Currency Wars, International Spillovers, and Guidelines for Policy Cooperation*

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Abstract
In an interconnected world, national economic policies regularly lead to international spillover effects that are politically contentious. This paper shows that many of these spillovers are nonetheless Pareto efficient. Furthermore, it narrows down inefficient spillovers to three specific sets of circumstances – if policymakers abuse market power, have imperfect policy instruments, or face imperfections in international markets. If none of these three sets of circumstances arise, we prove that spillovers from national economic policymaking are Pareto efficient and therefore not worth expending diplomatic efforts on. We give a number of examples of efficient spillovers, including from reserve accumulation, fiscal policy, exchange rate stabilization, and current account intervention to combat a liquidity trap.

JEL Codes: F34, F41, H23
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1 Introduction
In a globally integrated economy, national economic policies lead to international spillover effects.\(^1\) These are frequently large and lead to considerable controversy. For

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\(^1\)For a detailed review of both the theoretical and empirical literature on spillovers, see for example Kalemli-Ozcan et al. (2013).
example, the Brazilian finance minister Guido Mantega has used the term “currency wars” to describe the effects of US monetary easing on the country’s exchange rate (see Wheatley and Garnham, 2010). Other recent national policies that have led to international controversy include the large reserve accumulation by China and other Asian countries, the capital flow management policies by emerging market economies such as Brazil, as well as spillovers from monetary and exchange rate policy in Japan, Switzerland, the euro area and China. However, even though spillover effects may be controversial, this does not necessarily mean that they are inefficient.

The contribution of this paper is twofold. First, we show that we can narrow down the conditions under which spillovers are inefficient to three specific categories of problems: inefficiency only arises if policymakers (i) abuse market power, (ii) have imperfect policy instruments to influence external transactions, or (iii) face imperfections in international markets. This allows us to present a well-defined set of circumstances that are worth expending diplomatic efforts on. For each of the three categories of inefficiency, we provide general guidelines and examples for how cooperation can improve welfare. We also analyze when it is sufficient to coordinate the use of external policy instruments on trade and financial flows and when it is necessary to coordinate the use of domestic policies.

Secondly, if none of these three categories of problems are present, we show that spillovers from national economic policymaking are Pareto efficient and therefore not worth expending diplomatic efforts on – it is impossible to generate mutually beneficial cooperative agreements if an allocation already is Pareto efficient. This provides useful guidelines for when policy cooperation has a chance to bear fruit. Furthermore, even though the three idealized conditions are often not strictly met in practice, we will show in a number of examples below that the fundamental driving forces behind many contentious spillovers in recent years were likely Pareto efficient.

We analyze these questions in a multi-country framework that can flexibly nest a broad class of open economy macro models and that is able to capture a wide range of domestic market imperfections and externalities. We assume that each country consists of optimizing private agents and a policymaker who has policy instruments to affect the domestic and international transactions of the economy. When the policymaker in a given country changes her policy instruments, she influences the actions of domestic agents, which in turn may lead to general equilibrium adjustments that entail spillover effects to other economies.

We show that the spillovers from national economic policies are Pareto efficient so that there is no need for global cooperation under three sufficient conditions: (i) national policymakers refrain from monopolistic behavior, i.e. they act as price-takers in the international market, (ii) they possess a set of instruments to control the country’s external transactions that is effectively complete, and (iii) there are no

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2For detailed discussions on these subjects, see e.g. Gallagher et al. (2012), Jeanne et al. (2012), Ostry et al. (2012) and Stiglitz (2012).
imperfections in international markets. Under these three conditions, we can view national policymakers as competitive agents in a well-functioning global market, and the first welfare theorem applies. International spillover effects constitute pecuniary externalities that are mediated through market prices and are Pareto efficient.

We obtain our results by observing that, under quite general conditions, we can condense the welfare function of each country into a reduced-form welfare function $V(\cdot)$ that only depends on the country’s international transactions. Condition (i) ensures that there are no monopolistic distortions; condition (ii) guarantees that each domestic policymaker can actually implement her desired external allocation; condition (iii) ensures that the marginal rates of substitution of all countries are equated in equilibrium. As a result, we can apply the first welfare theorem at the level of national policymakers, interpreting $V(\cdot)$ as the utility functions of competitive agents in a complete market. Our model is general enough to allow for a wide range of domestic market imperfections, including price stickiness, financial constraints, incentive/selection constraints, missing or imperfect domestic markets, and imperfect domestic policy instruments.

Our framework offers clear guidelines for how policy cooperation can improve welfare when one or several of the conditions are violated: (i) Cooperation must ensure that countries refrain from monopolistic behavior and act with “benign neglect” towards international variables. (ii) If countries have incomplete external instruments, cooperation aims to expand the set of instruments or to use the existing set more efficiently, for example by making countries with better instruments assist those with worse instruments. (iii) If imperfections in international markets lead to inefficiency, global coordination is necessary since the imperfections are outside of the domain of individual national policymakers. We provide examples that characterize the scope for coordination in each of the three cases.$^3$

In the next part of our paper, we analyze a number of contentious policy measures that have led to spillovers in recent years and show that the fundamental driving forces behind these policies and the resulting spillovers were likely efficient. Our first application, chosen both because it captures some of the forces behind the reserve accumulation in Asia and because it provides the simplest illustration of our results, is an economy in which exports generate positive learning externalities. Reserve accumulation (or subsidizing exports) internalizes such externalities and is perfectly Pareto efficient from a global perspective. We extend this example to domestic learning-by-doing externalities and show that our efficiency result continues to hold if first-best policy measures to internalize the externalities are not available and if reserve accumulation is just a second-best measure.$^3$

$^3$We can also categorize recent examples of international cooperation along these lines: For example, Basel III provides policymakers with new counter-cyclical capital buffers (better instruments), recent IMF proposals allow countries to use capital controls (new instruments; see Ostry et al, 2011), and swap lines between advanced economy central banks provide new forms of liquidity insurance (more complete markets).
Our second application describes an economy that suffers from a shortage of aggregate demand that cannot be corrected using domestic monetary policy because of a binding zero-lower-bound on nominal interest rates. In such a situation, a national economic policymaker finds it optimal to impose controls on capital inflows or to subsidize capital outflows in order to mitigate the shortage in demand. Under the three conditions discussed earlier, this behavior leads, again, to a Pareto efficient global equilibrium.

Our third application analyzes the incentives for the optimal level of fiscal spending. If a domestic policymaker chooses how much fiscal spending to engage in based on purely domestic considerations, we show that the resulting global equilibrium is Pareto efficient under the three discussed conditions. In the context of fiscal policy, it is of particular importance that the domestic policymaker needs to act competitively, i.e. with benign neglect towards the international price effects of her policy actions. In particular, if the policymaker strategically reduces her stimulus in order to manipulate its terms-of-trade, then the resulting equilibrium is inefficient and generally exhibits insufficient stimulus. During the Great Recession, this observation has led many policymakers to argue that it is necessary to coordinate on providing fiscal stimulus since part of the stimulus spills over to other countries.

Our fourth application analyzes an economy in which domestic agents face incomplete risk markets and cannot insure against fluctuations in the country’s real exchange rate. Intervening in the economy’s capital account to stabilize the economy’s real exchange rate can serve as a second-best policy tool to insure domestic agents and improve welfare. Again, the outcome is Pareto efficient under the three conditions identified above. This illustrates that our results on global Pareto efficiency continue to hold even if a national planner intervenes in the current account to pursue purely domestic distributive objectives or to implement domestic political preferences.

In each of the described applications, we show that the spillover effects from policy intervention are efficient under the three benchmark conditions for efficiency. Any arguments about the desirability of global policy coordination therefore needs to be made on the basis of deviations from these conditions.

The final part of our paper examines the three areas of international policy coordination that are highlighted by our efficiency conditions: (i) When policymakers act monopolistically, coordination should aim to restrict monopolistic behavior so as to maximize gains from trade. The basic idea has been well understood at least since the rebuttal of mercantilism by Smith (1776). We add to this literature by providing general conditions for the direction of monopolistic intervention that help
to distinguish monopolistic intervention from intervention to correct domestic market imperfections. However, we also show that there are ample circumstances under which it is difficult to distinguish monopolistic from corrective intervention. Furthermore, we show that a national policymaker would never use domestic policies for monopolistic reasons unless she faces restrictions on her external policy instruments.

(ii) A country’s external policy instruments are effectively incomplete if the country does not have sufficient instruments to target her external transaction and would like to target them because they give rise to externalities. In that case, domestic policymakers do not have sufficient control over their private agents to choose her desired allocations, and international policy cooperation can improve outcomes. (Importantly, this condition applies only to the policy instruments targeting external transactions – it is perfectly fine for domestic policy instruments to be incomplete or restricted.) Generally speaking, we find that cooperation implies that countries with better instruments or better-targeted instruments assisting those without. For example, if a country experiences externalities from capital inflows but has no instrument to control them, welfare is improved if other countries control their capital outflows.

Policy cooperation under incomplete instruments has a rich intellectual tradition, going back to the targets and instruments approach of Tinbergen (1952) and Theil (1968). They observed in a reduced-form setting without private agents that incomplete instruments may give rise to a role for economic policy cooperation. Our contribution to this literature is to embed the Tinbergen-Theil approach into a general equilibrium framework in which optimizing individual agents interact in a market setting. This leads to a number of novel findings. First, we show that many of the spillover effects that would suggest a role for cooperation in the Tinbergen-Theil framework actually constitute efficient pecuniary externalities. Once the optimizing behavior of private agents is taken into account, a wide range of spillovers can be considered as efficient. Secondly, monopoly power and incomplete markets create independent roles for cooperation even if policy instruments are complete – a fact that was not considered by Tinbergen and Theil.5

(iii) When the international market is subject to imperfections, the first welfare theorem no longer applies and a global planner can generally improve outcomes. Borrowing from Leo Tolstoy’s quote on unhappy families, each imperfect market is imperfect in its own way. In the given paper, we limit our attention to two examples of international market imperfections.6 We refer to the rich literature on market imperfections in general equilibrium models with individual optimizing agents.7 By

5In the more recent literature, Jeanne (2014) provides an interesting example where the coordination of macroprudential policies is warranted because of missing policy instruments.

6Recent applications in which international market imperfections create a case for cooperation include Bengui (2013) who analyzes the need for coordination on liquidity policies when global markets for liquidity are incomplete, and Jeanne (2014) who analyzes a world economy in which agents are restricted to trading bonds denominated in the currency of a single country.

7This literature includes, for example, Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986) who discuss the implications of incomplete markets for the efficiency of equilibrium,
reinterpreting national economic policymakers as individual agents who interact in the international market, the inefficiency results of this literature and the lessons for the desirability of intervention can be applied to a setting of national planners to make a case for global cooperation.

The remainder of the paper is structured as follows: Section 2 illustrates the main forces at work in a simple example. Section 3 introduces our general model setup and examines its welfare properties, stating the general conditions under which spillovers are efficient. Section 5 provides a number of illustrations of efficient spillovers. Sections 6 to 8 examine the case for international policy coordination to rule out monopolistic behavior or to address incomplete external policy instruments and imperfections in international markets.

2 An Example

We start our formal analysis with a simple example that illustrates the main effects at work in our analysis. Assume a two-country two-period economy in which there is a single consumption good and intertemporal trade. The countries are denoted by $i = A, C$ (as in America and China) and are each inhabited by a unit mass of identical consumers and a national policymaker. Our key assumption is that country $C$ experiences growth externalities from learning-by-exporting.

In the absence of intervention, consumers in each country solve the optimization problem

$$\max_{c^i_1, c^i_2, m^i_1, m^i_2} U^i = u(c^i_1) + u(c^i_2) \quad \text{s.t.} \quad c^i_1 = y^i_1 + m^i_1$$

$$c^i_2 = y^i_2 + m^i_2$$

$$m^i_1 + m^i_2/R \leq 0$$

where $c^i_t$ and $y^i_t$ denote consumption and output, $m^i_t \geq 0$ denotes net imports (or, if negative, exports) of the consumption good or equivalently capital inflows (outflows), and $1/R$ is the relative intertemporal price of period 2 consumption goods or equivalently the inverse gross world interest rate. The period utility functions satisfy $u''(c^i) < 0 < u'(c^i)$ as well as $\lim_{c \to 0} u'(c) = \infty$. The optimization problem of each country is subject to two period budget constraints that capture the domestic budget constraint for a given level of imports, plus a dynamic budget constraint that reflects the intertemporal external budget constraint of country $i$ consumers.

or Farhi and Werning (2013) for a general treatment of the effects of price stickiness.

There is a considerable theoretical literature that postulates that such effects are important for developing countries in the phase of industrialization. See for example Rodrik (2008) and Korinek and Servén (2010). For a survey of the associated empirical literature see Giles and Williams (2000). We also provide a more detailed analysis of such externalities in an infinite horizon setup in Appendix 5.
In the following, we use lower-case variables to denote the allocations of individual consumers and upper-case letters to denote aggregate allocations, for example $M_i^t$ for aggregate net imports. Since consumers are identical, individual and aggregate allocations coincide in equilibrium, for example $m_i^t = M_i^t$, but it is useful to distinguish the two in our notation to allow for externalities, which we introduce next.

We assume that output in country $A$ is a constant exogenous endowment $y_A^1 = y_A^2$. In country $C$, period 1 output $y_C^1$ is also exogenous, but period 2 output is a function of aggregate period 1 imports $M_C^1$ that satisfies $y_C^2(0) = y_C^1$ and that is continuous and decreasing $y_C^2(M_C^1) < 0$. In short, higher aggregate net exports increase growth, capturing learning-by-exporting externalities in reduced form. For symmetry of notation, we denote $y_A^2(M_A^1) = y_A^2$ which satisfies by definition $y_A^2(M_A^1) = 0$.

**Reduced-Form Welfare Functions** In the following, we express the problem of each country $i$ in reduced form in a manner that will prove convenient throughout the remainder of the paper. Let us collect the period net imports of country $i$ in column vector format, $m_i = (m_i^1, m_i^2)^T$, and similarly for $M_i$ and the remaining allocations. We call the pair of vectors $(m_i, M_i)$ the external allocation of country $i$, and $(c_i, C_i)$ the domestic allocation of country $i$. We also define the international price vector $Q = (1, 1/R)$. We denote the reduced-form utility of a representative agent in country $i$ for a given external allocation $(m_i, M_i)$ by the function

$$V_i(m_i, M_i) = u(y_i^1 + m_i^1) + u(y_i^2(M_i^1) + m_i^2).$$

Observe that the marginal utility of private net imports is $V_i = \partial V_i/\partial m_i = (u'(c_i^1), u'(c_i^2))^T$ and the uninternalized social marginal utility is $V_i^s = \partial V_i/\partial M_i = (y_i^2(M_i^1), u'(C_i^2), 0)^T$.

**Laissez-Faire Equilibrium** Private agents in country $i$ take the aggregate allocation $M_i$ as given and solve the optimization problem

$$\max_{m_i} V_i(m_i, M_i) \quad \text{s.t.} \quad Q \cdot m_i \leq 0$$

Assigning shadow price $\lambda_i$, this yields the optimality condition

$$V_i^* = \lambda_i Q^T \quad \text{or, equivalently,} \quad \frac{u'(c_i^1)}{u'(c_i^2)} = R$$

(1)

The first equation describes optimality in vector notation: the marginal utility of each type of imports equals its market price times the (scalar) shadow price of wealth. The second equation is obtained by substituting out the shadow price: private agents equate their marginal rates of substitution (MRS) to the common world interest rate.

The allocation $m_i = M_i = (0, 0)^T$ together with the price vector $Q = (1, 1)$ represents an equilibrium of the system since the endowment of both countries is constant, implying perfect consumption smoothing for private agents under zero net imports. However, the laissez-faire equilibrium is sub-optimal since private agents neglect the potential gains from learning externalities.
National Planner Allocation  Let us next analyze the optimization problem of a national planner in country $i$ who internalizes learning externalities but acts competitively on world markets.$^9$ The planner internalizes that $m^i = M^i$ and solves the optimization problem

$$\max_{M^i} V^i(M^i, M^i) \quad \text{s.t.} \quad Q \cdot M^i \leq 0$$

Assigning shadow price $\Lambda^i$, the planner obtains the optimality condition

$$V^i_m + V^i_M = \Lambda^i Q^T \quad \text{or, equivalently,} \quad \frac{u'(c^i_1)}{u'(c^i_2)} = R - y^i_2(M^i_1)$$  \hspace{1cm} (2)$$

The first equation states that the described national planner equates the sum of the private and uninternalized social marginal utility of imports to the world market price. In the second equation, this is re-written in terms of the MRS of consumers – recall that $y^C_1 < 0$ so the social planner in country $C$ increases the MRS of consumers or, equivalently, encourages private agents to export in period 1, in order to benefit from the learning externalities.

The country $i$ planner can implement the described allocation using price or quantity interventions. In the first case, the planner imposes a vector of taxes $\tau^i$ on net imports that enter the external budget constraint of private agents in the form $\frac{Q}{1-\tau^i} \cdot m^i \leq T^i$ and $T^i$ rebates the revenue to domestic agent. (All vector divisions are element-by-element.) The optimal tax vector satisfies

$$\tau^C = -\left(\frac{V^C_M}{V^C_m}\right)^T = \left(-y^C_2 \cdot \frac{u'(c^C_2)}{u'(c^C_1)}, 0\right)$$  \hspace{1cm} (3)$$

so the planner subsidizes period 1 exports $\tau^C_1 > 0$ (or, equivalently, taxes imports) and sets $\tau^C_2 \equiv 0$ in period 2. There is no role for intervention in country $A$ so $\tau^A = (0, 0)$.

The planner could also use a quantity intervention and set net exports to the optimal level $M^C_1 < 0$ to internalize the growth externalities. In practice, this is typically achieved by closing the capital account to private transactions and accumulating $-M^C_1 > 0$ in foreign reserves.

Spillovers  In the resulting global equilibrium, country $C$ will be a net exporter in period 1 and a net importer in period 2, and vice versa for country $A$, so $m^C_1 = -m^A_1 < 0$ and $m^C_2 = -m^A_2 > 0$. Furthermore, the world interest rate will decline below the laissez-faire level $R < 1$. These quantity and price adjustments represent spillovers from the intervention of the planner in country $C$.$^{10}$

$^9$ We describe conditions under which such behavior is optimal on page 19. Furthermore, we devote the entire Section 6 to an analysis of the effects of non-competitive behavior.

$^{10}$ In the described example, country $A$ happens to be better off from the intervention of country $C$. We could easily describe examples in which country $A$ is worse off: if $y^A_1 > y^A_2$ holds sufficiently so that $A$ is a net lender in the national planning allocation, then a decline in the interest rate would hurt $A$, representing a negative spillover effect.
Global Optimum  The key question of our paper is under what conditions the equilibrium among national planners is socially efficient. To answer this question in the current example, we will compare the equilibrium among national planners with the allocations that would be chosen by a global planner.

A global planner maximizes the sum of worldwide welfare for a given set of Pareto weights $\phi^A$ and $\phi^C$. Substituting the global market-clearing condition $M^A = M = -M^C$, the planner’s problem can be described as

$$\max_M \phi^AV^A(M, M) + \phi^C V^C(-M, -M)$$  \hspace{1cm} (4)

with associated optimality condition

$$\phi^A [V^A_m + V^A_M] = \phi^C [V^C_m + V^C_M]$$  \hspace{1cm} (5)

We can find out whether the national planning allocation described by optimality condition (2) is Pareto efficient if a global planner will implement the same allocation as the national planners for an appropriate set of welfare weights. We find:

**Proposition 1** The competitive equilibrium among national planners is Pareto efficient.

**Proof.** Consider a national planning allocation that satisfies the optimality conditions (2) for $i = A, C$. Combine the conditions for the two countries and observe $V^A_M \equiv 0$ to obtain

$$\frac{V^A_m}{\Lambda^A} = Q^T = \frac{V^C_m + V^C_M}{\Lambda^C}$$

It can be easily seen that the optimality conditions of the national planners coincide with the optimality conditions of a global planner (5) with welfare weights $\phi^i = 1/\Lambda^i$ for $i = A, C$. The national planning allocation also satisfies global market clearing and is therefore globally Pareto efficient. 

By contrast, observe that the laissez faire equilibrium described by condition (1) for $i = A, C$ will never be globally efficient – combining the optimality conditions of private agents, we obtain $V^A_m/\Lambda^A = Q^T = V^C_m/\Lambda^C$. This is inconsistent with the planner’s optimality condition (5) no matter what set of welfare weights $(\phi^A, \phi^C)$ the global planner employs since the first element of the vector $V^C_{M,1} \neq 0$ but the second element $V^C_{M,2} = 0$.

Intuitively, the national planners described in the example ensure that each country equates the social marginal benefit of transacting with the rest of the world to the common vector of world market prices. Since (i) the described national planners act competitively, (ii) they have sufficient policy instruments and (iii) the international market is complete, the outcome is Pareto efficient. Even though the intervention of
country C has spillover effects on country A, these effects are Pareto efficient; in fact, they are necessary for the efficient functioning of the market.

Our example can also be used to illustrate how deviations from the three conditions required for efficiency lead to Pareto inefficient equilibria that call for global cooperation. In the following, we illustrate the case for global cooperation in each of the three cases:

**Deviating from Condition (i): Monopolistic National Planner** Let us consider a planner, w.l.o.g. in country C, who takes into consideration that her international transactions \( M^C \) will affect the world interest rate and that global market clearing requires \( M^A + M^C = 0 \). The Euler equation of private agents in country A implies that

\[
R(D_M^C) = u_0^1(y_A^1 M_C^1) = u_0^2(y_A^2 M_C^2)
\]

where

\[
\frac{\partial R}{\partial M_C^1} = u_0^1(C_A^1) = u_0^2(C_A^2)
\]

\[
\frac{\partial R}{\partial M_C^2} = u_0^2(C_A^2) R/u'(C_A^2)
\]

or, in vector notation, \( Q(M^C) = (1, 1/R(M^C)) \). A monopolistic planner in country C will solve the optimization problem

\[
\max_{M^C} V^C(M^C, M^C) \quad \text{s.t.} \quad Q(M^C) \cdot M^C \leq 0
\]

The associated optimality condition is

\[
V_i^i + V_m^i = \Lambda^i Q^T (1 - Q^{i, M})
\]

where \( Q_{i, M} = -[\partial Q/\partial M^C \cdot M^C]/Q^T \) is a vector of demand elasticities of world prices which satisfies \( Q_{i, M_1} < 0 < Q_{i, M_2} \), with the division performed element-by-element. The expression captures that the planner in country C internalizes that manipulating her import and export decisions enables her to improve the country’s terms-of-trade vis-à-vis country A. The resulting allocation can be implemented by setting the external policy instruments to

\[
\left(1 - \hat{\tau}_{1C}^C\right) = \frac{1 + V_M^i}{1 - Q_{i, M}} = \left(1 + \frac{y_A^2}{w(C_A^2)} \cdot \frac{w^0(C_A^1)}{w^0(C_A^2)} \right) / \left(1 - \frac{w^0(C_A^1) M^C}{R^2}ight)
\]

where all divisions are performed element-by-element. This implies that \( \hat{\tau}_{1C}^C > 0 > \hat{\tau}_{2C}^C \) – in addition to internalizing the growth externalities, the planner recognizes that restricting exports in period 1 and restricting imports in period 2 increases the world interest rate, which allows country C to earn a higher return on its savings.

Interestingly, the monopolistic national planner subsidizes exports in period 1 at a lower rate than a competitive national planner (as captured by the denominator in the expression for the period 1 tax rate), i.e. she forgoes part of the benefit of internalizing the learning externalities in order to manipulate the world interest rate. The spillovers created by the monopolistic national planner are thus smaller than those created by a price-taking (efficient) national planner. As a result, country A benefits less from valuable intertemporal trading opportunities with country C.
Since the planner imposes monopolistic wedges, the allocation is clearly not Pareto efficient and worldwide welfare is reduced. The deviation from price-taking behavior creates a clear scope for global coordination: global policymakers can increase worldwide welfare by forestalling monopolistic behavior.

**Deviating from Condition (ii): Incomplete Instruments** Let us return to the national planner allocation without monopolistic distortions but assume that country C has imperfect external policy instruments. For simplicity, assume that the country is unable to affect the external allocations of private agents so \( \tau^C = (0, 0) \) but country A has a full set of external policy instruments \( \tau^A \).

In the national planning allocation, the policymaker in country C does not engage in policy intervention because she is not able to; the policymaker in country A does not engage in policy intervention and sets \( \tau^A = (0, 0) \) because she does not see any domestic rationale to intervene in markets. The resulting allocation is identical to the global laissez-faire allocation. As we showed earlier, it is not Pareto efficient because it neglects the learning externalities.

Again, there is a clear scope for global coordination: the global optimum described in Proposition 1 requires that the social marginal products of the two countries are equated, \( (V^C_m + V^C_m) / \Lambda^C = V^A_m / \Lambda^A \). This allocation can be replicated if the policymaker in country A sets her policy instruments to

\[
1 - \hat{\tau}^A = \frac{1}{1 - \tau^C} \quad \text{or} \quad \hat{\tau}^A = \left( 1 - \frac{1}{1 + y^U \cdot \frac{u'(c^C_2)}{u'(c^C_1)}} \right)
\]

to internalize the externality of country C and country C provides a transfer to finance the policy intervention.\(^{11}\) In short, instead of the policymaker in country C subsidizing exports, she pays the policymaker in country A to subsidize imports. The world price vector adjusts to \( \hat{Q} = (1/(1-\tau^C_1), 1/R) \). At this new price vector and given the transfer payment, the original optimal allocation of Proposition 1 is feasible for both countries and the social marginal products of the two countries are equated since \( V^C_m / \Lambda^C = \hat{Q} = (1 - \hat{\tau}^A_1)V^A_m / \Lambda^A \). Intuitively, it does not matter if country C subsidizes exports or country A subsidizes imports in period 1 – the resulting allocation is the same.

**Deviating from Condition (iii): International Market Imperfections** The third area that requires global coordination are international market imperfections.

\(^{11}\)The transfer is of the exact same magnitude as what country C would have used to finance its own export subsidy if that instrument was available, so no extra government revenue is required. However, transfers may be politically contentious. If we rule out transfers, a global planner would still want to use the policy instruments of country A to internalize any externalities of country C with incomplete instruments, but the allocation of Proposition 1 can no longer be replicated. In other words, implementing globally efficient allocations would generally go hand in hand with redistributions.
To illustrate a relevant example, loosely inspired by Jeanne (2014), let us assume that international financial transactions are restricted to take place in the currency of country $A$, which is in a liquidity trap and faces a zero interest rate so the international price vector is $Q = (1, 1)$. Furthermore, as is common in the New Keynesian literature, assume that period 1 output in country $A$ is demand-determined and adjusts so as to clear the market. In other words, when country $C$ increases exports, the world interest rate cannot decline, but country $A$ imports more and experiences a decline in demand for domestic output and thus a decline in $y_A^1$. Furthermore, assume that period 1 output in country $A$ is produced at a continuously differentiable convex utility cost $d(y_A^1)$ that satisfies $d(0) = d'(0) = 0$ and $d'(1) = 1$.

When the zero-lower-bound is binding, period 1 output is determined by the Euler equation

$$u'(C_A^1) = u'(C_A^2) \quad \text{or} \quad y_A^1(M_A) = y_A^2 - M_A^1 + M_A^2$$

(6)

The reduced-form utility function of country $A$ is

$$V^A(m^A, M_A) = u(y_A^1(M_A) + m_A^1) - d(y_A^1(M_A)) + u(y_A^2 + m_A^2)$$

The national planner in country $A$ recognizes that imports lead to aggregate demand externalities and sets her external policy instruments to

$$\tau^A = \frac{-V_m^A}{V_m^A} = \left(1 - \frac{d'(y_A^1)}{u'(C_A^1)} \right) \left(1, -u'(C_A^1)/u'(C_A^2) \right)$$

We can interpret the term $1 - d'(y_A^1)/u'(C_A^1) > 0$ as the analogon of the labor wedge in New Keynesian models, i.e. as the cost of the demand shortage – an additional unit of output would cost $d'(y_A^1)$ but bring utility benefit $u'(C_A^1)$. The national planner in country $A$ would thus tax period 1 imports which take away from domestic demand and subsidize period 2 imports, which create a future boom and by implication boost today’s output [see equ. (6)]. The national planner in country $C$ would continue to operate as in the baseline example (3) above. Given the sticky price vector $\bar{Q} = (1, 1)$, the resulting global equilibrium is described by the equilibrium condition

$$\frac{V_m^A + V_m^A}{\Lambda^A} = \bar{Q} = \frac{V_m^C + V_m^C}{\Lambda^C}$$

(7)

From this condition, it is apparent that the price mechanism cannot play its usual role of efficiently allocating goods across countries – prices do not reflect the relative social valuation of goods, but are given exogenously.

A global planner solves the optimization problem (4) with optimality condition (5). It can easily be seen that the equilibrium described by (7) can be improved

\[12\] For a detailed derivation of how a typical New-Keynesian setup with a binding zero-lower-bound determines output see below in Section ZZZZ.
upon: at the described uncoordinated allocation, country C internalizes learning externalities by equating the marginal benefit of imports/exports in the two periods to the fixed world price vector; however, period 1 exports from C create negative demand externalities for country A. A marginal reduction in period 1 exports from country C would come at a second-order cost for country C (since the country was at the point of optimality, given prices \( \bar{Q} \)) but would provide a first-order benefit of \( u'(C_1^A) - d'(y_1^A) > 0 \) to country A.

In the following section, we develop a significantly more general model to show that the basic insights provided by our example are robust across a wide range of open economy macro models.

3 General Model Setup

Our general model describes a multi-country economy in which each country consists of a continuum of private agents as well as a domestic policymaker. Both maximize domestic welfare subject to a set of constraints on their domestic allocations and a standard budget constraint on their external transactions. In order to create an interesting role for policy intervention, we distinguish between individual and aggregate allocations so as to capture the potential for externalities. An individual takes aggregate allocations as given, whereas the domestic planner employs her policy instruments to affect aggregate allocation so as to correct for domestic externalities. Our framework nests a wide range of open economy macro models in which there is a case for policy intervention, as we illustrate in a number of examples below.

Countries We describe a world economy with \( N \geq 2 \) countries indexed by \( i = 1, \ldots, N \). The mass of each country \( i \) in the world economy is \( \omega^i \in [0, 1] \), where \( \sum_{i=1}^{N} \omega^i = 1 \). A country with \( \omega^i = 0 \) corresponds to a small open economy.

Private Agents In each country \( i \), there is a continuum of private agents of mass 1. We denote the allocations of individual private agents by lower-case variables and the aggregate allocations of the country by upper-case variables.

A representative private agent obtains utility according to a function

\[
U^i(x^i)
\]

(8)

where \( x^i \) is a column vector of domestic variables that includes all variables relevant for the utility of domestic agents, for example the consumption of goods or leisure. We assume that \( U^i(x^i) \) is increasing in each element of \( x^i \) and quasiconcave. To keep notation compact, we assume that \( x^i \) encompasses all domestic variables, including those that do not directly yield utility but that we want to keep track of, for example the capital stock \( k^i \) in models of capital accumulation. For such variables, \( \partial U^i/\partial k^i = 0 \).
**External Budget Constraint**  We denote the international transactions of private agents by a column vector of net imports \( m^i \) that are traded at an international price vector \( Q \). Private agents may be subject to a vector of tax/subsidy instruments \( \tau^i \) that is imposed by the domestic policymaker. We denote by \( \frac{Q}{1-\tau^i} \), the element-by-element (Hadamard) division of the price vector \( Q \) by the tax vector \( (1-\tau^i) \), which are both row vectors, and by \( Q \cdot m^i \) the inner product of the price and quantity vectors. The external budget constraint of domestic agents is then

\[
\frac{Q}{1-\tau^i} \cdot m^i \leq T^i \tag{9}
\]

Any tax revenue is rebated as a lump sum transfer \( T^i = \frac{\tau^i Q}{1-\tau^i} \cdot m^i \). If taxes are zero, then the budget constraint reduces to the *country i external constraint* \( Q \cdot m^i \leq 0 \).

**Domestic Constraints**  We assume that the representative agent in country \( i \) is subject to a collection of constraints, which encompass domestic budget constraints and may include financial, incentive/selection, or price-setting constraints as well as restrictions imposed by domestic policy measures,

\[
f^i \left( m^i, x^i, M^i, X^i, \zeta^i \right) \leq 0 \tag{10}
\]

where \( \zeta^i \) is a collection of domestic policy instruments such as taxes, subsidies, government spending, or constraints on domestic transactions. In some of our applications below, we will define \( Z^i \) as a collection of exogenous state variables, for example endowments, productivity shocks or initial parameters. The domestic constraint depends on these state variables, but for compactness of notation and since they are exogenous, we will omit them as arguments of the function \( f^i(\cdot) \).

Observe that we include both the individual external and domestic allocations \( (m^i, x^i) \) and aggregate allocations \( (M^i, X^i) \) in the constraint to capture the potential for externalities from aggregate allocations to the choice sets of individuals, which we will analyze further in the coming sections. The choice variables of the representative agent are \( (m^i, x^i) \) and he takes all remaining variables in the constraint as given.

In summary, the optimization problem of a representative domestic agent is to choose the optimal external and domestic allocations \( (m^i, x^i) \) so as to maximize utility \( (8) \) subject to the collection of domestic and external constraints,

\[
\max_{m^i, x^i} U^i \left( x^i \right) \quad \text{s.t.} \quad (9), (10) \tag{11}
\]

**Domestic Policymaker**  The domestic policymaker sets the domestic policy instruments \( \zeta^i \) and external policy instruments \( \tau^i \) in order to maximize the utility \( (8) \) of domestic private agents subject to the domestic and external constraints \( f^i(\cdot) \leq 0 \) and \( M^i \cdot Q \leq 0 \). The policymaker internalizes the consistency requirement that the allocations of the representative agent coincide with the aggregate allocations so
m^i = M^i and x^i = X^i. Furthermore, the policymaker internalizes that these allocations \((m^i, x^i)\) have to solve the optimization problem of domestic agents as described by problem (11).

**Definition 1 (Feasible Allocations)** We define a feasible allocation in country \(i\) for given world prices \(Q\) as a collection \((X^i, M^i, \zeta^i)\) that satisfies the country \(i\) domestic and external constraints \(f^i(M^i, X^i, M^i, X^i, \zeta^i) \leq 0\) and \(M^i \cdot Q \leq 0\).

Furthermore, we define a feasible global allocation as a collection \((X^i, M^i, \zeta^i)_{i=1}^N\) that satisfies the domestic constraints \(f^i(\cdot) \leq 0 f^i(\cdot) \leq 0 \forall i\) and global market clearing \(\sum_{i=1}^N \omega^i M^i \leq 0\).

To make our setup a bit more concrete, the following examples illustrate how a number of benchmark open economy macro models map into our framework:

**Example 3.1 (Infinite Horizon Model of Capital Flows)** Our first example is a simple model of capital flows between neoclassical endowment economies \(i = 1, ..., N\) with a single consumption good in infinite discrete time. Assume that the domestic variables in each economy \(i\) consist of a vector of consumption goods \(x^i = \{(c^i_t)_{t=0}^\infty\}\) and that the vector of external transactions \((m^i_t)_{t=0}^\infty\) denotes the imports of the consumption good in each period, which is equivalent to the trade balance. Since there is a single good, we can also interpret \(m^i_t\) as the net capital inflows in period \(t\). Furthermore, there are no domestic policy instruments so \(\zeta^i = \emptyset\) and the vector of exogenous variables \(Z^i\) consists of an exogenous endowment process \((y^i_t)_{t=0}^\infty\). Assume the utility function in each country is given by \(U^i(x^i) = \sum_i \beta^i u(c^i_t)\) and the domestic constraints contain one budget constraint for each time period so \(f^i(\cdot) = \{f^i_t(\cdot)\}_{t=0}^\infty\) where \(f^i_t(\cdot) = c^i_t - y^i_t - m^i_t \leq 0\). If we normalize \(Q = 1\) then each element of the vector \(Q_t\) represents the price of a discount bond that pays one unit of consumption good in period \(t\), and the external budget constraint of the economy is given by (9). This fully describes the mapping of a canonical open economy model into our baseline setup.

The policymaker’s vector of external policy instruments \(\tau^i = (\tau^i_t)_{t=0}^\infty\) can be interpreted as capital controls according to the following categorization:

<table>
<thead>
<tr>
<th>(m^i_t)</th>
<th>(\tau^i_t)</th>
<th>(\tau^i_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0 (net saving)</td>
<td>outflow tax</td>
<td>inflow tax</td>
</tr>
<tr>
<td>&gt; 0 (net borrowing)</td>
<td>inflow subsidy</td>
<td>outflow subsidy</td>
</tr>
</tbody>
</table>

**Table 1:** Interpretation of capital control \(\tau^i_t\)

For example, if a country is a net importer \(m^i_t > 0\) (i.e. experiences net capital inflows) in period \(t\), a positive tax rate \(\tau^i_t > 0\) raises the cost of the imports/inflows so the measure represents an inflow tax on capital.
It is common in the open economy macro literature to keep track of the external wealth position $w_t^i$ of a country over time and denote the external budget constraint (9) by the law of motion of the external wealth position together with a transversality condition $\lim_{t \to \infty} Q_t w_t^i = 0$. This law of motion can be represented by the series of period-by-period constraints

$$
\frac{(1 - \xi_{t+1}^i)}{1 + r_{t+1}} w_{t+1}^i = w_t^i - m_t^i + T_t^i \quad \forall t
$$

where the interest rate $r_{t+1}$ corresponds to the relative price of discount bonds in two consecutive periods, $1 + r_{t+1} = Q_t/Q_{t+1}$, and the period capital control $\xi_{t+1}^i$ corresponds to the increase in the cumulative controls, $1 - \xi_{t+1}^i = (1 - \tau_t^i) / (1 - \tau_{t+1}^i)$. The revenue from any controls is rebated in lump sum fashion $T_t^i$ in the period it is raised. The Arrow-Debreu formulation and the period-by-period description of our setup are equivalent, and we will use both in our applications below.

**Example 3.2 (Stochastic Economy)** To extend our example to a stochastic economy, all that is required is to define a set of states of nature $\Omega$ and a series of probability spaces $(\Omega, \mathcal{F}_t, P_t)$ such that $\mathcal{F}_t$ defines a sigma-algebra of states measurable at time $t$ and satisfying $\mathcal{F}_t \subseteq \mathcal{F}_{t+1}$ and $P_t$ is a probability measure defined over each $\mathcal{F}_t$ with an associated expectations operator $E_t[.] = E[.]|\mathcal{F}_t$. Then we can label all variables by both time $t$ and state of nature $s \in \Omega$, for example $x^i_t = \{ (c_{t,s}^i, \ell_{t,s}^i, i_{t,s}^i, k_{t,s}^i)_{t=0,s \in \Omega} \}$, and define utility as $U^i(x^i) = E_0[\sum_t \beta^t u(c_{t,s}^i)]$. Importantly, net imports $m^i_t = \{ (m_{t,s}^i)_{t=0,s \in \Omega} \}$ are now also state-contingent. Everything else remains unchanged.

**Example 3.3 (Production Economy)** Next we describe capital flows in a world of neoclassical production economies. We build on example 1 and include leisure, investment, and capital in the collection of domestic variables so $x^i = \{ (c_{t,s}^i, \ell_{t,s}^i, i_{t,s}^i, k_{t,s}^i)_{t=0,s \in \Omega} \}$ where labor in a given period is $1 - \ell_{t,s}^i$. The vector of exogenous variables contains the initial capital stock and the path of productivity, $Z^i = \{ k_0, (A_{t,s}^i)_{t=0}^{\infty} \}$. We extend the utility function to include leisure $U^i(x^i) = \sum_t \beta^t u(c_{t,s}^i, \ell_{t,s}^i)$. The collection of domestic constraints consists of a budget constraint $f_{t,c}^i(\cdot)$ and a capital accumulation constraint $f_{t,k}^i(\cdot)$ each period,

$$
f_{t,c}^i(\cdot) = c_t^i + i_t^i - A_t^i(k_{t,s}^i)^\alpha (1 - \ell_t^i)^{1-\alpha} - m_t^i \leq 0
$$

$$
f_{t,k}^i(\cdot) = k_{t+1}^i - (1 - \delta) k_{t,s}^i - i_t^i \leq 0
$$

As in example 3.1, $m_t^i$ equivalently captures net imports and capital inflows, and the policy measure $\tau_t^i$ can be interpreted as import taxes or as capital controls as in Table 1. The framework can be extended to stochastic shocks as described before.

**Example 3.4 (Multiple Consumption Goods)** The framework can also be extended to multiple consumption goods by defining the variables $c_t^i$, $m_t^i$, $y_t^i$ and $\tau_t^i$ in
each time period as vectors of size \( K \) capturing the consumption, net imports, endowment and inflow taxes on each good \( k = 1 \ldots K \) in period \( t \). Utility can then be written as \( U^i (x^i) = \sum_t \beta^t u (c^i_{t,1}, \ldots, c^i_{t,K}) \). If some of the consumption goods are non-traded, we omit them from the vector of net imports \( m^i_t \) so that \( \dim m^i_t = K_T < K \).

Assuming that \( \tau^i_t \) is a vector of equal size as \( m^i_t \) supposes that the planner can set differential tax/subsidy rates on each traded good \( k = 1 \ldots K_T \). Alternatively, assuming that the planner can only differentiate taxes by time period would amount to a restriction on the set of instruments \( \tau^i_{t,1} = \ldots = \tau^i_{t,K_T} \forall t \). We will analyze such restrictions in Section 7.

4 Equilibrium

Definition 2 (Global Competitive Equilibrium) An equilibrium in the described world economy consists of a feasible global allocation \( (X^i, M^i, \zeta^i)_{i=1}^N \) and set of external policy measures \( (\tau^i)_{i=1}^N \) together with a vector of world market prices \( Q \) such that

- the individual allocations \( x^i = X^i \) and \( m^i = M^i \) solve the optimization problem of private agents in country \( i \) for given prices \( Q \), aggregate allocations \( (X^i, M^i, \zeta^i) \) and external policy measures \( (\tau^i) \) for each \( i = 1, \ldots N \),

- the aggregate allocations \( (X^i, M^i, \zeta^i) \) and external policy measures \( (\tau^i) \) solve the optimization problem of the policymaker in country \( i \) for each country \( i = 1, \ldots N \) and given prices \( Q \), and

- markets for international transactions clear, \( \sum_i^N \omega^i M^i = 0 \).

A formal description of the optimization problems of private agents and domestic policymakers is provided in appendix A.1.

In the following two subsections, we separate the analysis of equilibrium in the world economy into two steps. The first step is the domestic optimization problem of each economy for a given external allocation \( (m^i, M^i) \) and is described in the next subsection. We show that the welfare of each country can be expressed as a reduced-form utility function \( V^i (m^i, M^i) \) that greatly simplifies our analysis. The second step solves for the optimal external allocation of the economy given \( V^i (m^i, M^i) \) for each country and is described in the ensuing subsection. A formal lemma establishes that the described two-step procedure solves the full optimization problem.

4.1 Domestic Optimization Problem

Domestic Agents We describe the domestic optimization problem of a representative agent in country \( i \) for given external allocations \( (m^i, M^i) \), domestic aggregate
allocations $X^i$ and domestic policy variables $\zeta^i$. We define the reduced-form utility of the representative agent as

$$v^i \left(m^i, M^i, X^i, \zeta^i\right) = \max_{x^i} U^i \left(x^i\right) \quad \text{s.t.} \quad f^i \left(m^i, x^i, M^i, X^i, \zeta^i\right) \leq 0 \quad (12)$$

Denoting the shadow prices on the domestic constraints $f^i$ by the row vector $\lambda^i_d$, the collection of domestic optimality conditions of a representative agent is

$$U^i_x = f^i_x \lambda^i_d^T \quad (13)$$

where $U_x$ denotes a column vector of partial derivatives of the utility function with respect to $x^i$, and $f^i_x$ is the Jacobian of derivatives of $f^i$ with respect to $x^i$ and is a matrix of the size of $f^i(\cdot)$ times the size of $x^i$.

**Domestic Policymaker** For a given aggregate external allocation $M^i$, a domestic policymaker in economy $i$ chooses the optimal domestic policy measures $\zeta^i$ and aggregate choice variables $X^i$ subject to the consistency conditions $x^i = X^i$ and $m^i = M^i$ as well as the implementability constraint (13). The planner’s problem is

$$\max_{X^i, \zeta^i, \lambda^i_d} U^i \left(X^i\right) \quad \text{s.t.} \quad f^i \left(M^i, X^i, M^i, X^i, \zeta^i\right) \leq 0, \quad (13) \quad (14)$$

We assign the row vector of shadow prices $\lambda^i_d$ to the collection of domestic constraints $f^i$ and $\mu^i_d$ to the collection of domestic implementability constraints. The solution to this problem defines the optimal domestic policy measures $\zeta^i(M^i)$ and aggregate domestic choice variables $X^i(M^i)$.

**Optimal Domestic Allocation** For a given aggregate external allocation $M^i$, the optimal domestic allocation in country $i$ consists of a consistent domestic allocation $x^i = X^i$ and domestic policy measures $\zeta^i$ that solve the domestic optimization problem (14) of a domestic policymaker and, by implication, the domestic optimization problem (12) of private agents in country $i$ since the policymaker observes the implementability constraint (13).

**Definition 3 (Reduced-Form Utility)** We define the reduced-form utility function of a representative agent in economy $i$ for a given pair $(m^i, M^i)$ by

$$V^i \left(m^i, M^i\right) = v^i \left(m^i, M^i, X^i(M^i), \zeta^i(M^i)\right) \quad (15)$$

The reduced-form utility function $V^i \left(m^i, M^i\right)$ is also defined for off-equilibrium allocations in which $m^i$ and $M^i$ differ since individual agents are in principle free to choose any allocation of $m^i$. In equilibrium, however, $m^i = M^i$ will hold.
For the remainder of our analysis, we will focus on the case where the partial derivatives of this reduced-form utility function satisfy \( V_{m} > 0 \) and \( V_{M} + V_{M}^t > 0 \) \( \forall i \): ceteris paribus, a marginal increase in individual imports \( m_i \) or a simultaneous marginal increase in both individual and aggregate imports \( m^i = M^i \) increases the welfare of a representative consumer. These are fairly mild regularity conditions that hold for the vast majority of open economy macro models. For instance, the reduced-form utility function in Example 3.1 is \( V^i(m^i, M^i) = \sum_t \beta^t u(y_t^i + m_t^i) \), satisfying the above marginal utility conditions since \( V_{m,t}^i = V_{m,t}^i + V_{M,t}^i = \beta^t u'(c_t^i) > 0 \forall i, t. \)

The reduced-form utility function \( V^i(m^i, M^i) \) contains all the information that is required to describe external allocations and the global equilibrium.

### 4.2 External Allocations

**Representative Agent** Given the reduced-form utility function \( V^i(m^i, M^i) \), an international price vector \( Q \), a vector of tax instruments \( \tau^i \) on external transactions, transfer \( T^i \) and aggregate external allocations \( M^i \), the second-step optimization problem of a representative agent in country \( i \) defines the agent’s reduced-form import demand function

\[
m^i(Q, \tau^i, T^i, M^i) = \arg \max_{m^i} V^i(m^i, M^i) \quad \text{s.t.} \quad (9)
\]

Assigning the scalar shadow price \( \lambda^i_e \) to the external budget constraint (9), the associated optimality conditions

\[
(1 - \tau^i)^T V_{m}^i = \lambda^i_e Q^T
\]

describe the excess demand for each component of the import vector \( m^i \) of the representative agent as a function of the vector of world market price \( Q \), where the tax vector \( (1 - \tau^i) \) pre-multiplies the column vector \( V_{m}^i \) in an element-by-element fashion.

We define the aggregate reduced-form import demand function \( M^i(Q, \tau^i) \) as the fixed point of the representative agent’s reduced-form import demand function \( M^i = m^i(Q, \tau^i, \frac{\tau^i Q}{1 - \tau^i}, M^i, M^i) \) that satisfies the government budget constraint \( T^i = \frac{\tau^i Q}{1 - \tau^i} \cdot M^i \).

**Laissez-Faire Equilibrium** We define the allocation that prevails when \( \tau^i = 0, T^i = 0 \) \( \forall i \) as the laissez-faire equilibrium. We assume that domestic policymakers use their domestic policy instruments \( \zeta^i \) optimally as described in problem (14).

**National Planner** Next we consider how a policymaker in country \( i \) optimally determines the policy instruments \( \tau^i \) on external transactions if she acts competitively on the world market in the sense of taking the price vector \( Q \) as given. We term this interchangably a competitive national planner.
There are several potential interpretations for such price-taking behavior: First, country \( i \) may be a small economy with \( \omega^i \approx 0 \) so that it is not possible for the planner to affect world market prices. Secondly, the planner may choose her optimal allocations while acting with benign neglect towards international markets. This may be the consequence of an explicitly domestic objective of the policymaker prescribed by domestic law. Third, the behavior may be the result of an explicit or implicit multilateral agreement to abstain from monopolistic behavior and disregard worldwide terms-of-trade effects, or because of enlightened pursuit of efficient allocations.\(^{13}\)

We devote Section 6 to analyzing the behavior of a monopolistic policymaker who internalizes her market power over world market prices \( Q \). There, we will also discuss how to distinguish between monopolistic and competitive behavior of policymakers in practice.

Since the planner has a full set of policy instruments to control \( M^i \), we can solve directly for the planner’s optimal allocations and set the instruments \( \tau^i \) to implement the desired allocation. (Section 8 will analyze monopoly power under incomplete external policy instruments.) A competitive planner who faces the reduced-form utility function \( V^i (m^i, M^i) \) solves

\[
\max_{M^i} V^i (M^i, M^i) \quad \text{s.t.} \quad Q \cdot M^i \leq 0
\]

Assigning shadow price \( \Lambda^i \) to the planner’s external budget constraint, the optimality condition is

\[
V^i_m + V^i_M = \Lambda^i Q^T
\]

**Lemma 1 (Implementation)** The planner can implement her optimal external allocation by setting the vector of policy instruments

\[
\tau^i = - \left( \frac{V^i_M}{V^i_m} \right)^T
\]

where the division \( V^i_M / V^i_m \) is performed element-by-element at the optimal allocation.

**Proof.** Substituting the optimal \( \tau^i \) from (20) into the optimality condition of private agents (17) yields the planner’s optimality condition (19). \( \blacksquare \)

For given world prices \( Q \), the lemma defines a function \( \tau^i (Q) \) that implements the optimal external allocation. According to this implementation, the planner does not intervene in time periods/states of nature/goods for which \( V^i_{M,i} = 0 \), i.e. for which the marginal benefit is fully internalized by private agents. By contrast, if there is

\(^{13}\)For example, the US Federal Reserve claims to follow a policy of acting with benign neglect towards external considerations such as exchange rates, as articulated by Bernanke (2013). Similarly, the G-7 Ministers and Governors proclaimed in a Statement after their March 2013 summit that “we reaffirm that our fiscal and monetary policies have been and will remain oriented towards meeting our respective domestic objectives using domestic instruments, and that we will not target exchange rates” (G-7, 2013).
an uninternalized benefit to inflows $V_{i}^{i} > 0$, then $\tau_{i} > 0$ so the planner subsidizes inflows of $m_{i}^{i}$, and vice versa for negative externalities $V_{i}^{i} < 0$.

Finally, we observe formally that the two-stage procedure that we followed to separate the problem into a domestic and external optimization problem indeed solves the general problem:

**Lemma 2 (Separability)** *The allocation that solves the two separate stages of the domestic and external optimization problem described by (14) and (18) solves the combined planning problem.*

**Proof.** See appendix A.1.

Since the planner has a full set of instruments to determine the optimal external allocation, the optimal domestic allocation can be determined without considering the interactions with the external allocation – there is no need to distort the domestic allocation in order to achieve external goals. Formally, the separability follows from the fact that the external implementability constraint on the planner is slack and, given $M^{i}$, can be ignored by the planner when solving for the optimal domestic allocations. We will show in section 7 that this no longer holds when the planner faces an imperfect set of external policy instruments. In that case, the planner has an incentive to use domestic policies to affect external allocations $M^{i}$ and therefore the planner’s domestic optimization problem is no longer described by problem (14). This result has an important practical implication:

**Corollary 1 (Separating Domestic and External Economic Policy)** *The two tasks of implementing the optimal domestic economic policy and optimal external allocations of an economy can be performed by two separate agencies without need for coordination.*

When the set of external policy instruments is complete, the agency responsible for domestic policy therefore does not need to coordinate with the agency that sets the country’s external policy instruments, it just observes the external allocation $M^{i}$ and acts optimally.

The following lemma establishes that the implementation of Lemma 1 is just one out of a continuum of alternative implementations that all lead to the same real allocation:

**Lemma 3 (Indeterminacy of Implementation)** *There is a continuum of alternative implementations for the external allocation of a country $i$ described by lemma 1, in which its policy instruments are re-scaled by a positive constant $k^{i} > 0$ s.t. $(1 - \tau^{i}) = k^{i} (1 - \tau^{i})$.*
Proof. The re-scaling of policy instruments does not affect the external budget constraint of the economy since revenue is rebated lump-sum. It simply rescales the shadow price $\Lambda^i_k$ in the optimality condition (19) by $1/k^i$. Therefore the old allocation still satisfies all the optimality conditions of the economy. ■

The intuition is that the incentive of a representative agent to shift consumption across time/states of nature/goods only depends on the relative price of goods in the vector $m^i$. Multiplying all prices by a constant and changing initial net worth by the corresponding amount is equivalent to changing the numeraire.

By the same token, if there exists a scalar $h^i \in (-1,\infty)$ s.t. $V^i_M = h^i V^i_m$, then it is not necessary for the country $i$ planner to intervene since the vector of policy instruments $\bar{\tau}^i = 0$ will implement the same equilibrium as the vector $\tau^i = -(V^i_M/V^i_m)^T$. This can easily be verified by setting $k^i = \frac{1}{1+h^i}$ and applying lemma 3. In the following, we will assume that $\nexists h^i$ that satisfies $V^i_M = h^i V^i_m$ when we speak of a country that exhibits externalities.

Sequential Trading The formulation of problem (16) assumed – in Arrow-Debreu fashion – that all intertemporal trade takes place in period 0. If trading occurs sequentially, i.e. period after period, as it did in our baseline model, then the planner can implement her optimal allocation by imposing the relative tax wedge $(1 - \tau^i_{t,t+s}) = (1 - \tau^i_t) / (1 - \tau^i_{t+s})$ between any two periods $t$ and $t + s$.

4.3 Welfare Properties of Equilibrium

We now turn to the welfare properties of the described global equilibrium. We start with a definition:

Definition 4 (Pareto Efficient Allocations) A Pareto efficient global allocation is a feasible global allocation $(M^i, X^i, \zeta^i)^N_{i=1}$ such that there does not exist $\nexists$ another feasible allocation $(\tilde{M}^i, \tilde{X}^i, \tilde{\zeta}^i)^N_{i=1}$ that makes at least one country better off, $U^i(\tilde{X}) \geq U^i(X) \forall i$, with at least one strict inequality.

Given this definition, we find:

Proposition 2 (Efficiency of Global Equilibrium) The global competitive equilibrium allocation as per Definition 2 is Pareto efficient.

Proof. An allocation is Pareto efficient if it maximizes the weighted sum of welfare of all countries for some vector of welfare weights $\{\phi^i \geq 0\}^N_{i=1}$ subject to the global resource constraint, the domestic constraints $f^i(\cdot)$ and the domestic implementability conditions.
constraint (13) of each country $i$. Given the complete set of external instruments, the planner can directly choose the external allocations $M^i$ and solve

$$\max \{M^i, X^i, \zeta^i\} \sum_i \phi^i \omega^i U^i (X^i) \quad \text{s.t.} \quad \sum_i \omega^i M^i = 0, \ (13),$$

$$f^i (M^i, M^i, X^i, X^i, \zeta^i) \leq 0 \ \forall i$$

By the definition of $V^i (m^i, M^i)$, we can restate this problem in terms of reduced-form utilities of the optimal external allocations $(M^i)_{i=1}^N$,

$$\max_{\{M^i\}} \sum_i \phi^i \omega^i V^i (M^i, M^i) \quad \text{s.t.} \quad \sum_i \omega^i M^i = 0$$

Assigning the shadow price $\nu$ to the vector of resource constraints, the optimality condition of the global planner is

$$\phi^i (V^i_m + V^i_M) = \nu^T \ \forall i$$

Any global competitive equilibrium that satisfies Definition 2 also satisfies these optimality conditions if we use the shadow price $\nu = Q$ and assign the welfare weights $\phi^i = 1/\lambda^i_e$ where $\lambda^i_e$ is the shadow price on the external budget constraints of the external optimization problem (18) of country $i$. Therefore any such equilibrium is Pareto efficient.

The proposition is a version of the first welfare theorem. Since the domestic competitive planner in each country $i$ has a complete set of external tax instruments $\tau^i$, she can fully determine the efficient excess demand $M^i$ of country $i$ given the world market price $Q$. If the planner acts competitively in determining $M^i$, then all the conditions of the first welfare theorem apply and the resulting competitive equilibrium is Pareto efficient. Given that the planner has internalized all domestic externalities, the excess demand $M^i$ of the country correctly reflects the country’s social marginal valuation of capital flows. The marginal rates of substitution of all domestic planners are equated across countries, and the resulting equilibrium is Pareto efficient.

**Inefficiency of Laissez-Faire Equilibrium** A straightforward corollary to Proposition 2 is that the laissez-faire equilibrium is generally not Pareto efficient if there are countries subject to externalities from international capital flows with $V^i_M \neq 0$.

Moving from the global laissez-faire equilibrium to the equilibrium with efficient interventions $\tau^i$ as characterized in Lemma 1 does create spillover effects since global prices $Q$ and quantities $(M^i)_{i=1}^N$ will adjust. Even if these spillover effects are large, they are not a sign of Pareto inefficiency. They constitute pecuniary externalities that are mediated by a complete market for $M^i$. As such, they generate redistributions between borrowing (importing) and lending (exporting) countries, but Pareto efficiency is independent of such redistributive considerations.
**Tatonnement and Arms Race** The equilibrium adjustment (tatonnement) process when the optimal interventions \( \tau_i \) are imposed may sometimes involve dynamics that look like an arms race. For example, assume that several countries experience negative externalities \( V_{Mi} < 0 \) from capital inflows \( M^i \) and that the absolute magnitude of these externalities increases in a convex fashion \( V_{MMi,t} < 0 \) in period \( t \). An exogenous shock that makes one country increase its optimal degree of intervention, leads to greater capital flows to all other countries. This increases the externalities in other countries and induces them to respond with greater intervention, which in turn deflects capital back into the original country, triggering further intervention there, and so on.\(^\text{14}\)

Such dynamics may give the appearance of an arms race but are nonetheless efficient. As long as the conditions of Proposition 2 are satisfied, this “arms race” is simply the natural mechanism through which an efficient equilibrium is achieved. In the described example, each successive round of spillovers will be smaller and the degree of intervention will ultimately converge towards its efficient levels, which involves greater intervention by all affected countries.

### 4.4 Pareto-Improving Intervention

If the objective of a global planner is not only to achieve Pareto efficiency but the more stringent standard of achieving a Pareto improvement compared to the laissez-faire allocation, then the imposition of policy instruments \((\tau^i)_{i=1}^N\) generally needs to be accompanied by cross-border transfers that compensate the countries that lose from changes in world prices/interest rates:

**Proposition 3 (Pareto-Improving Intervention with Transfers)** Starting from the laissez faire equilibrium, a global planner who identifies domestic externalities \( V_M \neq 0 \) can achieve a Pareto improvement by setting the interventions \( \tau_i = -V_M / V_m \) \( \forall i \) and providing compensatory international transfers \( T^i \) that satisfy \( \sum_i \omega^i T^i = 0 \).

**Proof.** Denote the net imports and world prices in the laissez faire equilibrium by \((M^{i,LF}, Q^{LF})\) and in the global planner’s equilibrium that results from imposing \((\tau^i)\) and transfers \((T^i)\) by \((M^{i,GP}, Q^{GP})\). Assume the planner provides cross-border transfers

\[
T^i = Q^{GP} \cdot (M^{i,LF} - M^{i,GP})
\]

These transfers satisfy \( \sum T^i = 0 \) since both sets of allocations \((LF\text{ and }GP)\) clear markets. Furthermore, given these transfers, consumers in each country \( i \) can still afford the allocation that prevailed in the laissez faire equilibrium. For non-zero interventions \((\tau^i)\), the allocation differs from the laissez faire equilibrium according to the optimality condition (17). Given that the old allocation is still feasible but is

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\(^{14}\text{See e.g. Giordani et al. (2014) for a detailed theoretical and empirical analysis of such capital flow deflection.}\)
not chosen, revealed preference implies that every country is better off under the new allocation.

In an international context, compensatory transfers may be difficult to implement. As an alternative, we show that a planner who can coordinate the policy instruments of both source and destination countries for \( M^j \) can correct the domestic externalities of individual economies while holding world prices and interest rates constant so that no wealth effects arise. As a result, the global planner’s intervention generates a global Pareto improvement at a first-order approximation.

The following lemma demonstrates how a global planner can manipulate world prices by simultaneously adjusting the instruments in all countries worldwide; then we show how this mechanism can be used to hold world prices fixed so as to avoid redistributive effects when correcting for externalities in a given country.

**Lemma 4** Consider a global competitive equilibrium with an external allocation \( (M^j)_j \), external policy instruments \( (\tau^j)_j \) and world prices \( Q \). A global planner can change world prices by \( dQ \) while keeping the external allocations of all countries constant by moving the policy instruments in each country \( j = 1 \ldots N \) by moving

\[
(d\tau^j)^T = - (M^j_r)^{-1} M^j_Q (dQ)^T \tag{21}
\]

**Proof.** We set the total differential of the net import demand function \( M^j (Q, \tau^j, w^j_0) \) of country \( j \) with respect to world prices and policy instruments to zero,

\[
dM^j = M^j_Q (dQ)^T + M^j_j (d\tau^j)^T = 0
\]

and rearrange to obtain equation (21). ■

In the following proposition, we assume an exogenous increase in the negative externalities \( dV^i_M < 0 \) to a country \( i \). If the country did not respond to this shock, its welfare would decline by \( dV^i_m \cdot M^i \). If the country responds by unilaterally increasing its policy instruments by \( d\tau^i = -dV^i_M / V^i_m > 0 \) as suggested by lemma 1, world market prices \( Q \) would change, and some countries would gain whereas others would lose from the resulting redistribution. The change in world prices and the redistribution can be avoided using the following policy:

**Proposition 4 (Pareto-Improving Intervention, No Transfers)** Assume an exogenous marginal increase in the externalities of country \( i \) that calls for an adjustment \( d\tau^i \) in the optimal unilateral taxes. A global planner can correct for the increase in externalities while keeping world prices constant \( dQ = 0 \) to avoid income and wealth effects by adjusting

\[
(d\tau^j)^T = - \omega^i (M^i_r)^{-1} M^i_Q (M_Q)^{-1} M^i_r (d\tau^i)^T
\]

and

\[
(d\tau^i)^T = \left[ I - \omega^i (M^i_r)^{-1} M^i_Q (M_Q)^{-1} M^i_r \right] (d\tau^i)^T
\]
where we define $M_Q \equiv \sum_j \omega^j M^j_Q$. In the resulting equilibrium, net imports $(M^j)_j$ are marginally altered but world prices are unchanged. By the envelope theorem, welfare is unchanged at a first-order approximation.

**Proof.** If the domestic planner implemented the unilaterally optimal change $d\tau^i$, then world prices would move by $(dQ)^T = -\omega^i (M_Q)^{-1} M^i (d\tau^i)^T$. According to Lemma 4, the move in world prices can be undone if the taxes of all countries $j = 1 \ldots N$ are simultaneously adjusted by $-(M^j)^{-1} M^j_Q (dQ)^T$, which delivers the first equation of the proposition. The second equation is obtained by adding the optimal unilateral change in intervention $d\tau^i$ plus the adjustment given by the first equation with $j = i$. In the resulting equilibrium, the change in the externality $d\tau^i$ is accounted for but world market prices are unchanged. Furthermore, by the envelope theorem, the change in welfare that results from a marginal change in $M^j$ is

$$dV^j\big|_{dQ=0} = \left(V^j_m + V^j_M\right)^T \cdot dM^i = 0$$

The intuition of this intervention is best captured by the following example:

**Example 4.1 (Pareto-Improving Intervention, Symmetric Countries)** Consider a world economy that consists of $N$ open economies as described in the baseline model of example 3.1 that are identical except in their size $\omega^i$. Since the economies are identical, observe that $M^i_Q \cdot (M_Q)^{-1} = I \forall j$. Assume country $i$ experiences a marginal increase in an externality that calls for a change $d\tau^i$ in its optimal unilateral external policy instruments. A global planner would achieve a Pareto improvement at a first-order approximation by instead setting

$$d\tau^i = (1 - \omega^i) d\tau^i$$
$$d\tau^j = \omega^i d\tau^i$$

As this example illustrates, a planner who aims for a Pareto improving intervention would share the burden of adjusting policy instruments between country $i$ and the rest-of-the-world according to their relative size. The larger country $i$, the greater the impact of its interventions on world prices and therefore the more of the intervention the planner would shift to other countries so as to keep world prices $Q$ constant and avoid redistributions. In the extreme case that $i$ is a small open economy, the planner would only intervene in country $i$ since its impact on world prices is negligible. If the countries differ in other aspects than size, the terms $M^i_Q$ and $M^i_i$ in proposition 4 account for their differential responses to changes in world prices and policy instruments.

---

For non-infinite changes in $\tau^i$, changes in net imports $\Delta M^j$ have second-order effects on welfare (i.e. effects that are negligible for infinitesimal changes but growing in the square of $\Delta M^j$) even if world prices are held constant. Under certain conditions, e.g. if there are only two types of countries in the world economy, a global planner can undo these second-order effects via further adjustments in the world prices $Q$. 

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5 Examples and Applications

This section investigates several examples of policy interventions that affect external allocations and that are therefore relevant for our normative analysis of spillover effects on other countries. To illustrate the insights of our general framework in a very simple setting, we start with an extension of the example in Section 2 in which we introduce a richer structure for the domestic economy and show that optimal policy actions to correct domestic externalities lead to Pareto efficient global equilibria, no matter if they are first-best or second-best interventions.

Then we analyze a number of examples of policy interventions and spillovers that have been subject to fierce debates about currency wars in recent years. In all cases, we show that a policymaker who operates under the three conditions that we outlined will entail spillover effects that are globally Pareto efficient: First, we consider aggregate demand externalities in an economy that experiences a liquidity trap and show that there is a role for export-promotion policies. Next, we investigate the role for exchange rate stabilization policies in an economy in which exchange rate fluctuations lead to undesirable redistributions between agents. Finally, we analyze the spillovers of fiscal policy. We do not take a position on the empirical relevance of each of the described imperfections – we simply delineate the efficiency implications. This is useful since each of the described cases triggered spillovers that led to debates in international policy circles.

5.1 Learning-by-Doing Externalities

The following example illustrates the power of our general framework by analyzing spillover effects from domestic policies as well as from second-best interventions in international markets. We build on Section 2 and enrich the structure of the domestic economy to allow for production and learning-by-doing externalities. The first-best policy instrument in such a setting is a subsidy to employment. If such an instrument is not available (for example, because of a lack of fiscal resources, a large informal sector, or the risk of corruption), a subsidy to exports or capital outflows constitutes a second-best instrument to improve welfare. We will show that both cases nest into our general model and therefore lead to Pareto efficient spillover effects under the three conditions required for efficiency.

Consider a two-period economy as in Section 2 but assume that output in each economy $i \in \{A, C\}$ is given by $y_i = A^i I_i$ where labor $I_i$ imposes a convex period disutility $d(I) = I^2/2$ on workers. W.l.o.g. assume furthermore that the period utility of consumption satisfies $u'(1) = 1$, which holds e.g. for all CRRA utility functions. To capture learning-by-doing externalities, period 2 productivity in country $C$ is a continuous and increasing function of aggregate period 1 employment, $A^C_2 = \psi^C(L^1_1)$, that satisfies $\psi^C(1) = 1$ and $0 < \psi^C(\cdot) < 1$. Productivity in country $A$ and in period 1 of country $C$ is fixed and exogenous $A^A_1 = A^C_1 \equiv 1$, but for symmetry of notation
we define the constant function $\psi^A(L) \equiv 1$.

We nest this structure into our general model by defining the vector of domestic variables $x^i = (c^i_1, c^i_2, \ell^i_1, \ell^i_2)$ and the utility function $U^i(x^i) = \sum [u(c^i_t) - d(\ell^i_t)]$, and we include the law-of-motion for technology, $f^i_2(\cdot) = A_2 - \psi^i(L^i_1)$, in the set of domestic constraints, where aggregate labor $L^i_1$ is taken as exogenous by individual private agents.

**Laissez-Faire Equilibrium** In the absence of policy action, it is easy to verify that the economy will replicate the autarky allocation as in Section 2 with $c^i_t = \ell^i_t = 1$ and $m^i_t = 0$.

**First-Best Policy: Domestic Labor Subsidy** The first policy intervention that we consider is a subsidy $s^i_t$ to labor. In this case, we denote the vector of domestic policy instruments as $\zeta^i = (s^i_1, s^i_2)$ and the set of domestic period budget constraints by $f^i_t(\cdot) = c^i_t - (1 + s^i_t)A^i_t\ell^i_t + T^i_t - m^i_t \leq 0$, where the term $T^i_t$ reflects a lump-sum tax that needs to be set to $T^i_t = s^i_tA^i_tL^i_t$ so the policymaker’s budget is balanced. The domestic optimality condition for the labor supply of private agents [equivalent to (13)] for $t = 1, 2$ can be written as

$$A^i_tu'(c^i_t) [1 + s^i_t] = d' (\ell^i_t)$$

(22)

Since the planner has complete instruments for the domestic labor market, these implementability constraints can always be satisfied by setting $s^i_t$ to the appropriate level and can be omitted from the planner’s optimization problem. As a result, the planner’s optimal labor supply conditions are

$$A^i_1u'(C^i_1) + \psi''(L^i_1)L^i_1u'(C^i_2) = d' (L^i_1)$$

(23)

$$A^i_2u'(C^i_2) = d' (L^i_2)$$

(24)

The second term on the left-hand-side of (23) reflects the learning externalities from period 1 labor supply. Substituting the period budget constraints, the planner’s optimality conditions (23) and (24) implicitly define the optimal labor supply function $L^i(M^i)$. The optimal labor subsidies to internalize the externalities and satisfy the implementability constraints (22) are

$$s^i_1(M^i) = \frac{\psi''(L^i_1)L^i_1u'(C^i_2)}{A^i_1u'(C^i_1)} > 0 \quad \text{and} \quad s^i_2(M^i) \equiv 0.$$

The period 1 subsidy reflects the learning externalities $\psi''(L^i_1)L^i_1u'(C^i_2)$ normalized by the marginal utility product of labor – it is zero in country $A$ since $\psi^A \equiv 0$ and positive in country $C$. In period 2, there is no role for subsidies in either country.

Observe that $V^i_M = 0$ – the planner has no incentives to intervene in the current account itself since she can internalize the learning-by-doing externalities using the
first-best instruments precisely in the market where they arise. This follows the optimal targeting principle established by Bhagwati and Ramaswami (1963).

The domestic subsidy $s_1^C$ increases period 1 output and net exports, leading to international spillover effects on country $A$ in the form of a price adjustment $R < 1$ and quantity adjustment $m_1^C = -m_1^A < 0$. However, in accordance with Proposition 2, these spillover effects are efficient. Given the described allocation, there is no role for cooperation to increase welfare.

**Second-Best Policy: Current Account Intervention** Next we consider the case that labor subsidies, the first-best instrument, are not available so $\zeta = \emptyset$. This may reflect the inability of developing countries to raise sufficient fiscal revenue for subsidies, or administrative limitations that make it difficult to ensure that subsidies would reach the intended recipients (see Korinek and Serven, 2010, for a discussion). As a result, the planner’s implementability constraint (22) is replaced with the individual labor supply condition

$$A_i^t u'(A_i^t L_i^t + M_i^t) = d' (L_i^t)$$

(25)

This constraint for $t = 1, 2$ pins down aggregate labor supply $L^t(M^i)$, which satisfies $\frac{\partial L_i^t}{\partial M_i^t} < 0$ since increasing $M_i^t$ reduces the marginal utility of consumption, which leads to lower labor supply $L_i^t$. Although a labor subsidy is not available, we will see that the planner can use this mechanism as a second-best device to internalize learning-by-doing externalities.

The reduced-form utility function is

$$V^i(m^i, M^i) = \max_{\hat{c}_i^i} u(A_i^t \hat{c}_i^i + m_i^t) - d(L_i^1) + u \left( \psi^i \left( L_i^1 (M^i) \right) \hat{c}_2^i + m_2^i \right) - d(\hat{c}_2^i)$$

This function satisfies $\frac{\partial V^i}{\partial m_i^t} = u'(c_i^t), \frac{\partial V^i}{\partial M_i^t} = u'(c_i^t)\psi^i(L_i^1)\frac{\partial L_i^1}{\partial M_i^t}\hat{c}_2^i < 0$ and $\frac{\partial V^i}{\partial M_2^i} = 0$ since $\frac{\partial L_i^1}{\partial M_2^i} = 0$. Following Lemma 1, it is optimal for country $i$ to tax capital inflows (or subsidize outflows) in period 1 at rate

$$\tau_i^1 = -\frac{V_{M,1}^i}{V_{m,1}^i} > 0$$

This serves as a second-best device to internalize the learning-by-doing externalities. There is no need to intervene in period 2.

The spillover effects of this optimal domestic policy are to increase net exports in period 1, $m_1^C = -m_1^A < 0$, and reduce the world interest rate, $R < 1$. Proposition 2 implies that the described application of second-best capital controls (or import controls) leads to a globally Pareto efficient outcome. Even though the intervention is only a second-best instrument, it is chosen to equate the marginal social benefit from triggering the LBD-externality to the marginal social cost, given the restriction on the set of available instruments. Since a global planner does not have superior instruments, he cannot do better than this and chooses an identical allocation.
5.2 Aggregate Demand Externalities at the ZLB

Next we study the multilateral implications of capital controls to counter aggregate demand externalities at the zero lower bound (ZLB) on nominal interest rates. We develop a stylized framework that captures the essential nature of such externalities in the spirit of Krugman (1998) and Eggertsson and Woodford (2003), adapted to an open economy framework as in Jeanne (2009).\footnote{A complementary analysis of prudential (as opposed to stimulative) capital controls in a small open economy due to aggregate demand externalities at the ZLB is provided in Section 5.2 of Farhi and Werning (2013).}

Assume that a representative consumer in country $i$ derives utility from consuming $c^i_t$ units of a composite final good and experiences disutility from providing $\ell^i_t$ units of labor. Collecting the two time series in the vectors $c^i$ and $\ell^i$, we denote

$$U^i(c^i, \ell^i) = \sum \beta^t [u(c^i_t) - d(\ell^i_t)]$$

As is common in the New Keynesian literature, we assume that there is a continuum $z \in [0, 1]$ of monopolistic intermediate goods producers who are collectively owned by consumers. They each hire labor to produce an intermediate good of variety $z$ according to the linear function $y^i_z = \ell^i_z$, where labor market clearing requires $\int \ell^i_z dz = \ell^i_t$. All the varieties are combined in a CES production function to produce final output

$$y^i_t = \left( \int_0^1 (y^i_z)^{\frac{\varepsilon - 1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

where the elasticity of substitution is $\varepsilon > 1$. We assume that the monopoly wedge arising from monopolistic competition is corrected by a proportional subsidy $\frac{1}{\varepsilon - 1}$ that is financed by a lump-sum tax on producers. This implies that the wage income and profits of the representative agent equal final output, which in turn equals labor supply $w_t \ell^i_t + \pi^i_t = y^i_t = \ell^i_t$. In real terms and vector notation, the period budget constraints of a representative agent and the external budget constraint are given by

$$c^i = w^i \ell^i + \pi^i + m^i = y^i + m^i \quad \text{and} \quad \frac{Q}{1 - \tau^i} \cdot m^i - T^i \leq 0$$

The condition for the optimal labor supply of the representative agent is

$$d^i(\ell^i_t) = w^i_t u'(c^i_t)$$

We assume that the nominal price of one unit of consumption good follows an exogenous path $P^i_t = (1, P^i_2, P^i_3, ...)$ that is credibly enforced by a central bank (see e.g. Korinek and Simsek, 2014, for further motivation). This assumption precludes the central bank from committing to a future monetary expansion or future inflation
in order to stimulate output in the present period. The corresponding gross rate of inflation is given by \( \Pi_{t+1} = P_{t+1}^i / P_t^i \) or by \( \Pi^i = P / L (P) \) in vector notation with lag operator \( L (\cdot) \). One example is a fixed inflation target \( \Pi_{t+1}^i = \bar{\Pi}^i \forall t \).

We denote the gross real interest rate between periods \( t \) and \( t + 1 \) by \( R_{t+1} = Q_t / Q_{t+1} \) and the gross nominal interest rate by \( 1 + i_{t+1} = R_{t+1} \Pi_{t+1} \). The zero-lower-bound (ZLB) on the nominal interest rate implies that \( i_{t+1} \geq 0 \) needs to be satisfied – this constraint is commonly motivated by the existence of cash that delivers zero net returns and is always available as a savings vehicle. Combining the ZLB with the period \( t \) Euler equation of consumers and substituting for \( R_{t+1} \), the ZLB imposes a ceiling on aggregate period \( t \) consumption,

\[
 u' (C_t^i) \geq \frac{\beta}{\Pi_{t+1}^i} u' (C_{t+1}^i) \quad \forall t
\]

Intuitively, a binding ZLB implies that consumption is too expensive in period \( t \) compared to consumption in the following period, limiting aggregate demand in period \( t \) to the level indicated by the constraint.

In the laissez-faire equilibrium, this constraint is slack if world aggregate demand for bonds and by extension the world interest rate is sufficiently high, i.e. if \( R_{t+1} \geq 1 / \Pi_{t+1}^i \). Then the market-clearing wage \( W_t^i = 1 \) will prevail and output \( Y_t^i \) is at its efficient level determined by the optimality condition \( u' (C_t^i) = d' (L_t^i) \). We call this output level potential output \( Y_{t,i}^* \).

If worldwide aggregate demand declines and the world real interest rate hits the threshold \( R_{t+1} = 1 / \Pi_{t+1}^i \), then the domestic interest rate cannot fall any further. Instead, any increase in the world supply of bonds will flow to economy \( i \), which pays a real return of \( 1 / \Pi_{t+1}^i \) by the feature of offering liabilities with zero nominal interest rate. Given the high return on nominal bonds, consumers in economy \( i \) find that today’s consumption goods are too expensive compared to tomorrow’s consumption goods and consumers reduce their demand for today’s consumption goods. Since output is demand-determined, \( Y_t^i \) falls below potential output \( Y_{t,i}^* \) in order to satisfy equation (26). The wage falls below its efficient level \( w_t^i < 1 \) to clear the labor market. This situation captures the essential characteristic of a liquidity trap: at the prevailing nominal interest rate of zero, consumers do not have sufficient demand to absorb both domestic output and the capital inflow \( M_t^i \). Intermediate producers cannot reduce their prices and need to adjust output so that demand equals supply.

At the ZLB, a planner finds it optimal to erect barriers against capital inflows or encourage capital outflows in order to stimulate domestic aggregate demand. We substitute the domestic period budget constraint and the consistency condition \( x_t^i = X_t^i \) and denote the reduced-form utility maximization problem of a planner in country

\footnote{It is well known in the New Keynesian literature that the problems associated with the zero lower bound could be avoided if the monetary authority was able to commit to a higher inflation rate. See e.g. Eggertsson and Woodford (2003).}
Assigning the shadow prices \( \beta_t \mu_t \) and \( \beta_t \gamma_t \) to the two constraints, the associated optimality conditions are

\[
FOC(L^i_t) : u'(C^i_t) - d'(L^i_t) + \left[ \mu_t - \mu_{t-1}/\Pi^i_{t+1} \right] u''(C^i_t) - \gamma_t \left[ d''(L^i_t) - u''(C^i_t) \right] = 0
\]

When the ZLB constraint is loose, the shadow prices \( \mu_t \) and \( \gamma_t \) are zero. If the ZLB is binding in period \( t \), then \( \mu_t = \frac{u'(C^i_t) - d'(L^i_t)}{-u''(C^i_t)} > 0 \) reflects the labor wedge in the economy created by the lack of demand; the second constraint is trivially satisfied in period \( t \) so \( \gamma_t = 0 \). If the ZLB is loose in the ensuing period \( t + 1 \), then the planner would like to commit to stimulate output in that period as captured by the term \( -\mu_t u''(C^i_{t+1})/\Pi^i_{t+1} \) so as to relax the ZLB constraint at date \( t \), but we imposed the second constraint to reflect that the planner cannot commit to do this. Therefore \( u'(C^i_{t+1}) = d'(L^i_{t+1}) \) in that period and the shadow price \( \gamma_{t+1} \) adjusts so that the optimality condition is satisfied \( \gamma_{t+1} = \frac{-\mu_t u''(C^i_{t+1})/\Pi^i_{t+1}}{d''(L^i_{t+1}) - u''(C^i_{t+1})} > 0 \).

The externalities of capital inflows in periods \( t \) and \( t + 1 \) in such an economy are given by the partial derivatives

\[
\begin{align*}
V_{m,t} (\cdot) &= \beta_t u'(C^i_t) \\
V_{M,t} (\cdot) &= \beta_t \left[ \mu_t - \mu_{t-1}/\Pi^i_t + \gamma_t \right] u''(C^i_t)
\end{align*}
\]

If the economy experiences a liquidity trap in period \( t \) but has left the trap in period \( t + 1 \), then \( V_{M,t} = \beta_t \mu_t u''(C^i_t) = -\beta_t [u'(C^i_t) - d'(L^i_t)] < 0 \) – the externality from a unit capital inflow is to reduce aggregate demand by one unit, which wastes valuable production opportunities as captured by the labor wedge \( u'(C^i_t) - d'(L^i_t) \). It is optimal to set the policy instrument \( \tau^i_t = 1 - \frac{d'(L^i_t)}{u'(C^i_t)} > 0 \) to internalize this social cost, i.e. to restrict capital inflows and encourage outflows.

\[\text{Footnote: The domestic optimization problem of private agents is trivial since private agents receive both labor income and profits from domestic production and face the same incentives as a domestic planner. Technically, the planner’s implementability constraint on domestic labor supply is slack. Therefore we can write the planner’s problem directly in terms of aggregate labor employed, which equals aggregate output } Y^i_t = L^i_t.\]
It is also beneficial to commit to setting $\tau_{t+1} < 0$ in the following period so as to subsidize capital inflows and achieve a future consumption boom since

$$V_{M,t+1} = -\beta^{t+1} \mu_t / \Pi_{t+1} \left[ \frac{d'' \left( L_{t+1}^i \right)}{d'' \left( L_{t+1}^i \right) - u'' \left( C_{t+1}^i \right)} \right] u'' \left( C_{t+1}^i \right) > 0$$

This has the effect of raising future consumption, which stimulates consumption during the liquidity trap by relaxing the ZLB constraint (26).\(^{19}\)

Note that the capital account interventions of a planner in this setting are second-best policies since the first-best policy would be to restore domestic price flexibility to abolish the ZLB constraint. The planner solves the optimal trade-off between foregoing profitable opportunities for intertemporal trading with foreigners and wasting profitable production opportunities because of the ZLB. Nonetheless, the setup nests into our general setup; therefore the spillovers created by the planner’s intervention are Pareto efficient and there is no scope for global coordination.

5.3 Exchange Rate Stabilization Policy

Next we analyze a developing economy in which a large fraction of agents derive their income exclusively from the traded or non-traded sector. As a result, fluctuations in the country’s exchange rate lead to fluctuations in income for these agents. We assume that they do not have access to financial markets to insure against their income risk, and that frictions in factor markets prevent them from redeploying their factors between the traded and non-traded sector in response to exchange rate fluctuations. As a result, a policymaker may find it optimal to stabilize the exchange rate as a second-best device to provide insurance to the population.

Formally, consider an economy $i$ with a traded and a non-traded intermediate good as well as a final good. There are two categories of agents, which we call the “financial elite” and the “people.” The financial elite $E$ obtain an endowment of $\alpha y_{T,t}$ traded goods and $\alpha y_{N,t}$ non-traded goods every period $t$. The people obtain their income in one of the two intermediate sectors and are therefore made up of two types $j \in \{N,T\}$: the people in the traded sector $T$ obtain an endowment of $(1 - \alpha) y_{T,t}$ traded goods; the people in the non-traded sector $N$ obtain $(1 - \alpha) y_{N,t}$ non-traded goods every period $t$. For simplicity, we assume that the endowments $(y_{T}, y_{N})$ are constant and equal to each other – this is straightforward to generalize, as we will discuss below. Each type of the people as well as the elite are a continuum of mass 1 each.

Every period there is a spot market in which all agents exchange traded and non-traded goods at relative prices $p_{N,t}$. After having traded, each agent consumes traded

\(^{19}\)In a time-consistent setting for capital account interventions, the planner would not be able to commit to future policy actions. The intervention during a liquidity trap would still be given by the same expression $V_{M,t} = d' \left( L_{t}^i \right) - u' \left( C_{t}^i \right)$, but after the liquidity trap has passed the planner would find $V_{M,t+1} = 0$ and no further intervention would occur.
$c_{T,t}$ and non-traded goods $c_{N,t}$, which enter their period utility as a function of a Cobb-Douglas index $c = c_T^\sigma c_N^{1-\sigma}$ where we set w.l.o.g. $\sigma = \frac{1}{2}$ to maintain symmetry between the two sectors. Assuming CES intertemporal preferences, the utility of each type of agent $j \in \{N, T, E\}$ is

$$U_j = \sum \beta^t \frac{(c_T^{\sigma} c_N^{1-\sigma})^{1-\gamma}}{1-\gamma}$$

**(The People)** do not have access to financial markets so they cannot borrow, save, or insure; their decision problem is purely intratemporal: they collect their endowment and trade in the spot market to maximize utility. We denote the period $t$ wealth of people of type $j \in \{N, T\}$ by $w_{j,t}$ so $w_{N,t} = p_{N,t} (1-\alpha) y_{N,t}^j$ and $w_{T,t} = (1-\alpha) y_{T,t}^j$. Given the Cobb-Douglas consumption index, the period $t$ utility of type $j$ is described by the indirect utility function

$$v_{j,t} = v(p_{N,t}, w_{j,t}) = \frac{(\kappa w_{j,t}/p_{N,t}^{1-\sigma})^{1-\gamma}}{1-\gamma}$$

where $\kappa = \sigma^{\sigma} (1-\sigma)^{1-\sigma}$ is a constant. For each type of the people, utility is increasing in the quantity and relative price of their endowment good. An appreciation (increase) in the real exchange rate $p_{N,t}^i$ benefits the people in the nontraded sector at the expense of those in the traded sector, and vice versa for a depreciation. This is of concern in our model since the people lack access to both formal and informal insurance markets and are limited in their sectoral mobility.\textsuperscript{20} The combined period $t$ welfare of the people is

$$v_P(p_{N,t}) = v_{T,t} + v_{N,t} = \frac{[\kappa (1-\alpha)]^{1-\gamma}}{1-\gamma} \left[ \left( \frac{y_{T,t}^i}{p_{N,t}^{1-\sigma}} \right)^{1-\gamma} + \left( \frac{p_{N,t}^i y_{N,t}^j}{p_{N,t}^{1-\sigma}} \right)^{1-\gamma} \right]$$

This combined welfare of the people is thus concave in $p_{N,t}$ for typical parameter values.\textsuperscript{21} Given the symmetry between the two sectors, it reaches a maximum at the autarky exchange rate $p_{N,aut}^t = 1$.

**(The Financial Elite)** is connected to international financial markets, allowing for intertemporal trade. Their optimization problem is

$$\max_{\left\{ c_{T,t}, c_{N,t}, m_t^i \right\}_{t=0}^\infty} U_E \text{ s.t. } c_{T,t} - \alpha y_{T,t} + p_{N}^i (c_{N,t} - \alpha y_{N,t}) - m_t^i \leq 0 \ \forall t$$

\textsuperscript{20}For comparison, in a first-best economy, the people in the traded and non-traded sectors would perfectly insure each other against fluctuations in the real exchange rate.

\textsuperscript{21}The analytic condition for this is $\gamma > 1$, which rules out the Cole-Obstfeld effect and is satisfied in standard calibrations. For $\sigma \neq \frac{1}{2}$, the condition that guarantees that the welfare of the people is concave around the autarky exchange rate is $[tk]$. 
and subject to the standard intertemporal budget constraint (9). Using the indirect utility function (28), the problem can equivalently be stated as

$$\max_{\{m_i\}_{t=0}^{\infty}} \sum \beta^t v \left( p_{N,t}, \alpha (y_T + p_{N,t} y_N) + m_i^t \right) \quad \text{s.t.} \quad \frac{Q}{1 - \tau_i} \cdot m^i \leq T^i$$

and yields an optimality condition

$$\left(1 - \tau_i^t\right) \beta^t v_w (\cdot) = \lambda_e^i q_t$$

If world interest rates correspond to the autarky interest rate in the economy, \(q_t = \beta^t\), and if there are no capital account interventions \(\tau_i = 0\), then the financial elite does not engage in intertemporal trade and the exchange rate will be at its autarky level \(p_{N,aut}\), which also maximizes the welfare of the people.

However, if there is a motive for international trade for the elite (\(\lambda_e^i \text{ s.t. } \lambda_e^i q_t = \beta^t\)), the elite will receive inflows \(m_i^t > 0\) in periods of comparatively low world interest rates and export \(m_i^t < 0\) in periods with high world interest rates. Domestic market clearing requires that

$$p_i^{N,t} (M_i^t) = \frac{1 - \sigma y_{T,t} + M_i^t}{\sigma y_{N,t}}$$

(29)

The real exchange rate moves in parallel with capital inflows and outflows – when there are inflows, the people in the non-traded sector benefit at the expense of the traded sector, and vice versa.

A planner can lean against these redistributions by smoothing capital flows and the resulting fluctuations in the real exchange rate. Specifically, assume a national planner who places a weight of \(\varphi\) on the elite and \((1 - \varphi)\) on the people. The planner will maximize the weighted sum of welfare

$$V^i (m^i, M^i) = \sum \beta^t \left[ \varphi v \left( p_{N,t} (M_i^t), \alpha (y_T + p_{N,t} (M_i^t) y_N) + m_i^t \right) + (1 - \varphi) v_p \left( p_{N,t} (M_i^t) \right) \right]$$

subject to the intertemporal budget constraint.

**Proposition 5 (Capital Flow Management as Insurance)** The greater the weight \(\varphi\) that the planner places on the people versus the elite, the closer the allocation will be to the autarky allocation with perfect exchange rate stabilization.

**Proof.** See discussion above. \(\blacksquare\)

As the planner varies the welfare weight \(\varphi\) from zero to one, we go from the autarky allocation – which provides perfect insurance to the people – to the allocation that optimizes the welfare of the elite. In the more general case in which the endowments \((y_{T,t}, y_{N,t})\) are time-varying, capital inflows may also provide insurance against domestic endowment shocks to the people – for example, the non-traded sector will be better off by allowing capital inflows in states of nature with low \(y_{T,t}\). As a result,
the planner has to trade off the optimal insurance arrangement among all three sets of agents.

More generally, our example illustrates that capital flow management may be a second-best insurance device. The less the people have access to insurance (either in a market setting or from social insurance) and the less flexible factor markets are (so that the people can switch the sector in which they earn their income), the more of a role there is for capital flow management. The described exchange rate stabilization policy may thus be of particular interest to developing and emerging economies.

Returning to the main theme of the paper, capital flow management in the described economy is optimal not only for the national planner but also for a global planner who faces the same imperfections in domestic insurance markets and in factor mobility. The described economy can easily be mapped into the general model of section 3. As emphasized in proposition 2, the described interventions are therefore efficient and do not generate a role for global coordination.

5.4 Level of Fiscal Spending

Our findings also apply to multilateral considerations about the optimal level of fiscal spending. Such considerations have, for example, been at the center of the political debate on coordinating fiscal stimulus in the aftermath of the Great Recession (see e.g. the G-20 Leaders’ Declaration, Nov. 2008; Spilimbergo et al., 2008). We demonstrate how uncoordinated fiscal policy decisions lead to an efficient equilibrium as long as the three conditions underlying our efficiency result are met. In the context of fiscal policy, price-taking behavior is particularly relevant since fiscal spending has important terms-of-trade effects. The ensuing example illustrates how price-taking behavior leads to globally efficient allocations. In Section 6 below, we will demonstrate that a policymaker who exerts monopoly power finds it optimal to intervene in external transactions but does not distort the relative mix of private vs. public consumption. In Section 7, we will show that a monopolistic policymaker only distorts this mix if she has an incomplete set of external policy instruments.

Example 5.1 (Fiscal Spending) Consider an economy with two goods indexed by \( t = 1, 2 \), which can be interpreted as goods in subsequent time periods or as different goods within the same time period. Private agents obtain the endowments \((y^1_i, y^2_i)\) of the two goods and derive utility from consumption and government spending

\[
U^i(x^i) = u(c^i_1) + u(c^i_2) + \alpha u(g^i_1) + u(g^i_2)
\]

where, for simplicity, we use the same strictly increasing and concave period utility \(u(\cdot)\) for both types of spending. In our baseline, we set \(\alpha = 1\); below we will vary the parameter for comparative statics on the desirability of government spending.

To nest the setup into our general framework, we define the vectors of domestic variables \(x^i = (c^i_1, c^i_2, g^i_1, g^i_2)\) and policy instruments \(\zeta^i = (G^i_1, G^i_2)'\) together with a set
of domestic constraints

\[ f_1^i (\cdot) = c^i - y^i - m^i + G^i \leq 0 \]

\[ f_2^i (\cdot) = g^i - G^i = 0 \]

The first constraint captures that government raises the revenue necessary for its spending via lump-sum taxes in the amounts of \( G^i_1 \) and \( G^i_2 \). The second constraint reflects that fiscal spending is determined by the choices of the policymaker not by private agents. Given these constraints, private agents solve their usual optimization problem. In our baseline with \( \alpha = 1 \), a domestic policymaker who takes international prices as given sets fiscal spending such that

\[ u' (C^i_t) = u' (G^i_t) \]

or, equivalently, \( C^i_t = G^i_t \) for \( t = 1, 2 \).

**Spillovers** Let us parameterize our example by setting \( u(c^i) = \log c^i \) and assuming that country \( i \) faces a vector of international prices that is given by \( Q = (1, Q^i_2) \), i.e. the first good is the numeraire good and \( Q^i_2 (M^i_1, M^i_2) \) is the relative price of the \( t = 2 \) good. Assuming both goods are ordinary, the price satisfies \( \partial Q^i_2 / \partial M^i_1 < 0 < \partial Q^i_2 / \partial M^i_2 \). If we interpret \( t = 1, 2 \) as subsequent time periods, then \( Q^i_2 = 1/R \) is the price of discount bonds or the inverse interest rate. Substituting \( M^i_2 = -M^i_1 / Q^i_2 \) from the budget constraint and assuming a standard Marshall-Lerner condition for the demand of the rest-of-the-world, we can express this relative price as a function \( Q^i_2 (M^i_1) \) that satisfies \( Q'_i_2 (M^i_1) < 0 \), i.e. the interest rate increases the more the country borrows in period 1, or the relative price of \( t = 2 \) goods declines the more \( t = 1 \) goods are imported.

The log-utility function implies that the optimal mix of private and public spending satisfies \( G^i_1 = \alpha C^i_1 \) and \( G^i_2 = C^i_2 \), and the Euler equation is given by \( C^i_1 = Q^i_2 C^i_2 \).

Period 1 imports are \( M^i_1 = (1 + \alpha) C^i_1 - y^i_1 \), and the external budget constraint implies

\[ [(1 + \alpha) C^i_1 - y^i_1] + [2C^i_1 - y^i_2 Q^i_2 ((1 + \alpha) C^i_1 - y^i_1)] = 0 \]

To generate a fiscal shock in economy \( i \), we assume an exogenous increase \( d\alpha > 0 \) in the usefulness of government spending. This can be interpreted as capturing a sudden requirement to reinvest in broken infrastructure, or as the benefits of fiscal stimulus to offset a sudden aggregate demand shock. Implicitly differentiating the expression above, we find that \( dC^i_1 / d\alpha < 0 \), \( dG^i_1 / d\alpha = d[\alpha C^i_1] / d\alpha > 0 \) and \( dM^i_1 / d\alpha = d[(1 + \alpha) C^i_1 - y^i_1] / d\alpha > 0 \) – in short, the shock reduces private consumption but (unsurprisingly) increases government spending at \( t = 1 \). Furthermore, the shock

\[ \text{37} \]
leads to spillovers in the form of greater $t = 1$ imports and, by implication, a lower relative price $Q_2$.

In the intertemporal interpretation of the model, the spillovers of the spending shock consist of higher period 1 imports (lower exports) and a higher world interest rate. This hurts other borrowing countries but benefits other lenders. If we interpret the model as capturing multiple goods and $t = 1$ is the country 1 export good ($M_1 < 0 < M_2$), then the shock captures that it is desirable to reallocate spending towards domestic goods. The resulting spillovers consist of raising the price of exports and lowering the relative price of imports, i.e. of improving the country’s terms of trade. Since the described policies satisfies the three conditions required for efficiency, all these spillovers are nonetheless Pareto efficient.

6 Monopolistic Behavior

6.1 Optimization Problem

Assume next that there is a monopolistic planner in country $i$ with positive mass $\omega^i > 0$ that maximizes the utility of the representative consumer $U^i$ and internalizes that she has market power over world prices $Q$. We solve the problem of the monopolistic planner under the assumption that the remaining countries $j \neq i$ behave according to the competitive planning setup in section 4.2. The excess demand of the rest-of-the-world excluding country $i$ is then given by the function

$$M^{-i}(Q) = \sum_{j \neq i} \omega^j M^j(Q, \tau^j(Q))$$

The function can be inverted to obtain an inverse rest-of-the-world excess demand function $Q^{-i}(M^{-i})$.

A monopolistic planner recognizes that global market clearing requires $\omega^i M^i + M^{-i}(Q) = 0$ and that her external allocations $M^i$ affect world prices since $Q = Q^{-i}(-\omega^i M^i)$. She solves the optimization problem

$$\max_{M^i} V^i(M^i, M^i) \quad \text{s.t.} \quad Q^{-i}(-\omega^i M^i) : M^i \leq 0$$

leading to the optimality conditions

$$V^i_m + V^i_M = \lambda^i Q^T (1 - \mathcal{E}^i_{Q,M}) \quad \text{with} \quad \mathcal{E}^i_{Q,M} = \omega^i Q^{-i}_{M^i} M^i / Q^T$$

where the column vector $\mathcal{E}^i_{Q,M}$ represents the inverse demand elasticity of imports of the rest of the world and consists of four elements: the country weight $\omega^i$ reflects

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\[23\] Our findings can easily generalized to the case where other countries engage in monopolistic behavior or operate under laissez-faire. The only important assumption for our problem is that each country $j$ has a well-defined and continuous demand function. This rules out, for example, discontinuous trigger strategies.
the country’s market power in the world market; the Jacobian square matrix \( Q_M^{-i} = \frac{\partial Q^{-i}}{\partial M^{-i}} \) captures how much world market prices respond to absorb an additional unit of exports from country \( i \). The column vector \( M^i \) post-multiplies this matrix to sum up the marginal revenue accruing to country \( i \) from the different goods as a result of monopolistically distorting each good, where the vector \( M^i \) is normalized element-by-element by the price vector \( Q \) to obtain elasticities.

**Lemma 5 (Monopolistic Capital Account Intervention)** The allocation of the monopolistic planner who internalizes her country’s market power over world prices can be implemented by setting the vector of external policy instruments to

\[
1 - \hat{\tau}^i = \frac{1 + V_M^i / V_m^i}{1 - \varepsilon_{Q,M}^i} \tag{33}
\]

where all divisions are performed element-by-element.

**Proof.** The tax vector \( \hat{\tau}^i \) ensures that the private optimality condition of consumers (17) replicates the planner’s Euler equation (32). ■

Returning to equation (32), a monopolistic planner equates the social marginal benefit of imports \( V_M^i + V_m^i \) to the marginal expenditure \( Q^T (1 - \varepsilon_{Q,M}^i) \) rather than to the world price \( Q^T \) times a factor of proportionality \( \Lambda^i \). She intervenes up to the point where the marginal benefit of manipulating world prices – captured by the elasticity term \( \Lambda^i Q^T \varepsilon_{Q,M}^i \) – equals the marginal cost of giving up profitable consumption opportunities \( \Lambda^i Q^T (V_m^i + V_M^i) \). However, giving up profitable consumption opportunities creates a deadweight loss – the planner introduces a distortion to extract monopoly rents from the rest of the global economy. The intervention constitutes a classic inefficient beggar-thy-neighbor policy:

**Proposition 6 (Inefficiency of Exerting Market Power)** An equilibrium in which a domestic planner in a country with \( \omega^i > 0 \) exerts market power is Pareto-inefficient.

**Proof.** The result is a straightforward application of proposition 2 that optimality requires competitive behavior. ■

To provide some intuition, assume that the matrix \( Q_M^{-i} \) in a country with \( \omega^i > 0 \) is a diagonal matrix and that there are no domestic externalities so \( V_M^i = 0 \). The diagonal entries, indexed by \( t \), satisfy \( \partial Q^{-i} / \partial M^{-i} < 0 \), reflecting that greater rest-of-the-world imports \( M_t^{-i} \) of good \( t \) require a lower world price \( Q_t \). If the country is a net importer \( M_t^i > 0 \) of good \( t \) then the elasticity \( \varepsilon_{Q,M,t}^i \) is negative and the optimal monopolistic tax on imports \( \hat{\tau}_t^i > 0 \) is positive. Similarly, for goods that are net exports \( M_t^i < 0 \) the planner reduces the quantity exported by a tax \( \hat{\tau}_t^i < 0 \). This captures the standard trade-reducing effects of monopolistic interventions.

\[24 \text{This is the case, for example, if the reduced-form utility } V (m^i, M_t^i) \text{ is Cobb-Douglas in } m^i.\]
6.2 Distinguishing Competitive and Monopolistic Behavior

The spillover effects of a policy intervention in external allocations are the same, no matter what the motive for intervention. However, the effects on Pareto efficiency and thus the scope for global cooperation depend crucially on whether policymakers correct for domestic distortions (as described in lemma 1) or exert market power (as described in lemma 5).

Unfortunately there is no general recipe for distinguishing between the two motives for intervention. It is easy for policymakers to invoke market imperfections, domestic objectives or different political preferences to justify an arbitrary set of policy interventions in the name of domestic efficiency, and it is difficult for the international community to disprove them. Specifically, for any reduced-form utility function \( V^i(m^i, M^i) \) and monopolistic interventions \( \hat{\tau}^i \), we can construct an alternative reduced-form utility function \( \hat{V}^i(m^i, M^i) \) such that \( \hat{\tau}^i \) implements the optimal competitive planner allocation under that utility function,

\[
\hat{V}^i(m^i, M^i) = V^i(m^i, M^i) - \hat{\tau}^i \cdot (V^i_{m^i}(M^i))
\]

The reduced-form utility function \( \hat{V}^i(\cdot) \) can in turn be interpreted as deriving from a fundamental utility function \( \hat{U}^i(x^i) \) and a set of constraints \( \hat{f}^i(\cdot) \) that justify it.

Nonetheless, the direction of optimal monopolistic policy interventions is often instructive to determine whether it is plausible that a given intervention is for efficient or monopolistic reasons. In the following, we describe optimal monopolistic capital account interventions along a number of dimensions. If the observed interventions of a policymaker are inconsistent with these observations, then they are likely not for monopolistic reasons. Recall the definition of the elasticity \( \varepsilon^i_{Q,M} = -\omega^i Q^{-i}_M (M^i/Q)^T \), and let us discuss the various parameters:

**Country Size** \( \omega^i \) The optimal monopolistic intervention is directly proportional to the country’s weight \( \omega^i \) in the world economy. Larger countries have a greater impact on the rest of the world since market clearing requires \( M^{-i} = \omega^i M^i \).

For example, if a small open economy with \( \omega^i \approx 0 \) and undifferentiated exports regulates capital in- or outflows, the reason cannot be monopolistic.

**Responsiveness of Price** \( Q^{-i}_M \) Monopolistic intervention requires that world market prices are sufficiently responsive to changes in consumption. If there are, for example, close substitutes to the goods traded by country \( i \), this is unlikely to be the case.

**Direction and Magnitude of Flows** \( M^i \) The intervention to manipulate a given price \( Q_t \) is directly proportional to the magnitude of a country’s net imports \( M^i_t \) in that time period/good/state of nature. The larger \( M^i_t \) in absolute value, the greater the revenue benefits from distorting the price \( Q_t \). By contrast, if \( M^i_t \approx 0 \), the optimal monopolistic intervention is zero.
The direction and magnitude of flows has the following implications for monopolistic behavior:

- **Intertemporal trade:** In our example 3.1 with a single consumption good per time period, the elements of $M^i$ capture net capital flows or equivalently the trade balance. In time periods in which the trade balance is close to zero $M^i_t \approx 0$, it is impossible to distort intertemporal prices. By contrast, monopolistic reasons may be involved if a country with a large deficit $M^i_t > 0$ taxes inflows $\tau^i_t > 0$ to keep world interest rates lower, or vice versa for a country with a large surplus $M^i_t < 0$. If capital accounts are closed to private agents, optimal monopolistic intervention consists of reduced/increased foreign reserve accumulation.\(^{25}\)

- **Risk-sharing:** In the stochastic extension of example 3.1, $M^i_t (s_t)$ denotes different states of nature. Each country has – by definition – monopoly power over its own idiosyncratic risk. Optimal risk-sharing implies greater inflows (imports) in bad states and greater exports in good states of nature. A planner who exerts monopoly power would restrict risk-sharing so as to obtain a higher price for the country’s idiosyncratic risk and to reduce the price of insurance from abroad. By contrast, if a country encourages insurance (e.g. by encouraging FDI and forbidding foreign currency debt; see Korinek, 2010), then the motive is unlikely to be monopolistic.

- **Intratemporal trade:** Exercising monopoly power in intratemporal trade consists of tariffs $\tau^i_{t,k} > 0$ on imported goods $k$ with $M^i_{t,k} > 0$ and taxes on exports $\tau^i_{t,k} < 0$ for $M^i_{t,k} < 0$, as is well known from a long literature on trade policy (see e.g. Bagwell and Staiger, 2002).

In all these cases, observe that the optimal monopolistic intervention typically reduces the magnitude of capital or goods flows but does not change their direction.

It is straightforward that any price intervention $\tau^i$ can also be implemented by an equivalent quantity restriction $\bar{M}^i$, for example by imposing a quota rather than a tax on inflows.

### 6.3 Monopolistic Use of Domestic Policy Instruments

A policymaker could in principle also use domestic policies $\zeta^i$ to monopolistically distort her country’s terms of trade. However, if the set of external policy instruments

\(^{25}\)For example, when policymakers reduce reserve accumulation because they are concerned that they are pushing down the world interest rate too much, this is classic non-competitive behavior and is equivalent to monopolistic capital controls. This corresponds to statements by some Chinese policymakers that were concerned about pushing down US Treasury yields because of their reserve accumulation.
\( \tau^i \) is complete, then it can be shown that it would always be suboptimal to distort domestic policies for monopolistic reasons – it is more efficient to only external policy instruments for this purpose when they are available:

**Lemma 6 (Undistorted Domestic Policy, Monopolistic Planner)**  A monopolistic planner who has a complete set of external instruments \( \tau^i \) will only use her external instruments and will not distort domestic policies \( \zeta^i \) to exert market power.

**Proof.** The proof follows from the separability result in appendix A.1. After extending the planner’s optimization problem to internalize the dependence of world market prices on country \( i \) imports, \( Q = Q^{-i}(-\omega^i M^i) \), we observe that the implementability constraint on external transactions is still slack when the planner has complete instruments and therefore \( \mu_e^i = 0 \). As a result, the optimality condition for domestic instruments \( \zeta^i \) is unchanged.

The intuition for this result is closely related to the optimal targeting principle established by Bhagwati and Ramaswami (1963): if the goal of a policymaker is to distort international prices to her country’s benefit, then it is most efficient to directly target the variables under the policymaker’s control that affect international prices in the most direct way possible – and these are net imports \( M^i \). It is undesirable to introduce any distortions in domestic optimality conditions, given the optimal monopolistic external allocation.

Our result provides an interesting new perspective on the literature on international policy coordination with countries that internalize their market power: in much of the literature on fiscal or monetary policy cooperation, it is assumed that countries distort their fiscal or monetary instruments to internalize their terms-of-trade effects in the absence of coordination. This fails to consider that these domestic policies are only second-best instruments that would not be used for monopolistic objectives if other instruments to directly target external transactions are available. Although this may be an appropriate assumption in many cases, it is rarely given explicit attention. Our results here imply that it is inefficient to distort domestic policies if external instruments are available.

Conversely, when the set of external policy instruments \( \tau^i \) is incomplete, for example because of international agreements that restrict \( \tau^i \) or because of technical difficulties in targeting or implementing such measures, it may be desirable to use the domestic policy instruments \( \zeta^i \) for monopolistic reasons.

To illustrate this, we consider the extreme case of a monopolistic planner who cannot use external policy instruments at all so \( \tau^i \equiv 0 \). Furthermore, for simplicity, assume that the planner faces no domestic targeting problems so that she can choose

\(^{26}\) We could also place a more general set of restrictions on the set of policy instruments, e.g. \( \Xi(\tau^i) \leq 0 \). Our main insight that a monopolistic planner would use domestic policy instruments to distort external allocations would continue to hold.
$X^i$ without any restrictions. This allows us to ignore the domestic optimization problem of private agents and treat the vector $X^i$ as part of the vector of policy instruments $\xi^i = \{X^i, \xi^i\}$ for ease of notation. A monopolistic planner under these assumptions solves

$$\max_{M^i, \xi^i, X^i} U^i(\xi^i) \quad \text{s.t.} \quad f^i(M^i, M^i, \xi^i) \leq 0,$$

$$Q^{-i}(-\omega^i M^i) \cdot M^i \leq 0,$$

$$\lambda_{d}^{i} f_{m}^{i} = -\lambda_{e}^{i} Q^{-i}(-\omega^i M^i)$$

The domestic constraint and external budget constraint in the first two lines are the usual ones, but the planner internalizes the effects of her external allocations on world market prices $Q^{-i}(-\omega^i M^i)$. The implementability constraints in the third line reflect that the planner lacks policy instruments to create a wedge in the external optimality condition of private agents. We assign the usual shadow prices $\Lambda_{d}^{i}$, $\Lambda_{e}^{i}$ and $\mu_{e}^{i}$ to these constraints and obtain the optimality conditions

$$FOC \left( M^i \right) : 0 = \Lambda_{d}^{i} f_{m}^{i} + \Lambda_{e}^{i} Q \left(1 - \omega^i \mathcal{E}_{Q,M}^{-i}\right) + \mu_{e}^{i} \left[\lambda_{d}^{i} f_{m}^{i} - \omega^i \lambda_{e}^{i} Q^{-i}\right]$$

$$FOC \left( \xi^i \right) : \lambda_{e}^{i} = \Lambda_{d}^{i} f_{m}^{i} + \mu_{e}^{i} \lambda_{d}^{i} f_{mc}^{i}$$

$$FOC \left( \lambda_{e}^{i} \right) : 0 = \mu_{e}^{i} \cdot Q^T$$

where we denote the sum of partial derivatives as $f_{m}^{i} = \frac{\partial f}{\partial m}$ and so on to condense notation. The optimality condition on $\lambda_{e}^{i}$ implies that the price-weighted sum of shadow prices on the external implementability constraint (IC) is zero so $\sum_{k} \mu_{e,k}^{i} Q_{k} = 0$. With the exception of knife-edge cases (e.g. no monopoly power because $\omega^i = 0$), some of the shadow prices on the external IC will therefore be positive and some will be negative. In particular, if we denote the first term in the optimality condition on $M^i$ by the vector $-V_{m}^{i}$, we can write the condition as

$$\mu_{e}^{i} = A^{-1} \left[V_{m}^{i} - \Lambda_{e}^{i} Q \left(1 - \omega^i \mathcal{E}_{Q,M}^{-i}\right)\right]$$

where $A = \lambda_{d}^{i} f_{m}^{i} - \omega^i \lambda_{e}^{i} Q^{-i}$ is a negative semidefinite matrix. To interpret this expression, consider an $A$ that is close to diagonal and assume that there are no domestic externalities so $V_{m}^{i} = V_{m}^{i}$. In the absence of monopolistic behavior, the square brackets in this expression would be zero by the private optimality condition of consumers $V_{m}^{i} = \lambda_{e}^{i} Q$. By contrast, for a monopolistic planner, the vector of shadow prices $\mu_{e}^{i}$ captures the monopolistic cost (benefit) of increasing imports of each good and is therefore negative for net imports and positive for net exports.

In setting the optimal domestic allocation and policy measures $\xi^i$ according to $FOC \left( \xi^i \right)$, the planner internalizes that the marginal benefit of consumption needs to be equated to the marginal cost $\Lambda_{d}^{i} f_{m}^{i}$ complemented by the monopolistic benefits or costs $\mu_{e}^{i} \lambda_{d}^{i} f_{mc}^{i}$.

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27This assumption holds exactly if $m^i$ enters in Cobb-Douglas fashion. In the general case, the diagonal elements of the matrix are likely to be an order of magnitude larger than the off-diagonal elements as long as goods aren’t strong complements or substitutes.
6.4 Examples of Monopolistic Policy Intervention

Example 6.1 (Monopolistic Fiscal Policy, Complete Instruments) We continue the Example 5.1 on fiscal policy but consider a policymaker who takes into account monopolistic considerations. First, we assume the policymaker has complete external instruments $\tau^i$ and maximizes utility (30) subject to the intertemporal budget constraint. (For simplicity, we continue to assume $\alpha = 1$.) In that case, it is easy to see that the mix of private and public spending is determined by the same optimality condition (31) as for a competitive policymaker, $u'(C^i_t) = u'(G^i_t)$ or $C^i_t = G^i_t$ for $t = 1, 2$. This illustrates the findings of Lemma 6. However, the monopolistic planner will impose tax wedges (33) on the external transactions of private agents of $E^{i}_{Q;M} = \frac{\partial Q^2}{\partial M^i_1} - \frac{\partial Q^2}{\partial M^i_2}$, where

\[ 1 - \tau^i = \frac{1}{1 - E^{i}_{Q;M}} \quad \text{where} \quad E^{i}_{Q;M} = \left( \frac{\partial Q^2}{\partial M^i_1} - \frac{\partial Q^2}{\partial M^i_2} \right) \cdot M^i_2 \]

Consider w.l.o.g. the case that $M^i_1 > 0 > M^i_2$, i.e. the country imports $t = 1$ goods and exports $t = 2$ goods. In that case, we find $(E^{i}_{Q;M})_1 > 0 > (E^{i}_{Q;M})_2$ and therefore $\tau^i_1 < 0 < \tau^i_2$, i.e. the country taxes both $t = 1$ imports and $t = 2$ exports in order to push up the relative price $Q^2$ of its exports. In the intertemporal interpretation of the model, the monopolistic intervention pushes down the interest rate $R = 1/Q^2$ at which the country is borrowing in period 1.

Example 6.2 (Monopolistic Fiscal Policy, No External Instruments) In the absence of external instruments (i.e. if we restrict $\tau^i \equiv 0$), a monopolistic policymaker distorts government spending to achieve the same objective in a less efficient manner. Specifically, the policymaker maximizes utility (30) subject to the intertemporal budget constraint and the implementability constraint given by the private Euler equation $Q^2 u'(C^i_1) = u'(C^i_2)$ or $C^i_1 = Q^2 C^i_2$ under log-utility, to which we assign shadow price $\mu$. The planner’s optimality conditions can be combined to yield

\[ u' \left( C^i_1 \right) = u' \left( G^i_1 \right) - \mu \]
\[ u' \left( C^i_2 \right) = u' \left( G^i_2 \right) + \mu Q^2 \]

The shadow price satisfies $\mu > 0$ and implies that the planner distorts the optimal mix between private and public spending to further monopolistic objectives. In particular, $t = 1$ government spending is reduced compared to private spending, $G^i_1 < C^i_1$, and vice versa, $G^i_2 > C^i_2$, for $t = 2$ spending. Both interventions serve to push up the relative price $Q^2$ or, in the intertemporal interpretation, to push down the world interest rate $R = 1/Q^2$ at which the country borrows. Since the planner’s optimization problems in Examples 6.1 and 6.2 are identical except for the additional implementability constraint in the latter, welfare in Example 6.1 is strictly higher.
7 Imperfect External Policy Instruments

In practice policymakers do not have the set of perfect external policy instruments that we have depicted in our earlier analysis (see e.g. Carvalho and Marcio, 2006, in the context of capital controls). This section analyzes under what circumstances imperfections in the set of external policy instruments create a case for global policy coordination. We first discuss circumstances under which the set of external policy instruments is effectively complete even if there are limitations on actual instruments. Then we analyze two types of such imperfections: implementation costs and imperfect targeting. We formalize both and analyze when and how a global planner can achieve a Pareto improvement by coordinating the imperfect external policy instruments of different countries.

To be clear, it is only incomplete external policy instruments (i.e. of the instruments \(i\) that target the external transactions \(M_i\)) that justify global coordination; imperfect domestic policy instruments (i.e. the instruments to set domestic variables \(X_i\) to their desired levels) by themselves are irrelevant and do not create inefficiencies that can be addressed via global coordination, as captured by our main proposition 2.

7.1 Completeness of Set of External Instruments

Definitions A policymaker in country \(i\) has a complete set of external policy instruments \(\tau_i\) if the policymaker has sufficient instruments to implement any feasible external allocation as a decentralized allocation, i.e. \(\forall M^i\) s.t. \(Q \cdot M^i = 0\), \(\exists \tau^i\) s.t. \(M^i = M^i (Q, \tau^i)\). In the baseline setup of section 3, the set of policy instruments is complete by construction.

Conversely, a policymaker in country \(i\) has an incomplete set of external policy instruments \(\tau_i\) if there are feasible external allocations that the planner cannot implement as a decentralized allocation, i.e. \(\exists M^i\) with \(Q \cdot M^i = 0\) s.t. \(\not\exists \tau^i\) s.t. \(M^i = M^i (Q, \tau^i)\). Clearly, the two definitions are exhaustive and mutually exclusive, i.e. every set of external policy instruments is either complete or incomplete.

A policymaker in country \(i\) has an effectively complete set of external policy instruments \(\tau_i\) if the policymaker has sufficient instruments to implement the optimal allocation that she would choose if she had a complete set of external instruments, i.e. \(\exists \tau^i\) s.t. \(M^i (Q, \tau^i) = \arg \max_{M^i} V (M^i, M^i)\) s.t. \(Q \cdot M^i = 0\). If a policymaker’s set of external instruments is complete, it is also effectively complete, but not vice versa – it is possible that the planner has sufficient instruments to implement the optimal allocation but not all other feasible allocations.

Effective Completeness and Efficiency Our efficiency result in proposition 2 requires that the policymakers in each country have at least an effectively complete set of external policy instruments. In that case, each policymaker can choose her desired
allocations, and the logic of the first welfare theorem applies. Observe that only effective completeness in external instruments is required. No matter how incomplete the set of domestic policy instruments is, our efficiency proposition holds since a global planner cannot improve on the allocation given the restrictions on domestic instruments.

The case $V^i_M = 0$ of no externalities in international transactions represents a benchmark case in which the set of instruments of the policymaker in country $i$ is always effectively complete. In that case, incompleteness in actual policy instruments is irrelevant since the laissez faire external allocation implements the efficient allocation.

**Effective Completeness at the Global Level** The set of external policy instruments $(\tau^i)_{i=1}^N$ is *effectively complete at the global level* if a global planner can implement a Pareto efficient equilibrium for a given set of welfare weights. A given set $(\tau^i)_{i=1}^N$ may be effectively complete at the global level even if the set of policy instruments $\tau^i$ in some country $i$ is not effectively complete. In that case a global planner can implement a Pareto efficient equilibrium that is unattainable without global cooperation. Effective completeness at the global level is thus a weaker condition than effective completeness in each country $i$. The reason for the discrepancy between the two concepts is captured by lemma 3: there is a continuum of ways of implementing a given global allocation, but only one of these implementations corresponds to a global competitive equilibrium in which national policymakers act unilaterally. A global planner may be able to employ one of the other implementations if the implementation through a competitive equilibrium is unavailable.

**Example 7.1 (Effectively Complete Instruments at the Global Level)** Consider a world economy with $N = 2$ countries that are described by the reduced-form utility functions $V^i(m^i, M^i)$. Assume that country 1 suffers from externalities to capital inflows so $V^1_M \neq 0$ but does not have any policy instruments to correct for them. Furthermore, assume that country 2 does not suffer from externalities $V^2_M \equiv 0$ but has a complete set of external instruments $\tau^2$.

If country 1 had the instruments to do so, it would impose an optimal tax on capital inflows $\tau^1_0 = -(V^1_M/V^1_m)^T$ and the resulting equilibrium would satisfy $(1-\tau^i)V^1_m/\lambda^i_c = Q = V^2_m/\lambda^2_c$. However, since the country has no policy instruments, the global competitive equilibrium coincides with the laissez-faire equilibrium – country 2 has no incentive to intervene in its external transactions.

By contrast, under cooperation, country 2 would set its policy instrument $1-\tau^2_0 = \frac{1}{1-\tau^1_0}$, i.e. it would tax capital outflows to correct for the negative externalities of inflows in country 1 and implement the same (efficient) equilibrium that would prevail if country 1 could use its own instruments.
7.2 Imperfections in Set of External Instruments

To formally capture imperfections in the set of instruments of a policymaker in country \(i\), we assume that there is a cost to imposing the set of policy instruments \(\tau^i\) given by a function \(\Gamma^i(\tau^i)\) that is non-negative, satisfies \(\Gamma^i(0) = 0\), is convex and continuously differentiable, and that negatively enters the planner’s objective \(U^i(X^i) - \Gamma^i(\tau^i)\).

This formulation is able to capture a large number of imperfections in the set of instruments: First, if we index the elements of vector \(\tau^i\) by the letter \(t\), a cost function of the simple quadratic form \(\Gamma^i(\tau^i) = \sum_t \gamma^i_t (\tau^i_t)^2 / 2\) may capture implementation costs that arise from policy intervention and that grow in the square of the intervention. The chosen specification allows for this cost to vary across different elements of the vector \(\tau^i\) by adjusting \(\gamma^i_t\), for example \(\gamma^i_t = \beta^t \tau^i_t\). Secondly, if we assume the limit case \(\gamma^i_t \to \infty\) for some \(t\), the cost function captures that instrument \(\tau^i_t\) is not available. In the extreme case of \(\gamma^i_t \to \infty\) \(\forall t\), country \(i\) has no external policy instruments. Thirdly, if we index the vector \(\tau^i\) both by the letters \(t\) and \(k\), for example to capture that there are different goods \(k = 1 \ldots K\) in each time period \(t\) as in example 3.4, a cost function of the form \(\Gamma^i(\tau^i) = \sum_t \sum_{k=2}^K \gamma (\tau^i_{t,k} - \tau^i_{t,1})^2 / 2\) with \(\gamma \to \infty\) captures that the planner is unable to differentiate her policy instruments across different goods and has to set \(\tau^i_{t,k} = \tau^i_{t,1} \forall k\). Fourth, a similar specification \(\Gamma^i(\tau^i) = \sum_t \sum_{s \in \Omega} \gamma (\tau^i_{t,s} - \tau^i_{t,0})^2 / 2\) captures restrictions on the ability of the planner to target flows in different state of nature \(s \in \Omega\). For example, the planner may be unable to differentiate between capital flows with different risk profile. Fifth, for \(\Gamma^i(\tau^i) \equiv 0\), the setup collapses to our benchmark model of section 3.

7.3 Model of Imperfect Instruments

In the following, we focus on a simplified model version without domestic policy instruments and constraints \(\zeta^i\) in which we assume \(X^i = M^i\) so there is no role for independent domestic allocations (as for instance in the example of Section 2). The problem of a country is fully described by the reduced-form utility function \(V^i(m^i, M^i)\).

A competitive national planner maximizes \(V^i(M^i, M^i)\) subject to the collection of external implementability constraints \((1 - \tau^i)^T V^i_m = \lambda^i_e Q^T\) and the external budget constraint (9), to which we assign shadow prices \(\mu^i_e\) and \(\Lambda^i_e\) respectively. The planner’s optimality condition \(FOC(M^i)\) can be written to express the shadow prices on the implementability constraint \(\mu^i_e\), which captures the extent of mis-targeting,

\[
\mu^i_e = \frac{V^i_{m+M}^T - \Lambda^i_e Q}{1 - \tau^i} \cdot [V^i_{m(m+M)}]^{-1}
\]

28The setup is isomorphic to one in which the cost is a resource cost that is subtracted from the external budget constraint \(Q \cdot M^i + \Gamma^i(\tau^i) \leq 0\).

29Similar results can be derived if the cost of policy intervention depends on quantities transacted, e.g. \(\Gamma^i(\tau^i, M^i)\), which may for example capture the costs associated with attempts at circumvention.
The mis-targeting indicator $\mu^i_{e,t}$ is negative for those elements of the import vector $M^i$ that are less than optimal, i.e. for which the marginal social value is greater than the market price, $V^i_{m+M,t} > \Lambda^i_e Q_t$, and positive in the converse case, since the matrix $V^i_{m(m+M)}$ is negative semi-definite. For those elements of $M^i$ for which the planner has perfect instruments, she can set $V^i_{m+M,t} = \Lambda^i_e Q_t$ and the mis-targeting is $\mu^i_{e,t} = 0$. If the planner has no external policy instruments (i.e. in the limit case that using the instruments is infinitely expensive), then the mistargeting is reflected in $\mu^i_e$ but there is nothing that the planner can do about it.

If policy instruments $\tau^i$ are available but costly, then the planner sets the instruments such that the marginal cost of the policy instruments $\tau^i$ equal the benefit from reducing the mis-targeting, as captured by the optimality condition $FOC(\tau^i)$,

$$\Gamma^i(\tau^i) = \mu^i_e V^i_m T$$

If there are excessive flows of a good so $\mu^i_{e,t} > 0$, the planner imposes a positive tax $\tau^i > 0$ that leads to a positive marginal distortion $\Gamma^i(\tau^i) > 0$; conversely, if flows of a good are insufficient $\mu^i_{e,t} < 0$, the planner imposes a subsidy.

The optimality condition $FOC(\lambda^i_e)$ requires that the price-weighted average mis-targeting is zero, $\mu^i_e \cdot Q^T = 0$. Compared to the laissez-faire equilibrium, the marginal valuation of wealth $\Lambda^i_e$ in equation (34) adjusts to ensure that this condition is satisfied. If externalities $V^i_M$ are on average positive, then the planner’s marginal valuation $\Lambda^i_e$ of will be above $\lambda^i_e$ and vice versa. The optimality condition implies the following result for how the planner chooses to employ her policy instruments:

**Proposition 7 (Implementation with Imperfect Instruments)** Under the constrained optimal allocation of a competitive planner, the average marginal cost of the policy instruments $\tau^i$ is zero,

$$\Gamma^i(\tau^i) \cdot (1 - \tau^i)^T = 0 \quad (35)$$

**Proof.** The result is obtained by combining the optimality conditions on $\tau^i$ and $\lambda^i_e$ with the external implementability constraint. □

Intuitively, given the constraints on her instruments, the planner chooses her allocations such that flows are too low in some states and too high in others compared to the complete instruments case. The intuition goes back to lemma 3 on the indeterminacy of implementation: under complete instruments, the efficient allocation can be implemented using a continuum of policy instruments that satisfy $(1 - \tilde{\tau}^i) = k (1 - \tau^i)$ for some $k > 0$. Under imperfect instruments, the planner picks the implementation from within this continuum that minimizes total implementation costs. Whenever the planner finds it optimal to tax some flows, she will subsidize others, such that the weighted average distortion is zero.
The optimal tax formula under imperfect instruments satisfies

\[ 1 - \tau^i = \frac{1}{\frac{\alpha^i}{\lambda^i} + \frac{\Gamma^u}{(V^i_{m})} V^i_{m(m+M)}} \]

If the set of instruments is effectively complete, observe that the denominator is one and the expression reduces to equation (20). Under incomplete instruments, the term \( \frac{\alpha^i}{\lambda^i} \geq 1 \) in the denominator adjusts the average level of controls so that proposition 7 is satisfied. The term \( \Gamma^u (\tau^i) \) reduces/increases the optimal level of policy intervention to account for the costs of intervention.

**Example 7.2 (Costly Instruments)** Consider a country similar to country C in Section 2 in which inflows in period 1 decrease output in period 2 due to an externality so \( y_2^i (M_1^i) = y_1^i - \eta_i^i M_1^i \). Assume there is no discounting, world prices satisfy \( Q = (1, 1) \) and external policy instruments impose a quadratic utility cost of implementation \( \Gamma^i (\tau^i) = \gamma^i \cdot \tau^i T / 2 \). The reduced-form utility function net of implementation costs is

\[ V^i (m^i, M^i) = u \left( y_1^i + m_1^i \right) + u \left( y_1^i - \eta_i^i M_1^i + m_2^i \right) - \Gamma^i (\tau^i) \]

Let us start from an equilibrium in which \( \eta_i^i = 0 \) and assume a small increase \( d\eta_i^i > 0 \) in the externality. Given the costly instruments, the planner will set \( \tau_1^i = d\eta_i^i / 2 = -d\tau_2^i \) such that condition (35) is satisfied. The planner taxes the externality-generating inflows (or, equivalently, subsidizes outflows) in period 1 but subsidizes inflows (and taxes outflows) in period 2. The planner internalizes that higher \( m_2^i \) implies lower \( m_1^i \) by the external budget constraint, and that she can correct the externality while saving on implementation costs by spreading her intervention across both periods.

### 7.4 Global Coordination with Imperfect Instruments

We next determine under what conditions the equilibrium in which national planners impose capital controls according to equation (35) is constrained Pareto efficient. In other words, if national planners follow the described rule, can a global planner achieve a Pareto improvement on the resulting equilibrium?

We analyze a global planner who faces the same set of imperfect instruments and maximizes the weighted sum of welfare

\[
\begin{align*}
\max_{(M^i, \tau^i, \lambda^i)_{i=1}^N} \sum_i \phi^i \omega^i V^i (M^i, M^i) - \Gamma^i (\tau^i) \\
\text{s.t.} \quad \sum_i \omega^i M^i = 0, \quad (1 - \tau^i)^T V^i_{m} = \lambda^i Q^T, \quad \forall i
\end{align*}
\]

\(^{30}\)We obtain the expression by subtracting the implementability constraint from the FOC \((M^i)\) and rearranging terms.
for a given set of welfare weights \((\phi^i)_{i=1}^N\). As we vary the welfare weights, we trace the entire constrained Pareto frontier. We assign the vectors of shadow prices \(\nu_i\) and \(\mu_e^i\) to the resource and implementability constraints and observe that the market price \(Q\) is now a choice variable of the global planner. The implementability constraints thus capture that the post-tax private marginal products \((1 - \tau^i)V^i_m\) of different countries must be proportional to each other.

We describe the extent of mis-targeting using the shadow prices \(\mu_e^i\) by re-arranging the global planner’s optimality conditions \(FOC (M^i)\) to obtain

\[
\mu_e^i = \frac{\nu_i^i}{1 - \tau^i} \cdot \left[ V^i_m (m+M^i) \right]^{-1} \quad \forall i
\]

As in our analysis of national planners, a positive element \(\mu_{e,t}^i\) means that the planner would like more inflows of \(M^i_t\) but cannot implement this due to the imperfect instruments. Also, the optimality condition \(FOC (\tau^i)\) implies that the planner sets the instruments \(\tau^i\) such that the marginal cost is proportional to the extent of mis-targeting \(\Gamma^i (\tau^i) = \mu_e^i V^i_m \tau^i\).

**Proposition 8 (Global Coordination with Imperfect Instruments)** *In a constrained efficient equilibrium, the average marginal cost of the policy instruments \(\tau^i\) in a given country is zero,*

\[
\Gamma^i (\tau^i) \cdot (1 - \tau^i)^T = 0 \quad \forall i
\]

*Furthermore, the weighted average mis-targeting across countries is zero for each good,*

\[
\sum_{i=1}^{N} \omega^i \phi^i \Gamma^i (\tau^i) (1 - \tau^i) = 0 \quad (37)
\]

**Proof.** The first (second) result is obtained by combining the optimality conditions on \(\tau^i\) and \(\lambda^i_e\) (on \(\tau^i\) and \(Q\)) with the external implementability constraint. ■

As before, the weighted average marginal distortion across different goods in the same country \(i\) is zero. However, unlike in the competitive allocation, a global planner also sets the average marginal distortion for each good across countries equal to zero. This implies that if some countries impose taxes on inflows for certain \(m^i_t\), others must impose taxes on outflows. In short, the planner spreads her intervention across inflow and outflow countries in proportion to their cost of intervention.

We illustrate our findings in the following examples:

**Example 7.3 (Wasteful Competitive Intervention)** Assume a world economy that consists of \(N\) identical countries with reduced-form utility functions \(V^i (m^i, M^i)\) that experience externalities \(V^i_M \neq 0\) and that suffer from implementation costs...
\( \Gamma^i(\tau^i) = \gamma^i \tau^i \cdot \tau^iT/2 \). Following proposition 7, the national planners in all countries impose the same non-zero policy instruments \( \tau^i \neq 0 \) and incur the same costs \( \Gamma^i(\tau^i) > 0 \).

A global planner who puts equal weight on the countries would recognize that the competitive interventions in all countries are wasteful – since the countries are identical, there is no trade \( M^i = 0 \forall i \) and all countries could save the cost \( \Gamma^i(\tau^i) \) without changing global allocations by coordinating to reduce their policy instruments to zero. Technically, the global competitive equilibrium violates condition (37) since the policy instruments of all countries have the same sign. The only way the planner can satisfy this optimality condition is to move all controls to zero. Observe that the set of instruments is effectively complete at the global level even though all \( N \) countries have incomplete/imperfect instruments.

**Example 7.4 (Sharing the Regulatory Burden)** Consider a world economy consisting of \( N = 2 \) economies as described in example 7.2 with utility functions given by (36) and cost functions \( \Gamma^i(\tau^i) = \frac{\gamma^i \tau^i \cdot \tau^iT}{2} \). Assume an equilibrium with \( \eta^1 = \eta^2 = 0 \) and consider the effects of a small increase \( d\eta^1 > 0 \) in country 1’s externality. In the global competitive equilibrium, the planner in country 1 would behave as described in example 7.2, but the resulting allocation would violate condition (37) since \( \tau^1_0 > 0 > \tau^1_1 \) but \( \tau^2_0 = 0 \).

A global planner with equal welfare weights would share the regulatory burden among both countries to minimize the total cost of intervention. Specifically, the planner would set the policy instruments in accordance with the relative cost of intervention,

\[
d\tau^1_0 = -\frac{\gamma^2 d\eta}{2(\gamma^1 + \gamma^2)} = -d\tau^1_1 \quad \text{and} \quad d\tau^2_0 = -\frac{\gamma^1 d\eta}{2(\gamma^1 + \gamma^2)} = -d\tau^2_1
\]

This guarantees that the sum of interventions equals the increase in the externality \( d\eta \) and that both conditions (35) and (37) are satisfied.\(^{31}\) If the cost of intervention is equal among the two countries, then the fractions are \( 1/4 \) and the planner corrects one quarter of the externality in each time period in each country to implement the constrained efficient equilibrium.

For given \( \gamma^1 > 0 \), we can analyze two interesting limit cases: first, if it becomes cost-less to impose external policy instruments in country 2 \( (\gamma^2 \to 0) \), then the planner would only intervene in country 2 and fully correct the externality there, leaving \( \tau^1_0 = 0 \). Conversely, if it becomes prohibitively costly to employ policy instruments in country 2 \( (\gamma^2 \to \infty) \), then the planner would only intervene in country 1 and leave \( \tau^2_0 = 0 \).

\(^{31}\)Ghosh et al. (2014) provide empirical evidence for the practical feasibility of burden-sharing between inflow and outflow countries.
In summary, our examples illustrate that the rationale for global cooperation is to save on implementation costs by (i) avoiding wasteful competitive regulation and (ii) shifting the regulatory burden towards those who can implement regulation in the most cost-effective way.

**Scope for Pareto Improvement** One caveat to the cooperative agreements described in this section is that our planning setup implicitly assumes that lump-sum transfers are available. Sharing the regulatory burden generally involves a shift in world market prices that may create winners and losers. This will be the case especially when a “helping” country taxes outflows or subsidizes inflows.

Although Pareto-improvements are not always possible, there are two constellations under which cooperation does constitute Pareto-improvements: (i) if there is no trade as in our example 7.3 on wasteful competition and (ii) if the savings from the marginal cost of an instrument $\Gamma_i^i (\tau_i^i)$ in the country that suffers the terms-of-trade loss is greater than the loss.

### 7.5 Domestic Policy under Imperfect External Instruments

We now return to the full setup of our baseline model with domestic instruments and policy measures $(X^i, \zeta^i)$ in order to study the effects of incomplete external policy instruments on domestic allocations.

**Lemma 7 (Non-Separability)** (i) If a domestic planner faces a set of external policy instruments that is not effectively complete, she will generically distort her domestic policy choices $\zeta^i$ as a second-best device to target external transactions.

(ii) If the set of policy instruments in a world economy is not effectively complete at the global level, a global planner will also coordinate the use of domestic policies to achieve a superior global allocation.

**Proof.** For (i), we add the utility cost $-\Gamma_i^i (\tau_i^i)$ to the optimization problem of national planners in appendix A.1 and observe that $\mu_e^i \neq 0$ if the planner’s set of external policy instruments is not effectively complete; therefore the optimality condition $\text{FOC} (\zeta^i)$ of a domestic planner is affected by the imperfect targeting of external transaction (except in knife-edge cases when $f_m^i = 0$ happens to hold, i.e. when domestic policy has no effect whatsoever on external transactions). For (ii) we proceed in the same manner, and the global planner faces the same optimality conditions $\text{FOC}(\zeta^i)$. The shadow prices $\mu_e^i$ on the global planner’s external implementability constraints are non-zero if external policy instruments are not complete at the global level. Therefore the global planner internalizes how domestic choices affect external transactions.

Intuitively, the planners in both cases consider not only domestic objectives in setting the domestic policy instruments $\zeta^i$ but also how their choices will improve
external allocations, which they can only imperfectly target with the incomplete set of external instruments. For example, if $\zeta^i_t$ is complementary to $m^i_t$ and $m^i_t$ is excessive, then the planner will reduce $\zeta^i_t$ to bring down $m^i_t$.

Example 6.2 illustrated this mechanism in the case of fiscal policy under incomplete external instruments and monopolistic considerations – a monopolistic planner distorted fiscal spending as a second-best device to influence external transactions. It is straightforward to apply this logic to cases in which a domestic planner wants to intervene in external allocations to correct inefficiencies. For example, if we introduce fiscal spending and incomplete external policy instruments in our introductory example of learning-by-exporting externalities (section 2), then the planner in country $C$ would reduce fiscal spending in period 1 to reduce imports and internalize the country’s growth externalities.

8 Imperfect International Markets

This section discusses how global cooperation can improve outcomes if there are imperfections in international markets. We already noted in the introduction, borrowing from Leo Tolstoy’s quote on unhappy families, that each imperfect market is imperfect in its own way, i.e. that there is a myriad of different forms of market imperfections. Unlike in the case of monopoly power and imperfect instruments, we must therefore limit our focus on specific instances of international market imperfections.

In the following, we consider two specific examples that cover a wide range of the literature in the area: market incompleteness and price stickiness in the international arena. We first show that if each country has a full set of external policy instruments $\tau^i$, cooperation to deal with these two problems will be limited to adjusting the external policy instruments of countries, and there is no need to coordinate domestic policy measures $\zeta^i$. This result is likely to hold for many other examples of international market imperfections, but a formal proof depends on the specific context. Then we provide several examples of imperfections in international markets and how cooperation can improve outcomes.

8.1 Cooperation with Complete External Instruments

To capture international market incompleteness and price stickiness in a general way, we follow Farhi and Werning (2013) and impose a set of constraints on international transactions and prices of the form

$$\Phi \left( (M^i)^N_{i=1}, Q \right) \leq 0 \quad (38)$$

To provide a few of examples for this constraint, suppose the market for a good $t$ is missing, this corresponds to a constraint $M^i_t = 0 \forall i$. If risk markets in period $t$ are absent, this is captured by a constraint $M^i_{t,s} = M^i_{t,s'}$ for any two states $s, s'$ in period
t. Price stickiness can be captured by constraints of the form $Q_t = \bar{Q}_t$ and so on. An important assumption inherent in (38) is that the constraint does not directly depend on domestic variables. Nonetheless, the constraint covers a wide range of models of international market imperfections.

We add this constraint to our baseline model in appendix A.1 and observe:

**Lemma 8 (Domestic Policy under Imperfect International Markets)** If international markets are imperfect as captured by constraint (38) but external policy instruments are effectively complete at the global level, then a global planner will only coordinate the use of external policy instruments $\tau^i$ not domestic policies $\zeta^i$.

**Proof.** We add the constraint (38) to the planner’s optimization problem in appendix A.1 and observe that the constraint only affects the optimality conditions for $M^i$ and $Q$ but not for domestic policies. As long as external policy instruments are effectively complete, the implementability constraint on external transactions is slack and $\mu^i_e = 0$. As a result, the optimality conditions for domestic instruments $\zeta^i$ are unchanged.

Intuitively, the result reflects a pecking order of instruments on which countries should coordinate: if available, it is desirable to use only external instruments to correct international market imperfections. Conversely, if external policy instruments are incomplete, then it is desirable to also coordinate on the use of domestic policy measures. (Technically, if external instruments are incomplete, implementability constraints will be binding so $\mu^i_e \neq 0$ and optimal domestic policy choices depend on external objectives.) The result is again a reflection of our findings on the separability between domestic and external choices under complete external instruments.

When the set of external instruments is not effectively complete, then a global planner will also coordinate the use of domestic policies, following the same intuition that we first explored in Section 6.3 on the use of domestic instruments to achieve external objectives – if a global planner lacks an external instrument to increase imports of good $t$ (reflected in a shadow price $\mu^i_{e,t} > 0$), then he will increase domestic policies that are complementary to imports of good $t$, for example government spending that fall partially on good $t$ or monetary policy that stimulates relative expenditure on good $t$.

### 8.2 Examples

**Example 8.1 (Missing Markets for Idiosyncratic Risk)** Consider a two period world economy with $N > 1$ countries that

**Missing Risk Markets**
9 Conclusions

This paper investigates under what conditions the coordination of national economic policies is desirable from a global welfare perspective. We develop a benchmark under which national policymaking delivers efficient outcomes and spillovers constitute pecuniary externalities that cancel out at the global level. This benchmark depends on three conditions: (i) national policymakers act as price-takers in the international market, (ii) they possess an effectively complete set of instruments to control external transactions so they can implement their desired allocations and (iii) international markets are free of imperfections. Under these conditions, we can apply an analogue of the first welfare theorem at the level of national economic policymakers.

We establish this benchmark result in a very general framework that nests a wide range of open economy macro models. We provide a number of examples in which our efficiency results apply, counter to the intuition of many commentators and policymakers. Then we investigate each of the three conditions required for efficiency in more detail. We show how violations of the conditions generally lead to inefficiency and provide guidelines for how global cooperation among national policymakers can improve outcomes. Furthermore, we also discuss guidelines for how to ascertain whether the conditions for our benchmark result are satisfied, for example how to detect monopolistic behavior and how to identify symptoms of targeting problems.

An important research area is how to implement Pareto-improving cooperation in practice. Willett (1999) discusses a range of political economy problems that make international policy cooperation difficult. Bagwell and Staiger (2002), among others, provide an analysis of how to achieve agreements to abstain from monopolistic beggar-thy-neighbor policies if countries are sufficiently symmetric.

References


A Mathematical Appendix

A.1 Combined Optimization Problem

Proof of Lemma 2 (Separability) We describe the Lagrangian of the combined optimization problem of an individual agent as

\[ w^i (m^i, x^i; M^i, X^i, \zeta^i, T^i) = \max_{m^i, x^i} U^i (x^i) - \lambda_d^i \cdot f^i (m^i, x^i, M^i, X^i, \zeta^i) - \lambda_e^i \left[ \frac{Q}{1 - \tau^i} \cdot m^i - T^i \right] \]

The optimality conditions are given by the vector equations

\[ \text{FOC} (x^i) : U^i_x = f^i_x \lambda_d^i \]
\[ \text{FOC} (m^i) : \lambda_d^i f^i_m + \lambda_e^i \frac{Q}{1 - \tau^i} = 0 \]
These two conditions represent domestic and external implementability constraints on the national planner’s problem, which is given by the Lagrangian

$$L^i = \max_{M^i, X^i, \xi^i, \tau^i, \lambda^i_d, \lambda^i_e} \ U^i (X^i) - \Lambda^i_d \cdot f^i (M^i, X^i, M^i, X^i, \zeta^i)$$

(A.1)

$$-\Lambda^i_e [Q \cdot M^i]$$

(A.2)

$$-\mu^i_d \cdot \left[ U^i_x - f^i_x \lambda^i_d T \right]$$

(A.3)

$$-\mu^i_e \cdot \left[ \lambda^i_d f^i M^i + \lambda^i_e \frac{Q}{1 - \tau^i} \right]^T$$

(A.4)

and delivers the associated optimality conditions (with all multiplications and divisions in the FOC (\(\tau^i\)) calculated element-by-element)

$$FOC (X^i) : U^i_x = (f^i_x + f^i_k)^T \Lambda^i_d T + U^i_{xx} T \mu^i_d T - \sum_k \left[ \mu^i_{d,k} (f^i_{xx} + f^i_x X^i_k)^T \lambda^i_d T - \mu^i_{e,k} (f^i_{mx} + f^i_m X^i_k)^T \lambda^i_d T \right]$$

(A.5)

$$FOC (M^i) : 0 = (f^i_m + f^i_M)^T \Lambda^i_d T + \Lambda^i_e Q^T - \sum_k \left[ \mu^i_{d,k} (f^i_{mm} + f^i_m M^i_k)^T \lambda^i_d T - \mu^i_{e,k} (f^i_{mm} + f^i_m M^i_k)^T \lambda^i_d T \right]$$

(A.6)

$$FOC (\zeta^i) : 0 = f^i_\zeta T \Lambda^i_d T - \sum_k \left[ \mu^i_{d,k} f^i_{\zeta k} T \lambda^i_d T - \mu^i_{e,k} f^i_{\zeta m} f^i M^i_k T \lambda^i_d T \right]$$

(A.7)

$$FOC (\tau^i) : 0 = \mu^i_e \left[ \frac{\lambda^i_e Q}{(1 - \tau^i)^2} \right]$$

(A.8)

$$FOC (\lambda^i_d) : 0 = -\mu^i_d f^i T + \mu^i_e f^i T$$

(A.9)

$$FOC (\lambda^i_e) : 0 = \mu^i_e \cdot \left( \frac{Q}{1 - \tau^i} \right)^T$$

(A.10)

Given the complete set of external instruments \(\tau^i\), the system of optimality conditions

$$FOC (M^i, \tau^i, \lambda^i_d)$$

implies that the vector of shadow prices on the external implementability constraint satisfies \(\mu^i_e = 0\) – the planner sets the vector \(\tau^i\) to whichever levels she wants without facing trade-offs. By implication, the last term in the other five optimality conditions drops out, allowing us to separate the problem into two blocks.

The optimality conditions (A.5), (A.7) and (A.9) with \(\mu^i_e = 0\) replicate the optimality conditions of the optimal domestic planning problem (14) in section 4.1. Together with the domestic constraint (A.1) and the domestic implementability condition (A.3), these five conditions are identical to the five conditions that pin down the optimal domestic allocation for given \(M^i\) in section 4.1 and yield identical solutions for the five domestic variables \(X^i, \zeta^i, \lambda^i_d, \Lambda^i_d, \mu^i_d\).

Given the envelope theorem, the optimality condition (A.6) can equivalently be written as

$$\partial \mathcal{L}^i / \partial M^i = dV^i / dM^i = 0,$$

where the latter condition coincides with the optimality
condition (19) defining the optimal external allocation in section 4.2. The optimality conditions (A.8) and (A.10) for $\mu_e^i = 0$ capture that the planner can set the product $\lambda_e^i (1 - \tau^i)$ such as to precisely meet the constraints (A.4) where the planner has one scalar degree of freedom, as we also emphasized in section 4.2. The three optimality conditions together with the two constraints yield an identical set of solutions for the five external variables $(M^i, \tau^i, \lambda_e^i, \Lambda_e^i, \mu_e^i)$ as we described in section 4.2. This shows that the two-step procedure that separates domestic and external optimization as described in section 3 yields identical solutions as the combined optimization problem described here.

B Further Examples and Applications

B.1 Learning-by-Exporting Externalities – Infinite Horizon

Assume a representative agent in a canonical open economy $i$ that behaves as in our baseline infinite-horizon model of capital flows (Example 3.1), except that the endowment income $y_{t+1}^i$ is a function $\varphi_t(\cdot)$ of the economy’s aggregate net imports $M_t^i$ that satisfies $\varphi_t(0) = 0$ and that is continuous and decreasing $\varphi'_t(M_t^i) \leq 0$ to capture that higher net exports increase growth,

$$y_{t+1}^i = y_t^i + \varphi_t(M_t^i)$$

(B.1)

The reduced-form utility of a representative agent in country $i$ is

$$V^i(m^i, M^i) = \sum_t \beta^t u \left( y_0^i + \sum_{s=0}^{t-1} \varphi_s(M_s^i) + m_t^i \right)$$

with marginal utility of private and aggregate capital inflows of $V_{m,t}^i = \beta^t u'(C_t^i)$ and $V_{M,t}^i = \varphi'_t(M_t^i) \beta^t v_{t+1}^i$ where $v_{t+1}^i = \sum_{s=t+1}^{\infty} \beta^{s-t} u'(C_s^i)$ is the PDV of one extra unit of output growth at time $t + 1$, capturing the growth externalities from exporting. Following lemma 1, a planner can implement the socially efficient allocation in economy $i$ by imposing capital controls

$$\tau_t^i = -V_{M,t}^i / V_{m,t}^i = -\frac{\varphi'_t(M_t^i) v_{t+1}^i}{u'(C_t^i)} \geq 0$$

(B.2)

The planner subsidizes exports/capital outflows and taxes imports/capital inflows in periods in which net exports generate positive externalities. We can also translate this into controls on period-by-period bond accumulation of $1 - \xi_t^i = (1 - \tau_{t+1}^i) / (1 - \tau_t^i)$ as described in Example 3.1.

It is typically argued that learning externalities are only relevant during transition periods in developing economies (see e.g. Rodrik, 2008). In that case, the externality term $\varphi'_t(M_t^i)$ would at first be negative and would gradually converge to zero.

Since our infinite horizon model of learning-by-exporting externalities nests into the general model of section 3, our welfare results from Proposition 2 apply.
B.2 Learning-by-Doing Externalities – Infinite Horizon

Assume that the output of a representative worker in economy \( i \) is given by \( y^i_t = A^i_t \ell^i_t \), where labor \( \ell^i_t \) imposes a convex disutility \( d(\ell^i_t) \) on workers. We capture learning-by-doing externalities by assuming that productivity growth \( A^i_{t+1} \) in the economy is a continuous and increasing function of aggregate employment \( \psi_t(L^i_t) \) that satisfies \( \psi'_t(\cdot) \geq 0 \) so that

\[
A^i_{t+1} = A^i_t + \psi_t(L^i_t) = A^i_0 + \sum_{s=0}^{t} \psi_s(L^i_s) \tag{B.3}
\]

In the described economy, the first-best policy instrument to internalize such learning effects would be a subsidy \( s^i_t \) to wage earnings in the amount of

\[
s^i_t = \psi^i_t(L^i_t) v^i_{A,t+1}/[u'(c^i_t) A^i_t] \text{ where } v^i_{A,t+1} = \sum_{s=t+1}^{\infty} \beta^{s-t} u'(c^i_s) L^i_s \text{ is the PDV of one unit of productivity growth starting period } t+1.
\]

In the absence of a policy instrument to target the labor wedge, a planner faces the implementability constraint

\[
A^i_t u' (A^i_t L^i_t + M^i_t) = d^i (L^i_t) \tag{B.4}
\]

which reflects the optimal labor supply condition of individual workers. Observe that reducing \( M^i_t \) in this constraint is akin to a negative wealth effect and increases the marginal utility of consumption, which in turn serves as a second-best instrument to raise \( L^i_t \) and trigger learning-by-doing externalities.

Accounting for this implementability constraint and imposing the consistency condition \( \ell^i_t = L^i_t \), a constrained planner recognizes that the reduced-form utility of the economy is

\[
V(m^i, M^i) = \max_{L^i_t} \sum_{t} \beta^t \left\{ u(A^i_t L^i_t + m^i_t) - d(L^i_t) \right\} \text{ s.t. (B.3), (B.4)}
\]

with marginal utility of private and aggregate capital inflows of \( V^i_{m,t} = \beta^t u'(C^i_t) \) and \( V^i_{M,t} = -\lambda^i_t \beta^t A^i_t u''(C^i_t) < 0 \) where \( \lambda^i_t \) is the shadow price on the implementability constraint (25) and is given by

\[
\lambda^i_t = \frac{\psi^i_t(L^i_t) v^i_{A,t+1}}{d''(L^i_t) - (A^i_t)^2 u''(C^i_t)} > 0
\]

In this expression, the positive learning externalities (in the numerator) are scaled by a term that reflects how strongly labor supply responds to changes in consumption (in the denominator).\(^{32}\) If the economy has outgrown its learning externalities, the term drops to zero.

\(^{32}\)Given that there are no first-best policy instruments available, the PDV of one unit of productivity growth \( v^i_{A,t} \) also includes the effects of higher productivity on future labor supply decisions: on the one hand, higher productivity increases incentives to work, on the other hand it makes the agent richer and reduces the incentive to work via a wealth effect. The two effects are captured by
Following lemma 1, the planner can implement this second-best solution by imposing capital controls of

$$\tau_i^t = -\frac{\lambda_i A_i^t u''(C_i^t)}{u'(C_i^t)} = \frac{\psi_i^t (L_i^t) v_{A,t+1}}{d' (L_i^t) \left(1 + \frac{\eta_L}{\sigma_C}\right)}$$  \hspace{1cm} (B.5)

This control reduces capital inflows and stimulates domestic production to benefit from greater learning-by-doing externalities. The numerator in the expression is analogous to the optimal capital control (B.5) under learning-by-exporting. In the denominator term, $\eta_L$ and $\sigma_C$ are the Frisch elasticity of labor supply and the intertemporal elasticity of substitution: the second-best intervention is more desirable the less responsive the marginal disutility of labor (low $\eta_L$) and the less responsive the marginal utility of consumption (high $\sigma_C$). Again, Proposition 2 applies and the application of the described second-best capital controls leads to a globally Pareto efficient outcome.

The right-hand side of the expression can be obtained by substituting for $\lambda_i^t$ and observing that $A_i^t u' (C_i^t) = d' (L_i^t)$ and

$$\frac{A_i^t u'' (C_i^t)}{d'' (L_i^t) - (A_i^t)^2 u'' (C_i^t)} = -\frac{1}{A_i^t \left(1 + \frac{\eta_L}{\sigma_C}\right)}$$

The two expressions in square brackets,

$$v_{A,t+1}^i = \sum_{s=t+1}^{\infty} \beta^{s-t} \left\{ u' (C_s^i) L_s^i + \lambda_s^i \left[ u' (C_s^i) + A_s^i u'' (C_s^i) L_s^i \right] \right\}$$

\textsuperscript{33}