Abstract

I consider a New Keynesian model of a small open economy where international financial markets are imperfect and the exchange rate is determined by capital flows. I use this framework to study the effects of exchange rate fluctuations driven by capital flows and characterize the optimal foreign exchange intervention. Capital flows shocks cause inefficient exchange rate fluctuations that trigger boom-bust cycles in the domestic economy. The optimal policy response is to partially stabilize these fluctuations using both foreign exchange intervention and monetary policy. The optimal foreign exchange intervention leans against the wind and stabilizes the path of the exchange rate: following an increase (decrease) in the foreign demand for domestic assets, the central bank increases (decreases) their net supply and accumulates (de-cumulates) foreign reserves. By doing so the central bank stabilizes the path of the exchange rate and smooths out fluctuations in domestic consumption. Simultaneously, the central bank reduces (increases) the nominal interest rate in order to reduce the relative price of domestic goods and mitigate the output gap. Foreign exchange intervention is not a mere substitute for monetary policy. Rather, the two tools complement each other.


Keywords: Capital Flows, Foreign Exchange Intervention, Monetary Policy
Flexible exchange rates have been praised in economic theory as a mechanism that facilitates relative price adjustments between countries and helps them achieve efficient outcomes. The countercyclical response of exchange rates to relative demand and supply shocks triggers expenditure-switching effects that smooth out output volatility and stabilizes the economy. According to this view, fluctuations in the real exchange rate are efficient responses to fundamental shocks and must therefore be welcomed. However, an alternative explanation for fluctuations in the real exchange rate is the presence of shocks arising in international financial markets that move the nominal exchange rate and are then passed on to the real exchange rate. These fluctuations could drive the real exchange rate away from its efficient level and be the source of, rather than the cure for, economic disruption.

The view that exchange rates might deviate from their efficient levels as a consequence of non-fundamental shocks has a long tradition among policymakers. It was certainly at the heart of the Plaza and Louvre agreements and it is probably a component of the well known “fear of floating” documented by Calvo and Reinhart (2002) in emerging economies. This irreconcilable dichotomy lies at the core of the debate around the exchange rate policies of emerging economies, and it is the reason why the consensus among policymakers has frequently changed. The 1970s and 1980s were characterized by a large variety of pegs while the bipolar (float or fix) view was predominant during the 1990s. However, following the series of balance of payments crises that plagued emerging markets in the 1990s, many policymakers have settled for a floating exchange rate regime combined with inflation targeting monetary policy. This consensus seems to be on the verge of shifting once more.

The evolution of financial integration and the increased openness of both advanced and emerging economies to international capital flows have raised new concerns among policymakers about exchange rate fluctuations driven by financial forces rather than fundamentals. As the participation of foreign investors in domestic asset markets increases, their portfolio allocation decisions become significant drivers of overall capital flows. Shifts in foreign investors’ portfolios can drive exchange rates away from their efficient levels and potentially affect domestic output and inflation. These concerns are especially strong for emerging market economies, where the relative shallowness of foreign exchange markets and the limited volume of trades strengthen the link between capital flow shocks and exchange rate volatility.

The volatility of international capital flows has increased substantially since the global financial crisis of 2008-2010. To lean against the wind of capital inflows, many policymakers around the world have relied on heterodox policies such as non-conventional monetary policies, capital controls and foreign exchange interventions. According to the IMF\(^1\), the last decade has seen an

\(^1\)International Monetary Fund (2012), Annual Report on Exchange Arrangements and Exchange Restrictions, Washington, DC.
increasing number of countries actively managing their exchange rates. Brazil, Chile, Colombia, Turkey, and other emerging markets with announced inflation targeting regimes have increased both the frequency and the size of their interventions.\(^2\) Along them, also developed economies such as Israel and Switzerland have engaged in considerable intervention of their exchange rates and have accumulated substantial reserves. \(^3\)

Exchange rate interventions are typically attributed to precautionary or mercantilist motives. Accumulating foreign reserves might provide a cushion of liquidity if the flow of short-term debt is interrupted. Alternatively, currency interventions can be used to undervalue the domestic currency and boost the competitiveness of the export sector. Recent evidence (see for example Adler and Mora (2011) and Daude et al. (2014), among others) documents the prevalence of an alternative reason for intervening: that of stabilizing the exchange rate and reducing its volatility. The decision to intervene is increasingly driven by the goal of limiting what the policymaker perceives as unwarranted deviations of the exchange rates from their fundamental levels. Interventions correlate negatively with exchange rate pressures, i.e. they “lean against the wind”, and positively with foreign financial conditions and capital flows. Central bankers appear to be particularly worried about the negative effects of a strong currency on the competitiveness of their exports and ultimately on domestic output. Hence their attempt to fight non-fundamental appreciating pressures.\(^4\)

However popular, the logic behind such interventions has not been formally tested. The goal of this paper is to characterize the optimal use of sterilized foreign exchange intervention in response to exchange rate fluctuations driven by capital flow shocks. I consider a New Keynesian model of a small open economy, as originally formulated by Gali and Monacelli (2005) and later extended to continuous time by Farhi and Werning (2012). I depart from the assumption of frictionless markets and assume that international financial markets are imperfect. Following the model developed by Gabaix and Maggiori (2014), I assume that agents in each country have limited access to international capital markets. International financiers absorb any imbalance between demand and supply of assets denominated in different currencies and clear the markets. In order to do so they require a premium in the form of expected currency appreciation/depreciation. I use this model to study the effects of shocks to foreign investors’ demand for domestic assets. When international asset markets are imperfect, portfolio flow shocks generate boom-bust cycles in the domestic economy.

---

\(^2\)The Central Bank of Brazil (BC) raised foreign reserves from $54 billion at the end of 2005 to $350 billion at the end of 2011. Source: Central Bank of Brazil

\(^3\)Between March 2008 and August 2011 the Bank of Israel (BOI) accumulated $28.1 billion in foreign reserves, through daily purchases initially of $25 million and later raised to $100 million. From August 2009 to July 2011 the BOI continued to occasionally purchase foreign exchange to bring total reserves to $77.9 billion, an increases of almost 170% with respect to their level in March 2008. Source: Flug and Shpitzer (2013)

\(^4\)Former Israel central bank governor Stanley Fisher remarked: “I have no doubt that the massive purchases [of foreign exchange] we made between July 2008 and into 2010 [...] had a serious effect on the exchange rate which I think is part of the reason that we succeeded in having a relatively short recession”. Source: Levinson (2010)
For example, a positive portfolio flow shock, that is an increase in the foreign demand for domestic assets, appreciates the domestic currency and reduces the domestic real interest rate. Capital flows in the domestic economy and fuel a consumption boom while output falls. The dynamics of consumption and output are reversed during the bust phase of the cycle. When foreign demand for domestic assets return to its steady state level their return increases and the external debt accumulated by domestic households must be reabsorbed. Consumption falls below its steady state level and output rises until the Home country pays off its debt. The optimal foreign exchange intervention leans against the wind and stabilizes the path of the exchange rate. Following an increase in the demand for domestic assets, the central bank increases their net supply and accumulates foreign reserves. By doing so the central bank absorbs part of the capital inflow and reduces its impact on the exchange rate. By leaning against the wind the central bank achieves two purposes. First, it reduces the initial appreciation of the domestic currency and sustains foreign demand, therefore reducing the output gap. This is the “monetary” aspect of the intervention: the central bank depreciates the nominal exchange rate in order to help domestic exporters to reduce their foreign currency prices. Second, by absorbing part of the capital inflows, it increases the domestic real interest rate and smooths out the consumption boom. Smoothing out consumption fluctuations increases welfare as it reduces the output gap caused by the wealth effect on labor supply and it allows the planner to exploit the monopsonistic/monopolistic power that the country has on its own terms of trade. An increase in domestic consumption appreciates the terms of trade. It is never optimal for the central bank to fully stabilize domestic consumption. The reason is that, by creating a wedge between the return of domestic and foreign assets, portfolio flow shocks are akin to positive wealth shocks for the Home country. Foreign exchange interventions are costly since they reduce this wedge and its benefits. In determining the optimal amount of stabilization the central bank trades off the benefits that arise from the positive wealth shock and the inefficiencies caused by the exchange rate fluctuations. The optimal policy mix required to deal with portfolio flow shocks includes both foreign exchange intervention and monetary policy. The two tools are complements rather than substitutes. Foreign exchange intervention is the appropriate tool to smooth out consumption fluctuations as it allows the planner the choose the welfare-maximizing consumption path. However, when foreign exchange intervention is used also to reduce the output gap this leads to a stabilization bias. Monetary policy can help in reducing the output gap and reduce this burden. However, monetary policy can only move the allocation along an inefficient IS curve. Without a state-contingent labor subsidy, foreign exchange intervention is still used to shift the IS curve closer to efficiency. This conclusion is in sharp contrast with the standard view of foreign exchange interventions which emphasizes their monetary aspect.

The focus of this paper is the desirability of foreign exchange intervention in response to currency misalignments arising in financial markets. Questions regarding its effectiveness are the
focus of a vast and lively literature. Most of the studies from the late 80s found that the coor-
dinated interventions, conducted after the Plaza and Louvre agreements, were effective. Dominguez
and Frankel (1993) analyze interventions using data from U.S. dollar, German mark and Swiss
franc between 1982 and 1988. Their study points to the presence of a significant portfolio chan-
nel in the U.S. Dollar and German mark markets. More recently, Sarno and Taylor (2001) survey
central bank interventions conducted in the 90s and conclude that foreign exchange intervention
can be effective especially if it is publicly announced and provided that it is consistent with the
underlying stance of monetary and fiscal policy. Menkhoff (2010) and Menkhoff (2013) survey
the literature focusing on studies that use high-frequency data for both developed and emerging
economies. The author finds that the evidence corroborates the hypothesis that interventions move
the exchange rate level in the desired direction, especially in emerging market economies where
higher reserves and shallow markets give central banks more leverage. Adler and Mora (2011)
study interventions in Latin America between 2004 and 2010. Their quantitative estimation sug-
gests that interventions can be successful in lowering the pace of appreciation but that the effects
decrease with the degree of capital account openness. Although the evidence of its effectiveness is
far from conclusive, it suggests that foreign exchange intervention can be a useful tool for central
banks. The contribution of this paper is to study how to use it.

The rest of the paper is organized as follows: section 1 reviews the relevant literature; section 2
describes the model and its equilibrium; section 3 describes the log-linearized dynamics around the
steady state and the planner problem; in section 4 I study optimal foreign exchange intervention;
section 5 study the interaction between foreign exchange intervention and monetary policy; section
6 concludes.

1 Literature Review

This paper is related to three broad streams of literature: the international macroeconomic literature
on the volatility of capital flows, the literature on international financial intermediation and the
literature on optimal policy and welfare in open economies.

The volatility of capital flows and their effects on open economies are at the center of a large
literature. Starting with the seminal work of Calvo (1998) most of the literature has focused on
“sudden stops”, episodes in which the inflow of foreign capitals suddenly reverses. See for ex-
ample Caballero and Krishnamurthy (2004), Mendoza (2010), Bianchi (2009), Bianchi and Men-
doza (2010), Jeanne and Korinek (2010) and Korinek (2011). These papers emphasize the role of
domestic borrowing constraints and pecuniary externalities in amplifying shocks to the external
financing conditions. More recently Farhi and Werning (2013a) study the effects of foreign risk
premia shocks in a small open economy with nominal rigidities. All these papers focus on capital
controls as a tool to manage capital flows and stabilize the domestic economy. I contribute to this literature by studying the optimal use of an alternative tool, namely sterilized foreign exchange intervention, in response to capital flow shocks. A related strand of literature has focused on the role of capital flows in the transmission of financial conditions across countries. Rey (2013), Agrippino and Rey (2013), Shin (2013) and Blanchard et al. (2014) provide evidence of a global financial cycle driving capital flows, asset prices and credit growth across countries. The cycle might constraints and render ineffective domestic monetary policy if the capital account is not managed. Consistent with this conjecture, I find that monetary policy independence increases when foreign exchange intervention is used to deal with currency misalignments caused by capital flows.

This paper also relates to the literature on international financial intermediation and exchange rate determination in the presence of frictions. The earlier and most prominent strand of the literature on asset demand and exchange rate determination in general equilibrium has focused on complete markets. See for example Lucas Jr (1982), Backus, Kehoe and Kydland (1992), Backus and Smith (1993), Pavlova and Rigobon (2007) Verdelhan (2010), Colacito and Croce (2011), Stathopoulos (2012) and Farhi and Gabaix (2014). An other stream of literature has analyzed the role of market incompleteness in the determination of international portfolios. For recent examples see Chari, Kehoe and McGrattan (2002), Corsetti et al. (2008) and Rigobon and Pavlova (2011).

As shown by Backus and Kehoe (1989) in these classes of models sterilized intervention is not an extra policy instrument available to the central banks. When portfolio decisions are frictionless, the imperfect substitutability between assets postulated by the portfolio balance channel is not enough for sterilized intervention to have an effect on prices and quantities. In light of this result, in this paper I follow a different approach based on frictions in the intermediation process of international capital flows. Important contributions in the study of international financial frictions include Caballero and Krishnamurthy (2001), Jeanne and Rose (2002), Evans and Lyons (2002), Hau and Rey (2006) and Bruno and Shin (2013). A related stream of literature has focused on the effects of domestic financial frictions and market segmentation on the dynamics of portfolio flows and exchange rates. See for example Alvarez et al. (2009), Bacchetta and Van Wincoop (2010) and Maggiori (2012). Bruno and Shin (2014) develop a model of the international banking system where global and local funding constraints interact in the transmission of financial conditions across borders. Gabaix and Maggiori (2014) build an analytically tractable 2-period general equilibrium model where constrained international financiers intermediate capital flows across countries. They provide a novel micro-foundation to the portfolio balance channel and analyze the welfare effects of heterodox policies such as foreign intervention and capital controls. I build on their analysis and incorporate the intermediation friction in a standard New Keynesian small open economy. This allows me to provide a complete characterization of the optimal foreign

---

5The theoretical underpinnings of the portfolio balance channel date back to Kouri (1976) and Kouri (1981).
exchange intervention policy and its interaction with standard monetary policy.

Finally, this paper relates to the literature on optimal policy and welfare in New Keynesian open economies. Aguiar et al. (2009), Schmitt-Grohé and Uribe (2011), Farhi and Werning (2012; 2012b; 2013b) and Farhi et al. (2014) provide innovative analysis of the effects of conventional policies such as capital controls, fiscal transfers and fiscal devaluations on the welfare of a small open economy. This paper is most closely related to the literature on the desirability of stabilizing exchange rates. Much of this literature is concerned with investigating the conditions under which it is optimal to include the exchange rate in the reaction function of monetary policy. See for example Sutherland (2005), Corsetti and Pesenti (2005), Benigno (2009), De Paoli (2009) and the comprehensive review of Engel (2014). Engel (2011) characterizes the optimal response of monetary policy to currency misalignments arising from differences between home and foreign prices due to Local Currency Pricing (LCP). The focus of this paper is on currency misalignments arising in financial markets. There are few existing papers that allow for a non-trivial role for sterilized intervention. Some notable exceptions are Benes et al. (2013), Montoro and Ortiz (2013) and Devereux and Yetman (2014).6 These papers analyze the welfare effects of different exogenously specified intervention policies aimed at stabilizing the exchange rate. My contribution to this literature is to derive the optimal foreign exchange intervention policy and characterize its implementation.

2 A Small Open Economy Model

Consider a continuous-time model with infinite horizon. The world economy is composed of a continuum of measure one of countries indexed by $i \in [0, 1]$. Every country is inhabited by a continuum of measure one of identical households, and each household is composed of a measure one of family members with identical preferences over consumption paths. By assumption, there is perfect consumption insurance within the household because all members pay out their earnings to be shared equally across the entire household. In each country there is also a measure one of monopolistically competitive firms that produce a continuum of differentiated tradable goods. International financial markets are incomplete and segmented. The only assets available in the world economy are a continuum of riskless nominal bonds denominated in different currencies. Each bond pays one unit of the currency of a specific country. Households can freely trade domestic assets, i.e. bonds denominated in domestic currency, but they are constrained in their holdings of foreign assets. Hence, imbalances in financial markets might arise. The excess demand or supply of assets is absorbed, at some premium, by global financial firms. Financial intermediaries are allowed to invest in bonds denominated in different currencies and clear the markets.

---

6 See also the reduced-form model proposed by Ostry et al. (2012).
Uncertainty is represented by a filtered probability space \((\Omega, \mathcal{F}, \{\mathcal{F}(t)\}, \mathbb{P})\) on which is defined a standard \(n\)-dimensional Brownian motion \(Z(t)\) for \(t \in [0, \infty)\). All stochastic processes are assumed adapted to \(\{\mathcal{F}(t)\}_{t=0}^{\infty}\), the augmented filtration generated by \(Z\). In what follows I assume, without stating explicitly, all regularity conditions that ensure all processes introduced are well defined.

Since the focus of this paper is on the policy of a single economy, I will describe the model from the point of view of one country, which I call Home or domestic country, and can be thought of as a particular value \(H \in [0, 1]\). To simplify the analysis I will also assume that all foreign countries are at their symmetric equilibrium at any point in time. I refer to them as rest of the world and denote their variables with a star superscript. This assumption allows me to keep track of only one set of international prices rather than a continuum of bilateral prices.

I will start by defining some relative prices. Define the Home terms of trade as the Home price of imported goods divided by the price of Home goods, such that an increase in the terms of trade represents a deterioration in the Home terms of trade: \(S \equiv P_F/P_H\). Let \(\mathcal{E}\) be the nominal exchange rate between the Home country and the rest of the world, defined as the Home currency price of one unit of any foreign currency, such that a decrease in \(\mathcal{E}\) is an appreciation of the domestic currency. Assume that its law of motion is

\[
\frac{d\mathcal{E}(t)}{\mathcal{E}(t)} = \mu_\mathcal{E}(t) \, dt + \sigma_\mathcal{E}(t) \, dZ(t)
\]

where \(\mu_\mathcal{E}\) and \(\sigma_\mathcal{E}\) are the drift and volatility of \(\mathcal{E}\). Finally, define the real exchange rate as the relative price of one unit of foreign consumption in terms of domestic consumption: \(Q \equiv \mathcal{E}P^*/P\). A decrease in \(Q\) increases the real purchasing power of the Home currency.

In the next sections, I describe in detail the problem facing households and firms located in the Home country. Unless otherwise noted, the problems facing foreign agents are symmetric. I then describe the decision of financial intermediaries and the instruments available to the domestic policymaker.

### Households

The Home country is inhabited by a continuum of measure one of identical households. Each household chooses consumption and hours of labor for all its family members. The representative household maximizes

\[
\mathbb{E} \left[ \int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) \, dt \right]
\]

where \(\rho > 0\) is the time discount factor, \(L\) is the amount of labor supplied by each family member, \(M\) is money holdings and \(P\) is the domestic Consumer Price Index (CPI), such that \(\frac{M}{P}\) is real...
money balances. The consumption index, $C$, is an aggregator of Home and imported goods, given by

$$C_T(t) \equiv \frac{C_H(t)^{1-\alpha} C_F(t)^{\alpha}}{\alpha^\alpha (1 - \alpha)^{1-\alpha}}$$

where $\alpha \in [0, 1]$ measures the degree of Home bias in consumption and is therefore a natural index of the openness of the Home economy. As $\alpha \to 0$ the share of expenditure on foreign goods vanishes and the representative household consumes only goods produced domestically. The economy is effectively closed. As $\alpha \to 1$ the share of expenditure on domestically produced goods vanishes and the representative household consumes only goods produced abroad. Since each country has zero measure, this case describes a very open economy. The imported goods index $C_F$ is itself an aggregator of goods produced in different countries and it is defined by $C_F(t) \equiv \exp \left( \int_0^1 \ln C_i(t) \, di \right)$, where $\varsigma$ is the elasticity of substitution between them. Each country produces a continuum of varieties of both tradable and non tradable goods. Therefore each $C_i$, for $i \in [0, 1]$, and $C_{NT}$ are indexes of consumption of all varieties defined by

$$C_i(t) \equiv \left[ \int_0^1 C_{i,j}(t)^{\frac{\epsilon-1}{\epsilon}} \, dj \right]^{\frac{\epsilon}{\epsilon-1}} \quad i \in [0, 1]$$

$$C_{NT}(t) \equiv \left[ \int_0^1 C_{NT,j}(t)^{\frac{\epsilon-1}{\epsilon}} \, dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ measures the elasticity of substitution across different varieties.

International financial markets are incomplete and segmented. The representative Home household can only invest in three assets: Home currency, a Home bond paying one unit of the domestic currency, and a continuum of foreign bonds each paying one unit of a foreign currency. However, while it can freely trade domestic assets, its holding of foreign bonds is constrained. Let $D(t)$ denote the value of Home bonds held by the representative household and $F(t) \equiv \int_{i \neq H} F_i(t) \, di$ the foreign-currency value of its portfolio of foreign bonds.\(^7\) For simplicity, I will model the demand for foreign assets as an exogenous process. The portfolio flow $F$ can be thought of as an inelastic demand coming from noise family members. It can also be motivated as a liquidity shock or as the result of time-varying portfolio constraints. Understanding the determinants of international portfolios is a key research question in international finance, and the focus of a developing literature\(^8\), but it goes beyond the scope of this paper.

\(^7\)Since foreign countries are identical at any point in time, the actual composition of the portfolio is indeterminate. Only its aggregate value matters.

\(^8\)See for example Pavlova and Rigobon (2010), Rigobon and Pavlova (2011) and Maggiori (2012), among others.
representative household. Its dynamic budget constraint is
\[ dA(t) = (A(t) - M(t)) i(t) dt + \mathcal{E}(t) F(t) \left[ (i^*(t) - i(t)) + \mu_\mathcal{E}(t) \right] dt + \sigma_\mathcal{E}^T(t) d\mathbf{Z}(t) \]
for all \( i \) is the domestic nominal interest rate, \( i^* \) is the nominal interest rate in the rest of the world, \( W \) is the nominal wage, \( \Pi \) represents nominal profits received from domestic firms and \( T \) is a nominal lump-sum component of income which includes all taxes paid to and transfer received from the government.

The optimal allocation of expenditure across different goods yields the following demand functions
\[ P_H(t) C_H(t) = (1 - \alpha) P(t) C(t) \quad P_F(t) C_F(t) = \alpha P(t) C(t) \]
\[ P_i(t) C_i(t) = P_F(t) C_F(t) \quad C_{i,j}(t) = \left( \frac{P_{i,j}(t)}{P_i(t)} \right)^{-\epsilon} C_i(t) \]
\[ C_{H,j}(t) = \left( \frac{P_{H,j}(t)}{P_H(t)} \right)^{-\epsilon} C_H(t) \quad C_{NT,j}(t) = \left( \frac{P_{NT,j}(t)}{P_{NT}(t)} \right)^{-\epsilon} C_{NT}(t) \]
for all \( i \in [0, 1] \) and \( j \in [0, 1] \). Home CPI is defined as \( P \equiv P_H^{1-\alpha} P_F^\alpha \). The Imported goods Price Index (IPI) is \( P_F \equiv \exp \left( \int_0^1 \ln P_i dt \right) \) while the price index for tradable goods produced in country \( i \) is \( P_i \equiv \left( \int_0^1 P_{i,j}^{1-\epsilon} dj \right)^{^{1/\epsilon}} \).

The problem of the representative household is to choose consumption, labor and investment policies to maximize (1) subject to the budget constraint (2), given the exogenous demand for foreign assets \( F \). In what follows I will specialize the period utility function to take the form
\[ U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) \equiv \ln C(t) - \frac{L(t)^{1+\varphi}}{1+\varphi} + \zeta(t) \ln \frac{M(t)}{P(t)} \]
Under this specification the optimal consumption/saving policy is described by the Euler equation
\[ \frac{dC(t)}{C(t)} = \left[ i(t) - \mu_P(t) - \rho + \sigma_P^T(t) \sigma_P(t) + \sigma_C^T(t) \sigma_P(t) + \sigma_C^T(t) \sigma_C(t) \right] dt + \sigma_C^T(t) d\mathbf{Z}(t) \]
where \( \mu_P \) and \( \sigma_P \) are drift and diffusion of the CPI, such that \( dP(t) / P(t) = \mu_P(t) dt + \sigma_P^T(t) d\mathbf{Z}(t) \). The labor supply policy is
\[ L(t)^{\varphi} C(t) = \frac{W(t)}{P(t)} \]
while money demand is given by \( i(t) \frac{M(t)}{\zeta(t)} = C(t) P(t) \).

Foreign households have symmetric preferences and solve a similar problem. With a slight abuse of notation, I denote with \( F^*_H(t) = \int_{i \neq H} F^*_{H,i}(t) \, di \) the aggregate foreign-currency value of Home bonds held by foreign households. As for \( F \), I model foreign demand for domestic assets as an exogenous process. Since each country has measure zero, Home assets represents a negligible share of foreign households’ portfolios.\(^9\) Thus, shocks to the foreign demand for Home bonds do not affect foreign prices nor aggregate quantities.

**Firms**

Each country produces a domestic variety of tradable goods. Each country’s tradable good is produced in a measure one of varieties by a continuum of monopolistically competitive firms, indexed by \( j \in [0, 1] \). Firms use the same technology, described by the production function

\[
Y_{H,j}(t) = L_{H,j}(t)
\]

for \( j \in [0, 1] \). The profits generated by a generic firm \( j \) are given by

\[
d\Pi_{H,j}(t) = \left[ P_{H,j}(t) C_{H,j}(t) + \mathcal{E}(t) P^*_{H,j}(t) C^*_{H,j}(t) - (1 - \tau_H) W(t) L_{H,j}(t) \right] \, dt
\]

where \( P_{H,j} \) is the Home-currency price of good \( j \) when it is sold in the Home country while \( P^*_{H,j} \) is the foreign-currency price of the good when it is sold abroad. \( C_{H,j} \) represents sales of the good in the Home country while \( C^*_{H,j} \) represents its export. The Home policymaker is assumed to have only limited fiscal instruments, a constant sector-specific labor subsidy \( \tau_H \) which is chosen to maximize domestic welfare in steady state.

Firms face an identical isoelastic demand schedule for their own goods and set prices infrequently \( \text{à la Calvo} \) (Calvo (1983)). Each firm is allowed to reset its price only at stochastic dates determined by a Poisson process with intensity \( \theta \). Firms set their prices in domestic currency and the law of one price holds:

\[ P^*_{H,j}(t) \equiv \frac{P_{H,j}(t)}{\mathcal{E}(t)} \]

Under this assumption there is perfect exchange rate pass-through as exchange rate shocks are transmitted one-to-one to import prices.\(^10\) The firm’s objective is to maximize the present discounted value of its stream of profits, subject to the sequence of domestic and foreign demand

---

\(^9\)Provided \( |F^*(t)| < \infty \) for all \( t \in [0, T] \)

\(^10\)This price-setting assumption is also known as Producer Currency Pricing (PCP) in contrast with Local Currency Pricing (LCP). Under the latter, firms choose their domestic and foreign prices independently and the law of one price does not hold.
schedules. A tradable good firm that is allowed to reset its prices at time $t$ solves the following problem

$$\max_{\hat{P}_{H,j}(t)} \mathbb{E} \left[ \int_t^\infty \theta e^{-(\rho+\theta)(k-t)} \frac{C(t) P(t)}{C(k) P(k)} \left\{ \hat{P}_{H,j}(t) Y_{H,j}(k|t) - (1 - \tau_H) W(k) L_{H,j}(k|t) \right\} dk \right]$$

subject to (4), where $Y_{H,j}(k|t) = \left( \frac{\hat{P}_{H,j}(t)}{P_H(k)} \right)^{-\epsilon} Y_H(k)$.

**International Financial Intermediaries**

Households can freely trade domestic assets, i.e. domestic currency and bonds denominated in domestic currency, but they are constrained in their holdings of foreign assets. Hence, imbalances in financial markets might arise. In the global financial markets operate a mass one of financial firms, that can invest in assets denominated in different currencies and are therefore able to absorb any excess demand or supply. In order to do so, however, they require a premium in the form of expected currency appreciation/depreciation. Financial intermediaries are owned by foreign households and managed by their family members.\(^{11}\) Formally, each foreign household owns a continuum of measure one of financial intermediaries, each managed by one of its family members. Each intermediary is specialized in trading assets issued by a specific country. That is, the financial intermediary specialized in country $i$’s assets is restricted to hold a portfolio composed only of such assets and domestic bonds.\(^ {12}\)

In order to keep the analysis tractable I assume that the management of financial firms is a short-term job. At time $t$, family members are randomly assigned to financial intermediaries. Each manager then chooses a self-financed portfolio of domestic assets and assets issued by the country in which her intermediary is specialized in. At time $t + dt$, with $dt \downarrow 0$, they collect profits and pay them back to the household, where they will be shared equally among all family members. Managers are then randomly reassigned to new intermediaries and the cycle starts over. Consider the problem facing the managers of financial intermediaries specialized in trading with the Home country living in country $i$. At time $t$, each of them chooses a portfolio of assets $\{Q_{H,i}^*(t), Q_{i,i}^*(t)\}$ subject to the balance sheet constraint

$$\frac{Q_{H,i}^*(t)}{E(t)} + Q_{i,i}^*(t) = 0$$

where $Q_{H,i}^*$ is the Home-currency value of Home assets and $Q_{i,i}^*$ is the foreign-currency value of

\(^{11}\)This assumption is made only to simplify the exposition and it is without loss of generality.

\(^{12}\)This arrangement guarantees that in each market, domestic households and foreign investors have comparable masses.
domestic assets held by the intermediary. At time $t + dt$, they collect returns and return to their own households. Their instantaneous net portfolio return is

$$dV^*_i(t) = \frac{Q^*_{H,i}(t)}{E(t)}(i(t) - i^*(t) - \mu_E(t) + \sigma^*_E(t) \sigma_E(t)) dt - \frac{Q^*_{H,i}(t)}{E(t)}\sigma^*_E(t) dZ(t)$$

Following Gabaix and Maggiori (2014), I will assume that the borrowing process is subject to an agency friction that limits the size of the balance sheet of the intermediary and prevents perfect arbitrage between assets denominated in different currencies. At time $t$, after taking positions $\{Q^*_{H,i}(t), Q^*_{i,i}(t)\}$, managers can divert borrowed funds at rate $\left[\frac{Q^*_{H,i}(t)}{E(t)}\sigma^*_E(t) \sigma_E(t)\right]^v \Gamma$ per unit of time, where $\Gamma, v \geq 0$.

Hence, the total amount of divertable funds is $\left[\frac{Q^*_{H,i}(t)}{E(t)}\right]^2 \left[\sigma^*_E(t) \sigma_E(t)\right]^v \Gamma dt$. Notice that the manager, and implicitly the household owners of the intermediary, has limited commitment before returns are realized but full commitment after that. At $t + dt$ the financier is not allowed to default and any loss is absorbed by the household. Since that of the financier is a short-term job, and there is perfect consumption insurance within the household, managers will only care about the instantaneous expected return of their portfolio. Hence, they will choose not to divert funds if the following incentive compatibility constraint holds

$$\mathbb{E}[dV^*_i(t)] \geq \left[\frac{Q^*_{H,i}(t)}{E(t)}\right]^2 \left[\sigma^*_E(t) \sigma_E(t)\right]^v \Gamma dt$$

(5)

Creditors correctly anticipate the incentives of the financier to divert funds and are willing to lend as long as 5 holds. Therefore, financiers solve the following problem:

$$\max_{Q^*_{H,i}(t)} \mathbb{E}[dV^*_i(t)] = \frac{Q^*_{H,i}(t)}{E(t)}[i(t) - i^*(t) - \mu_E(t) + \sigma^*_E(t) \sigma_E(t)] dt$$

subject to $\mathbb{E}[dV^*_i(t)] \geq \left[\frac{Q^*_{H,i}(t)}{E(t)}\right]^2 \left[\sigma^*_E(t) \sigma_E(t)\right]^v \Gamma dt$

Since the value function of the manager is linear in $Q^*_{H,i}$, while the credit constraint is convex, the constraint will always bind and the solution to the manager’s problem yields the following demand for Home bonds: $\frac{Q^*_{H,i}(t)}{E(t)} = \frac{i(t) - i^*(t) - \mu_E(t) + \sigma^*_E(t) \sigma_E(t)}{\Gamma\sigma^*_E(t) \sigma_E(t)}$. Since all foreign countries are identical, simple aggregation across countries yields the aggregate financial sector demand schedule for Home

---

13 Since the financial intermediary has no capital, the foreign-currency value of the funds borrowed is always equal to $\left[\frac{Q^*_{H,i}(t)}{E(t)}\right]$, regardless of the positions the manager is taking.

14 For the constraint to make economic sense it must be the case that $\left[\frac{Q^*_{H,i}(t)}{E(t)}\right] \left[\sigma^*_E \sigma_E\right]^v \Gamma \leq 1$. That is, the financier cannot steal more than 100% of the funds borrowed. In what follows I assume that the parameters of the model are such that this condition always holds.
The financiers’ demand for Home assets is increasing in the excess return of Home bonds vis-à-vis foreign bonds while it is decreasing in the volatility of the exchange rate. The parameter \( \Gamma \) determines the size of the balance sheet of the financiers and is therefore a (inverse) measure of their risk-bearing capacity. The higher is \( \Gamma \), the lower the financiers’ ability to sustain the currency risk of their portfolio and the higher the required compensation per unit of risk. As \( \Gamma \uparrow \infty \) then \( \frac{Q^*}{E} \to 0 \) and the financiers are unable to absorb any imbalance. Vice versa, as \( \Gamma \downarrow 0 \) then \( i - i^* - \mu_E + \sigma_E^\top \sigma_E \to 0 \) and all bonds have the same expected rate of return, once measured in the same currency. Financiers’ risk-bearing capacity is so high that they arbitrage away any excess return. The parameter \( \upsilon \) governs the sensitivity of financiers’ demand to the volatility of the exchange rate. As \( \upsilon \downarrow 0 \) the sensitivity decreases and financiers only care about excess return.

**Central Bank**

The Home central bank has two instruments to stabilize the domestic economy: monetary policy and foreign exchange intervention. As is standard in the New Keynesian literature, it is convenient to work with the cashless limit of the economy. Let \( \zeta(t) \equiv i(t) \tilde{\zeta}(t) \) and take the limit for \( \{M(t), \tilde{\zeta}(t)\} \downarrow 0 \), such that \( \tilde{M}(t) \equiv M(t)/\tilde{\zeta}(t) \) is finite. Then the optimal money demand can be combined with the Euler equation to obtain \( \mathbb{E} \left[ \frac{d\tilde{M}(t)}{\tilde{M}(t)} \right] = \left\{ i(t) - \rho(t) + [\sigma_P(t) + \sigma_C(t)]^\top ([\sigma_P(t) + \sigma_C(t)]) \right\} dt \). In the rest of the paper I will specify monetary policy in terms of an interest rate rule, with the implicit assumption that the central bank chooses the appropriate path of money supply required to implement it.

The second tool, currency intervention, is the main focus of this paper. The Home central bank can intervene in the asset market by buying and selling domestic and foreign bonds. Unlike households, the central bank is not constrained in his holdings of foreign assets. Let \( X(t) \equiv \int_{\not H} X_i(t) di \) denote the foreign-currency value of the portfolio of foreign bonds held by the central bank, i.e. its foreign reserves. The central bank funds its holding of foreign reserves by issuing domestic bonds. Let \( X_H(t) \) denote the amount of Home bonds held by the central bank. Without loss of generality, since Ricardian equivalence holds, assume that the central bank has no capital and rebates all profits and losses generated by its portfolio of assets to domestic households. Its balance sheet equation is

\[
\mathcal{E}(t) X(t) + X_H(t) = 0
\]

A (sterilized) foreign exchange intervention is any purchase/sale of foreign assets that alters the relative supply of Home bonds. Through foreign exchange purchases the central bank increases

\[
\frac{Q^*_{H,t}}{\mathcal{E}(t)} = \int_0^1 Q^*_{H,i}(t) di = \frac{i(t) - i^*(t) - \mu_E(t) + \sigma_E^\top (t) \sigma_E(t)}{\Gamma \left[ \sigma_E^\top (t) \sigma_E (t) \right]^\upsilon} \tag{6}
\]
foreign reserves \((X(t) \uparrow)\) and increases the net supply of assets denominated in Home currency \((X_H(t) \downarrow)\). Vice versa, foreign exchange sales reduce the amount of foreign reserves held by the central bank \((X(t) \downarrow)\) and decrease the net supply of Home-currency bonds \((X_H(t) \uparrow)\). Notice that, by focusing on the cashless limit of the economy, I am ignoring the currency component of the balance sheet of the central bank. The central bank cannot increase (decrease) its holding of foreign assets by issuing (purchasing) domestic currency. Thus, foreign exchange interventions considered in this paper are sterilized interventions. Non-sterilized interventions can be modeled as a combination of sterilized intervention and interest rate policy.

The net transfer to domestic households includes central bank’s revenues/losses and taxes raised to finance the labor subsidy. It is given by

\[
dT(t) = [X_H(t)(i(t) - i^*(t) - \mu_E(t)) - W(t)(\tau_NT_{NT}(t) + \tau_HL_H(t))] dt - X_H(t)\sigma_E(t) d\mathbf{Z}(t)
\]

### Uncovered Interest Parity

This section characterizes the link between portfolio flows and exchange rates. The time-\(t\) market clearing condition for Home bonds is:

\[
D(t) + F_{H}^*(t) + Q_{H}^*(t) + X_{H}(t) = 0
\]

Using financial intermediaries’ aggregate demand 6 I obtain the following arbitrage condition between Home and foreign bonds

\[
i(t) - i^*(t) - \mu_E(t) + \sigma_E(t)\sigma_E(t) = -\Gamma [\sigma_E(t)\sigma_E(t)]^{\text{tr}} \left[ \frac{D(t)}{E(t)} + \frac{F_{H}^*(t)}{E(t)} + \frac{X_{H}(t)}{E(t)} \right]
\]

This equation provides a link between the demand for Home-currency assets and the exchange rate. The Home currency risk premium, on the left hand side of the equation, is proportional to the volatility of the exchange rate and the excess demand for Home-currency bonds that is absorbed by the financial sector. When international asset markets are imbalanced, the relative return of Home and foreign bonds must adjust in order to induce financial intermediaries to absorb the imbalance. The nominal exchange rate is the relative price between assets denominated in different currencies and provides the adjustments necessary to clear the assets market. In the model deviations from the Uncovered Interest rate Parity (UIP) are endogenous and proportional to the imbalances in demand and supply of assets denominated in different currencies. When an excess demand for Home assets arises \(\left(\frac{D(t)}{E(t)} + \frac{F_{H}^*(t)}{E(t)} + \frac{X_{H}(t)}{E(t)} > 0\right)\) the Home currency risk premium must fall in order to induce international financiers to take short positions denominated in domestic currency. This is achieved through an expected depreciation of the Home currency \((\mu_E(t) > 0)\). Vice versa, when the supply
of Home bonds exceeds their demand \( \left( D(t) + \frac{F_H(t)}{\bar{e}(t)} + \frac{X_H(t)}{\bar{e}(t)} < 0 \right) \), an expected appreciation of the domestic currency raises its risk premium and induces international financiers to absorb the excess supply of domestic assets.

By using the balance sheet equation of Home households, the excess demand for Home bonds can be further decomposed as follows:

\[
i(t) - i^*(t) - \mu_E(t) + \sigma_E^T(t) \sigma_E(t) = -\Gamma \left[ \sigma_E^T(t) \sigma_E(t) \right]^\text{CA Flow} + \frac{F_H^*(t)}{\bar{e}(t)} - F(t) + \frac{X_H(t)}{\bar{e}(t)} \]

This decomposition highlights three sources of demand for Home assets. The first component is associated with capital account, or net foreign assets, flows. These flows are induced by the desire of the Home households to reallocate their consumption intertemporally and they are accompanied by symmetric trade flows. The second component of the demand for Home assets is associated with portfolio flows. Portfolio flows are induced by the desire of domestic or foreign households to alter the composition of their portfolio of assets and therefore alter the country’s gross external position. Without any form of asset markets segmentation, portfolio flows don’t have any impact on the exchange rate as households in other countries would be willing to take the other side of the portfolio as long as uncovered interest parity holds. However, when asset markets are segmented the relative demand for domestic and foreign assets matter, and changes in countries’ gross external position affect the path of the exchange rates. Shocks to portfolio flows will be the main focus of this paper. Without loss of generality, I will assume \( F(t) = 0, \forall t \in [0, T] \) and consider shocks to the foreign demand for domestic assets, \( F_H^* \). Although I model these shocks as exogenous, shifts in foreign investors’ demand for Home bonds capture a broad range of situations: they might be triggered by shocks to the perceived risk of investing in the home country, or by shocks to the overall riskiness of the investor’s portfolio which induce yield-searching or safe-heaven type of flows, or they may represent investor’s preferences for a particular country’s bonds. Importantly, these shocks induce exchange rate fluctuations that would not arise in standard models with frictionless asset markets. In this sense, they capture exchange rate movements that are not driven by standard macroeconomic fundamentals. The last component of the demand for Home assets is given by the central bank intervention. By expanding and contracting its balance sheet, the Home central bank can alter the relative supply of assets and therefore the size of the imbalance that financiers must absorb. As we have seen above, this affects the Home currency risk premium and therefore path of the exchange rate. Formally, foreign exchange intervention is central bank intermediation of international flows that coexists and complements intermediation provided by financial intermediaries. Foreign exchange intervention affects the exchange rate to the extent there exists limits to arbitrage.
in private intermediation, that is $\Gamma > 0$.\textsuperscript{15}

**Equilibrium**

Following the literature on open economies with incomplete markets, define the consumption wedge as the marginal utility of a unit of Home currency for foreign households relative to Home households. Formally

$$\Lambda (t) \equiv \frac{1}{Q(t)} \frac{C(t)}{C^*} \quad (9)$$

If international asset markets were complete and frictionless ($\Gamma = 0$), marginal utilities in the Home country and in the rest of world would grow at the same rate once converted in the same units. Thus, $\Lambda$ would be constant for all $t$ and the real exchange rate would be proportional to their ratio: $Q(t) \propto \frac{C(t)}{C^*}$. This is the well-known Backus-Smith condition (Backus and Smith (1993)) when there is perfect risk sharing between Home and the rest of the world. This condition fails in the present model since not only markets are incomplete but households have to trade with constrained financiers. Fluctuations in $\Lambda$ introduce a time-varying wedge between the marginal rate of substitution between foreign and domestic consumption and their marginal rate of transformation, the real exchange rate. Using the Euler equations of domestic and foreign agents, we can derive the law of motion of the consumption wedge:

$$\frac{d\Lambda (t)}{\Lambda (t)} = \left[ i(t) - i^*(t) - \mu (t) + \sigma^\top \sigma (t) + \sigma^\top \Lambda (t) (\sigma (t) + \sigma (t)) \right] dt + \sigma^\top \Lambda (t) dZ(t) \quad (10)$$

where $\sigma^\Lambda \equiv \sigma^P + \sigma^C - \sigma^\xi$. This equation establishes the link between the UIP condition of the model, equation 8, and the dynamics of the consumption wedge. A fall in the Home currency risk premium increases the consumption wedge, since it tends to reduce its drift $\left( \mathbb{E} \left[ \frac{d\Lambda (t)}{\Lambda (t)} \right] < 0 \right)$. Either domestic consumption increases relative to foreign consumption or the real exchange rate appreciates, or both. The domestic currency is overvalued as the real exchange rate falls below its efficient level, defined as the value that would arise under complete and frictionless markets given the current relative consumption. Vice versa, an increase in the Home currency risk premium reduces the consumption wedge and the domestic currency is undervalued, as the real exchange rate raises above the level that would arise under complete and frictionless markets given the current relative consumption. It must be stressed that the term efficient here is used to denote the globally efficient allocation, i.e. the one that arises under complete markets. My focus however will be on

\textsuperscript{15}In closed economy, similar forms of intervention have been studied in the context of large scale asset purchases and unconventional monetary policy. See for example Gertler and Karadi (2011), Gertler et al. (2012) and Gertler and Karadi (2012)
the Home country. The Home policymaker might prefer allocations in which the real exchange rate is undervalued/overvalued and thus differ from the global optimum.

Let \( Y_k \equiv \left[ \int_0^1 Y_{k,j} (t) \mu_j d j \right] \) for \( k = H, NT \) be aggregate domestic output in each sector. Market clearing in the goods markets require

\[
Y_H (t) = C_H (t) + C_H^* (t)
\]

The labor market clearing condition is

\[
L (t) = \int_0^1 L_{H,j} (t) d j + \int_0^1 L_{NT,j} (t) d j = \Delta_H (t) Y_H (t)
\]

where \( \Delta_H (t) \equiv \int_0^1 \left( \frac{P_{H,j} (t)}{P_H (t)} \right) d j \) is an index of price dispersion. The nominal wage level is determined by the households labor supply schedule: \( W (t) = L (t)^{\beta} C (t) P (t) \). The law of motion of Home net foreign assets, evaluated in foreign currency, is

\[
dA^s (t) = \left[ A^s (t) i^* + (A^s (t) + X^s_H (t)) \left( i (t) - i^* - \mu (t) + \sigma^T (t) \sigma (t) \right) \right] dt + \frac{Y_H (t) - C (t) S (t)}{S (t)} P^* dt - (A^s (t) + X^s_H (t)) \sigma (t) dZ (t)
\]

where a dollar superscript denotes the foreign-currency value of the variable. This is the Home country aggregate budget constraint. The real exchange rate is given by \( Q (t) = S (t)^{1-\alpha} \). Finally, market clearing in the Home bonds market yields the UIP equation described in the previous section

\[
i (t) - i^* - \mu (t) + \sigma^T (t) \sigma (t) = -\Gamma \sigma^T (t) \Gamma (A^s (t) + F_H^s (t) + X^s_H (t))
\]

The model is closed by the firms pricing policies which determine the laws of motion of prices and their dispersions. I will consider three different price-setting assumptions. First I will consider the case of flexible prices. When firms are allowed to reset their prices at every \( t \) then the optimal pricing setting decisions are \( P_{H,j} (t) = \frac{1}{\epsilon} (1 - \tau_H) W (t) \). Second I will consider the extreme case of rigid prices. Under this assumption firms are not able to change their prices at any point in time. Therefore \( P_{H,j} (t) = P_{H,j} (0), \forall t \in [0, \infty) \). Finally, I will consider the intermediate case of staggered pricing. When firms are allowed to reset their prices only infrequently, PPI inflation is given by \( \pi_H (t) = \frac{1}{1-\epsilon} \left[ \left( \frac{\Psi_H (t)}{Y_H (t)} \right)^{1-\epsilon} - 1 \right] \), where \( \Psi_H \) and \( Y_H \) are expected present discounted values of future revenues and costs in sector \( k \), respectively. Their equations, together with the law of motion of price dispersion, are reported in the appendix.

The next lemma characterizes the symmetric steady state of the economy and derives the opti-
mal labor subsidy.

**Lemma 1.** Suppose prices are flexible, and there are no portfolio flows: $F^s_H = 0$. Then the symmetric steady state of the economy has $\Lambda = 1$, $A^s = X^s_H = 0$ and the optimal labor subsidy is $\tau_H = 1 - \frac{1}{M(1-\alpha)}$.

At a symmetric steady state, the central bank of the Home country does not manipulate the exchange rate. This should not be surprising since at a symmetric steady state there are no capital flows between the Home country and the rest of the world. Therefore financial market imperfections are mute in equilibrium. This result, however, highlights the fact that the use of foreign exchange intervention in this model is not driven by mercantilist motives. The policymaker does not have any incentive to sustain a permanently undervalued currency with the goal of increasing foreign demand for domestic goods.

### 3 The Log-Linearized Model

In order to study the effects of portfolio flow shocks on the small open economy and characterize the optimal policy response I use a log-linearized version of the equilibrium conditions described in the previous section. Before doing so, however, in order to preserve the financial frictions that is at the heart of the model I take the limit for $\nu \downarrow 0$. I characterize optimal policies by considering a second order approximation of the welfare function around the deterministic steady state. The optimal policies obtained from the solution of the linear-quadratic problem are linear and in many cases an analytical solution can be derived. In what follows lowercase letters denote log-deviations from steady state, if the variable has a non-zero steady state value. Otherwise, they denote simple deviations from steady state.

The equations that approximate the local dynamics of the equilibrium around its steady state are the log-linearized analogues of the stochastic differential equations described in the previous section. The consumption wedge and net foreign assets evolve as follows:

$$d\lambda(t) = -\Gamma \left( a^s(t) + f^s_H(t) + x^s_H(t) \right) dt$$  \hspace{1cm} (14)

$$da^s(t) = \left( \rho a^s(t) - \eta \lambda(t) \right) dt$$  \hspace{1cm} (15)

where $\eta = \alpha (1 - \beta) P^sC^s$, subject to the initial condition $a^s(0) = 0$ and the terminal condition $\lim_{t \to \infty} e^{-\rho t} a^s(t) = 0$. The laws of motion of domestic output and PPI inflation depend on the price setting assumption. If prices are sticky then domestic output follows

$$dy_H(t) = \left( i(t) - \rho - \pi_H(t) - \alpha \lambda(t) \right) dt$$  \hspace{1cm} (16)
while PPI inflation evolves according to

\[ d\pi_H (t) = [\rho \pi_H - \kappa (1 + \varphi) y_H (t) - \alpha \kappa \lambda (t)] dt \]  

(17)

where \( \kappa \equiv \theta (\rho + \theta) \). These are the log-linearized IS and New Keynesian Philips curves of the model. Notice that both the IS curve and the NKPC include an additional term that captures the effect of the consumption wedge on tradable output and inflation. When prices are completely flexible domestic output is proportional to the consumption wedge and it is given by \( y_H (t) = -\alpha \frac{1 + \psi}{1 + \varphi} \lambda (t) \). Finally, when prices are completely rigid output is determined by \( y_H (t) = e (t) + (1 - \alpha) \lambda (t) \).

The loss function for the Home policymaker is derived from a second order approximation to the domestic households’ utility, given in equation (1), and a second order approximation to the Home country budget constraint, given by equation 13. Details of the derivation are provided in the appendix. The policymaker wishes to minimize

\[
\mathbb{L} = \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \phi_{yH} y_H (t)^2 + \phi_{sH} s_H (t)^2 + \phi_\lambda \lambda (t)^2 + \phi_x x_H (t)^2 \right] dt
\]

(18)

where

\[
\phi_{yH} = (1 - \alpha) (1 + \varphi) \quad \phi_{sH} = (1 - \alpha) \frac{\xi}{\kappa} \\
\phi_\lambda = \alpha (1 - \alpha)^2 \quad \phi_x = 2 \alpha (2 - \alpha) \frac{\Gamma}{\eta}
\]

There are four sources of welfare losses. The first term captures the welfare cost of deviating from the efficient level of output. The second term captures the welfare losses induced by inflation through its effect on prices dispersion and domestic total factor productivity. The third term captures the welfare cost caused by an inefficient intertemporal allocation of consumption relative to the path of the real exchange rate.\(^{16}\) The log-linearized consumption wedge is \( \lambda (t) = c (t) - q (t) \). A positive consumption wedge is associated with an appreciation of the real exchange rate relative to domestic consumption, while a negative consumption wedge is associated with a depreciation of the real exchange rate relative to domestic consumption. By iterating the budget constraint forward and using initial and terminal conditions I obtain \( \int_0^\infty e^{-\rho t} \lambda (t) dt = 0 \). Thus, periods with positive consumption wedge must be followed or preceded by periods with negative consumption wedge. These fluctuations are inefficient since they reduce the present discounted value of the domestic consumption stream. Domestic consumption is too high when the domestic consumption bundle is relatively more expensive (appreciated real exchange rate) and too low when it is relatively

\(^{16}\)An additional term related to imperfect international risk-sharing appears in the quadratic loss function of many models with incomplete markets. See for example De Paoli (2009) for a small open economy model and Corsetti et al. (2010) for two-country model. In all these models, however, this additional term disappears when the elasticity of intertemporal substitution and the elasticity of substitution between domestic and foreign goods are unitary.
cheaper. A reallocation of consumption from periods with positive consumption wedge to periods with negative consumption wedge would raise it’s present discounted value. Finally, the last term in the loss function comes from a second order approximation of the Home budget constraint, given by equation (13), and captures the monetary cost of using foreign exchange interventions. The monetary cost arises from the fact that by intervening in the foreign exchange market the central bank moves the excess return of domestic bonds vis-à-vis foreign bonds against its own positions. For example, a foreign exchange purchase increases foreign reserves and reduces the central bank holdings of Home bonds. By increasing the supply of Home-currency bonds, however, the central bank depreciates the exchange rate and increases their return relative to foreign bonds, therefore increasing the cost of funding reserves. Vice versa, a foreign exchange sale increases the central bank holding of Home-currency assets and appreciates the exchange rate, reducing their return. Since financiers must take the opposite position in order for markets to clear, returns adjust in their favor and against the central bank.

The loss function is derived without specific assumptions about how prices are set (indeed whether prices are sticky or not). This highlight the fact that the loss in welfare arises not specifically from price stickiness but from prices that do not deliver the efficient allocation. In what follows I study the effects of and characterize the optimal policy response to a shock to the foreign demand for Home assets, $f^*$. The next proposition defines the log-linearized planning problem

**Problem 1.** The planning problem is to choose an interest rate policy $i(t)$ and a foreign exchange intervention policy $x_H^S(t)$ that minimize (18) subject to (14), (15), (16), (17), and

$$df_H^S(t) = -\rho f_H^S(t) dt$$

(19)

given initial conditions $f_H^S(0) = \varepsilon$, $a_S^S(0) = 0$ and the terminal condition $\lim_{t \to \infty} e^{-\rho t} a_S^S(t) = 0$.

## 4 Optimal Foreign Exchange Intervention

In this section I characterize the optimal use of foreign exchange intervention alone. That is, I assume that the central bank commits to maintaining the nominal interest rate at its steady state level ($i(t) = \rho$) and uses only foreign exchange interventions. I will study the planner problem under flexible and rigid prices as these two extreme cases provide similar analytical solutions which deliver sharp intuition for the underlying mechanism of the model. The intermediate case of sticky prices will be analyzed in the next section when I will allow the central bank to use both foreign exchange intervention and monetary policy.
4.1 Optimal Foreign Exchange Intervention with Flexible Prices

In this section I characterize the optimal foreign exchange intervention policy under flexible prices. With flexible prices, the planner problem can be simplified to

\[
\min_{x_H} \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \phi^F \lambda(t)^2 + \phi_x x_H(t)^2 \right] dt
\]

subject to 14, 15 and 19, given initial conditions \(f_H^*(0) = \varepsilon, a^*(0) = 0\) and the terminal condition \(\lim_{t \to \infty} e^{-\rho t} a^*(t) = 0\). The relative weight on the consumption wedge is given by \(\phi^F = \phi_y H(\alpha_1 + \phi)\), and therefore includes both the welfare cost induced by output gaps and that induced by an inefficient intertemporal allocation of consumption. I will consider the case where the central bank can credibly commit at time \(t = 0\) to the entire future path of foreign reserves. The next proposition characterizes the allocation implemented by the optimal foreign exchange intervention policy.

**Proposition 1.** When prices are flexible and the central bank uses foreign exchange intervention, the equilibrium of the model is

\[
\lambda(t) = \frac{\Gamma}{\Delta} + \Delta \left[ \frac{\rho - \mu}{\Delta} + \frac{\rho + \varphi}{\Delta} \right]^{\top} \left[ \begin{array}{c} x_H^*(t) \\ a^*(t) \\ f_H^*(t) \end{array} \right]
\]

where the states \(\begin{bmatrix} x_H^* & a^* & f_H^* \end{bmatrix}\) evolve as

\[
\begin{bmatrix}
\dot{x}_H^* \\
\dot{a}^* \\
\dot{f}_H^*
\end{bmatrix} = \begin{bmatrix}
\frac{\Delta - \Delta}{\Delta + \Delta - \Delta}
\\
\frac{\Delta - \Delta}{\Delta + \Delta - \Delta}
\\
\frac{\Delta - \Delta}{\Delta + \Delta - \Delta}
\end{bmatrix}
\begin{bmatrix}
-x + \nu - \mu
\\
-x + \nu + \mu
\\
-x + \nu + \mu
\end{bmatrix}
\begin{bmatrix}
x_H^* \\
a^* \\
f_H^*
\end{bmatrix} dt
\]

with \(\begin{bmatrix} x_H^* & a^* & f_H^* \end{bmatrix} = \begin{bmatrix} 0 & 0 & \varepsilon \end{bmatrix}\) and \(\varphi > \nu > 0, \Delta > \Delta > 0\).

In order to describe the effects of portfolio flow shocks and the optimal policy response I will focus on the case of an increase in the foreign demand for domestic assets, that is \(\varepsilon > 0\). Since the solution is linear, both the effects and the optimal policy response to a decrease in the demand for domestic assets \(\varepsilon < 0\) are symmetric. Figure 1 plots the allocation implemented by the optimal foreign exchange intervention under flexible prices, in response to a time zero positive shock to foreign investors demand for domestic assets, and compares it with the allocation that arises without intervention. The analytical solution for the allocation without intervention is
Impulse responses for the model with flexible prices to a positive shock to foreign demand for Home assets. The blue line represents the allocation implemented by the optimal FX intervention, while the red line represents the allocation without intervention. Parameter values: $\rho = 0.01$, $\alpha = 0.4$, $\varphi = 3$, $\beta = (1 - \alpha) \frac{1}{1+\varphi}$, $\Gamma = 1$, $\varepsilon = 0.05$, $\varrho = 0.2$, $\phi_x = 0.03$. 
reported in the appendix. The numerical simulation is meant to describe the qualitative properties of the allocation and should not be thought of as a serious calibration exercise, for which the model is clearly too stylized.

At \( t = 0 \) foreign households want to increase their holdings of Home bonds. The excess demand for Home-currency assets generates an imbalance in the asset market that financiers will have to absorb. The Home currency appreciates on the spot \( e(0) < 0 \) and depreciates in expectation \( \dot{e}(0) > 0 \) such that the price of domestic assets increases and their foreign-currency return falls. The excess return of foreign assets vis-à-vis domestic assets induces financiers to short Home bonds until the imbalance is absorbed. The appreciation of the Home currency induce domestic producer to reduce their prices and increases domestic PPI inflation, while the expected depreciation of the domestic currency increases import prices inflation. Both effects increase CPI inflation and reduce the real borrowing cost for Home agents therefore inducing domestic household to accumulate debt. Foreign capitals flow into the Home economy \( \dot{a}(0) < 0 \) and finance a consumption boom \( c(0) = \alpha \frac{1+\varphi(2-\alpha)}{1+\varphi} \lambda(0) \).

The appreciation of the exchange rate makes domestic goods relatively more expensive with respect to foreign goods. Thus, both domestic and foreign demand for domestic goods decrease. On the other hand, the fall in the real interest rate faced by Home households increases domestic consumption and raises domestic demand. Domestic producers react to the loss in competitiveness by reducing their prices and increasing output. Even though prices are fully flexible, output falls \( y_H(0) = -\alpha \frac{1+\varphi(1-\alpha)}{1+\varphi} \lambda(0) \). This is due to a wealth effect on labor supply: the increase in domestic consumption pushes up wages and prevent prices from falling enough to restore full employment. Therefore, a positive portfolio flow shock generates an immediate consumption boom and a fall in output. As exports fall and imports increases, domestic terms of trade appreciate \( s(0) = -\frac{1+\varphi(1-\alpha)}{1+\varphi} \lambda(0) \) and the trade balance turns negative \( nx(0) = -\beta \lambda(0) \). This is however not the end of the story, as the boom part of the cycle is followed by a bust. As domestic agents accumulate debt and foreign demand for domestic assets subsides, the imbalance in the asset market changes sign and an excess supply of Home bonds arises. Financial intermediaries must absorb the debt accumulated by the Home country and their portfolio shifts towards Home-currency assets. The exchange rate falls below its steady state value and the initial dynamics are reversed. Consumption falls below its steady state level, the trade balance turns positive and a positive output gap emerges. Thus, portfolio flow shocks generate inefficient boom-bust cycles.

The optimal foreign exchange intervention leans against the wind and stabilizes the path of the exchange rate. Following an increase in the demand for Home-currency assets, the central bank increases the net supply of Home bonds \( \dot{x}_H^s(0) < 0 \) and accumulates foreign reserves. By doing so the central bank absorbs part of the imbalances arising in the international asset market and reduces the impact of the portfolio flow shock on the exchange rate. The intervention reduces the
initial appreciation of the domestic currency and moderates its expected depreciation. This, in turn, increases the domestic real interest rate and smooths out the consumption boom. Smoothing out consumption fluctuations increases domestic welfare for two reasons: it stabilizes domestic output and it maximizes the present discounted value of consumption.

The optimal level of employment is chosen by the planner by maximizing (3) subject to the technological constraint \( Y_H(t) = L(t) \) and the market clearing condition, equation (11), given the consumption/output possibility set implied by the value of the consumption wedge \( \Lambda \). In the special case of \( \gamma = \eta = \varsigma = 1 \), the optimal allocation implies a constant employment \( N(t) = (1 - \alpha)^{\frac{1}{1+\phi}} \). The possibility of altering the domestic terms of trade through foreign exchange intervention does not alter the well known result, derived by Gali and Monacelli (2005) in a model with complete markets, that under a Cole-Obstfeld parametrization a constant level of employment is optimal. The presence of imperfections in international financial markets, however, affects the ability of the planner to implement the optimal allocation. The labor subsidy that implements the efficient level of employment is

\[
\tau^{OPT}(t) = 1 - \frac{\alpha + (1 - \alpha) \Lambda(t)}{M(1 - \alpha) \Lambda(t)}
\]

If markets are frictionless then \( \Lambda \) is constant and a constant labor subsidy is enough to render the flexible price equilibrium level of employment efficient. When domestic consumption increases, a depreciation of the exchange rate increases foreign demand for domestic goods. The increase in foreign demand offsets the wealth effect on labor supply triggered by the increase in domestic consumption. Hence employment remains constant. When international financial markets are imperfect and the exchange rate is determined by financial forces, this expenditure-switching mechanism breaks down. A positive portfolio flow shock simultaneously appreciates the exchange rate and causes a consumption boom in the Home economy. The wealth effect on labor supply pushes up wages and domestic prices while the appreciation of the exchange rate reduces foreign demand. As a result, output and employment fall. In order to restore the efficient level of employment domestic prices must fall, therefore the optimal labor subsidy increases. If fiscal policy is not flexible enough to correct these distortions through an appropriate time-varying subsidy - as it is assumed in this paper - fluctuations in consumption driven by portfolio flow shocks result in inefficient levels of output. By stabilizing these fluctuations, the central bank reduces wage inflation/deflation and mitigate the welfare losses caused by the associated output gaps.

The second reason for stabilizing consumption fluctuations has to do with dynamic terms of trade management. An increase in Home consumption appreciates the terms of trade via two channels: the wealth effect on labor supply described above and the Home bias effect on domestic demand. Since Home consumption is biased toward domestic goods, and the law of one price holds, an increase in domestic consumption raises domestic demand for Home goods and appre-
ciates the terms of trade. Therefore, domestic households consume more exactly when the cost of their consumption bundle is high. The domestic planner would like to exploit its monopsonistic power by reducing consumption and depreciating the terms of trade. Vice versa, a decrease in domestic consumption below its steady state level depreciates the terms of trade and reduce export revenues. The domestic planner would like to exploit its monopolistic power by increasing consumption and appreciating the terms of trade. This logic is similar to Costinot et al. (2014), where the planner wishes to manipulate the dynamic path of the terms of trade in order to maximize the present discounted value of a time-varying endowment.

Despite the inefficiencies induced by fluctuations in domestic consumption, it is never optimal for the central bank to fully stabilize the path of the exchange rate. The reason is that, by creating a wedge between the return of domestic and foreign assets, portfolio flow shocks are akin to positive wealth shocks for the Home country. Although domestic households cannot fully exploit the arbitrage opportunity provided by deviations from uncovered interest parity, the central bank can. By leaning against the wind the central bank earns a positive return on its portfolio of assets which is decreasing in the size of the intervention. As the central bank fully absorbs the demand for Home assets, uncovered interest rate parity holds and domestic and foreign assets have the same rate of return. Therefore it is never optimal to completely offset the premium generated by the portfolio flow shock. In determining the optimal amount of stabilization the central bank trades off the benefits that arise from the arbitrage opportunity and the inefficiencies described above.

4.2 Optimal Foreign Exchange Intervention with Rigid Prices

When prices are rigid the domestic terms of trade are given by \( s(t) = e(t) \). If the central bank keeps the nominal interest rate at its steady state level \( (i(t) = \rho) \), then \( e(t) = -\lambda(t) \) and the output gap is \( y_H(t) = -\alpha \lambda(t) \). The planner problem becomes

\[
\min_{x^s} \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \phi^R \lambda(t)^2 + \phi_x x^s(t)^2 \right] dt
\]

subject to 14, 15 and 19, given initial conditions \( f^x_H(0) = \varepsilon, \alpha^s(0) = 0 \) and the terminal condition \( \lim_{t \to \infty} e^{-\rho t} \alpha^s(t) = 0 \). The relative weight on the consumption wedge is given by \( \phi^R \equiv \alpha^2 \phi_{y_H} + \phi \lambda \).

The structure of planner problem is identical to the one solved under flexible prices. Therefore also the solution has the same structure and it is reported in the appendix. The only difference between the two problems is the relative weight that the planner attaches to the stabilization of the consumption wedge. As showed in the previous section, when prices are flexible producers react to a positive portfolio flow shock by reducing their prices in order to regain the competitiveness lost.
Impulse responses for the model with rigid prices to a positive shock to foreign demand for Home assets. The blue line represents the allocation implemented by the optimal FX intervention, while the red line represents the allocation without intervention. In both cases monetary policy is idle ($i = \rho$). Parameter values: $\rho = 0.01$, $\alpha = 0.4$, $\varphi = 3$, $\theta = 0.7$, $\epsilon = 6$, $\beta = (1 - \alpha)^{\frac{1}{1+\varphi}}$, $\Gamma = 1$, $\varepsilon = 0.05$, $\delta = 0.2$, $\phi_x = 0.03$. 

27
due to the appreciation of the exchange rate. By doing so they increase production and mitigate the negative impact of the shock on domestic output. When prices are rigid this mechanism is shut off. Now domestic prices remain too high not only because of the wealth effect on labor supply triggered by the consumption boom, but also because of the nominal friction that prevents domestic producers from adjusting them downwards. As a result of price rigidity, domestic demand remain constant since the increase induced by the consumption boom perfectly offsets its fall due to the appreciated terms of trade. Foreign demand, on the other hand, falls sharply as the foreign price of domestic goods increases one-for-one with the nominal exchange rate. The fall in output is more severe and the welfare cost of a portfolio flow shock increases. Notice, however, that the path of the consumption wedge is unaltered by the presence of nominal rigidities. Thus, the increase in the welfare cost of the shock comes entirely from wider fluctuations in output.

With rigid prices the central bank responds to an increase in the foreign demand for Home-currency assets by increasing their supply more aggressively. When prices are flexible the main goal of the intervention is to mitigate the depreciation of the exchange rate that follows the portfolio flow shock. Since producers are allowed to adjust their prices freely, the nominal appreciation of the exchange rate \textit{per se} does not generate any welfare loss. When prices are rigid, however, the nominal appreciation reduces the competitiveness of domestic goods abroad and causes a bigger fall in output. Now the central bank has an additional goal as it wishes to stabilize not only the dynamic of the exchange rate, but also its level. By fighting the appreciation, the intervention reduces the foreign price of domestic goods and sustains foreign demand. This is the “monetary” aspect of foreign exchange interventions: the central bank leans against the wind in order to mitigate the effect of the portfolio flow shock on the competitiveness of domestic exports.

### 4.3 Optimal Foreign Exchange Intervention Rule

Under both flexible and rigid prices the planner problem has the following structure

\[
\min_{x_H} \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \phi \lambda(t)^2 + x_H(t)^2 \right] dt
\]

subject to 14, 15 and 19, given initial conditions \( f_H^s(0) = \varepsilon, \ a_H^s(0) = 0 \) and the terminal condition \( \lim_{t \to \infty} e^{-\rho t} a_H^s(t) = 0 \). In order to gain more intuition on how the central bank uses foreign exchange intervention, in the next lemma I characterize the optimal intervention rule in terms of endogenous variables.

**Lemma 2.** The optimal foreign exchange intervention rule is

\[
dx_H^s(t) = \Psi \begin{bmatrix} \lambda(t) \\ a_H^s(t) \\ x_H(t) \end{bmatrix} dt
\]

with

\[
\begin{align*}
\lambda(t) \\
a_H^s(t) \\
x_H(t)
\end{align*}
\]
\( x_H^s (0) = 0 \), where

\[
\Psi_\lambda = -\beta \frac{\Delta - \Delta (\nu - \nu) [(\rho + \nu + \nu) (\rho + \rho + \nu) + \Delta (\rho + \rho + \nu)]}{\Delta (\rho + \rho + \nu) + \Delta (\rho + \rho + \nu)} < 0
\]

\[
\Psi_a = \frac{\Delta - \Delta (\nu - \nu) (\rho + \rho) (\rho + \nu) (\rho + \nu)}{\Delta (\rho + \rho + \nu) + \Delta (\rho + \rho + \nu)} > 0
\]

\[
\Psi_x = \frac{\nu \Delta (\rho + \rho + \nu) + \nu \Delta (\rho + \rho + \nu)}{\Delta (\rho + \rho + \nu) + \Delta (\rho + \rho + \nu)} > 0
\]

Furthermore

\[
\frac{\partial |\Psi_\lambda|}{\partial \phi} > 0 \quad \frac{\partial |\Psi_a|}{\partial \phi} > 0 \quad \frac{\partial |\Psi_x|}{\partial \phi} < 0
\]

and, as \( \delta \downarrow 0 \)

\[
\frac{\partial |\Psi_\lambda|}{\partial \phi} > 0 \quad \frac{\partial |\Psi_a|}{\partial \phi} > 0 \quad \frac{\partial |\Psi_x|}{\partial \phi} > 0
\]

\[
\frac{\partial |\Psi_\lambda|}{\partial \Gamma} < 0 \quad \frac{\partial |\Psi_a|}{\partial \Gamma} < 0 \quad \frac{\partial |\Psi_x|}{\partial \Gamma} > 0
\]

while for \( \delta \uparrow \infty \)

\[
\frac{\partial |\Psi_\lambda|}{\partial \phi} > 0 \quad \frac{\partial |\Psi_a|}{\partial \phi} > 0 \quad \frac{\partial |\Psi_x|}{\partial \phi} < 0
\]

\[
\frac{\partial |\Psi_\lambda|}{\partial \Gamma} < 0 \quad \frac{\partial |\Psi_a|}{\partial \Gamma} > 0 \quad \frac{\partial |\Psi_x|}{\partial \Gamma} > 0
\]

Lemma 2 shows how the central bank can implement the optimal foreign exchange intervention. The central bank uses foreign reserves as a state variable \( x_H^s (0) = -x_H^s (0) = 0 \) and commits to alter the supply of Home-currency bonds as a function three implicit targets: the consumption wedge \( \lambda \), Home net foreign assets \( a^s \) and foreign reserves \( x^s \). The first target of the intervention, the consumption wedge \( \lambda \), captures the goal of macroeconomic stabilization pursued by the central bank. Since \( \Psi_\lambda < 0 \), the central bank commits to increase the net supply of domestic bonds \( x_H^s \) when the consumption wedge is positive and to reduce it \( x_H^s \) when it is negative. The stabilizing effect of such policy can be understood by looking at equation (14). By iterating it forward we obtain

\[
\lambda (t) = \Gamma \int_t^\infty \left( a^s (u) + x_H^s (u) + f_H^s (u) \right) du
\]

The term in parenthesis is the size of the time-\( u \) imbalance in the Home assets market that is absorbed by the financial sector. Thus, the consumption wedge at time \( t \) is equal to the present discounted value of all future imbalances, where the discount term is constant and given by \( \Gamma \). If
If \( \lambda(t) > 0 \) then the present discounted value of future demands for domestic bonds exceeds the present discounted value of future supplies. Financial intermediaries expect to be, on average, short Home assets. The exchange rate is below its steady state value and is expected to depreciate in order to compensate them. The fall in the return of Home assets reduces the borrowing cost for domestic households and a positive consumption wedge arises. By committing to increase the supply of domestic bonds the central bank reduces the present discounted value of future imbalances and reduces the consumption wedge. Vice versa when a negative consumption wedge arise, the Home central bank stabilizes the return of domestic assets by committing to decrease their supply. The second target of the optimal foreign exchange intervention, net foreign assets, captures its precautionary aspect. Since \( \Psi_a > 0 \), the central bank commits to increase the net supply of domestic assets when the country is a net debtor and to decrease it when the country is a net creditor. By doing so the central bank moves the return of domestic assets against the position held by domestic agents. Ceteris paribus, an increase in the net supply of domestic assets increases their return making borrowing more expensive for Home households. Vice versa, a decrease in the net supply reduces their return and the incentive to save. The logic behind this type of intervention is purely precautionary. By committing to increase the cost (reduce the profitability) of their positions, the central bank discourages domestic households from accumulating assets. Once the portfolio flow subsides, these assets will have to be absorbed by financial intermediaries causing the same types of inefficiencies generated by the initial phase of the cycle. Thus, by discouraging a build up in imbalances the central bank mitigates the severity of the bust phase of the cycle. Finally the third target of the intervention, the level of foreign reserves, captures the monetary cost of the intervention. Since \( \Psi_x > 0 \), the central bank commits to increase (decrease) the supply of domestic bonds and accumulate (decumulate) foreign reserves when they are below (above) their steady state level. The monetary cost of the intervention arises from the fact that when the central bank intervenes in the asset market it moves returns against its own positions. Notice also that, since holding reserves is costly, the central bank optimally smooths out the intervention rather then intervening aggressively at the time of the shock. At \( t = 0 \), following a positive portfolio flow shock, the central bank keeps reserves at their steady state level and increases the supply of domestic bonds only gradually over time.

The comparative statics results reported in Lemma 2 shed some lights on the determination of the weights that the central bank attributes to the three implicit targets. As the portfolio flow shock becomes more persistent \( (\delta \downarrow) \), the central bank reduces the weights on the consumption wedge and net foreign assets while it increases the weight on foreign reserves. This reduces the duration of the intervention vis-a-vis the persistency of the shock. Vice versa, when the shock is temporary the central bank reacts more aggressively to the consumption wedge and the net foreign assets position of the Home country, while simultaneously increasing the persistency of the intervention. As the
desire for stabilization increases \((\phi \uparrow)\), the central bank reacts more aggressively to deviations of the consumption wedge and net foreign assets, while its effect on the duration of the intervention depends on the persistence of the shock. When the portfolio flow shock is very persistent \((\delta \downarrow 0)\) an increase in the desire for stabilization leads to a more aggressive but less persistent intervention. Vice versa, when the portfolio flow shock mean-reverts quickly \((\delta \uparrow \infty)\) an increase in the desire for stabilization increases the duration of the intervention. Intuitively, in the former case it is optimal for the central bank to front-load the intervention since its cost will be relatively constant across time, while in the latter case it is optimal to smooth out the intervention since its cost is lower in the future. Finally, a decrease in the risk-bearing capacity of the financial sector \((\Gamma \uparrow)\) increases the cost of the intervention. As a result, the central bank will reduce its weight on the consumption wedge and reduce the duration of the intervention \((\Psi_x \uparrow)\). If the portfolio flow shock is very persistent, the central bank also reduces the weight placed on net foreign assets. Vice versa, if the shock is temporary the central bank stabilizes capital account imbalances more aggressively.

5 Optimal Monetary Policy and Foreign Exchange Intervention

5.1 Optimal Monetary Policy

Before characterizing the optimal currency intervention with sticky prices, it is instructive to understand the role played by monetary policy. In this section I characterize the optimal use of monetary policy in response to portfolio flow shocks, assuming that the central bank does not intervene in the foreign exchange market. With sticky prices, the planner’s problem is the one described in Problem 1 with \(x^S_H = 0\). The next proposition characterizes the allocation implemented by optimal monetary policy under full commitment. For simplicity, I will ignore the issue of the zero lower bound on the nominal interest rate.

**Proposition 2.** When prices are sticky and the central bank uses monetary policy only, the equilibrium of the model is

\[
\begin{bmatrix}
\lambda(t) \\
\pi_H(t)
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{\phi y}{\beta \\
\frac{\phi y}{\beta} & \frac{\phi y}{\beta} + \frac{\phi y + \nu}{\beta} + \frac{\phi y + \nu}{(\rho + \nu + \phi)(\rho + \nu + i)}
\end{bmatrix}^\top
\begin{bmatrix}
y_H(t) \\
a^S(t) \\
f^S_H(t)
\end{bmatrix}
\]
where the states \([y_H, a^s, f^{sS}_H]\) evolve as

\[
d \begin{bmatrix} y_H(t) \\ a^s(t) \\ f^{sS}_H(t) \end{bmatrix} = \begin{bmatrix} -l -\frac{\phi_x \alpha_n \nu + \rho}{\phi_y + \beta + \nu + \tau} - \frac{\phi_n \alpha_n \nu + \rho}{(\rho + \nu + \tau)} \\ 0 -\nu \\ 0 \end{bmatrix} \begin{bmatrix} y_H(t) \\ a^s(t) \\ f^{sS}_H(t) \end{bmatrix} \ dt
\]

with \([y_H(0), a^s(0), f^{sS}_H(0)] = \begin{bmatrix} 0 & 0 & \varepsilon \end{bmatrix}\) and \(\nu > \nu > \nu > 0, \ i > 0\).

In the previous sections we have seen that a positive portfolio flow shock appreciates the exchange rate and reduces the domestic real interest rate. The latter effect triggers a consumption boom which reduces output and the present discounted value of the consumption stream. The appreciation of the domestic currency, on the other hand, increases the foreign price of domestic goods which, if prices are sticky, depresses domestic output even further. One might naively guess that, in analogy with the use of foreign exchange intervention, the central bank should tighten monetary policy in order to mitigate the consumption boom and its negative effects on domestic welfare. This logic however does not take into account the effects of such a policy on the price of domestic assets and their returns. An increase in the nominal interest rate would make domestic assets even more attractive for foreign investors. In order for markets to clear, the exchange rate would have to appreciate even more, depressing foreign demand further. Therefore, by simultaneously reducing domestic consumption and domestic demand, a tighter monetary policy would cause a bigger fall in output.

A quick inspection of the equations in Proposition 2 reveals that the consumption wedge \(\lambda\) and the foreign-currency value of the Home net foreign asset position \(a^s\) are independent of the degree of price stickiness and of the output gap. Hence monetary policy is unable to affect them. By using the domestic and foreign demand schedules we can rewrite the consumption wedge as the gap between domestic and foreign demand:

\[
\lambda(t) = c_H(t) - c^{s*}_H(t)
\]

Monetary policy moves domestic and foreign demand in the same direction, therefore it has limited power in reducing the tension between them generated by the portfolio flow shock. In the special case considered in this paper where all elasticities of substitution equal to one \((\gamma = \eta = \varsigma = 1)\), the effects of monetary policy on domestic and foreign demand have exactly the same magnitude. Thus, their ratio is independent of the path of the nominal interest rate chosen by the central bank. This result has two striking implications. First of all, monetary policy cannot be used to maximize the present discounted value of the path of domestic consumption as it cannot simultaneously reduce consumption and depreciate the terms of trade. Second, its ability to reduce the output
Impulse responses for the model with sticky prices to a positive shock to foreign demand for Home assets. The blue line represents the allocation implemented by the optimal monetary policy, while the red line represents the allocation with flexible prices and no FX intervention. Parameter values: $\rho = 0.01$, $\alpha = 0.4$, $\varphi = 3$, $\theta = 0.7$, $\epsilon = 6$, $\beta = (1 - \alpha)^{1/3}$, $\Gamma = 1$, $\varepsilon = 0.05$, $\varrho = 0.2$, $\phi_x = 0.03$. 
gap is limited by wages inflation triggered by the consumption boom. Figure 3 plots a numerical simulation of the allocation implemented by the optimal monetary policy and compares it with the flexible prices allocation when the central bank does not intervene in the foreign exchange market. In response to a positive portfolio flow shock, the central bank reduces the nominal interest rate and reduces the attractiveness of domestic assets. This depreciates the exchange rate and helps domestic producers to reduce the foreign price of domestic goods. The expansionary monetary policy also increases domestic consumption and therefore domestic demand. This reduces the output gap even further but exacerbates wages inflation triggered by the portfolio flow shock and eventually leads to positive PPI inflation. Portfolio flow shocks act as markup shocks for the small open economy. Hence monetary policy cannot simultaneously set the output gap and inflation to zero.

5.2 Optimal Monetary Policy and Foreign Exchange Intervention

In this section I consider the case where the central bank uses both tools available: foreign exchange intervention and monetary policy. The planner’s problem is the one described in Problem 1. Unfortunately the problem does not admit a closed form solution. Figure (4) depicts the optimal allocation with both foreign exchange intervention and optimal interest rate policy, together with the allocation with foreign exchange intervention only (and $i(t) = \rho$).

In dealing with the effects of portfolio flow shocks the central bank optimally uses both instruments. Foreign exchange intervention and monetary policy are complementary tools rather than substitutes. In the previous sections I showed how Foreign exchange intervention is the appropriate tool to smooth out consumption fluctuations. It allows the planner the choose the welfare-maximizing consumption path by trading off the benefits and costs of the real interest rate shocks. However, when foreign exchange intervention is used also to reduce the output gap this leads to an excessive stabilization. Monetary policy can help in reducing the output gap and mitigate this burden. However, as I’ve explained before, monetary policy can only move the allocation along an inefficient IS curve. Without a state-contingent labor subsidy, foreign exchange intervention is still used to shift the IS curve closer to efficiency. The two tools are devoted to disjoint objectives only in two cases. When $\theta \to 0$ and prices become fully rigid, the central bank can use monetary policy to fully stabilize output and use foreign exchange intervention to select the efficient path of domestic consumption. This is true also when an appropriate state-contingent labor subsidy is in place. In this case monetary policy is used to deal with nominal rigidities as it can simultaneously stabilize inflation and the output gap. This is in sharp contrast with the standard view of foreign exchange interventions which emphasizes their “monetary” aspect.
Impulse responses for the model with sticky prices to a positive shock to foreign demand for Home assets. The blue line represents the allocation implemented by the optimal joint use of FX intervention and monetary policy, while the red line represents the allocation implemented by the optimal FX intervention alone ($i = \rho$). Parameter values: $\rho = 0.01$, $\alpha = 0.4$, $\varphi = 3$, $\theta = 0.7$, $\epsilon = 6$, $\beta = (1 - \alpha)^{\frac{1}{1+\varphi}}$, $\Gamma = 1$, $\varepsilon = 0.05$, $\varrho = 0.2$, $\phi_x = 0.03$. 
6 Conclusions

I consider a New Keynesian model of a small open economy where international financial markets are imperfect and the exchange rate is determined by capital flows. I use this framework to study the effects of exchange rate fluctuations driven by capital flows and characterize the optimal foreign exchange intervention. Capital flows shocks cause inefficient exchange rate fluctuations that trigger boom-bust cycles in the domestic economy. The optimal policy response is to partially stabilize these fluctuations using both foreign exchange intervention and monetary policy. The optimal foreign exchange intervention leans against the wind and stabilizes the path of the exchange rate: following an increase (decrease) in the foreign demand for domestic assets, the central bank increases (decreases) their net supply and accumulates (decumulates) foreign reserves. By doing so the central bank stabilizes the path of the exchange rate and smooths out fluctuations in domestic consumption. Simultaneously, the central bank reduces (increases) the nominal interest rate in order to reduce the relative price of domestic goods and mitigate the output gap. Foreign exchange intervention is not a mere substitute for monetary policy. Rather, the two tools complement each other.
References

Adler, Gustavo and Camilo E Tovar Mora, Foreign exchange intervention: a shield against appreciation winds?, International Monetary Fund, 2011.

Agrippino, Silvia Miranda and Hélène Rey, “Funding Flows and Credit in Carry Trade Economies,” Liquidity Funding Markets, 2013, p. 211.


Benes, Jaromir, Mr Andrew Berg, Mr Rafael A Portillo, and Mr David Vavra, Modeling sterilized interventions and balance sheet effects of monetary policy in a New-Keynesian framework, International Monetary Fund, 2013.

Benigno, Pierpaolo, “Price stability with imperfect financial integration,” Journal of Money, Credit and Banking, 2009, 41 (s1), 121–149.


Farhi, Emmanuel and Ivan Werning, “Fiscal unions,” 2012b.


Flug, Karnit and Amir Shpitzer, “Rethinking Exchange Rate Policy in a Small Open Economy: The Israeli Experience During the Great Recession,” BIS Paper No. 73n, 2013.


_ and _, “A Framework for Analyzing Large Scale Asset Purchases as a Monetary Policy Tool,” 2012.


Stathopoulos, Andreas, “Asset Prices and Risk Sharing in Open Economies,” *Available at SSRN 1364082*, 2012.


Appendix

...