods of nearly 400 professional exchange rate forecasters. The data shows that the vast majority of forecasters use both technical analysis (chart patterns of past exchange rate movements) and fundamental economic data to construct their forecasts. Another study of survey data by ter Ellen, et al. (2014) finds evidence that large wholesale investors in the foreign exchange market employ both chartist and fundamentals-based forecasting strategies.

To capture the above ideas, we postulate that agents’ perceived law of motion (PLM) for the exchange rate is given by

$$s_t = s_{t-1} + \alpha u_t, \tag{11}$$

where  $u_t$  represents “fundamental news,” as measured by the innovation to the AR(1) fundamental driving process (8). There is only one parameter  $\alpha$  that agents in the model can readily estimate by running a regression of  $\Delta s_t$  on  $u_t$ . The PLM can be interpreted as being comprised of both chartist and fundamentalist elements. The presence of the lagged exchange rate  $s_{t-1}$  reflects the chartist element while the news term  $\alpha u_t$  reflects the fundamentalist element. The PLM also captures the idea that a fundamental news shock  $u_t$  can induce a significant and immediate jump in the exchange rate, consistent with the findings of Andersen, Bollerslev, and Diebold (2003) who employ high frequency data.

The PLM is used by agents to construct a subjective forecast  $\widehat{E}_t s_{t+1}$  which takes the place of the rational forecast  $E_t s_{t+1}$  in the no-arbitrage condition (6). Following Yu (2013), p. 476. our solution procedure implies that the no-arbitrage condition “holds ex ante under investors’ perception.” But ex post, the exchange rate evolves according to the actual law of motion (ALM) to be derived below.<sup>21</sup> Since the no-arbitrage condition implies that  $s_t$  depends on the agent’s own subjective forecast, it is questionable whether an agent could make use of the contemporaneous value  $s_t$  when constructing a forecast in real-time. To deal with this timing issue, models that employ adaptive learning or other forms of boundedly-rational expectations typically assume that agents can only make use of the lagged realization of the

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<sup>21</sup>Ilut (2012) develops a rational model where the UIP condition holds ex ante under agents’ endogenously pessimistic beliefs, but the UIP condition fails ex post.

forecast variable (in this case  $s_{t-1}$ ) when constructing their subjective forecast at time  $t$ .<sup>22</sup> A lagged-information setup avoids simultaneity in the determination of the actual and expected values of the forecast variable.

Here we wish to maintain generality regarding agents' information. Motivated by the discussion in Froot and Thaler (1990), we postulate that a fraction  $\lambda$  of market participants (labeled Type-1 agents) employ contemporaneous information about the exchange rate while the remaining fraction  $1 - \lambda$  (labeled Type-2 agents) employ lagged information, where  $0 \leq \lambda < 1$ .<sup>23</sup> As in the RE solution, we assume that both Type-1 and Type-2 agents have access to contemporaneous information about the fundamental driving variable  $x_t$ . According to the definition (7),  $x_t$  does not depend on  $s_t$  and hence there is no controversy about including it in the information sets of both Type-1 and Type-2 agents. A long history of observations of  $x_t$  would allow both types of agents to discover the stochastic process (8). Given this knowledge, both types of agents could infer the fundamental news  $u_t$  from sequential observations of  $x_t$  and  $x_{t-1}$ .

From the PLM (11), the subjective forecast of Type-1 agents is given by

$$\widehat{E}_{1,t} s_{t+1} = s_t, \quad (12)$$

which coincides with a random walk forecast because the Type-1 agents' knowledge of fundamentals implies  $\widehat{E}_{1,t} u_{t+1} = 0$ . Since Type-2 agents employ lagged information about the exchange rate, their PLM must be iterated ahead two periods to obtain

$$\begin{aligned} \widehat{E}_{2,t} s_{t+1} &= \widehat{E}_{2,t} [s_t + \alpha u_{t+1}], \\ &= \widehat{E}_{2,t} [s_{t-1} + \alpha u_t + \alpha u_{t+1}], \\ &= s_{t-1} + \alpha u_t, \end{aligned} \quad (13)$$

which makes use of the lagged exchange rate  $s_{t-1}$ . Finally, we assume that the aggregate market forecast is given by the following population-weighted average of the two agent types:

$$\begin{aligned} \widehat{E}_t s_{t+1} &= \lambda \widehat{E}_{1,t} s_{t+1} + (1 - \lambda) \widehat{E}_{2,t}, \\ &= \lambda s_t + (1 - \lambda) (s_{t-1} + \alpha u_t), \\ &= s_{t-1} + \lambda \Delta s_t + (1 - \lambda) \alpha u_t, \end{aligned} \quad (14)$$

where  $\lambda \Delta s_t$  is a momentum term that derives from aggregating across agents with different information sets.

Substituting the aggregate market forecast (14) into the no-arbitrage condition (6) and solving for  $s_t$  yields the following actual law of motion for the exchange rate

$$s_t = \left[ 1 - \frac{(1 - \kappa)(1 - b)}{1 - b\lambda} \right] s_{t-1} + \frac{b(1 - \lambda)\alpha}{1 - b\lambda} u_t + \frac{x_t}{1 - b\lambda}, \quad (15)$$

<sup>22</sup>For an overview of these methods, see Evans and Honkapohja (2001), Hommes (2013), and Hommes and Zhu (2014).

<sup>23</sup>Adam, Evans and Honkapohja (2006) employ a similar generalization of agents' information sets in a model of adaptive learning applied to hyperinflation.

where  $x_t$  is governed by (8). Notice that the form of the ALM is similar, but not identical, to the RE model solution from Proposition 1. Recall that we previously showed that the equilibrium coefficient on the lagged exchange rate in the RE model must be a small positive number such that  $a_s \simeq 0$ . In the CE model, the parameter restriction  $0 \leq \kappa < 1$  implies that the equilibrium coefficient on the lagged exchange rate has a lower bound of  $b(1 - \lambda) / (1 - b\lambda)$  when  $\kappa = 0$ . This lower bound is close to unity when  $b \simeq 1$ . Since the equilibrium coefficient on  $s_{t-1}$  in the CE model is near unity, the agents' perception of a unit root in the exchange rate via the PLM (11) turns out to be close to self-fulfilling. Hence we can say that agents are forecasting in a way that, ex post, appears to be near-rational.

### 3.2.1 Defining the Consistent Expectations Equilibrium

We now define a “consistent expectations equilibrium” along the lines of Hommes and Sorger (1998) and Hommes and Zhu (2014). Specifically, the parameter  $\alpha$  in the agents' PLM (11) is pinned down using the moments of observable data. Since the PLM presumes that  $s_t$  exhibits a unit root, agents in the model can readily estimate  $\alpha$  as follows

$$\alpha = \frac{Cov(\Delta s_t, u_t)}{\sigma_u^2}, \quad (16)$$

where  $Cov(\Delta s_t, u_t)$  and  $\sigma_u^2 = Var(u_t)$  can be computed from observable data. An analytical expression for the observable covariance can be derived from the ALM (15) which implies:

$$\Delta s_t = -\frac{(1 - \kappa)(1 - b)}{1 - b\lambda} s_{t-1} + \frac{b(1 - \lambda)\alpha}{1 - b\lambda} u_t + \frac{x_t}{1 - b\lambda}, \quad (17)$$

$$Cov(\Delta s_t, u_t) = \left[ \frac{b(1 - \lambda)\alpha + 1}{1 - b\lambda} \right] \sigma_u^2. \quad (18)$$

Equations (16) and (18) can be combined to form the following definition of equilibrium.

**Definition 1.** *A consistent expectations (CE) equilibrium is defined as a perceived law of motion (11), an aggregate market forecast (14), an actual law of motion (15), and a subjective forecast parameter  $\alpha$ , such that the equilibrium value  $\alpha^*$  is given by the unique fixed point of the linear map*

$$\begin{aligned} \alpha &= T(\alpha) \equiv \frac{b(1 - \lambda)\alpha + 1}{1 - b\lambda}, \\ \alpha^* &= \frac{1}{1 - b}, \end{aligned}$$

where  $b \equiv 1 / [1 + (1 - \theta)g_s] < 1$  is the effective discount factor and  $\lambda$  is the fraction of agents who employ contemporaneous information about the exchange rate.

Interestingly, the fixed point value  $\alpha^*$  depends only on the effective discount factor  $b$  and not on  $\lambda$  which measures the fraction of Type-1 agents who employ contemporaneous

information about  $s_t$ . But as we shall see,  $\lambda$  does influence other results, such as the theoretical slope coefficient from a UIP regression. The slope of the  $T(\alpha)$  map determines whether the equilibrium is stable under learning. The slope is given by  $T'(\alpha) = b(1 - \lambda)/(1 - b\lambda)$ . Since  $0 < T'(\alpha) < 1$ , the CE equilibrium is globally stable. In Section 4, we demonstrate that a real-time learning algorithm always converges to the vicinity of the theoretical fixed point  $\alpha^*$  regardless of the shock sequences or the starting value for  $\alpha$ .

### 3.2.2 Implications for the Forward-Premium Anomaly

The unconditional moments implied by the actual laws of motion (15) and (17) turn out to be quite complicated, as shown in Appendix C. It is useful to consider what happens to these moments when the effective discount factor approaches unity. When  $b \rightarrow 1$  the equilibrium exchange rate exhibits a unit root. From equation (17), the actual law of motion becomes  $\Delta s_t = \alpha u_t + x_t/(1 - \lambda)$  where  $x_t = -(i_t - i_t^*)$  from equation (9). In this case, the analytical slope coefficient from a UIP regression is given by

$$\beta_1 = \lim_{b \rightarrow 1} \frac{Cov(\Delta s_{t+1}, i_t - i_t^*)}{Var(i_t - i_t^*)} = -\frac{\rho}{1 - \lambda}, \quad (19)$$

which demonstrates that the CE model can deliver a negative slope coefficient, thus reproducing the well-documented forward-premium anomaly. When all agents are Type-2 (employ lagged information), we have  $\lambda = 0$  and the slope coefficient equals  $-\rho$ . However, when some agents are Type-1 (employ contemporaneous information), we have  $0 < \lambda < 1$  and the slope coefficient can become more negative than  $-\rho$ . When  $0 < b < 1$ , the slope coefficient remains negative, but is smaller in magnitude than in the limiting case of  $b \rightarrow 1$ . Two out of the three empirical slope coefficients reported in Table 1 are below  $-1$  which would suggest a calibration with  $\lambda > 0$ . We will confirm these results numerically in the quantitative analysis presented in Section 5.

The intuition for the forward-premium anomaly in the CE model is not complicated. For simplicity, let us consider the case where all agents are Type-2 such that  $\lambda = 0$ . From equation (14), the aggregate market forecast becomes  $\widehat{E}_t s_{t+1} = s_{t-1} + \alpha u_t$ . Subtracting  $s_t$  from both sides of this expression yields  $\widehat{E}_t s_{t+1} - s_t = -\Delta s_t + \alpha u_t$ . In contrast, any rational expectations model implies  $E_t s_{t+1} = s_{t+1} - \varepsilon_{t+1}$ , where  $\varepsilon_{t+1}$  is the white noise rational forecast error. Again subtracting  $s_t$  from both sides yields  $E_t s_{t+1} - s_t = \Delta s_{t+1} - \varepsilon_{t+1}$ . The UIP condition (5) relates the forecasted change in the exchange rate to the prior interest rate differential  $i_t - i_t^*$ . By introducing positive weight on  $s_{t-1}$  in the aggregate market forecast, the CE model shifts the temporal relationship between the forecasted change of the exchange rate and the interest rate differential, thus flipping the sign of the slope coefficient in the UIP regression.

Additional insight can be obtained by substituting the ALM for  $\Delta s_t$  (17) into the Taylor-rule based interest rate differential (9) to obtain the following equilibrium expression for the

interest rate differential:

$$\begin{aligned}
i_t - i_t^* &= -\frac{1}{b}x_t + \left(\frac{1-b}{b}\right) \underbrace{\left\{ \left[1 - \frac{(1-\kappa)(1-b)}{1-b\lambda} - \kappa\right] s_{t-1} + \frac{b(1-\lambda)\alpha}{1-b\lambda} u_t + \frac{x_t}{1-b\lambda} \right\}}_{s_t - \kappa s_{t-1}}, \\
&= (1-\lambda) \left[ \frac{(1-b)(1-\kappa)}{1-b\lambda} s_{t-1} + \frac{(1-b)\alpha}{1-b\lambda} u_t - \frac{x_t}{1-b\lambda} \right], \\
&= (1-\lambda)(-\Delta s_t + \alpha u_t) \quad (\text{CE model}), \tag{20}
\end{aligned}$$

where the last expression again makes use of (17). From equation (1), the sign of the UIP slope coefficient  $\beta_1$  is governed by the sign of  $Cov(\Delta s_{t+1}, i_t - i_t^*)$ . Iterating equation (20) ahead one period and then solving for  $\Delta s_{t+1}$  yields  $\Delta s_{t+1} = -(i_{t+1} - i_{t+1}^*) / (1-\lambda) + \alpha u_{t+1}$  which in turn implies

$$Cov(\Delta s_{t+1}, i_t - i_t^*) = \frac{-1}{1-\lambda} Cov(i_{t+1} - i_{t+1}^*, i_t - i_t^*) \quad (\text{CE model}). \tag{21}$$

The right-side of the above expression will be negative so long as  $0 \leq \lambda < 1$  and the interest rate differential exhibits positive serial correlation, as it does in both the model and the data. Intuitively, since the interest rate differential depends on the exchange rate via the Taylor-type rule, and the exchange rate depends on agents' expectations via the no-arbitrage condition (6), a departure from rational expectations that involves lagged information can shift the dynamics of the variables that appear on both sides of the UIP regression equation (1).

The corresponding derivation for the RE solution is

$$\begin{aligned}
i_t - i_t^* &= -\frac{1}{b}x_t + \left(\frac{1-b}{b}\right) \underbrace{[(a_s - \kappa)s_{t-1} + a_x x_t]}_{s_t - \kappa s_{t-1}}, \\
&= \frac{(1-b)(a_s - \kappa)}{b} s_{t-1} + (a_s + \rho - 1)a_x x_t, \\
&= (a_s - 1)a_s s_{t-1} + (a_s - 1)a_x x_t + a_x \rho x_t \\
&= \Delta s_{t+1} - a_x u_{t+1} \quad (\text{RE model}), \tag{22}
\end{aligned}$$

where we have made use of the definition of  $a_x$  in going from line 1 to line 2, the definition of  $a_s$  in going from line 2 to line 3, and finally the definitions of  $s_t$  from Proposition 1 and of  $x_{t+1}$  to obtain the final result. Solving equation (22) for  $\Delta s_{t+1}$  yields  $\Delta s_{t+1} = (i_t - i_t^*) + a_x u_{t+1}$  which in turn implies

$$Cov(\Delta s_{t+1}, i_t - i_t^*) = Var(i_t - i_t^*) \quad (\text{RE model}), \tag{23}$$

such that the right-side is always positive and the slope coefficient in the UIP regression is unity.

## 4 Applying the Model's Methodology to the Data

Using the definition of the fundamental driving variable  $x_t$  in equation (7), we construct time series for  $x_t$  in Canada, Japan, and the U.K. using monthly data on the consumer price index, industrial production, and the short-term nominal interest rate differential relative to the U.S. The interest rate differential is computed using 3-month government bond yields. Our data are from the International Monetary Fund's International Financial Statistics (IFS) database and covers the period January 1974 through October 2012. To construct measures of the output gap for each country, we estimate and remove a quadratic trend from the logarithm of the industrial production index.<sup>24</sup> In constructing the time series for  $x_t$ , we use the following calibrated values for the Taylor-rule parameters:  $\theta = 0.9$ ,  $g_\pi = 1.5$ ,  $g_y = 0.5$ ,  $g_s = 0.2$ , and  $\kappa = 0.98$ . These values are consistent with those typically employed or estimated in the monetary literature.<sup>25</sup> Empirical estimates of the interest rate smoothing parameter  $\theta$  typically imply  $\theta \simeq 0.8$  for quarterly data. Since our model employs monthly data, we choose  $\theta = 0.9$ . Given the Taylor-rule parameters, the effective discount factor in our model is  $b \equiv 1/[1 + (1 - \theta)g_s] = 0.9804$ .

Table 2 reports summary statistics for the nominal interest rate differential (relative to the U.S.) and the constructed time series for  $x_t$ . As noted earlier in the discussion of equation (9), the equilibrium dynamics for  $i_t - i_t^*$  will be very similar to the equilibrium dynamics for  $-x_t$  whenever  $b \simeq 1$ , as is the case here.

Table 2. Summary Statistics of Data Fundamentals

	Canada	Japan	U.K.
<i>Std Dev</i> ( $i_t - i_t^*$ )	1.62%	2.35%	2.18%
<i>Corr</i> ( $i_t - i_t^*$ , $i_{t-1} - i_{t-1}^*$ )	0.956	0.972	0.953
<i>Std Dev</i> ( $x_t$ )	1.64%	2.52%	2.30%
<i>Corr</i> ( $x_t$ , $x_{t-1}$ )	0.889	0.892	0.871
<i>Corr</i> ( $i_t - i_t^*$ , $-x_t$ )	0.964	0.957	0.955

Note: Sample period is from 1974.m1 to 2012.m10. The fundamental driving variable  $x_t$  is defined by equation (7).

Before proceeding with simulations from the theoretical model, we wish to examine the performance of the postulated forecast rules (12) and (13) using the exchange rate data for Canada, Japan, and the U.K. Table 3 reports the average root mean squared forecast errors (RMSFE) for four different forecast rules. The RMSFE statistics are computed sequentially using a 15-year rolling sample period and then averaged. The initial sample period is from 1974.m1 to 1988.m1. The first two rows of the table show the results for our postulated forecast

<sup>24</sup>Similar results are obtained if industrial production is detrended using the Hodrick-Prescott filter. Removing a stochastic trend defined using the method of Beveridge and Nelson (1981) would result in a less-volatile output gap (see Cogley 2001) and hence a less-volatile fundamental driving variable, further exacerbating the excess volatility puzzle.

<sup>25</sup>See, for example, Lubik and Schorfheide (2007) and Justiniano and Preston (2010). In particular, they estimate values for the exchange rate response coefficient  $g_s$  in the range of 0.07 to 0.29.

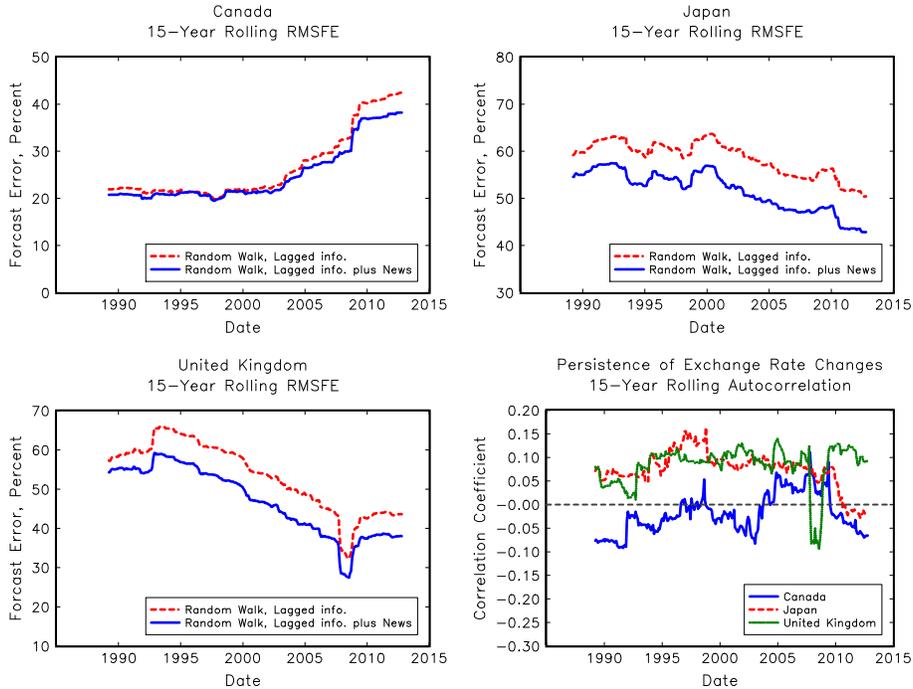


Figure 4: For Canada, Japan and the U.K., the use of fundamental news can improve forecast performance relative to a random walk forecast with lagged information. In all three countries, exchange rate changes exhibit low persistence, consistent with the near-random walk behavior of exchange rates.

rules (12) and (13) which are associated with the Type-1 and Type-2 agents, respectively.<sup>26</sup> The third row shows the results of a random walk forecast using only lagged-information about the exchange rate, i.e., no fundamental news. The fourth row shows the results for a fundamentals-only forecast where the coefficients  $f_{0,t}$  and  $f_{1,t}$  are estimated sequentially for each rolling sample period using data for each country's fundamental driving variable  $x_t$ .

Table 3. Average 15-year Rolling RMSFEs

Forecast Rule	Canada	Japan	UK
$\hat{E}_{1,t} s_{t+1} = s_t$	19.0%	39.8%	36.2%
$\hat{E}_{2,t} s_{t+1} = s_{t-1} + \alpha_{t-1} u_t$	25.1%	51.6%	46.9%
$\hat{E}_t s_{t+1} = s_{t-1}$	26.6%	58.6%	53.3%
$\hat{E}_t s_{t+1} = f_{0,t} + f_{1,t} x_t$	125%	358%	159%

Notes: Root mean squared forecast errors (RMSFE) are computed sequentially using a 15-year rolling sample period and then averaged. The initial sample period is from 1974.m1 to 1988.m1.

Consistent with many previous studies, the top row of Table 3 shows that a random walk

<sup>26</sup>To construct the Type-2 agent's forecast, we sequentially estimate the law of motion for fundamentals (8) for the rolling sample period and use the resulting parameter estimates to identify a sequence of fundamental innovations  $u_t$  for that sample period. We then estimate the response coefficient to fundamental news using only lagged information about the exchange rate, i.e.,  $\alpha_{t-1} = Cov(\Delta s_{t-1}, u_{t-1}) / \sigma_u^2$ .

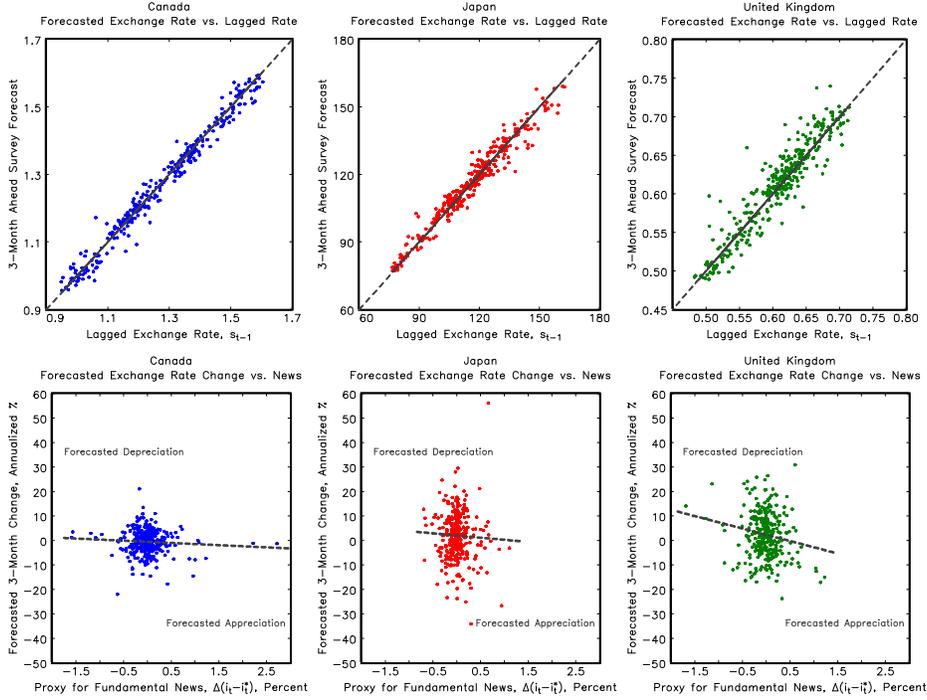


Figure 5: Top panels: Professional survey forecasts of 3-month ahead exchange rates track well with the lagged exchange rate, consistent with the Type-2 agents’ subjective forecast rule (13) in the CE model. Bottom panels: Survey forecasts appear to respond to changes in the interest rate differential (a proxy for fundamental news), again consistent with the Type-2 agents’ subjective forecast rule.

forecast using contemporaneous information (Type-1 agent forecast) has the lowest RMSFE and thus outperforms the other three forecast rules. The fundamentals-only forecast in the bottom row of the table exhibits the worst performance with the highest RMSFE. For this exercise, our main interest is asking whether the use of fundamental news can improve the performance of a random walk forecast that uses lagged information about the exchange rate. The answer is yes. Comparing the second and third rows of Table 3 shows that the use of fundamental news substantially improves forecast performance (lower RMSFE) when averaged over the entire sequence of rolling sample periods. In addition, Figure 4 also shows that the use of fundamental news can lower the RMSFE statistics (relative to a random walk forecast with lagged information) for all three country pairs using a rolling 15-year sample period. This result is particularly strong for Japan and UK. In the case of Canada, the use of fundamental news can still improve forecast performance, particularly in the more recent 15-year sample periods. The weaker results for Canada can be traced to the fact that Canada’s exchange rate behaves closest to a pure random walk of the three countries. Evidence of this can be seen in the bottom right panel of Figure 4 where we plot the 15-year rolling autocorrelation of exchange rate changes  $Corr(\Delta s_t, \Delta s_{t-1})$ . The rolling autocorrelation statistic for Canada remains closest to zero over the entire sample period, indicating the near-random walk behavior

of Canada’s exchange rate.

While the preceding exercise showed that the use of fundamental news can improve a lagged-information random walk forecast, it is interesting to consider some direct evidence on whether professional forecasters behave in a way that is consistent with the CE model. We noted earlier that the vast majority of professional forecasters report that they use both chart patterns and fundamentals when constructing their exchange rate forecasts. Figure 5 provides some additional evidence that professionals construct exchange rate forecasts in a way that is consistent with the Type-2 agents in the CE model.

The top panel of Figure 5 plots the 3-month ahead survey forecasts for the three countries’ exchange rates versus the lagged value of the actual exchange rates, i.e., the exchange rate at time  $t - 1$ .<sup>27</sup> For all three countries, the survey forecasts are clustered along the 45-degree line, suggesting that a lagged information random walk forecast captures much of the behavior of the professional forecasters.<sup>28</sup> The bottom panel of Figure 5 shows scatter plots of the forecasted 3-month ahead exchange rate changes from the survey versus the change in the interest rate differentials—a proxy for fundamental news. Specifically, a positive change in the interest rate differential  $i_t - i_t^*$  is a proxy for  $-u_t$  since, as shown earlier, the fundamental driving variable  $x_t$  is negatively correlated with the interest rate differential. The scatter plots show that survey respondents tend to forecast a negative change in the exchange rate (a forecasted appreciation) in response to a positive change in the interest rate differential, which is a proxy for  $-u_t$ . When  $\alpha > 0$ , the Type-2 agents’ subjective forecast rule (13) also predicts a negative change in the exchange rate in response to a negative realization of  $u_t$ .

## 5 Quantitative Analysis

### 5.1 Numerical Solution for the Equilibrium

We now examine a numerical solution for the consistent expectations equilibrium by plotting the map  $T(\alpha)$  from Definition 1. As noted earlier, the calibrated Taylor-rule parameters imply an effective discount factor of  $b = 0.9804$ . The parameters of the fundamental driving process (8) are chosen to achieve  $Std\ Dev(x_t) = 0.02$  (i.e., 2%) and  $Corr(x_t, x_{t-1}) = 0.95$ , which are close to the values observed in the data (see Table 2). This procedure yields  $\sigma_u = 0.00624$  and  $\rho = 0.95$ . We calibrate the fraction of Type-1 agents who employ the contemporaneous observation  $s_t$  to be  $\lambda = 0.5$ . We find that different values of  $\lambda$  primarily influence the size of the UIP regression slope coefficient in the CE model with only minor effects on the model’s other quantitative properties. From equation (19) with  $\lambda = 0.5$  and  $\rho = 0.95$ , the CE model predicts a slope coefficient of  $\beta_1 = -1.9$  which falls within the range of values obtained from

<sup>27</sup>The survey forecasts for the Canada/U.S., Japan/U.S., and U.K./U.S. exchange rate are the consensus values from a monthly survey of a large number of financial institutions, as compiled by FX4casts.com. The survey dates cover the period 1986.m8 to 2012.m10.

<sup>28</sup>A similar pattern can be observed when plotting survey forecasts for U.S. inflation versus lagged actual inflation. See Lansing (2009).

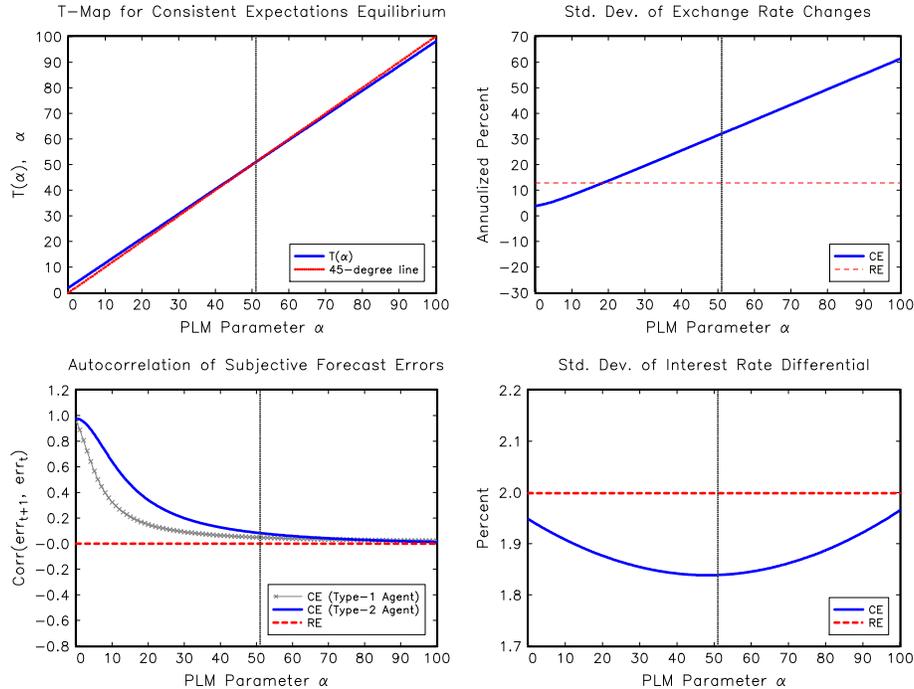


Figure 6: The T-map for the consistent expectations equilibrium lies very close to the 45-degree line. At the unique fixed point equilibrium, the volatility of exchange rate changes in the consistent expectations model (CE) is more than twice that of the solution to the rational expectations (RE) model. In equilibrium, the forecast errors observed by Type-1 and Type-2 agents in the CE model are both close to white noise, making it difficult for agents to detect any misspecification of their subjective forecast rules.

the data (see Figure 3). As a point of comparison, Carroll (2003) uses survey data on U.S. inflation forecasts to estimate a value of 0.27 for the fraction of agents who update their forecasts to the most recent information. One might expect the fraction of up-to-date agents to be higher in foreign exchange markets which are characterized by large transaction volumes and high frequency trading.

The top-left panel of Figure 6 plots the map  $T(\alpha)$  from Definition 1 using the calibrated parameter values. The map crosses the 45-degree line at  $\alpha^* = 1/(1-b) = 51$ . The slope of the map is  $T'(\alpha) = b(1-\lambda)/(1-b\lambda) = 0.96$ . Since the slope is less than unity, the fixed point is stable under learning, as we demonstrate in the real-time learning simulations below. Still, convergence to the equilibrium can be slow since the T-map lies very close to the 45-degree line. The top-right panel of Figure 6 plots the theoretical standard deviation of  $\Delta s_t$  in the CE model as  $\alpha$  varies from 0 to 100. The horizontal line shows the corresponding standard deviation implied by the rational expectation solution. At the equilibrium value of  $\alpha^* = 51$ , the CE model delivers excess volatility, i.e., the volatility of  $\Delta s_t$  in the CE model is nearly three times higher than the corresponding volatility in the RE model. Recall that the agents'

subjective forecasts in the CE model embed a unit root assumption. Due to the self-referential nature of the no-arbitrage condition (6), the agents’ subjective forecasts influence the dynamics of the object that is being forecasted. In equilibrium, the agents’ perception of a unit root becomes close to self-fulfilling.

The bottom-left panel of Figure 6 shows the persistence of the forecast errors for the Type-1 and Type-2 agents. At the fixed point equilibrium, both subjective forecast rules perform well in the sense that they produce near-zero autocorrelations, making it extremely difficult for the agents to detect any misspecification errors.

The bottom-right panel of Figure 6 plots the theoretical standard deviation of the interest rate differential  $i_t - i_t^*$ . For all values of  $\alpha$ , the volatility of  $i_t - i_t^*$  in the CE model is slightly below the volatility implied by the RE solution. This result is robust to different values of  $\lambda$ . Hence, the CE model’s excess volatility in  $\Delta s_t$  is not being driven by excess volatility in the interest rate differential but rather is driven solely by agents’ expectations.

Table 4 shows the theoretical moments of  $\Delta s_t$  in the CE model as predicted by the perceived law of motion (11) and the actual law of motion (15). The actual moments observed by the agents are very close to those predicted by their perceived law of motion, giving no indication of any misspecification. In addition, the autocorrelation structure of  $\Delta s_t$  remains close to zero at all lags, reinforcing the agents’ perception of unit root in the law of motion for  $s_t$ .

Table 4. Theoretical Moments of  $\Delta s_t$

Statistic	Rational Expectations		Consistent Expectations	
	PLM = ALM	PLM (predicted)	ALM (observed)	
<i>Std Dev</i> ( $\Delta s_t$ )	12.79%	31.85%	32.06%	
<i>Corr</i> ( $\Delta s_t, \Delta s_{t-1}$ )	-0.0059	0	0.0478	
<i>Corr</i> ( $\Delta s_t, \Delta s_{t-2}$ )	-0.0243	0	0.0454	
<i>Corr</i> ( $\Delta s_t, \Delta s_{t-3}$ )	-0.0235	0	0.0430	

Notes: Parameter values are  $b = 0.9804$ ,  $\kappa = 0.98$ ,  $\rho = 0.95$ ,  $\sigma_u = 0.00624$ ,  $\lambda = 0.5$ ,  $\alpha = \alpha^* = 51$ .

## 5.2 Forecast Error Comparison

Proposed departures from rational expectations are often criticized on the grounds that intelligent agents would eventually detect the misspecification of their subjective forecast rule. We counter this criticism in two ways. First, we show that the autocorrelation structure of the forecast errors observed by the Type-1 and Type-2 agents in the CE model are close to white noise, making it difficult for them to detect any misspecification. Second, we show that the forecast performance of both types of agents is nearly as good as that of a hypothetical “Type-3” agent who is endowed with knowledge of the actual law of motion (15). Hence, there is very little reward available in the model for an individual agent who may wish to expend resources searching for a better forecast rule.

From equations (12) and (13), the forecast errors observed by the Type-1 and Type-2

agents in the CE model are given by

$$\begin{aligned} err_{1,t+1} &= s_{t+1} - \widehat{E}_{1,t} s_{t+1}, \\ &= s_{t+1} - s_t, \end{aligned} \tag{24}$$

$$\begin{aligned} err_{2,t+1} &= s_{t+1} - \widehat{E}_{2,t} s_{t+1}, \\ &= s_{t+1} - s_{t-1} - \alpha u_t, \end{aligned} \tag{25}$$

where the evolution of  $s_t$  is governed by the ALM (15).

Now consider an atomistic Type-3 agent who understands that the evolution of  $s_t$  is governed by the ALM. The hypothetical Type-3 agent cannot influence the evolution of  $s_t$  but is tasked only with making forecasts. The forecast error observed by the Type-3 agent is given by

$$err_{3,t+1} = s_{t+1} - \left[ 1 - \frac{(1-\kappa)(1-b)}{1-b\lambda} \right] s_t - \frac{\rho x_t}{1-b\lambda}. \tag{26}$$

Given the forecast error expressions, it is straightforward to compute the associated autocorrelations and the root mean squared forecast errors, as given by  $RMSFE_i \equiv \sqrt{E[(err_{i,t+1})^2]}$  for  $i = 1, 2, 3$ . In all cases, the forecasts are unbiased such that  $E(err_{i,t+1}) = 0$ .

Within the RE model, the forecast errors observed by the fully-rational agent are given by

$$\begin{aligned} err_{re,t+1} &= s_{t+1} - E_t s_{t+1} \\ &= a_s s_t + a_x x_{t+1} - (a_s s_t + a_x \rho x_t) \\ &= a_x u_{t+1}, \end{aligned} \tag{27}$$

where we have made use of the solution from Proposition 1. The above expression implies  $RMSFE_{re} = a_x \sigma_u$ .

Table 5 compares the moments of the various forecast errors defined above, as computed from model simulations. In the CE model, the autocorrelations of the forecast errors are near zero at all lags for both Type-1 and Type-2 agents, giving no significant indication of a misspecification. A comparison of the  $RMSFE$  values shows that the RE model exhibits the most accurate forecasts (lowest  $RMSFE$ ), as expected. However, it is important to realize that an individual agent inhabiting the CE model could never achieve this level of forecast performance unless all of the other agents decided to switch to the fully-rational forecast rule. This is because the actual law of motion for the exchange rate in the CE model is permanently shifted by the presence of the Type-1 and Type-2 agents.

In the last column of Table 5, we see that the the ALM-based forecast of the hypothetical Type-3 agent has a slightly lower  $RMSE$  of 31.8% in comparison to the values of 32.0% and 32.7% for the Type-1 and Type-2 agents, respectively. Hence, there is little room for an

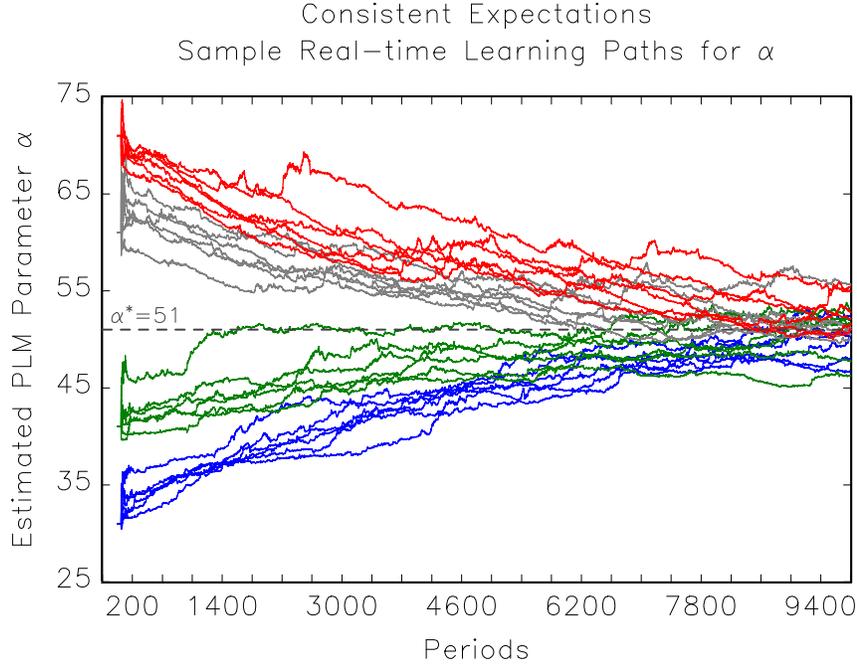


Figure 7: The figure plots twenty-four separate real-time learning paths (grouped by starting value) for the PLM parameter  $\alpha$ . The simulations confirm that the consistent expectations equilibrium is learnable; the estimated value of  $\alpha$  eventually converges to the vicinity of the theoretical fixed point value  $\alpha^* = 51$ , regardless of the shock sequences or the starting value for  $\alpha$ .

individual agent in the CE model to improve forecasting performance by employing more sophisticated (and presumably more costly) econometric methods to discover the true underlying law of motion for the exchange rate.

Table 5. Comparison of Forecast Errors

	CE Model			
	RE Model	Type-1 Agent	Type-2 Agent	ALM-based Forecast
$Corr(err_{t+1}, err_t)$	-0.002	0.048	0.084	-0.001
$Corr(err_{t+1}, err_{t-1})$	0.012	0.059	0.127	0.010
$Corr(err_{t+1}, err_{t-2})$	0.001	0.046	0.111	0.008
$\sqrt{Mean(err_{t+1}^2)}$	12.62%	32.0%	32.7%	31.8%

Notes: Forecast errors are defined by equations (24) through (27). Model statistics are computed from a 10,000 period simulation. RE = rational expectations, CE = consistent expectations, ALM = actual law of motion.

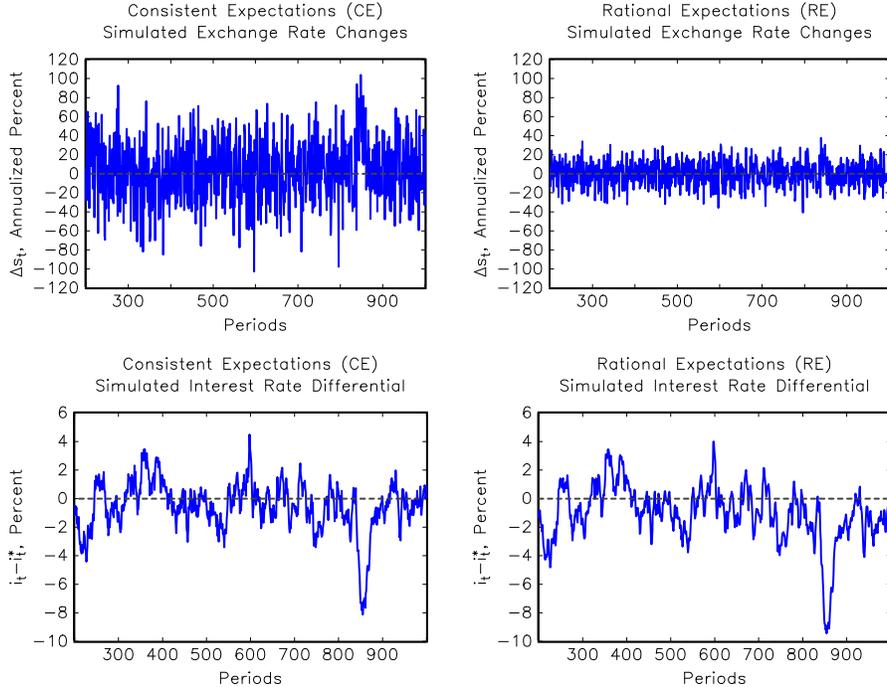


Figure 8: The consistent expectations (CE) model generates much more volatility in the exchange rate change  $\Delta s_t$  than the rational expectation (RE) model. Both model solutions exhibit similar volatility for the interest rate differential  $i_t - i_t^*$ .

### 5.3 Model Simulations

Using the same baseline parameter values shown in Table 4, Figure 7 plots twenty-four separate real-time learning paths (grouped by starting value) for the fundamental news response parameter  $\alpha$ . We employ four separate starting values  $\alpha_0 \in \{31, 41, 61, 71\}$  that initially enter the ALM (11), with each learning path subject to a different sequence of draws for the fundamental innovation  $u_t$ . Each period, a new value for  $\alpha$  is computed from past observable data using equation (16) and then substituted into the ALM and so on. To speed up the learning process, we assume that agents in the CE model compute the relevant covariance in equation (16) using a 30-year (360-month) rolling sample period. The simulations confirm that the consistent expectations equilibrium is learnable; the estimated value of  $\alpha$  eventually converges to the vicinity of the theoretical fixed point value  $\alpha^* = 51$ , regardless of the starting value.

Figure 8 plots simulated values for  $\Delta s_t$  and  $i_t - i_t^*$  for both models. The simulations confirm the theoretical results presented earlier in Table 4; the CE model generates considerably more volatility in  $\Delta s_t$  than the RE model. This is true despite the fact that CE model exhibits slightly lower volatility for the interest rate differential relative to the RE model, as shown by the bottom panels of Figure 8.

Table 6 compares the moments observed in the data to those generated by the model simulations. The CE model does a remarkably good job of matching all of the data moments. The CE model and the RE model can both generate near-random walk behavior of the exchange rate such that  $Corr(\Delta s_t, \Delta s_{t-1}) \simeq 0$  and  $Corr(\Delta^2 s_t, \Delta^2 s_{t-1}) \simeq -0.5$ , where  $\Delta^2 s_t \equiv \Delta s_t - \Delta s_{t-1}$  is the second difference of the exchange rate. However, the RE model generates much lower volatility in  $\Delta s_t$  and  $\Delta^2 s_t$ .

The middle rows of Table 6 confirm that the moments of the interest rate differential  $i_t - i_t^*$  in both models are close to those observed in the data. This is true even though we have not attempted to customize the values of the Taylor rule parameters to each country. Finally, the bottom row of Table 6 shows that the CE model produces a negative correlation between  $\Delta s_{t+1}$  and the prior interest rate differential  $i_t - i_t^*$ . The negative correlation is consistent with the data in all three countries and with the typical negative sign of the slope coefficient in the empirical UIP regressions, as discussed further below.

Table 6. Unconditional Moments: Data versus Models

	Canada	Japan	U.K.	CE Model	RE Model
<i>Std Dev</i> ( $\Delta s_t$ )	22.7%	38.3%	35.6%	32.0%	12.76%
<i>Corr</i> ( $\Delta s_t, \Delta s_{t-1}$ )	-0.054	0.055	0.093	0.048	-0.009
<i>Std Dev</i> ( $\Delta^2 s_t$ )	33.0%	52.5%	48.0%	44.2%	18.1%
<i>Corr</i> ( $\Delta^2 s_t, \Delta^2 s_{t-1}$ )	-0.535	-0.493	-0.467	-0.504	-0.496
<i>Std Dev</i> ( $i_t - i_t^*$ )	1.62%	2.35%	2.18%	1.86%	2.04%
<i>Corr</i> ( $i_t - i_t^*, i_{t-1} - i_{t-1}^*$ )	0.956	0.972	0.954	0.952	0.975
<i>Corr</i> ( $\Delta s_{t+1}, i_t - i_t^*$ )	-0.021	-0.115	-0.074	-0.117	0.153

Notes: Data sample period is from 1974.m1 to 2012.m10. CE = consistent expectations. RE = rational expectations. Model statistics are computed from a 10,000 period simulation.

## 5.4 UIP Regressions

Table 7 compares the results of UIP regressions using country data to those using model-generated data. In addition to the full-sample estimates for Canada, Japan, and the U.K. reported earlier in Table 1, we now also report the mean results from 15-year rolling regressions for each country. Figure 9 presents the model-generated analogs to the country data plots that we showed earlier in Figures 1 and 3. Specifically, the top panels of Figure 9 show scatter plots of the simulated exchange rate change  $\Delta s_{t+1}$  versus the prior interest rate differential  $i_t - i_t^*$  in both models. The dashed lines show the fitted relationships from the full-sample UIP regressions, with details reported in Table 7. The bottom panels of Figure 9 plot the estimated slope coefficients from UIP regressions run over a rolling 15-year sample period using model-generated data.

Table 7 shows that the estimated UIP slope coefficients for Canada, Japan, and the U.K. are consistently negative, illustrating the forward-premium anomaly. The  $R^2$  statistics from these regressions are all close to zero, illustrating the exchange rate disconnect puzzle. Similar

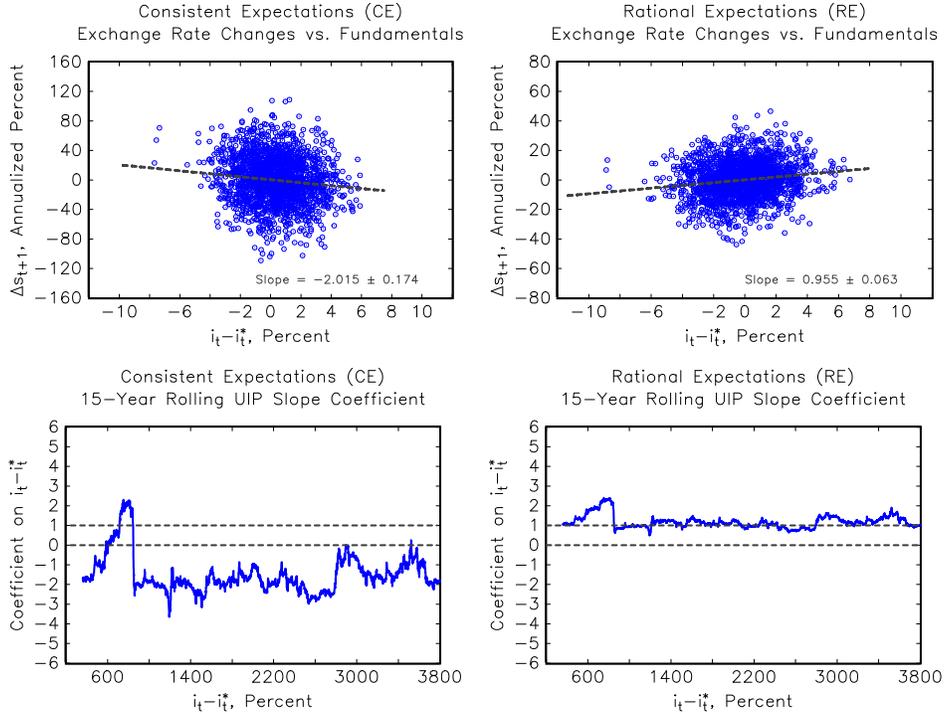


Figure 9: The fitted relationship between  $\Delta s_{t+1}$  and  $i_t - i_t^*$  in the consistent expectations (CE) model exhibits a negative slope coefficient, similar to the country data plotted in Figure 1. The estimated UIP slope coefficient in the CE model exhibits considerable time variation, similar to the data estimates shown in Figure 3. In contrast, the solution to the rational expectations (RE) model exhibits a positive UIP slope coefficient with much less time variation.

to the country data, the CE model produces negative UIP slope coefficients in both the full-sample and rolling regressions. The 95% confidence interval around the full-sample point estimate of  $-2.015$  would lie entirely in negative territory. The  $R^2$  statistics are very low, similar to those in the data. Another realistic feature of the CE model, evident in the bottom-left panel of Figure 9, is the considerable time variation in the estimated slope coefficient when the UIP regressions are run using a 15-year rolling sample period. Similar time variation in the estimated slope coefficients can be seen in the country-data UIP regressions plotted earlier in Figure 3. In contrast, the rolling-regression estimates for the RE model are always in positive territory, exhibit much smaller time variation, and cluster around unity. Notice that the rolling regressions all produce larger standard errors in comparison to the corresponding full-sample regressions. This is partly due to the smaller sample size and partly due to a correction of the estimation bias that can arise in the presence of a persistent regressor.<sup>29</sup>

Interestingly, the RE model can account for the exchange rate disconnect puzzle as it

<sup>29</sup>See Stambaugh (1986) and Nelson and Kim (1993) for detailed discussions of estimation bias in the presence of a persistent regressor.

produces a very small  $R^2$  statistic in both the full-sample and rolling regressions. This result is consistent with the arguments put forth by Engel and West (2005, 2006) who show that a fully-rational model with highly-persistent fundamentals and a discount factor close to unity can deliver near-random walk behavior of the exchange rate such that changes in the exchange rate are nearly unpredictable using fundamentals. Still, the RE model fails to account for the forward premium anomaly and woefully underpredicts the observed volatility of exchange rate changes in the data (Table 6).

Table 7. UIP Regressions: Data versus Models

Full-Sample Estimates	Canada	Japan	U.K.	CE Model	RE Model
$\hat{\beta}_1$	-0.29	-1.86	-1.21	-2.02	0.96
<i>Std Error</i>	(0.65)	(0.75)	(0.76)	(0.17)	(0.06)
$R^2$	0.007	0.013	0.005	0.014	0.023

Mean 15-year Rolling Estimates	Canada	Japan	U.K.	CE Model	RE Model
$\hat{\beta}_1$	-1.68	-2.79	-0.78	-1.40	1.21
<i>Std Error</i>	(1.08)	(1.48)	(1.44)	(0.99)	(0.36)
$R^2$	0.024	0.020	0.020	0.013	0.030

Notes: The table shows the results of an OLS regression in the form of equation (1). The data covers the period from 1974.m1 to 2012.m10. CE = consistent expectations. RE = rational expectations. Model statistics are computed from a 10,000 period simulation.

## 6 Concluding Remarks

A reading of the last three decades of academic research on exchange rates highlights numerous puzzles and anomalies that have stubbornly resisted explanation, particularly in the context of models where all agents are fully-rational. This paper showed that a plausible small departure from full rationality—in the form of boundedly-rational agents who augment a lagged-information random walk forecast with contemporaneous news about Taylor-rule fundamentals—can account for several empirical exchange rate puzzles, including the apparent disconnect from fundamentals, the extremely high volatility of exchange rate changes relative to that of cross-country interest rate differentials, and the failure of exchange rates to satisfy the UIP condition. Given that real-world exchange rates exhibit near-random walk behavior, it makes sense that agents would adopt a forecast rule that allows for a unit root. Our model setup is also consistent with survey data which shows that: (1) professional exchange rate forecasts track well with the lagged value of the exchange rate, and (2) professional forecasts respond to movements in fundamental news, as measured by changes in cross-country interest rate differentials.

## A Appendix: Proof of Proposition 1

The characteristic equation of the stochastic difference equation (6) is given by

$$f(r) = r^2 - \frac{1}{b}r + \frac{\kappa(1-b)}{b} = 0, \quad (\text{A.1})$$

We denote the two roots of equation (A.1) as  $r_1$  and  $r_2$ . Inserting these roots into the equation and applying some algebra yields the following relationships

$$r_1 r_2 = \frac{\kappa(1-b)}{b} > 0 \quad (\text{A.2})$$

$$r_1 + r_2 = \frac{1}{b} > 0 \quad (\text{A.3})$$

$$\begin{aligned} (r_1 - 1)(r_2 - 1) &= r_1 r_2 - (r_1 + r_2) + 1 \\ &= \frac{\kappa(1-b)}{b} - \frac{1}{b} + 1 \\ &= \frac{(\kappa - 1)(1-b)}{b} < 0 \end{aligned} \quad (\text{A.4})$$

Since  $(r_1 - 1)(r_2 - 1) < 0$ , one root is greater than one and the other is less than one. Also because  $r_1 r_2 > 0$  and  $r_1 + r_2 > 0$  it must be true that both roots are positive. Therefore we can conclude that one root is greater than unity and the other root is positive but less than unity. Since we have one initial condition that pins down  $s_{t-1}$ , there exists a unique rational expectation solution (Blanchard and Kahn 1980).

We postulate that the solution to equation (6) takes the form:  $s_t = a_s s_{t-1} + a_x x_t$ , where  $a_s$  and  $a_x$  are undetermined coefficients. Iterating the postulated solution ahead one period and taking the rational expectation yields  $E_t s_{t+1} = a_s s_t + a_x \rho x_t$ . Substituting this expectation into (6) and collecting terms yields

$$s_t = \underbrace{\frac{\kappa(1-b)}{1 - ba_s}}_{= a_s} s_{t-1} + \underbrace{\frac{1 + ba_x \rho}{1 - ba_s}}_{= a_x} x_t, \quad (\text{A.5})$$

which shows that the postulated form is correct. The value of  $a_s$  must satisfy

$$b(a_s)^2 - a_s + \kappa(1-b) = 0, \quad (\text{A.6})$$

which has two solutions. The stable solution is the one that delivers  $|a_s| < 1$ . This is given by  $a_s = \left[ 1 - \sqrt{1 - 4\kappa b(1-b)} \right] / (2b)$ . Given  $a_s$ , the value of  $a_x$  must satisfy  $a_x = 1 / [1 - b(a_s + \rho)]$ . ■

## B Appendix: Moments with Rational Expectations

Using the law of motion for  $s_t$  from Proposition 1, straightforward computations yield the following unconditional moments.

$$\frac{Var(s_t)}{Var(x_t)} = \frac{a_x^2}{(1 - a_s^2)} \left[ \frac{1 + a_s \rho}{1 - a_s \rho} \right], \quad (\text{B.1})$$

$$\frac{Var(\Delta s_t)}{Var(x_t)} = \frac{2a_x^2(1 - \rho)}{(1 + a_s)(1 - a_s \rho)}, \quad (\text{B.2})$$

$$Corr(s_t, s_{t-1}) = \frac{a_s + \rho}{1 + a_s \rho}, \quad (\text{B.3})$$

$$\begin{aligned} \frac{Cov(\Delta s_{t+1}, i_t - i_t^*)}{Var(x_t)} &= \frac{a_x}{b} \left\{ \frac{(a_s - 1)(1 - b)(1 - \kappa a_s) a_x (1 + a_s \rho)}{(1 - a_s^2)(1 - a_s \rho)} - \rho \right\} + \\ &\quad \frac{a_x}{b} \left\{ \frac{[-\rho \kappa a_x (1 - b) - 1](a_s - 1) + \rho a_x (1 - b)(1 - \rho \kappa)}{1 - a_s \rho} \right\}, \end{aligned} \quad (\text{B.4})$$

$$\frac{Var(i_t - i_t^*)}{Var(x_t)} = \frac{Cov(\Delta s_{t+1}, i_t - i_t^*)}{Var(x_t)}, \quad (\text{B.5})$$

where  $Var(x_t) = \sigma_u^2 / (1 - \rho^2)$ .

## C Appendix: Moments with Consistent Expectations

Using the actual laws of motion (15) and (17), straightforward but tedious computations yield the following unconditional moments.

$$\frac{Var(s_t)}{Var(x_t)} = \frac{1 + [b^2(1 - \lambda)\alpha^2 + 2b\alpha] (1 - \lambda)(1 - \rho^2) + \frac{2\rho[(1 - b\lambda) - (1 - \kappa)(1 - b)][b\alpha(1 - \lambda)(1 - \rho^2) + 1]}{(1 - \rho)(1 - b\lambda) + \rho(1 - \kappa)(1 - b)}}{2(1 - \kappa)(1 - b)(1 - b\lambda) - (1 - \kappa)^2(1 - b)^2} \quad (\text{C.1})$$

$$\begin{aligned} \frac{Var(\Delta s_t)}{Var(x_t)} &= \frac{2 \{ 1 + [b^2(1 - \lambda)\alpha^2 + 2b\alpha] (1 - \lambda)(1 - \rho^2) \}}{(1 - b\lambda) [2(1 - b\lambda) - (1 - \kappa)(1 - b)]} - \\ &\quad \frac{2(1 - \kappa)(1 - b)\rho [1 + b\alpha(1 - \lambda)(1 - \rho^2)]}{[2(1 - b\lambda) - (1 - \kappa)(1 - b)] (1 - b\lambda) [(1 - \rho)(1 - b\lambda) + \rho(1 - \kappa)(1 - b)]} \end{aligned} \quad (\text{C.2})$$

$$\begin{aligned}
Corr(s_t, s_{t-1}) &= \frac{(1-b\lambda) - (1-\kappa)(1-b)}{1-b\lambda} + \\
&\frac{\rho [1 + b\alpha(1-\lambda)(1-\rho^2)]}{(1-b\lambda) [(1-\rho)(1-b\lambda) + \rho(1-\kappa)(1-b)]} \frac{Var(x_t)}{Var(s_t)}, \tag{C.3}
\end{aligned}$$

$$\begin{aligned}
\frac{Cov(\Delta s_{t+1}, i_t - i_t^*)}{Var(s_t)} &= -\frac{(1-\kappa)(1-b)^2}{b(1-b\lambda)} [1 - \kappa Corr(s_t, s_{t-1})] + \\
&\left\{ \frac{(1-b) [\rho(1-\rho\kappa) + (1-\kappa)] [b(1-\lambda)\alpha(1-\rho^2) + 1]}{b(1-b\lambda) [(1-\rho)(1-b\lambda) + \rho(1-\kappa)(1-b)]} - \frac{\rho}{b(1-b\lambda)} \right\} \frac{Var(x_t)}{Var(s_t)}, \tag{C.4}
\end{aligned}$$

$$\begin{aligned}
\frac{Var(i_t - i_t^*)}{Var(s_t)} &= \frac{(1-b)^2}{b^2} [1 + \kappa^2 - 2\kappa Corr(s_t, s_{t-1})] + \\
&\frac{1}{b^2} \left[ 1 - 2(1-b)(1-\rho\kappa) \frac{b(1-\lambda)\alpha(1-\rho^2) + 1}{(1-\rho)(1-b\lambda) + \rho(1-\kappa)(1-b)} \right] \frac{Var(x_t)}{Var(s_t)}, \tag{C.5}
\end{aligned}$$

where  $\alpha = \alpha^* = 1/(1-b)$  in equilibrium.

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