

Credit Rating Inflation and Firms' Investment Behavior

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Abstract

Why do potentially inflated credit ratings affect rational, well-informed creditors' bond investment decisions? What are credit ratings' effects on firms' behavior? We resolve these questions by modeling a credit rating agency (CRA) as a certified expert who transmits information to creditors with coordination incentives and dispersed beliefs. In the unique equilibrium, high credit ratings are potentially inflated but serve as public signals that truncate supports of creditors' posterior beliefs, owing to corporate credit ratings' partial verifiability. Such public signals change well-informed investors' decisions, because of their coordination incentives and dispersed beliefs. Depending on the firm's fundamentals, inflated ratings may mitigate or aggravate the firm's moral hazard. We show that if the CRA commits to a rating strategy, the optimal committed rating strategy is time-consistent. Both CRAs' rating standards and credit rating inflation are endogenous in our model, and we show that laxer rating standards are not necessarily accompanied by higher rating inflation. Ultimately, we discuss how certain policies, such as verifying the firm's investment and providing precise unbiased public signals, can regulate the credit rating industry and mitigate CRAs' adverse effects.

Keywords: Credit rating agency, rating inflation, real effect, persuasion, global game

JEL Classification Codes: D82, D83, G24, G32

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1 Introduction

Credit rating agencies (CRAs) have come under intense scrutiny in the wake of the 2007 – 2009 financial crisis. They are accused of knowingly assigning overgenerous ratings to both financial products, such as mortgage-backed securities (MBSs), and corporates, including financial institutions that played critical roles in the financial crisis (Partnoy, 2009).¹ These criticisms of CRAs center on the conflicts of interest caused by the “issuer-pays” business model prevailing in the credit rating industry: because CRAs are paid by issuers of securities they are assessing, they will have strong incentives to assign overgenerous ratings, in order to charge higher fees and attract new clients. The overgenerous credit ratings are usually called *credit rating inflation*, as they stand for better credit qualities than issuers actually have.

Despite of the well-known potential rating inflation, corporate credit ratings have been significantly affecting firms’ behavior (see, for example, Kisgen (2006), Sufi (2009), and Bannier, Hirsch, and Wiemann (2012)). Researchers attribute corporate credit ratings’ effects to investors’ imperfect rationality or their lack of information.² However, investors are now much better educated about credit ratings, and they can access information from various sources. Indeed, SEC and even CRAs themselves keep emphasizing that investors should make investment decisions mainly based on their own private information and use credit ratings as secondary opinions only. Then in an environment with rational investors who have precise information, it is less obvious why and how potentially inflated credit ratings can still affect investors’ decisions and thus affect firms’ behavior.

In this paper, we analyze corporate credit ratings’ effects on rational, well-informed investors’ investment decisions and on firms’ behavior. We model a CRA as a certified expert who transmits information about a firm to a continuum of creditors who are deciding whether to roll over the

¹The Financial Crisis Inquiry Report concludes that “the failures of credit rating agencies were essential cogs in the wheel of financial destruction.” Empirical studies have documented inflated credit ratings assigned to mortgage-backed securities, for example, Griffin and Tang (2012) and He, Qian, and Strahan (2012). Corporate credit rating inflation is also significant. Jiang, Stanford, and Xie (2012) show that Standard and Poor’s (S&P) assigns higher bond ratings after it switches from the “investor-pays” business model to the “issuer-pays” business model. By comparing ratings by CRAs adopting “issuer-pays” business model and those adopting “investor-pays” business model, Strobl and Xia (2012) and Cornaggia and Cornaggia (2013) demonstrate that the conflicts of interest lead to credit rating inflation.

²Some studies also argue that corporate credit ratings can make effects through regulations tied to ratings. For example, pension funds and insurance companies cannot invest in securities, unless the securities are assigned high ratings by “nationally recognized statistical rating organizations” (NRSROs). But while high ratings may allow firms to access more financial markets, whether credit ratings can encourage investors in those markets to invest still depends on the ratings’ informational role.

firm's short-term debts; the aggregation of creditors' rollover decisions will determine the firm's financial cost, which in turn determines the firm's investment project choice; different projects have different credit qualities, so the firm's project choice will affect the CRA's ratings. Hence, our model features feedback effects between corporate credit ratings and firms' creditworthiness through creditors' responses to the credit ratings.

Our model captures two critical features of corporate credit ratings. First, because some of a firm's decisions are observable and verifiable, credit ratings are subject to a *partial verifiability constraint*. Specifically, we assume that after observing the financial cost, which is determined by the firm's fundamentals and the aggregation of creditors' rollover decisions, the firm may strategically opt for an early default, or invest either in a viable project (VP) with a high success probability and a positive expected NPV, or a risky project (HR) with a negative expected NPV due to a low probability of success. Importantly, whereas the public can observe a firm's early default, its choice of VP or HR is hidden and unverifiable. Therefore, the firm's early default will be the evidence against the CRA in any lawsuit or cause huge reputation costs to the CRA, if the CRA assigns a high credit rating to the firm. Hence, the CRA would never assign a high rating when it knows that the firm will default early. Put differently, if the CRA assigns a high rating, the public can infer that the firm's fundamentals are not extremely bad, although a high rating cannot warrant an investment in VP because of the possible credit rating inflation.

Second, a firm's short-run debt market features coordination incentives among creditors, as emphasized by Boot, Milbourn, and Schmeits (2006). If more creditors choose to roll over, the firm's financial cost is lower, leading to a higher probability that the firm will invest in the viable project and have a higher credit quality. This will cause more creditors to roll over.

Our model has a unique equilibrium. In the equilibrium, the conflicts of interest imply that the CRA always employs a lax rating strategy, which assigns all firms, which invest in HR, the credit rating standing for the VP's credit quality. However, high credit ratings generated by the lax rating strategy are not purely cheap talks. Due to the partial verifiability constraint, a high credit rating provides creditors with a public signal about the firm. This public signal is endogenous and takes a different form from that in Morris and Shin (2002): it truncates the supports of creditors' interim beliefs from below. This will shift up creditors' beliefs about the firm's fundamentals, no matter how precise investors' private signals are. Although such a belief shift may not change any separate individual creditor's decision as she possesses very precision information, it causes more creditors to roll over in our model because of creditors' coordination incentives.

Although a high credit rating delivers a positive public signal to creditors and thus lowers the firm's financial costs, it does not necessarily have positive real effects. Indeed, we uncover credit

ratings' non-monotonic effects on the social welfare.³ As the firm's fundamentals improve from a very bad starting point, the CRA assigns a low rating, leading all creditors to run. This will prompt the firm's efficient default, which is socially optimal when the firm's fundamentals are bad. When the firm's fundamentals increase above a threshold, the CRA will assign a high rating. Then, firms with fundamentals just above the threshold will invest in HR, instead of defaulting early when there is no CRA; but, firms with some even better fundamentals will invest in VP, instead of HR when there is no CRA.

The real effects of corporate credit ratings are consequences of credit ratings' effects on the firm's moral hazard, which result from their effects on creditors' investment decisions. When there is no CRA, some firms' financial costs will be so high that they will choose to default early; but, with a good credit rating, more creditors roll over the debts, so those firms' financial costs decrease such that they want to take risks rather than default early. That is, the CRA provides firms with incentives to "gamble for resurrection."⁴ This adverse effect is a consequence of the firm's aggravated moral hazard. Similarly, when there is no CRA, some firms' financial costs will be so high that they will choose the HR; but when they receive a high credit rating, their financial cost decreases, leading to the "risk-shifting" investment from the HR to the VP.

If the CRA commits to an increasing rating strategy before knowing the firm's fundamentals, the optimal committed rating strategy is the same as the equilibrium rating strategy and thereby time-consistent. Such a time-consistency property comes from the CRA's aim to maximize the average credit ratings subject to the partial verifiability constraint. Because of the time-consistency property, we can view the CRA as a certified expert who tries to persuade creditors to roll over the debt, so that the firm can enjoy a lower financial cost. As a result, credit ratings can be viewed as "straightforward signals" (Gentzkow and Kamenica, 2011), with the low credit rating meaning "run" and the high credit rating meaning "rollover."

In our setting, both the CRA's rating strategy (or rating standard) and the rating inflation are endogenous, and we show that laxer rating standards are not necessarily accompanied by higher rating inflation. This is due to the feedback effect between of credit ratings and the firm's investment and dramatically different from credit ratings of MBSs, where laxer credit rating standards will lead to higher credit rating inflation. We show this claim in several comparative

³Because the VP has a positive expected NPV, and the HR has a negative NPV, the VP is the socially optimal investment, whereas the HR leads to the lowest ex-ante social welfare.

⁴Freixas, Parigi, and Rochet (2004) use the term "gambling for resurrection" to reference the behavior of "zombie" savings and loans during the 1980s U.S. S&L crisis. A famous business story about "gambling for resurrection" regards Fred Smith, the founder of Fedex, who took the company's last \$5000 to Las Vegas and won necessary money for the life of the nowadays successful Fedex.

static analyses, which also provide us with new empirical predictions about CRAs' credit rating standards and credit rating inflation. In particular, a decrease in the firm's transparency, or an increase in upside returns of risky projects will lead to laxer rating standards (assigning high ratings to more firms). These are consistent with the observations in the financial crisis: CRAs employ laxer rating standards for financial institutions, which were lack of transparency and took risky investments. We also show that a decrease in the measure of creditors who encounter liquidity shocks will also lead to laxer rating standards. However, such changes of economic environments do not necessarily cause higher rating inflation. In fact, we show that a decrease in the firm's transparency has an ambiguous effect on rating inflation, an increase in upside returns of risky projects will cause higher rating inflation, and a decrease in the measure of creditors who encounter liquidity shocks leads to lower rating inflation.

Analyzing how corporate credit ratings affects well-informed investors' decisions and thus firms' behavior helps shed light on credit rating industry regulation. According to our model, the CRA's adverse effects result from both the partial verifiability and creditors' belief dispersion, so potential policies that eliminate one of these two conditions may help mitigate the CRA's adverse effects. We describe two possible policies that may mitigate the CRA's adverse effects by dealing with the partial verifiability and the belief dispersion, respectively. The first possible policy is to verify the firm's investment choice; this is an ex-post monitoring method that affects the CRA's incentives. Because the CRA must avoid being caught lying, such a policy can force the CRA to be "self-disciplined." We show that a "self-disciplined" CRA leads to multiple equilibria. In a certain equilibrium, the CRA can help prevent the firm taking HR, which maximizes social welfare.

A second potential regulatory policy is to provide a precise, unbiased public signal about the firm's fundamentals, which is an ex-ante information method that weakens the CRA's informational effects. This shares some similar spirits with Fong, Hong, Kacperczyk, and Kubik (2014), who find that security analysts may discipline CRAs. If unbiased public signals are sufficiently precise, they can resolve investors' coordination problem: creditors can confidently rely on the public signals to make investment decisions, because they know that other creditors are also rely on such public signals to make investment decisions.

Our paper is closely related to the pioneer works about the feedback between credit ratings and the firm's behavior by Boot, Milbourn, and Schmeits (2006) and Manso (2013). Boot, Milbourn, and Schmeits (2006) uncover the CRA's coordination role when the firm has moral hazard problems. In their model, a credit rating sets a focal point for equilibrium selection when there exist multiple equilibria in the game between the firm and investors. Manso (2013) analyzes the feedback effect between the firm's endogenous default decision and the CRA's ratings. In these

two models, there is no asymmetric information; therefore, the CRA does not play an information role. Moreover, CRAs in these two models always want to provide accurate credit ratings, so there is no conflict of interests between CRAs and investors. Our paper contributes to this literature by studying credit ratings' informativeness and real effects when there are information asymmetries, and conflicts of interest may cause CRAs to provide overgenerous credit ratings. Furthermore, in Boot, Milbourn, and Schmeits (2006), firms default exogenously only and in Manso (2013), firms default endogenously only. But in our paper, firms may default both endogenously and exogenously, which generates credit ratings' partial verifiability constraint.

Our paper also contributes to the literature about the CRA's rating inflation. In Bolton, Freixas, and Shapiro (2012), firms' credit shopping, combined with the existence of a large number of naive investors who take credit ratings at face value, causes rating inflation. Skreta and Veldkamp (2009) assume that investors do not take into account that issuers "shop for ratings," so although any individual CRA's rating is not necessarily inflated, the reported rating is inflated. In these two papers, investors' irrationality plays an important role in credit ratings' effects. Frenkel (2014) uncovers the "double reputation" of CRAs: a public reputation for accurate information, and a private one for pleasing issuers. He assumes that investors solely rely on credit ratings to make investment decisions, and concludes that the private reputation for pleasing issuers causes rating inflation. Opp, Opp, and Harris (2013) attributes rating inflation and ratings' effects to ratings-based regulation in a rational framework. In all these papers, the CRA's effects are due to investors' bounded rationality, their lack of information, or the regulations tied to ratings. Our paper complements to this literature by analyzing CRAs' roles in environments where investors are purely rational and have very precise private information. We also contribute to this literature by focusing on the feedback between credit ratings and firms' credit qualities. Importantly, for corporate credit ratings, laxer rating strategies are not necessarily accompanied with higher credit rating inflation, because both corporate credit ratings and credit rating inflation are endogenously determined.

We model a CRA as a certified expert who tries to persuade creditors to roll over a firm's short-run debts. This is an important application of theoretical models about disclosure and persuasion (see, for example, Gentzkow and Kamenica (2011, 2014)). In particular, credit ratings' partial verifiability constraint is an important application of the verification assumption in general Bayesian persuasion theories, which means that experts' claims may be more or less informative, but cannot be verified to be false. In addition, the equilibrium credit rating strategy is actually a straightforward information transmission mechanism, with a low rating meaning a recommendation of run and a high rating meaning a recommendation of rollover. Our paper contributes to

this literature by analyzing a persuasion problem, when there are multiple audiences who have coordination incentives and dispersed private beliefs. Our paper also contributes to experts' disclosure literature. Lizzeri (1999) considers the information manipulation of perfectly informed information intermediaries. He shows that a monopoly intermediary's optimal disclosure choice is to reveal that the quality remains above a minimum standard. Our conclusion shares the same spirit: a high rating does not mean the firm will invest in VP, but implies that the firm will not default early. However, the lowest type of the firm that receives a high rating is endogenously determined by how the creditors interpret the inflated rating.

In our model, investors play a global game (Carlsson and van Damme (1993) and Morris and Shin (2003)). Our model differs from the traditional global games mainly in that the creditors' dominant region of run is endogenous, which is determined by the CRA's rating strategy. In fact, off the equilibrium path, if the CRA employs an extremely conservative rating strategy, the firm will choose VP even if no creditor roll over the short-term debt. That is, if there is absolutely no rating inflation, the dominant region of run disappears. Our model's inclusion of endogenous information relates it to the growing literature of global games with endogenous information (see, for example, Angeletos, Hellwig, and Pavan (2006, 2007) and Huang (2014)). Nevertheless, our model has a unique equilibrium, because a high credit rating, the endogenous information, does not completely remove creditors' dominant region of run.

We organize the rest of the paper as follows. In Section 2, we describe our model. In Section 3, we analyze a benchmark model without a CRA and derive the social welfare in the benchmark model. Section 4 analyzes the informativeness and real effects of corporate credit ratings. Section 5 shows that the optimal committed rating strategy is time-consistent, if the CRA commits to a rating strategy before knowing the firm's fundamentals. In Section 6 and Section 7, we show the empirical implications and policy implications, respectively. Section 8 concludes. All proofs appear in the Appendix.

2 A Model of corporate credit ratings

We study a model of a CRA that is assigning credit ratings to a firm who needs to roll over short-term debt to continue its investment in a long-term project. There are three dates, $t = 0, 1, 2$. At the beginning of date 0, the firm's outstanding short-term debt is mature, so it needs to repay \$1 to each of its creditors. At date 0, the CRA assigns credit ratings to the firm. Observing the CRA's rating, creditors simultaneously decide whether to roll over the debt or to run. At date 1, depending on the financial cost, the firm may choose to default or to continue investing. In

the latter case, the firm needs to finance the deficit by withdrawing money from a pre-committed bank credit line. If the firm decides to continue investing, the cash flow is realized at date 2, and, if possible, creditors are paid in full.

2.1 Investments

The firm’s existing investment generates an intermediate asset at the beginning of date 0. The intermediate asset’s liquidation value is \$1. Following Boot, Milbourn, and Schmeits (2006), we assume that at date 1, the firm can either invest the intermediate asset in a viable (i.e., low-risk) project VP, or a high-risk alternative HR. VP generates a cash flow $V > 0$ with probability $p \in (0, 1)$ at date 2; however, it fails with probability $1 - p$. HR generates a cash flow $H > V$ with probability $q \in (0, p)$; but, HR fails with probability $1 - q$. The firm will receive zero cash flow if the project it invests in fails. We assume

$$pV > 1 > qH, \tag{1}$$

which implies that VP has a positive expected NPV, whereas HR’s expected NPV is negative. Since both VP and HR fail with positive probabilities, the firm’s investment choice between VP and HR is unobservable and unverifiable.⁵

At date 1, instead of investing in VP or HR, the firm may choose to default, in which case, any unliquidated part of the intermediate asset becomes valueless, and the game ends. Because the firm’s default decision at date 1 is publicly observable, early default is verifiable.

2.2 Financing

The firm financed the intermediate asset with short-term debts. At the beginning of date 0, the firm’s outstanding short-term debt is mature, and is held by a continuum of creditors with measure one. The creditors are uniformly distributed over $[0, 1]$ and indexed by i . According to the short-term debt contract, the firm needs to pay \$1 to each creditor at date 0. Therefore, if the firm liquidates the whole intermediate asset, it can repay all the creditors.

At date 0, the creditors can roll over the debt or run. In particular, $\gamma \in (0, 1)$ fraction of creditors become “impatient” and need to run for exogenous liquidity reasons. All “patient” creditors with measure $1 - \gamma$ will then simultaneously decide whether to roll over the debt. According to the original short-term debt contract, if a creditor decides not to roll over the debt, he will receive

⁵In practice, creditors may know the name of the project the firm invests in, but they usually lack the professional knowledge to judge whether the project is HR or VP.

payoff 1. For creditors who roll over the debt, the promised payment by the new short-term debt contract is $F > 1$ at date 2, so long as the firm does not default either endogenously at date 1 or exogenously at date 2.⁶

We denote the measure of creditors who roll over the debt by W , so the firm needs to finance $1 - W$ from non-debt sources to prevent liquidating any fraction of the intermediate asset. We assume that the firm has a pre-committed bank credit line. The firm can withdraw up to \$1 from the credit line with the constant marginal cost $f(\theta)$. Here, θ represents the firm's capacity to manage the rollover risk and is drawn by nature from the real line \mathbb{R} . (We also call θ the fundamentals of the firm.) The function $f(\cdot)$ is differentiable and strictly decreasing. When the firm's fundamentals are extremely good, the cost of the credit line financing approaches the payment of the new short-term debt; that is, $\lim_{\theta \rightarrow +\infty} f(\theta) = F$. However, when the firm's fundamentals are extremely bad, borrowing from the credit line is extremely expensive, so $\lim_{\theta \rightarrow -\infty} f(\theta) = +\infty$.⁷ Therefore, if the firm decides to invest in either VP or HR, the firm's financial cost is

$$K(\theta) = WF + (1 - W)f(\theta). \quad (2)$$

2.3 Firm's Payoff and Social Welfare

The firm has limited liability. If it defaults, whether endogenously at date 1 or exogenously at date 2 (when the project fails), its payoff is zero. If the firm generates a positive cash flow at date 2, the firm needs to repay the creditors according to the new short-term debt contract. Therefore,

⁶To focus on the role of the credit rating agency, we take the new short-term debt contract as exogenously given, as in He and Xiong (2012). One might argue that a higher promised payment F' can attract more patient investors, so the firm can reduce its financial cost by offering F' . However, an endogenously promised payment $F' > F$ will have signaling effects. Because it immediately rules out all θ 's such that $f(\theta) < F'$, a higher promised payment F' may be self-defeated, so its effect is ambiguous. Furthermore, there exists a pooling equilibrium, in which the firm will always offer the promised payment F , with the off-equilibrium path belief that any firm choosing $F' \neq F$ will invest in HR.

⁷Credit lines are important means of corporate liquidity management. The firm has to pay the bank a commitment fee, in order to obtain the right to withdraw urgent money (up to a limit) in the future. So a credit line is like a put option to the firm. Terms of a credit line are usually determined in the negotiation between the firm and the bank, so we can interpret θ as the firm's bargaining power in the negotiation. Please refer to the survey by Almeida, Campello, Cunha, and Weisbach (2014) for more details of credit lines.

the firm's payoff U depends on its own investment choice and its financial cost:

$$U = \begin{cases} 0, & \text{if the firm defaults at date 1;} \\ p[V - WF - (1 - W)f(\theta)], & \text{if the firm invests in VP;} \\ q[H - WF - (1 - W)f(\theta)], & \text{if the firm invests in HR.} \end{cases} \quad (3)$$

Social welfare is ranked by the firm's investment decisions.⁸ When the firm invests in VP, the social welfare is $pV - 1$; when the firm invests in HR, the social welfare is $qH - 1$; and if the firm defaults at date 1, the social welfare is at least $\gamma - 1$, since there are at least γ measure creditors who run. We assume that it is very unlikely for HR to generate a positive cash flow, so that $qH < \gamma$. Therefore, whereas VP is the social optimal investment project, HR leads to an even lower ex-ante social welfare than the early default.

For each individual patient creditor, if he knows the firm will invest in VP, he will roll over the debt. So we assume $pF > 1$. However, the probability that HR is successful is so low ($qF < qH < \gamma < 1$) that a patient creditor will run, if he knows that the firm will invest in HR. Obviously, if the creditor knows that the firm will default early, he will also choose to run.

2.4 Information Structure

The firm's liquidity management ability, θ , is the firm's private information, which remains unknown to creditors. Before deciding whether to roll over the firm's debt, each patient creditor i observes a private signal $x_i = \theta + \xi_i$, where $\xi_i \sim \mathcal{N}(0, \beta^{-1})$ is independent of θ and independent across all creditors. In our model, creditors are well-informed if and only if β is sufficiently large, and we aim to analyze credit ratings' effects on rational, well-informed creditors, so in this paper, we focus on the case when β is sufficiently large.

Besides their private signals, creditors will also observe a public credit rating by a credit rating agency (CRA). Denoted by $a_i \in \{0, 1\}$ creditor i 's rollover decision, where $a_i = 1$ means creditor i rolls over the bond, while $a_i = 0$ means creditor i runs.

2.5 Credit Rating Agency

The CRA will assign the firm a credit rating R .⁹ Following Boot, Milbourn, and Schmeits (2006), we restrict the space of ratings to $R \in \{0, q, p\}$. Early default at date 1 means the firm

⁸This notion of social welfare is in fact the sum of all agents' ex-ante payoffs at date 0, including that of the bank.

⁹This may also be interpreted as a revision of the previous credit rating.

will certainly default, and thus the accurate credit rating should be 0. Similarly, the firm investing in HR has a credit quality q , and the firm investing in VP has a credit quality p .

We assume that the CRA knows θ , so that we can separate credit rating bias due to the conflicts of interest from that caused by the CRA’s capacity to acquire precise information. Because the CRA knows θ , there is no aggregate shock. In addition, we consider pure strategies, so the CRA can perfectly predict the firm’s choice and its corresponding default probability at date 0. Hence, our model captures an important feature of credit ratings — forward-looking.

The CRA always has incentives to assign the firm a high credit rating, due to the credit rating industry’s current “issuer-pays” business model. Because the CRA is paid by issuers, it is incentivized to please issuers by assigning them high credit ratings. The CRA’s incentives to assign high credit ratings may also come from issuers’ rating shopping (Bolton, Freixas, and Shapiro, 2012), or the CRA’s reputation for being nice to issuers (Frenkel, 2014).

However, a partial verifiability condition constrains the CRA’s rating. Although the CRA wants to assign the firm the highest rating, the CRA must also beware of lawsuits resulting from verifiable frauds; that is, the CRA never wants to be caught lying. Consequently, for any given θ , the CRA wants to assign the firm the highest possible rating, provided that it cannot be verified as wrong.

2.6 Timeline and Equilibrium

We summarize the model’s timeline in Figure 1 below.

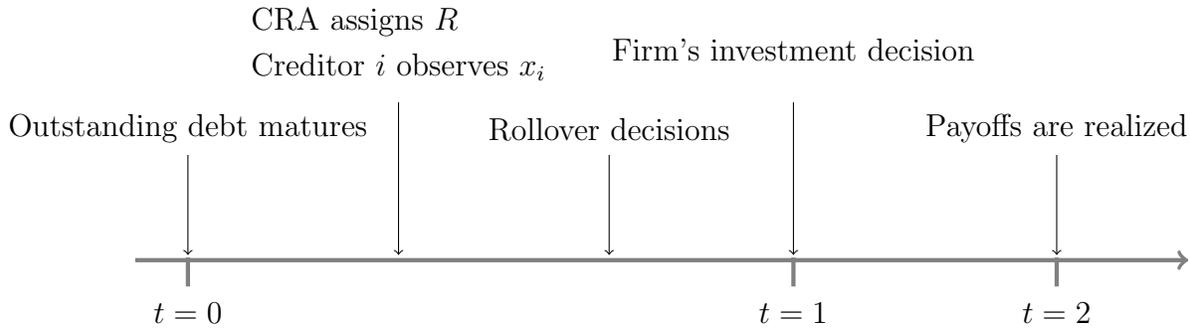


Figure 1: Timing

The CRA’s rating strategy maps the firm’s liquidity management ability to the rating space $\{0, q, p\}$; the firm’s strategy maps its fundamentals, the CRA’s rating, and the measure of creditors

rolling over the debt to investment choices; and creditors' strategies map their own private signals and the CRA's rating to their rollover decisions.

We are interested in monotone equilibria.

Definition 1. *The CRA's rating strategy, the firm's investment strategy, and creditors' rollover strategies constitute a monotone equilibrium, if*

1. *given the firm's investment strategy and creditors' rollover strategies, the CRA maximizes the nominal rating $R(\theta)$ for all $\theta \in \mathbb{R}$ subject to the partial verifiability constraint;*
2. *given financial costs in Equation (2), the firm's investment strategy will maximize the firm's expected profits;*
3. *given the CRA's rating strategy, the firm's investment strategy, and other creditors' strategies, any creditor i 's strategy is monotonic in his private signal x_i and will maximize his expected payoff; and*
4. *creditors use Bayes' rule to update their beliefs.*

Because there is no aggregate shock, given the firm's investment strategy and the creditors' rollover strategies, the CRA knows precisely the firm's equilibrium investment choice. If the CRA believes that the firm will default at date 1, the CRA has no choice but to assign a rating $R = 0$, because the early default choice can be publicly observed; however, if the CRA believes the firm will continue the investment to date 2, then, since the firm's choice between VP and HR cannot be verified, the CRA will assign $R = p$. Following this argument, the rating $R = q$ cannot appear in an equilibrium.¹⁰ Following the literature of credit rating inflation, we formally define rating inflation below.

Definition 2. *A credit rating assigned to the firm with fundamentals θ is inflated, if in an equilibrium, the firm chooses HR, but the CRA assigns $R(\theta) = p$. In addition, a rating strategy is inflated, if credit ratings assigned according to the rating strategy are inflated for a non-negligible subset of fundamentals; and a credit rating strategy is more inflated, if for a larger measure of fundamentals, credit ratings assigned according to the rating strategy are inflated.*

¹⁰On the off-equilibrium path following $R = q$, the creditors form the belief that the firm will choose to continue to invest in HR. In Section 7.1, we analyze a self-disciplined CRA. The rating $R = q$ may appear in some equilibria.

3 The Benchmark: No CRA

In order to analyze the CRA's role, we set up a benchmark. We analyze a model that excludes the CRA, so that when deciding whether to roll over the short-term debt, all patient creditors make choices solely based on their own private information. After observing the measure of creditors who roll over the debt, the firm makes its investment choice. The model is similar to the debt-run model by Morris and Shin (2004), with the key difference being that the firm has a moral hazard problem.

Let's first analyze the firm's behavior in such a benchmark model. Denote the measure of creditors who roll over the firm's debt by W . If the firm chooses to default at date 1, the firm's payoff is 0; if the firm invests in HR, the firm's expected payoff is $q[H - WF - (1 - W)f(\theta)]$; and if the firm invests in VP, the firm's expected payoff is $p[V - WF - (1 - W)f(\theta)]$. Since $H > V$, the firm will default early, if and only if,¹¹

$$WF + (1 - W)f(\theta) > H. \quad (4)$$

Conditional on that the firm decides to continue investing, it invests in VP rather than HR, if and only if,

$$\begin{aligned} p[V - WF - (1 - W)f(\theta)] &\geq q[H - WF - (1 - W)f(\theta)] \\ \Rightarrow WF + (1 - W)f(\theta) &\leq \frac{pV - qH}{p - q}. \end{aligned} \quad (5)$$

The firm's choice between VP and HR is the same as in Boot, Milbourn, and Schmeits (2006). To enhance the model's interest, we designed it so that if all creditors, patient and impatient, roll over the debt, so that the firm can reach its lowest possible financial cost F , the firm will choose VP. That is, we maintain the assumption that

$$F < \frac{pV - qH}{p - q}.$$

As a result, given the measure of creditors who roll over the debts, the firm's optimal investment strategy is

$$\begin{cases} \text{early default,} & \text{if } WF + (1 - W)f(\theta) > H; \\ \text{HR,} & \text{if } WF + (1 - W)f(\theta) \in \left(\frac{pV - qH}{p - q}, H \right]; \\ \text{VP,} & \text{if } WF + (1 - W)f(\theta) \leq \frac{pV - qH}{p - q}. \end{cases} \quad (6)$$

¹¹We assume that the firm will default at date 1, if its financial cost is larger than the highest possible cash flow the firm can generate. This can be justified by a punishment or a reputation loss to the firm's manager, if the firm's investment is successful but the firm still defaults.

From the properties of the function $f(\cdot)$, we know that when the firm's fundamentals are extremely good ($\theta \rightarrow +\infty$), $f(\theta)$ is very close to the par value of the firm's debt, F ; then $WF + (1 - W)f(\theta)$ is strictly less than $(pV - qH)/(p - q)$, implying that the firm will invest in VP. When the firm's fundamentals are extremely bad ($\theta \rightarrow -\infty$), $f(\theta)$ is extremely large, so that $WF + (1 - W)f(\theta) > H$, implying that the firm will default early.

Since impatient creditors with measure $\gamma > 0$ will not roll over the debt for exogenous liquidity reasons, the firm has to withdraw some funds from the credit line, if it decides to invest in either VP or HR. As a result, the firm's financial cost depends crucially on its fundamentals, θ . Hence, as shown in the global game literature, in such a benchmark model, all patient creditors have both the dominant regions of rolling over and running. That is, when creditor i 's private signal x_i is extremely negative, he believes that the firm's fundamentals are weak, so that even if all other creditors roll over the debt, the firm's financial cost of investing in one project is beyond its highest possible cash flow H , and thus the firm will default at date 1. Therefore, creditor i will run, even when all other patient creditors roll over the debt. This establishes creditors' dominant region of running. Conversely, creditors also have a dominant region of rollover. If creditor i 's private signal x_i is extremely positive, he believes that the firm's fundamentals are extremely good, and thus the firm will choose VP; as a result, creditor i will roll over the debt, even when all other patient creditors run. Therefore, as in other global game models, in a monotone equilibrium, any patient creditor will employ a cutoff strategy with the threshold \tilde{x} , such that he rolls over the debt, if and only if $x_i \geq \tilde{x}$.

Given θ and the creditors' cutoff strategy, the measure of creditors who roll over the debt is

$$(1 - \gamma) \Pr(x \geq \tilde{x}|\theta) = (1 - \gamma) \left\{ 1 - \Phi \left[\sqrt{\beta}(\tilde{x} - \theta) \right] \right\},$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. Then θ -firm's financial cost is

$$\begin{aligned} K(\theta) &= (1 - \gamma) \left\{ 1 - \Phi \left[\sqrt{\beta}(\tilde{x} - \theta) \right] \right\} F + \left[\gamma + (1 - \gamma)\Phi \left[\sqrt{\beta}(\tilde{x} - \theta) \right] \right] f(\theta) \\ &= [(1 - \gamma)F + \gamma f(\theta)] + (1 - \gamma)\Phi \left[\sqrt{\beta}(\tilde{x} - \theta) \right] (f(\theta) - F). \end{aligned} \quad (7)$$

The first term in Equation (7) is the financial cost resulting from the exogenous liquidity shocks to creditors, whereas the second term in Equation (7) is the endogenous financial cost resulting from creditors' strategic complementarities.

As θ increases, that is, as the firm's fundamentals improve, the cost of withdrawing funds from the credit line decreases (since $f(\theta)$ is a strictly decreasing function), and the measure of patient creditors who roll over the debt increases. Therefore, given creditors' cutoff strategies, the firm's financial cost strictly decreases in its fundamentals. In contrast with classical global games, in this

benchmark model the firm has two indifference conditions. First, given the creditors' strategies, the firm will choose to default early if and only if $\theta < \tilde{\theta}_1$. This implies that

$$K(\tilde{\theta}_1) = \left[(1 - \gamma)F + \gamma f(\tilde{\theta}_1) \right] + (1 - \gamma)\Phi \left[\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right] \left(f(\tilde{\theta}_1) - F \right) = H. \quad (8)$$

Because $K(\theta)$ is strictly decreasing, for any $\theta < \tilde{\theta}_1$, the firm's financial cost will be greater than H , the upside cash flow of HR; as a result, the firm would likely default at date 1. But if $\theta \geq \tilde{\theta}_1$, the firm can at least choose HR in order to receive a non-negative expected payoff due to the limited liability assumption, and thus the firm will not default early.

When the firm's fundamentals are $\theta \geq \tilde{\theta}_1$, the firm needs to choose between VP and HR. From Equation (6) and the fact that $K(\theta)$ is strictly decreasing in θ , there must be a $\tilde{\theta}_2 > \tilde{\theta}_1$, such that the firm will choose VP if and only if $\theta \geq \tilde{\theta}_2$. Hence,

$$K(\tilde{\theta}_2) = \left[(1 - \gamma)F + \gamma f(\tilde{\theta}_2) \right] + (1 - \gamma)\Phi \left[\sqrt{\beta}(\tilde{x} - \tilde{\theta}_2) \right] \left(f(\tilde{\theta}_2) - F \right) = \frac{pV - qH}{p - q}. \quad (9)$$

Following the above arguments, in a monotone equilibrium, the firm will default early if $\theta < \tilde{\theta}_1$, invest in HR if $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2)$, and invest in VP if $\theta \geq \tilde{\theta}_2$.

Any creditor i , receiving a private signal x_i about θ , first updates his belief about θ according to Bayes' rule:

$$\theta | x_i \sim \mathcal{N}\left(x_i, \frac{1}{\beta}\right).$$

Then, given the firm's strategy described above, creditor i calculates his return from rolling over the debt:

$$\left\{ \Phi \left[\sqrt{\beta}(\tilde{\theta}_2 - x_i) \right] - \Phi \left[\sqrt{\beta}(\tilde{\theta}_1 - x_i) \right] \right\} qF + \left\{ 1 - \Phi \left[\sqrt{\beta}(\tilde{\theta}_2 - x_i) \right] \right\} pF.$$

Because any creditor will receive the payoff 1 if he runs, the creditor with private signal \tilde{x} would be the marginal creditor who is indifferent about running or rolling over the debt. As a result, the creditor's indifference condition is

$$\left\{ \Phi \left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x}) \right] - \Phi \left[\sqrt{\beta}(\tilde{\theta}_1 - \tilde{x}) \right] \right\} qF + \left\{ 1 - \Phi \left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x}) \right] \right\} pF = 1. \quad (10)$$

Proposition 1 below characterizes the equilibrium of the benchmark model.

Proposition 1 (The Unique Equilibrium in the Benchmark Model). *There exists a $\tilde{\beta} > 0$, such that for all $\beta > \tilde{\beta}$,¹² the benchmark model without a CRA has a unique equilibrium described by $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$, where $\tilde{\theta}_1 < \tilde{\theta}_2$. In particular,*

¹²In this paper, we focus on the case that β is sufficiently large. There are two reasons. First, as in the global game literature, sufficiently precise private signals can guarantee equilibrium uniqueness, which is critical for the equilibrium analysis. More importantly, the aim of this paper is to study the credit ratings' effects when creditors have very precise private information, which refers to sufficiently large β in the model.

1. the firm invests in VP, if and only if $\theta \geq \tilde{\theta}_2$;
2. the firm invests in HR, if and only if $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2)$;
3. the firm defaults at date 1, if $\theta < \tilde{\theta}_1$; and
4. any creditor i will roll over the firm's short-term debt if and only if $x_i \geq \tilde{x}$.

One significant property of the equilibrium is that \tilde{x} strictly increases in $\tilde{\theta}_1$. For any given private signal x_i , an increase in $\tilde{\theta}_1$ means that the firm is more likely to default early. This hurts creditors, and thus they are less likely to roll over the debt, leading to a higher rollover threshold.

We conclude our benchmark model analysis by examining social welfare. We define social welfare as the expected investment results, and denote it by Ω . If the firm defaults early, the unliquidated fraction of the intermediate asset becomes valueless. Therefore, the social loss equals the measure of creditors who roll over the short-term debt. If the firm invests in HR or VP, social welfare will comprise the difference between the expected cash flows and the liquidation value of the intermediate asset, which is 1.

Proposition 2 (Social Welfare in the Benchmark Model). *In the equilibrium of the benchmark model, social welfare is non-monotonic in the firm's fundamentals. In particular,*

$$\tilde{\Omega} = \begin{cases} -(1 - \gamma) [1 - \Phi(\sqrt{\beta}(\tilde{x} - \theta))], & \text{if } \theta < \tilde{\theta}_1; \\ qH - 1, & \text{if } \theta \in [\tilde{\theta}_1, \tilde{\theta}_2); \\ pV - 1, & \text{if } \theta \geq \tilde{\theta}_2. \end{cases} \quad (11)$$

Therefore, in the benchmark model, when $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2)$, social welfare is at its lowest, because of the assumption $qH < \gamma$. For $\theta < \tilde{\theta}_1$, social welfare is strictly decreasing in θ ; that is, given that the firm will default early, the better the firm's fundamentals, the lower the social welfare. This is because, as the firm's fundamentals improve, more creditors roll over the debt, the value of which is destroyed due to the firm's early default.

4 Credit Ratings' Informativeness and Real Effects

In this section, we analyze how corporate credit ratings affect perfectly rational, well-informed creditors' decisions and the firm's project choice. We first discuss the informativeness of credit ratings, assuming that there are extreme conflicts of interest between the CRA and creditors due

to the current “issuer-pays” business model. Although the CRA always has incentives to assign overgenerous ratings, its rating strategy is subject to the partial verifiability constraint: the event of the firm’s early default is publicly observable and thus verifiable. As a result, in the equilibrium, the CRA will assign a high rating if and only if the firm does not default at date 1. We will show that this partial verifiability constraint plays a critical role in determining the informativeness of the CRA’s ratings.

We then analyze credit ratings’ real effects. In our model, credit ratings affect social welfare only if they can impact the firm’s investment decisions. The firm’s investment decision depends on the firm’s financial cost, which is in turn determined by the aggregated creditors’ decisions. This creates strategic complementarities among creditors. The precision of the creditors’ private signals, which also measures the dispersion of the creditors’ beliefs, will affect creditors’ behavior in our model, which features strategic complementarities, and will significantly impact credit ratings’ real effects.

4.1 Rating Strategy is always Inflated

We first derive possible equilibrium rating strategies. From Equation (2), given the measure of creditors who roll over (W), the firm’s financial cost is strictly decreasing in its fundamental θ , because $f(\theta)$ is strictly decreasing. Furthermore, in a monotone equilibrium (if exists), given the same rating, the better the firm’s fundamentals, the larger the measure of creditors rolling over. Therefore, if the firm with θ does not default at date 1, the firm with any better fundamentals $\theta' > \theta$ will not default early either. This argument leads to Lemma 1 below, which characterizes all possible equilibrium rating strategies and simplifies our analysis much.

Lemma 1. *In an equilibrium (if exists), the CRA’s rating strategy can be described by a threshold $\hat{\theta}$, such that*

$$R(\theta) = \begin{cases} 0, & \text{if } \theta < \hat{\theta}; \\ p, & \text{if } \theta \geq \hat{\theta}. \end{cases} \quad (12)$$

When $\hat{\theta}$ decreases, the CRA will assign more firms with the high rating p . So for two rating strategies R_1 with the threshold $\hat{\theta}_1$ and R_2 with the threshold $\hat{\theta}_2$, we posit that the rating strategy R_2 is *laxer* than the rating strategy R_1 if and only if $\hat{\theta}_2 < \hat{\theta}_1$.

Lemma 1 implies that following the rating $R = 0$, all creditors run, ignoring their own private signals. They do so because the rating $R = 0$ means that the CRA believes the firm will default at date 1, which is due to the early default’s verifiability. But if $R = p$, creditors all know that $\theta > \hat{\theta}$, which implies Corollary 1 below.

Corollary 1 (Creditors' belief supports following $R = p$). *Following the credit rating $R = p$, independent of his private signal x_i , the support of any creditor i 's interim belief about θ is truncated from below by $\hat{\theta}$.*

We now derive more conditions for the equilibrium $\hat{\theta}$. Let's first consider the game after $R = 0$. Intuitively, because the CRA has incentives to inflate the rating, it will assign the rating $R = p$ whenever the firm does not default early, given creditors' strategies. Therefore, the fact that the CRA assigns a rating $R = 0$ implies that the CRA believes the firm will default early. Then, following $R = 0$, no creditor will roll over the debt, leading to the financial cost $K(\theta) = f(\theta)$, if the firm invests in either VP or HR. For the rating strategy described in Equation (12) to be part of an equilibrium, we must have $K(\theta) = f(\theta) > H, \forall \theta < \hat{\theta}$. Then, by the continuity of $f(\cdot)$, we have the first condition for the equilibrium below.

$$f(\hat{\theta}) \geq H. \tag{13}$$

Now, consider $R = p$. If the rating strategy in Equation (12) is part of an equilibrium, all creditors believe that the firm will not default early. But, following $R = p$, the firm may choose HR. That is, the CRA may employ a rating strategy with inflation. The following Lemma 2 shows that there is no equilibrium in which the CRA's rating is not inflated.

Lemma 2 (No Equilibrium without Rating Inflation). *There is no equilibrium in which the firm invests in VP whenever $R(\theta) = p$.*

The rating inflation inevitably appears in the equilibrium, both because the conflicts of interest caused by the "issuer-pays" business model, and because the firm's investment in HR or VP is unverifiable. However, since all creditors understand such a fact, they will make their debt rollover decisions based on their own private information as well.

4.2 Following the Rating $R = p$

Lemma 2 implies that, in an equilibrium, when θ is greater than but very close to $\hat{\theta}$, the firm will invest in HR. Since $qF < qH < \gamma < 1$, the creditors with extremely negative signals will not roll over the firm's short-term debt, because they hold extremely strong beliefs that the firm will invest in HR. Therefore, in a monotone equilibrium, given the belief about the CRA's rating strategy described by $\hat{\theta}$, any creditor i will roll over the debt if and only if x_i lands above some threshold x^* . Then, as in the benchmark model, because the firm's financial cost strictly decreases in its fundamentals, the firm will invest in HR if and only if θ is less than a threshold θ^* . Hence,

given a possible equilibrium $\hat{\theta}$, a monotone equilibrium following $R = p$ would be described by (x^*, θ^*) , such that

1. $\theta^* > \hat{\theta}$;
2. creditor i rolls over the short-term debt if and only if $x_i \geq x^*$; and
3. θ -firm chooses VP if $\theta \in [\theta^*, +\infty)$, and it chooses HR if $\theta \in [\hat{\theta}, \theta^*)$.

Given the creditors' cutoff strategy with the threshold x^* , $(1-\gamma) [1 - \Phi(\sqrt{\beta}(x^* - \theta))]$ measure of creditors will roll over the debt, for any $\theta \geq \hat{\theta}$. Consequently, if the θ -firm decides to invest in either VP or HR, its financial cost is

$$\begin{aligned} K(\theta) &= (1-\gamma) \left[1 - \Phi(\sqrt{\beta}(x^* - \theta)) \right] F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^* - \theta)) \right] f(\theta) \\ &= [(1-\gamma)F + \gamma f(\theta)] + (1-\gamma)\Phi(\sqrt{\beta}(x^* - \theta))(f(\theta) - F). \end{aligned}$$

This is precisely the same as Equation (7). Because the firm invests in VP if and only if $K(\theta) \geq (pV - qH)/(p - q)$, and θ^* -firm is indifferent between HR and VP, the firm's indifference condition, given the CRA's rating strategy, is

$$(1-\gamma) \left[1 - \Phi(\sqrt{\beta}(x^* - \theta^*)) \right] F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^* - \theta^*)) \right] f(\theta^*) = \frac{pV - qH}{p - q}. \quad (14)$$

Let's consider a creditor i 's decision. With his private signal x_i , creditor i 's interim belief about θ given the CRA's rating strategy $\hat{\theta}$ would be a normal distribution with mean x_i and precision β , truncated below by $\hat{\theta}$. This truncation is due to creditors' belief about the CRA's rating strategy that $R(\theta) = p$ if and only if $\theta \geq \hat{\theta}$. Then, given the firm's strategy, creditor i 's expected payoff from rolling over the debt is

$$\frac{\Phi[\sqrt{\beta}(\theta^* - x_i)] - \Phi[\sqrt{\beta}(\hat{\theta} - x_i)]}{1 - \Phi[\sqrt{\beta}(\hat{\theta} - x_i)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta^* - x_i)]}{1 - \Phi[\sqrt{\beta}(\hat{\theta} - x_i)]} pF.$$

Because refraining from rolling over the short-term debt always brings a creditor a payoff 1, a marginal creditor with the private signal x^* must have

$$\frac{\Phi[\sqrt{\beta}(\theta^* - x^*)] - \Phi[\sqrt{\beta}(\hat{\theta} - x^*)]}{1 - \Phi[\sqrt{\beta}(\hat{\theta} - x^*)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta^* - x^*)]}{1 - \Phi[\sqrt{\beta}(\hat{\theta} - x^*)]} pF = 1. \quad (15)$$

Lemma 3 (Debt Financing Following $R = p$). *There exists a $\hat{\beta} > 0$, such that for any $\beta > \hat{\beta}$, if an equilibrium exists, given the CRA's rating strategy $\hat{\theta}$, following $R = p$, there is a unique solution (θ^*, x^*) with $\theta^* > \hat{\theta}$ to Equation (14) and Equation (15).*

In the analysis of the interaction between the firm and the creditors above, the CRA's rating strategy $\hat{\theta}$ is given. Lemma 4 below shows how the CRA's rating strategy affects the creditors' debt rollover decisions and the firm's moral hazard.

Lemma 4 (Laxer Rating Strategy). *For any $\beta > \hat{\beta}$, both x^* and θ^* are strictly decreasing in $\hat{\theta}$.*

When $\hat{\theta}$ is lower, the CRA's rating strategy is laxer. In this scenario, creditors discount the good rating by increasing their debt rollover threshold. Because more creditors refrain from rolling over the debt, the firm's financial cost is higher for any θ ; as a result, the threshold that the firm chooses VP is also higher.

Note that, in an equilibrium, the firm defaults early if and only if $\theta < \hat{\theta}$. Therefore, the threshold $\hat{\theta}$ is analogous to the threshold $\tilde{\theta}_1$ in the benchmark model. However, $\hat{\theta}$ and $\tilde{\theta}_1$ have opposite effects on the creditors' rollover decisions. When the CRA employs a laxer rating strategy (a smaller $\hat{\theta}$), more firms will receive good ratings; consequently, creditors' rollover threshold will increase (an increase in x^*). Conversely, in the benchmark model, a decrease in $\tilde{\theta}_1$ implies that less firms will default at date 1. To the firm's creditors, early default is even worse than HR, because they receive nothing when the firm defaults at date 1. As a result, a decrease in $\tilde{\theta}_1$ causes a decrease in creditors' debt rollover threshold (an decrease in \tilde{x}).

4.3 Equilibrium Rating Strategy

Lemma 4 shows that creditors' responses to the CRA's rating inflation produce a trade-off for the CRA. On one hand, the CRA wants to assign the rating $R = p$ to more firms, due to the issuer-pays business model; on the other hand, when the CRA employs a laxer rating strategy, creditors discount the high credit rating by increasing their rollover threshold, leading to higher financial costs to the firm and thus a higher probability of early default. Therefore, through the firm's investment decisions, creditors' responses to the CRA's ratings affect the CRA's ratings.

Because of the firm's limited liability, the firm will choose to endogenously default only if the financial cost is higher than H , which is the upside return from investing in HR. Therefore, the CRA will choose $\hat{\theta}$ such that $\hat{\theta}$ -firm is indifferent between early default and HR. That is,

$$(1 - \gamma) \left[1 - \Phi(\sqrt{\beta}(x^* - \hat{\theta})) \right] F + \left[\gamma + (1 - \gamma)\Phi(\sqrt{\beta}(x^* - \hat{\theta})) \right] f(\hat{\theta}) = H. \quad (16)$$

Proposition 3 below shows that the model has a unique equilibrium, in which the CRA's rating, the firm's investment decision, and the creditors' rollover decisions interact with one another.

Proposition 3 (Unique Equilibrium). *There is a $\hat{\beta} > 0$, such that when $\beta > \hat{\beta}$, the model has a unique equilibrium. The equilibrium is characterized by $(\hat{\theta}, \theta^*, x^*)$ with $\theta^* > \hat{\theta}$, such that*

1. the CRA will assign a rating $R = p$, if the firm's fundamentals $\theta \in [\hat{\theta}, +\infty)$; and it will assign a rating $R = 0$, if the firm's fundamentals $\theta < \hat{\theta}$;
2. if $R = 0$, all creditors run, and the firm defaults at date 1;
3. if $R = p$, a patient creditor rolls over the debt if and only if his private signal lands above x^* , and the firm will choose HR if $\theta \in [\hat{\theta}, \theta^*)$ and VP if $\theta \in [\hat{\theta}, +\infty)$; and
4. $(\hat{\theta}, \theta^*, x^*)$ solves Equations (14), (15), and (16).

Proposition 3 not only characterizes the equilibrium rating strategy, but also shows how a high rating $R = p$ is informative given the extreme conflicts of interest. Because of the partial verifiability constraint, the CRA will not assign the rating $R = p$ when θ is less than $\hat{\theta}$. As a result, the rating $R = p$ can deliver a signal to creditors that $\theta \geq \hat{\theta}$. Such a signal will truncate the support of creditors' updated beliefs from below, therefore, it is a positive signal about the firm.

In such a rational model, all creditors understand that the CRA is strongly incentivized to employ an inflated rating strategy; thus, they will use their own private signals to correct the rating bias. Although such a correction is never complete because of creditors' coordination incentives, creditors' responses to credit ratings lead to much lower credit rating inflation in the equilibrium than when creditors naively interpret the rating $R = p$ as a guarantee that the firm will invest in VP. When creditors naively take the credit ratings' face value, all patient creditors will roll over the debt following the credit rating $R = p$, leading the firm's financial cost to be $\gamma f(\theta) + (1 - \gamma)F$. Then, the CRA will employ a rating strategy with $\hat{\theta}'$ such that $\gamma f(\hat{\theta}') + (1 - \gamma)F = H$. Since $f(\theta)$ is strictly decreasing, and Proposition 3 implies $\gamma f(\hat{\theta}) + (1 - \gamma)F < H$, we know that $\hat{\theta} < \hat{\theta}'$, demonstrating that the equilibrium rating inflation is lower than that if the creditors naively interpret the rating $R = p$ as the investment in VP.

4.4 Credit Ratings' Real Effects

Because credit ratings affect creditors' decisions, which determine the firm's financial cost and thus the firm's investment choice, credit ratings' real effects are significant. In the equilibrium, $\theta^* > \hat{\theta} > -\infty$ implies that a set of firms with fundamentals between $\hat{\theta}$ and θ^* will invest in HR. This theoretical result is consistent with the idea of "gambling for resurrection." On the one hand, when the firm's fundamentals are lower than θ^* , the firm's financial cost is so high that VP is not as profitable as HR, given the firm's limited liability; on the other hand, when the firm's

fundamentals are greater than $\hat{\theta}$, the firm's financial cost is lower than the upside return of HR, following the rating $R = p$. Therefore, when $\theta \in [\hat{\theta}, \theta^*)$, the firm does not want to await its doom, and will decide to gamble by investing in HR. This implies the credit ratings' effects on social welfare, as shown in Proposition 4 below.

Proposition 4 (Social Welfare in the Model with a CRA). *In the equilibrium of the model with a CRA, the social welfare is*

$$\Omega = \begin{cases} 0, & \text{if } \theta < \hat{\theta}; \\ qH - 1, & \text{if } \theta \in [\hat{\theta}, \theta^*); \\ pV - 1, & \text{if } \theta \geq \theta^*. \end{cases} \quad (17)$$

Then, what are the effects of a CRA on social welfare? Let's first analyze how the CRA affects the firm's and the creditors' equilibrium behavior. The CRA's rating $R = p$, though commonly known to be possibly inflated, provides the market with a positive signal, because in the equilibrium, $R = p$ implies $\theta \geq \hat{\theta}$. Therefore, any creditor becomes more optimistic about the firm's fundamentals, which increases patient creditors' incentives to roll over the debt, leading to lower financial costs for the firm. Then, the firm is more willing to invest in either VP or HR. By comparing the equilibrium behavior in the benchmark model to that in the model with a CRA, we can see the CRA's effect on the firm's equilibrium investment behavior, which are summarized in Proposition 5 below.

Proposition 5. *Comparing the equilibrium of the model with a CRA (described in Proposition 3) to that of the benchmark model without a CRA (described in Proposition 1), we have $\hat{\theta} < \tilde{\theta}_1$, $\theta^* < \tilde{\theta}_2$, and $x^* < \tilde{x}$. However, the sign of $\theta^* - \tilde{\theta}_1$ is undetermined.*

The CRA's effects on the firm's equilibrium behavior imply its dramatic effects on social welfare. By comparing the equilibrium in the benchmark model with that of the model with a CRA, we uncover the CRA's non-monotonic effects on social welfare, which is formally presented in Proposition 6 below.

Proposition 6. *There are two cases of the analysis of the CRA's effects on social welfare.*

1. *If $\theta^* > \tilde{\theta}_1$, the CRA improves social welfare when $\theta < \hat{\theta}$, hurts social welfare when $\theta \in [\hat{\theta}, \tilde{\theta}_1)$, and improves social welfare when $\theta \in [\theta^*, \tilde{\theta}_2)$. For all other firms, the CRA has no effect on social welfare.*

2. If $\theta^* \leq \tilde{\theta}_1$, the CRA improves social welfare when $\theta < \hat{\theta}$, hurts social welfare when $\theta \in [\hat{\theta}, \theta^*)$, and improves social welfare when $\theta \in [\theta^*, \tilde{\theta}_2)$. For all other firms, the CRA has no effect on social welfare.

The CRA can improve social welfare when the firm's fundamentals are very weak ($\theta < \hat{\theta}$), because the CRA provides creditors with new information, which guides the creditors to make optimal rollover choices. In the equilibrium, when $\theta < \hat{\theta}$, the CRA will assign a rating $R = 0$, implying that the firm will default early. Then, no creditor will roll over the debt based on such a signal, so the early default leads to no social loss.

When the firm's fundamentals $\theta \in [\hat{\theta}, \min\{\tilde{\theta}_1, \theta^*\})$, the CRA will assign a rating $R = p$, which is a positive signal to the market, and yet, surprisingly, social welfare is harmed. That is, when $\theta \in [\hat{\theta}, \min\{\tilde{\theta}_1, \theta^*\})$, the firm will default early without a CRA but will invest in HR with the credit rating $R = p$.

The reason for such an adverse effect of the CRA is that it incentivizes the firm to gamble for resurrection. Two key forces drive this result. First, the credit rating is partially verifiable, so the credit rating $R = p$ is a noisy, but informative positive signal. If the credit rating is completely unverifiable, the CRA's rating becomes a purely cheap talk; hence, given the conflicts of interests, no creditor will pay attentions to the CRA's rating. As a result, the CRA should have no effect on creditors' and firm's behavior, and thus no effects on social welfare. At the other extreme, if the credit rating is completely verifiable, the CRA will provide creditors with an accurate signal about the firm's equilibrium investment choice. Then, with perfect information about the firm's investment choice, creditors and the firm will make correct decisions, presumably avoiding the adverse effect. However, based on the realistic partial verifiability constraint, the CRA can only provide creditors with an incomplete positive signal.

Second, creditors have coordination incentives. When the CRA assigns a high credit rating to the firm, creditors, even those whose beliefs about the firm's fundamentals are hardly changed by the rating, believe that some creditors will choose to roll over. As a result, creditors expect a lower financial cost to the firm and thus a higher probability of investing in VP. Consequently, they are more likely to roll over. Therefore, the firm with $\theta \in [\hat{\theta}, \min\{\tilde{\theta}_1, \theta^*\})$ will have lower financial costs because of the good credit rating and thus will take a gamble by investing in HR instead of early default.

Similarly, because of credit ratings' partial verifiability and creditors' coordination incentives, when $\theta \in [\max\{\tilde{\theta}_1, \theta^*\}, \tilde{\theta}_2)$, the CRA will have positive effects on the social welfare. In such a case, the CRA assigns the rating $R = p$, causing more creditors to roll over the debt. Hence, the

rating $R = p$ decreases the firm’s financial cost, so that the firm, originally investing in HR, is not willing to take risks but will invest in VP.

5 Time-consistent Rating Strategy

In the model described in Section 2, we assume that the CRA tries to maximize the credit rating assigned to the firm after observing the firm’s fundamentals. That is, the CRA does not need to commit to a rating strategy ex-ante. Hence, even if the CRA claims that it will assign accurate ratings, when it finds that the firm will invest in the HR following the rating $R = p$, it will issue an inflated rating rather than an accurate one. Therefore, after the recent financial crisis, there are many reform proposals that aim to improve transparency of credit ratings, by requiring CRAs to disclose information about the assumptions underlying their methodology on a rating-by-rating basis.

Such proposals are essentially requiring the CRA to commit to a rating strategy before analyzing firms’ credit qualities, because with information about the assumptions underlying its methodology on a rating-by-rating basis, investors can recover the rating strategy that the CRA employs.

However, can such reforms better regulate the credit rating industry? Motivated by this question, we assume that the CRA commits to a rating strategy at the beginning of the game, without the knowledge of θ .

Specifically, we assume that the common prior belief about the firm’s fundamentals is $\theta \sim \mathcal{N}(\bar{\theta}, \mu^{-1})$. The CRA commits to and announces a rating strategy $R : \mathbb{R} \rightarrow \{0, q, p\}$ before learning the realized θ . We restrict the space of rating strategies to all increasing rating functions, because no firm will accept a low rating if the CRA assigns high ratings to other firms with worse fundamentals. Denote by Σ the space of increasing rating strategies. The CRA’s maximization problem is¹³

$$\max_{R \in \Sigma} \int_{-\infty}^{+\infty} R(\theta) d\Phi [\sqrt{\mu} (\theta - \bar{\theta})], \quad (18)$$

subject to the partial verifiability constraint. Following an assigned rating, the firm and creditors play the exactly same debt-run game described in Section 2. If in the subgame following a rating, there are multiple equilibria, we select the one with the largest measure of creditors rolling over. That is, we are selecting the “Sender-preferred subgame perfect equilibrium” as in Gentzkow and

¹³With the common prior belief, the CRA’s payoff is well-defined.

Kamenica (2011): when more creditors roll over, the firm’s financial cost is lower, and it is less likely to default early.

We first show that when the CRA commits to a rating strategy before analyzing the firm and getting to know θ , the rating $R = q$ will not be assigned in an equilibrium. Because investors are more optimistic about the firm’s fundamentals following the rating $R = p$ than following the rating $R = q$, if the firm with a rating $R = q$ does not default, it will not default if it receives a rating $R = p$. This result is formally stated in Lemma 5 below.

Lemma 5. *When the CRA commits to a rating strategy before knowing θ , in an equilibrium (if exists), there is no open interval (θ', θ'') with $\theta'' > \theta'$ such that $R(\theta) = q$ for all $\theta \in (\theta', \theta'')$.*

Lemma 5 implies that the optimal committed rating strategy must be in the form of

$$R(\theta) = \begin{cases} 0, & \text{if } \theta < \theta'; \\ p, & \text{if } \theta \geq \theta'. \end{cases} \quad (19)$$

This is the same as in Equation (12), so if the CRA commits to a rating strategy by setting $\theta' = \hat{\theta}$, following the rating $R = p$, (x^*, θ^*) is the unique equilibrium in the subgame. Then, Proposition 7 below shows that the optimal committed credit rating strategy is time-consistent.

Proposition 7. *When the CRA commits to a rating strategy before knowing θ , there is $\hat{\beta} > 0$ such that for all $\beta > \hat{\beta}$, there is a unique equilibrium, which is characterized in Proposition 3. Hence, the optimal committed rating strategy is time-consistent.*

The time-consistency of the CRA’s optimal committed rating strategy has great implications. First, this provides us with a new opinion about credit rating agencies. We can view the CRA as a certified expert, who tries to persuade creditors to roll over the debt to reduce the firm’s financial cost. Therefore, the equilibrium ratings p and 0 are “straightforward” signals (Gentzkow and Kamenica, 2011), with p meaning rollover and 0 meaning run. Second, from the policy perspective, the transparency requirement about CRAs will not change the CRA’s effects on creditors’ behavior, the firm’s behavior, and thus social welfare.

6 Empirical Predictions

The theory we develop in this paper provides several new empirical predictions about CRAs’ rating strategies and credit rating inflation. In this section, we analyze how a CRA’s rating strategy and the rating inflation vary when the economic environment changes. That is, we

perform comparative static analysis to provide empirical predictions about CRAs' rating strategies and credit rating inflation.

Here, we emphasize that the CRA's rating strategy (described by $\hat{\theta}$) and the credit rating inflation (described by $\theta^* - \hat{\theta}$) are two different concepts and are both endogenously determined in our model. Recently, Alp (2013) and Baghai, Servaes, and Tamayo (2014) find that CRAs become more conservative by using stricter rating standards (strategies). However, from Lemma 2, though the CRAs' rating strategies may become stricter in recent years, credit rating inflation always exists. This is consistent with the empirical findings in Strobl and Xia (2012). Furthermore, changes of the economic environment that lead to stricter rating strategies may not reduce credit rating inflation, because the changes of the exogenous environment can directly affect the firm's investment decision.

We first analyze how a firm's transparency can affect the CRA's rating strategy. The conflicts of interests between the CRA and creditors due to the "issuer-pays" business model are at the center of debates about CRAs' performances and potential regulations. However, surprisingly, although conflicts of interests have been recognized for a long time, they didn't attract much attention until the recent 2007-2009 subprime crisis. Therefore, it is important to consider how CRAs played a role in the crisis, which included the downfall of many financial institutions due to excessive borrowing, risky investments, and most significantly, lack of transparency. As a result, we first analyze equilibrium rating strategy and credit rating inflation when the firm under evaluation is opaque. We model the transparency of a firm or a financial institution by the precision of creditors' private signals, β .

To get some ideas about how the precision of creditors' private signals affects credit ratings, let's fix the CRA's rating strategy, $\hat{\theta}$. When β is sufficiently large so that $\beta > \hat{\beta}$ as in Proposition 3, the cutoff of the firm choosing VP instead of HR, θ^* , is strictly greater than $\hat{\theta}$. When we consider the θ^* -firm, most creditors expect that the firm's fundamentals will be very close to θ^* , before their beliefs are truncated below by $\hat{\theta}$. Then, by the properties of the mean of truncated normal distribution, creditors' beliefs about the firm's fundamentals will be strictly decreasing in β . That is, if we fix the CRA's rating strategy, an increase in the precision of private signals makes creditors more pessimistic about the firm's fundamentals. Consequently, creditors will increase their rollover threshold, leading to a higher financial cost for the firm. Because the firm's financial cost increases for any given fundamental, the CRA has to employ a stricter rating strategy. The Proposition 8 below formally proves our above intuition.

Proposition 8. *When β is large enough, in the equilibrium, $\hat{\theta}$ is strictly increasing in β .*

While Proposition 8 implies that CRAs will employ stricter rating strategies for more trans-

parent firms, it does not imply that credit ratings assigned to more transparent firms are more inflated. Since more creditors will increase their rollover threshold when the firm is more transparent, the firm's financial cost is higher, causing a higher θ^* , which is the firm's VP investment threshold. That is, given the CRA's rating strategy, more firms will invest in HR. Therefore, an increase in the firm's transparency has two effects on the credit rating inflation: the CRA will assign the high rating $R = p$ to less firms, but more firms will invest in HR. As a result, whether the rating inflation for a more transparent firm is higher or lower depends on which of these two effects dominate. This in turn depends on other parameters of the model.

Proposition 8 predicts that credit ratings for firms lacking transparency are laxer. Financial institutions are usually more opaque than corporations, especially because it is very difficult for their creditors to estimate their capacities to manage liquidity risks.¹⁴ As a result, creditors' private signals about the firm's liquidity management ability would be very noisy. Due to the partial verifiability constraint, the CRA provides creditors with a noisy but positive signal, so creditors' expectations about the firm's fundamentals increase, leading to the firm's lower financial cost. Therefore, some financial institutions will choose not to immediately default, which provides the CRA more room to inflate its rating.¹⁵

In our model, the precision of creditors' private signals also represents creditors' belief dispersion. For any fixed fundamentals of the firm, the smaller the precision of creditors' private signals, the more dispersed creditors' beliefs. Therefore, Proposition 8 also implies that the CRA's rating strategy is laxer if creditors' beliefs about the firm's fundamentals are more dispersed.

Second, cross-sectionally, firms differ in the upside returns of their available projects. Equation (16) suggests that the highest upside return among all available projects may determine the credit rating assigned to the firm. Hence, it is interesting to consider how the firm's upside return from HR affects the CRA's rating strategy. An increase in H does not directly affect creditors' behavior, because creditors' payoffs are solely determined by the debt contract, which does not involve the cash flow to the firm, conditional on the success of the investment. Yet, H has direct effects on both the firm's investment and the CRA's rating strategy. On one hand, an increase in H

¹⁴Financial institutions, such as Bear Stearns and Lehman Brothers, were operating with extraordinarily thin capital. They financed mainly by very short-term debt, such as commercial paper or repos. Therefore, outside creditors lacked professional knowledge to predict how expensive those financial institutions financed from resources other than short-term debt.

¹⁵For example, after the two Bear-sponsored hedge funds declared bankruptcy on July 31, 2007, former Bear Stearns treasurer Robert Upton spoke with the three main credit rating agencies, because even a threat of a downgrade by a rating agency would make Bear's financing extremely expensive. Obviously, Bear Stearns would not go bankrupt in August 2007, so credit rating agencies maintained their top ratings then.

increases the firm's incentives to invest in HR, conditional on that the firm does not default early. On the other hand, an increase in H decreases the firm's incentives to default early, because the firm has limited liability, and it can always choose to take the risk, even with a small probability of success. As a result, for fixed creditors' strategies, when H increases, the CRA's rating strategy will be laxer and the firm is more likely to invest in HR rather than VP. These arguments are summarized in Proposition 9 below.

Proposition 9. *When β is sufficiently large, in the equilibrium, $\hat{\theta}$ is strictly decreasing in H , θ^* is strictly increasing in H , and thus $\theta^* - \hat{\theta}$ is strictly increasing in H .*

Hence, cross-sectionally, the CRA's rating strategy for firms with higher upside returns from high risky projects is laxer and involves higher inflation.

Finally, suppose the measure of impatient creditors, γ , increases. The direct effect of an increase in γ is that the firm's financial cost surely will increase, because the firm needs to finance more money from expensive non-debt sources. In addition, an increase in γ will lead to less patient creditors rolling over the debt, due to the strategic complementarities among patient creditors. This further increases the firm's financial cost. Then, the firm's threshold of investing in VP will increase, implying that more firms will invest in HR given the CRA's credit rating strategy. In the meanwhile, the higher financial cost of the firm means that more firms may default early. As a result, the CRA would want to employ a stricter rating strategy. As we show in Proposition 10 below, as γ increases, the measure of firms that shift from VP to HR due to the increased financial cost is greater than the measure of firms that are downgraded to the credit rating $R = 0$ and thus default early, so an increase in γ leads to higher rating inflation.

Proposition 10. *In the equilibrium, both $\hat{\theta}$ and $\theta^* - \hat{\theta}$ is strictly increasing in γ .*

The comparative static analyses about β , H , and γ provide us with good examples how corporate credit ratings differ from those on mortgage-backed securities. For mortgage-backed securities, if changes of different exogenous parameters lead to laxer rating strategies, they will all cause higher rating inflation. However, as we show above, a decrease in the firm's transparency, an increase in upside returns of risky projects, and a decrease in the measure of creditors who run for exogenous reasons all lead to laxer rating strategies. However, they have different effects on the rating inflation: the decrease in the firm's transparency has ambiguous effects on the rating inflation, an increase in upside returns of risky projects causes higher rating inflation, and a decrease in the measure of creditors who run for exogenous reasons leads to lower rating inflation.

Such a significant difference between corporate credit ratings and those on mortgage-backed securities is due to the feedback between credit ratings and firms' investment behavior. When

securities' credit qualities are exogenously given, for example, mortgage-backed securities, laxer rating strategies lead to higher rating inflation. But when securities' credit qualities are endogenously determined, like corporate bonds, laxer rating strategies may not be accompanied by higher credit rating inflation.

7 Policy Implication

In our model, the CRA's rating strategy is always inflated in the equilibrium. Although inflated credit ratings provide creditors with positive signals, they may have adverse effects on social welfare, because inflated credit ratings may incentivize firms to gamble for resurrection. The two forces driving the CRA's adverse effects are the credit rating's partial verifiability and the dispersion of creditors' beliefs. As a result, in order to mitigate the adverse effects of the CRA, policies could be designed according to these two driving forces. While we agree that any policy is costly, as technology develops, the benefits of such policies will outweigh the costs. However, because the costs of implementing the policies are not modeled in this paper, we do not aim to solve the optimal policy design problem here.

7.1 Verifying Investment

Given the credit rating industry's current business model, CRAs are always incentivized to inflate their ratings, and are constrained by their desire to avoid being caught lying. If evidence exists that a CRA is lying, it may be sued. Hence, a natural regulatory policy for the credit rating industry would be to verify the firm's investment choice.

Because the government will verify the firm's ex-post investment choice, the CRA, who can perfectly predict the firm's choice, has to assign a forward-looking rating, which reflects the firm's investment choice. That is, knowing θ , the CRA has to assign a rating, which reflects the true probability that the firm will default. Hence, the policy of verifying the firm's investment makes a CRA "self-disciplined."

The range of the rating is still $\{0, q, p\}$. Suppose the CRA's rating is $R = 0$. Since the CRA is self-disciplined, all creditors believe that the firm will surely default early. Then, after $R = 0$, no creditor will roll over the debt, leading to the firm's financial cost $K(\theta) = f(\theta)$. Thus, the firm chooses to default early following $R = 0$ if and only if

$$f(\theta) \geq H. \tag{20}$$

Let z_0 be the solution to Equation (20). Then, following the rating $R = 0$, the firm will default early if $\theta \leq z_0$.

Suppose $R = q$; then, in the equilibrium, all creditors believe that the firm will invest in HR. Because $qF < 1$, all creditors run. Thus, we also have the financial cost $K(\theta) = f(\theta)$. For the firm to choose HR, we must have

$$\frac{pV - qH}{p - q} \leq f(\theta) < H. \quad (21)$$

Denote the solution to the equation $\frac{pV - qH}{p - q} = f(\theta)$ by z_q . Because $f(\theta)$ is decreasing, and $H > \frac{pV - qH}{p - q}$, we have $z_q > z_0$. Then, the firm invests in HR following the rating $R = q$, if and only if $\theta \in (z_0, z_q]$.

Finally, suppose $R = p$; then, in the equilibrium, all patient creditors roll over the debt, because they believe that the firm will invest in VP. This implies that the firm's financial cost is

$$K(\theta) = (1 - \gamma)F + \gamma f(\theta).$$

Denote the solution to the equation $(1 - \gamma)F + \gamma f(\theta) = \frac{pV - qH}{p - q}$ by z_p . Then, the firm invests in VP following the rating $R = p$, if and only if $\theta \geq z_p$. Note that $(1 - \gamma)F + \gamma f(\theta) < f(\theta)$ for all θ . Therefore, $z_q > z_p$.

The arguments above prove Proposition 11 below, which characterizes the equilibria of the model with a self-disciplined CRA.

Proposition 11. *In the model with a self-disciplined CRA, there are multiple equilibria. In particular, for any $\bar{\theta} \in (\max\{z_0, z_p\}, z_q)$, there is an equilibrium in which*

$$R(\theta) = \begin{cases} 0, & \text{if } \theta \leq z_0; \\ q, & \text{if } \theta \in (z_0, \bar{\theta}_1]; \\ p, & \text{if } \theta \in (\bar{\theta}_1, +\infty). \end{cases} \quad (22)$$

In addition, if $z_p < z_0$, for any $\bar{\theta}' \in [z_p, z_0]$, there is an equilibrium in which

$$R(\theta) = \begin{cases} 0, & \text{if } \theta \leq \bar{\theta}'; \\ p, & \text{if } \theta > \bar{\theta}'. \end{cases} \quad (23)$$

Proposition 11 implies great economic effects emerging from the investment verification policy. In fact, because the government commits to verifying the firm's investment, the CRA becomes self-disciplined. Consequently, the self-disciplined CRA's rating can eliminate inefficiencies in the

benchmark model without a CRA, and in the model with a normal CRA to the largest degree. Specifically, Equation (23) shows that when $z_p < z_0$, the CRA can eliminate all inefficiencies: for weak firms, no creditor rolls over the debt and there is zero social loss, while all other firms will choose the socially optimal investment project. Even if $z_p > z_0$, except $\theta \in (z_0, z_p)$ when the firm invests in HR in any event, given all other θ 's, firms attain social optimal allocations.

With a self-disciplined CRA, the rating is unbiased, so creditors will ignore their private signals. That is, a self-disciplined CRA can coordinate creditors' behavior, as shown by Boot, Milbourn, and Schmeits (2006). The self-disciplined CRA model also provides a structural foundation for the strategic complementarity between the CRA and the firm discovered by Manso (2013). For example, given any $\theta \in (\max\{z_0, z_p\}, z_q)$, a rating $R = p$ coordinates all patient creditors' rollover decisions, leading to a firm's low financial cost. Consequently, the firm will invest in VP, which is the high credit quality project. But, for the same θ , if the CRA assigns a rating $R = q$, all creditors will believe that the firm intends to invest in HR, so they will run, which leads to a high financial cost for the firm. Consequently, the firm will invest in HR, the low credit quality project.

7.2 Providing Public Signals

Another key reason for the CRA's adverse effects is the creditors' dispersed beliefs about the firm's fundamentals. One potential policy for reducing the dispersion of creditors' beliefs is to provide precise unbiased public signals. When the public signal are higher-order more precise than each individual creditor's private signal, the belief dispersion problem will be resolved. In particular, if we fix the precision of creditors' private signals and set the precision of the public signal to infinity, the model converges to the model with one public signal only. In this section, we analyze the policy of providing public signals by studying the CRA's role when all investors share a public signal (or a big investor).¹⁶ Such an analysis will also show the roles of creditors' coordination incentives and belief dispersion in our core model.

To formalize the idea, let's assume that the public signal leads to the belief $\theta \sim \mathcal{N}(\theta_s, \alpha^{-1})$. We will consider the case when α is sufficiently large. We maintain the assumption that the measure of patient creditors is $1 - \gamma$ and consider symmetric equilibria. Because patient creditors will

¹⁶In the core model, we assume for simplicity an improper uniform prior. This assumption is without losing any generality, because we consider the case in which investors' private signals are sufficiently precise. Adding a normal prior belief or a normal public signal to the core model will not change the results, but then the ex-ante economic effects of the CRA depend on the prior mean of the firm's fundamentals. In this subsection about the policy of providing public signals, also without losing any generality, we ignore investors' private signals, because we assume the public signal is higher-order more precise than investors' private signals.

employ a symmetric strategy in an equilibrium, the firm's financial cost of investing in either HR or VP will be

$$K(\theta) = \begin{cases} (1 - \gamma)F + \gamma f(\theta), & \text{if patient creditors roll over the debt;} \\ f(\theta), & \text{if patient creditors run.} \end{cases}$$

When creditors choose to roll over the debt, the firm's optimal investment choice is

$$\begin{cases} \text{Default early,} & \text{if } (1 - \gamma)F + \gamma f(\theta) > H; \\ \text{HR,} & \text{if } (1 - \gamma)F + \gamma f(\theta) \in \left(\frac{pV - qH}{p - q}, H \right]; \\ \text{VP,} & \text{if } (1 - \gamma)F + \gamma f(\theta) \leq \frac{pV - qH}{p - q}. \end{cases}$$

Denoting the solution to the equation $(1 - \gamma)F + \gamma f(\theta) = H$ by y_1 and that to the equation $(1 - \gamma)F + \gamma f(\theta) = (pV - qH)/(p - q)$ by y_2 , the firm's optimal investment choice when patient creditors roll over the debt can be written as

$$\begin{cases} \text{Default early,} & \text{if } \theta < y_1; \\ \text{HR,} & \text{if } \theta \in [y_1, y_2); \\ \text{VP,} & \text{if } \theta \geq y_2. \end{cases}$$

Similarly, we denote the solution to the equation $f(\theta) = H$ by y'_1 and that to the equation $f(\theta) = (pV - qH)/(p - q)$ by y'_2 . The firm's optimal investment choice when patient creditors run can be written as

$$\begin{cases} \text{Default early,} & \text{if } \theta < y'_1; \\ \text{HR,} & \text{if } \theta \in [y'_1, y'_2); \\ \text{VP,} & \text{if } \theta \geq y'_2. \end{cases}$$

Because $f(\theta) > (1 - \gamma)F + \gamma f(\theta)$ for any θ , we have $y_1 < y'_1$ and $y_2 < y'_2$. When α is sufficiently large, creditors will mainly rely on the public signal to make the debt rollover decision. Because creditors' behavior determines the firm's investment choice, the public signal and the creditors' behavior determine the CRA's credit rating. Proposition 12 below shows the equilibrium credit rating strategy when all patient creditors receive a homogeneous, precise public signal.

Proposition 12 (Credit Ratings Affected by Public Signals). *There exists $\bar{\alpha} > 0$, such that for all $\alpha > \bar{\alpha}$, the public signal determines the CRA's equilibrium rating strategy. Specifically,*

1. when $\theta_s \geq y'_2$, the CRA will employ the rating strategy $\hat{\theta} = y_1$;

2. when $\theta_s < y_2$, the CRA will employ the rating strategy $\hat{\theta} = y'_1$; and
3. when $\theta_s \in [y_2, y'_2)$, the CRA will set $\hat{\theta} = y_1$ if patient creditors roll over the debt after $R = p$, while the CRA will set $\hat{\theta} = y'_1$ if patient creditors run after $R = p$.

Significantly, we observe in Proposition 12 that, when the public signal is very positive ($\theta_s \geq y'_2$), the CRA employs a laxer rating strategy, meaning that the good rating is a less positive signal. When the public signal is very negative ($\theta_s < y'_2$), the CRA employs a stricter rating strategy, meaning that the good rating is a more positive signal. Such a “substitution” results from creditors relying more heavily on the public signal when making decisions. When the public signal is in the medium range, there will be multiple equilibria: if the creditors roll over the debt, the CRA will employ a more inflated credit rating strategy; and if the creditors refrain from rolling over the debt, the CRA will employ a more conservative rating strategy.

Proposition 12, together with the analysis above about the firm’s investment decisions, shows that when the government provides extremely precise public signals, the CRA does not have adverse effects. In fact, it has almost no effect on social welfare. This is so because creditors mainly rely on the public signal to make investment decisions.

This policy of providing public signals differs from the firm investment verification policy, which pressures the CRA to provide more accurate information, and thereby enhances the CRA’s informational role. Yet the policy of providing public signals eliminates the CRA’s adverse effects by weakening the CRA’s informational role.

8 Conclusion

The current “issuer-pays” business model prevailing in the credit rating industry leads to conflicts of interest, which prevent CRAs from sending a perfect signal about issuers’ credit quality. This is the center of most criticisms of credit ratings, both on mortgage-backed securities and on corporate bonds. However, different from mortgage-backed securities whose credit qualities are exogenously given, corporate bonds’ credit qualities are endogenously determined by firms’ investment choices, which result from creditors’ investment decisions and thus CRAs credit ratings.

In this paper, we analyze the informativeness and real effects of corporate credit ratings, where there is striking feedback between credit ratings and credit qualities of securities under assessments. We model a CRA as an certified expert who are persuading creditors to roll over the firm’s short-run debts, and our model captures two important features of corporate credit ratings. First, because some of the firm’s investment decisions can be observable and verifiable, the CRA’s

rating is subject to a partial verifiability constraint. Second, creditors in the corporate bonds market have coordination incentives and dispersed beliefs.

We show that credit ratings, though commonly known to be possibly inflated, are still informative positive signals, so they dramatically change creditors' and the firm's behavior. Inflated credit ratings may lead to adverse effects on social welfare because of the partial verifiability constraint and investors' belief dispersion.

We emphasize that credit rating standards and credit rating inflation are two different concepts, and they are both endogenously determined. Therefore, changes of economic environments that lead to laxer rating strategies do not necessarily cause higher rating inflation.

Our paper offers applied and theoretical contributions. From the applied perspective, we provide a rational framework, enabling us to analyze the causes and consequences of credit rating inflation. Our model generates several testable empirical predictions and two reasonable policy suggestions. The partial verifiability constraint can be applied to many other scenarios, such as financial advising, auditing, marketing, and academic recommendation. From the theoretical perspective, we analyze an expert persuasion model with multiple audiences, who have coordination incentives and dispersed beliefs. More importantly, the expert's message will endogenously affect the fundamentals signaled by the message, which may motivate new research on general theoretical persuasion models.

A Proofs of lemmas and propositions

Proof of Proposition 1. To show there is a unique equilibrium in this benchmark model, we only need to show that there is a unique solution $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$ to Equations (8), (9), and (10).

We first solve \tilde{x} from Equation (9). Define

$$\tilde{\Delta} = \frac{\frac{pV-qH}{p-q} - \left[(1-\gamma)F + \gamma f(\tilde{\theta}_2) \right]}{(1-\gamma) \left(f(\tilde{\theta}_2) - F \right)}.$$

Because $f(\theta)$ is strictly decreasing, $\tilde{\Delta}$ is strictly increasing in $\tilde{\theta}_2$. Then we have

$$\tilde{x} = \tilde{\theta}_2 + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\tilde{\Delta}), \quad (24)$$

and \tilde{x} is strictly increasing in $\tilde{\theta}_2$.

Plugging \tilde{x} as a function of $\tilde{\theta}_2$ into Equation (10), we have

$$\tilde{\Delta}(pF - qF) + \Phi \left[\sqrt{\beta} \left(\tilde{\theta}_2 - \tilde{\theta}_1 + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\tilde{\Delta}) \right) \right] qF = 1. \quad (25)$$

The left-hand side of Equation (25) strictly increases in $\tilde{\theta}_2$ and strictly decreases in $\tilde{\theta}_1$. So, we have $\partial \tilde{\theta}_2 / \partial \tilde{\theta}_1 > 0$ and $\partial \tilde{x} / \partial \tilde{\theta}_1 > 0$.

Let's finally consider Equation (8). The derivative of the left-hand side of Equation (8) is

$$\frac{\partial K}{\partial \tilde{\theta}_1} + \frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1},$$

where

$$\begin{aligned} \frac{\partial K}{\partial \tilde{\theta}_1} &= \left[\gamma + (1-\gamma) \Phi \left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) \right] f'(\tilde{\theta}_1) \\ &\quad - (1-\gamma) \sqrt{\beta} \phi \left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) \left(f(\tilde{\theta}_1) - F \right) < 0; \\ \frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} &= (1-\gamma) \sqrt{\beta} \phi \left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) \left(f(\tilde{\theta}_1) - F \right) \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0. \end{aligned}$$

Note that $\tilde{\Delta}$ is between 0 and 1. From Equation (25), we have $\beta \rightarrow +\infty$, $\tilde{\Delta}$ is bounded away from both 0 and 1. To see this, suppose $\tilde{\Delta} \rightarrow 1$ first. Then, the left-hand side of Equation (25) goes to pF , which is greater than 1, the right-hand side of Equation (25). Similarly, if $\tilde{\Delta} \rightarrow 0$, the left-hand side of Equation (25) is strictly less than 1.

Hence, from Equation (24), $\tilde{x} \rightarrow \tilde{\theta}_2$. In addition, as $\beta \rightarrow +\infty$, $\tilde{\theta}_2$ cannot converge to $\tilde{\theta}_1$; otherwise, Equation (8) and Equation (9) cannot hold at the same time. Therefore, as $\beta \rightarrow +\infty$, $\tilde{x} - \tilde{\theta}_1$ is bounded away from 0. This implies that

$$\lim_{\beta \rightarrow +\infty} \sqrt{\beta} \phi \left(\sqrt{\beta} (\tilde{x} - \tilde{\theta}_1) \right) = 0.$$

Therefore, though $\frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0$, when β is large enough, $\frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1}$ is very close to 0. For the term of $\frac{\partial K}{\partial \tilde{\theta}_1}$, it will not go to 0 as β goes to ∞ , because $f'(\tilde{\theta}_1) < 0$. Therefore, there is a $\tilde{\beta} > 0$, such that for all $\beta > \tilde{\beta}$, the left-hand side of Equation (8) strictly decreases in $\tilde{\theta}_1$. The left-hand side of Equation (8) converges to F when $\tilde{\theta}_1$ goes to $+\infty$ and diverges to $+\infty$ when $\tilde{\theta}_1$ goes to $-\infty$. Then by the continuity of function $f(\cdot)$, there is a unique $\tilde{\theta}_1$. Then, there is a unique solution to Equation (8), (9), and (10). □

Proof of Lemma 1. Suppose there is an equilibrium, in which the firm will the fundamentals θ does not default early. So the CRA will assign the rating $R(\theta) = p$, because the CRA wants to maximize the nominal rating $R(\theta)$. Let $W(p)$ be the measure of creditors who choose to roll over, after observing the credit rating $R(\theta)$ and their own private signals. Then the assumption that θ -firm does not default early implies

$$K(\theta) = W(p, \theta)F + (1 - W(p, \theta)) f(\theta) < H.$$

Now, let's consider any θ' -firm with $\theta' > \theta$. Again, because the CRA wants to maximize the nominal rating $R(\theta')$, if and only if the θ' -firm does not default early, the CRA will assign $R(\theta') = p$. In a monotone equilibrium, any creditor i 's strategy is monotonic in his private signal x_i , and any creditor's private signal conditional on θ' First-order Stochastic dominates that conditional on θ . So $W(p, \theta') > W(p, \theta)$. Recalling that $f(\theta) > F$ for all θ , we have

$$\begin{aligned} K(\theta') &= W(p, \theta')F + (1 - W(p, \theta')) f(\theta') \\ &< W(p, \theta)F + (1 - W(p, \theta)) f(\theta') \\ &< W(p, \theta)F + (1 - W(p, \theta)) f(\theta) \\ &< H. \end{aligned}$$

Therefore, if θ -firm does not default early, θ' -firm does not default early either, implying that in the equilibrium, $R(\theta') = p$.

Furthermore, independent of creditors' decisions, when θ is very negative, the firm will default early, and when θ is very positive, the firm will not default early. As a result, in any equilibrium (if exists), the CRA's rating strategy must be in the form described by Equation (12). \square

Proof of Lemma 2. Suppose there is an equilibrium in which the firm invests in VP for all θ such that $R(\theta) = p$. All patient creditors will roll over the debt, leading to the firm's financial cost

$$\gamma f(\theta) + (1 - \gamma)F.$$

For the firm to choose VP if and only if $\theta \geq \hat{\theta}$, we must have

$$(1 - \gamma)F + \gamma f(\hat{\theta}) \leq \frac{pV - qH}{p - q} < H.$$

But because $f(\cdot)$ is continuous and strictly decreasing, there exists $\hat{\hat{\theta}} < \hat{\theta}$ such that,

$$\frac{pV - qH}{p - q} < \gamma f(\hat{\hat{\theta}}) + (1 - \gamma)F < H.$$

That is, there is a positive measure subset of θ 's that are greater than $\hat{\hat{\theta}}$ but very close to $\hat{\hat{\theta}}$, the firm will invest in HR. Since the firm's investment choice HR is unverifiable, a deviation to the rating strategy with $\hat{\hat{\theta}}$ is profitable to the CRA. Therefore, the rating strategy with $\hat{\theta}$ such that $(1 - \gamma)F + \gamma f(\hat{\theta}) \leq \frac{pV - qH}{p - q}$ cannot be part of an equilibrium. Therefore, if an equilibrium exists, in an equilibrium, the rating strategy must be inflated. \square

Proof of Lemma 3. For a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the left-hand side of Equation (14) is strictly decreasing in θ . When $\theta \rightarrow +\infty$, the LHS of Equation (14) goes to F , which is assumed strictly less than $\frac{pV - qH}{p - q}$. However, if when $\theta = \hat{\theta}$, the LHS is still greater than $\frac{pV - qH}{p - q}$, the firm will always choose VP after the rating $R = p$. This contradicts Lemma 2. Therefore, for a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, there is a unique $\theta^* > \hat{\theta}$, such that Equation (14) holds. Then we can solve for x^* from Equation (14)

$$x^* = \theta^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} \left[\frac{\frac{pV - qH}{p - q} - ((1 - \gamma)F + \gamma f(\theta^*))}{(1 - \gamma)[f(\theta^*) - F]} \right].$$

Denote

$$\Delta = \frac{\frac{pV-qH}{p-q} - ((1-\gamma)F + \gamma f(\theta^*))}{(1-\gamma)[f(\theta^*) - F]},$$

so $x^* = \theta^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)$. Because $f(\cdot)$ is strictly decreasing, Δ is strictly increasing in θ^* , and thus x^* is strictly increasing in θ^* .

Then, plugging x^* as a function of θ^* into Equation (15), we have

$$\frac{\Delta}{\Phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right]} (pF - qF) = 1 - qF. \quad (26)$$

Differentiating the left-hand side of Equation (26), the sign of this derivative would be the same as the sign of

$$\begin{aligned} & \frac{\partial \Delta}{\partial \theta^*} \frac{\Phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right]}{\Delta} - \phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \left(\sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \theta^*} \right) \\ &= \frac{\partial \Delta}{\partial \theta^*} \frac{pF - qF}{1 - qF} - \phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \left(\sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \theta^*} \right). \end{aligned}$$

The first term is positive for any β . The second, though is negative, will converge to 0 as $\beta \rightarrow +\infty$. This is because $\phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right]$ will converge to 0 higher order faster than $\sqrt{\beta}$. We need to consider three cases to prove this argument. First, as $\beta \rightarrow +\infty$, $\Delta \rightarrow 1$. In this case, it is trivially that $\phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \sqrt{\beta}$ converges to 0. Second, as $\beta \rightarrow +\infty$, Δ is bounded away from both 0 and 1. Then $x^* - \theta^* \rightarrow 0$. But because $\theta^* - \hat{\theta}$ is positive and bounded away from 0, $\phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \sqrt{\beta} = \phi \left[\sqrt{\beta}(x^* - \hat{\theta}) \right] \sqrt{\beta}$ must converge to 0. Finally, as $\beta \rightarrow +\infty$, $\Delta \rightarrow 0$. Then from Equation (26), we must have $\Phi \left[\sqrt{\beta}(x^* - \hat{\theta}) \right] \rightarrow 0$ and thus $\sqrt{\beta}(x^* - \hat{\theta}) \rightarrow -\infty$ as $\beta \rightarrow +\infty$. By L'Hôpital's rule, we have

$$\lim_{\beta \rightarrow +\infty} \frac{1}{\sqrt{\beta}(x^* - \hat{\theta})} = \lim_{\beta \rightarrow +\infty} \frac{\beta^{-\frac{1}{2}}}{(x^* - \hat{\theta})} = \lim_{\beta \rightarrow +\infty} \frac{1}{2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta}} = 0$$

Therefore, $\lim_{\beta \rightarrow +\infty} 2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta} = +\infty$. Then, simple algebra can lead to the result that

$\phi \left[\sqrt{\beta} \left(\theta^* - \hat{\theta} + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \sqrt{\beta}$ converges to 0, as $\beta \rightarrow +\infty$. Therefore, there is a $\hat{\beta}$ such that when $\beta > \hat{\beta}$, the left-hand side of Equation (26) is strictly increasing in θ^* .

Note by definition, Δ must be a number in $[0, 1]$. Therefore, there are $\bar{\theta}$ and $\underline{\theta}$ such that, $\hat{\theta} < \underline{\theta} < \bar{\theta} < +\infty$, $\Delta(\bar{\theta}) = 1$, and $\Delta(\underline{\theta}) = 0$. Then when $\theta^* \rightarrow \bar{\theta}$, the left hand side of Equation (26) is strictly greater than $1 - qF$; when $\theta^* \rightarrow \underline{\theta}$, the left hand side of Equation (26) is close to 0 and thus strictly smaller than $1 - qF$.

Therefore, there is a unique θ^* , and thus there is a unique x^* . □

Proof of Lemma 4. The left-hand side of Equation (26) is strictly increasing in $\hat{\theta}$, fixing θ^* . Combined with the fact that the left-hand side of Equation (26) is strictly increasing in θ^* , the Implicit Function Theorem implies that θ^* is strictly decreasing in $\hat{\theta}$. Since x^* is strictly increasing θ^* , x^* is strictly decreasing in $\hat{\theta}$ (given θ^* , x^* is determined by Equation $x^* = \theta^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}[\Delta]$). □

Proof of Proposition 3. When β is sufficiently large, Proposition 3 shows that, for a fixed $\hat{\theta}$, there is a unique solution, (x^*, θ^*) , to Equation (14) and Equation (15) following $R = p$. Then, by Lemma 4, we only need to show that there is a unique $\hat{\theta}$ such that Equation (16) holds, given x^* as a function of $\hat{\theta}$. When β is sufficiently large, by Lemma 4, x^* is strictly decreasing in $\hat{\theta}$. Then, the derivative of the left hand side of Equation (16) with respect to θ is

$$\begin{aligned} & \left[\gamma + (1 - \gamma)\Phi[\sqrt{\beta}(x^* - \theta)] \right] f'(\theta) \\ & - (1 - \gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}(f(\theta) - F) + (1 - \gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}\frac{\partial x^*}{\partial \theta} < 0. \end{aligned}$$

We know that when $\theta \rightarrow +\infty$, $f(\theta) \rightarrow F$, the left hand side of Equation (16) converges to F , which is less than H ; when $\theta \rightarrow -\infty$, $f(\theta) \rightarrow +\infty$, the left hand side of Equation (16) diverges to $+\infty$, which is greater than H . Therefore, the solution to Equation (16) exists and is unique.

Because $H > \frac{pV - qH}{p - q}$, Equation (16) and Equation (14) imply that $\theta^* > \hat{\theta}$. In addition, Equation (16) also implies that $f(\hat{\theta}) > H$, because $f(\theta) > F$ for all $\theta \in \mathbb{R}$. These complete the proof of the uniqueness of the equilibrium of the model. □

Proof of Proposition 5. Recall that the three equations determining the equilibrium of the benchmark model are

$$[(1 - \gamma)F + \gamma f(\theta_1)] + (1 - \gamma)\Phi[\sqrt{\beta}(x - \theta_1)](f(\theta_1) - F) = H \quad (27)$$

$$[(1 - \gamma)F + \gamma f(\theta_2)] + (1 - \gamma)\Phi[\sqrt{\beta}(x - \theta_2)](f(\theta_2) - F) = \frac{pV - qH}{p - q} \quad (28)$$

$$\left\{ \Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)] \right\} qF + \left\{ 1 - \Phi[\sqrt{\beta}(\theta_2 - x)] \right\} pF = 1; \quad (29)$$

and the three equations determining the equilibrium of the model with the CRA are

$$[(1 - \gamma)F + \gamma f(\theta_1)] + (1 - \gamma)\Phi \left[\sqrt{\beta}(x - \theta_1) \right] (f(\theta_1) - F) = H \quad (30)$$

$$[(1 - \gamma)F + \gamma f(\theta_2)] + (1 - \gamma)\Phi \left[\sqrt{\beta}(x - \theta_2) \right] (f(\theta_2) - F) = \frac{pV - qH}{p - q} \quad (31)$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} pF = 1 \quad (32)$$

The difference between the equilibrium in the benchmark model and that in the model with the CRA stems from the difference between Equation (29) and Equation (32). That is, the creditors' indifference conditions differ. If we change Equation (29) by dividing both sides by the term $1 - \Phi[\sqrt{\beta}(\theta_1 - x)]$, we have

$$\begin{aligned} & \frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} pF \\ &= \frac{1}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} \\ &> 1. \end{aligned} \quad (33)$$

Solve x as a function of θ_2 from Equation (28) or Equation (31), and plug it into Equation (33) and Equation (32). Then, for a same θ_1 , θ_2 in Equation (33) is greater than that in Equation (32), because the left-hand sides of these two equations are strictly increasing in θ_2 . Hence, x in Equation (33) is greater than x in Equation (32). Furthermore, because θ_1 in the benchmark model is positively correlated to θ_2 , while θ_1 in the model with the CRA is negatively correlated to θ_2 , we know $\hat{\theta} < \tilde{\theta}_1$. Moreover, we have $\tilde{\theta}_2 > \theta^*$ and $\tilde{x} > x^*$.

However, the sign of $\theta^* - \tilde{\theta}_1$ is undetermined. Consider Equation (27) and Equation (31). Both $\tilde{\theta}_1$ and θ^* are strictly increasing functions of x . While we have shown that $\tilde{x} > x^*$, the right-hand side of Equation (27) is greater than that of Equation (31). Therefore, without specifying parameters' values, we cannot determine the sign of $\theta^* - \tilde{\theta}_1$. □

Proof of Proposition 6. In both cases, when $\theta < \hat{\theta}$, $\Omega = 0 > \gamma + (1 - \gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)) - 1 = \tilde{\Omega}$. So, in both cases, when $\theta < \hat{\theta}$, the CRA leads to a higher social welfare. When $\theta \geq \tilde{\theta}_2$, $\Omega = \tilde{\Omega} = pV - 1$, so the CRA has no effect on the social welfare.

In case 1, where $\theta^* > \tilde{\theta}_1$, for $\theta \in [\hat{\theta}, \tilde{\theta}_1)$, $\Omega = qH - 1 < \gamma + (1 - \gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)) - 1$, because qH is assumed to be less than γ . For $\theta \in [\theta^*, \tilde{\theta}_2)$, $\Omega = pV - 1 > qH - 1 = \tilde{\Omega}$. For all other θ 's,

the firm's investment choice is the same in both the benchmark model and the model with the CRA; thus, CRA has no effect on the social welfare for such firms.

In case 2, where $\theta^* \leq \tilde{\theta}_1$, for $\theta \in [\hat{\theta}, \theta^*)$, $\Omega = qH - 1 < \gamma + (1 - \gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)) - 1$, because qH is assumed to be less than γ . For $\theta \in [\theta^*, \tilde{\theta}_2)$, $\Omega = pV - 1 > \gamma + (1 - \gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)) - 1 > qH - 1 = \tilde{\Omega}$. For all other θ 's, the firm's investment choice is same in both the benchmark model and the model with the CRA; thus, CRA has no effect on the social welfare for such firms. □

Proof of Lemma 5. Suppose there is an equilibrium in which the CRA commits to a rating strategy R with $R(\theta) = q$ for all $\theta \in (\theta', \theta'')$, where $\theta'' > \theta'$. Because $R(\theta)$ is increasing, without losing any generality, we assume the equilibrium rating strategy is

$$R(\theta) = \begin{cases} 0, & \text{if } \theta < \theta'; \\ q, & \text{if } \theta \in [\theta', \theta'']; \\ p, & \text{if } \theta > \theta''. \end{cases} \quad (34)$$

Because we consider monotone equilibrium in subgames following any credit rating, we denote by (θ_q, x_q) the equilibrium in the subgame following the credit rating $R = q$.

Suppose $x_q = +\infty$. That is, no creditor rolls over when the CRA assigns a rating $R = q$ to the firm. However, the firm that receives such a rating does not default, because of the partial verifiability constraint. Hence, $f(\theta) \leq H$ for all $\theta \in [\theta', \theta'']$. Then, if the CRA deviates to a rating strategy R' with $R'(\theta) = p$ when $\theta \geq \theta'$ and $R'(\theta) = 0$ when $\theta < \theta'$, the firm that is assigned a rating p will not default early, since $f(\theta) \leq H$. Hence, such a deviation does not violate the partial verifiability constraint and thereby is a profitable deviation.

Now, suppose $x_q = -\infty$; that is, all patient creditors roll over. This implies that

$$K(\theta') = \gamma f(\theta') + (1 - \gamma)F \leq \frac{pV - qH}{p - q},$$

which in turn implies that $K(\theta) \leq \frac{pV - qH}{p - q}$ for all $\theta \geq \theta'$. Let's again consider the deviation to the rating strategy R' with $R'(\theta) = p$ for all $\theta \geq \theta'$ and $R'(\theta) = 0$ for all $\theta < \theta'$. Given such a committed rating strategy, when the firm is assigned the rating $R' = p$, all investors know that $K(\theta) \leq \frac{pV - qH}{p - q}$. So all patient investors believe that the firm will invest in the VP, if they all roll over. Hence, following the rating $R' = p$, all investors roll over, and the firm with the fundamentals $\theta \geq \theta'$ will not default early. Hence, R' is a profitable deviation.

Finally, suppose $x_q \in \mathbb{R}$, then $\theta_q \in (\theta', \theta'')$. Because the creditor with the private signal x_q is the marginal creditor, following the credit rating $R = q$, her indifference condition is

$$\begin{aligned} & \frac{\Phi \left[\sqrt{\beta + \mu} \left(\theta_q - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right] - \Phi \left[\sqrt{\beta + \mu} \left(\theta' - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right]}{\Phi \left[\sqrt{\beta + \mu} \left(\theta'' - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right] - \Phi \left[\sqrt{\beta + \mu} \left(\theta' - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right]} qF \\ & + \frac{\Phi \left[\sqrt{\beta + \mu} \left(\theta'' - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right] - \Phi \left[\sqrt{\beta + \mu} \left(\theta_q - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right]}{\Phi \left[\sqrt{\beta + \mu} \left(\theta'' - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right] - \Phi \left[\sqrt{\beta + \mu} \left(\theta' - \frac{\beta x_q + \mu \bar{\theta}}{\beta + \mu} \right) \right]} pF = 1 \end{aligned} \quad (35)$$

The θ_q -firm's indifference condition is

$$\left[\gamma + (1 - \gamma) \Phi \left(\sqrt{\beta} (x_q - \theta_q) \right) \right] F + (1 - \gamma) \left[1 - \Phi \left(\sqrt{\beta} (x_q - \theta_q) \right) \right] f(\theta) = \frac{pV - qH}{p - q}. \quad (36)$$

From Equation (36), we can solve x_q as an strictly increasing function of θ_q . Substitute such a function into Equation (35), and apply the implicit function theorem, we can show that both x_q and θ_q are strictly decreasing in θ'' . That is, if the CRA deviates from the rating strategy R by increasing θ'' , more investors will choose to roll over. This implies that when the CRA deviates to the rating strategy R' with $R'(\theta) = p$ for all $\theta \geq \theta'$ and $R'(\theta) = 0$ for all $\theta < \theta'$, the investors' rollover threshold decreases. Therefore, given the committed rating strategy R' , following the rating $R' = p$, more investors will roll over, so the financial cost of the firm with the fundamental $\theta = \theta'$ decreases. Given that the θ' -firm does not default early following the rating $R = q$ under the rating strategy R , it does not default early following the rating $R' = p$ under the rating strategy R' . Consequently, for any $\theta \geq \theta'$, θ -firm does not default early following $R' = p$ given the committed rating strategy R' . Therefore, R' is a profitable deviation.

In sum, the rating strategy described in Equation (34) cannot be an equilibrium rating strategy. \square

Proof of Proposition 7. In this proof, we denote the committed rating strategy by R' and the equilibrium rating strategy without commitment by R .

Suppose the equilibrium committed rating strategy R' has the threshold $\theta' > \hat{\theta}$. Then, the CRA can profitably deviate to the rating strategy R with the threshold $\hat{\theta}$: given the rating strategy R , no firm with the rating $R = p$ will default early, and the CRA assigns the high rating to a superset of θ' 's.

Now, suppose the equilibrium committed rating strategy has the threshold $\theta' < \hat{\theta}$. From Lemma 4, investors' rollover threshold following the rating $R' = p$ is $x' > x^*$. This implies that

following the rating $R' = p$, fewer investors roll over, and thus the firm's financial cost is higher under the committed rating strategy R' . Therefore, θ' -firm's financial cost under R' is strictly higher than that of $\hat{\theta}$ -firm under R . However, $\hat{\theta}$ -firm has the financial cost H , implying that θ' -firm's financial cost under R' is strictly greater than H . So when the firm's financial cost is very close to θ' , the firm will default early, which implies that the partial verifiability constraint is violated. So R' with $\theta' < \hat{\theta}$ cannot be an equilibrium rating strategy.

Therefore, in equilibrium rating strategy must have $\theta' = \hat{\theta}$. Indeed, it is an equilibrium that the CRA employs the rating strategy R with the threshold $\hat{\theta}$, and investors and the firm play the equilibrium (x^*, θ^*) . Since $(\hat{\theta}, x^*, \theta^*)$ is an equilibrium when the CRA does not commit to a rating strategy, we conclude that the optimal committed rating strategy is time-consistent. \square

Proof of Proposition 8. Recall that

$$x^* = \theta^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} [\Delta],$$

where

$$\Delta = \frac{\frac{pV - qH}{p - q} - [(1 - \gamma)F + \gamma f(\theta^*)]}{(1 - \gamma)[f(\theta^*) - F]},$$

so $\partial\Delta/\partial\theta^* > 0$.

Substitute x^* as a function of θ^* into Equation (15) and Equation (16), and denote $\sqrt{\beta}(\theta^* - \hat{\theta}) + \Phi^{-1}(\Delta) = \Psi$ for simplicity, we have

$$\Delta(pF - qF) - \Phi \left[\sqrt{\beta}(\theta^* - \hat{\theta}) + \Phi^{-1}(\Delta) \right] (1 - qF) = 0 \quad (37)$$

$$(1 - \gamma)\Phi \left[\sqrt{\beta}(\theta^* - \hat{\theta}) + \Phi^{-1}(\Delta) \right] \left[f(\hat{\theta}) - F \right] + \gamma f(\hat{\theta}) = H - (1 - \gamma)F \quad (38)$$

Total differentiation of the above two equations with respect to θ^* , $\hat{\theta}$, and β , we have

$$A \begin{bmatrix} \frac{\partial\theta^*}{\partial\beta} \\ \frac{\partial\hat{\theta}}{\partial\beta} \end{bmatrix} = \begin{bmatrix} \phi(\Psi)(1 - qF) \frac{\theta^* - \hat{\theta}}{2\sqrt{\beta}} \\ -(1 - \gamma)\phi(\Psi) \frac{\theta^* - \hat{\theta}}{2\sqrt{\beta}} \left[f(\hat{\theta}) - F \right] \end{bmatrix}, \quad (39)$$

where

$$A = \begin{bmatrix} \frac{\partial\Delta}{\partial\theta^*}(pF - qF) - \phi(\Psi) \left(\sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\theta^*} \right) (1 - qF) & -\phi(\Psi)\sqrt{\beta}(1 - qF) \\ (1 - \gamma)\phi(\Psi)[f(\hat{\theta}) - F] \left(\sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\theta^*} \right) & -(1 - \gamma)\phi(\Psi)[f(\hat{\theta}) - F]\sqrt{\beta} + [\gamma + (1 - \gamma)\Phi(\Psi)]f'(\hat{\theta}) \end{bmatrix} \quad (40)$$

As we have shown in the proofs of Lemma 3, when β is large enough, $\phi(\Psi)\sqrt{\beta}$ is very close to 0. Therefore, when β is sufficiently large, the determinant of the matrix A is close to

$$\frac{\partial\Delta}{\partial\theta^*}(pF - qF) [\gamma + (1 - \gamma)\Phi(\Psi)] f'(\hat{\theta}) < 0,$$

because $f'(\hat{\theta}) < 0$.

Then further algebra shows that when β is sufficiently large, the sign of $\partial\hat{\theta}/\partial\beta$ is the same as that of

$$\frac{\partial\Delta}{\partial\theta^*} \frac{pF - qF}{1 - qF},$$

which is positive. Therefore, $\hat{\theta}$ is strictly increasing in β . □

Proof of Proposition 9. Total differentiation of Equation (37) and Equation (38) with respect to θ^* , $\hat{\theta}$, and H , we have

$$A \begin{bmatrix} \frac{\partial\theta^*}{\partial H} \\ \frac{\hat{\theta}}{\partial H} \end{bmatrix} = \begin{bmatrix} -(pF - qF) \frac{\partial\Delta}{\partial H} + \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial H} (1 - qF) \\ 1 - (1 - \gamma) \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial H} [f(\hat{\theta}) - F] \end{bmatrix},$$

where A is defined in Equation (40).

Note that $\phi(\Psi)\sqrt{\beta}$ is very close to 0 when β is sufficiently large, we have

$$\text{sign} \begin{bmatrix} \frac{\partial\theta^*}{\partial H} \\ \frac{\hat{\theta}}{\partial H} \end{bmatrix} = \text{sign} \begin{bmatrix} [\gamma + (1 - \gamma)\Phi(\Psi)] f'(\hat{\theta}) \frac{\partial\Delta}{\partial H} \\ -\frac{\partial\Delta}{\partial\theta^*} \end{bmatrix}.$$

Because $f'(\hat{\theta}) < 0$, $\partial\Delta/\partial H < 0$, and $\partial\Delta/\partial\theta^* > 0$, we have $\partial\hat{\theta}/\partial H < 0$ and $\partial\theta^*/\partial H > 0$. Therefore, $\partial(\theta^* - \hat{\theta})/\partial H > 0$. □

Proof of Proposition 10. Similar to the proof of Proposition 9, the total differentiation of of Equation (37) and Equation (38) with respect to θ^* , $\hat{\theta}$, and γ , we have

$$A \begin{bmatrix} \frac{\partial\theta^*}{\partial H} \\ \frac{\hat{\theta}}{\partial H} \end{bmatrix} = \begin{bmatrix} -(pF - qF) \frac{\partial\Delta}{\partial\gamma} + \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\gamma} (1 - qF) \\ - \left[1 - \Phi(\Psi) - (1 - \gamma) \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\gamma} \right] (f(\hat{\theta}) - F) \end{bmatrix},$$

where A is defined in Equation (40).

Note that $\phi(\Psi)\sqrt{\beta}$ is very close to 0 when β is sufficiently large. In addition, $(1 - \Phi(\Psi))/\phi(\Psi) \rightarrow 0$ when $\beta \rightarrow +\infty$ and $\partial\Delta/\partial\gamma > 0$. Then simple algebra will show that $\partial\hat{\theta}/\partial\gamma > 0$ and $\partial\theta^*/\partial\gamma > 0$. Furthermore, because when β is sufficiently large, $\partial\hat{\theta}/\partial\gamma$ is close to 0, while $\partial\theta^*/\partial\gamma > 0$ is bounded away from 0, we have $\partial(\theta^* - \hat{\theta})/\partial\gamma > 0$. □

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