Optimal Portfolio Choices and the Determination of Housing Rents in the Context of Housing Price Uncertainty

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Abstract

This paper develops a utility indifference-based model to investigate the pricing issue of house rents under housing price uncertainty. Our model not only allows for the crucial features in the housing market, such as illiquidity, market incompleteness, and idiosyncratic property risks, but also the interaction of investors' house tenure choices with their financial asset holdings. Our model provides interesting insights into the hedging of house resale risk and determination of housing rental prices. In addition to the parameters describing the expected changes and volatility on stock and house prices, we also show that the investors' precautionary savings motive, idiosyncratic property risks, and the correlation between stock and housing price have important implication for the determination issue of housing rentals. We empirically test the model predictions using the data from major Asian markets and the results overall support the model predictions.

Key Words: Tenure Choice, Resale Risk, Reservation Rental Prices, Utility Maximization, Incomplete Markets

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1. Introduction

Owner-occupied housing has been widely thought of as one of the most important components of household asset portfolios for most homeowners (e.g., Flavin and Yamashita, 2002), and homeownership is also therefore often viewed as one important channel to create household wealth (e.g., Beracha and Johnson, 2012). However, the recent house crisis in the U.S. have aggregated our concern about the effect of housing price uncertainty on household housing decisions due to the high concentration of homeowners' household wealth on residential real estate in this country. According to the 2010 Survey of Consumer Finances (SCF), about 67.3% of U.S. households possess their primary residences, but the average value of the primary residences of these U.S. homeowners, in effect, dropped 17.6% during the period from 2007 to 2010. Figure 1 further demonstrates that although the average housing price in the U.S. had experienced a continuously increasing process prior to 2005, the price fluctuated obviously and showed a downward trend after 2005. However, it is noteworthy that the average housing rent did not go down with the housing price after 2005 and, instead, still maintain a steady growth. This suggests the importance and necessity of investigating the role of house renting in hedging against housing price uncertainty and of exploring the implication of housing price uncertainty for the determination of housing rental prices in the context of household asset portfolios. As a result, our study extends Merton's (1969) optimal portfolio model to examine the determination of housing rental prices in this context.

Theoretically, one might think that homeowners can determine their housing rents according to the user costs of their homes based on the rental equivalent approach, whereby the rental price of a home can usually be considered to be equivalent to its user costs. However, recent research has noticed the marked divergence between

housing rents and user costs (Verbrugge (2008) and Díaz and Luengo-Prado, 2008), implying the lack of usefulness of this approach. The increasing uncertainty of real estate market facilitates improving our understanding for the usefulness lack and also poses a new challenge for incorporating the uncertainty of real estate market into the evaluation of housing rental prices. On the other hand, recently Beracha and Johnson (2012) demonstrate that the investment performance of house renting is actually superior over house buying during most of the 1978-2009 period. Given that housing is usually one of the most important components of household asset portfolios, this suggests that it is possible to improve the performance of household asset portfolio by taking renting housing services into consideration. For these reasons, our study extends the real options based approach to look at the determination of housing rental prices under housing price uncertainty.

In examining the crucial role of housing in determining optimal household portfolios, existing studies usually pay attention to the interaction of owner-occupied housing with traded financial asset holdings [see, e.g., Brueckner (1997), Flavin and Yamashita. (2002), Cocco (2005), Hu (2005), and Yao and Zhang (2005)]. Our study also attempts to investigate the implication of house tenure choice of an investor who is exposed to substantial housing price risk for his household asset holdings, while we develop a dynamic asset portfolio model by allowing for the stochastic evolution of both stock and housing price. Although homeownership allows the investor to lock in future housing costs and hedge against fluctuations in future rent payments, he also has to be faced with housing price risk. Higher house price risk probably lowers this investor's willingness to own a house and increase his renting likelihood. So far, however, little research has looked at the hedging of housing price risk and its interaction with house tenure choice in the context of stochastic evolution of both stock and housing price. Such model specification can make us better capture the crucial feature of housing price risk in order to shed new light on its important implication for household asset portfolios.

Homeownership can bring benefits to households in numerous countries in terms of taxes, and is also found to play an important role in hedging against rent fluctuations (e.g., Englund et al. 2002; Sinai and Souleles, 2005). However, how to hedge against house price risk has also been one key research issue that cannot be ignored in the existing real estate literature. Ortalo-Magné and Rady (2002) show that financial securities such as real estate stocks are a poor financial device hedging the idiosyncratic risk associated with residential real estate, while household home probably plays an important role in hedging against adverse fluctuations in housing prices and rents [see also Díaz and Luengo-Prado (2008)]. Englund et al. (2002) stress the necessity of incorporating home price derivatives as a category of potential hedging instruments into real estate risk-management strategies, while they also demonstrate the importance of financial assets such as stocks, bonds and t-bills in hedging residential property price risk over longer holding periods. Meyer and Wieand (1996) find that idiosyncratic risk in the housing market is an important determinant for diversifying and hedging housing market risk, and that housing rental strategies can exert a noticeable role in hedging against housing price risk. More recently, Yao and Zhang (2005) emphasize that since investors can partition their housing consumption from their housing investment, renting housing services can be viewed as an important strategy against house price risks [see also Meyer and Wieand (1996) Voicu and Seiler (2013)].[**] The recent real estate crisis also has further exacerbated our concerns on developing and using housing rental strategies to avoid house price risks. Of course, other hedging strategies might be useful in hedging against housing price uncertainty, but there is little evidence supporting the usefulness in hedging against housing price uncertainty.

Renting house services provides a hedge against substantial housing price risk at the time of resale.[††] Conventionally, the options-based pricing approach has been

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[**] In sharp contrast, Sinai and Souleles (2005) also provide evidence that housing rent volatility has a significant positive impact of the demand for homeownership so that home owning can reduce a household's exposure to future rent fluctuations.

[††] The 2007 Survey of Consumer Finances (SCF) demonstrates that about 31.4% of US households consume housing services via renting whereas for the rest households their consumption of housing services are realized

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employed for pricing various lease contracts due to its potential in allowing for uncertainties involved in the leasing activities.[‡‡] However, while asset lease contracts can be viewed as compound options, standard option pricing models are still subject to the following problems in examining rental choices in the housing market. First, suppose that capital markets are complete and frictionless, and financial assets are traded "continuously", such that any contingent claims built on these assets can meet the spanning condition. We can readily price these claims based on non-arbitrage arguments. However, housing markets are largely different from the capital markets, and characterized by the market frictions mentioned above. Consequently, it is difficult to find a trading strategy to completely replicate the payoffs of contingent claims built on residential real estate, and no-arbitrage arguments will no longer hold true. Second, in the standard models contingent claims are evaluated in the risk-neutral world where all the investors are risk preference-free. As a result, the models do not explicitly consider the effect of investors' subjective degree of risk aversion, whereas Shilling (2003) provides supportive evidence that real estate investors are extremely risk averse. Third, the option-based models usually only calculate and predict the "fair" market value of the contingent claims, and it is difficult in the complete market setting to analyze the bid and ask prices of real estate lease trades, while these trades are typically reached through a bargaining process.

Suppose that a house renter or investor seek to maximize their expected utility of terminal wealth, and the present model evaluates various housing rental choices based on the principle of expected wealth utility equivalence instead of no arbitrage arguments due to the reasons discussed above. Similar to Yao and Zhang (2005), we incorporate the renting-versus-owning decision into optimal portfolio choices, and are concerned about which of the two tenure options yields a higher expected wealth utility. Specifically, if the renter chooses to rent a house rather than own, the expected

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through homeownership.

[‡‡] See, e.g., Smith (1979), McConnell and Schallheim (1983), Grenadier (1995,1996, 2005), Ambrose, Hendershott and Klosek (2002), Hendershott and Ward (2003), Stanton and Wallace (2002), Buttimer and Ott (2007), and Cho and Shilling (2007).

utility he can obtain from his optimal wealth portfolio under this choice should be at least not lower than that he can derive conditional on purchasing the house or another comparable house; *vice versa.* When he is indifferent, in the sense of expected wealth utility, between renting and purchasing this house, we can derive his housing reservation rental price.

The contribution of this study to the relevant literature is apparent. It develops a utility indifference-based framework to value reservation housing rental prices by allowing for a homeowner's and renter's optimal wealth portfolio choice and the effective hedging of house resale price risk. Rosen *et al.* (1984) find that housing price uncertainty plays an important role in a household's housing tenure choice decision, and higher uncertainty in housing prices relative to rents could lead to the reduction of the proportion of homeownership.[§§] Our model further explores that the effect of housing price uncertainty on the determination of several housing rental prices. Although the conventional present value model implies that housing price movements should be interpreted by changes in housing rents, empirical results usually reject the implication (e.g., Poterba, 1991). Our model is also parallel to the indifference pricing theory associated with the valuation of contingent claims on non-tradeable or illiquid assets (Musiela and Zariphopoulou, 2004a, 2004b). It not only explicitly takes account of investors' precautionary savings motive, but also investigates the impact of the price correlation between the traded risky asset and the nontradable property. The expected change and volatility of both the traded asset price and the housing price are also identified to be important determinants for the agent's housing reservation rental prices.[***] Idiosyncratic risk in the housing market is also shown to play an important role in hedging against house price risk and determining housing rental choices. Our model first investigates the optimal choice problem by focusing on the

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[§§] A large body of economic literature, both theoretical and empirical, has also paid attention to the important implication of uncertainties in other influential factors on housing tenure status and consumption. See, e.g., Haurin (1991), Fu (1995), Robst *et al.* (1999), Chung and Haurin (2002), Ortalo-Magné and Rady (2002), and Díaz and Luengo-Prado (2008).

[***] Buttimer and Ott (2007) have shown that property reservation lease prices play an important role in the valuation of real estate and real estate leases.

non-institutional economic aspects, and then incorporate the consideration of other factors such the tax system, as in Henderson and Ioannides (1983).

The remainder of this article is organized as follows. Section 2 develops a theoretical model for valuing housing rental contracts, by considering the determination of market clearing rental prices. Section 3 describes the data and empirical designs. Section 4 presents the empirical results. Section 5 summarizes this article and draws relevant conclusions.

2. Utility Indifference Models

2.1 Model Setup

This section develops a utility indifference-based model framework for pricing house rental contracts based on the equivalent principle proposed above. Since a risk-neutral pricing approach built on the assumption of market completeness is inappropriate to be utilized for the purpose of examining our research problem, we resort to a stochastic dynamic optimization approach. Suppose that a risk-averse representative agent could be a house buyer or renter, whose optimization problem is to maximize his expected utility of terminal wealth. We also assume that the agent faces a house choice problem on two mutually exclusive modes of either purchasing a given house or renting this space in order to satisfy his consumption or investment demand. As a result we can define the reservation housing rental price as the amount which the agent is willing to pay for renting a piece of available housing space for a pre-specified period of time so that he is indifferent in the sense of expected wealth utility towards buying and renting this property.

While purchasing this given house can be regarded as a typical good consumption behavior, it likewise is also a real estate investment choice. Suppose that in addition to the house investment opportunity, the agent's wealth is held in the form of financial

assets consisting of a risk-free bond and a risky traded stock. Without loss of generality, the agent's utility function is assumed to satisfy the constant absolute risk aversion (CARA) form

$$u(x) = -\frac{1}{\gamma} \exp(-\gamma x), \tag{1}$$

where $\gamma > 0$ represents his absolute risk aversion level.[†††] Such a utility specification allows us to derive closed-form solutions to the agent's optimization problem in order to deeply investigate the effects of uncertainty arising from capital and real estate markets on the pricing of housing rental contracts.

Suppose that the price process of the risk-free bond is governed by

$$dR_t = rR_t dt, \qquad R_0 = 1 \quad t \le T, \tag{2}$$

where r is the risk-free rate. That is, the risk-free bond price changes with time at the rate rR_t. However, the price of the traded stock evolves in a geometric Brownian motion

$$d S_t = \mu\ S_t\ dt + \sigma\ _t S \iota, \quad S_0 = s > 0 \qquad t \le T, \tag{3}$$

where both coefficients μ and σ are given positive constants, and the process B_t stands for a standard Brownian motion.

Under the specifications above, the agent with an initial endowment x, given at time $t \ge 0$, faces an optimization problem of choosing a house investment opportunity and allocating the remaining endowment between the risk-free bond and the traded stock in order to maximize his expected utility of terminal wealth. Merton (1969) considers a similar but simplified optimal investment problem, where a risk averse individual needs to choose an optimal investment allocation strategy between a riskless bond and a risky traded asset for maximizing his expected wealth utility. Following his

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[†††] $\gamma > 0$ is also the efficient of absolute prudence, and therefore measures the precautionary savings motive (Miao and Wang, 2007).

framework, let $(\theta_s)_{t \le s \le T}$ denote the agent's investing strategy, where θ_s is the amount he invest in the traded stock at time s. Given that the agent seeks to maximize his expected utility of wealth at some future time T, we define his value function as

$$V(x,t) = \max_{(\theta_s)_{t \le s \le T}} E[u(X_T) \mid X_t = x], \tag{4}$$

where $\{X_s : t \le s \le T\}$ represents the wealth process. In the absence of the real estate investment opportunity, by a standard argument $V(x,t)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation

$$V_t + rxV_x + \max_{\theta} \left[(\mu - r) \theta V_x + \frac{1}{2} \theta^2 \sigma^2 V_{xx} \right] = 0, \tag{5}$$

subject to $V(x,T) = u(x)$. We can derive the following analytic solution to this equation[‡‡‡]

$$\overline{V}(x,t) = -\frac{1}{\gamma} \exp\left(-\gamma x e^{r(T-t)} - \frac{(\mu - r)^2 (T-t)}{2\sigma^2} \right). \tag{6}$$

Correspondingly, the optimal investment strategy can be identified through the first-order condition for (5)

$$\theta_t^* = \frac{(\mu - r) e^{-r(T-t)}}{\gamma \sigma^2}. \tag{7}$$

Given that in the exponential utility function the absolute risk aversion γ is constant and independent of the wealth process, the strategy θ_t^* is readily found to be a deterministic decreasing function of the trading horizon T, the volatility σ^2 of the traded stock and the risk aversion coefficient γ, and does not depend on the wealth or stock dynamics.

Given that investing in real estate is also a possible investment choice for this agent, we extend Merton's (1969) analytical framework to incorporate this additional choice into his investment portfolio. Since uncertainty on housing prices is a key determinant

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[‡‡‡] See, e.g., Young and Zariphopoulou (2002).

for rent-versus-buy decisions as discussed above, let the value of the given dwelling evolve in the following Brownian process

$$dP = a(P_t, t)dt + b(P_t, t)dW_t \qquad (8)$$

where $a(\cdot, \cdot)$ and $b(\cdot, \cdot)$ are the drift and diffusion coefficients for the value process, and W_t is a new Brownian motion correlated to B_t with coefficient $\rho \in [-1,1]$. As a result, the wealth dynamics of the agent satisfies the following controlled diffusion process

$$dY_t = \theta_t \frac{dS_t}{S_t} + r(Y_t - \theta_t)dt. \qquad (9)$$

Under the consideration, the value function of this agent can therefore be rewritten as

$$U(y, p, t) = \max_{(\theta_s)_{t \le s \le T}} E[u(Y_T + P_T) \mid Y_t = y, P_t = p]. \qquad (10)$$

Alternatively, the agent can rent the house for meeting his consumption demand rather than buy. Let L be the payout rate of rents and θ_t be the amount allocated to the traded stock at time t. The wealth dynamics at any time t satisfy the following controlled diffusion process

$$dX_t^L = \theta_t \frac{dS_t}{S_t} + r(X_t^L - \theta_t)dt - Ldt \qquad (11)$$

Correspondingly, with an initial endowment x, the value function can be specified as

$$V(x, L, t) = \max_{(\theta_s)_{t \le s \le T}} E[u(X_T^L) \mid X_t^L = x]. \qquad (12)$$

To build a utility indifference-based model, we give the following definition

Definition 1: *The rental reservation price for the agent at time t is defined as the amount $L(x, p, t)$ such that*

$$V(x, L(x, p, t), t) = U(x - p, p, t) \qquad (13)$$

That is, the agent is willing to pay $L(x, p, t)$ for renting the residential space at time t such that he is indifferent towards renting it or purchasing it with the cost p at time t.

By this definition, if he expects that the left hand side of equation (13) will be greater

than the right side, the agent will choose to rent in that renting helps him realize greater wealth utility than buying. On the contrary, if he thinks that purchase can bring him a greater expected wealth utility, the agent will choose to buy rather than rent.

A. Complete Markets

For expositional convenience, we first take into account a special case where there is an instantaneous perfect positive correlation between the two Brownian motions B_t and W_t, implying $dB_t = dW_t$. Based on the principle of dynamic programming, the HJB equation with regard to the value function V can be written as follows:

$$V_t - LV_x + rxV_x + \max_\theta \left[(\mu - r)\theta V_x + \frac{1}{2}\theta^2\sigma^2 V_{xx} \right] = 0 \tag{14}$$

subject to $V(x,T) = u(x)$. We can obtain the optimal strategy based on the first-order condition for this equation

$$\theta_t^* = \frac{-(\mu - r)V_x}{\sigma^2 V_{xx}} \quad . \tag{15}$$

Substituting (15) into equation (14) produces the following HJB equation

$$V_t - LV_x + rxV_x - \frac{V_x^2(\mu - r)^2}{2V_{xx}\sigma^2} = 0 \quad . \tag{16}$$

Since the value function V is smooth, it is the unique smooth solution of the HJB equation. As a result, we can give the follow proposition about this unique solution.

Proposition 1: *Given a perfect positive correlation between the stock price and housing price, under the optimal investment strategy (15) and the house renting choice, the value function, namely the solution to HJB equation (16), is given by*

$$V(x,L,t) = -\frac{1}{\gamma}\exp\left(-\gamma(x - \frac{L}{r})e^{r(T-t)} - \frac{(\mu-r)^2(T-t)}{2\sigma^2} \right)\exp(-\gamma\frac{L}{r}) . \tag{17}$$

Proof: See Appendix A.

Compared with the solution (6) to Merton's (1969) problem, this result implies that if the agent chooses to rent and make rental payments with the rate L per unit time, he needs to set aside an amount of $\dfrac{L}{r}$ for rental payments at time t, which can be explained as the capitalized present value of the payable rent stream. As a result, the agent's value function is differentiated from (6) derived from the Merton problem, because it is also determined by the remaining amount $(x - \dfrac{L}{r})$ instead of x and adjusted by a risk aversion-related term $\exp(-\gamma\dfrac{L}{r})$.

On the other hand, the perfect correlation implies the completeness of the house market. In other words, the price risk in this house market can be completely hedged with the traded stock. Consequently, one can easily derive the certainty equivalent value for the given house at time t by discounting its expected value under the risk neutral probability

$$CE(P_T, t) = E_t\left[\zeta_t P_T\right],\tag{18}$$

where ζ is usually referred to as the state price density and can be expressed as[§§§]

$$\zeta_t = e^{-r(T-t)} \exp\left[-\frac{1}{2}(\frac{\mu-r}{\sigma})^2(T-t) - (\frac{\mu-r}{\sigma})(B_T - B_t)\right]\tag{19}$$

Based on the derived certainty equivalent value, we can convert the agent's optimization problem with the house investment into the classic Merton (1969) problem. Accordingly, his value function in this case is rewritten as

$$U(y, p, t) = \bar{V}\left(y + CE(P_T, t), t\right)\tag{20}$$

where $\bar{V}$ is the standard value function of the Merton (1969) problem defined in (6). In the end, setting $t = 0$ as the beginning time we have the following proposition.

Proposition 2: *Under a perfect positive correlation between the stock price and*

[§§§] Under the risk neutral probability $\tilde{P}$, all the financial assets has a fixed rate of return equal to r such that $d\tilde{P}_t = \zeta_t dP_t$ and $CE(P_T, t) = \tilde{E}_t[P_T] = E_t\left[\zeta_t P_T\right]$. For more details, see Shreve (2003, chapter 5).

housing price and the optimal investment strategies (15) and (7), the rental reservation price for the agent is given by

$$L = \frac{r\left(p - CE(P_T, 0)\right)}{1 - e^{-rT}}. \tag{21}$$

Proof: Substituting (18) into (20) and then both the resultant relationship and (17) into (13), it is straightforward to derive the analytical solution with regard to L through simple manipulation.

This proposition clearly shows that the rental reservation price is independent of the risk aversion coefficient γ. This implies that under the perfect correlation, the housing rental price is unique and independent of individual risk preference, and all the potential users in the housing market are willing to offer the same price for renting the residential space. Furthermore, the price change of the given house, $p - CE(P_T, 0)$, is found to be a crucial determinant for the housing rental price over the time period (0, T). The economic implication is more apparent if equation (21) is rearranged as follows

$$\frac{L\left(1 - e^{-rT}\right)}{r} = p - CE(P_T, 0). \tag{22}$$

The left hand side of equation (22) represents the present value of the payable rental reservation price stream over the time period (0, T), while the right side reflects the price change of the given house during this period. When $p > CE(P_T, 0)$, the agent is exposed to higher house resale price risk, and renting house services is therefore picked as a feasible hedging strategy against the risk. As a consequence, the housing rental contract can be priced according to the rental payment rate L. However, if $p < CE(P_T, 0)$, equation (22) will no longer hold true, and the agent will prefer to directly own this house.

B. Incomplete Markets

Owner-occupied housing is a typical class of differentiated commodities such that it is difficult, in practice, to find a traded financial asset whose value is completely correlated with the underlying house value. As a result, we relax the assumption of the perfect relatedness specified above and allow the traded stock to be imperfectly correlated with the given house. Based on the principle of dynamic programming, the value function (10) satisfies the following nonlinear HJB equation

$$U_t + ryU_y + a(p,t)U_p + \frac{1}{2}b(p,t)^2 U_{pp} + \max_\theta \left[(\mu - r)\theta U_y + \frac{1}{2}\theta^2\sigma^2 U_{yy} + \rho\theta\sigma b(p,t)U_{yp} \right] = 0.$$

(23)

Differentiating this equation with respect to θ produces

$$\theta_t^{**} = \frac{-(\mu - r)U_y}{\sigma^2 U_{yy}} - \frac{b(y,t)\rho U_{yp}}{\sigma U_{yy}}$$

(24)

The first term of the right side of (24) is of the same form as the optimal investment strategy in the Merton (1969) problem, while their value functions may be different. However, the second term in the optimal strategy represents the strategy hedging the price risk of the house market. If the correlation coefficient between B_t and W_t, ρ, is zero implying that the capital and house markets are independent of each other, then the hedging term vanishes because house price risk is idiosyncratic and cannot be hedged using the traded stock. If $0 < |\rho| < 1$, the house price risk can be partially hedged using the traded stock. This term therefore suggests that the volatility of the housing price is another important determinant for this hedging strategy in addition to ρ. Given $U_{yy} < 0$, a decrease in the volatility leads to a less hedging demand for offsetting the price risk in the house market.

Under the optimal strategy (24), HJB equation (23) can be reduced to

$$U_t + ryU_y + a(p,t)U_p + \frac{1}{2}b(p,t)^2 U_{pp} - \frac{\left[(\mu - r)U_y + \rho\sigma b(p,t)U_{yp}\right]^2}{2\sigma^2 U_{yy}} = 0$$

(25)

subject to $U(y,p,T) = -\frac{1}{\gamma}e^{-\gamma(y+p)}$. Then we have the following proposition.

Proposition 3: *Given a less perfect correlation between the housing price and stock price and the optimal investment strategy (24), the value function solution to HJB equation (25) can be expressed as*

$$U(y,p,t) = -\frac{1}{\gamma}\exp\left[-\gamma ye^{(r(T-t))}\right]e^{-\frac{(\mu-r)^2}{2\sigma^2}(T-t)}\left[E\left(\psi_t e^{-\gamma(1-\rho^2)P_T}\right)\right]^{\frac{1}{1-\rho^2}} \quad (26)$$

where $\psi_t = \exp\left[-\frac{1}{2}(\frac{(\mu-r)\rho}{\sigma})^2(T-t) - \frac{(\mu-r)\rho}{\sigma}(Z_T - Z_t)\right]$.

Proof: See Appendix B.

Based on the result of Proposition 3, we can further derive the following proposition.

Proposition 4: *Given a less perfect correlation between the house price and stock price and the optimal investment strategies (15) and (24), the rental reservation price for the agent can be found to be*

$$L = \frac{r}{1-e^{-rT}}\left(p + e^{-rT}\frac{\ln E\left[\psi_0 e^{-\gamma(1-\rho^2)P_T}\right]}{\gamma(1-\rho^2)}\right). \quad (27)$$

Proof: Substituting (26) and (17) into (13), we can readily derive the analytical solution with regard to L through simple manipulation.

Given that the term $-e^{-rT}\frac{\ln E\left[\psi_0 e^{-\gamma(1-\rho^2)P_T}\right]}{\gamma(1-\rho^2)}$ is the certainty equivalent of the resale price P_T of the house at time 0, we can interpret the crucial role of the price change of the given house over the time period $(0, T)$ in determining the rental reservation price L as in Proposition 2. However, compared with Proposition 2, we find that the housing rental price is no longer independent of the risk aversion degree or precautionary savings motive, and is affected by the price correlation between the house and stock. Nevertheless, when $|\rho| \to 1$ and $\gamma \to 0$, the certainty equivalent

value approaches that under the perfect correlation such that the rental reservation price is also close to the one given in (21).

Equation (27) can be also rearranged as

$$\frac{L\left(1-e^{-rT}\right)}{r} = \left(p + e^{-rT}\frac{\ln E\left[\psi_0 e^{-\gamma(1-\rho^2)P_T}\right]}{\gamma(1-\rho^2)} \right). \tag{28}$$

This suggests that the value of housing rental contracts is not only associated with the parameters describing the interest rate, rental term, expected return and volatility of the traded stock, expected growth and volatility of the housing price, but also with the agent's precautionary savings motive and the price correlation between the traded stock and house.

C. A Special Case

For expositional convenience, this subsection assumes that the housing price evolves in an arithmetic Brownian motion.[****] We also first take into account that simplified situation where there is an instantaneous perfect price correlation between the housing and stock. As a result, equation (8) is rewritten as

$$dP_t = \alpha dt + \upsilon dB_t \tag{29}$$

According to (18), the time-0 certainty equivalent of the resale price P_T of the house is given by

$$CE(P_T, 0) = E\left(\zeta_0 (p + \alpha T + \upsilon(B_T - B_0)) \right). \tag{30}$$

We can evaluate (30) based on the risk neutral pricing argument in that the housing price risk can be hedged completely by trading the stock and bond. Under the risk neutral probability, $\tilde{B}_t = B_t - B_0 + \frac{\mu - r}{\sigma}t$ is a Brownian motion, and therefore

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[****] Stochastic differential equation (8) includes several special processes for asset prices, such as the geometric Brownian motion, arithmetic Brownian motion and mean-reverting process. These processes have been widely utilized in the economic literature. We keep to the arithmetic Brownian motion to highlight the contributions in our paper without causing unnecessary complications that throw no additional insights.

$$CE(P_T,0) = e^{-rT} \tilde{E} \left[p + \alpha T + \upsilon \tilde{B}_T - (\frac{\mu - r}{\sigma})\upsilon T \right]$$

$$= e^{-rT} \left[p + \alpha T - (\frac{\mu - r}{\sigma})\upsilon T \right] \qquad (31)$$

where $\tilde{E}$ denotes the expected operator under the risk neutral probability.

By proposition 2, we can derive the following proposition.

Proposition 5: *Given that the housing price follow an arithmetic Brownian motion process and there is a perfect positive correlation between the stock price and housing price, under the optimal investment strategy (15) the rental reservation price of the agent is given by*

$$L = \frac{r}{1 - e^{-rT}} \left[(1 - e^{-rT})p + e^{-rT} \left((\frac{\mu - r}{\sigma})\upsilon T - \alpha T \right) \right]. \qquad (32)$$

Proof: Substituting (31) into (21), it is straightforward to derive the analytical solution with regard to L.

This proposition demonstrates that the rental reservation price is dependent on the parameters describing the interest rate, rental term, expected return and volatility of the traded stock, expected growth and volatility of the housing price. Differentiating equation (32) with respect to the key parameters produces the following comparative static results:

$$\frac{\partial L}{\partial \alpha} < 0, \frac{\partial L}{\partial \upsilon} > 0, \ \frac{\partial L}{\partial \mu} > 0, \frac{\partial L}{\partial \sigma} < 0, \frac{\partial L}{\partial p} > 0.$$

The exact comparative static derivatives are given in Appendix C. Holding other things unchanged, it is shown that the rental reservation price L decreases in the expected return on the property, but increases in the volatility of the property return. The results are not surprising in that higher expected returns on the property suggest housing investment becoming more attractive than leasing, while increasing volatility of the housing price implies a higher property resale risk and therefore increases L.

On the other hand, the results also show that a higher expected return on the traded stock leads to the higher rental reservation price, while increasing volatility of the stock price has a negative effect on the rental reservation price. This is because higher expected stock returns make investing in the traded stock more attractive than investing in the house and therefore raises the agent's willingness to rent, while increasing stock volatility decreases this willingness. One can also readily find that L is an increasing function of the initial price p of the house, as higher initial prices discourage the agent from housing investment. Also we notice that the rental reservation price is not directly dependent on P_T, which seems to be different from our intuition. Since the housing price follows a Markov process, the future housing price is only determined by p, αT and υW_T.

Now we turn to the incomplete market scenario, and allow for an imperfect correlation between B_t and W_t. In this scenario, the certainty equivalent value of the house at time 0 is given by

$$CE(P_T,0) = -e^{-rT} \frac{\ln E\left[\psi_0 e^{-\gamma(1-\rho^2)P_T}\right]}{\gamma(1-\rho^2)},\tag{33}$$

where

$$\psi_0 = \exp\left[-\frac{1}{2}(\frac{(\mu-r)\rho}{\sigma})^2 T - \frac{(\mu-r)\rho}{\sigma}(B_T - B_0)\right]$$

and

$$P_T = p + \alpha T + \upsilon W_T.$$

Given the correlation coefficient ρ, we have $dB_t = \rho dW_t + \sqrt{1-\rho^2}\,dZ_t$, where Z_t is a new Brownian motion independent of W_t. As a consequence, equation (33) can be reduced to

$$CE(P_T,0) = e^{-rT}\left[p + \alpha T - \frac{\mu-r}{\sigma}\rho^2\upsilon T - \frac{1}{2}\gamma(1-\rho^2)\upsilon^2 T\right]\tag{34}$$

where $\dfrac{\mu - r}{\sigma}$ is the price of market risk in the financial market. According to proposition 4, we have the following result.

Proposition 6: *Given the less perfect correlation between the housing price and stock price and the optimal investment strategies (15) and (24), if the housing price evolves in an arithmetic Brownian motion, then the rental reservation price for the agent can be found to be*

$$L = \frac{r}{1-e^{-rT}}\left[(1-e^{-rT})p + e^{-rT}\left(\frac{\mu-r}{\sigma}\rho^2 \upsilon T + \frac{1}{2}\gamma(1-\rho^2)\upsilon^2 T - \alpha T \right)\right]. \qquad (35)$$

Proof: Substituting (34) into (27), it is straightforward to derive the analytical solution with regard to L.

Compared with (32), it is shown that the rental reservation price is also determined by the agent's precautionary savings motive and the price correlation between the stock and house as well as those parameters identified from (32). However, when $\rho = 1$ and $\gamma = 0$, (34) can be reduced to (32).

Differentiating (35) with respect to the key parameters, we have the following comparative static results:

$$\frac{\partial L}{\partial \alpha} < 0, \frac{\partial L}{\partial \upsilon} > 0, \; \frac{\partial L}{\partial \mu} > 0, \frac{\partial L}{\partial \sigma} < 0, \frac{\partial L}{\partial p} > 0, \frac{\partial L}{\partial \gamma} > 0, \frac{\partial L}{\partial \rho^2} \gtrless 0.$$

The exact partial derivatives are reported in Appendix D. These results show that the effects of changes in μ, σ, α, and υ, on the rental reservation price L are consistent with those identified above. In addition, our results also show that an increase in the initial house price decreases the agent's willingness to buy, and hence raises L. Higher precautionary savings motive discourages the agent from investing in the residential property, and therefore increases L. However, the effect of varying correlation between the two risky assets on the lease reservation price is ambiguous, and an increase in the correlation is likely to produce opposite impacts on the lease reservation price. More specifically, it is shown that the lease reservation price

increases in the correlation when $\dfrac{(\mu-r)}{\sigma} > \dfrac{1}{2}\gamma\upsilon$, where $\dfrac{(\mu-r)}{\sigma}$ can be explained as

the market price of risk. Under this condition, Figure 1 clearly shows that the lease

reservation price rises as the correlation increases.

[Insert Figure 1]

In addition, if $\gamma = 0$, we can directly obtain the risk-neutral house rental price from

(35)

$$ L = \frac{r}{1-e^{-rT}}\left[(1-e^{-rT})p + e^{-rT}\left(\frac{\mu-r}{\sigma}\rho^2\upsilon T - \alpha T\right)\right]. $$

Define the idiosyncratic risk premium of the house market as the difference between a

rental reservation price and the risk-neutral rental price. Then the risk premium can be

derived as follow[††††]

$$ I = \frac{r}{1-e^{-rT}}\left[e^{-rT}\frac{1}{2}\gamma(1-\rho^2)\upsilon^2 T\right]. \tag{36} $$

This implies that even though investors chooses to rent housing services, the risk

premium is still a remarkable factor determining their house rental reservation prices

due to the effect of their exposure to resale price risk in the housing market in owning

a house. Equation (36) also shows that the risk premium increases with the agent's

precautionary savings motive, and volatility of housing price, but decreases with the

price correlation between the house and traded stock.

2.2 Market Clearing Rental Price

Since a house rental contract is usually reached through a bilateral bargaining process,

this section investigates the determination of the market clearing rental price. To

address this issue, we require allowing for the second market agent in this model,

namely, the owner of the given house. Suppose that the owner is also likewise facing

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[††††] A similar definition can also be found in Miao and Wang (2007).

two different options at initial time 0: either selling or renting out her house. If she chooses to rent out, the owner would receive regular rental incomes derived from her property tenant, namely, the first agent. On the other hand, the owner probably decides to sell up her property and receives a lump sum payment at time 0. As a consequence, the rental reservation price for the owner is that amount which she is willing to regularly accept for renting out her property over a pre-specified period of time so that she is indifferent in the sense of expected wealth utility towards renting or selling this house. We assume that the agent's utility function also satisfies the constant absolute risk aversion (CARA) form

$$u_2(x) = -\frac{1}{\gamma_2}\exp(-\gamma_2 x), \tag{37}$$

where $\gamma_2 > 0$ represents her absolute risk aversion level.

To evaluate the owner's rental reservation price, suppose that if the owner sells her residential property at initial time 0, she receives a lump sum payment $P_0 = p$; if she chooses to rent out her property, her payoff will be composed of two components: the rental incomes derived from the tenant over the time period from 0 to T, and the resale price P_T of the given property at time T. Therefore, we have the following relationship:

$$p = \int_0^T L_2 e^{-rt} dt + CE_2(P_T, 0) \tag{38}$$

where L_2 denotes the owner's rental reservation price, and $CE_2(P_T, 0)$ represents the time-0 certainty equivalent of the property resale price.

As a result, we give the following proposition

Proposition 7: *Given the definition of the owner's rental reservation price, the rental reservation price is given by*

$$L_2 = \frac{r(p - CE_2(P_T, 0))}{1 - e^{-rT}}; \tag{39}$$

when there is a less perfect correlation between the housing price and stock price, the

rental reservation price can be found to be

$$L_2 = \frac{r}{1-e^{-rT}}\left(p + e^{-rT} \frac{\ln E\left[\psi_0 e^{-\gamma_2(1-\rho^2)P_T} \right]}{\gamma_2(1-\rho^2)} \right). \tag{40}$$

Proof: We can directly obtain (39) via a simple manipulation for (38); since

$CE_2(P_T,0)$ can be written as the form of (33), we can derive (40) by substituting

$CE_2(P_T,0)$ into (39).[‡‡‡‡]

One can readily find that the owner's rental reservation price have the same function

form as that of the tenant. For simplicity, suppose that the two agents have different

precautionary savings motives, but have common beliefs about the price evolutions of

the risk-free bond, and traded stock and house.[§§§§] As a result, let L_1 be the tenant's

rental reservation price, and if the rental transaction is carried out we might expect the

following inequality holds:

$$L_2 < \bar{L} < L_1, \tag{41}$$

where $\bar{L}$ is the market clearing rental price. This inequality suggests a range of

rental price within which both the agents are willing to enter into a rental contract. If

$L_2 > \bar{L}$, the owner would prefer to directly sell out her residential property rather than

enter into a rental transaction. If $L_1 < \bar{L}$, the tenant would choose to purchase this

house instead of renting.

Since inequality (41) specifies the upper and lower bounds for the market clearing

rental price, this implies that the market power of these two agents play a crucial role

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[‡‡‡‡] Alternatively, we can also directly solve the HJB equations analytically in order to price the owner's rental reservation prices.
[§§§§] A similar assumption can also be found in Wang (1996).

in determining the market clearing rental price. Let $Q \in [0,1]$ represent the owner's deterministic market power, that is, her ability to raise the market rental price independently. Consequently, the market clearing rental price can be expressed as follows

$$\bar{L} = QL_2 + (1-Q)L_1. \tag{41}$$

When the market power Q is equal to unity, the owner has a full market power such that the highest rental price that the tenant is willing to accept is the market clearing rental price. Reversely, when Q is equal to zero, the full market power is possessed by the tenant rather than the owner, and the market clearing rental price is therefore the lowest rental price that the owner is willing to accept. If $0 < Q < 1$, the market clearing rental price is determined by the interaction between supply and demand forces in the house rental market.

2.3 Other Considerations

The previous sections examine the choice problem by only focusing on the identical residential unit available to the buyer or renter and the effect of the non-institutional economic aspects. We can extend the examinations within the framework developed above by allowing the housing units with different characteristics and the impact of the maintenance costs and institutional factors such as the tax system.

Since individuals might have different investment and consumption demands for housing (Henderson and Ioannides, 1983; Fu, 1991), we incorporate the difference into the present dynamic portfolio model. Suppose that if the representative agent chooses to consume house services through homeownership, then he will buy one housing unit from this owner. However, if he prefers to acquire housing services only by renting, this agent will lease a κ-unit house due to his different demand for housing in the scenario.[*****] Under this consideration, his rental payment will be

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[*****] For simplicity, while a house unit can be described by many housing characteristics, we make use of κ to represent the consumption demand for housing, which can also be a vector of housing characteristics.

κL according to the previous discussion. As a result, his wealth dynamics now becomes

$$dX_t^L = \theta_t \frac{dS_t}{S_t} + r(X_t^L - \theta_t)dt - \kappa Ldt. \tag{45}$$

Following the same proving procedure of proposition 1, we may derive the value function for this agent under renting housing services

$$V(x,L,t) = -\frac{1}{\gamma}\exp\left(-\gamma(x - \frac{\kappa L}{r})e^{r(T-t)} - \frac{(\mu - r)^2(T-t)}{2\sigma^2}\right)\exp(-\gamma\frac{\kappa L}{r}). \tag{46}$$

Substituting (26) and (46) into (13), we find that the agent's rental reservation price can be rewritten as

$$L = \frac{r}{\kappa(1 - e^{-rT})}\left(p + e^{-rT}\frac{\ln E\left[\psi_0 e^{-\gamma(1-\rho^2)P_T}\right]}{\gamma(1-\rho^2)}\right). \tag{47}$$

This suggests that holding other things unchanged, the rental reservation price per unit is not necessarily increasing in the demand for renter-occupied housing, which usually reflects individuals' real consumption demand for housing.

In addition, we also investigate the effects of the maintenance costs and property tax factors, which usually distort house choice problem. If the agent chooses to rent a house, he does not require allowing for these effects. However, if he decides to purchase a house unit, then his wealth process and value function will differ from those discussed above due to these impacts. Suppose that both the property tax and maintenance costs be proportional to the price of the house, which are paid continuously for $0 < t < T$. Let c_1 and c_2 represent the property tax rate and maintenance cost, respectively, and then the wealth process follows

$$dY_t = \theta_t \frac{dS_t}{S_t} + r(Y_t - \theta_t)dt - (c_1 + c_2)P_t dt. \tag{48}$$

Correspondingly, the agent's value function can be defined as

$$U(y,p,t) = \max_{(\theta_s)_{t \le s \le T}} E[u(Y_T + (1-c_1-c_2)P_T) \mid Y_t = y, P_t = p]. \tag{49}$$

It can be shown that the value function satisfies the following expression

$$U(y,p,t) = -\frac{1}{\gamma}\exp\left[-\gamma y e^{(r(T-t))}\right]e^{-\frac{(\mu-r)^2}{2\sigma^2}(T-t)}\left[E\left(\psi_t e^{-\gamma(1-\rho^2)(1-c_1-c_2)P_T}\right)\right]^{\frac{1}{1-\rho^2}} \tag{50}$$

where

$$\psi_t = \exp\left[-\frac{1}{2}(\frac{(\mu-r)\rho}{\sigma})^2(T-t) - \frac{(\mu-r)\rho}{\sigma}(Z_T-Z_t)\right].$$

Substituting (50) and (17) into (13) yields the following rental reservation price

$$L = \frac{r}{\kappa(1-e^{-rT})}\left((1-c_1-c_2)p + e^{-rT}\frac{\ln E\left[\psi_0 e^{-\gamma(1-\rho^2)(1-c_1-c_2)P_T}\right]}{\gamma(1-\rho^2)}\right).$$

It can be shown that the effects of changes in the property tax rate, c_1, and maintenance cost, c_2, on the rental reservation price are unexpected. An increase in these two parameters discourages the agent from buying this house, and therefore enhances his rental reservation price. However, when the expected price growth on this property is high enough to compensate the adverse impact, the increase cannot lower his rental reservation price.

2.4 The Model Predictions

The model and its extensions generate interesting implications for the relations among housing rent, the housing risk, the co-movement of the assets and investment. In the complete market, equation (17) shows that in the presence of house investment, the agent's value is a function of rental payments $\frac{L}{r}$ (decreasing function) and further adjusted by a risk aversion-related term $\exp(-\gamma\frac{L}{r})$. Both equation (21) and (22) show that the rental reservation price is a function of the interest rate, rental term, expected return and volatility of the traded stock, expected growth and volatility of the housing

price.

These equations predict the following. The rental reservation price *L* decreases in the expected return on the property and the volatility of the stock price, but increases in the volatility of the property return and expected return on the traded stock. Higher expected stock returns make investing in the traded stock more attractive than investing in the house and therefore raises the agent's willingness to rent, while increasing stock volatility decreases this willingness. The above relations generate the following hypotheses for household investment:

H1: The rental reservation price is negatively associated with expected return on the property and positively associated with the volatility of the property return.

H2: The rental reservation price is negatively associated with the volatility of the stock price and positively associated with the expected return on the traded stock.

In the incomplete markets, both equation (27) and (28) suggest that the value of housing rental contracts is further dependent on the agent's precautionary savings motive and the co-movement between the traded stock and house. Equation (36) shows that the idiosyncratic risk premium increases with the agent's precautionary savings motive, hence the rental price. Although investors chooses to rent housing services, the idiosyncratic risk premium is still a remarkable factor determining their house rental reservation prices due to the effect of their exposure to resale price risk in the housing market in owning a house. However, the effect of varying correlation between the two risky assets on the lease reservation price seems ambiguous, it is shown that the lease reservation price increases in the correlation when the market price of risk $\frac{(\mu-r)}{\sigma}$ satisfies $\frac{(\mu-r)}{\sigma} > \frac{1}{2}\gamma\upsilon$. Hence empirically we hypothesize:

H3: The rental reservation price is positively associated with the idiosyncratic risk premium.

H4: The rental reservation price is positively associated with the correlation between the two risky assets.

3 The sample

We empirically test the model predictions using the data from several major Asian markets. We focus on the following Asian markets: Taiwan, Hong Kong, Korea, and Singapore, given the transparency and completeness of these real estate markets. Also, unlike the U.S., these countries or regions have made less preferential tax policies associated with homeownership. The sample to be used is from 1994 to 2010. The housing data are collected from their respective statistics bureaus, and macro data are retrieved from DataStream.

To test the model's implications on the rental price, we regress the rental price on the assets attributes and macro characteristics in the panel:

$$Rental\ Price_{it} = a_i + b * AssetAttributes_{it} + c * controls_{it-1} + w_{it} \qquad (51)$$

where *RentalPrice* is the rental price collected from Statistics Bureau of respective region reported on quarterly basis (in logarithm), a_i is region fixed effect, and *AssetAttributes* takes *Property Volatility, Property Return, Stock Volatility, Stock Return, Asset Co-movement,* and *Idiosyncratic Risk Premium* respectively.

We measure *Property Volatility* as the standard deviation of the housing price within the quarter, a time-series measure for the volatility of the property return. In similar vein, we calculate the standard deviation of the stock price index within the quarter and define it as the volatility of the traded stock return (*Stock Volatility*). We use the realized return in the next quarter as the proxy for the expected return on the property (*Property Return*) and the traded stock (*Stock Return*). We include the contemporary housing price (*Property Price*) as the proxy for the initial housing price in the model,

since the model shows that the rental reservation price is not directly dependent on the current housing price P_T, but determined by the initial housing price.

To measure the varying correlation between different assets, we calculate the correlation on a quarterly basis between the housing price index returns and stock index returns for each region using monthly observations, named *Asset Co-movement*.

To capture the idiosyncratic risk premium, we use quarter-specific unexpected fluctuation to capture the time-series variation in the idiosyncratic risk of households. To do so, we first orthogonalize the excess returns of housing price index returns to the excess market returns:

$$R_{re,t} - R_f = \alpha_0 + \alpha_1(R_{mk,t} - R_f) + \varepsilon_t$$

(20)

where R_{re} is the return on the housing price index for each region. R_{mk} is the stock price index return for each region. Both index returns are measured in excess of the risk-free rate R_f on the 1-month Treasuries for the respective region. The regression is conducted with monthly data over the full sample period, 1994 to 2010. The estimated residuals are the housing market specific returns orthogonal to the stock market returns, which we define as the idiosyncratic risk premium (*Idiosyncratic Risk Premium*). The estimated coefficient on the idiosyncratic risk premium from this regression, measures the change of rental prices in response to the increase in the idiosyncratic risk premium of the households.

Macro control variables include *GDP, Interest Rate, CPI,* and *Consumption. GDP is the* log difference of gross domestic product (GDP), *Interest Rate* is the 3-month deposit rate, *CPI* is the log difference of CPI, and *Consumption* is the log difference of private consumption.

In the robustness check, we analyse the impact of assets attributes on the rental price using the vector autoregressive model for each region and estimate the impulse response function of different asset attribute to the rental prices.

4 Empirical Results

This section reports the empirical evidence on rental price, the results of which overall support the model's predictions on the determinants of the rental price.

In Figure 4, we plot the housing rent and housing price for HongKong, Korea, Singapore and Taiwan. All figures show a salient shift around the Asian financial crisis. Both housing price volatility and rental price volatility before 2000 are relatively smaller than it is afterwards in Korea and Taiwan, vice versa for Hong Kong and Singapore. The house price in all these regions fluctuated obviously. However, it is noteworthy that the average housing rent did not fluctuate as much as the housing price. This reinforces the importance of house renting in hedging against housing price uncertainty.

[Insert Figure 4]

Table 1 reports the results on how the property attributes affect the rental prices. Column (1) reports the results with the simplest estimation with only control variables, and they explain about 11% of rental prices. Column (2) shows that the contemporary property prices, as a proxy for the initial housing price, is significantly and positively associated with the rental price. For each 1% increase in the housing price, the rental price is further increased by 41.6%. Column (3) shows that the volatility of the property return (*Property Volatility*) is significantly and positively associated with the rental prices. When the volatility of the property return increases by 1%, the rental prices are shrunk by 2.75%. In column (4), the expected property return (*Property Return*) is significantly and negatively associated with the rental prices. For each 1% of increase of the expected property return, the rental prices decreases by 13.7%.

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Overall, the results in Table 1 support the hypotheses that the rental reservation price is negatively associated with expected return on the property and positively associated with the volatility of the property return. (H1).

[Insert Table 1]

Table 2 reports the results on how the asset attributes affect the rental prices. Column (1) and Column (2) show that the rental reservation price is negatively associated with the volatility of the stock price and positively associated with the expected return on the traded stock (H2). The coefficients are all significant at 95% confidence level. For each 1% of increase of the volatility of traded stock return, the rental prices decreases by 10%. For each 1% of increase of the traded stock's expected return, the rental prices increases by 2.9%. If the investor decides to rent house services rather than purchase, his wealth portfolio is composed of those liquid financial assets, and he is therefore not exposed to the house price risk. The results show that the investor' spot rental reservation price is an increasing function of the expected return on traded risky assets, but a decreasing function of traded asset volatility. This is possibly because of the effective hedging of house resale price risk. In Column (3), we further add the correlation between house and traded stock into the specification, which shows a significant and positive impact on the rental prices. Finally, in Column (4), *Idiosyncratic Risk Premium* is found to be significantly and positively associated with the rental prices. For each 1% of increase of the idiosyncratic risk premium, the rental prices increases by 2.6%. This is consistent with the agent's precautionary savings motive, as the idiosyncratic risk premium increases with the agent's precautionary savings motive, hence the rental price.

Overall, the results in Table 2 support the hypotheses that the rental reservation price is dependent on the asset attributes like the traded stock volatility, the varying correlation between the assets class and the idiosyncratic risk (H2, H3, H4).

[Insert Table 2]

The robustness tests discussed in the previous Section generate similar results to those reported, which are available upon request.

5 Conclusions

This paper proposes a utility indifference-based model for analyzing the joint decisions of household portfolio selection, house price risk hedging and housing rental behavior under asset price uncertainties. We obtain closed-form solutions to the optimal problem and carry out comparative static analysis based on the solutions. Our model can provide interesting insights into the joint decisions and testable predictions on the determination of housing rental prices.

Our results show that the investor' rental reservation price is an increasing function of the expected return on traded risky assets, but a decreasing function of traded asset volatility. However, the varying expected return on residential real estate is found to have a reverse impact on the rental reservation price, while the effect of increasing real estate volatility is shown to be positive. In particular, we find that higher precautionary savings motive makes this investor enhance his rental reservation price, while the effects of changes in the correlation between the traded asset and residential real estate on the reservation price are ambiguous. In addition, it is also shown that idiosyncratic risk premium in the housing market is also a major consideration for the investor to determine rental reservation price, while under the situation of renting housing services he is not faced with idiosyncratic property risk. Moreover, our model is also extended to allow for the impacts of the maintenance costs and institutional factors such as the tax system, which can distort the above findings. Neglecting the impacts can result in an inaccurate forecast for the housing rental prices. Finally, we empirically verify the model's predictions using the panel data from several Asians

market. The empirical evidence supports all the predictions from the model.

This study contributes to understanding of the determination of housing rental prices and household portfolio choices due to considering optimal household portfolio and the effective hedging of house resale price risk. When an investor is indifferent between owing and renting a house, he can choose to consume housing services by owning, and stock and bond investments are correspondingly crowed out in his wealth portfolio. This investor is also exposed to house price risk. Very little is known about how the household hedges the house price risk. Our paper fills the gap with both theoretical modeling and empirical evidence.

Appendix A

Proof of Proposition 1

To solve HJB equation (16) analytically, suppose that the solution form of (16) can be expressed as

$$V(x,t) = \bar{V}(x,t)\psi(t), \tag{A1}$$

where $\bar{V}(\cdot,\cdot)$ is defined by (6), which is a solution to the following HJB equation

$$V_t + rxV_x - \frac{V_x^2(\mu-r)^2}{2V_{xx}\sigma^2} = 0 \tag{A2}$$

subject to $V(x,T) = -\frac{1}{\gamma}\exp(-\gamma x)$.

By substituting (A1) into (16), we can obtain

$$\bar{V}\psi' - L\bar{V}_x\psi + \psi\bar{V}_t + rx\bar{V}_x\psi - \frac{\psi\bar{V}_x^2(\mu-r)^2}{2\bar{V}_{xx}\sigma^2} = 0 \tag{A3}$$

Since $\bar{V}$ solves (A2), the third, fourth and fifth terms on the left side of equation (A3) can be cancelled out by each other. Consequently, we find that ψ satisfies the following relationship:

$$\frac{\psi'}{\psi} = L\frac{\bar{V}_x}{\bar{V}} = -L\gamma\exp(r(T-t)), \tag{A4}$$

which can be easily solved using the technique of separation of variables. The solution of ordinary differential equation (A4) is given by

$$\psi = c\exp(\gamma\frac{L}{r}e^{r(T-t)}) \tag{A5}$$

where c is a constant, which is determined by the final condition of (16)

$$V(x,T) = -\frac{1}{\gamma}\exp(-\gamma x).$$

Substituting the resultant ψ into (A1) produces (17).

<div align="right">Q.E.D.</div>

Appendix B

Proof of Proposition 3

To derive an analytical solution to HJB equation (25), we follow Herson's (2005) approach and write the value function as

$$U(Y_t, P_t, t) = -\frac{1}{\gamma} \exp\left(-\gamma Y_t e^{(r(T-t))}\right) g(P_t, T-t) \tag{B1}$$

Since $U(Y_t, P_t, t)$ is a martingale under the optimal strategy, applying Ito's formula to the above expression produces

$$g_t - (a - \frac{(\mu - r)b\rho}{\sigma})g_p - \frac{1}{2}b^2 g_{pp} + \frac{(\mu - r)^2}{2\sigma^2}g + \frac{b^2\rho^2 g_p^2}{2g} = 0 \tag{B2}$$

where $g(p, 0) = e^{-\gamma p}$. Let the solution of equation (B2) be of the following form

$$g(p, T-t) = v(p, T-t)^\beta e^{\alpha t} \tag{B3}$$

where $\alpha = -\frac{(\mu - r)^2}{2\sigma^2}$ and $\beta = \frac{1}{1 - \rho^2}$. Substituting (B3) into (B2) reduces the

nonlinear partial differential equation into a linear one as follows

$$v_t + \left(a - \frac{(\mu - r)b\rho}{\sigma}\right)v_k + \frac{1}{2}b^2 v_{kk} = 0 \tag{B4}$$

where $v(p, T) = \exp\left(-\gamma p(1 - \rho^2)\right)$. Suppose that all the coefficients are smooth enough and satisfy the regularity assumptions, and then by Feynman-Kac theorem we can find the unique solution to (B4)

$$v(p, t) = E[\psi_t e^{-\gamma(1 - \rho^2)P_T}] \tag{B5}$$

where $\psi_t = \exp\{-\frac{1}{2}(\frac{(\mu - r)\rho}{\sigma})^2(T - t) - \frac{(\mu - r)\rho}{\sigma}(Z_T - Z_t)\}$. Substituting the above results back to (B1), we can readily obtain the analytical solution to HJB equation (25).

Q.E.D.

Appendix C

Given $L = \dfrac{r}{1-e^{-rT}}\left[(1-e^{-rT})p + e^{-rT}\left((\dfrac{\mu-r}{\sigma})\upsilon T - \alpha T\right)\right]$, we have the following exact

partial derivatives

$$\frac{\partial L}{\partial \alpha} = -\frac{rTe^{-rT}}{1-e^{-rT}} < 0$$

$$\frac{\partial L}{\partial \upsilon} = \frac{rT(\mu-r)e^{-rT}}{\sigma\left(1-e^{-rT}\right)} > 0$$

$$\frac{\partial L}{\partial \mu} = \frac{rTe^{-rT}}{\sigma\left(1-e^{-rT}\right)} > 0$$

$$\frac{\partial L}{\partial \sigma} = -\frac{rT\upsilon(\mu-r)e^{-rT}}{\sigma^2\left(1-e^{-rT}\right)} < 0$$

$$\frac{\partial L}{\partial p} = r > 0.$$

Appendix D

Given (34), we can derive the following exact partial derivatives

$$\frac{\partial L}{\partial \alpha} = -\frac{rTe^{-rT}}{1-e^{-rT}} < 0$$

$$\frac{\partial L}{\partial \upsilon} = \frac{re^{-rT}}{1-e^{-rT}} \left[\frac{\mu - r}{\sigma} \rho^2 T + \gamma \left(1 - \rho^2\right) \upsilon T \right] > 0$$

$$\frac{\partial L}{\partial \mu} = \frac{r\rho^2 Te^{-rT}}{\sigma\left(1 - e^{-rT}\right)} > 0$$

$$\frac{\partial L}{\partial \sigma} = -\frac{r\rho^2 \upsilon T(\mu - r)e^{-rT}}{\sigma^2 \left(1 - e^{-rT}\right)} < 0$$

$$\frac{\partial L}{\partial p} = r > 0$$

$$\frac{\partial L}{\partial \gamma} = \frac{re^{-rT}}{1-e^{-rT}} \left[\frac{1}{2}\left(1 - \rho^2\right)\upsilon^2 T \right] > 0$$

$$\frac{\partial L}{\partial \rho^2} = \frac{re^{-rT}}{1-e^{-rT}} \left[\frac{(\mu - r)}{\sigma}\upsilon T - \frac{1}{2}\gamma\upsilon^2 T \right] \gtrless 0.$$

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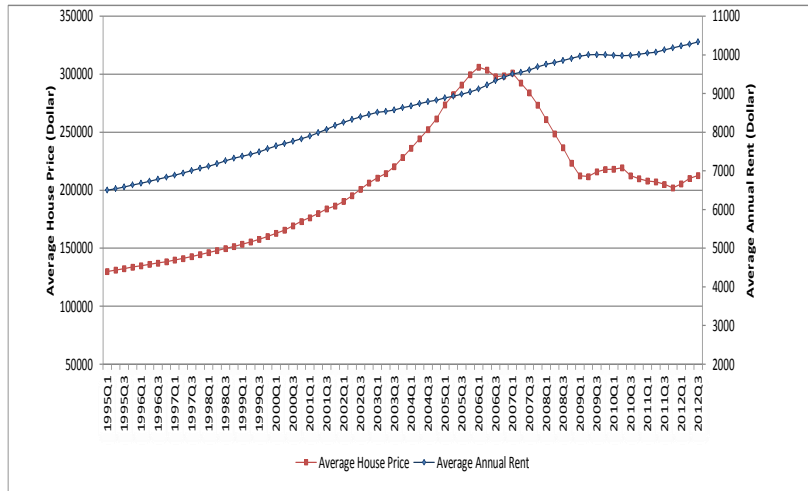
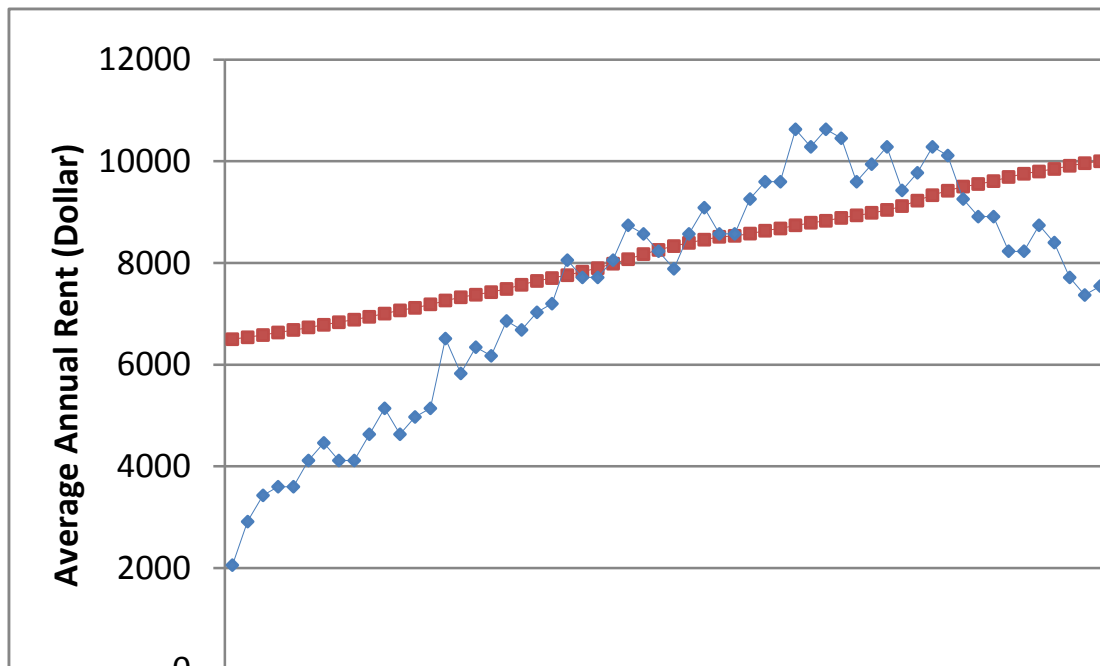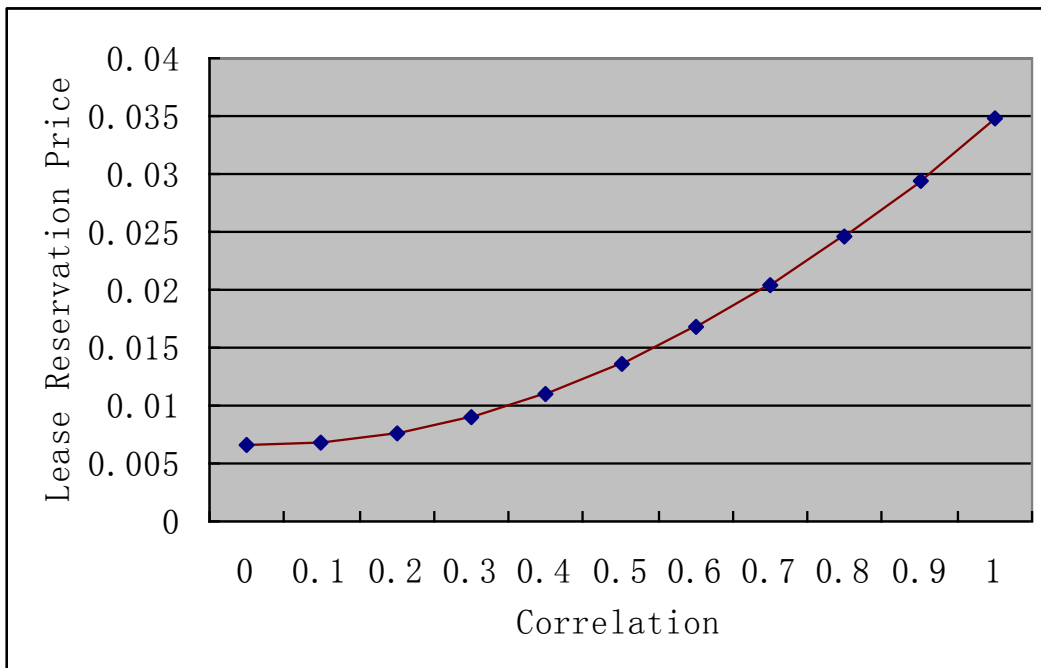Figure 1 Average House Price vs. Average Annual Rent

Figure 2 Average Annual Rent vs. Homeownership Rate
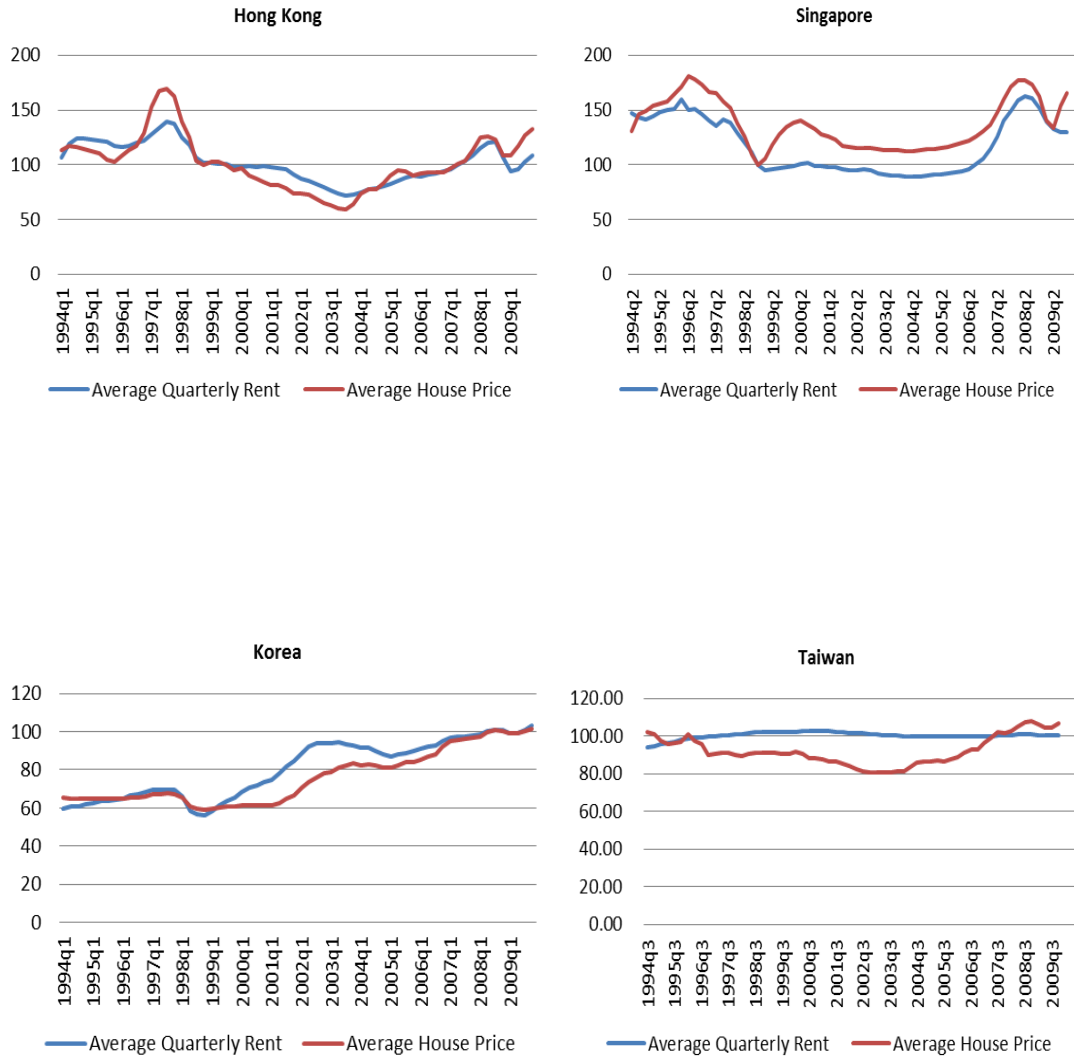
Figure 3. Lease Reservation Price vs. Correlation

Note: The relevant parameter values are r=0.04, p=1, T=1, μ=0.07, σ=0.08, α=0.08, v=0.2, γ=1.

Figure 4 Average House Price vs. Average Annual Rent in major Asian Markets

Hong Kong

Singapore

Korea

Taiwan

Table 1 The property attributes and the rental prices

This table presents the relation between the property attributes and the rental prices. The dependent variable is *Rental Price*, measured as the average rental price from Statistics Bureau of respective region reported on quarterly basis (in logarithm). The variables of interest are *Property Volatility, Property Return, and Property Price*. The independent variables are *GDP, Interest Rate, CPI,* and *Consumption*.*, ** and *** represent the 10%, 5% and 1% significance levels, respectively. Coefficients and standardized coefficients (elasticity) for the variables of interest are presented in sequence, and T-statistics are included in parentheses.

	Model Predictions	(1)	(2)	(3)	(4)
Property Volatility	+			0.0275***	
				(7.28)	
Property Return	**-**				-0.137***
					(-3.06)
Property Price	+		0.416***		0.495***
			(12.28)		(11.25)
Interest Rate		0.0149***	0.0127***	0.0136***	0.0103**
		(2.85)	(3.07)	(2.86)	(2.49)
GDP		0.104	0.101**	0.0830	0.119**
		(1.64)	(2.01)	(1.43)	(2.40)
CPI		-0.00141	-0.00189	-0.00121	0.184
		(-0.49)	(-0.82)	(-0.46)	(1.19)
Consumption		0.207***	0.115*	0.180**	0.131**
		(2.68)	(1.86)	(2.55)	(2.10)
Constant		Yes	Yes	Yes	Yes
Region fixed effect		Yes	Yes	Yes	Yes
No. of Obs		261	261	257	257
r2		0.124	0.449	0.280	0.472
Adjusted r2		0.110	0.438	0.265	0.459
F stat		9.027	41.58	19.48	37.26

Table 2 The assets attributes and the rental prices

This table presents the relation between the property attributes and the rental prices. The dependent variable is *Rental Price*, measured as the average rental price from Statistics Bureau of respective region reported on quarterly basis (in logarithm). The variables of interest are *Stock Volatility, Stock Return, Asset Co-movement,* and *Idiosyncratic Risk Premium*. The independent variables are *GDP, Interest Rate, CPI,* and *Consumption.**, ** and *** represent the 10%, 5% and 1% significance levels, respectively. Coefficients and standardized coefficients (elasticity) for the variables of interest are presented in sequence, and T-statistics are included in parentheses.

	Model Predictions	(1)	(2)	(3)	(4)
		*Rental Price*			
Stock Volatility	**-**	-0.100**			
		(-2.21)			
Stock Return	+		0.0290**		
			(2.39)		
Co-movement	+			0.0444**	
				(2.29)	
IdioRisk	+				0.0258***
					(5.60)
Interest Rate		0.0131**	0.0146***	0.0176	0.0175
		(2.50)	(2.82)	(0.48)	(0.50)
GDP		0.0992	0.0979	-0.0560	0.0309
		(1.58)	(1.56)	(-0.12)	(0.07)
CPI		-0.000854	-0.00171	0.00683	0.00517
		(-0.30)	(-0.59)	(0.33)	(0.26)
Consumption		0.213***	0.195**	-1.370**	-1.494***
		(2.76)	(2.53)	(-2.49)	(-2.85)
Constant		Yes	Yes	Yes	Yes
Region fixed effect		Yes	Yes	Yes	Yes
No. of Obs		261	261	257	257
r2		0.140	0.143	0.0532	0.141
Adjusted r2		0.123	0.126	0.0343	0.124
F stat		8.304	8.497	2.819	8.231