The Dynamics of Multinational Activity: Evidence from U.S. Corporations

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This Paper

Introduction

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- ContributionLiterature

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Model

- A theoretical and empirical investigation of the dynamic behavior of foreign affiliate sales.
- Using a panel of US MNEs from the BEA, we trace the evolution of horizontal, vertical, and export platform FDI within firms over time.
 - 1. How do affiliate sales volumes evolve over their life cycle?
 - 2. How does the organization of affiliates sales across destinations depends on affiliate age?

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 - How do affiliate sales volumes evolve over their life cycle?
 New affiliates have flat sales growth profiles.
 - 2. How does the organization of affiliates sales across destinations depends on affiliate age?

Affiliates start specialized in a core activity, usually horizontal FDI, and diversify over time.

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Affiliates start specialized in a core activity, usually horizontal FDI, and diversify over time.

• We propose a dynamic model of FDI consistent with these stylized facts, and use it for counterfactual analysis (IN PROGRESS).

Contribution

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- This analysis improves our understanding of the activities of MNEs, the largest players in the global economy.
- Observations from the time series help simplifying the complex problem of MNEs' location decisions.
- Dynamic features of the data inform us about the nature of FDI costs: country and mode dependent, fixed versus sunk. Crucial to quantify the welfare gains from openness.

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Literature

On the complex strategies of MNEs:

Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2014), Ramondo, Rodríguez-Clare, and Tintelnot (2105), Tintelnot (2105).

• Trade dynamics:

Roberts and Tybout (1997), Das, Roberts and Tybout (2007), Alessandria and Choi (2007), Ruhl and Willis (2105), Impullitti, Irarrazabal, and Opromolla (2013).

• FDI dynamics:

Russ (2007), Fillat, Garetto, and Oldenski (2015), Gumpert et al. (2015).

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Summary Statistics

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- Diversification
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- BEA affiliate-level data on the operations of U.S. MNEs, 1987-2011.
- Sales originating at each foreign affiliate and at the U.S. parent.
- Affiliate-level sales broken down into horizontal, vertical, and export platform based on their destination:
 - Horizontal FDI: sales to the country where the affiliate is located.
 - Vertical FDI: sales back to the Home country (the U.S.).
 - **Export Platform FDI**: sales to third countries.
- Restrict the sample to manufacturing affiliates born during the sample period and that survive for at least 10 years.



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Summary Statistics

Activity type	Horizontal	Vertical	Export-platform
No. of observations	34,181	34,181	34,181
with positive sales	32,335	15,869	20,127
	(94.6%)	(46.43%)	(58.88%)
of which:	11,072	523	639
pure type	(32.39%)	(1.53%)	(1.87%)

Note: Observations at the affiliate-year level. A pure-type affiliate is an affiliate with all its sales in only one activity type.



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Summary Statistics (II)

Activity type	Horizontal (as a sha	Vertical are of total	Export-platform affiliate sales)
average std dev	0.723 0.343	0.080 0.200	0.197 0.291
25 pc	0.540	0.000	0.000
50 pc	0.897	0.000	0.022
75 pc	1.000	0.044	0.310
90 pc	1.000	0.244	0.723
95 pc	1.000	0.529	0.905

Note: Distribution of affiliate sales by activity type. Observations at the affiliate-year level. Averages of the 11 firms around the indicated percentile are reported to preserve confidentiality.



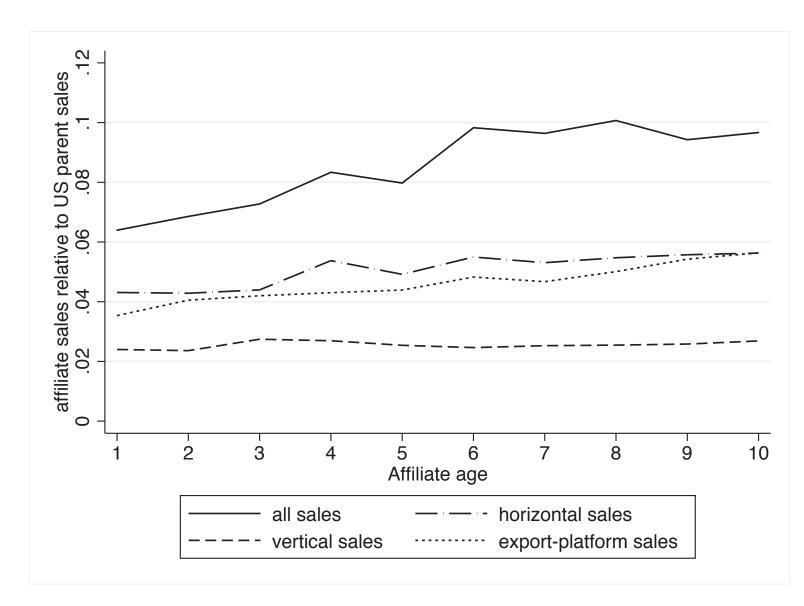
Fact 1: New Affiliates Have Flat Sales Growth Profiles

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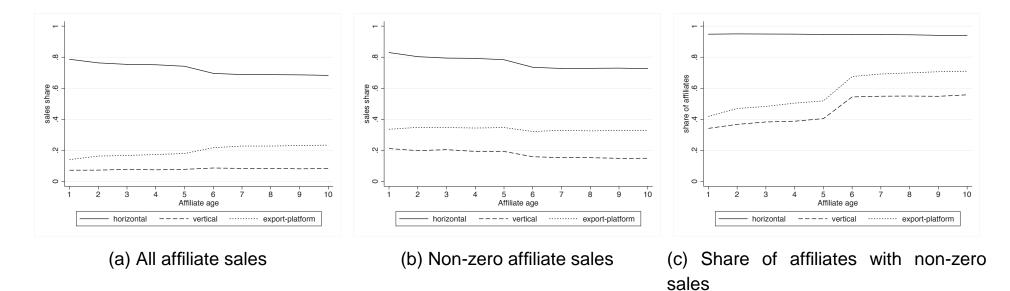
Model



 $[\Rightarrow$ Go to Regressions]

Fact 2: From Specialization to Diversification

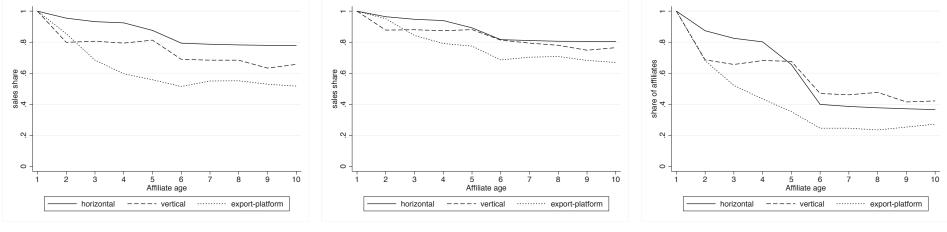
Figure 1: Intensive and extensive margins, by activity type. All new affiliates.



 $[\Rightarrow$ Go to Regressions]

From Specialization to Diversification in Pure Type Affiliates

Figure 2: Intensive and extensive margins, by activity type. Pure-type-born affiliates.



(a) All affiliate sales

(b) Non-zero affiliate sales

(c) share of pure-type affiliates

 $[\Rightarrow$ Go to Regressions]

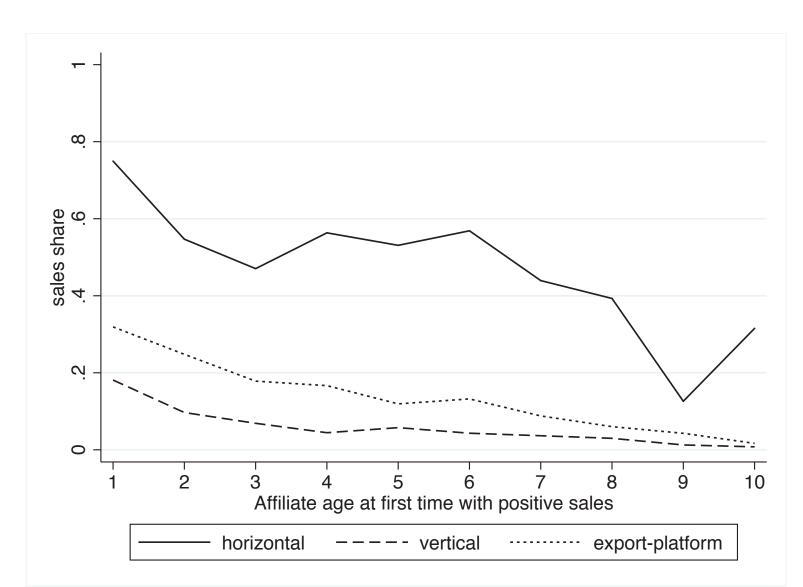


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Sale Shares by Entry Age



Notes: Average sales shares in each activity for affiliates that enter that particular activity at age 1, 2, ..., 10.

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Stylized Facts: Summary

- Affiliate sales, both to the local market to other countries, grow very little over the life cycle of the affiliate.
- Affiliates tend to specialize in a core activity at birth, which persists as the main activity during their life cycle.
- Some diversification of affiliate sales across modes is observed later in life, particularly from horizontal to export platform activities.

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Model Setup

- N+1 countries.
- Preferences:

$$U = \int_0^\infty e^{-\rho t} Q(t) dt, \text{ where } Q(t) = \left[\int q_i(t)^{1-1/\eta} dt \right]^{\eta/(\eta-1)}$$

- A continuum of monopolistically competitive Home country firms, with firm-level productivity φ and linear technologies.
- All firms sell domestically, and decide whether, when and how to enter foreign markets:
 - Assume that firms first set up a horizontal affiliate, then decide about vertical and platform sales.
 - Setting up a horizontal affiliate in j involves sunk entry costs F_j^h and fixed costs f_j^h .
 - A horizontal affiliate in j may start to export, which involves sunk entry costs F_j^e , fixed costs f_j^e , and iceberg costs τ_j^e .
 - There is an exogenous death rate δ .

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Shock Structure

- Firm-level productivity: $\varphi \equiv z \cdot Z$;
- $z \sim G(z)$ is a firm-specific component;
- $Z = e^X$ is an aggregate component, where X is a standard Brownian motion with drift:

$$dX = \mu dt + \sigma dz$$

for $\mu \in \Re$ and $\sigma > 0$.

• The state of the economy is the vector of country-specific productivity shocks $\bar{X} = (X_1, X_2, ..., X_{N+1})$.



Bellman Equations

Value of the firm:

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$\mathcal{V}(\varphi, \bar{X}) = V_d(\varphi, \bar{X}) + \sum_{j=1}^N \max\left\{V_j^o(\varphi, \bar{X}), V_j^h(\varphi, \bar{X}), V_j^e(\varphi, \bar{X})\right\}$

where:

$$\begin{split} V_{d}(\varphi,\bar{X}) &= \frac{1}{1+(\rho+\delta)\Delta t} \left[\pi_{d}(\varphi,\bar{X})\Delta t + E[V_{d}(\varphi,\bar{X}'|\bar{X})] \right] \\ V_{j}^{o}(\varphi,\bar{X}) &= \max\left\{ \frac{1}{1+(\rho+\delta)\Delta t} E[V_{j}^{o}(\varphi,\bar{X}'|\bar{X})]; V_{j}^{h}(\varphi,\bar{X}) - F_{j}^{h} \right\} \\ V_{j}^{h}(\varphi,\bar{X}) &= \max\left\{ \frac{1}{1+(\rho+\delta)\Delta t} \left[\pi_{j}^{h}(\varphi,\bar{X})\Delta t + E[V_{j}^{h}(\varphi,\bar{X}'|\bar{X})] \right]; \dots \\ V_{j}^{e}(\varphi,\bar{X}) - F_{j}^{e} \right\} \\ V_{j}^{e}(\varphi,\bar{X}) &= \frac{1}{1+(\rho+\delta)\Delta t} \left[(\pi_{j}^{h}(\varphi,\bar{X}) + \pi_{j}^{e}(\varphi,\bar{X}))\Delta t + E[V_{j}^{e}(\varphi,\bar{X}'|\bar{X})] \right] \end{split}$$



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Value Functions

• Value of domestic sales:

$$V_d(\varphi, \bar{X}) = \frac{\pi_d(\varphi, \bar{X})}{\rho + \delta - \hat{\mu}}$$

where $\hat{\mu} = \mu(\eta - 1) - \frac{1}{2}\sigma^2(\eta - 1)^2$.



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where $\hat{\mu} = \mu(\eta - 1) - \frac{1}{2}\sigma^2(\eta - 1)^2$.

• Option value of opening an affiliate in j:

$$V_j^o(\varphi, \bar{X}) = B_o e^{\beta X}$$

where
$$B_o > 0$$
 and $\beta > 1$ solves: $\frac{1}{2}\sigma^2\beta^2 + \mu\beta - (\rho + \delta) = 0$.



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Value Functions

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where $B_o > 0$ and $\beta > 1$ solves: $\frac{1}{2}\sigma^2\beta^2 + \mu\beta - (\rho + \delta) = 0$.

• Value of a horizontal affiliate in *j*:

$$V_j^h(\varphi, \bar{X}) = B_j e^{\beta X} + \frac{\pi_j^h(\varphi, \bar{X})}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta}$$

where $B_j > 0$.



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Value Functions

• Value of domestic sales:

$$V_d(\varphi, \bar{X}) = \frac{\pi_d(\varphi, \bar{X})}{\rho + \delta - \hat{\mu}}$$

where $\hat{\mu} = \mu(\eta - 1) - \frac{1}{2}\sigma^2(\eta - 1)^2$.

• Option value of opening an affiliate in *j*:

$$V_j^o(\varphi, \bar{X}) = B_o e^{\beta X}$$

where $B_o > 0$ and $\beta > 1$ solves: $\frac{1}{2}\sigma^2\beta^2 + \mu\beta - (\rho + \delta) = 0$.

• Value of a horizontal affiliate in *j*:

$$V_j^h(\varphi, \bar{X}) = B_j e^{\beta X} + \frac{\pi_j^h(\varphi, \bar{X})}{\rho + \delta - \hat{\mu}} - \frac{f_j^h}{\rho + \delta}$$

where $B_j > 0$.

• Value of a "diversified" affiliate in country *j*:

$$V_j^e(\varphi, \bar{X}) = \frac{\pi_j^h(\varphi, \bar{X}) + \pi_j^e(\varphi, \bar{X})}{\rho + \delta - \hat{\mu}} - \frac{f_j^h + f_j^e}{\rho + \delta}$$



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Qualitative Predictions of the Model

- Growth profiles.
 - In the data: the ratio of affiliate to parent sales is "flat", and affiliates are born as sizable entities.
 - In the model: aggregate shocks to domestic productivity imply that domestic and foreign sales are hit by the same shocks, so their ratio is flat. Sunk costs imply that affiliates "start big".



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• From specialization...

- In the data: almost all affiliates have some horizontal sales, and a negligible share of affiliates are pure type vertical or platform.
- In the model: firms start FDI either as pure horizontal or as a combination of horizontal and other modes. There are no pure-type vertical/platform affiliates.



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Qualitative Predictions of the Model

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• From specialization...

- In the data: almost all affiliates have some horizontal sales, and a negligible share of affiliates are pure type vertical or platform.
- In the model: firms start FDI either as pure horizontal or as a combination of horizontal and other modes. There are no pure-type vertical/platform affiliates.
- ...to diversification.
 - **In the data:** the organization of affiliates is persistent over time, but most affiliates diversify over time towards a mix of the three modes.
 - In the model: unit-root shocks drive persistence. Positive growth induces affiliates to expand in third countries.



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Conclusions and Next Steps

A lot of work still to do:

- Compute equilibrium for each firm, aggregation and industry equilibrium. GE?
- How well does this simple model matches the stylized facts?
- Quantitative framework: allow for multiple third countries (not observable in the data), endogenous exit, etc.

Once we are "happy" with the framework:

- Quantify frictions to different modes of FDI.
- Welfare counterfactuals and comparison with static models.



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• Value Functions: Notes

New Affiliates Have Flat Sales Growth Profiles

affiliate sales^{*i*}_{*a*}/parent sales = $\beta_a age + FE + \varepsilon_a^i$.

Dep var	affil	$iate\ sales$	$s_a^i/parent$	sales
	i = all	i = H	i = V	i = E
aff. age	(1)	(2)	(3)	(4)
	0.002	0.002	0.0002	0.0004
	(0.002)	(0.001)	(0.0007)	(0.001)
ind fe	yes	yes	yes	yes
ctry-yr fe	yes	yes	yes	yes
aff. fe	no	no	no	no
Obs	34,179	32,335	15,869	20,127
R-sq	0.0174	0.0138	0.0398	0.0229
within	0.0197	0.0125	0.0632	0.0273
between	0.0015	0.0452	0.0000	0.0144

Note: The dependent variable $affiliate \ sales_a^i/parent \ sales$ refers to the sales of type i = H, V, E for a new affiliate of age a relative to the domestic sales of the U.S. parent. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted ***p < 0.01, **p < 0.05, and *p < 0.1.



From Specialization to Diversification

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affiliate sales^{*i*}_{*a*}/total affiliate sales_{*a*} = $\beta_a age + FE + \varepsilon_a^i$

Dep var		S	$ales^i_a/tot$	$al \ sales_a$		
	<i>i</i> =	= <i>H</i>	i =	= V	i =	= <i>E</i>
	(1)	(2)	(3)	(4)	(5)	(6)
aff age	-0.002	-0.014***	0.001	0.001**	0.001	0.01***
	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
ind fe	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes
Obs	34,181	34,181	34,181	34,181	34,181	34,181
R-sq	0.094	0.006	0.076	0.047	0.000	0.008
within	0.095	0.025	0.067	0.056	0.001	0.026
between	0.104	0.000	0.098	0.044	0.001	0.001

Note: $sales_a^i/total \ sales_a$ refers to sales of type i = H, V, E as a share of total affiliate's sales for a new affiliate of age a_i . Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted ***p < 0.01, **p < 0.05, and *p < 0.1.

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Dep var	$sales^i_a(sales^i_a > 0)/total\ sales_a$					
	<i>i</i> =	= H	<i>i</i> =	= V	i = E	
	(1)	(2)	(3)	(4)	(5)	(6)
aff age	-0.002	-0.015***	-0.002	-0.003**	-0.005**	0.005***
	(0.002)	(0.0015)	(0.002)	(0.0013)	(0.002)	(0.0015)
ind fe	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes
Obs	32,335	32,335	15,869	15,869	20,127	20,127
R-sq	0.080	0.006	0.093	0.005	0.046	0.007
within	0.081	0.035	0.080	0.003	0.066	0.005
between	0.069	0.000	0.198	0.008	0.024	0.021

From Specialization to Diversification (II)

Note: $sales_a^i(sales_a^i > 0)/total \ sales_a$ refers to sales of type i = H, V, E for a new affiliate of age a, conditional on having positive sales in type i, as a share of total affiliate's sales. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted ***p < 0.01, **p < 0.05, and *p < 0.1.

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From Specialization to Diversification (III)

Dep var	$D(sales_a^i > 0)$						
	<i>i</i> =	i = H		i = V		i = E	
	(1)	(2)	(3)	(4)	(5)	(6)	
aff age	-0.001	-0.0003	0.006***	0.035***	0.014***	0.042***	
	(0.001)	(0.0009)	(0.002)	(0.003)	(0.003)	(0.003)	
ind fe	yes	no	yes	no	yes	no	
ctry-yr fe	yes	yes	yes	yes	yes	yes	
aff fe	no	yes	no	yes	no	yes	
Obs	34,181	34,181	34,181	34,181	34,181	34,181	
R-sq	0.044	0.000	0.088	0.008	0.067	0.033	
within	0.037	0.000	0.059	0.053	0.042	0.062	
between	0.067	0.001	0.186	0.006	0.124	0.011	

Note: $D(sales_a^i > 0)$ is a dummy variable equal to one if sales of type i = H, V, E for a new affiliate of age a are positive. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** p < 0.01, ** p < 0.05, and *p < 0.1.

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From Specialization to Diversification: Put	re Type Affiliates
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Dep var	$sales^i_a/total\ sales_a$					
	i =	= H	<i>i</i> =	= V	<i>i</i> =	= E
	(1)	(2)	(3)	(4)	(5)	(6)
aff age	-0.013***	-0.021***	-0.004	-0.042***	-0.030	-0.059***
	(0.002)	(0.002)	(0.015)	(0.011)	(0.019)	(0.009)
ind fe	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes
Obs	16,908	16,908	657	657	683	683
R-sq	0.132	0.023	0.035	0.059	0.090	0.08
within	0.065	0.078	0.273	0.124	0.429	0.191
between	0.176	0.001	0.001	0.003	0.008	0.004

Note: The sample is restricted to affiliates born with 100 percent of their sales in type i = H, V, E, with $sales_a^i/total \ sales_a$ being sales of type i = H, V, E, as a share of total affiliate's sales, for an affiliate of age a. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted $^{***}p < 0.01$, $^{**}p < 0.05$, and $^*p < 0.1$.

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From Specialization to Diversification: Pure Type Affiliates (II)

Dep var	$sales^i_a(sales^i_a > 0)/total\ sales_a$					
	i = H	i = V	i = E			
	(1)	(2)	(3)			
aff age	-0.012***	0.008	-0.006			
	(0.002)	(0.009)	(0.011)			
ind fe	yes	yes	yes			
ctry-yr fe	yes	yes	yes			
aff fe	no	no	no			
Obs	16,556	588	549			
R-sq	0.13	0.059	0.030			
within	0.06	0.226	0.564			
between	0.14	0.002	0.012			

Note: The sample is restricted to pure-type born affiliates with positive sales in the respective activity type. Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted *** p < 0.01, ** p < 0.05, and *p < 0.1.

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From Specialization to Diversification: Pure Type Affiliates (III)

Dep var	$D(sales_a^i/total\ sales_a = 1)$					
	i =	= <i>H</i>	<i>i</i> =	= V	<i>i</i> =	= <i>E</i>
	(1)	(2)	(3)	(4)	(5)	(6)
aff age	-0.041***	-0.063***	-0.013	-0.069***	-0.051**	-0.086***
-	(0.004)	(0.004)	(0.015)	(0.011)	(0.020)	(0.010)
ind fe	yes	no	yes	no	yes	no
ctry-yr fe	yes	yes	yes	yes	yes	yes
aff fe	no	yes	no	yes	no	yes
Obs	16,908	16,908	657	657	683	683
R-sq	0.239	0.067	0.063	0.068	0.123	0.108
within	0.081	0.160	0.282	0.198	0.386	0.239
between	0.368	0.000	0.000	0.009	0.026	0.028

Note: $D(sales_a^i/total\ sales_a=1)$ is a dummy variable equal to one if the affiliate at age a is of pure-type i=H,V,E (i.e., the share of sales of type i in total sales is 1). Robust standard errors, clustered at the parent level, are in parenthesis. Levels of significance are denoted ***p < 0.01, **p < 0.05, and *p < 0.1.



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Value Functions: Notes

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- Still to be determined: parameters B_o , B_j and productivity thresholds that induce each firm to change status $X_j^{oh}(z)$, $X_j^{he}(z)$, $\forall j$.
- For each firm and for each affiliate host country *j*, impose a system of value-matching and smooth pasting conditions to recover parameters and thresholds:

$$\begin{array}{rcl}
V_{j}^{o}(\varphi,\bar{X}) &=& V_{j}^{h}(\varphi,\bar{X})\\
V_{j}^{h}(\varphi,\bar{X}) &=& V_{j}^{e}(\varphi,\bar{X})\\
\frac{\partial V_{j}^{o}(\varphi,\bar{X})}{\partial X} &=& \frac{\partial V_{j}^{h}(\varphi,\bar{X})}{\partial X}\\
\frac{\partial V_{j}^{h}(\varphi,\bar{X})}{\partial X} &=& \frac{\partial V_{j}^{e}(\varphi,\bar{X})}{\partial X}.
\end{array}$$

• Analytical properties of the solution:

$$\frac{\partial B_o(z)}{\partial z} \ge 0 \quad \text{and} \quad \frac{\partial B_o(z)}{\partial z} \ge 0$$
$$\frac{\partial X_j^{oh}(z)}{\partial z} \le 0 \quad \text{and} \quad \frac{\partial X_j^{he}(z)}{\partial z} \le 0.$$